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THE PHYSICS OF CRYSTALLINE BEAMS

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The possibility of seeing ordering phenomena in particle beams that are cooled to very low temperatures has been discussed for some time[1-5]. The types of ordering depend on the linear particle density: as the density increases, the expected structures change from one dimensional strings to two dimensional zig-zag patterns, then to three-dimensional arrangements on the surface of a cylinder, growing into a multiple shell structure of concentric cylinders[4,5]. All of these shapes are seen in simulations where the "temperature" of the system is allowed to reach a low value; they are illustrated in figure 1.
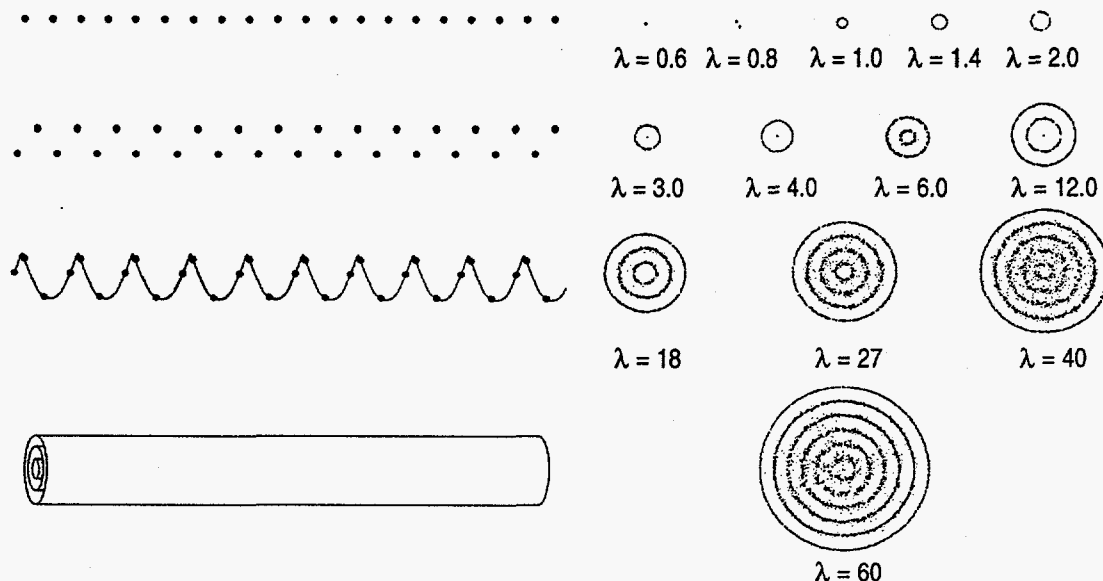

Figure 1. Configurations of ions in a cold beam for various values of the linear particle density λ. On the left are schematic pictures for the one-dimensional "string", two-dimensional "zig-zag", and three-dimensional simple helix and multi-shell configurations. On the right the output of Molecular Dynamics simulations for these various configurations is shown.

Fundamentals

What quantities matter in considering a cold beam? For cylindrically symmetric focusing, the shell structure is determined by a single parameter, the dimensionless linear particle density[4] λ: $\lambda \equiv (N/C) a_{SW}$, where N is the number of ions, C the circumference of the ring, and a_{SW} the Seitz-Wigner radius $a_{SW} = [3mq^2/\omega_\beta^2]^{3/2}$, with m the ion's mass, q its charge, and ω_β the betatron frequency. Note that a_{SW} also enters in the plasma coupling parameter: $\Gamma \equiv (q^2/a_{SW})/kT$. A system with $\lambda < 0.9$ will be a "string" for $0.9 < \lambda < 1.3$ it is a two-dimensional "zig-zag", for $1.3 < \lambda < 2.4$ it forms a single shell, and for $\lambda = 27$,

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for instance there would be four shells -- the number of shells increases as the $\sqrt{\lambda}$. Asymmetric focusing introduces an additional parameter that influences the shell structure. Finite length beam bunches become elongated spheroids, and an example is shown in figure 2.
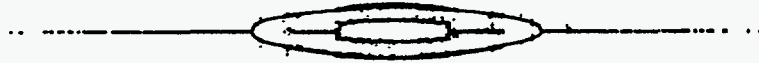

Figure 2. Simulations of a bunched beam, of 20,000 ions with length on the order of 20 cm and the maximum radius on the order of 100 microns.

