CONF-9605100--1

SAND96-0405C

SIMULATION OF NONLINEAR STRUCTURES WITH ARTIFICIAL NEURAL NETWORKS

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Abstract

Structural system simulation is important in analysis, design, testing, control, and other areas, but it is particularly difficult when the system under consideration is nonlinear. Artificial neural networks offer a useful tool for the modeling of nonlinear systems, however, such modeling may be inefficient or insufficiently accurate when the system under consideration is complex. This paper shows that there are several transformations that can be used to uncouple and simplify the components of motion of a complex nonlinear system, thereby making its modeling and simulation a much simpler problem. A numerical example is also presented.

Introduction

Artificial neural networks (ANNs) have been applied to the recurrent (autoregressive) modeling of nonlinear systems. Investigations have shown that nonlinear structures can be modeled with ANNs, at least in the case of simple systems. (See, for example, Yamamoto, 1992.) In principle, complicated systems can also be modeled using ANNs. This can be done directly (i.e., without any substantial transformation of the input or output data) using many types of ANNs. As the complexity of the system increases, an ANN that can naturally and efficiently accommodate a large number of inputs must be used for system simulation. When a mechanical system is modeled using a recurrent ANN to directly simulate motions at a large number of degrees of freedom, a very large number of exemplars of motion will be required to train the ANN to accurately represent the system. The reason is that it takes a large number of exemplars to populate a high dimensional input space.

This paper shows how the ANN modeling of nonlinear structures can be made more efficient and accurate when using data measured during experimental vibration. There are a number of operations that can be performed on the data to accomplish these goals. These are: (1) principal component analysis, (2) elimination of statistically dependent components of motion, and (3) transformation of the components of motion to statistically independent, standard normal random signals. These

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This work was sponsored by the Department of Energy under contract No. DE-AC04-94AL85000.

operations are briefly described in the following sections, along with the modeling of the components with an ANN known as the CNLS net. An example is included.

Principal Component Analysis

Principal component analysis of complex structural system motions is aimed at decomposing the motions into their essential constituent parts. A special example of this is the modal decomposition of linear systems, and analogous decompositions can be defined for nonlinear systems using, for example, singular value decomposition (SVD), or a principal component analysis ANN.

SVD is described in detail, for example, in Golub and Van Loan (1983). It can be used to decompose linear or nonlinear structural motions in the following way. Let X be an $N \times n$ matrix representing the motion of a structural system at n transducer locations and at N consecutive times. The form of the SVD is

$$X = UWV^T \cong uwv^T \qquad (1)$$

V and W describe characteristic shapes present in X and their corresponding amplitudes. The columns of U are filtered versions of the motions in X. The approximate equality on the right indicates that some components of the representation can be eliminated, and still maintain a good approximation to X. It is the evolution of the columns in u (a reduced form of U) which we seek to simulate.

Elimination of Statistically Dependent Components

Under certain circumstances, some of the components produced during principal component analysis are completely statistically dependent upon others. For the sake of efficiency, we seek to eliminate statistically dependent components from the set to be modeled, then reintroduce these components during physical system simulation. In this way, ANN modeling of structural behavior is simplified.

When a dependency exists, it can be characterized using the conditional expected value of the variables in one column of u given values in another column of u. This requires approximation of a joint probability density function (pdf) of the data, and this can be obtained using the kernel density estimator. (See Silverman, 1986.)

The effect of eliminating components of motion that are completely dependent on other components is to eliminate some columns in the matrix u. Denote the reduced matrix u_r; our objective is to model the evolution of the columns of u_r with an ANN.

Rosenblatt Transform

The previous step produces a description of the motion of a complex structure in terms of a set of components, none of which is completely statistically dependent on others. We can further transform the components, the columns of u_r, into signals that are statistically independent with Gaussian distributions. The transformation that accomplishes this is the Rosenblatt transform. (See Rosenblatt, 1952.) The complete form of the transform cannot be written here, but it can be denoted

$$z = T(u_r) \qquad u_r = T^{-1}(z) \qquad (2)$$

The second expression indicates that the Rosenblatt transformation is uniquely invertible. The matrix z is the same size as the matrix u_r with the same number of nonzero columns. It is the evolution of the values in these columns that is to be simulated with ANNs. Because the columns in z are statistically independent, we need only to create ANN models for signals in individual columns.

Modeling of Component Motion with the CNLS Net

Our objective at this point is to simulate the components of system motion obtained using the decompositions and transformations described above. An ANN suitable for this simulation is the connectionist normalized linear spline (CNLS) network. We cannot provide the complete form of the ANN here, but that is given in Jones, et.al., (1990). To simulate of a column in z using a CNLS net, we configure the net in a recurrent (autoregressive) framework. This configuration uses as inputs previous response values and the independent excitation, and yields on output, the current response. To specify a simulator CNLS net, we select its parameters so that it simulates the evolution of a column of z with minimum error. A following section provides an example of the use of one of the transformations described above and the modeling of the components of motion with CNLS nets.

Summary

Figure 1 summarizes the decomposition, simulation, and modeling of structural motion described in the previous sections.

