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NONLINEAR DYNAMICS AND PLASMA TRANSPORT

Final Progress Report
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Principal Investigators
C. S. Liu, R. Sagdeev, T. Antonsen, J. Drake, A. Hassam, P. N. Guzdar

Institute for Plasma Research
University of Maryland
College Park, MD 20742-3511

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DISCLAIMER

1 INTRODUCTION AND PROGRAM OUTLINE

This progress reports work done on a program in nonlinear dynamical aspects of plasma turbulence and transport funded by DOE from 1992–1995. The purpose of this program has been to promote the utilization of recent pathbreaking developments in nonlinear science in plasma turbulence and transport and to fully utilize the scientific expertise of Russian fusion and plasma community in collaboration with our group to address outstanding fusion theory problems.

In the work reported in our progress report, we have studied simple models which are motivated by observation on actual fusion devices. The models focus on the important physical processes without incorporating the complexity of the geometry of real devices. We have also studied linear stability problems which incorporated important physics issues related to geometry involving closed field lines and open field lines. This allows for a deeper analysis and understanding of the system both analytically and numerically.

The strong collaboration between the Russian visitors and the US participants has lead to a fruitful and strong research program that taps the complementary analytic and numerical capabilities of the two groups.

Over the years several distinguished Russian visitors have interacted with various members of the group and set up collaborative work which forms a significant part of proposed research. Dr. Galeev, Director of the Space Research Institute of

Moscow and Dr. Novakovskii from the Kurchatov Institute are two such ongoing collaborations.

2 PROGRESS REPORT

In this section we summarize the work completed over the last three years in three different areas of research:

a. shear flow generation in plasmas and fluids,

b. nonlinear dynamics and visualization of 3D flows,

c. self-consistent MHD behavior in the presence of chaotic field lines.

2.1 Shear Flow Generation in Plasmas and Fluids (Drake, Finn, Guzdar, Rogalsky)

2.1.1 Nonlinear Interaction of Rayleigh-Taylor and Shear Instabilities

Through strong collaborative effort of Shapiro, Sagdeev and our group, an important process of shear flow generation by tilting and reconnecting an array of vortices was first identified in our joint paper.[1] Based on these approches the nonlinear behavior of the Rayleigh-Taylor or interchange instability with very weak magnetic shear was obtained andconsequent developments of shear flow by the shear or peeling instability were obtained.[2] It was found that the shear flow is generated at sufficient amplitude to reduce greatly the convective transport. For high viscosity, the time-asymptotic state consists of an equilibrium with shear flow and vortex flow (with islands, or "cat's eyes"), or a relaxation oscillation involving an interplay between the shear instability and the Rayleigh-Taylor instability in the presence of shear. For low viscosity, the dominant new feature we found is a high-frequency nonlinear standing wave consisting of convective vortices (islands) localized near the top and bottom boundaries. The localization of these vortices is due to the smaller shear near the boundary regions. The convective transport is largest around these convective vortices near the boundary and there is a region of good confinement near the center. For very small viscosity μ and thermal conduction coefficient κ, these vortices or islands are strongly localized near the boundaries. For intermediate μ and κ, these islands were found to overlap, giving chaotic $\mathbf{E} \times \mathbf{B}$ advection and enhanced transport. The possible relevance of this behavior to the H mode and edge-localized modes (ELM's) in the tokamak edge region has been explored. We have suggested that the equilibria with shear may correspond to the H-mode, the low frequency relaxation oscillations may correspond to ELM's and the high frequency oscillations may correspond to "grassy ELM's". This work appeared in Phys. Fluids B **5**, 415 (1993).[2]

2.1.2 2D Nonlinear Dynamics of Four Driven Vortices

In a tokamak plasma the number of vortices in the edge plasma are large since high poloidal mode numbers are unstable. Thus a simple model which elucidates the physics of generation of shear flow and the self-interaction among the vortices is a set of four vortices. Thus the interaction of four, alternately driven counter-rotating vortices in a two dimensional box, with inpenetrable free-slip boundary conditions in the x direction and periodic boundary conditions in the y direction, has been studied numerically.[3] For viscosity above a critical value the nonlinear state consists of four alternately counter-rotating vortices. For a lower value of the viscosity the system evolves to a nonlinear steady state consisting of four vortices and shear flow generated by the "peeling instability" [Drake et al., Phys. Fluids B **4**, 447 (1992)].[1] For a still lower viscosity the steady state nonlinear state undergoes a Hopf bifurcation. The periodic state is caused by a secondary instability associated with vortex pairing. However, the vorticity of the shear flow, though periodic, has a definite sign. With a further decrease in the viscosity, a global bifurcation gives rise to a periodic state during which the vorticity of the shear flow changes sign. At even lower viscosity, there is a transition to a steady state, involving dominantly shear flow and a two-vortex state. Finally, this state undergoes a bifurcation to a temporally chaotic state, with the further decrease of viscosity. The results were compared to some recent experiments in fluids with driven vortices [P. Tabeling et al, J. Fluid Mech. **215**, 511 (1990)].

