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COMPARISON OF SIMPLIFIED AND STANDARD SPHERICAL HARMONICS IN THE VARIATIONAL NODAL METHOD*

by

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Recently, the variational nodal method has been extended through the use of the Rumyantsev interface conditions[1] to solve the spherical harmonics (P_N) equations of arbitrary odd order.[2] Here, we generalize earlier x-y geometry work[3] to fit the corresponding simplified spherical harmonics (SP_N) equations into the variational nodal framework. Both P_N and SP_N approximations are implemented in the multigroup VARIANT code at Argonne National Laboratory in two- and three- dimensional Cartesian and hexagonal geometries. The availability of angular approximations through P_5 and SP_5, and of flat, linear and quadratic spatial interface approximations allows investigation of both spatial truncation and angular approximation errors. Moreover, the SP_3 approximation offers a cost-effective method for reducing transport errors.

The even-parity SP_N approximations are derived by first writing the slab geometry P_N approximation for odd order N. Let ψ and χ be vectors of length (N+1)/2 of the even and odd parity flux moments. Then

$$\mathbf{E}\frac{\partial}{\partial x}\chi + \sigma\psi = \mathbf{b}[\sigma_s\phi + S]$$

and

$$\mathbf{O}\frac{\partial}{\partial x}\psi + \sigma\chi = 0\,,$$

where $b_i = \delta_{1i}$ and $\mathbf{E}$ and $\mathbf{O}$ are two-striped lower and upper triangular matrices, respectively. The even parity equation obtained by eliminating χ is then

$$-\frac{\partial}{\partial x}\frac{1}{\sigma}\mathbf{H}\frac{\partial}{\partial x}\psi + \sigma\psi = \mathbf{b}[\sigma_s\phi + S]\ ,$$

where $\mathbf{H} = \mathbf{E}\,\mathbf{O}$, and ψ and χ are related by

$$-\frac{1}{\sigma}\mathbf{H}\frac{\partial}{\partial x}\psi = \mathbf{E}\chi\,.$$

The SP_N equations are obtained simply by letting $\frac{\partial}{\partial x} \to \vec{\nabla}$ and allowing ψ and χ to become functions of the x, y and z. Thus

$$-\vec{\nabla}\frac{1}{\sigma}\mathbf{H}\vec{\nabla}\psi + \sigma\psi = \mathbf{b}[\sigma_s\phi + S] \qquad (1)$$

and

$$-\frac{1}{\sigma}\mathbf{H}\hat{n}\cdot\vec{\nabla}\psi = \mathbf{E}\chi\,. \qquad (2)$$

The following functional may be shown to have Eq. 1 as its Euler Lagrange equations within the node and Eq. 2 as an interface condition

$$F_V[\psi,\chi] = \int_V dV\left[\vec{\nabla}\psi^T\frac{1}{\sigma}\mathbf{H}\vec{\nabla}\psi + \sigma\psi^T\psi - \sigma_s\phi^2 - 2\phi S\ \right] + 2\sum_\gamma \int_\gamma d\Gamma\psi^T\mathbf{E}\chi_\gamma$$

From here on, the procedure is the same as published prvieously.[4] Spatial polynomial approximations are used for ψ and χ ; a Ritz procedure is applied, and the resulting equations are cast in response matrix form.

Studies have been undertaken to compare the relative performance of SP_N and P_N approximations in two and three dimensions. In model fixed-source problems SP_N closely mimic the corresponding P_N solutions where large numbers of interfaces are not present. In criticality problems, the results shown in Fig 1 for the "rods-in" Takada Benchmark II in x-y-z geometry[5] are indicative of the eigenvalue errors which are found. In all cases studied the spatial truncation errors - which may be isolated by comparing flat, linear and quadratic interface conditions with the same angular approximation - are found to be positive. Errors attributable to the angular approximations - which may be isolated by comparing the spatially converged quadratic approximations - are negative. Thus, in some configurations, going from a lower to a higher order space or angular approximation may produce an accuracy loss as a result of the decreased error cancellations.

