A PEDAGOGICAL APPROACH TO THE BACH TWO PART INVENTIONS

THESIS

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By

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PREFACE

Since the nineteenth century the Two Part Inventions have become standard repertoire for piano students. However, piano teachers have often failed to give serious consideration to the suitable selection of Inventions for study. Piano students have commonly formed a dislike for Bach's piano music because of an ungratifying initial experience.

There is little material written in English dealing with the Two Part Inventions. Those studies of the Inventions which do exist consist mainly of brief articles in periodicals and limited discussions in piano literature books. Therefore, there seemed to be a need for a systematic collection of pedagogical ideas concerning the Two Part Inventions. In addition, it was felt that an analysis of the problems in each piece and a subsequent graded list of the Inventions would be a step toward a more intelligent and more knowledgeable approach in the teaching of these compositions.

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CHAPTER I

STATEMENT OF THE PROBLEM

The Purpose

The purpose of this study was to analyze the Bach Two Part Inventions and to prepare a graded list for pedagogical purposes.

Sub-Problems

Analysis of the problem statement led to subordinate questions, which may be stated as follows:

1. What performance problems are identifiable through an analysis of the linear characteristics in the Two Part Inventions?

2. What performance problems are identifiable through an analysis of the textural characteristics of these pieces?

3. What performance problems are identifiable through an analysis of the rhythmic characteristics in the Inventions?

4. What performance problems are identifiable through an analysis of the formal design of these pieces?

5. What system can be devised to grade the Inventions in order of difficulty?

Definition of Terms

1. The term "performance problems" refers to difficulties encountered in the playing of a composition by an intermediate student on the piano. These problems may be divided into two basic categories: technical difficulties and interpretative difficulties.

a. The term "technical difficulties" refers to the demands placed upon the mechanical skill of the performer. They include coordination, voice projection, balance, touch, tempo, and execution of embellishments.

1) The word "coordination" refers to the harmonious functioning of muscles in producing complex, and sometimes, dissimilar, movements (15, p. 325).

2) The term "voice projection" refers to the predominance in intensity of one part.

3) The term "voice balance" refers to the equality of intensity between the parts.

4) The word "touch" refers to "the physical approach to the key as the conditioning factor to obtain a variety of tonal qualities, ranging from the soft and lyrical to the harsh and percussive" (1, p. 528). This word also includes legato and staccato playing.

5) The phrase "execution of embellishments" refers to the realization in performance of ornaments.

b. The term "interpretative difficulties" refers to the problems in the rendering of a musical composition according to one's conception of the composer's idea (9, p. 415). Interpretative difficulties include phrasing, articulation, slurring, dynamic variation, mood, tempo, and style.

1) The word "phrasing" refers to the "clear rendering in musical performance of the . . . natural division of the melodic line. . . . This is achieved by the interpolation of a slight rest . . . with the last note which, therefore, will have to be shortened (depending upon the speed and loudness of the passage)" (1, pp. 572-573).

2) The term "articulation" refers to "a feature of musical performance which includes all those more or less minute breaks in the continuity of sound that contribute, together with accent, to give it shape and render in intelligible" (5, p. 235).

3) The term "slurring" refers to "an effect . . . whereby the second of two notes is curtailed and weakened . . . " (5, p. 840).

4) The term "dynamic variation" refers to changes in the degrees of sound-volume in the music.

5) The word "mood" refers to the general tone, or dominant emotional character, of the music.

6) The word "tempo" refers to the rate of speed of the music.

7) The word "style" refers to the manner of performing that is distinctive or characteristic of a period or individual artist.

2. The term "graded list" refers to the ranking of the Two Part Inventions beginning with the easiest and continuing to the most difficult.

3. The music "Bach Two Part Inventions" refers to "fifteen small keyboard pieces--each in two parts and each developing a single idea--composed by Johann Sebastian Bach" (3, p. 526).

4. The term "linear characteristics" refers to the distinct qualities in the horizontal arrangement of pitch and duration. Analysis of the linear characteristics includes a study of the motive and the episodes

a. The word "motive" refers to the melodic statement--generally one to two measures--which is the basic element in the structure of an Invention.

b. The word "episode" refers to the intermediate development sections between the exposition and counter-exposition(s).

c. For purposes of this paper the terms "first voice" and "upper voice" are synonymous. Likewise, the terms "second voice" and "lower voice" are synonymous.

5. The term "textural characteristics" refers to the distinctive qualities of the resultant sound of the combination of the two lines or "parts." Textural characteristics include implied harmonies, harmonic progressions, cadences, tonality, intervalic relationships, consonance and dissonance, and rhythmic relationships.

6. The term "rhythmic characteristics" refers to the distinctive features of the accentual structure of the music. Rhythm includes meter--duple, triple, simple, or compound--and rhythmic patterns--prominence, kind, frequency.

7. The term "formal design" refers to the structural scheme of a piece. Analysis of the structure will include examination of the exposition, imitation, and sections.

a. For purposes of this paper the term "exposition" refers to the initial section of the Invention containing the statement and the imitation of the motive.

b. The word "imitation" refers to the restatement in close succession of the motive in different voice parts. The motive in imitation may appear in an altered form.

c. The word "sections" refers to general divisions within the Invention which are determined by changes in tonality and use of motivic and countermotivic material.

Delimitations

1. The study is concerned with the Bach Two Part Inventions only because (a) Bach's total contribution in Inventions was considered to be too large to be included in one investigation--either the Two Part or Three Part Inventions constituted sufficient area for this thesis; and (b) the Two Part Inventions were thought to be the more widely used by pedagogues.

2. The Ludwig Landshoff edition of the Two Part Inventions was used for analysis and reference in this study. This edition was chosen as "an excellent example" of an Urtext edition. The Urtext edition is, in the final analysis, the only trustworthy edition (14, pp. 12-14).

3. Ornamentation as presented in the Landshoff edition was accepted for this study. However, for purposes of this paper, only those ornaments appearing in ordinary type were considered. The embellishments are those that appeared in the original script of 1723 which were clearly authored by Bach. Any additional ornaments appearing in the original autograph and other authentic sources appear in smaller type.

4. The study did not investigate the problems related to the performance of the Inventions on the harpsichord, clavichord, or any keyboard instrument other than piano.

Basis Hypothesis

The basic hypothesis of this study was that the Two Part Inventions do vary in the degree of difficulty and that the difficulties could be determined.

Basis Assumptions

The two following assumptions seemed basic to this study:

1. It was assumed that through the process of analysis and from the evaluation of six pedagogues the difficulties of each Invention could be determined.

2. It was assumed that the Ludwig Landshoff edition was widely recognized among pedagogues as being the most acceptable.

Background for the Study

J. S. Bach: Two Part Inventions

"To learn . . . to acquire good ideas (Inventions) but also to work them out themselves . . . and at the same time, to gain a strong predilection for composition," wrote J. S. Bach in a letter in 1723 in reference to the purpose of students practicing the Inventions (11, p. 470).

The Two and Three Part Inventions, thirty keyboard pieces by Johann Sebastian Bach (1685-1750), were written about 1720 when he was conductor of the orchestra at Cöthen. They were intended as exercises for his son, Wilhelm Friedemann Bach,

then nine years and two months old, and were a part of his Clavier-Buchlein. According to Geiringer, the first two and the last eight of the Praeambula, as the Two Part Inventions were first termed by Bach, were written by the father, numbers three through seven by Friedemann (4, p. 271).

Forkel gives more detailed information concerning the circumstances surrounding the composition of the Inventions in the following passage:

> I will first speak of his instructions in playing. The first thing he did was to teach his scholars his particular mode of touching the instrument. For this purpose, he made them practice, for months together, nothing but isolated exercises for all the fingers of both hands, with constant regard to this clear and clean touch.
>
> For some months, none could get excused from these exercises; and, according to his firm opinion, they ought to be continued, at least, for from six to twelve months. But if he found that anyone, after some months of practice, began to lose patience, he was so obliging as to write little connected pieces, in which those exercises were combined together. Of this kind are the six little Preludes for Beginners, and still more the fifteen Two-Part Inventions (3, pp. 16-17).

The preludes by Johann Kuhnau and J. Kaspar Ferdinand Fischer served as models for Bach's Inventions. What Bach created, however, was new (4, p. 267). Geiringer continues his discussion in the following statements:

> No other composer had ever considered imbuing clavier compositions of such small dimensions with a content of similar significance. They are studies in independent part writing using all the devices of fugue and canon, double and triple counterpoint, but without strict adherence to any of them. Bach offers fantasias

> in the realm of polyplony, freely blending all known techniques, and creating forms which are held together by logic, and the iron consistency, of his musical thought (4, p. 267).

Authorities disagree on the reason and the source of the designation of these compositions as "Inventions." Bach may have acquired the title from his Italian contemporary, A. F. Bonporti, who some years earlier (1714-1715) published Peace: Inventions: or Ten Partitas for Violin and Figured Bass. Four of these, being found in Bach's handwriting, were actually printed in the German Bach Society's Edition as his compositions (11, p. 470). Perhaps Bach took the name from Vitali, who used the term "inventioni curiose" as a title for pieces involving special tricks (1, p. 364). Another possibility concerning the origin of the term "invention" lies with the seventeenth century publications of two-part "ricercares." It is suggested that "invention" was a translation of "ricercare." Or perhaps the vivid dialogue between the two parts of the pieces persuaded Bach to substitute the name "Invention" for the rather vague "Praembulum" he had originally used (1, p. 364).

The educational purposes of the Inventions are indicated in the inscription carried on a later autograph of the pieces;

> Honest Guide
> by which lovers of the clavier, and particularly those desirous of learning are shown a plain way not only

> (1) to play neatly in two parts, but also, as they progress, (2) to treat three obligato parts correctly and well, and, at the same time, to acquire good ideas and properly to elaborate them, and most of all to learn a singing style of playing, and simultaneously to obtain a strong foretaste of composition (4, p. 274).

