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Closed-form solutions for a disk, a filter, and a window for an x-ray beamline at the Advanced Photon Source

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Several temperature closed-form solutions are developed and presented in this paper. The heating on a one-dimensional thin disk and a two-dimensional thin plate are analyzed. Parametric studies are also included to determine the optimized temperature for the designs.

1. INTRODUCTION

The Advanced Photon Source (APS) is a 7-GeV synchrotron facility that will produce a high-powered beam of x-rays. A commissioning filter and window assembly has been designed [1] and analyzed [2, 3] for the APS insertion-device front ends. It consists of a 300-μm-thick graphite filter, a fixed mask (made of Glidcop), a multi-purpose transmitting filter/beam position monitor (BPM) (made of a 127-μm-thick, 1-inch-diameter diamond disk), and a window (made of 250-μm-thick beryllium). While the use of sophisticated finite element codes is the norm for such analysis, closed-form solutions, if they can be formulated, are very handy for doing parametric trend analysis. Using a closed-form solution that we developed for a thin disk or a plate subject to the Gaussian photon beam, we present here an extensive trend analysis on the filter, diamond disk, and window of the commissioning window assembly.

The formulation starts from a general, one-dimensional analytical solution for a thermal problem for a thin disk subject to an axisymmetrical, Gaussian-distributed heat flux and goes to a two-dimensional generalized analytical solution. This solution helps to predict the temperature/stress [4] fields as well as helps to optimize the design for the filter, the diamond disk, and the window.

II. NOMENCLATURE

Temperature	T	[°C]
Thermal conductivity	k	[W/(m K)]
Fixed polar coordinate	R	
Fixed Cartesian coordinate	X, Y	
Max. heat flux	q_0	[W/m²]
Convective heat transfer coefficient	h	[W/m²°C)]
Thickness	t	[m]
Ambient temperature	T_0	[0 °C]
Total power	T_p	[W]
Radius of the thin disk	a	[m]
Width of the thin plate in X direction	x_0	[m]
Width of the thin plate in Y direction	y_0	[m]
Standard deviation for an axisymmetrical, Gaussian-distributed heat flux	σ_o	[m]
Standard deviation in X direction for a two-dimensional Gaussian-distributed heat flux	σ_x	[m]
Standard deviation in Y direction for a two-dimensional Gaussian-distributed heat flux	σ_y	[m]

III. THIN DISK WITH AN AXISYMMETRICAL, GAUSSIAN-DISTRIBUTED HEAT FLUX: TEMPERATURE PRESCRIBED COOLING AT THE BOUNDARY

The governing equation for a thin disk with an axisymmetrical, Gaussian-distributed heat flux is

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{q_0}{kt}\exp\left(-\frac{r^2}{2\sigma_0^2}\right)=0.$$

The boundary conditions (in Fig. 1a) are

T = 0 when r = a, and T = finite when r = 0.

The solution for temperature T is

$$T=\frac{q_0\sigma_0^2}{2kt}\left[E_1\left(\frac{a^2}{2\sigma_0^2}\right)-E_1\left(\frac{r^2}{2\sigma_0^2}\right)+\log\left(\frac{a^2}{2\sigma_0^2}\right)-\log\left(\frac{r^2}{2\sigma_0^2}\right)\right]$$

where $E_1(x)$ is an Exponential Integral defined as

$$E_1(x)=\int_x^\infty\frac{e^{-t}}{t}dt,$$

and the Gaussian standard deviation σ_0 is defined as

$$\sigma_0=\sqrt{\frac{P_T}{2\pi q_0}}.$$

IV. THIN DISK WITH AN AXISYMMETRICAL, GAUSSIAN-DISTRIBUTED HEAT FLUX: WATER COOLING AT THE EDGE

The boundary conditions are (in Fig. 1b)

$$k\frac{\partial T}{\partial r}=-hT \quad \text{when } r = a, \text{ and } T = \text{finite when } r = 0.$$

