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Structure Functions of Electroweak Boson and Leptons[*]

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STRUCTURE FUNCTIONS OF ELECTROWEAK BOSONS AND LEPTONS*

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Abstract

The QCD structure of the electroweak bosons is reviewed and the lepton structure function is defined and calculated. The leading order splitting functions of electron into quarks are extracted, showing an important contribution from γ-Z interference. Leading logarithmic QCD evolution equations are constructed and solved in the asymptotic region where $\log^2$ behaviour of the parton densities is observed. Possible applications with clear manifestation of 'resolved' photon and weak bosons are discussed.

We would like to present a problem on the edge of the electroweak and strong sectors of the Standard Model. From the point of view of this Institute the question is how far the electroweak objects get poluted by the QCD partons. It is known since long that the QCD structure of the photon ('resolved' photon) [1] is now clearly visible in high energy experiments [2]. Theoretical analysis of the quark-gluon component has been recently extended to weak bosons [3]. The transverse W and Z develop similar structure with some noticable differences — the calculated densities show strong spin and flavour dependence. In the experiment however one has rarely real weak bosons at one's disposal and in most experiments even the high energy photons are 'nearly on-shell' only. In physical processes, where the structure of electroweak bosons may contribute significantly, it is the lepton, initiating the process which is the source of the intermediate bosons. The standard procedure applied in such cases is to use the equivalent photon approximation [4] (extended also to the case of weak bosons [5]) and, as a next step, to convolute the obtained boson distributions with parton densities inside the bosons. For example the parton k density of the electron $F_k^{e^-}$ would read:

$$F_k^{e^-}(z, \hat{Q}^2, P^2) = \sum_B F_B^{e^-}(\hat{Q}^2) \otimes F_k^B(P^2) \tag{1}$$

where $(F \otimes G)(z) \equiv \int dx\, dy\, \delta(z - xy) F(y) G(x)$, $\hat{Q}^2$ is the maximum allowed virtuality of the boson, P^2 is the hard process scale and z - the momentum fraction of the parton k with respect to the electron (detailed definitions follow). In such an approximation several questions arise: how far off-shell can the intermediate bosons be, how large are their interference effects, what are the relations between the energy scales governing the consecutive steps, is the convolution Eq.(1) correct in general? We think that it is more precise to answer the direct question: what is the quark and gluon content of the incoming lepton or, in other words, what is the lepton structure function? The answer to the above question [6] brings several corrections to the standard procedure.

The construction of the lepton structure function can be divided into two steps. In the first we calculate the splitting of the lepton into a $q\bar{q}$ pair, in the second the quark-gluon

cascade is resummed with the use of the evolution equations [7]. To obtain the electron splitting functions let us consider inclusive scattering of a virtual gluon off an electron. In the lowest order in the electromagnetic and strong coupling constants (α and α_s) the electron couples to $q\bar{q}$ pair as shown in Figure 1. The incoming electron e carries 4-momentum l and the off-shell gluon G^* of 4-momentum p with large $P^2 \equiv -p^2$, serves here as a probe of the electron. In the final state we have a massless quark q and antiquark $\bar{q}$ of 4-momenta k and k' and lepton ℓ' (electron or neutrino) of 4-momentum l'. The exchanged boson $B = \{\gamma, Z, W\}$ carries 4-momentum q ($Q^2 \equiv -q^2$).

