Model-Based Processing for Shallow Ocean Environments: The Broadband Problem

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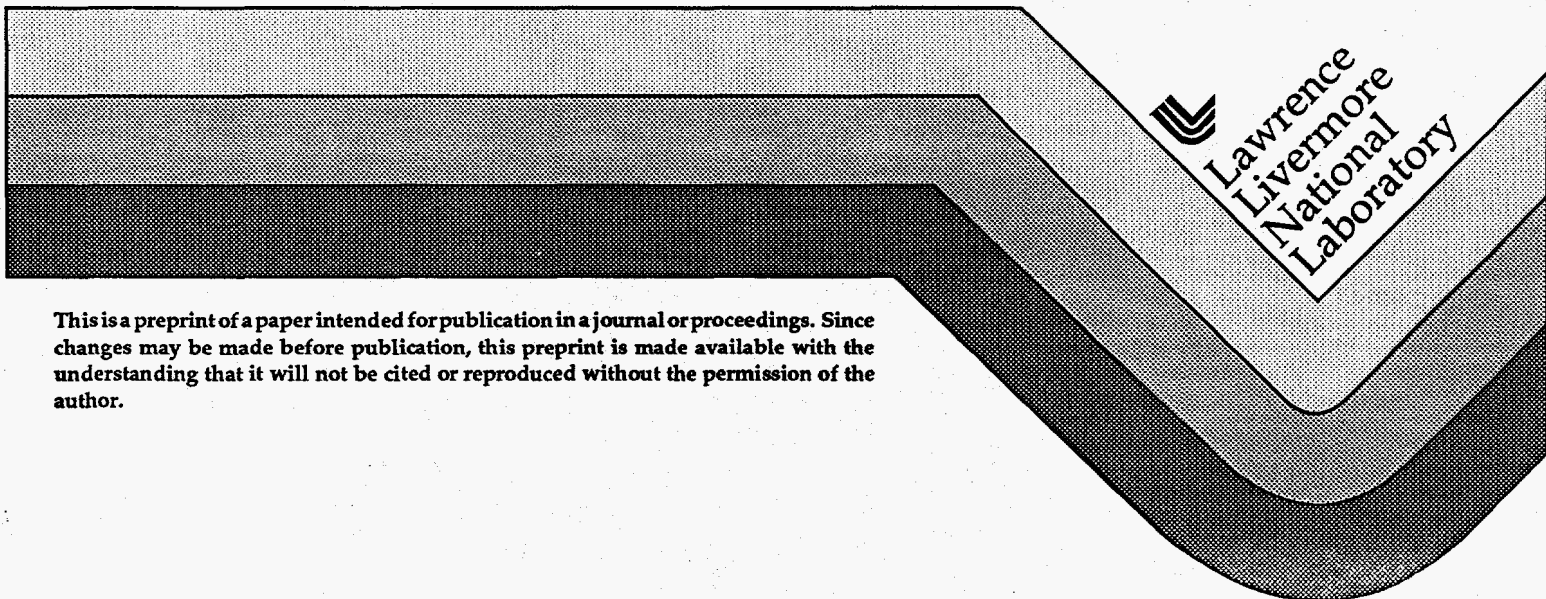Lawrence Livermore National Laboratory

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Model-based processing for shallow ocean environments: the broadband problem

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Most acoustic sources found in the ocean environment are spatially complex and broadband. When propagating in a shallow ocean these source characteristics complicate the analysis of received acoustic data considerably. The enhancement of broadband acoustic pressure-field measurements using a vertical array is discussed. Here a model-based approach is developed for a broadband source using a normal-mode propagation model.

1. INTRODUCTION

Acoustic sources found in the hostile ocean environment are complex both spatially and temporally being broadband rather than narrowband. When propagating in the shallow ocean these source characteristics complicate the analysis of received acoustic data considerably—especially in littoral regions providing an important challenge for signal processing [1-5]. It is this broadband or transient source problem that leads us to a model-based signal enhancement solution.