Intrabeam Scattering -- Good or Bad?

In the normal thinking about cooling beams in a storage ring, intrabeam scattering is seen as a limit to low temperatures. Such scattering arises from the Coulomb field between particles, and the Rutherford scattering between particles on different betatron trajectories is the perturbing problem. However, the ordering between particles in a very cold beam, just discussed, exists only because of the same Coulomb interaction between the particles in the beam, an interaction that requires that in the cold limit the ions in the beam be as nearly equally spaced as possible. The factors limiting beam cooling (as, for instance, mentioned in the announced purpose of this Workshop) are usually considered to be in intrabeam scattering. This thinking must be reconsidered in this cold limit. This subject is the main purpose of this talk. The extent to which the periodic transverse oscillations from the focusing elements and the shear from the bending elements in the accelerator lattice[2,6,7] couple into "heating" a beam (in the sense of moving it away from an ordered configuration) is perhaps the best measure of intrabeam scattering (in the damaging sense) in a cold beam -- it is not quite the same as the usual intrabeam scattering and will depend sensitively on many details.

The concept of beam "temperature" poses no problems for a beam that is moving in a straight line and is focused continuously. The problem is that this is not what one is dealing with in real beams, and the definition of temperature that is needed to approach ordering is not the same as the state that is imposed on a beam circulating in a storage ring. The subject is further complicated by the fact that neither the bending nor the focusing of the beam is achieved continuously, but by discrete focusing elements.

I would therefore like to discuss in a little more detail what is needed for a cold beam, and what should be meant by temperature. Much of this has been stated before[2,3], but I feel that clarification of these issues is at the heart of what is keeping us from attaining an ordered state in a circulating beam.

Bending with Cooling to *Fixed Linear Velocity*

First consider the case of a beam, uniformly bent and with uniform focusing. When the beam is hot, the particles undergo independent betatron oscillation in the focusing field and their interactions (intrabeam scattering) are small. But in the other limit, when a beam is at very low temperatures at the space-charge limit, the betatron oscillations

have been frozen out. In this cold equilibrium state the particles are pushed together by the focusing force to the point that the focusing force is exactly canceled by the space-charge repulsion. If the linear density of particles is sufficiently low for the beam to be a one-dimensional string, or a zig-zag pattern in the vertical plane, there is no problem. But when the number of particles is sufficiently high to condense into one or more cylinders, then the particles that are on the inside of the curving beam *must* travel more slowly than their neighbors, and the particles on the outside faster. This is illustrated schematically in figure 3. Since current cooling techniques force particles to travel with the *same linear velocity* the "cooling" forces in a storage ring enforce a velocity distribution that is different from the one required for an ordered state.
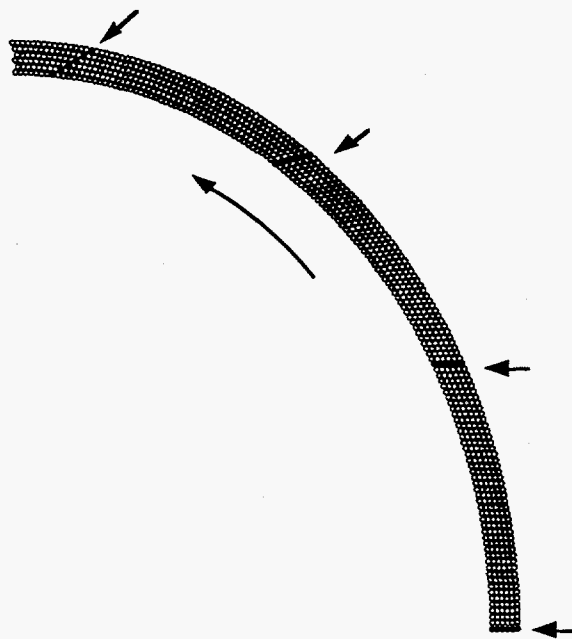

Figure 3. Schematic illustrating the shear that is experienced by a beam where particles are moving with a constant linear velocity. The curved arrow indicates the direction of motion, the short arrows and the darkened points indicate the relative positions of ions that start out, at the lowest arrow, at the same z-coordinate.