2.1.3 Neoclassical Theory of Plasma Rotation (Novakovskii, Galeev, Hassam, Liu, and Sagdeev)

The rotation of plasmas in different neoclassical collisionality regimes has been studied. Earlier work has many controversial results particularly in the banana and plateau regimes (see, for example, the introduction of Hsu et al.[5]) We have developed an analytical theory for the time evolution of poloidal and toroidal rotation, using time-dependent drift-kinetic equations. We have obtained general results for the magnetic pumping rate, the neoclassical Stringer spin-up rate and the effective plasma inertia. The Stringer spin-up[6], which was initially studied in the Pfirsch-Schlüter regime survives in the banana/plateau regime. We have shown that the enhancement in inertia in the plateau regime has an additional term dependent as $\nu_*^{-1/3}$, where ν_* is the effective collisional frequency, due to the effects of barely untrapped, resonant particles with $v_{||} \approx 0$. This brings to conclusion the neoclassical theory of rotation in the plateau regime.

2.1.4 New Unstable Branch of Drift Resistive Ballooning Modes in Tokamaks (Novakovskii, Guzdar, Drake and Liu)

During the last two years we (McCarthy et al.[8] and Guzdar et al.[9]) have studied Drift Resistive Ballooning modes extensively as a leading cause of anomalous transport in the edge region of tokamaks. We have re-examined the linear stability of such modes[10]. We found a new branch of unstable mode which is more unstable for realistic tokamak edge parameters. Thus, there are basically two unstable branches of the resistive ballooning modes. One branch is the conventional which has been extensively studied by many authors using the two-space scale analysis. This conventional branch is more unstable that the second branch found by us in the weak shear limit. However for realistic $\hat{s} \sim 0(1)$ the conventional branch is found to become stable, while the new branch, which is more strongly ballooning, is robustly unstable. This work provides the linear theoretical basis for the strongly ballooning modes observed in our 3D simulations[9] and further reinforces our contention that these modes are a prime candidate for causing edge transport in tokamaks.

2.1.5 Drift Resistive Ballooning Modes in the Scrape-Off-Layer of Tokamak Plasmas (S. V. Novakovskii, P. N. Guzdar, E. A. Novakovskaia, J. F. Drake and C. S. Liu)

Analytical and numerical study of the stability of the resistive ballooning modes (RBM) in the scrape-off layer (SOL) of a tokamak plasma has been performed[11] to take into account the effect of limiter and divertor. It has been found that the stability of the RBM is controlled by the two parameters $\lambda = (m_e/m_i)^{1/2}\nu_{ei}qR/vT_e$ — the "effective strength" of the Debye sheath current, and $\hat{m} = mL_0/a$ the dimensionless poloidal number, where the characteristic scale L_0 is given in the main text as a function of the basic plasma edge plasma parameters. For $\lambda \ll 1$ the RBM are robustly unstable for a large range of $\hat{m}$, while for $\lambda \gg 1$ modes with $\hat{m} \leq 1$ are strongly stabilized. The general case of arbitrary λ and $\hat{m}$ has been studied numerically and the spectrum of the unstable RBM has been obtained. The influence of the diamagnetic effects has also been investigated. Thus, we have shown that for tokamaks with $\lambda \ll 1$, (DIII-D, TFTR) the RBMs are more unstable in the SOL compared to the region inside the LCFS. However, in the case $\lambda \geq 1$ (C-Mod, ITER design) the modes in the SOL as well as inside the LCFS have very comparable growth rates.

2.1.6 Improved Low-Order Model for Flow Shear Driven by Rayleigh-Benard Convection (Hermiz, Guzdar and Finn)

The Rayleigh-Benard convection in 2D (Finn[2] and Hermiz and Finn[12]) has provided an interesting paradigm for understanding the generation of shear flow by the Reynolds-Stress and the interplay between the convection rolls generated by the temperature gradient and the self-consistently shear-flow. Although this simple model does not capture the complicated geometry of tokamak edge plasmas, the nonlinear dynamics that occurs above the critical Rayleigh number for the onset of convection as well as shear flow generation, is very akin to the ELM phenomenon observed in tokamak plasmas. It is desirable to have a low dimensional model which can explain the dynamics obtained by solving the full 2D system of equations. The work of Howard and Krishnamurti[13] (HK) provided a simple low-dimensional model (using a truncated Fourier series) for studying the shear generated by the convection. However this low-order model failed to conserve average vorticity. We have developed an improved low-dimensional model (based on our earlier work Finn et al.[14]) which shows that the Rayleigh number for the onset of shear flow is a strong function of the aspect ratio of the convective cell unlike the original HK model. This is in agreement with 2D simulations which show a strong dependence of shear flow generation on the aspect ratio of the cells. We have also shown that such truncated models have very limited range in parameter space in which they can reproduce the dynamics obtained by solving the full 2D system of equations.

2.1.7 Shear Flow Generation by the Drift/Rossby Waves (Guzdar, Shapiro)

The generation of shear flow driven by a large amplitude drift wave represented by the Hasegawa-Mima-Charney (HMC) equation was investigated[15]. It is shown that besides the existence of a finite amplitude threshold for the shear flow instability[16], there is also a necessary condition for instability on the large amplitude drift wave. This condition arises from the conservation of average potential vorticity and was missed in earlier work[16]. Also a comprehensive comparison of the instability criterion for the HMC equation and the incompressible, inviscid hydrodynamic equation from our earlier work has been completed.