Uncertainty of the ocean medium motivates the use of stochastic models to capture the random nature of the phenomena ranging from ambient noise and scattering to distant shipping and the non stationary nature of this hostile environment [6-7]. Therefore, processors that do not take these effects into account are doomed to large estimation errors. When contemplating the broadband problem it is quite natural to develop temporal techniques especially if the underlying model is the full wave equation. However, if we assume a normal-mode propagation model, then it seems more natural to: (1) filter the broadband receiver outputs into narrow bands; (2) process each band with a devoted processor; and then (3) combine the narrowband results either coherently or incoherently to create a broadband processor. One apparent advantage to this approach is to utilize narrowband signal processing techniques thereby providing some noise rejection and intermediate enhancement for the next stage.

This is the approach we take in this paper to construct our broadband model-based processor (MBP), that is, we first decompose the problem into narrowbands and construct

a bank of model-based processors–one for each band. It has already been shown that the state-space representation can be utilized for signal enhancement to spatially propagate both modal and range functions [8]. Specifically, using the normal-mode model of the acoustic field, and a known source location, the modal functions and the pressure field can be estimated from noisy array measurements. We provide a brief discussion of the broadband problem in section 2 and develop the underlying state-space model structures. Finally, in section 3 we discuss the theoretical development of a broadband model-based processor.

2. BROADBAND STATE-SPACE PROPAGATORS

In this section we discuss the development of a broadband propagator eventually employed in a model-based scheme to enhance noisy pressure-field measurements from a vertical array of hydrophone sensors. We develop a state-space representation as a forward propagation scheme for eventual use in model-based processor design.

It is well-known [9] in ocean acoustics that the pressure-field solution to the Helmholtz equation under the appropriate assumptions can be expressed as the sum of normal modes for a narrowband harmonic source. This modal representation has been extended to include a broadband source, $s(t)$, with corresponding spectrum, $S_s(\omega)$ [1-5]. In this case, the ocean medium is specified by its *Green's* function and expressed in terms of the inherent normal modes spanning the water column. Thus, the normal-mode solution for the broadband source problem can be decomposed into a series of narrowband solutions, that is,

$$p(r, z, \omega_q) = \sum_{m=1}^{M_q} a_q \mathcal{H}_o \left(\kappa_r(m, q) r \right) \phi_m(z_s, \omega_q) \phi_m(z, \omega_q), \tag{1}$$

where p is the acoustic pressure-field at the temporal source frequency ω_q; $\mathcal{H}_o$ is the zeroth order Hankel function; ϕ_m is the mth modal function evaluated at z and source depth z_s; $\kappa_r(m, q)$ is the horizontal wavenumber associated with the mth mode; r is the horizontal range, and the source amplitude is $a_q = \Delta \omega |S(\omega_q)|$ with $S(\omega_q)$ interpreted as a single narrowband impulse at ω_q. Here the wave numbers satisfy the corresponding *dispersion relation*

$$\kappa_r^2(m, q) = \frac{\omega_q^2}{c^2(z)} - \kappa_z^2(m, q), \qquad m = 1, \ldots, M_q; \quad q = 1, \ldots, Q, \tag{2}$$

and κ_z is the vertical wave number with c the depth-dependent sound speed.

Suppose we further assume an L-element vertical sensor array, then $z \to z_\ell, \ell = 1, \cdots, L$ and therefore, the pressure-field at the array for the qth temporal frequency of Eqn. 1 can be written as

$$p(r, z_\ell, \omega_q) = \sum_{m=1}^{M_q} \beta_m(r, z_\ell, \omega_q) \phi_m(z_\ell, \omega_q), \tag{3}$$

where $\beta_m(r, z_\ell, \omega_q) := a_q \mathcal{H}_o \left(\kappa_r(m, q) r \right) \phi_m(z_s, \omega_q)$ is the mth-modal coefficient at the qth temporal frequency.