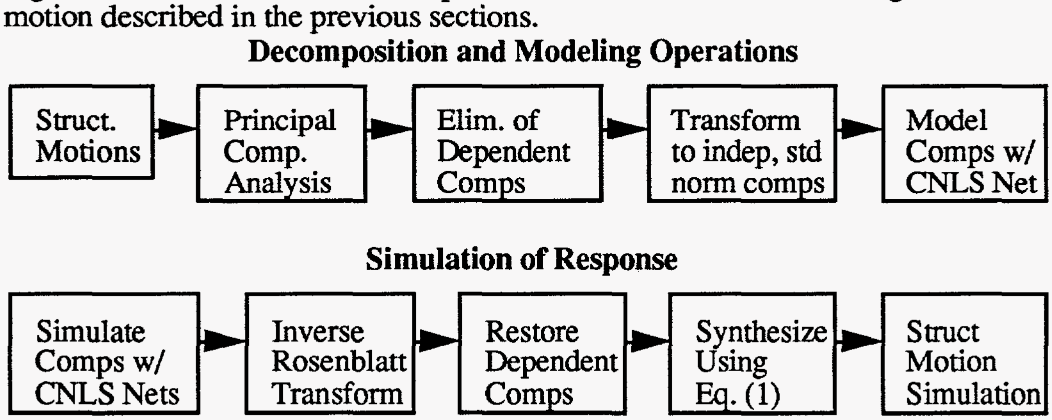

Figure 1.

Numerical Example

We simulate in this example the motion of a simple 10 degree-of-freedom system excited with a Gaussian white noise. Figure 2 shows the system. Training data for the CNLS net were generated by computing response over 6000 time steps; excitation and responses at ten locations were recorded. Figure 3 shows the response at one of the degrees of freedom. The responses were placed in a matrix X as referred to in the previous sections, and its SVD was computed. The singular values of the response indicate that accuracy of about 89% should be achieved by simulating the system response with its first three components. None of the three components were statistically dependent, so none was removed. The system is linear, so its response is Gaussian, therefore, the Rosenblatt transform was not used on the three components in the simulation. (Linearity is of no special benefit in the CNLS net modeling process since the framework for the CNLS net is nonlinear.) The first three components of the response were modeled with CNLS nets. The entire system was tested using data generated over 1000 steps of response computation. The initial conditions and excitation were used to start then execute a response simulation with CNLS nets in the space of $\boldsymbol{u}$. The first column of $\boldsymbol{u}$ from the test data and the first column from the simulated data are compared in Figure 4. This is the dominant component of the response. The match is good, particularly in view of the fact that the simulation is iterated. The simulated response is now reconstructed by substituting into Eq. (1) using the simulated $\boldsymbol{u}$, and the results are compared to the test response at one of the degrees of freedom. This is shown in Figure 5. Of course, the match is quite good since the dominant component is well simulated.

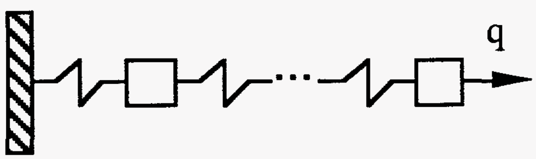

Figure 2. A simple system.

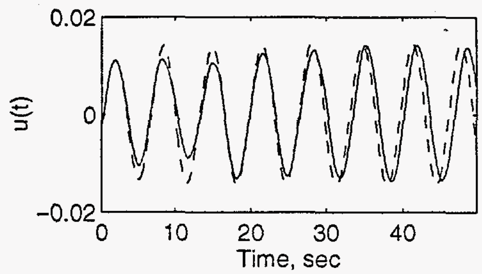

Figure 4. Comparison of test and CNLS simulated responses - Component 1.

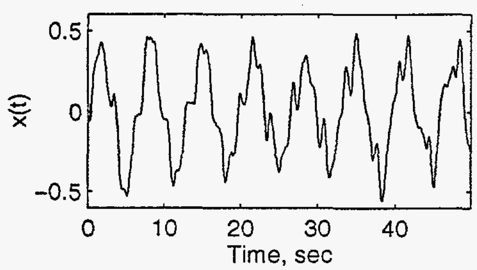

Figure 3. Response to white noise input.

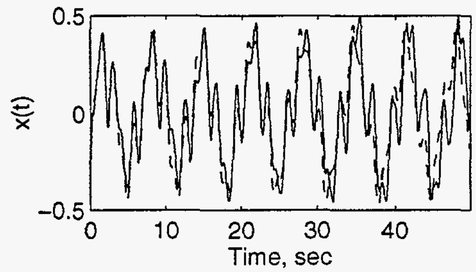

Figure 5. Test and simulated responses at a point in the simple system.

Conclusions

A sequence of operations leading to the simulation of structural system motions with ANNs is described in this paper. It is argued that if the motions can be decomposed and transformed into simple components, then the simulation will be simpler and more accurate. A numerical example confirms that relatively simple motions can, indeed, be modeled with ANNs and the CNLS net in particular.

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