A Tracking Code for Injection and Acceleration Studies in Synchrotrons *

E. Lessner, K. Symon†

Argonne National Laboratory
9700 Cass Ave., Argonne, IL 60439

Abstract. CAPTURE_SPC is a Monte-Carlo-based tracking program that simulates the injection and acceleration processes in proton synchrotrons. The time evolution of a distribution of charged particles is implemented by a symplectic, second-order-accurate integration algorithm. The recurrence relations follow a time-stepping leap-frog method. The time-step can be varied optionally to reduce computer time. Space-charge forces are calculated by binning the phase-projected particle distribution. The statistical fluctuations introduced by the binning process are reduced by presmoothing the data by the cloud-in-cell method and by filtering. Both the bin size and amount of filtering can be varied during the acceleration cycle so that the bunch fine structure is retained while the short wavelength noise is attenuated. The initial coordinates of each macro particle together with its time of injection are retained throughout the calculations. This information is useful in determining low-loss injection schemes.

INTRODUCTION

CAPTURE_SPC is a Monte-Carlo-based computer program developed for tracking simulation studies of the injection and acceleration processes in high intensity proton synchrotrons.

Charges in the bunch are represented by a distribution of macro particles whose time evolution follows a sympletic integration algorithm. The integrating time-step can be chosen so that it follows closely the dynamical process, or it can be made larger to reduce execution time, provided it is kept smaller than any relevant period of the motion. The initial particle phase-space configuration can be generated internally, according to some specified distribution, or can be read as input.

*Work supported by U. S. Department of Energy, Office of Basic Energy Sciences under Contract No. W-31-109-ENG-38.

†Also Dept. of Physics, University of Wisconsin-Madison, Madison, WI 53706

One of the main concerns in high intensity proton machines is the level of activation resulting from beam losses. The program provides information on when the losses occur and the dynamical conditions related to the rf voltage and space-charge at the time of loss. By pointer-bookeeping, the program provides also the initial phase-space coordinates and time of injection of the lost particles. This information is useful for reformulating the injection scheme to mitigate losses.

The code was tested extensively [1]. In the following, the algorithm used in the program and some of its capabilities are discussed.

TRACKING ALGORITHM

The equations of motion of a charged particle in a synchrotron describe the time-evolution of a particle subject to an accelerating force that depends on the rf phase and to forces that act on the particle. These equations can be modified to describe the motion of particles whose energies remain close to the energy of a synchronous particle, defined as that particle which remains in the equilibrium orbit determined by the magnetic field B(t). The equations so obtained are derivable from a Hamiltonian and suitable for rapid numerical integration by the leap-frog algorithm [2]. The coordinates of each particle, W and Φ, are advanced by a time-step τ according to:

$$W_{n+1/2} = W_{n-1/2} + \frac{eV_n\tau}{2\pi}(sin\Phi_n - sin\Phi_{s,n}) + \frac{e^2 g_0}{4\pi\varepsilon_0}\frac{h^2\tau}{R\gamma_{s,n}^2}(\frac{d\lambda(\Phi)}{d\Phi})_n, \quad (1)$$

$$\Phi_{n+1} = \Phi_n + h\tau(\frac{\eta_s\omega_s^2 W}{\beta_s^2 E_s})_{n+1/2} + \Phi_{s,n+1/2} - \Phi_{s,n-1/2}, \quad (2)$$

where $W = (E - E_s)/\omega_s$, E is the energy, the subscript s refers to the synchronous particle, and ω is the angular frequency. Φ denotes the phase of the gap voltage at the time the the particle crosses the accelerating cavity, V is the total rf gap voltage, h is the harmonic number, $\eta = (\gamma_{tr}^2)^{-1} - (\gamma^2)^{-1}$, and the subscript n indicates that a quantity is evaluated at a time $t = n\tau$. These equations are accurate to second order since the derivatives are calculated at the midpoint of each time interval. Moreover, they have the correct long-term behavior, because the phase-space area element $dWd\Phi$ is conserved to first order in W.

The last term in Eq. (1) is the electric field contribution from a continuous beam of cross-sectional radius a, moving in a smooth cylindrical pipe of radius b, and for which the wavelength associated with changes in the charge density is much greater than b/γ. $\lambda(\Phi)$ denotes the number of particles per unit rf-phase, and g_0 is a capacitive geometrical factor given by $g_0 = 1 + 2ln(b/a)$.

The numerical simulation of the injection, capture, and acceleration processes follows an ensemble of macro particles whose initial phase-space coordinates can be user-specified and read as input data, or generated by the program. In the latter case, the initial phase coordinates are derived from a random uniform distribution, whereby a stratified sampling in Φ is used to avoid clumping. The particle energies can be derived from either a cosine or a Gaussian distribution.