One may easily estimate the relative kinetic energies between particles in the perfect ordered beam: non-relativistically they are $\Delta E = E (\Delta r/R)^2$, where E is the kinetic energy of the circulating beam, R is the bend radius of the storage ring and Δr is the radial separation between particles, typically few x 10^{-3} cm in the ordered state. Clearly, what would be desirable, is some mechanism for introducing *graded cooling* which enforces the requisite velocity gradient for a constant *angular velocity* on the beam. At present there is no obvious technique for achieving this.

How should we think of a relevant "temperature" in the non-inertial frame that is co-rotating with the beam? It is the deviation from the velocity profile that corresponds to constant angular velocity. So if we had a beam with this profile, perfectly ordered, what would its "temperature" look like with the available measurement techniques, for instance

with a laser scan of the beam velocity spread? We can estimate the apparent "temperature" that would appear in such a scan as

$$T_{apparent} \approx \left(\frac{r_{beam}}{R_{ring}}\right)^2 E_{kin} = \frac{(2\pi)^2}{9}\lambda\frac{q^2}{a_{sw}}\frac{1}{v_\beta^2} \tag{1}$$

where r_{beam} is the radius of the beam, E_{kin} is its kinetic energy in the laboratory frame, q the charge of the ions, and v_β the betatron tune of the storage ring without space-charge effects. (With $r_{beam} = 0.6$ mm, $R_{ring} = 5$ m, $E_{kin} = 10$ MeV, $T_{apparent} \approx 1000°$ K. Note that q^2/a_{sw} is the energy numerator in the definition of the plasma parameter Γ.)

Conversely, one must remember that a beam that is at the space-charge limit is a strongly-coupled plasma and therefore the longitudinal and transverse degrees of freedom are strongly coupled -- with equilibration times on the order of $1/\omega_{plasma}$ where the plasma frequency is equal to the betatron frequency for a cold cylindrical beam and only slightly different when the beam is not cylindrically symmetric. When a constant linear velocity is forced onto the beam the "apparent temperature" defined above will be mixed into transverse (and random) velocities when the beam passes through bending magnets, thereby the above estimate for the apparent temperature, mixes into "heat" through a process, which is the form "intrabeam scattering" takes for cold beams in the space-charge limit. This heating can be avoided for very thin beams that become one- or two-dimensional, as long as they have no finite extent in the bend plane.

The parameter relevant to this heating is the rate at which the beam is sheared compared to the radius of the beam. The former is the cyclotron frequency, the latter is determined by the strength of the focusing whose measure is the betatron frequency. The ratio of the two frequencies is the betatron tune v_β. Equation (1) may then be rewritten in terms of the value of the plasma coupling parameter that one gets assuming the temperature is determined only by this source of heating as

$$\Gamma_{shear} \approx 0.7\frac{v_\beta^2}{\lambda} \tag{1a}$$

where the estimate of the apparent longitudinal temperature of (1) was divided by 3. This expression is an estimate of a limiting value of temperature for a multi-shell beam, say for $\lambda \gg 4$ or so, with constant linear-velocity "cooling" forced on the beam.

There are two other, not very realistic, options to conceivably deal with the shear problem. One of these, for a very stiff, strongly focused beam, will be discussed below. The other has been seen in simulations[3] but may be very difficult to realize in practice. If the beam cannot resist the imposed bending shear and the longitudinal cooling is applied continuously (easy in simulations not so easy in practice) then the particles segregate into a set of strings that slip by each other. The cooling needs to be strong because any transverse disorder would cause the particles to scatter from each other and as soon as that happens the system will heat up as discussed above. The strings themselves form an ordered

pattern within the beam envelope, a lattice of equilateral triangles -- a pattern that has also been seen in sheared colloids[8]. An example is shown in figure 4. But the stability of this type of order is very delicate against any type of change (e.g. the capture of an ion from the beam causing a lower particle density in one string and a migration of an ion from one string to the next would be a source of very strong heating until the system can settle down again. Simulations have not been done to test such instability.
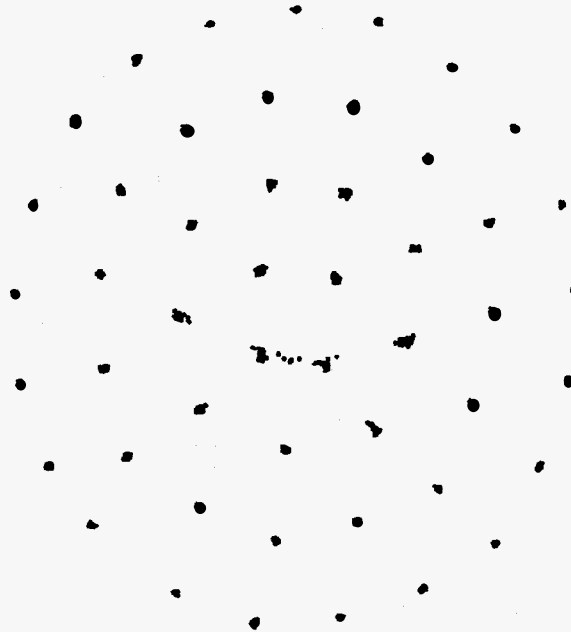