The pieces are intended as studies for performing and for composing, both of which Bach considered necessary. It is not clear whether Bach intended these pieces to be played on the clavichord or the harpsichord. Bodky discusses this enigma in the following passage:

> Some mystery darkens the title of the Inventions, dedicated by Bach to the "lovers of the clavier". . . . Had Bach meant here with the word "Clavir" the same as he meant in the title of the Well-tempered [sic] Keyboard, namely, to select for each piece one of the three keyboard instruments, he would have said more correctly "lovers of the keyboards". . . . Since all the Inventions show stylistically that they seem to be meant only for the clavichord, one would have to assume that in this instance Bach called the clavichord by the name "Clavir." As far as our knowledge goes, this term seems hardly to have been in general use before 1750 to designate the clavichord exclusively. In addition, the words that these pieces should be used "to achieve a singing touch" . . ., which Bach wrote in the preface, make the reservation of them for the clavichord almost one hundred per cent evident. Yet Wanda Landowska ascribes even these Inventions to the Harpsichord alone, stating that "since the clavichord is a singing instrument by nature long studies were not necessary to achieve expressive effects on it; therefore Bach must have had the harpsichord in mind, on which it is very difficult to play 'cantabile'" (2, p. 27).

Bodky concludes that, because of the reference to the "singing style of playing," the Inventions were intended to be performed on the clavichord (4, p. 274).

The Two Part Inventions are written in the following series of keys: C major and minor, D major and minor, E flat major, E major and minor, F major and minor, G major and minor, A major and minor, B flat major and b minor. Other keys were omitted for a specific reason. Because equal temperament had not yet been introduced, not all keys could be equally well tuned. Those least often used--those with many sharps and flats--were sacrificed in favor of the others. It was for this reason that Bach avoided all keys having more than four sharps or flats, and also F sharp minor, C sharp minor and A flat (6, p. 339).

Hutcheson concludes his discussion of the Inventions with the following statements:

> Pianists rightly regard the Inventions as the best possible preparation for the larger works of Bach, especially the preludes and fugues, but it would be wrong to underestimate their intrinsic value. They belong to that fine wholesome mass of musical literature which outlasts passing tastes and never palls (6, p. 30).

The first part of Bach's artistic career appeared bright and promising. In Weimar his fame as an organ virtuoso and expert spread rapidly. In Cöthen, where he composed the Inventions and other clavier music, he held a greatly respected position and enjoyed the friendship of Prince Leopold (4, p. 349).

However, when Bach, at the age of thirty-eight, moved to Leipzig, his music was greeted with little enthusiasm, and in the following years his compositions received less and less acclaim. Bach provided the culmination to the art of a vanishing era, and he refused to offer the uncomplicated, "natural" kind of music expected in the Rococo era of rationalist thinking (4, pp. 349-350).

It is due to a small group of admirers that Bach's compositions were not altogether forgotten after his death. A friend of Emanuel Bach introduced several of Bach's works to the Viennese classical composers. Joseph Haydn acquired the Well-Tempered Clavier and other works; Mozart is said to have studied at great length Bach's clavier works and fugues; Beethoven studied as a child the Well-Tempered Clavier. However, little direct reference is made to the use of the Two Part Inventions by teachers or students until Czerny.

Related Research

Three authors have made studies relating to the performance problems in the Inventions: (a) Platt explains the occurence of some of the startling dissonances in Bach's counterpoint as a result of the use of melodic figures; (b) Miller determines the degree of melodic independence in each of the Two Part Inventions through an analysis of the

motion of the counterpoint; and (c) Satterfield attempts to relate the emotional content in the Inventions to a measurable factor.

In the Platt study, "Melodic Patterns In Bach's Counterpoint," the following observation was made:

> In Bach's day certain groups of notes embellishing a single note or a chord had been used so often that they had come to be accepted as "patterns" or "figures," and Bach was able to use these patterns not as groups of separate notes, but as entities standing in place of the note or chord they represent. These "entities" from their very nature carry with them their own harmonic implications, and incidental clashes between notes comprising them and other parts may be disregarded provided that these implications are clear (10, pp. 48-49).

Although melodic figures may be found in the music of Purcell, Schutz, and Palestrina, it is in the music of Bach that "figuration plays such a consistently important part," and it is Bach who "has developed most highly this particular use of the accepted melodic figure as an entity with a significance of its own (10, p. 49).

Three instances of the use of melodic figures in the Two Part Inventions are given. The first is in Invention No. VII, in E minor.

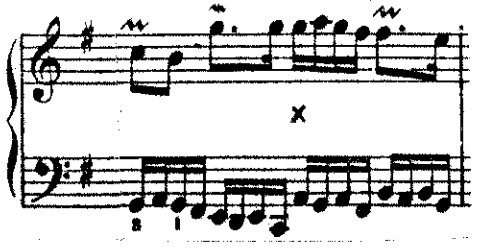

Fig. 1--Invention No. VII, melodic pattern, measure 21.

In this measure the a in the lower voice is ornamented by one figure while the g in the upper voice is ornamented by another. The result is three dissonances (and a "hidden" octave) (10, p. 50).

The second example given is from Invention No. I in C major.

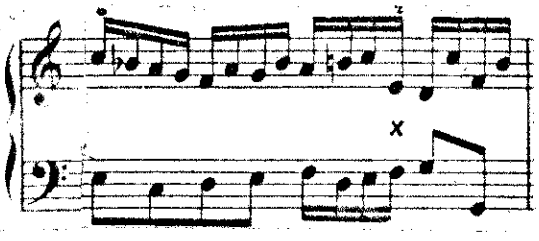

Fig. 2--Invention No. I, melodic pattern, measure 21

In this example the four sixteenth notes are a diminution of the preceding pattern. The last of these four sixteenth notes clashes with a harmony-note in the upper voice (10, p. 50).

The third example occurs in Invention No. XI, in G minor.

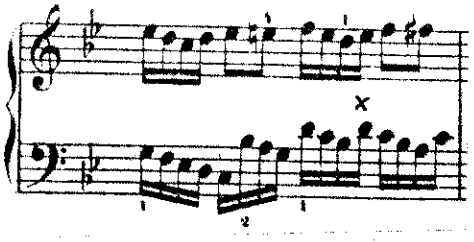

Fig. 3--Invention No. XI, melodic pattern, measure 4

This example contains a "clash of two easily grasped figures . . . . It is given added point by the two clashing parts consisting of the subject [motive] and an inversion of its countersubject" (10, p. 50).

In Miller's study, "Progression in Two-Part Counterpoint: A Method of Analysis," an attempt was made to isolate the factors which contribute to "what is commonly called melodic independence" (8, p. 289). "Contrapuntal progression," he wrote, consists of "movement in either or both of two concurrent melodic lines" (8, p. 289).

Four kinds of contrapuntal progression are identified. "Parallel motion" occurs "when both voices move together in either direction maintaining the same interval between them . . ." "Similar motion" happens "when both voices move in the same direction but progress in different intervals. . . ." "Oblique motion" results "when one voice moves in either direction while the other voice remains on the same tone, either sustaining or repeating it. . . ." "Contrary motion" occurs "when both voices move simultaneously in opposite directions . . . either converging or diverging" (8, p. 289).

According to Miller these four types of motion differ in regard to degree of melodic independence. There is no melodic independence in parallel motion, a somewhat greater amount in similar motion, and an even higher degree in oblique motion.

The greatest melodic independence, however, occurs in contrary motion. He then assigned numerical values to the four types according to the degree of melodic independence involved: parallel motion--zero, similar motion--one, oblique motion--two, and contrary motion--four. In the next step Miller classified all the progressions in the piece according to type and determined the percentage of the total number of progressions for each.

The degree of melodic independence was then determined "by multiplying the percentage of each type by the value of the type, totalling the result, and dividing by a 100" (8, p. 290). The resulting figure, the index of melodic independence, will be between zero and three. A number of one or less indicates low melodic independence. A number of two or more indicates unusually high melodic independence (8, p. 290). A table showing the frequency of each type of motion and the melodic index for each invention appears in Appendix A of this report.

The study revealed several facts. First, oblique motion constitutes more than half of all progressions in the Two Part Inventions. Most of the oblique motion occurs through the sustaining of a tone rather than tone repetition. Second, contrary motion is almost twice as frequent as parallel motion,

(Exceptions to this generalization are Inventions No. II, VIII, XI, and XIV). Third, similar motion forms a very small proportion of the total progressions. Fourth, there is a "considerable diversity" in the indices of melodic independence which range from a high of 2.1 in Invention No. XIII to a low of 1.6 in No. XIV. The average index is 1.9 (8, pp. 291-292).

Satterfield endeavored in his study to determine whether a measurable factor was related to emotional content in the Two Part Inventions. His first area of investigation was concerned with the relation of dissonance to emotional content. To begin his study he ranked the Inventions in order of poignancy from the most poignant to the least poignant. (The word "poignant" is defined as "piercingly effective or keenly touching." The degree of poignancy of expression was not intended to reflect the value of a composition) (11, p. 276). His order of Inventions is as follows: IX, XI, V, II, VI, VII, XV, XIV, XII, I, XIII, III, IV, VIII, X (11, p. 276).

Two of Satterfield's colleages were also shown the definition of poignancy and ask to rank the Inventions. Each person worked alone. The first subject produced the following arrangement from most poignant to least poignant: VI, IX, VII XIV, V, IV, II, XV, XI, XIII, I, XII, III, VIII, X. The ranking of the second subject was as follows: VI, IX, II, XI,

XIV, VII, V, IV, XV, XII, III, XIII, X, VIII, I. There was a good correlation among the orders set up by the three (11, p. 277).

Satterfield's next step was to quantify the accented dissonance content of the Inventions. He states; "The relative metric accent was treated as the unit, and the percentage of accented dissonant intervals was determined by the ratio of the number of accented dissonant intervals to the number of relative metric accents in the Invention under study" (11, p. 277). Bars in which only one voice sounded were excluded in the count of relative metric accents.

The intervals called dissonant in this study were all of those intervals except perfect unisons, fifths, and octaves, major and minor thirds and sixths. These intervals considered consonant.

The following list of the Inventions presents them according to percentages of relatively accented intervals, from greater to smaller: VI, XV, IV, I, III, II, V, XIII, X, VII, XI, VIII, XII, XIV, IX. (A table showing the number of intervals having relative metric accents, the number of relatively accented dissonant intervals, and the percentage of relatively accented dissonance in each of the Inventions appears in Appendix B). The coefficient of correlation between this

ranking and the subjective ranking based on poignancy was very low. Thus, it was concluded that dissonance as measured in this study had not been shown to be a factor in emotional content as identified in this investigation (11, p. 280).