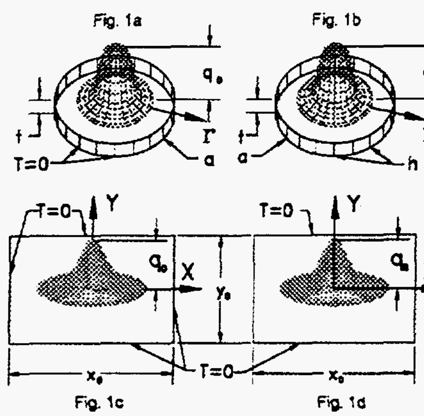

FIG.1 The geometry and boundary conditions for various closed-form solutions

The solution for temperature T is

$$T = \frac{q_0 \sigma_0^2}{2kt} \left[\begin{array}{l} E_1\left(\frac{a^2}{2\sigma_0^2}\right) - E_1\left(\frac{r^2}{2\sigma_0^2}\right) + \log\left(\frac{a^2}{2\sigma_0^2}\right) - \\ \log\left(\frac{r^2}{2\sigma_0^2}\right) + \frac{2k}{ah}\left[1 - \exp\left(\frac{a^2}{2\sigma_0^2}\right)\right] \end{array} \right].$$

V. THIN RECTANGULAR PLATE WITH GAUSSIAN-DISTRIBUTED HEAT FLUX: TEMPERATURE PRESCRIBED COOLING AT THE FOUR EDGES

A general, two-dimensional analytical solution to the thermal problem was also formulated for a thin, rectangular plate with Gaussian-distributed heat flux (Fig. 1c). The governing equation is

$$\frac{d^2T}{dx^2} + \frac{d^2T}{dy^2} + \frac{q_0}{kt}\exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right) = 0.$$

The boundary conditions are

$$\mathrm{T} = 0 \text{ when } x = \pm\frac{x_0}{2} \text{ and } \mathrm{T} = 0 \text{ when } y = \pm\frac{y_0}{2}.$$

The general solution is

$$T = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} a_{mn} \cos(\lambda_m x) \cos(\lambda_n y),$$

where

$$\lambda_m = \frac{(2m+1)\pi}{x_0},\ m = 0,1,2\cdots\cdots, \text{ and } \lambda_n = \frac{(2n+1)\pi}{y_0},\ n = 0,1,2\cdots\cdots$$

a_{mn} is found to be

$$a_{mn} = \frac{8\pi\,\sigma_x \sigma_y q_0}{(\lambda_m^2 + \lambda_n^2)\, x_0\, y_0\, kt} \exp\left(-\frac{\sigma_x^2 \lambda_m^2}{2} - \frac{\sigma_y^2 \lambda_n^2}{2}\right)$$
$$\mathrm{Re}\left[erf\left(\frac{x_0}{2\sqrt{2}\sigma_x} - \frac{i\sigma_x \lambda_m}{\sqrt{2}}\right)\right] \mathrm{Re}\left[erf\left(\frac{y_0}{2\sqrt{2}\sigma_y} - \frac{i\sigma_y \lambda_n}{\sqrt{2}}\right)\right].$$

VI. THIN RECTANGULAR PLATE WITH GAUSSIAN-DISTRIBUTED HEAT FLUX: TEMPERATURE PRESCRIBED COOLING AT THE TWO EDGES

The boundary conditions (in Fig. 1d) are

$$\mathrm{T} = 0 \text{ when } y = \pm\frac{y_0}{2} \text{ and } \frac{\partial T}{\partial x} = 0 \text{ when } x = \pm\frac{x_0}{2}.$$