Figure 4. Separation of particles into sliding strings shown in a simulations for a constant focusing field in which the particles are subjected to continuing shear. The simulation contains 2000 particles and is for a linear particle density $\lambda = 27$.

Periodic Focusing

The above considerations apply to beams for which both the focusing and bending is uniformly distributed. In a real storage ring both are periodic and the effects of this need to be taken into account. First, however, we need to consider, in general terms, what it implies about the properties of the beam when the focusing lattice consists of discrete elements. Forgetting for the moment about the alternating gradient (the fact that the beam is alternately focused and defocused) consider first what happens if the focusing of the beam is achieved only through periodic focusing lenses. This will mean that the beam particles are pushed together suddenly in focusing element and then are pushed apart gradually, because of their mutual space charge repulsion. In the steady focusing case the system was in equilibrium -- now it will undergo oscillations. Since deviations from equilibrium can be approximated with harmonic motion, the amplitude of the envelope oscillations of the beam between focusing elements will increase quadratically with the interval between focusing elements, and the amplitude of these oscillations determines the kinetic energy contained in them. This is illustrated schematically in figure 5. A perfectly
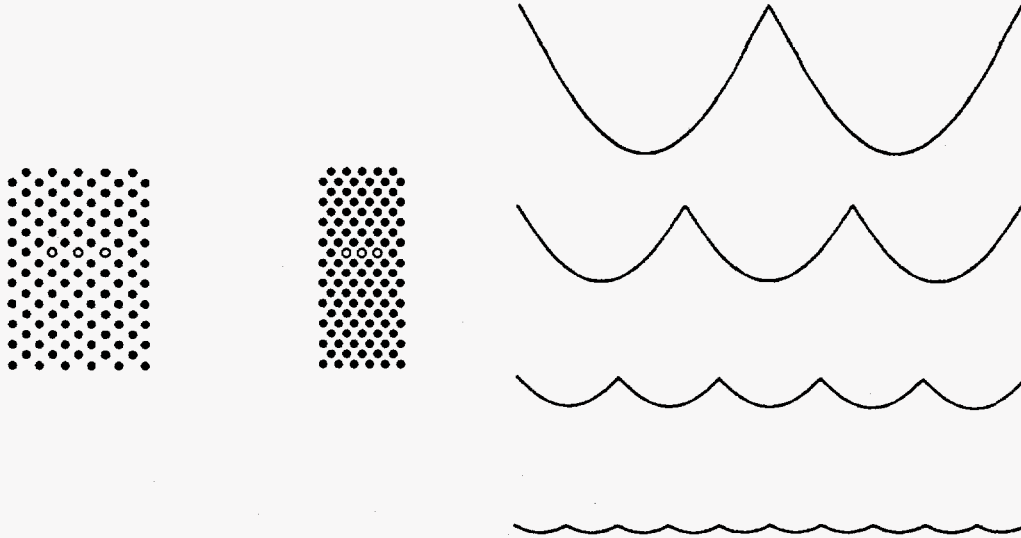
Figure 5. Schematic representation of a section of beam in a periodic focusing lattice with the beam traveling in the vertical direction. On the right are shown the excursions of the beam envelope in a simple lattice where the interval between focusing elements changes by a factor of two for each plot and the amplitude of oscillations changes quadratically with this interval.