The author made another investigation of dissonance as related to emotional content, this time altering his basis for determining the consonance or dissonance of an interval. A subsequent ranking of the Inventions again failed to produce a strong correlation between the degree of dissonance in an Invention and the degree of poignancy of that piece (11, p. 280).

Satterfield again attempted to determine whether a measurable factor was related to emotional content of the Two Part Inventions. In this study he investigated melodic and durational accents. The term "melodic accent" was used "to describe the accent attributed to the highest pitch in a series;" and the term "durational accent" referred to the longest pitch of a series (12, p. 173).

The first area of analysis included the relationship of melodic accents to metric accents. The term "metric accents" was defined as "those stresses normally 'heard into' a series of sounds because of psychological grouping of the numbers of the series into duple or triple combinations" (12, p. 173).

Satterfield describes his investigational procedure as follows:

> In making a first tabulation the following method has been used: for each voice of the Invention statistics have been made on the relative congruence of melodic and metric accents within each bar. Where these accents coincided, the bar was classified as smooth; where they did not coincide, the bar was categorized as rough. Percentages of roughness and smoothness were calculated on the basis of number of bars (12, p. 173).

This analysis revealed that "prevailing melodic roughness is demonstrated . . . to be extraordinarily characteristic of the style in the Two Part Inventions." The author adds that in this respect the music is "truly baroque in the original sense of the word" (12, p. 174).

The Inventions were then ranked "according to percentages of roughness in coordination of melodic accents with metric accents, from rougher to smoother" (12, p. 174). (A table showing the percentage of smooth and rough measures in each voice appears in Appendix C.) This list was compared with the subjective poignancy rating and the coefficient of correlation determined. Like the two proceeding studies this investigation failed to reveal a very strong possibility that roughness as measured here had any causal relationship to poignancy as measured here (12, pp. 174-175).

Next the author investigated the relationship of durational and metric accents. He states:

> To establish the relationship of durational accents and metric accents in the Two Part Inventions a like procedure has been used: for each voice statistics have been made on the relative congruence of these types of accents within each bar. Where the types coincide, the bar has been classified as smooth; otherwise, the bar was called rough (12, p. 175).

In the next step the writer combined the patterns of durational values of both the voices in a bar to establish one pattern, a "macrodurational pattern." Satterfield explains, "The combining is done by recording the shorter values when there is a difference in microdurational patterns" (the durational values of one voice) (12, p. 175).

The bars in each Invention were classified and the percentages for the amount of "rough" and "smooth" material. It was found that a high percentage of the music is "smooth" (12, p. 178). (A table showing the percentage of "rough" and "smooth" bars in each voice and in the macrodurational pattern appears in Appendix D). Again, the Inventions were arranged according to percentages of microdurational of roughness (roughness in each voice). The coefficient of correlation between this list and the list based on poignancy was found to be extremely high. Satterfield concludes that "Within limits, poignancy of emotional content appears to vary generally with the roughness in durational accents" (12, p. 179).

Several facts concerning the Two Part Inventions have been established by the Platt, Miller, and Satterfield studies. First, the use of melodic patterns is quite common in Bach's counterpoint and, as such, offers "a valuable clue to the less easily explained intricacies of his magnificent contrapointal technique" (10, p. 56). Second, oblique motion constitutes more than half of all progressions in the Inventions with the other types of motion in order of decreasing frequency being contrary, parallel, and similar. The average index of melodic independence was found to be 1.9, an indication of a high degree of melodic independence. Third, the emotional content of the Inventions is related neither to the dissonance content nor the roughness in melodic accents; it is, instead, related to the roughness in durational accents. In this study the Inventions were listed in order of poignancy and percentages of the amount of dissonance, consonance, roughness, and smoothness in each piece presented. The findings of these investigators provided a more comprehensive basis for the present study.

Methodology

Material for this study was collected from three sources: (a) a stylistic analysis of the music, (b) related research, and (c) an evaluation by pedagogues. These sources and the general procedure used are described in the following passage.

A preliminary step in preparing this report was to select a suitable edition of the Two Part Inventions. It was necessary to find an edition which represents our present-day state of knowledge of the text of the Inventions. The Landshoff edition corresponded to these requirements and provided the basis for the analysis and all discussion of performance problems.

The first source of material was a stylistic analysis of the characteristics of line, texture, rhythm, and formal design in the music. This study was accomplished in two manners. First, a graphic analysis of each piece, using a set of symbols based on those appearing in Essentials of Eighteenth-century Counterpoint by Mason (7, p. 73) was prepared. This graph indicates the formal design of the piece, the motive and the countermotive, the contrapuntal devices and the derivation of material in the developmental sections, the tonal centers, and the implied chordal structures in each measure. Second, the measurable factors contributing to the difficulty of the Inventions were determined. A system of statistical analysis to quantify these factors was then devised and each Invention investigated accordingly.

The second category of sources of material consisted of all available books, periodicals, and music reference books.

These were examined for related information. The material obtained is presented throughout this study.

The final source of information was an evaluation of the difficulty of the Inventions by six piano teachers (see Appendix E, this report). First, a rating sheet was devised. The pedagogues were asked to indicate the difficulty of technique, memorization and interpretation of each piece (see appendix F). Second, this rating sheet, accompanied by a sample analysis of two Inventions and a statement of the purpose of the thesis, was given to each of seven teachers. Six rating sheets were completed and returned. From the results of this survey a graded list of the Two Part Inventions was prepared.

Plan of This Report

Chapter Two, Stylistic Characteristics of the Bach Two Part Inventions, presents an analysis of the characteristics of line, texture, rhythm, and formal design in each Invention The discussion of each Invention is accompanied by a graphic analysis of the entire piece, with tables to show supporting evidence, and musical examples to identify melodic and textural features.

Chapter Three, Identification of Performance Problems and A Graded List of the Two Part Inventions, presents the

findings of the authorities concerning the difficulties in each Invention. A graded list based on these findings is shown in the chapter.

Chapter Four presents the summaries, conclusions and recommendations to researchers, teachers and pianists. The recommendations were based on an analysis of the findings in this investigation.

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CHAPTER II

A STYLISTIC ANALYSIS OF THE BACH TWO PART INVENTIONS

Introduction

"Only an infinitely fertile mind could venture to write thirty little pieces of the same style and the same compass, and, without the least effort, make each of them absolutely different from the rest" (16, p. 331). In order to identify the performance problems of the Two Part Inventions, "little pieces of the same style" and yet each "absolutely different from the rest," it was necessary to make an analysis of the sytlistic characteristics present in the music. Each Invention will be considered separately beginning with the first and continuing through the fifteenth. A graphic analysis of each, showing the form, the contrapuntal devices, and the implied harmonies, will be presented near the beginning of the discussion. The discussion will treat the characteristics of line, texture, rhythm, and formal design in the music.

An explanation will be given of the methods of procedure used in the following investigations: the analysis of the direction and size of the intervals of the motive and countermotive; the discussion of ornaments; the analysis of rhythmic

relationships and types of motion between the voices; the study of tonal relationships between the voices; and the discussion of the frequency of note values in each voice.

The following procedure was used in the analysis of the direction of the intervals in the motive and countermotive: (a) the number of intervals were counted; (b) the first interval was identified for the purpose of establishing the direction of the motivic line; (c) once the direction was established by the first two notes, successive intervals were analyzed and classified as either changes or continuances in direction. Where rests appeared within the motive the interval involving the rest and the interval immediately following were not included in the statistical analysis. All of the intervals (except those interrupted by rests) were examined for size and categorized as being a fifth or smaller or being larger than a fifth.

In the study of ornaments three classifications of trills were established. For purposes of this paper the term "pralltriller" refers to the three-note or four-note trill; the term "short trill" refers to a trill of less than two beats which is not a pralltriller; and the term "extended trill" refers to a trill of two or more beats.

In the study of texture all the material was classified in beat units into the appropriate type of counterpoint species.

A definition of the terms involved is as follows: (a) "First Species counterpoint" refers to a textural relationship of one note against one; (b) "Second Species counterpoint" refers to a textural relationship of two notes against one; (c) "Third Species counterpoint" refers to a textural relationship of four notes against one; (d) "Fourth Species counterpoint" refers to a textural relationship involving tied notes. Frequently a dotted note, or a syncopation, may be found for a tie which would have the same value (11, pp. 32-44); and (e) "Other counterpoint" refers to a textural relationship involving rests, pedal points, unevenly divided beats, trills--any relationships which cannot be classified in the first four types.

The procedure for determining "rough" and "smooth" counterpoint was examined earlier in this report (see page 20). "Parallel," "similar," "contrary," and "oblique" motion were also explained (see page 16 of this report).

In the discussion of tonal relationships the term "transient modulation" refers to a modulation lasting only a few beats or, possibly, only one beat. The term "major modulation" refers to a modulation of sufficient length so as to insure a definite impression of a new tonal center.

In the discussion of rhythmic characteristics the term "beat note" refers to that note value designated by the time

signature as receiving one beat. The classification "other note values and rests" appearing in the tables of note value frequency refers to rests and note values which are not listed in the table. This category also includes tied and dotted notes.

Invention No. I

Linear Characteristics

Motive and countermotive.--The motive is two-and-one-fourth beats long and has a range of only a fifth. The first seven notes are sixteenth notes and the last an eighth. The outstanding feature is the "somersaulting" tonal pattern of the last four sixteenth notes followed by the interval of the fifth. This movement is emphasized by the stepwise motion of the first three sixteenth notes.

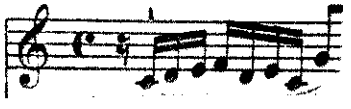

Fig. 4--Invention No. I, motive, measure 1 (8, p. 1)

Leonard ascribes symbolic meaning to this motive in the following statements: "Many of the themes in sixteenth notes are founded on the step motive of confidence or peace. . . .