The current matrix element squared for an unpolarized electron reads:

$$
\begin{aligned}
&\mathcal{J}(e^- \, G^* \to \ell' \, q_\eta \, \bar{q}_{-\eta}) = \mathcal{J}^\eta_{\mu\nu}(l,p) = \\
&\frac{1}{4\pi} \int d\Gamma_{l'} d\Gamma_k d\Gamma_{k'} (2\pi)^4 \delta_4(k + k' + l' - p - l) \\
&\times \langle e^- | J^\dagger_\mu(0) | \ell' q_\eta \bar{q}_{-\eta} \rangle \langle \ell' q_\eta \bar{q}_{-\eta} | J_\nu(0) | e^- \rangle \, ,
\end{aligned}
\tag{2}
$$

where

$$
d\Gamma_k = \frac{d^4 k}{(2\pi)^4} 2\pi \delta(k^2) \, .
\tag{3}
$$

For massless quarks we can decompose the current in the helicity basis:

$$
\mathcal{J}^\eta_\sigma(l,p) = \epsilon^{\mu*}_{(\sigma)}(p) \, \mathcal{J}^\eta_{\mu\nu}(l,p) \, \epsilon^\nu_{(\sigma)}(p) \, ,
\tag{4}
$$

where $\epsilon^\mu_{(\sigma)}(p)$ are polarization vectors of a spin-1 boson with momentum $p^\mu = (p_0, 0, 0, p_z)$:

$$
\epsilon^\mu_\pm = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0) \, ,
\tag{5}
$$

$$
\epsilon^\mu_0(q) = \sqrt{\frac{1}{|p^2|}} \, (p_z, 0, 0, p_0) \, .
\tag{6}
$$

In a frame where $\vec{q}$ is antiparallel to $\vec{p}$ contributions from different helicities of exchanged bosons do not mix and the current reads:

$$
\begin{aligned}
\mathcal{J}^\eta_\sigma(p,l) =& \frac{\alpha_s \alpha^2}{2\pi} \sum_{A,B,\rho} g^{Aq}_\eta g^{Bq}_\eta \int \frac{dy}{y} P^{e^-}_{A_\rho B_\rho}(y) \\
&\times \int_{Q^2_{\min}}^{Q^2_{\max}} \frac{Q^2 dQ^2}{(Q^2 + M_A^2)(Q^2 + M_B^2)} H^\eta_{\rho\sigma}(x, Q^2) \, ,
\end{aligned}
\tag{7}
$$

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where $P^{e^-}_{A_\rho B_\rho}(y)$ describes weak boson emission from the electhe γ-Z interference. As demonstrated below their contribution is substantial.

To answer our main problem of 'an electron splitting into a quark' we take the limit $Q^2 \ll P^2$ and keep the leading terms only. Within this approximation the kinematic variables x, y, z read

$$y = \frac{pq}{pl}, \quad z = xy = \frac{-p^2}{2pl}, \tag{8}$$

aquiring the parton model interpretation of the quark momentum fraction (z) and of boson momentum fraction (y), both with respect to the parent electron. The leading term of the hadronic part does not depend on the quark helicity η:

$$H_{\pm\mp}(x, Q^2) = x^2 \log \frac{P^2}{Q^2}, \tag{9}$$

$$H_{\pm\pm}(x, Q^2) = (1 - x)^2 \log \frac{P^2}{Q^2}, \tag{10}$$

with other components finite for $P^2/Q^2 \to 0$.

We also recognize $P^{e^-}_{A_\rho B_\rho}(y)$ as a generalization of the splitting functions of an electron into bosons [3]:

$$P^{e^-}_{A_\rho B_\rho}(y) = \frac{1}{2}(g^{Ae^-}_- g^{Be^-}_- yY_{-\rho} + g^{Ae^-}_+ g^{Be^-}_+ yY_\rho) \tag{11}$$

with

$$Y_+(y) = \frac{1}{y^2}, \tag{12}$$

$$Y_-(y) = \frac{(1 - y)^2}{y^2}, \tag{13}$$

$$Y_0(y) = \frac{2(1 - y)}{y^2}, \tag{14}$$

where $g^{Ae^-}_\pm$ is the electron to boson A_ρ coupling in the units of proton charge e.