The general solution becomes

$$T = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \cos(\lambda_m x) \sin(\lambda_n y) + \sum_{n=1}^{\infty} a_{0n} \sin(\lambda_n y),$$

where

$$\lambda_m = \frac{(2m+1)\pi}{x_0},\ m = 0,1,2\cdots\cdots, \text{ and } \lambda_n = \frac{(2n+1)\pi}{y_0},\ n = 0,1,2\cdots\cdots$$

a_{on} and a_{mn} are found to be

$$a_{0n} = \frac{2q_0}{\lambda_n^2 x_0 y_0 kt} \sqrt{2\pi}\,\sigma_x\, erf\left(\frac{x_0}{2\sqrt{2}\sigma_x}\right)$$
$$IMAG\left\{ \begin{array}{l} \frac{\sqrt{\pi}}{2}\exp\left(-\frac{iy_0\lambda_n + \sigma_y^2 \lambda_n^2}{2}\right) \\ \left[erf\left(\frac{\frac{1}{2}y_0 - i\sigma_y^2 \lambda_n}{\sqrt{2}\sigma_y}\right) + erf\left(\frac{\frac{1}{2}y_0 + i\sigma_y^2 \lambda_n}{\sqrt{2}\sigma_y}\right) \right] \end{array} \right\}.$$

$$a_{mn} = \frac{4q_0}{kt(\lambda_m^2 + \lambda_n^2)x_0 y_0}$$
$$\mathrm{Re}\left\{ \begin{array}{l} \frac{\sqrt{\pi}}{2}\exp\left(-\frac{ix_0\lambda_n + \sigma_x^2 \lambda_m^2}{2}\right) \\ \left[erf\left(\frac{\frac{1}{2}x_0 - i\sigma_x^2 \lambda_m}{\sqrt{2}\sigma_x}\right) + erf\left(\frac{\frac{1}{2}x_0 + i\sigma_x^2 \lambda_m}{\sqrt{2}\sigma_x}\right) \right] \end{array} \right\}$$
$$IMAG\left\{ \begin{array}{l} \frac{\sqrt{\pi}}{2}\exp\left(-\frac{iy_0\lambda_n + \sigma_y^2 \lambda_n^2}{2}\right) \\ \left[erf\left(\frac{\frac{1}{2}y_0 - i\sigma_y^2 \lambda_n}{\sqrt{2}\sigma_y}\right) + erf\left(\frac{\frac{1}{2}y_0 + i\sigma_y^2 \lambda_n}{\sqrt{2}\sigma_y}\right) \right] \end{array} \right\}.$$

The total power P_T is found to be

$$P_T = 2\pi\, q_o \sigma_x \sigma_y.$$

VII. PARAMETRIC STUDIES

From the parametric studies, the operational temperature for the graphite filter, diamond disk and beryllium window is predicted for any given size and thickness. The temperature of the graphite filter should be kept under 2000°C due to the vacuum consideration. The temperature of the diamond disk should be keep under 500°C because a few aluminum strips (melting at 675°C) are plated on the diamond disk to function as a BPM [1]. The parametric studies show the possible combinations of thickness, size, and thermal conductivity needed to achieve the design goal. The trend for all the low-Z components (in the commissioning window assembly) is that when the thickness increases and the size becomes smaller, the temperature is lower. The solutions discussed in Sections III and IV can be used for the diamond disk if it sits on a circular opening in the fixed mask in reference 4. The solutions discussed in Sections V and VI can be used for the graphite filter and beryllium window, which sit on a rectangular opening in the fixed mask.

All the analyses are for the APS Undulator A [5] which has positron beam energy (E) of 7 GeV, positron beam current (I) of 100 mA, pole number (N) of 72, period length (λ) of 3.3 cm, maximum deflection parameter (K) of 2.78. Code UA [5] was used to calculate P_T and q_o vs. thickness (see [2, 3] for more details). P_T and q_o were then fit into a function of thickness. Once the function for a given case is obtained, for example, various thicknesses of beryllium after 175-µm-thick diamond, the parametric studies can be performed for that given case. The q_o (W/mm^2) and P_T (W) for the graphite filter from 25 µm to 325 µm thick is

$$q_o = -4.1 \times t^4 \times 10^{-9} + 3.24 \times t^3 \times 10^{-6}$$
$$-9.7 \times t^2 \times 10^{-4} + 0.15 \times t + 3.45 \times t + 92$$
$$P_T = 1.44 \times t^3 \times 10^{-5} - 0.01 \times t^2 + 3.5 \times t + 92$$

Parametric studies were performed based on the following parameters and assumptions.