Symbols	Definitions
1. 3	Motive (M) in exposition (or any clearly defined statement of the motive). Number at beginning of symbol indicates scale step on which M starts.
2. xxxxxxxxxxxxxxx Motive B	Countermotive (CM) or second motive if indicated as "Motive B." If a countermotive and a second motive are both present, the second motive will be represented by a straight line and the designation "Motive B."
3. ///////////////	Important thematic material not derived from the motive or countermotive.
4. - - - - - - - -	Episodic material derived from the motive. With letters "inv."--by inversion, "aug."--by augmentation, and "dim"--by diminution. "B" indicates derivation from a second motive.
5. oooooooooooooo	Episodic material derived from the CM.
6. Pat. Seq.	Pattern and sequence indicated by brackets over symbol denoting derivation.
7. fffffffffffff	Free material (not obviously derived).
8. Pp5ppppppppppp	Pedal point. Number indicates scale step used as pedal point.
9. oooooooooooooo	Material derived from two sources simultaneously (for example, material relating to the rhythmic pattern of the CM and the tonal pattern of the M).
10. | | |	Measure bars
11. Imi.	Imitation

Fig. 5--Legend explaining symbols used in graphic analyses
(continued on next page)

Symbols	Definitions
12. Mod.	Beginning modulation to indicated key
13. Cad.	Cadence
14. Elis.	Elison
15. a, A, (a)	Small letters indicate minor tonalities; large letters indicate major tonalities. Letter in parenthesis indicates transient modulation

Fig. 5--Continued

Constant return to one or two notes indicates some form of the 'tumult' idea, whether it be mere gaiety or combat" (9, p. 645). He then includes Invention No. I in his listing of Inventions with the tumult motive.

In the eight-note pattern there are four changes of direction among the intervals of the motive, (67 per cent) and two continuations (33 per cent). All of the seven intervals are a fifth or smaller.

The countermotive of four notes is one-and-three-fourths beats long and progresses entirely by steps.

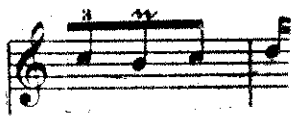

Fig. 6--Invention No. I, countermotive, measure 1-2 (8, p. 1)

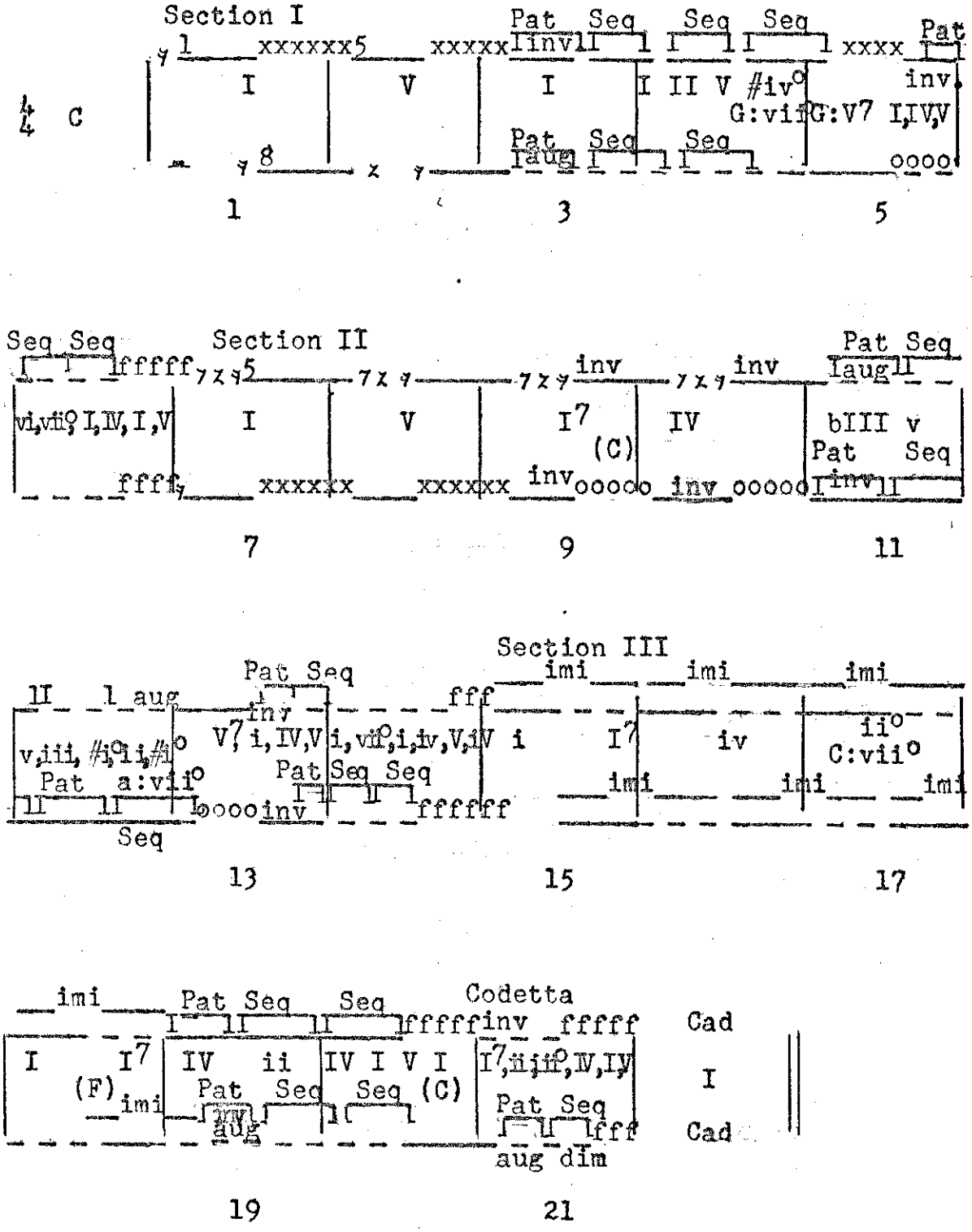

Fig. 7--Invention No. I, graphic analysis

There is one change of direction in the two note progressions under consideration and one continuation and the rhythm consists of three eighth notes and one sixteenth respectively.

Episodes--The rest of the Invention consists almost entirely of motivic and countermotivic development. The motive appears in its original form, in inversion, in sequence, and in augmentation.

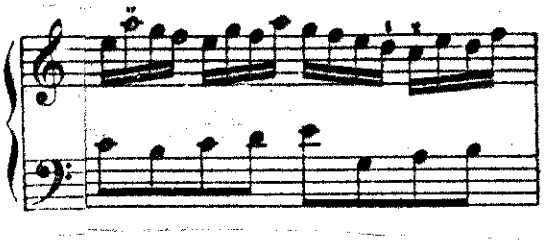

Fig. 8--Invention No. I, episodic pattern, measure 3

The countermotive is inverted when it follows the motive in inversion and is altered in the episodes.

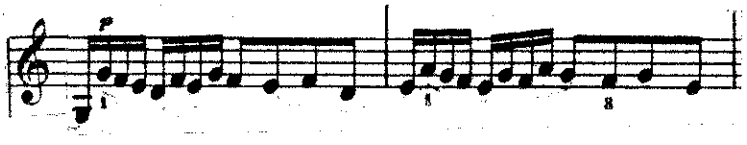

Fig. 9--Invention No. I, episodic pattern, measure 9-10

Only eleven of the total 176 beats (six per cent) were classified as free material and these occurred at cadence points. Thus, ninety-four per cent of the material is derived from the exposition.

Satterfield classifies 13.6 per cent of the bars "smooth" and 86.4 per cent "rough" in the upper voice. In the lower voice he classifies 18.2 per cent of the bars "smooth" and 81.8 per cent rough (14, p. 174).

Invention No. I is sparsely ornamented. There are four pralltrillers and two mordents.

Textural Characteristics

Rhythmic relationships.--This Invention is typical in that most of the material is in Second Species counterpoint. The following table shows the number of beats and the percentages of each type of Species of counterpoint.

TABLE I

FREQUENCY OF COUNTERPOINT SPECIES IN INVENTION NO. I

Species	Number of Beats	Per Cent of Total Beats
First	7	8
Second	47.5	54
Third	2	2
Fourth	3.5	4
Other	15.5	32

Within the First Species counterpoint twelve progressions (48 per cent) were in similar and parallel motion, twelve

progressions (48 per cent) were in contrary motion, and one progression (4 per cent) was in oblique motion.

Two authors offer other approaches. In his analysis of all the Inventions, Miller reports the following figures for the frequency of the types of motion: parallel--7 per cent, similar--10 per cent, oblique--48 per cent and contrary--25 per cent. The index of melodic independence is 2.0 (12, p. 291). A number of two or more indicates a high degree of melodic independence; a number of 1.0 or less indicates a low degree of melodic independence. In his analysis of the relationship of durational accents and metric accents in this Two Part Invention, Satterfield classifies nine per cent of the measures as "rough" and 91 per cent as "smooth" (14, p. 177). (The term "smooth" indicates a coincidence of melodic and metric accents; the term "rough" indicates a lack of coincidence in these accents) (14, p. 173).

Tonal relationships--There are 142 intervals having relative metric accents. Of these, thirty-six (25.3 per cent) are dissonant. The remaining eighty-six (74.7 per cent) are consonant (13, p. 278). An average of 2.6 identifiable chordal structures are implied in each measure. There are three major modulations--from tonic to dominant, to sub-mediant returning to tonic--and two transient modulations. Twenty-four altered tones occur in this Invention.

Rhythmic Characteristics

The meter signature is $\frac{4}{4}$. The most prominent rhythmic pattern is that of the motive itself which begins on the second of four divisions of the beat. Notable also is the similar positioning of the countermotive, which begins on the second half of the beat. These patterns appear continually through the Invention.

The following table shows the frequency of the note values appearing in Invention No. I.

TABLE II

FREQUENCY OF NOTE VALUES IN INVENTION NO. I

Note Values	Upper Voice Number of Beats	Per Cent of Total Beats
Quarter note (beat note)	0	0
Eighth note	13	15
Sixteenth note	49	56
Other note values and rests	26	29

Note Values	Lower Voice Number of Beats	Per Cent of Total Beats
Quarter note (beat note)	0	0
Eighth note	34	39
Sixteenth note	35.25	40
Other note values and rests	18.75	21

The majority of the movement occurs in sixteenth notes and none occurs in the beat note.

