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Status of Magnetically-Insulated Power Transmission Theory

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Abstract: The theory of magnetically-insulated power flow has improved dramatically over the last two decades since the early works of Creedon[1], Lovelace and Ott,[2] Ron, Mondelli, and Rostoker[3], and of Bergeron.[4] During the intervening years theoretical improvements included a complete general kinetic theory that involved distributions of electrons based on quasi-conserved canonical variables[5] and was used to study flow stability[6] and to analyze simulations[7] and pulsers with voltage adders.[8,9] The status of theory at this time allowed us to understand many features of these flows, but did not allow detailed analysis for design and data interpretation.

Recent theoretical advances have drastically changed this situation. Two recent static models[10,11] based on layered flows have allowed us to understand and to improve power coupling in voltage adders, current adders, plasma opening switches and in systems where the vacuum impedance varies along the flow. A dynamic model based upon electrons flowing in one or more thin layers has permitted detailed self-consistent time-dependent calculations which include electron flow.[12] This model accurately predicts experimental and simulational data.

Introduction: Models of static magnetically-insulated power flow have been studied for many years.[1-4] These are used to calculate voltage from current, and as a basis for stability analysis, but have not been very useful as a design tool. General kinetic theories included general static and dynamic flows.[5,7] These provided much understanding, and can be used for analyzing waves and stability,[6] and used for simulation diagnostics,[7,8] but kinetic theories have been of limited use as a design tool. Generally systems designers have used circuit codes that at most include only direct electron losses across the magnetically-insulated transmission line (MITL). These losses are important for slowing front velocities, but are not as important as electrons flowing parallel to the transmission line axis.

Recently new models have become available which include electron flow and can be useful for system design and data analysis. These include static models that can be used to optimize system components such as vacuum voltage adders and impedance transitions,[9-11] and a dynamic model that includes all important effects that are measured in experiments.[12]

Static Models: Illy, Kuntz, and Westermann used a layered model involving stratified, constant density layers to understand the voltage adder in the Kalif-Helia driver.[9] The model calculations were compared to simulations. Church and Sudan[10] used a similar model to study simulated layered flows. They concluded that multiple-layer flows can be modelled with two layers due to a tendency of layers away from the cathode to break into vortices. This had been noticed earlier in simulations done by Rosenthal.[8] Mendel and Rosenthal[11] found a somewhat simpler model with zero-thickness electron layers placed in appropriate positions. Electron layer position is determined by a parameter called flow impedance. Flow impedance was originally introduced for quantifying the performance of plasma-opening switches.[13] It was soon realized that the definition could be generalized so that its value described the position of the centroid of the electron charge in any transmission line.[11]

Most transmission line systems are axially symmetric. For this reason it is convenient to use cylindrical coordinates, and to transform the radius, r, to a new radial coordinate, R,

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given by $R=\pm(\mu_o/\varepsilon_o)^{1/2}/(2\pi)\ \ln(r/r_o)$ where r_o is an arbitrary positive constant. The sign of this definition is conveniently chosen to be negative for a negative outer conductor and positive for a positive outerconductor MITLs. In this coordinate system vacuum impedance is the distance between the cathode and the anode, and flow impedance is the distance between the centroid of the electronic charge and the anode. By working in this coordinate system, and by using enclosed current in place of the azimuthal magnetic field, and enclosed charge per unit length in place of radial electric field, the cylindrical problem has the same form as a planar problem.[11]

Magnetically-insulated flows at voltages relevant to pulsed-power systems involve important amounts of electron charge and current that should not be neglected. However, the electrons behave more simply than at lower voltages because field pressures dominate the electron pressure.[7] For this reason, models can assume that the net force on the electron cloud is zero. For axial forces this is equivalent to saying that the axial electric field is zero at the centroid of the electron cloud, i.e. at the position of the thin layer in the model. This is the same as saying that the electrons flow on equipotential lines, and is just the parapotential flow assumption that Creedon used to such great advantage.[1] The assumption that the radial force on the electron cloud is zero everywhere yields a pressure balance relationship relating the local values of enclosed current and charge.

An accurate model of these flows can be obtained by replacing the electron cloud by one or more thin layers of charge. The thin-layer model simplifies voltage adder calculations. Using the parapotential and pressure balance relationships, and assuming all input line voltages, vacuum impedance, and flow impedances are the same, the downstream flow impedance is calculated to be as shown in Fig. 1.