From kinematics $y \in [z, 1 - \mathcal{O}(m^2_e/P^2)]$ and the integration limits for Q^2 read

$$Q^2_{\min} = m^2_e \frac{y^2}{1 - y}, \quad Q^2_{\max} = P^2 \frac{z + y - zy}{z}, \tag{15}$$

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with m_e being the electron mass. Although smaller than the already neglected quark masses, it is the electron mass which must be kept finite in order to regularize collinear divergencies. The upper limit of integration requires particular attention. In general it is a function of P^2, however integration up to the maximum kinematically allowed value $Q^2_{\max}$ would violate the condition $Q^2/P^2 \ll 1$. For our approximation to work we integrate over Q^2 up to $\hat{Q}^2_{\max} = \epsilon P^2$ where $\epsilon \ll 1$ and generally depends on y and z. A similar condition is in fact used in phenomenological applications of the equivalent photon approximation [8]. Note that the maximum virtuality $\hat{Q}^2_{\max}$ can also be kept independent of P^2 when special kinematical cuts are arranged in experiment. We concentrate here on the fully inclusive case with the maximum virtuality being P^2-dependent. Integrating Eq.(7) over Q^2 within such limits and keeping only leading-logarithmic terms leads to

$$\mathcal{J}^\eta_\sigma(p,l) = \frac{\alpha_s \alpha}{6} \sum_{A,B,\rho} g^{Aq}_\eta g^{Bq}_\eta F^{e^-}_{A_\rho B_\rho}(P^2)$$
$$\otimes [P^\rho_{q\eta} \delta_{\eta,-\sigma} + P^\rho_{\bar{q}-\eta} \delta_{\eta,\sigma}] \log P^2 . \tag{16}$$

In the above equation $P^\rho_{q\eta}(x)$ and $P^\rho_{\bar{q}\eta}(x)$ are boson-quark (-antiquark) splitting functions [1,3]

$$P^\pm_{q\pm}(x) = P^\pm_{\bar{q}\pm}(x) = 3x^2, \; P^\mp_{q\pm}(x) = P^\mp_{\bar{q}\pm}(x) = 3(1-x)^2 \tag{17}$$

and $F^{e^-}_{A_\rho B_\rho}(y, P^2)$ is the density matrix of polarized bosons inside electron. Its transverse components read

$$F^{e^-}_{\gamma_\pm \gamma_\pm}(y) = \frac{\alpha}{2\pi} \frac{(1-y)^2 + 1}{2y} \log \mu_0 , \tag{18a}$$

$$F^{e^-}_{Z_+ Z_+}(y) = \frac{\alpha}{2\pi} \tan^2 \theta_W \frac{\rho^2_W (1-y)^2 + 1}{2y} \log \mu_Z , \tag{18b}$$

$$F^{e^-}_{Z_- Z_-}(y) = \frac{\alpha}{2\pi} \tan^2 \theta_W \frac{\rho^2_W + (1-y)^2}{2y} \log \mu_Z , \tag{18c}$$

$$F^{e^-}_{\gamma_+ Z_+}(y) = \frac{\alpha}{2\pi} \tan \theta_W \frac{\rho_W (1-y)^2 - 1}{2y} \log \mu_Z , \tag{18d}$$

$$F^{e^-}_{\gamma_- Z_-}(y) = \frac{\alpha}{2\pi} \tan \theta_W \frac{\rho_W - (1-y)^2}{2y} \log \mu_Z , \tag{18e}$$

$$F^{e^-}_{W_+ W_+}(y) = \frac{\alpha}{2\pi} \frac{1}{4 \sin^2 \theta_W} \frac{(1-y)^2}{y} \log \mu_W , \tag{18f}$$

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$$F_{W_-W_-}^{e^-}(y) = \frac{\alpha}{2\pi} \frac{1}{4\sin^2\theta_W} \frac{1}{y} \log\mu_W, \tag{18g}$$

where

$$\rho_W = \frac{1}{2\sin^2\theta_W} - 1, \tag{19}$$

θ_W is the Weinberg angle and

$$\log\mu_0 = \log\frac{\epsilon P^2}{m_e^2}, \; \log\mu_B = \log\frac{\epsilon P^2 + M_B^2}{M_B^2}. \tag{20}$$

All other density matrix elements (containing at least one longitudinal boson) do not develop logarithmic behaviour.