There is considerable disagreement among pedagogues concerning appropriate tempos for the Inventions. The tempos suggested by Bodky and Keller are presented in this chapter. (A list of these and other suggested tempos appears in Appendix G). Bodky suggests a tempo of ♩ = ±80 for Invention No. I and Keller recommends a tempo of ♩=63 (1, p. 345).

Formal Design

Invention No. I is in three sections (11, p. 78). The exposition contains four entrances, at the octave and at the fifth. A three-entrance counterexposition can be seen in the second section while the third section is dominated by an echo-like imitation sequence and closes with a one-measure codetta. The overall structure is unusually close knit in that the theme is so very prevalent throughout the piece (3, p. 339). Spitta describes Invention No. I in the following manner: "Of all fifteen this one has the most reserved and dispassionate character, and even the theme is somewhat conventional, only revealing its importance by degrees" (17, p. 61).

Invention II

Linear Characteristics

Motive and countermotive--The motive is seven-and-three-fourths beats in length and has a range of a tenth. The motive,

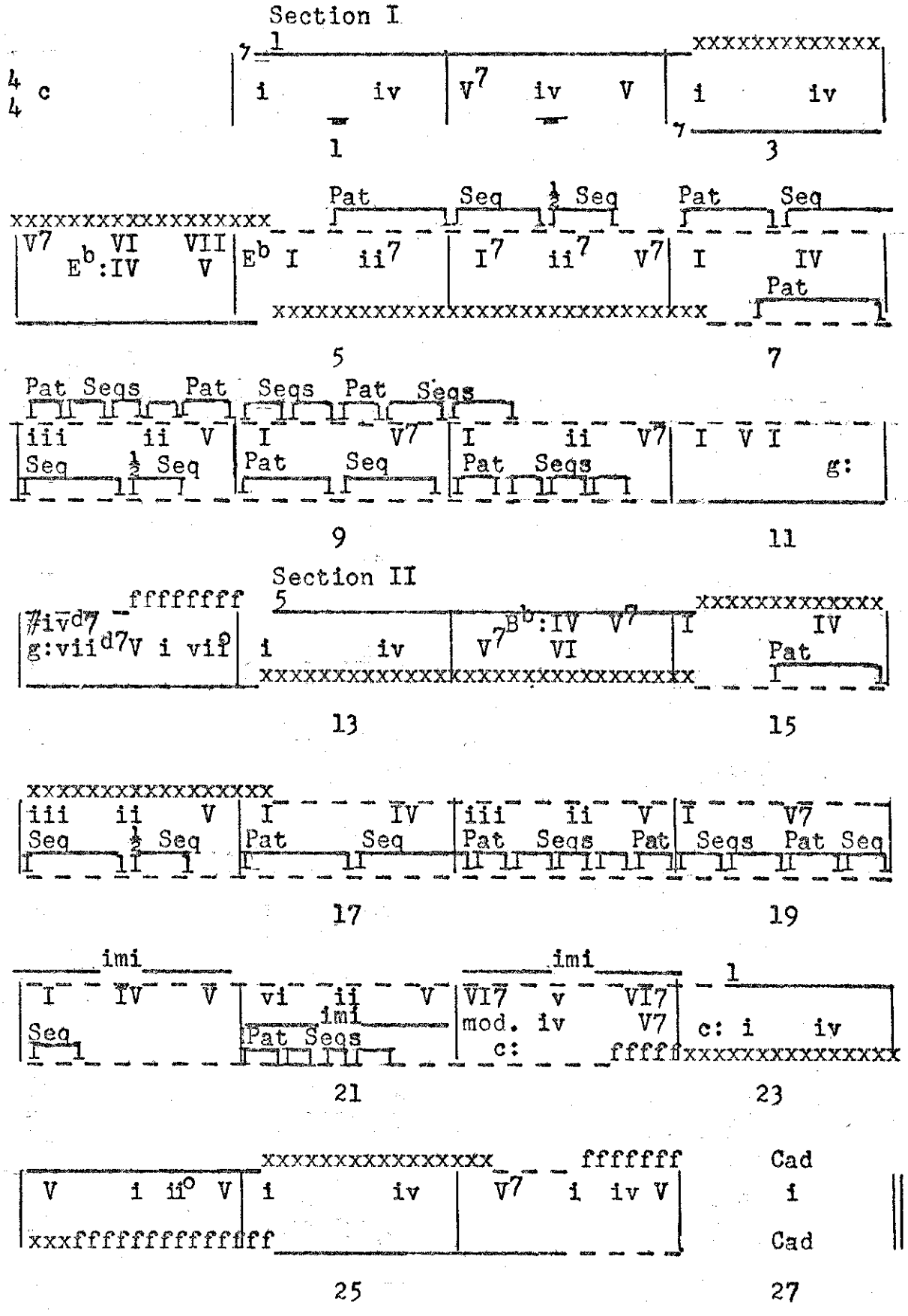

Fig. 10--Invention No. II, graphic analysis

like Invention No. I, moves primarily in sixteenth notes and, like Invention I, is classed as a "tumult" motive (9, p. 645). Whereas the motive in the First Invention is concise and compact, that of the Second Invention is rambling. The outstanding feature is wide leaps culminating in the interval of a diminished seventh in measure two. Also important is the trill on the dotted eighth note near the end of the motive.

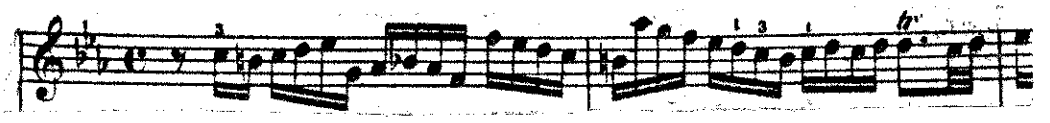

Fig. 11--Invention No. II, motive, measures 1-3 (8, p. 1)

Thirteen of the intervals (46 per cent) in the thirty-note motive change direction, fourteen (50 per cent) continue in the same direction, and one (4 per cent) involves note repetition. Twenty-six intervals (90 per cent) are a fifth or smaller and three (10 per cent) are larger than a fifth.

The countermotive is eight beats long and moves in sixteenth and eighth notes. It has a range of a ninth. It contains the "sigh motive," a repeated note followed by a falling second, which may be major, as it is here, but is usually minor. This figure expresses a "true moderato affect (we sigh only when we feel moderately badly; utter hopelessness and sighing do not go well together)" (1, pp. 116-117).

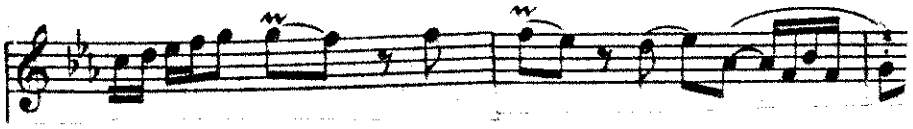

Fig. 12--Invention No. II, countermotive, measures 3-5

Six (50 per cent) of the note progressions in the countermotive change direction, four (33 per cent) continue in the same direction, and two (12 per cent) involve repition of notes. All of the intervals are a fifth or smaller.

Episodes--There are 8.75 beats of free material (4 per cent) in the Invention. Typically, the bulk of the material, 207.25 beats (96 per cent) is either related to the motive or countermotive or important thematically. Falling in the latter category is the following pattern which appears in measures five through ten in the upper voice, reappears in the lower voice, measures seven through ten and measures fifteen through twenty-one, and is seen again in the upper voice in measures seventeen through twenty-two.

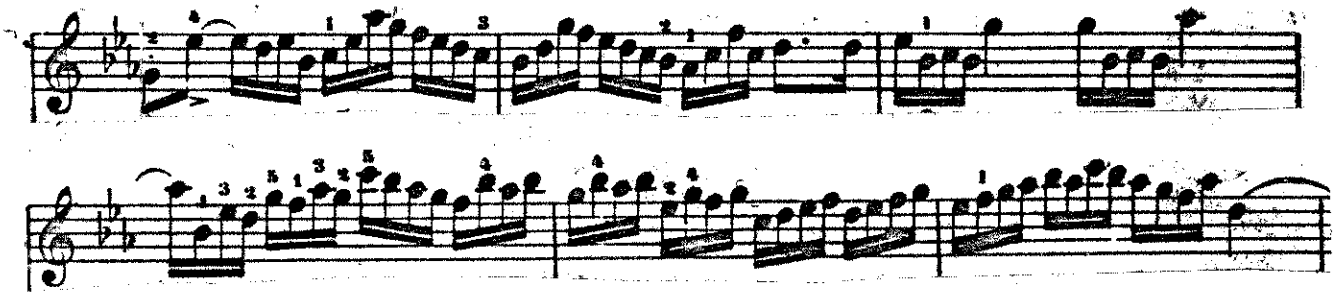

Fig. 13--Invention No. II, episodic pattern, measures 5-10

Satterfield classifies 29.6 per cent of the bars in the upper voice "smooth" and 70.4 per cent "rough" in melodic accents. In the lower voice 28.0 per cent are "smooth" and 72.0 per cent "rough" (14, p. 174). There are twelve prall-trillers and eight short trills.

Textural Characteristics

Rhythmic relationships.--The following table shows the number of beats and the percentage of each type of Species of counterpoint in Invention No. II.

TABLE III

FREQUENCY OF COUNTERPOINT SPECIES IN INVENTION NO. II

Species	Number of Beats	Per Cent of Total Beats
First	35	32
Second	26.5	25
Third	0	0
Fourth	28.25	26
Other	19.25	17

Atypically, First Species counterpoint represents the largest per cent of the types. Within the First Species counterpoint, a total of 134 note progressions, seventy-five (56 per cent) were found to be in similar and parallel motion, fifty-six (42 per cent) in contrary motion, and three (2 per cent) in oblique motion.

Two authors have made other textural studies. An analysis of the total Invention produces the following figures: parallel motion--21.5 per cent; similar motion--6.0 per cent; oblique motion--47.0 per cent; and contrary motion--25.5 per cent. An index of melodic independence of 1.8 was assigned to this piece (12, p. 291). Satterfield, in his analysis of the relationship of durational and metric accents between the voices classifies 11.1 per cent of the bars "rough" and 88.9 per cent "smooth" (14, p. 177).