1. No radiative cooling; this is not a realistic assumption if the resulting maximum temperature is around 2000°C or above (because radiative cooling becomes very effective).

2. The total absorption power and peak heat flux vs. thickness are calculated using code "UA" and were fit into a function of thickness. The peak power is assumed to be at the center of the disk or plate.
3. The thermal conductivity does not changed with the temperature.

A. THE GRAPHITE FILTER AS THE FIRST COMPONENT IN THE X-RAY PATH

This section will discuss the case in which the graphite filter as the first component in the x-ray path.

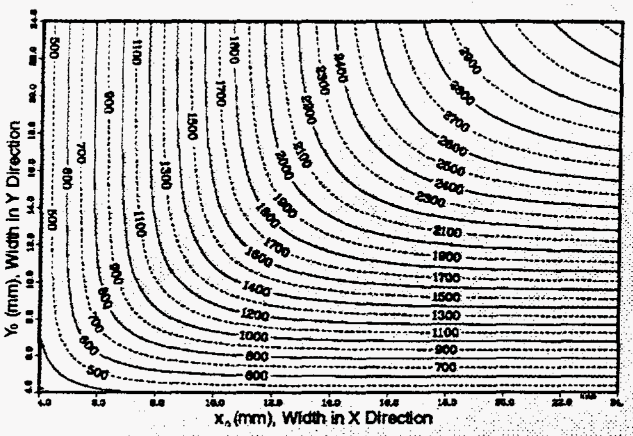

FIG. 2. Maximum temperature (°C) vs. x_0 and y_0 (mm) (300-μm-thick graphite filter, k = 180 W/(mK)

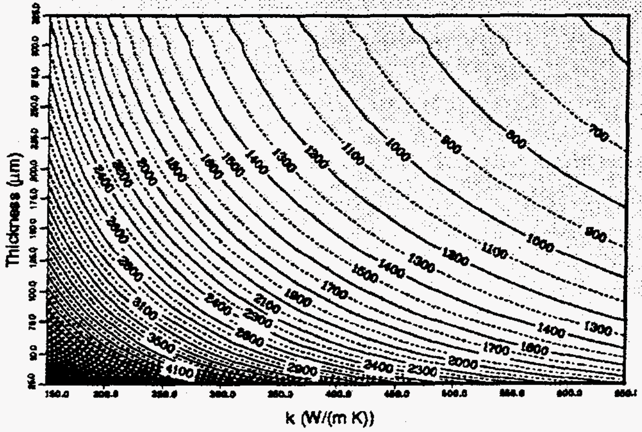

FIG. 3. The maximum temperature (°C) vs. thickness (μm) and conductivity (W/(mK)) for a graphite filter sits on a 35×12 mm^2 opening in the fixed mask.

Figure 2 shows the maximum temperature vs. x_0 (the width in the X direction) and y_0 (the width in the Y direction) for a 300-μm-thick graphite filter (k = 180 W/(m K)), which absorbs 606 W after a 7×16 mm^2 aperture, and with a q_o of 13.6 W/mm^2. If y_0 is fixed at 12 mm (in Fig. 2), the maximum temperature is about the same as that when $y_0 \geq 14$ mm; in other words, increasing the cooling area along the y direction will not help to decrease the maximum temperature. For a 300-μm-thick graphite filter sits on a 35×12 mm^2 opening in the fixed mask, in order to keep the temperature below 2000°C, the minimum thermal conductivity required is 220 W/(m K) as shown in Fig. 3. On the other hand, radiative cooling is very effective at that temperature which is not considered in the closed-form solutions. The ANSYS code is used to consider the effect of radiative cooling in references 2 and 3.