The time-step should be kept small whenever greater accuracy is needed, and the dynamical changes followed closely, e. g. , during injection and capture. It should be kept much smaller than the synchrotron period or any relevant period of the motion. Experience shows that a time-step less than 1/30 of the synchrotron period is quite adequate. The computer time required to track a fixed number of particles was shown to be 60% longer than the time to track the same number of particles during the same interval, with a time-step five times larger. With the larger time-step, the peak-current values are overestimated by 0.4%.

By using a system of pointers, the initial coordinates and time of injection of the lost particles can be determined. This knowledge is useful in exploring injection schemes and rf programs to minimize losses. Figure 1 shows an example of the initial phase-space coordinates of the particles uncaptured a few turns after the end of injection. The rf voltage is raised adiabatically. The loss pattern shown is typical of that obtained with adiabatic captures.

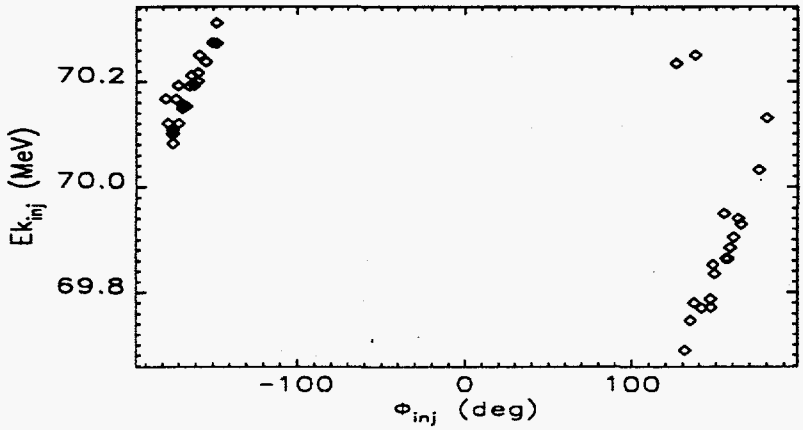

Figure 1: Phase-space coordinates of uncaptured particles, represented by the squares, soon after injection completion. The injected beam occupies uniformly a rectangle defined by $\Delta E = \pm 0.35$ MeV and $\Delta\Phi = \pm 180°$.

The stable phase-space area is determined with and without space-charge contributions. In cases when there are particles outside the rf bucket, as shown in Figure 2-(b), the space-charge forces shift the unstable fixed points. Then,

one of the turning points of the motion is calculated by searching the rightmost (below transition and $sin\Phi_s > 0$) intersection of the instantaneous total energy gain per turn, $\Delta E - \Delta E_s$, with the zero axis. In this example, 10,000 macro particles are used to represent the 1.0×10^{14} particles of a beam injected at 400 MeV. Due to the space-charges, the unstable fixed point is slightly displaced from 178.3° to 177.6°. The conjugated turning point is obtained from the usual expression relating the two extreme phases, including the space-charge potentials at both phases.

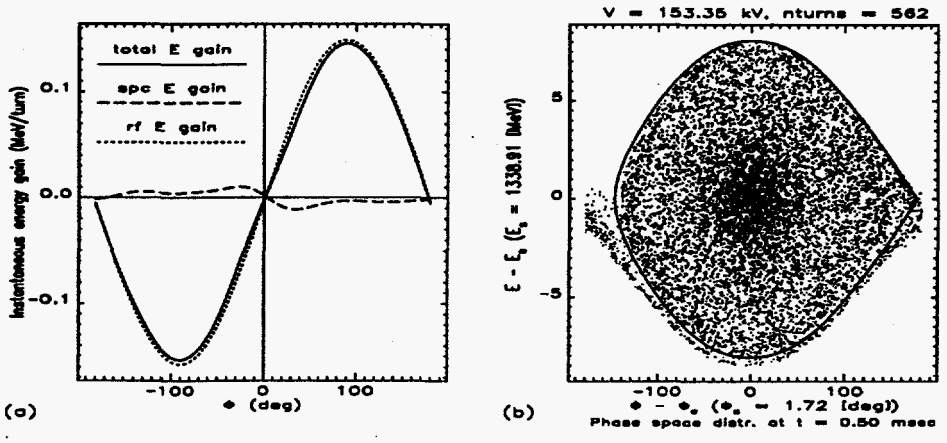

Figure 2: (a) Plot of the instantaneous energy gain per turn with the axes superimposed. (b) Bucket and phase-space distribution of the beam whose energy gain per turn is shown in (a).

The space-charge fields are calculated by using binning and data smoothing techniques that alleviate the statistical fluctuations introduced by the relatively small number of macro particles used in the simulations [3].