Other general observations are that space and angular approximations interact more strongly in P_N approximations, necessitating the refinement of the spatial approximation in tandem with increased P_N order. Conversely the accuracy of the SP_N approximations saturate as a result of the angular moments which are not included. The SP_3 approximation frequently offers substantial increases in accuracy at roughly double the cost of a corresponding nodal diffusion calculation, while full P_N calculations are substantially more expensive. On an IBM rs6000 the CPU times for the results in Fig. 1 were 78, 148 and 916 sec. for the P_1, SP_3 and P_3 calculations with linear interface conditions.

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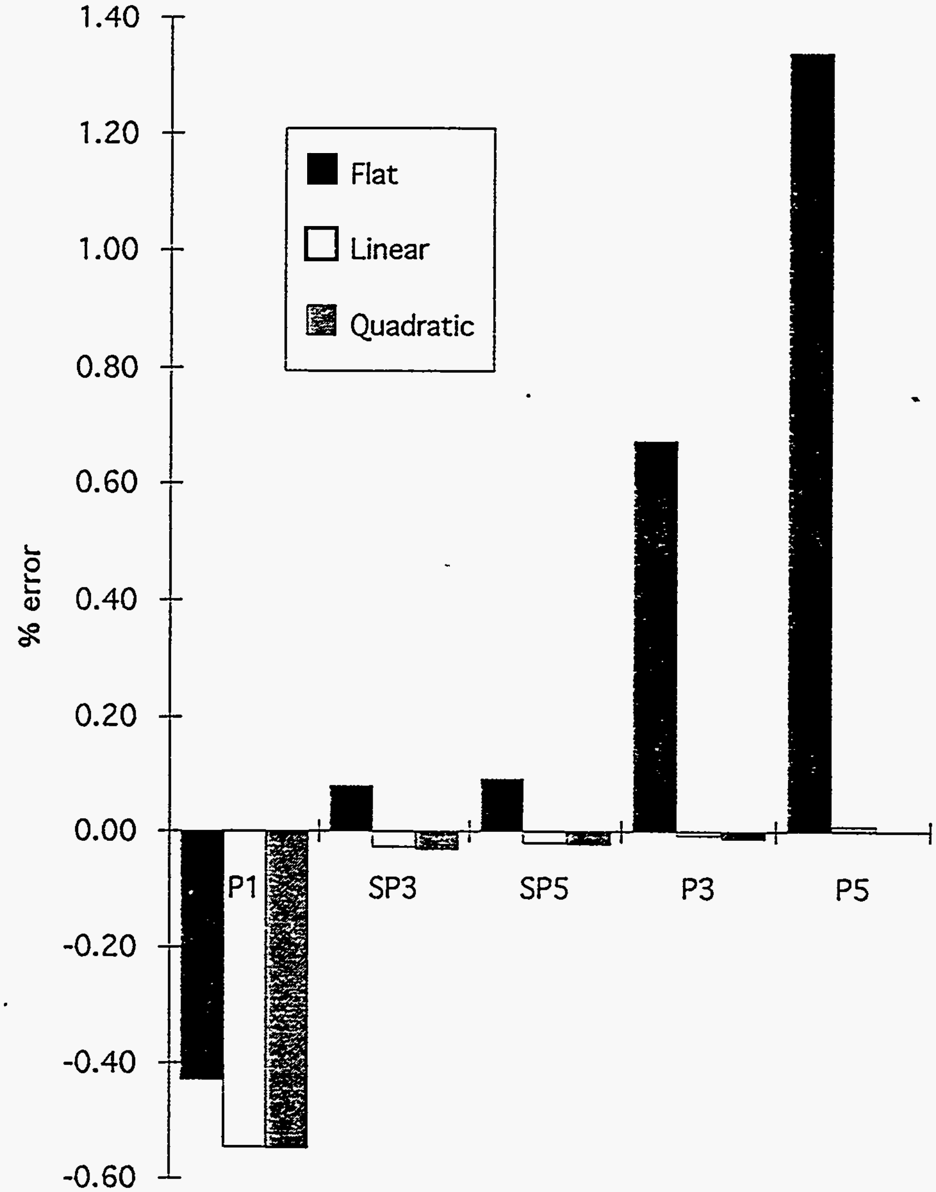

Figure 1. Eigenvalue Errors for the “Rods In” Takada Benchmark II
(reference k = 0.95954)