Dynamic Model: We have developed a dynamic model[12] capable of calculating the voltage, currents, and electrode charges at all points in MITL systems. The model inputs are the forward wave and impedance of the driver, the MITL vacuum and flow impedances at all axial locations, and the load current.

The model uses a single layer of electrons, and since there is one added conductor (the electron layer) there are three Telegrapher equations. These equations describe Faraday's law, continuity at the anode, and continuity at the cathode. When there is no electron flow the latter two equations are identical. Transverse currents are determined from data (experimental or simulational) for frontal velocities of step waves, plus space-charge limited cathode conditions. The frontal velocity data are general, and need to be determined only for one simple system and not repeated for each problem.

Figure 2 shows data taken behind forward and backward wave simulation data. The data are shown as A,V pairs where A is the magnetic flux per unit length and V is the electric voltage, both taken behind the step wave front. These data were used to determine two constants related to the electron losses in forward and reflected waves. These two constants do not change when the problem changes. Also shown in Fig. 2 are the model results taken for the same drive waves and MITL geometry. The load impedances were not the same since the simulation used an electron diode while the model used a resistor.

Figure 3 shows a simulated wave when it is in the transition region of a MITL that begins at 11.9 Ω and then increases to 24.3 Ω. The wave into the MITL input has a $\sin^2$ shape with a 6 ns wide base. Figure 4a shows the simulated and modelled line voltage versus axial position at one instant of time. Figure 4b shows the anode and cathode currents, also at 6 ns.

Three Interesting Observations: These models have proven valuable in interpreting experimental data, and have been used in the design of new pulsers. They also have been used to explain experimental data and provide some interesting design rules. The optimum adder design, shown in Fig. is given by a sequence of vacuum impedances proportional to $n^{1/2}$, rather that proportional to n as in the vacuum case. In such a design flow impedance is never below one-half vacuum impedance (Fig. 1). This reduces overall system inductance, and eliminates losses at adders during wavefronts.

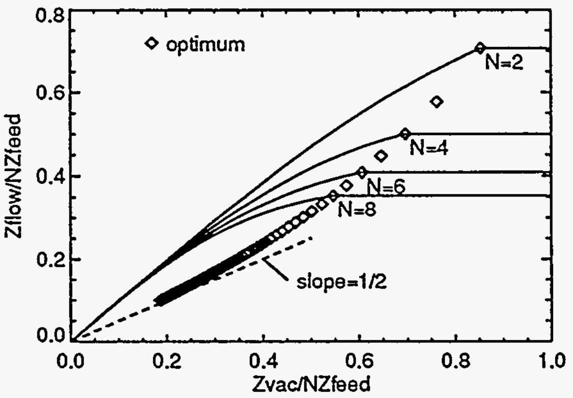

Figure 1. Downstream flow impedance versus vacuum impedance for multiple-line voltage adders.

Simulation and experimental data indicate that wave front velocity depends upon wave voltage and not geometry. Using Faraday's law, it can be shown that waves passing increases in vacuum impedance must lose electrons at the impedance transition in the proportion needed so that a MITL does not see a reflection from a vacuum impedance increase. This explains an observation of Rosenthal and Desjarlais.[13] Using simulations they noticed that a MITL seeing a load runs at the forward wave self-limited value until the load impedance is low enough to draw more than the self-limited cathode current.

Perhaps the most interesting prediction relates to flow impedance during forward waves downstream of voltage adders with more than four feed lines. If the line voltage is plotted versus the magnetic flux per unit length for a forward step wave, it must be a straight line in the A,V plane because of Faraday's law. The slope of this line is the frontal velocity. The frontal velocity is always faster than the electron drift velocity behind the front. As a result there is a loss front at the beginning of the wave, where magnetic field is low, followed by a region of emission where electrons are filling in behind the front as it moves away from the drifting electrons. Between these two regions is a point where insulation is marginal. Ahead of it electrons are not insulated, behind it they are.

Figure 5 shows such a wave plotted in the A,V plane. The straight line representing 1V(t) plotted versus A(t) starts at the point 0,0 (i.e. at the front of the wave), goes upward to the point where the electrons begin to be insulated, and finally onward to the final value on the generator side of the front. If this were done for all amplitudes of forward waves there would be two curves; the locus of insulation points and the locus of forward wave points.