At this point we are able to define the splitting functions of an electron into a quark at the momentum scale P^2 as

$$\mathcal{P}_{q_\eta}^{e^-}(P^2) = \sum_{AB} g_\eta^{Aq} g_\eta^{Bq} \sum_\rho F_{A_\rho B_\rho}^{e^-}(P^2) \otimes P_{q_\eta}^\rho. \tag{21}$$

The explicit expressions for quarks read

$$\begin{aligned}
\mathcal{P}_{q_+}^{e^-}(z, P^2) =& \frac{3\alpha}{4\pi}\{e_q^2\left[\Phi_+(z) + \Phi_-(z)\right]\log\mu_0 \\
&+ e_q^2\tan^4\theta_W\left[\Phi_+(z) + \rho_W^2\Phi_-(z)\right]\log\mu_Z \\
&- 2e_q^2\tan^2\theta_W\left[-\Phi_+(z) + \rho_W\Phi_-(z)\right]\log\mu_Z\},
\end{aligned} \tag{22a}$$

$$\begin{aligned}
\mathcal{P}_{q_-}^{e^-}(z, P^2) =& \frac{3\alpha}{4\pi}\{e_q^2\left[\Phi_+(z) + \Phi_-(z)\right]\log\mu_0 \\
&+ z_q^2\tan^4\theta_W\left[\Phi_-(z) + \rho_W^2\Phi_+(z)\right]\log\mu_Z \\
&+ 2e_q z_q\tan^2\theta_W\left[-\Phi_-(z) + \rho_W\Phi_+(z)\right]\log\mu_Z \\
&+ (1 + \rho_W)^2\Phi_+(z)\delta_{qd}\log\mu_W\},
\end{aligned} \tag{22b}$$

where

$$\Phi_+(z) = \frac{1-z}{3z}(2 + 11z + 2z^2) + 2(1+z)\log z, \tag{23}$$

$$\Phi_-(z) = \frac{2(1-z)^3}{3z}, \tag{24}$$

and

$$z_q = \frac{T_3^q}{\sin^2 \theta_W} - e_q \,, \tag{25}$$

with e_q and T_3^q being the quark charge and 3-rd weak isospin component, respectively. The splitting functions for antiquark of opposite helicity can be obtained from Eq.(22) by interchanging Φ_+ with Φ_-.

The splitting functions introduced above show two new features. The first one, already mentioned before, is the contribution from the interference of electroweak bosons (γ and Z only). The second is their P^2 dependence, which arises from the upper integration limit $\hat{Q}^2_{\max}$.

Having completed the first step of our procedure — the calculation of the splitting functions, we can proceed to the resumation of the QCD cascade using the evolution equations. We consider the evolution equations in the first order in electroweak couplings and leading logarithmic in QCD. Introducing $t = \log(P^2/\Lambda^2_{QCD})$ we can write (remembering that there is no direct coupling of the electroweak sector to gluons):

$$\frac{dF^{e^-}_{q_\eta}(t)}{dt} = \frac{\alpha}{2\pi} \mathcal{P}^{e^-}_{q_\eta}(t) + \frac{\alpha_s(t)}{2\pi} \sum_{k,\rho} P^{k_\rho}_{q_\eta} \otimes F^{e^-}_{k_\rho}(t) \tag{26a}$$

$$\frac{dF^{e^-}_{G_\lambda}(t)}{dt} = \frac{\alpha_s(t)}{2\pi} \sum_{k,\rho} P^{k_\rho}_{G_\lambda} \otimes F^{e^-}_{k_\rho}(t). \tag{26b}$$

We stress that the convolution of the equivalent boson distributions and boson-quark splitting functions, Eq.(21), occurs at the level of splitting functions. It is not equivalent to the usually performed convolution of the distribution functions because of the P^2 dependence of the boson distribution functions Eq.(18). Only in the case when the upper limit of integration $\hat{Q}^2_{\max}$ is kept fixed (P^2-independent), e.g. by special experimental cuts, are the convolutions equivalent at both levels.