The next question for the broadband problem is, "how can this approach be cast into a MBP solution?" Assuming that the source range is known ($r = r_s$), it is well-known [8] that the state-space propagators for the narrowband pressure-field can be obtained from the relationship

$$p(r_s, z, \omega_o) = \mathcal{H}_o(\kappa_r r_s)\phi(z)\delta(\omega - \omega_o). \tag{4}$$

Assuming a known range as before, then we have a "depth only" model [8], with the state-space forward propagator is given by

$$\frac{d}{dz}\phi(z) = A(z)\phi(z) + B(z)u(z), \tag{5}$$

with ϕ the state vector of modal functions and u the source function. We note that this model is valid for a single temporal frequency, ω_o and we would like to extend it to the broadband problem considering a set of temporal frequencies and associated amplitudes $[\{a_q\}, \{\omega_q\}]$. From the extension of the normal-mode, pressure-field solution to the broadband case previously, we see that not only do the temporal frequencies change, but so do the number of modes spanning the water column at each frequency as well as the corresponding wave numbers, $\{\kappa_r(m, q)\}, m = 1, \ldots, M_q; \quad q = 1, \ldots, Q$. Therefore, these variations imply that the state-space processor for the broadband case must also reflect these changes.

First, we investigate the state-space propagator for a single mode, m and temporal frequency, ω_q, then we expand these relations. The *broadband state-space propagator* for the mth-mode is given by the set of vector-matrix equations

$$\frac{d}{dz}\phi_m(z, \omega_q) = \begin{bmatrix} 0 & 1 \\ -\kappa_z^2(m, q) & 0 \end{bmatrix} \phi_m(z, \omega_q) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(z, \omega_q), \tag{6}$$

with corresponding pressure-field measurement model at the ℓth-sensor

$$p_m(r_s, z_\ell, \omega_q) = [\beta_m(r_s, z_s, \omega_q) \quad 0] \phi_{m1}(z_\ell, \omega_q), \tag{7}$$

for the state vector defined by $\phi_m^T(z, \omega_q) = [\phi_{m1}(z, \omega_q) \quad \phi_{m2}(z, \omega_q)]$ and $u(z, \omega_q) = \delta(z - z_s)\delta(\omega - \omega_q)$, and $\beta_m(r_s, z_s, \omega_q)$ is the mth modal coefficient defined in Eqn. 3 with $r = r_s$. We can rewrite these equations simply as

$$\begin{aligned} \frac{d}{dz}\phi_m(z, \omega_q) &= A_m(z, \omega_q)\phi_m(z, \omega_q) + B_m u(z, \omega_q) \\ p_m(r_s, z_\ell, \omega_q) &= C_m^T(r_s, z_s, \omega_q)\phi_m(z_\ell, \omega_q), \qquad m = 1, \cdots, M_q \end{aligned} \tag{8}$$

Expanding over each mode for the narrowband frequency ω_q, we have that

$$\frac{d}{dz}\Phi_q(z) = \begin{bmatrix} A_1(z, \omega_q) & \cdots & O \\ \vdots & \ddots & \vdots \\ O & \cdots & A_{M_q}(z, \omega_q) \end{bmatrix} \Phi_q(z) + \begin{bmatrix} B_1 \\ \vdots \\ B_{M_q} \end{bmatrix} u_q(z), \tag{9}$$

with corresponding sensor measurement

$$p_q(r_s, z_\ell) = C_q^T(r_s, z_s)\Phi_q(z_\ell), \tag{10}$$

where $\Phi_q^T(z) := \left[\phi_1^T(z,\omega_q) \cdots \phi_{M_q}^T(z,\omega_q) \right]$, and $C_q^T(r_s,z_s) := \left[C_1^T(r_s,z_s,\omega_q) \cdots C_{M_q}^T(r_s,z_s,\omega_q) \right]$.