For the single layer model these two lines are the same when the flow impedance is half the vacuum impedance. If it is less than half the vacuum impedance the insulation line

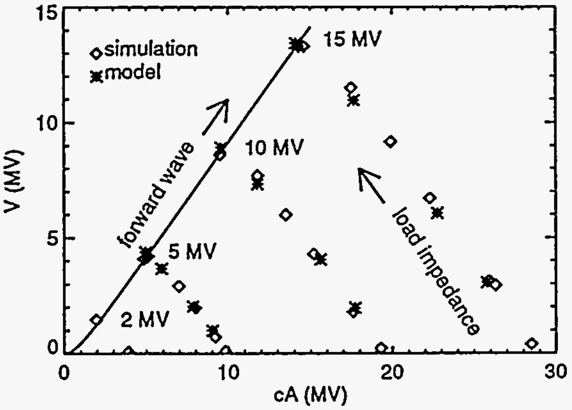

Figure 2. Step wave data taken behind forward a reflected waves from simulations and the model.

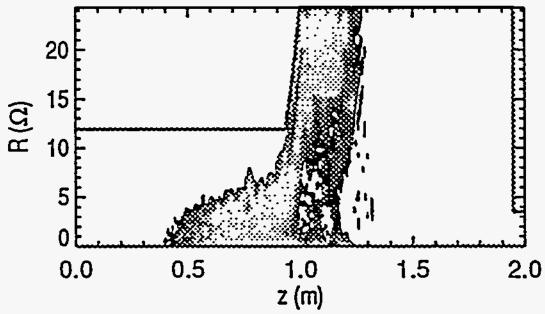

Figure 3. Electronic charge density in a simulated impedance change. The line is the electrode shape.

is to the right of the forward wave line, and the front is never insulated. Thus, if a device, for instance a vacuum-matched ten feed-line adder, tries to make the flow impedance less than half the vacuum impedance, electrons will be lost at the adder until the flow impedance is half the vacuum impedance. Thus the importance of using the optimum adder design.

Conclusions: The theory of magnetically-insulated electron flow in transmission lines is well understood, and models are now adequate for confident use for data analysis and system design.

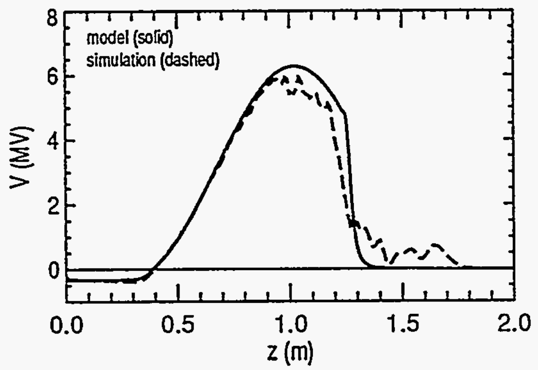

Figure 4a. V, at the time of Fig. 3.

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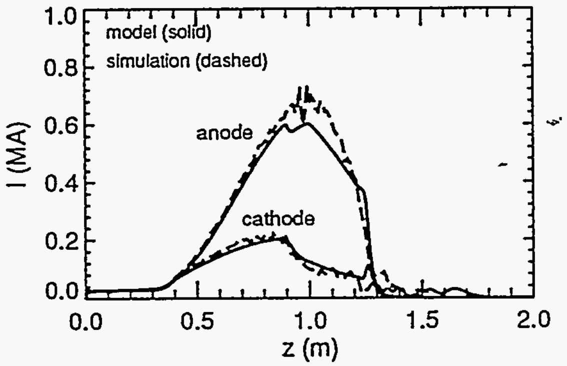

Figure 4b. Current at the anode and cathode at the same time as Fig. 3.

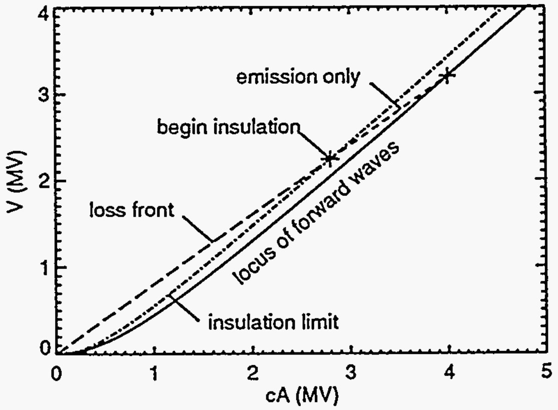

Figure 5. A step-forward wave in the A,V plane. When A,V is above the insulation line electrons can cross from cathode to anode.

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