Tonal relationships.--Of the 185 intervals having relative metric accents, forty-four (23.8 per cent) are dissonant. Consonances dominate, therefore, with a percentage of 77.2 (13, p. 278). Major modulations occur in this order: relative major, minor subdominant, and major leading tone returning to tonic. One transient modulation is present. There is an average of 2.6 implied harmonies in each measure. Thirty-six altered tones occur in this Invention.

Rhythmic Characteristics

The meter is quadruple simple. The following table shows the frequency of the note values appearing in the first voice.

TABLE IV

FREQUENCY OF NOTE VALUES IN INVENTION NO. II

Note Values	Upper Voice Number of Beats	Per Cent of Total Beats
Quarter note (beat note	0	0
Eighth note	11.5	11
Sixteenth note	69	64
Other note values and rests	27.5	25

Note Values	Lower Voice Number of Beats	Per Cent of Total Beats
Quarter note (beat note)	0	0
Eighth note	13.5	12
Sixteenth note	64	60
Other note values and rests	30.5	28

Most notably rhythmically is the continual movement in sixteenth notes.

Tempo suggestions vary among editors, performers and pedagogues. Keller suggests a tempo of ♩=52 and Bodky recommends a tempo of ♩=±80 (1, p. 267).

Formal Design

Invention No. II, a two-section composition, is a perfect canon with the imitation entering two measures later (6, p. 621). The design can be shown graphically. In the following figure each division marked by vertical lines represents a two-bar

section. The last division, which is marked by a double bar, represents only one measure.

Voices															
Upper	A	B	C	D	E	F	A	B	C	D	Episode	A	B	Cad	‖
Lower	-	A	B	C	D	A	B	C	D	E		B	A		

Fig. 14.--Invention No. II, formal plan (3, p. 339)

This particular form is unique among the Inventions (3, p. 339). This Invention is described, on one hand, as "passionate and eager" (17, p. 61) and "laughing and happy" (10, p. 26) while on the other hand others call it "composed" and "impertubable" (1, p. 230) or "contemplative" (2, p. 275).

Invention No. III

Linear Characteristics

Motives.--The first motive is seven beats in length and begins with an upbeat. Described as a "tumult" motive (9, p. 645) it has a tonal range of a fifth and moves entirely in sixteenth notes (the simple division of the beat). The striking feature is the rebounding movement of the tones within the fifth.

Fig. 15--Invention No. III, motive A, measures 1-2

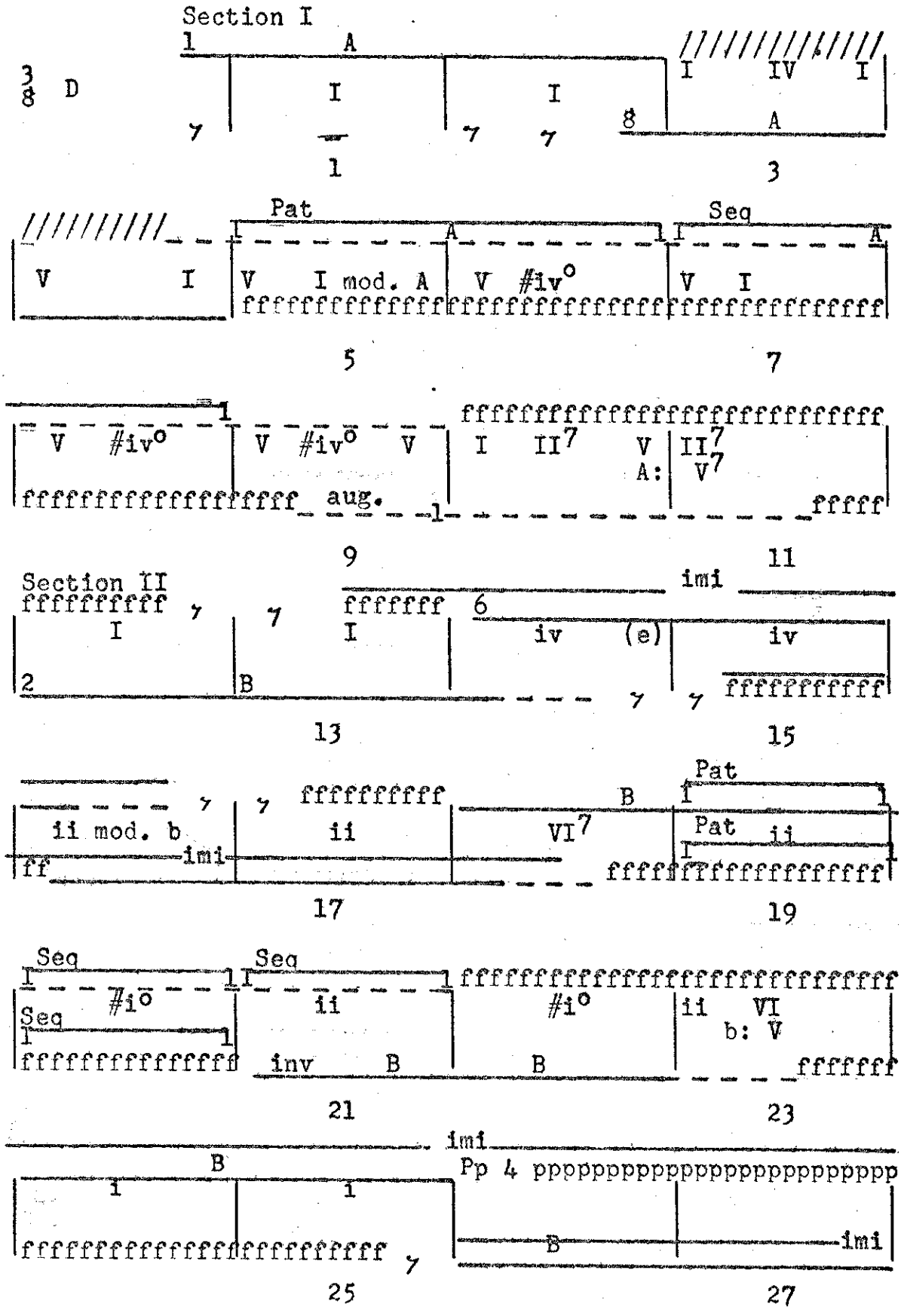

Fig. 16--Invention No. III, graphic analysis (continued on next page)

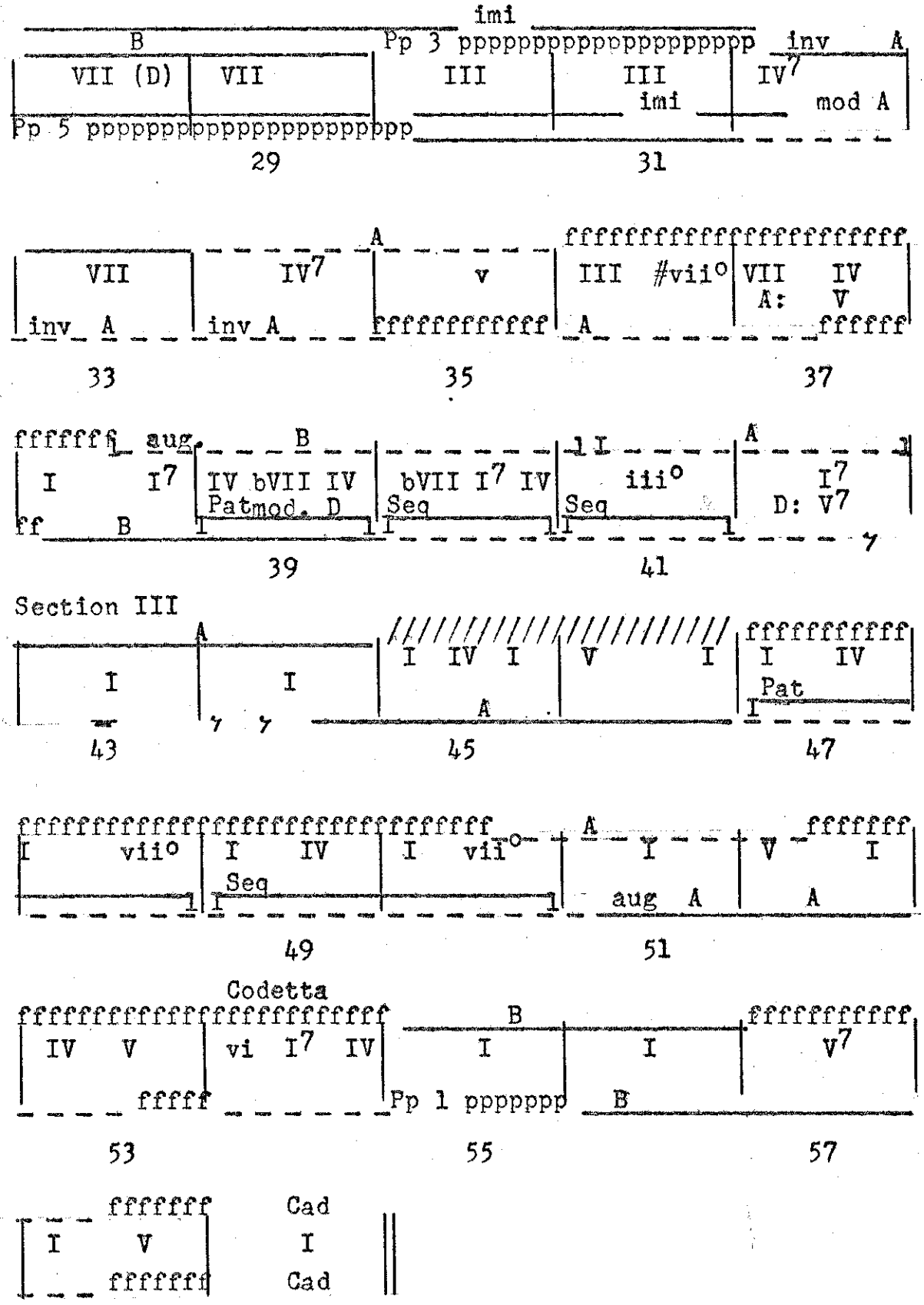

Fig. 16--Continued

Seven of the intervals (58 per cent) change directions while five (42 per cent) continue in the established direction. All of the thirteen intervals are a fifth or smaller. Bodky describes this motive as a "miniature 'Gloria' (the motive of the falling octave)" (1, p. 267).