ordered beam in the focusing lattice will undergo these oscillations and if we had a way of determining the spread in the transverse beam velocities there would be again an apparent temperature which may be written as

$$T_{apparent} \approx \left(\frac{\Delta r_{beam}}{r_{beam}}\right)^2 \left(\frac{r_{beam}}{R_{ring}}\right)^2 \frac{\nu_\beta^2 E_{kin}}{N_{f\beta}^2} = \frac{\pi^2}{6}\lambda\frac{q^2}{a_{sw}}\frac{1}{N_{f\beta}^2} \tag{2}$$

where Δr_{beam} is the change in the beam radius between focusing elements, and $N_{f\beta}$ is the number of focusing cells per betatron period. The important point is that these oscillations imposed by a periodic focusing field are not, by themselves, as destructive to ordered structure as the shear effect considered above, the ordered system can survive these oscillations as a rather plastic one and the restoring forces balance the perturbation so that an ordered array with superimposed envelope oscillations are obtained with no (or very little) coupling into random, "thermal" excitations[2,4,9]. The only caution is that the beam has normal modes[3,6] illustrated in figure 6 that should be avoided: for the pure radial volume oscillation the eigen-frequency of the beam is at the betatron frequency, while for the quadrupole shape oscillation it is $\sqrt{2}$ times that. So the period of focusing cells should avoid harmonics of these modes. Still, the kinetic energy associated with these oscillations is best minimized, and it is desirable to have as many focusing elements per betatron period as practical.

It should also be noted that any transverse cooling, should it become practical, would freeze these transverse oscillations at that point in the lattice and the location for transverse cooling in the lattice would need to be considered with caution.
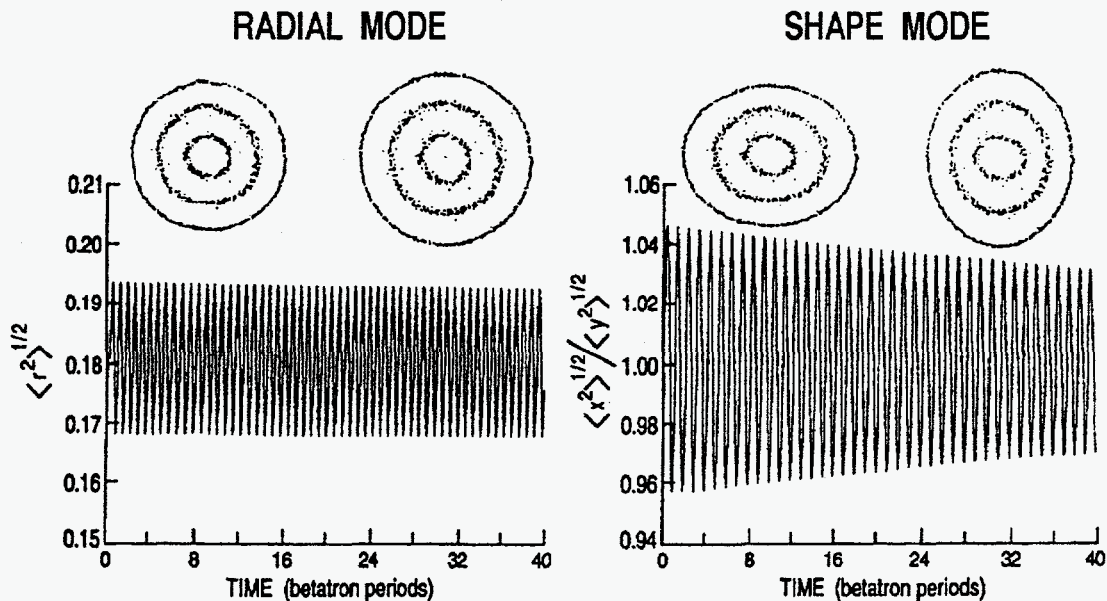
Figure 6. Normal modes of a beam in a constant focusing field are illustrated. On the left a radial "monopole" volume oscillation was excited; this seems to be a true eigen-mode of the system and does not couple into other degrees of freedom. On the right a quadrupole shape oscillation appears to be damped into more complicated modes. The simulation had 1000 ions.