Now including all of the temporal frequencies, $\{\omega_q\}$, $q = 1, \cdots, Q$, the overall *broadband state-space propagator* evolves given by

$$\frac{d}{dz}\Phi(z,\omega) = A(z,\omega)\Phi(z,\omega) + B(z,\omega)u(z)$$

$$p(z_\ell,\omega) = C^T(z,\omega)\Phi(z_\ell,\omega), \tag{11}$$

where $\Phi(z,\omega) \in \mathbf{R}^{2M \times 1}$, $A(z,\omega) \in \mathbf{R}^{2M \times 2M}$, $C^T(z,\omega) \in \mathbf{R}^{1 \times 2M}$ with $M := \sum_{q=1}^{Q} M_q$.

The internal structure of this overall processor admits the following decomposition:

$$\frac{d}{dz}\Phi(z,\omega) = \begin{bmatrix} A(z,\omega_1) & O & \cdots & O \\ O & A(z,\omega_2) & \cdots & O \\ & & \ddots & \\ O & O & \cdots & A(z,\omega_q) \end{bmatrix} \begin{bmatrix} \Phi_1(z,\omega) \\ \Phi_2(z,\omega) \\ \vdots \\ \Phi_Q(z,\omega) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_Q \end{bmatrix} u(z), \tag{12}$$

with $A(z,\omega_q) = diag \left[A_1(z,\omega_q) \cdots A_{M_q}(z,\omega_q) \right] \in \mathbf{R}^{2M_q \times 2M_q}$ and as before,

$$A_m(z,\omega_q) = \begin{bmatrix} 0 & 1 \\ -\kappa_z^2(m,q) & 0 \end{bmatrix}, \qquad m = 1, \cdots, M_q, \tag{13}$$

with pressure-field sensor measurement (asuming an incoherent sum over the temporal frequency bands)

$$p(z_\ell,\omega) = \begin{bmatrix} C_1^T(r_s,z_s) & \cdots & C_Q^T(r_s,z_s) \end{bmatrix} \begin{bmatrix} \Phi_1(z_\ell,\omega) \\ \Phi_2(z_\ell,\omega) \\ \vdots \\ \Phi_Q(z_\ell,\omega) \end{bmatrix}. \tag{14}$$

Clearly, from previous work on the large-scale narrowband problem [10], the broadband state-space propagator admits a parallel structure for real-time implementation and the processor for the vertical array still sequentially (one sensor at a time) possesses that same property. As before in previous work [8] this deterministic model can be extended to a stochastic Gauss-Markov representation given by

$$\frac{d}{dz}\Phi(z,\omega) = A(z,\omega)\Phi(z,\omega) + B(z,\omega)u(z) + \mathbf{w}(z,\omega)$$

$$p(z_\ell,\omega) = C^T(z,\omega)\Phi(z_\ell,\omega) + v(z,\omega), \tag{15}$$

where $\mathbf{w}$, v are additive, zero-mean gaussian noise sources with respective covariance matrices $\mathbf{R}_{ww}(z,\omega)$, and $\mathbf{R}_{vv}(z,\omega)$.

3. BROADBAND MODEL-BASED PROCESSING

In this section, we discuss the theoretical construction of processors for broadband state-space structures. The development of the corresponding MBP for the broadband shallow ocean problem follows directly from the structure of the broadband state-space forward propagator based on the well-known Gauss-Markov representation of normal-mode models [8]. The basic continuous-discrete recursive processor takes on a predictor-corrector form with the continuous (in z) *predictor* given by

$$\frac{d}{dz}\hat{\Phi}(z,\omega) = A(z,\omega)\hat{\Phi}(z,\omega) + B(z,\omega)u(z,\omega), \tag{16}$$

and the discrete (in z) *corrector* given by

$$\hat{\Phi}(z_\ell|z_\ell,\omega) = \hat{\Phi}(z_\ell|z_{\ell-1},\omega) + K(z_\ell,\omega)\epsilon(z_\ell,\omega), \tag{17}$$

where the corresponding *innovations* sequence is

$$\epsilon(z_\ell,\omega) = p(z_\ell,\omega) - \hat{p}(z_\ell|z_{\ell-1},\omega) = p(z_\ell,\omega) - C^T(r_s,z_s,\omega)\hat{\Phi}(z_\ell|z_{\ell-1},\omega). \tag{18}$$