The second motive, appearing in the second section (11, p. 82), is six-and-one-half beats in length and strongly resembles the first motive. It could be considered a motive variant. The second through the sixth beats of motive B correspond to the pattern found in beats one through five of motive A.

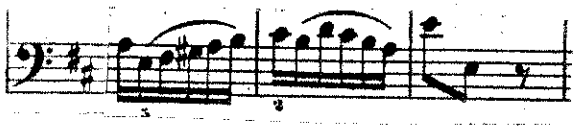

Fig. 17--Invention No. III, motive B, measures 12-14

Four of the intervals (40 per cent) change direction and six (60 per cent) continue in the same direction. Ten of the melodic intervals (91 per cent) are a fifth or smaller and one (9 per cent) is larger than a fifth. It has a range of an octave.

Episodes.--There are sixty-one beats (17 per cent) of free material and 295 beats (83 per cent) of material which

is either derived from the motives or establishes its thematic importance through use. The material in measures five through eleven appears in measures forty-seven through fifty-three with one significant alteration: the voices in the first four measures of the second section have been inverted so that the pattern which was in the first voice is seen in the second voice and vice versa. Beginning with the fifty-first measure the counterpoint resumes its original form.

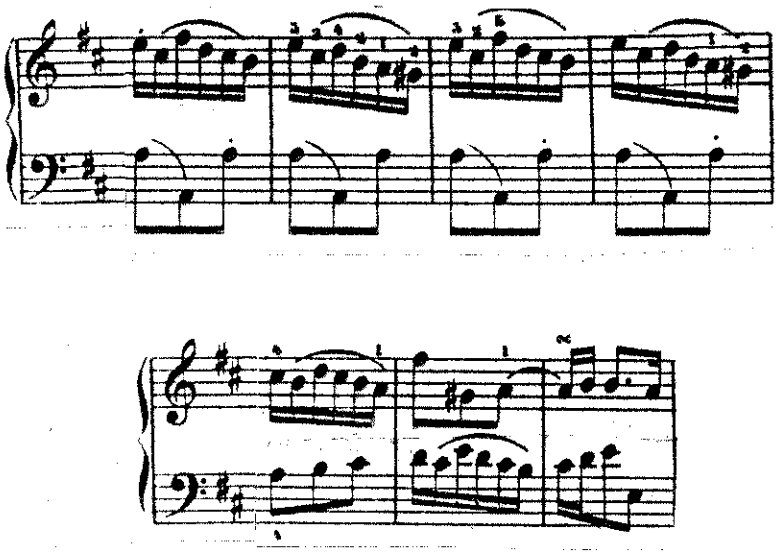

Fig. 18--Invention No. III, episodic pattern, measures 5-11

The counterpoint appearing in the lower voice in measures five through eight could be considered a pedal point.

The cadence pattern occuring in measures nine through eleven and fifty-one through fifty-three appears with some alteration in measures twenty-one through twenty-three, thirty-five through thirty-seven, and measures fifty-six through fifty-eight.

In the upper voice 29.6 per cent of the bars are "smooth" and 70.4 per cent are "rough." In the lower voice 28.0 per cent of the bars are "smooth" and 72.0 per cent "rough" (14, p. 174). There are six ornaments: four pralltrillers and two mordents.

Textural Characteristics

Rhythmic relationships.--The following table shows the number of beats and the percentage of each type of Species of counterpoint.

TABLE V

FREQUENCY OF COUNTERPOINT SPECIES IN INVENTION NO. III

Species	Number of Beats	Per Cent of Total Beats
First	29	16
Second	71	40
Third	0	0
Fourth	29	16
Other	49	28

Most of the counterpoint is in Second Species and none occurs in Third Species. Thirty-one of the fifty-six progressions in the First Species counterpoint (55 per cent) are in similar or parallel movement; twenty-two progressions (39 per cent) are in contrary motion; and three (6 per cent) are in oblique motion.

Two authors have made other studies of textural characteristics. An analysis of the whole piece reveals that most of the contrapuntal progressions are in oblique motion (57 per cent), with 25 per cent in contrary, 12 per cent in parallel, and 10 per cent in similar motion. The melodic index is 1.9 (12, p. 291). Regarding the relationship of durational and metric accents between the voices, the findings were as follows: 8.4 per cent of the bars are "rough" and 91.6 per cent "smooth" (14, p. 177).

Tonal relationships.--An analysis of the 155 intervals having relative metric accents revealed that thirty-eight (24.5 per cent) were dissonant and 117 (75.5 per cent) were consonant (13, p. 178). There are five major modulations and two transient modulations. An average of 1.5 harmonies are implied in each measure. There are twenty-one altered tones in this Invention.

Rhythmic Characteristics

The meter is triple simple. An analysis of the Invention showed the following frequencies of note values in the upper and lower voices.

TABLE VI

FREQUENCY OF NOTE VALUES IN INVENTION NO. III

Upper Voice Note Value	Number of Beats	Per Cent of Total Beats
Eighth note (beat note)	39	22
Sixteenth note	87	49
Other note values and rests	52	29
Lower Voice **Note Value**	**Number of Beats**	**Per Cent of Total Beats**
Eighth note (beat note)	44	25
Sixteenth note	98.5	56
Other note values and rests	34.5	19

Sixteenth notes are most common with the remainder of the motion fairly divided between the eighth note and other time values.

Authorities disagree on the correct tempo. Bodky suggests a tempo of ♩ = ±60 (♬ = ±180) and Keller recommends a tempo of ♬ = 138 (1, p. 364).

Formal Design

Mason classifies the D Major Invention formally as a three-part song form because of the return to the beginning in measure forty-three (11, p. 80). There is a counterexposition using the second motive in the second section, which begins in the key of A Major. This section contains well-defined cadences to b minor and to A Major and thus suggests subdivisions. The third section begins in the original key and closes with a six-measure codetta. This Invention is described as being "merry" in character (17, p. 61).

Invention No. IV

Linear Characteristics

Motive and countermotive.--The motive is seven beats long and moves in sixteenth notes altogether except for the last note, an eighth note. It has a range of a seventh. The most striking feature of the motive is a leap of a diminished seventh down followed by an identical leap upward. This leap contrasts sharply with the stepwise movement of the rest of the motive.

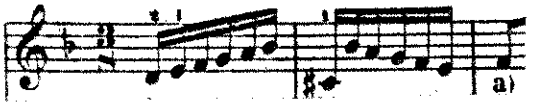

Fig. 19--Invention No. IV, motive, measures 1-3 (8, p. 2)

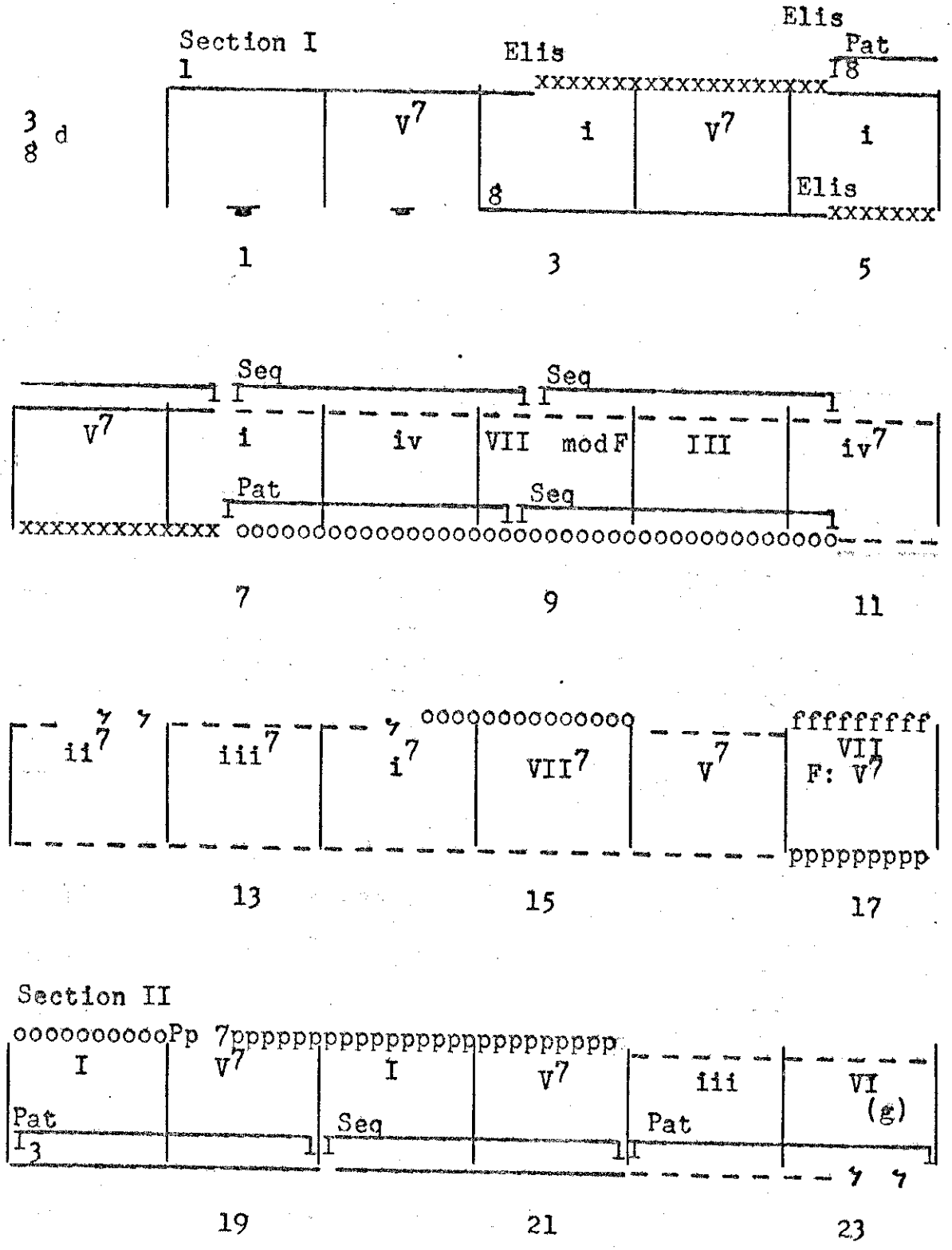

Fig. 20--Invention No. IV, graphic analysis (continued on next page)