The macro particles are binned first into an appropriate number of bins. The number of bins must be chosen so that the average number of macro particles per bin is not too small and the bin length not too large so as to reveal the bunch structure. The projected phase distribution is binned such that, for each particle, a weighted contribution is assigned to the two closest grid points, according to how far the particle is from these points. The data is then fast-Fourier-transformed and convolved with a $sink/k$ kernel, where k is a harmonic number. This smoothens the binned density and provides a finer grid from which the fields can be obtained by interpolation. Finally, the Fourier components are multiplied by a Lanczos convergence factor whose cutoff harmonic number can be varied. As the bunch length shortens during the cycle, higher cutoff frequencies should be used. The filtering process eliminates the ringing at the ends of the bunch, but has the disadvantage of overestimating the density at the ends of the bunch and underestimating its

peak. Figure 3 shows the linear particle density distributions obtained after application of the same kernel interpolation but with cutoff harmonics of 9 and 17. The coarsely-binned data, represented by the squares, is obtained by using 32 bins. The interpolated data is obtained with 512 bins. As shown in the figure, the cutoff at the 9^{th} harmonic underestimates the peak of the

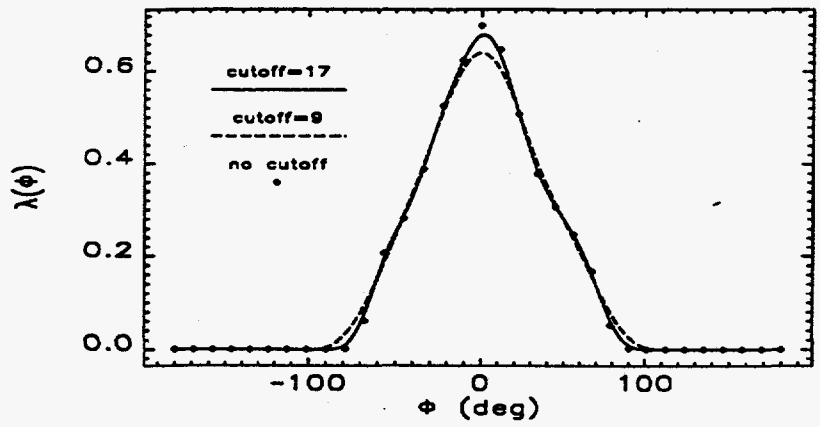

Figure 3: Linear particle density distribution obtained by using different filter cutoff frequencies.

distribution by 8% when compared with the peak of the distribution obtained from the 32-binned unfiltered data. The same cutoff overestimates the density at the ends of the bunch by 10%.

Input and Output Information

The program requires an input data file containing the time variation of the rf voltage to be tested. If desired, the variation of g_0 during the cycle can also be given as an input file. Two preprocessors can be used with CAPTURE_SPC. One preprocessor calculates the rf voltage program based on the desired rf bucket area. In this way, the bucket area can be tailored from injection through extraction to meet the design requirements of the accelerator. The reduction of the bucket area due to space-charge is not included in this calculation. The program allows the requirement for a fixed or ramped bucket area, linear or otherwise, through injection, and tailoring of the rf voltage at extraction to adjust the bunch length and momentum spread to meet the requirements for beam stability. The other preprocessor calculates the ratio of chamber size to average beam size as a function of beam energy. This allows comparisons of the effect of the space-charge impedance on the machine performance as a result of different injection phase-space painting schemes and different types of vacuum chambers.

Output files containing the energy and phase coordinates of the surviving particles, together with the separatrix coordinates, can be obtained at any specified time. The values of the energy and phase of the synchronous particle, the rf voltage, and number of turns at this time are also given, as can be seen in Figure 2-(b). As mentioned, information about the initial configuration of the uncaptured particles is available. Files containing the space-charge potential as a function of rf phase and the instantanous energy gain per turn, as depicted in Figure 2-(a), are also provided. The variation of the bucket area, bucket height, peak rf and space-charge potentials, and bunch harmonics is provided as a function of time.

FUTURE DEVELOPMENTS

Presently, only the image forces due to the space-charge fields account for the interactions of the beam with its environment through the capacitive factor g_0. Preliminary tests of the resistive-wall effects have been performed. These calculations did not include transient effects. Inclusion of general wake field contributions from elements in the ring will be implemented in a future version of the program. The algorithm will include corrections to the impedance calculations due to wake field decay times larger than the time it takes for beam parameters changes to occur. A detailed description of the algorithm is given elsewhere in these Proceedings [4].

ACKNOWLEDGEMENTS

The two preprocessors used with the program were designed and written by K. Harkay (ANL).

References

[1] Y. Cho, E. Lessner and K. Symon, "Injection and Capture Simulations for a High Intensity Proton Synchrotron," *Proc. of the European Particle Accelerator Conference*, London, p. 1228 (1994).

[2] K. R. Symon, "Synchrotron Motion with Space-Charge," ANL Report NSA-94-3 (April 4, 1994).

[3] E. Lessner, Y. Cho, K. Harkay, K. Symon, "Longitudinal Tracking Studies for a High Intensity Proton Synchrotron," *AIP Conference Proc. 377*, p. 375 (1996).

[4] K. R. Symon, "Longitudinal Wake Field Corrections in Circular Machines," *these Proceedings.*