2.2 Nonlinear Dynamics and Visualization of 3D Flows (Chernikov, Finn, Guzdar, Rogalsky, Usikov)

2.2.1 Diffusion of Stochastic Webs Near the Percolation Threshold

For a time dependent Hamiltonian which represents a linear oscillator perturbed by periodic kicks, the phase portrait of the oscillator reveals the formation of a stochastic web.[17] The rate of diffusion in the stochastic web was estimated to be the product of the global separatrix rate and the ratio of the phase space of the web to the total phase space. In the present work, exact results for scaling characteristics of the stochastic webs with quasi-crystal symmetrics, near the percolation threshold has been obtained.[18] This problem has application in plasmas and in hydrodynamics (stochastic heating of particles and passive particle advection or test particle diffusion) and solid state physics (electrical conduction in metal insulator composites).

2.2.2 3D Force-Free MHD Equilibria and their Visualization

Computations of three dimensional force-free MHD equilibria, $\nabla \times \mathbf{B} = \lambda \mathbf{B}$ with $\lambda = \lambda_0$, a constant (Taylor states) were performed. These equilibria were determined by boundary conditions of the normal component of $\mathbf{B}$ on a surface.[19] This surface corresponds to the z-electrodes in a flux core spheromak or, in space applications, to the solar photosphere or the earth's ionosphere. It was found that as λ_0 is increased, i.e. as the helicity is increased, the field lines become kinked, and for sufficiently large λ_0 develop knots. The relationship between the kinking and knotting properties of these equilibria and the presence of a kink instability and related loss of equilibrium was explored. Magnetic reconnection must be involved for an unknotted loop equilibrium to become knotted. It was concluded that there is indeed a loss of equilibrium process associated with the existence of an unstable kink mode, and that this process leads to the creation of a closed hyperbolic field line (X-line) about which this reconnection creating knotted field lines is centered. The field lines were visualized in 3D by AVS (Applications Visualization System) on the DEC-5000 Work-station at Maryland. We developed several modules for use with AVS for tracing field lines from numerical data and for computing Poincaré sections.

2.2.3 Loss of Equilibrium and Reconnection in Tearing of 2D Equilibria

Two-dimensional tearing-like behavior was studied in reduced resistive magnetohydrodynamics (MHD) with flux conserving boundary conditions on an elongated rectangular box.[20] This study was begun in order to understand the role of loss of equilibrium in axisymmetric MHD evolution in a tokamak, specifically the spontaneous splitting process in tokamaks with highly elongated cross sections. The tearing-like perturbations do not destroy the symmetries of the initial state, either discrete or continuous. In such cases linear instability is typically not directly observed. However, it was found that there can be a loss of equilibrium (a saddle-node bifurcation) associated with the existence of a tearing unstable state. The results in a very elongated tokamak, with pinching coils to elongate its flux surfaces, were compared to those in a model for the magnetotail or for solar arcades. The loss of equilibrium was demonstrated by means of a nonlinear energy functional. The importance of the fact that the splitting or tearing process occurs by means of a loss of equilibrium is that a large amount of free energy can be released, in the form of reconnection. Also, with this kind of bifurcation, there is a possibility of hysteresis, suggesting that a tokamak can be heated at the center by repeatedly forcing such a splitting and unsplitting process.

2.3 Self-Consistent MHD Behavior in the Presence of Chaotic Field Lines (Finn)

For equilibrium or low frequency behavior, chaotic field lines occur only in 3D. However, the beginnings of a systematic study of this general problem have been made in 2D by applying an external force which produces large amplitude Alfvén waves with resonance surfaces near the mode rational surface of a tearing mode.[21] It was found that the tearing mode saturates at a lower level or can be stabilized completely if the Alfvén wave amplitude is large enough. This stabilization is due in part to a quasilinear flattening of the current profile, by overlap of the Alfvén resonances with each other and with the tearing mode. However, another contributing factor in the stabilization is the velocity shear formed by the presence of the large amplitude Alfvén waves in the presence of a sheared background magnetic field.

2.3.1 Software Development (Usikov)

(a) Visualization of 3D Flows

We have improved on modules used in conjunction with AVS (Advanced Visualizations System) developed earlier for tracing 3D flow trajectories. The flow can be either given by an analytic expression or could be data generated by a code on a 3D grid. The improvement allows for dealing with arbitrary number of data points in each of the three directions as well as menu-driven options which makes the software more user-friendly.

(b) Wavelet Transforms Package

The Wavelet Transform has provided a new tool for studying multi-scale phenomenon especially in self-similar mathematical objects. We have developed an efficient menu-driven package for personal computers as well as work-stations which can compute the wavelet transform of a given time-series using a variety of piece-wise linear transform functions. This also provides a highly efficient algorithm for computing the fractal dimension of low-dimensional chaotic attractors, as well as obtaining the spectral index of a turbulent flow.

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