The equations Eq.(26) can be solved in the asymptotic t region where we approximate the strong coupling constant by

$$\alpha_s(t) \simeq \frac{2\pi}{bt} \,, \tag{27}$$

with $b = 11/2 - n_f/3$ for n_f flavours. The asymptotic (large t) solution to Eqs.(26) for the parton k of polarization ρ can be now parametrized as

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$$F_{k_\rho}^{e^-}(z,t) \simeq \frac{1}{2} \left(\frac{\alpha}{2\pi}\right)^2 f_{k_\rho}^{as}(z) t^2 \qquad (28)$$

resulting in purely integral equations

$$f_{i_\rho}^{as} = \hat{\mathcal{P}}_{i_\rho}^{e^-} + \frac{1}{2b} \sum_{k,\lambda} P_{i_\rho}^{k_\lambda} \otimes f_{k_\lambda}^{as}, \qquad (29)$$

where $\hat{\mathcal{P}}_{i_\rho}^{e^-}(z)$ are given by Eqs.(22) with all $\log \mu_A \equiv 1$.

Numerical solutions to the above equations with the method described in Ref. [3] are presented in Figure 2 for the unpolarized quark and gluon distributions. One notices significant contribution from the W intermediate state in the d-type quark density. The most surprising however is the γ-Z interference contribution which cannot be neglected, as it is comparable to the Z term. It violates the standard probabilistic approach where only diagonal terms are taken into account. This also stresses the necessity of introducing the concept of electron structure function in which all contributions from intermediate bosons are properly summed up. Due to the nature of weak couplings they turn out to be nonzero, even in the case of gluon distributions. Again the γ-Z interference term is important and the W contribution dominates in the asymptotic region.

One should keep in mind that at finite t the logarithms multiplying the photon contribution differ from the remaining ones (Eq.(20)). Being scaled by m_e, they lead to the photon domination at presently available P^2. The importance of the interference term remains constant relative to the Z contribution, as they are both governed by the same logarithm. But even at presently available momenta, where the 'resolved' photon dominates, one can see how the correct treatment of the scales changes the evolution. In Fig. 3 we present the asymptotic solutions of the evolution equations following from our procedure (ESF) compared to those following from naive application of the convolution Eq.(1) (FF). It is possible that the difference can be traced in the analysis of presently available data.

To summarize we have presented a construction of the electron structure functions which is the correct approach to lepton induced processes when the QCD partons are collinear with the lepton. The calculation has been done in the leading logarihtmic approximation to QCD

and leading order in electroweak interactions. One can definitely improve this approximation, in particular, treating the electroweak sector more precisely. The γ-Z interference, contrary to naive expectations, turns out to be important. Direct calculation of the splitting functions of an electron into quarks allows for precise control of the momentum scales entering the evolution. It also shows that the convolution of leptons, electroweak bosons and quarks should be made at the level of splitting functions rather than distribution functions. Unless forced otherwise by the experimental cuts, the electron splitting functions depend on the external scale P^2 and influence significantly the parton evolution.

Phenomenological applications of the above analysis require very high momentum scales in order to see the weak boson and interference contributions. Possible processes where these effects could show up include heavy flavour, large $p_\perp$ jet and Higgs boson production in lepton induced processes. At presently available momenta, where the photons dominate, the use of the electron structure function allows to treat correctly the parton evolution. It seems also more plausible to use one phenomenological function of the 'resolved' electron instead of parametrizing the equivalent bosons' and 'resolved' bosons' spectra separately.