B. THE DIAMOND DISK AS THE FIRST COMPONENT IN THE X-RAY PATH

This section will discuss the case in which the diamond disk (made of CVD diamond, k = 1000 W/(m K)) as the first component in the x-ray path; in other words, there is no graphite filter in front of the diamond disk. The q_o (W/mm^2) and σ_x (mm) (assuming $\sigma_x = \sigma_y$) for the diamond disk from 25 μm to 300 μm is fit as

$$q_o = 3.62 \times t^3 \times 10^{-7} - 2.5 \times t^2 \times 10^{-4} + 0.084 \times t + 4.3$$

$$\sigma_x = \sigma_y = 4.7 \times t^3 - 7.48 \times t^2 + 3.1 \times t + 2.7$$

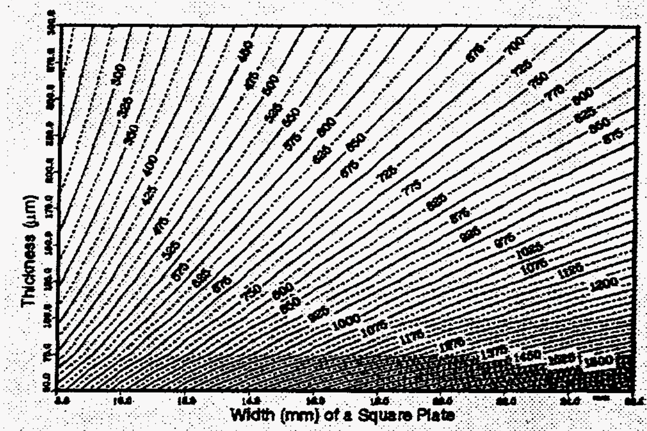

FIG. 4. The maximum temperature (°C) vs. width (mm) and thickness (μm) of a square plate ($x_0 = y_0$) (diamond, k = 1000 W/(mK), no filter)

Figure 4 shows the maximum temperature vs. thickness and width of a square plate. The maximum temperature decreases as the thickness increases, although both P_T and q_o are higher for a thicker plate. Assuming the size of the opening (in the fixed mask) is 12×12 mm^2, the minimum thickness required for the diamond disk is about 175 μm in order to keep the temperature below 500 °C. If a thinner diamond disk is requested, for example, 100 μm, the size of the opening (in the fixed mask) has to be 8.5×8.5 mm^2 or smaller. The minimum thickness required to avoid thermal buckling is discussed in references 3 and 4.

C. THE BERYLLIUM WINDOW AFTER A 175-μm-THICK DIAMOND DISK IN THE X-RAY PATH

A parametric study was performed for the case in which a beryllium window (k = 160 W/(m K)) is placed after a 175-μm-thick diamond disk without a filter. The q_o (W/mm^2) and P_T (W) for beryllium window from 100 μm to 400 μm is

$$q_o = -1.8 \times t^2 \times 10^{-6} + 6.9 \times t \times 10^{-3} + 6.4 \times 10^{-3}$$

$$P_T = 3.44 \times t^2 \times 10^{-5} + 0.12 \times t + 0.2$$

Figure 5 shows the maximum temperature vs. thickness and width of a square plate ($x_0 = y_0$). The width of the square plate is the only factor that affects the maximum temperature in the case when $x_0 = y_0 \leq 7$ mm. For a 250-µm-thick beryllium window, in order to keep the temperature of the beryllium below 150 °C, it should sit on a 7.5×7.5 mm² or smaller opening in the cooling block.

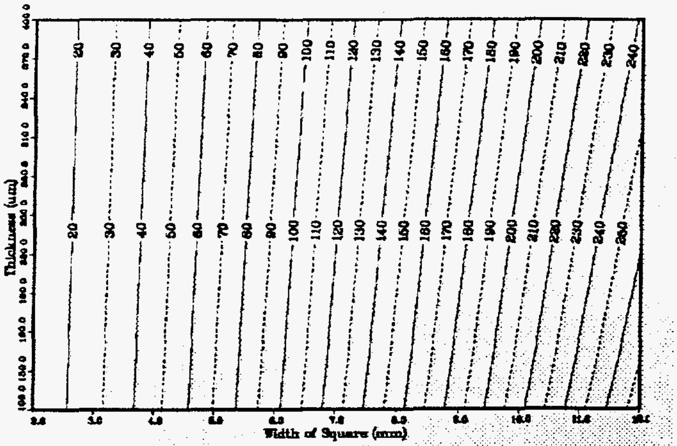

FIG. 5. The maximum temperature (°C) vs. width (mm) and thickness (µm) of a square plate ($x_0 = y_0$) (beryllium (k = 160 W/(mK) window after a 175 µm-thick diamond disk).