Periodic Bending with Cooling to _Constant Angular Velocity_ in the "Soft Beam" Limit

A technique that would cool the beam to a velocity profile, with a gradient appropriate to the bend radius, such that the particles all follow their trajectories isochronously, would be ideal for a lattice where bending is continuous -- all longitudinal shear effects would disappear. However, in a real lattice, that consists of discrete bending elements and straight sections, even such cooling can present problems. In the limit when the bending elements are short compared to the straight sections they will impose a shear on the particles; they will be sheared in one direction in the bending elements and gradually catch up in the straight sections. Figure 7 illustrates this. For instance, a 90° bend is pretty violent: for two particles separated radially by a distance d, such a bend would introduce a longitudinal displacement $\pi/2$ times d, in other words about 1.6 times the interparticle spacing. It is difficult to see how an ordered system could displace such violent effects. Of course, the ideal state would be one where the particles are displaced half that amount before the bend in one direction ($\pm \pi/4$), and then by the same amount in the other direction after the bend, but the displacement is till large. Clearly the more the bending elements are distributed the better -- a 60° bend would imply that the total displacement would be $\pi/3$ and from before to after it could be $\pm \pi/6$, so the excursions may become survivable. Simulations also confirm this in various ways. But any slight disorder in the beam will cause particles to rub against each other and become a source of heat.
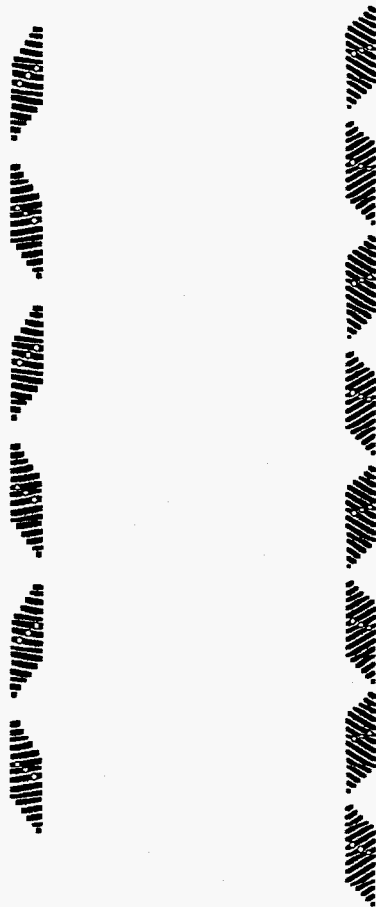
Figure 7. Schematic picture, in the same sense as figure 2, of a section of beam that is cooled to a constant angular velocity, and subjected to discrete bending elements. Each of the successive pairs represents a segment of beam before and after a bend (90° on the left and 60° on the right), with the change between segments taking place in the straight sections. The dark lines appear because of merging of the dots used to represent particles; three "adjacent" particles in the center of each segment are represented by open circles.

Bending of Beams in the "Stiff Beam" Limit

There is a further way out: *if* one can form the beam in an ordered state, then the crystalline structure produces a certain resistance to the shearing that is imposed on the beam by bends[2,3] -- in other words the particles *can* be forced to move faster or slower in the bends by the interparticle forces. The elastic shear modulus of these structures has been studied in simulations, though the results are somewhat contradictory. The results[3] seem to indicate that there is a limit to the resistance to shear that an ion beam, cooled to a constant linear velocity can sustain. This is shown in figure 8.

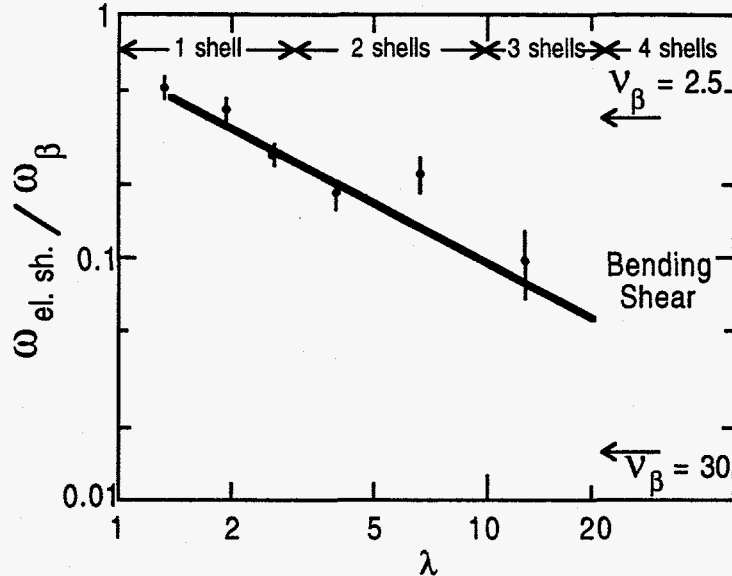### DISCLAIMER

ELASTIC SHEAR MODULUS IN CONDENSED BEAMS

Figure 8. Values of the elastic shear modulus (relative to the focusing strength, which is represented by the betatron frequency) of a cylindrical beam, extracted empirically from Molecular Dynamics simulations. When the imposed shear from the circular orbit, represented by the reciprocal of the betatron tune v_β is larger than the elastic shear modulus. Two values representing "small" and "large" storage rings are shown on the right. The elastic shear modulus is a function of the linear particle density λ, plotted on the bottom scale and the corresponding number of shells is shown on top.