Here the notation $\hat{p}(z_\ell|z_{\ell-1},\omega)$, implies estimator with the $\hat{}$ and the estimate at the ℓth sensor based on all of the data up to the $\ell-1$ sensor. The broadband weighting matrix or Kalman gain [11] is defined by $K(z_\ell,\omega)$, while the broadband system matrices $A(z,\omega)$ and $B(z,\omega)$, and the measurement matrix, $C^T(r_s,z_s,\omega)$ are defined in Eqn. 15 of the previous section. This completes the description of the basic broadband MBP. To implement this processor we must first narrowband filter the sensor temporal signals using the the DFT approach. Then using the DFT, each of the narrowband frequency samples can be separated by sensor and used as input to the appropriate MBP. The overall broadband pressure-field can be reconstructed replacing the noisy measurement $p(z_\ell,\omega)$ with the estimated or enhanced pressure-field, $\hat{p}(z_\ell|z_{\ell-1},\omega)$ predicted by the MBP. We depict the structure of the general broadband implementation of the MBP structure in Figure 1. If we merely use the processor to "enhance" the noisy pressure-field measurements, then the corresponding output of the MBP is simply

$$\hat{p}(z_\ell|z_{\ell-1},\omega) = C^T(r_s,z_s,\omega)\hat{\Phi}(z_\ell|z_{\ell-1},\omega), \tag{19}$$

with the extracted/enhanced modal function estimates given by $\hat{\Phi}$ and of course, the corresponding innovations used to monitor the performance of the processor (see [8] for details).

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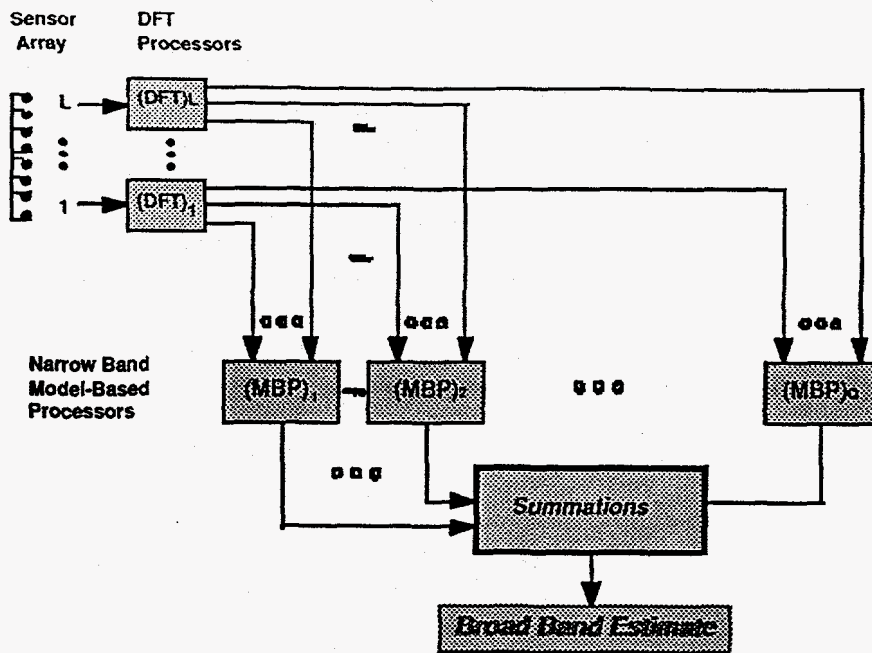
Figure 1: Structure of the Model-Based broadband Processor.

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