VIII. COMPARISON IN THE MAXIMUM TEMPERATURE BETWEEN CLOSED-FORM SOLUTION AND ANSYS

ANSYS code is used to simulate the real absorbed power shape and cooling scheme. The output from code "UA" is read by code "HIT" [6], and compared with the nodal coordinate (from ANSYS finite element model); then, the heat generation for each element in ANSYS input is generated. The power absorbed for a 300-µm-thick graphite filter in ANSYS modeling is shown in Fig. 6; the bottom left portion of Fig. 6 shows the power absorbed in the first 150 µm, the bottom right shows the power absorbed in the second 150 µm. The total absorbed power with a 16×7 mm² aperture is 606 W at 25 m from the Undulator A source.

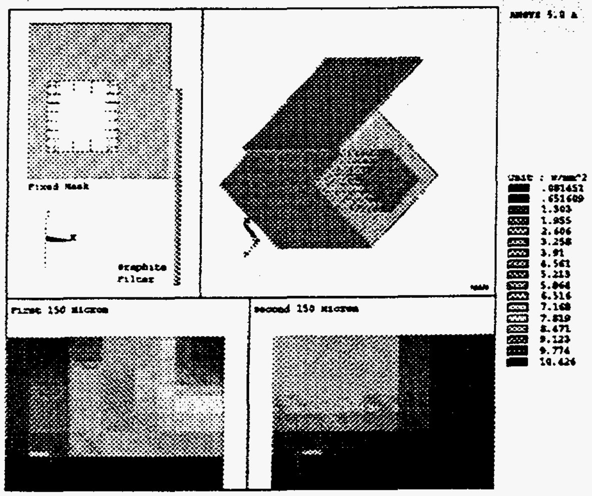

FIG. 6. The power absorbed for a 300-µm-thick graphite filter in ANSYS modeling.

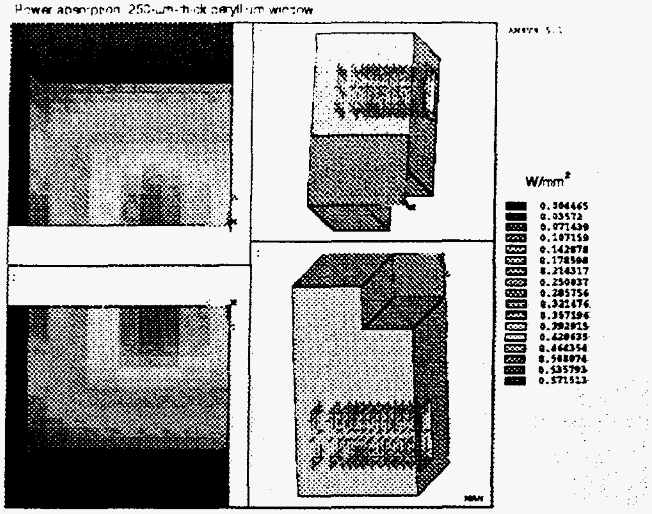

FIG. 7. The power absorbed for a 250-µm-thick beryllium window after a 175-µm-thick diamond disk in ANSYS modeling.

If the graphite filter in Fig. 6 brazed on a 35 × 12 mm² opening in the cooling block (in other words, x_o is 35 mm, and y_o is 12 mm in the closed-form solution), the maximum temperature is about 844°C if k = 500 W/(m K); the closed-form solution calculated at 825°C as shown in Fig. 3. The differences in the maximum temperature between closed-form solution and ANSYS (without considering the radiative cooling) is within 10% for both graphite filter and diamond disk. However, it is about 25% for the beryllium window due to its absorbed power shape as shown in Fig. 7, q_o is not located at the center of the plate which resulting lower temperature than that from the closed-form solution (which assumed q_o located at the center of the plate).

ACKNOWLEDGMENTS

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