As with any solid, the crystallized beam has a natural elastic resistance to shear. The elastic shear modulus may be characterized by a limiting natural shear frequency of the beam, and the requirement is that this natural frequency be larger than the driving frequency: $\omega_{shear} \geq \omega_{cyclotron}$. Simulations suggest that the values for this is $\omega_{shear} \approx \omega_\beta / \lambda$, in other words the condition for a beam to survive bending shear is that

$$v_\beta \geq \lambda \qquad (3)$$

With such a beam, cooling to constant linear velocity would be sufficient, for instance. Such lattices do exist in large storage rings, such as LEP and RHIC, or could have existed in the SSC, with the rate of bending relatively slow compared to the strength of the focusing -- and beams with 4-10 shells could survive in such rings. The value of v_β for smaller rings is typically around 2.5, corresponding to the single-shell regime.

Options

a)   Minimizing Heating from Bending Shear

1.   Find a method of cooling to <u>*constant angular velocity*</u>. (even a rough gradient will ease the problem), and/or

2. Find a method to *compensate for bends* by changing the velocity profile in special inserts (rf or electrostatic solutions suggested -- none seem viable at present).

3. Use *high v_β* and *high multiplicity* to increase the elastic shear modulus of the beam and minimize shear heating effects (appropriate in any case).

4. Put up with bending shear and settle for *"sliding strings"* (but this needs extremely strong cooling, which does not seem practical at present).

5. Stick to *sparse beams* (one-dimensional "strings" or single shells). (This may seem like giving up on a proper crystal -- also, it is not clear whether longitudinal cooling can be effective in this regime, since the transverse degrees of freedom will be very weakly coupled to the longitudinal ones.) If achievable, this could be a start.

b) Minimizing Heating from Discrete Focusing Elements

1. *Maximize* number of *focusing elements* in lattice per betatron period. (amplitude of oscillation decreases quadratically with number)

2. Focusing lattice and all aspects of ring should be as *evenly spaced* (and with as short a repeating cycle) as possible.

3. *Avoid harmonics* of the betatron frequency f_β —also harmonics of $\sqrt{2}f_\beta$ for equal vertical and horizontal frequencies (these are the frequencies of the normal modes and one should avoid driving them.) N.B. These values need to be modified for asymmetric focusing .

4. Any transverse cooling (should a practical scheme develop) will be most effective at a waist or maximum in the cold beam envelope.

Stability of the Ordered State and a Point about Simulations

I wish to make a point about simulations that may be important. There have been simulations in which configurations were found, with an accelerator lattice that included both periodic bending and "realistic" alternating-gradient focusing, similar but not identical to the solutions found with uniform focusing and bending. Now in such simulations the nature of the ordering, the cooling, and the stability of the configuration against perturbations is crucial. The principal point I wish to make is about the size of the sample that is followed in simulations.

In Molecular Dynamics simulations usually a section of the beam is treated, but then the question is what to do at the ends of this section. The usual procedure is to make the system self-enclosed -- that is let the particles at one end of the system interact with the forces from the other end. Thus the system must be periodic, at least with the length of the cell. In figure 9 there are examples for a simple case, showing that the configuration that comes from a simulation with Molecular Dynamics is strongly influenced by the choice of the length of the repeating cell, whether the number of ions in it is divisible by 2, by 3, or