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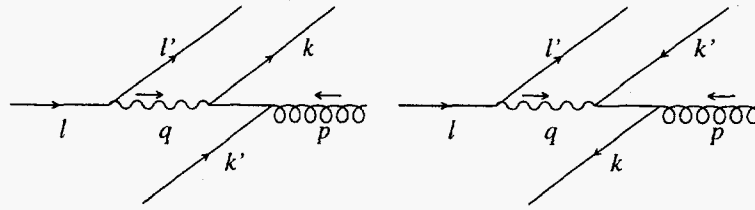
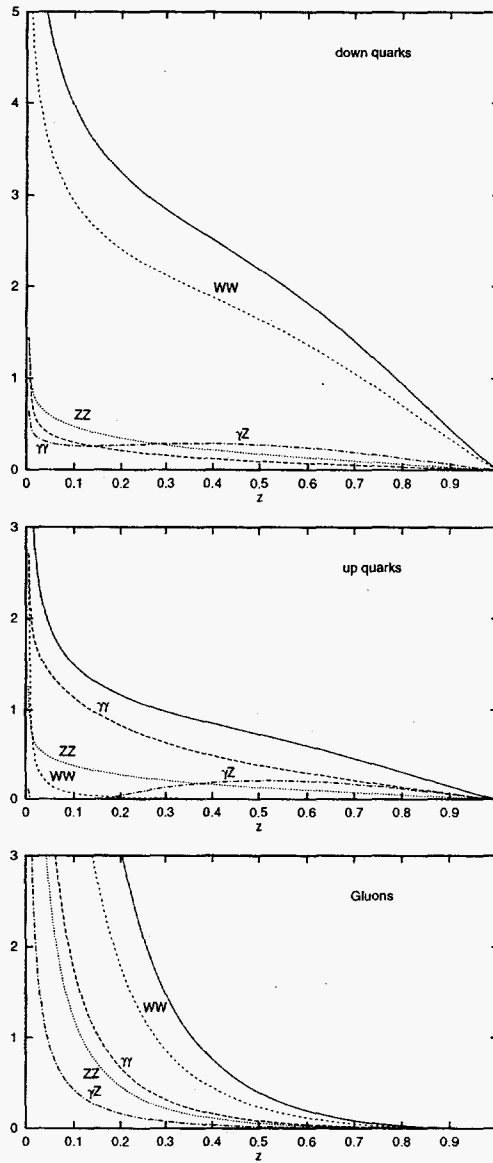FIG. 1. Lowest order graphs contributing to the process: $e + G^* \rightarrow \ell' + q + \bar{q}$.

FIG. 2. Unpolarized quark and gluon distributions $zf^{as}(z)$ — solid line. The other lines show contributions from different electroweak bosons.
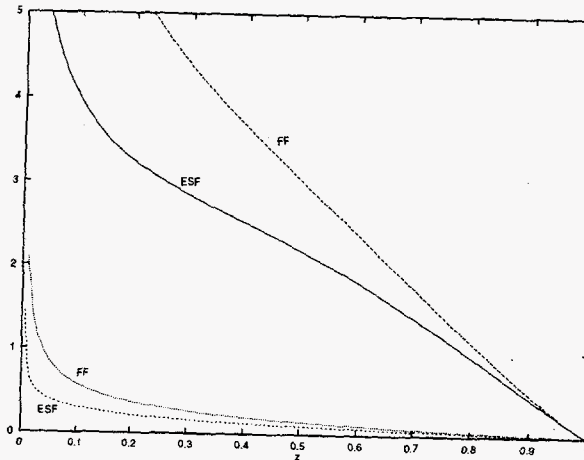
FIG. 3. Comparison of unpolarized d-quark distributions $zf^{as}(z)$ calculated by ESF and FF methods. The upper two lines result from contributions from all electroweak bosons while the lower two from $\gamma\gamma$ only.

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