by neither, even when the cell length is many times the repeating length for the lowest state as may be seen in figure 9. This can be particularly important for beams with many more ions, where there are multiple shells. A short repeating cell will have a symmetry forced
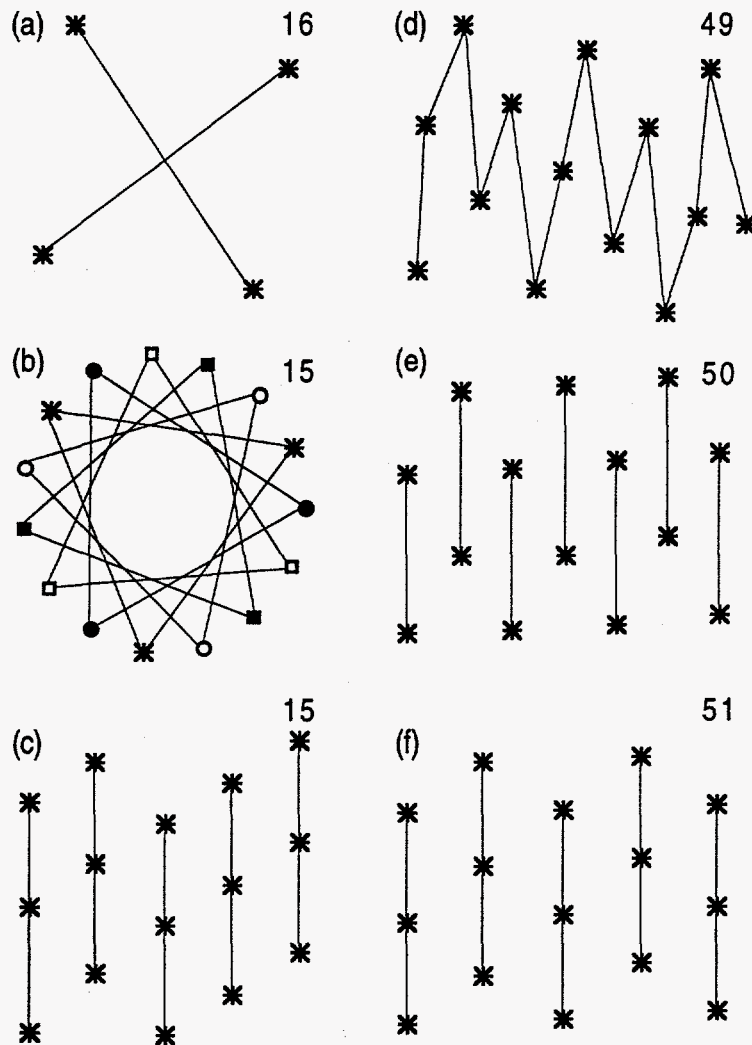

Figure 9. The effect of choice of repeating cell is shown on simulations of a beam with a fixed linear particle density $\lambda = 3.0$. The patterns seen are determined by the length of the cell. (a) shows the head on view for 16 ions in the cell -- they are lined up in pairs, along two perpendicular direction, (b) and (c) show results for 15 ions in the x-y plane and the x-z plane respectively -- the pattern is different with sets of three particles at a given z-coordinate along the beam forming triangles and the triangles rotating in orientation by 144 degrees; (d), (e), and (f) show a portion of the x-z plane for 49, 50, and 51 ions respectively.

on it that may not be the actual lowest state for a long beam, and the extent to which a particular configuration is stable under the repeating bending shear and focusing may very much depend on the symmetry of the configuration. It is therefore important to do simulations with as long a repeating unit cell as possible.

Summary

It seems that the time has come in the pursuit of lower and lower beam temperatures to start focusing more detailed attention to the reality of storage rings -- conventional cooling techniques and measures of temperature are generally not the appropriate ones at the lowest temperatures. Finding solutions to these serious problems does not appear to be impossible, but these considerations <u>must</u> be kept in mind in designing new storage rings with the aim to approach the regime of ordered three-dimensional beams. In particular, such rings will have to

a.   Use calculations of the lattice <u>*with the full effects of space charge included*</u>. (N.B. averaged over time, space charge exactly cancels the focusing fields for a cold beam and therefore must be explicitly included.)

b.   Find technical solutions and incorporate several of:

    i.   cooling to introduce a longitudinal <u>*velocity gradient*</u> and favor <u>*constant angular velocity;*</u>

    ii.   <u>*high multiplicity*</u> in bending and focusing elements;

    iii. stronger focusing <u>*(high betatron tune)*</u>;

    iv. <u>*high symmetry*</u> in the ring design.

c.   Finally, simulations should try to incorporate as much realism as possible, with larger repeating cells and more detailed descriptions of the lattice.

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