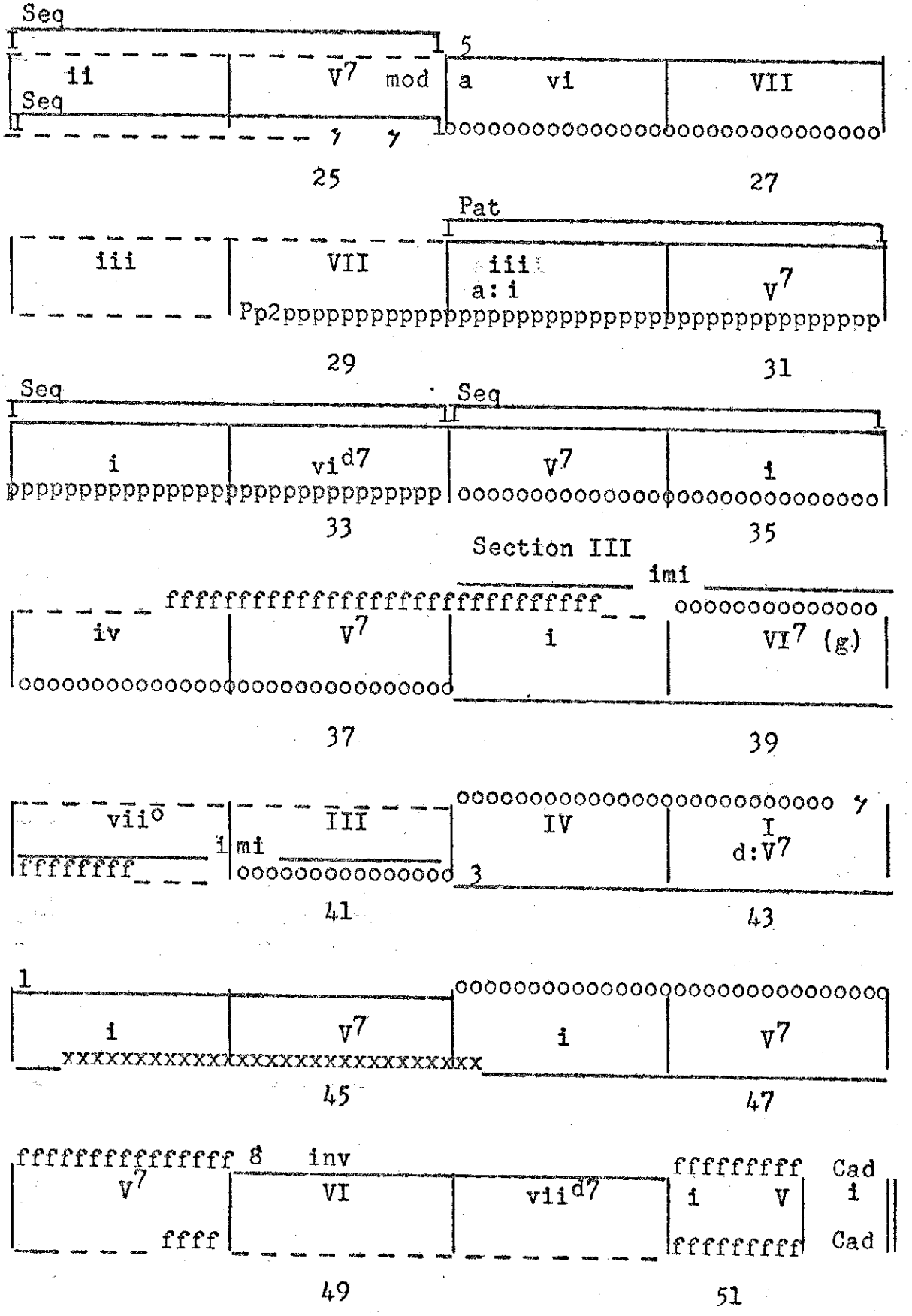

Fig. 20--Continued

There is little change of direction in the twenty-note motivic line. Three intervals (27 per cent) change direction and the remainder (73 per cent) continue in the same direction. Ten of the intervals (83 per cent) are a fifth or smaller and two (17 per cent) are larger than a fifth. Like those of the first three Inventions, this motive symbolizes "tumult" (9, p. 645).

An elision occurs on the last note of the motive and the first note of the countermotive, a simple chordal outline of a i^6 chord and a vii^{o6} chord in eighth notes. The length is six and one-half beats and the tonal range a seventh.

Fig. 21--Invention No. IV, countermotive, measures 3-5 (8, p. 2)

Two of the intervals (40 per cent) of the seven-note progression change direction while the remaining three (60 per cent) continue in the same direction. All of the intervals are a fifth or smaller.

Episodes.--The episodic material generally is derived from the motive and the countermotive. The ten-and-one-half beats of free material in the first voice and the thirteen beats of

free material in the second voice represent only 10 per cent of the total material.

Several patterns are used frequently in the episodes. The pattern appearing in measure eight (See figure 22) reappears in measures ten, fifteen, thirty-four, thirty-five and thirty-six (8, p. 2).

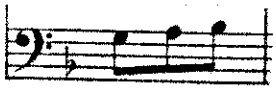

Fig. 22--Invention No. IV, episodic pattern, measure 8

The pattern first seen in measures eleven and twelve (See figure 23) also occurs in measures thirteen and fourteen, twenty-two and twenty-three, and twenty-four and twenty-five.

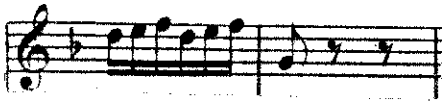

Fig. 23--Invention No. IV, episodic pattern, measure 11

A slightly-altered form of the motive appearing in the first voice in measures seven through ten and imitated in the second voice in measures eleven through sixteen is seen again in the first voice in measures forty to forty-one.

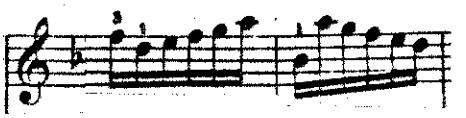

Fig. 24--Invention No. IV, episodic pattern, measures 7-8

In the upper voice, 28.8 per cent of the bars are "smooth" and 71.2 per cent are "rough." In the lower voice 24.0 per cent of the measures are "smooth" and 76.0 per cent "rough" (14, p. 174).

Invention No. IV contains few embellishments. Two extended trills are indicated: one in the first part for nine beats and one in the second voice for fifteen beats.

Textural Characteristics

Rhythmic relationships.--Most of the counterpoint in Invention No. IV is in Second Species. The following table shows the frequency of each Species in beats and in percentage.

TABLE VII

FREQUENCY OF COUNTERPOINT SPECIES IN INVENTION NO. IV

Species	Number of Beats	Per Cent of Total Beats
First	32	22
Second	68	44
Third	0	0
Fourth	26.5	17
Other	26.5	17

Within the First Species counterpoint thirty-two progressions (51 per cent) move in parallel and similar motion, thirty (48 per cent) in contrary motion and one (1 per cent) in oblique motion.

Two authors have made other textural studies of this Invention. Based on an analysis of all the progressions in Invention No. IV, Miller gives the following figures for the types of motion: parallel--12 per cent, similar--10 per cent, oblique--53 per cent, and contrary--25 per cent. The index of melodic independence is 1.9 (12, p. 291). From a textural approach Satterfield finds 9.6 per cent of the measures "rough" and 93.6 per cent "smooth" in durational accents (14, p. 177).

Tonal relationships.--There are 140 intervals having relative metric accents. Of these, thirty-six (25.7 per cent) are dissonant and 104 (74.3 per cent) are consonant (13, p. 278).

An average of one identifiable chordal structure is implied in each measure. There are three well defined modulations and two transient modulations. Forty-nine altered tones occur in Invention No. IV.

Rhythmic Characteristics

The meter is triple simple and the time signature $\frac{3}{8}$. The following table shows the frequencies of the note values in this Invention.

TABLE VIII

FREQUENCY OF NOTE VALUES IN INVENTION NO. IV

Upper Voice Note Values	Number of Beats	Per Cent of Total Beats
Eighth note (beat note)	30	19
Sixteenth note	106	68
Thirty-second note	0	0
Other note values and rests	20	13
Lower Voice **Note Values**	**Number of Beats**	**Per Cent of Total Beats**
Eighth note (beat note)	53	34
Sixteenth note	72	46
Thirty-second note	0	0
Other note values and rests	31	20

The great majority of the movement is in sixteenth notes.

Two rhythmic patterns are obvious: a succession of sixteenth notes as in the motive and a succession of eighth notes as in the countermotive. Generally, the Invention is based on these patterns.

There is considerable disagreement concerning the correct tempo. However, Keller and Bodky both suggest a tempo of ♩. = 60 (1, p. 364). This is the only Invention of the fifteen with identical tempo suggestions by these two authors.

Formal Design

Invention No. IV is in three-sections (11, p. 79). The exposition contains three entrances of the motive occurring at the octave. There is no counterexposition in the second section but rather only a free imitation of the motive in the second voice. A strong cadence to the key of A minor in this section suggests a subsection. The third section begins with a quasi-exposition statement of the motive and ends with a four measure codetta. Invention No. IV is described as "gloomy" in nature (17, p. 61), as "contemplative" (1, p. 231) and as having "sad colors of gray and lavender" (7, p. 452).

Invention No. V

Linear Characteristics

Motive and countermotive.--The motive is fifteen-and-three-fourths beats long and has a range of a tenth. Rhythmically, it begins on the second half of the first beat and ends on the first quarter of the seventeenth beat. Classed as a "tumult" motive (9, p. 645) it may be divided into two parts, an antecedent group and a consequent group. The first part, consisting of two measures, presents a strong rhythmic pattern which is repeated one step higher in measure two. The consequent group answers by continuing the momentum with new

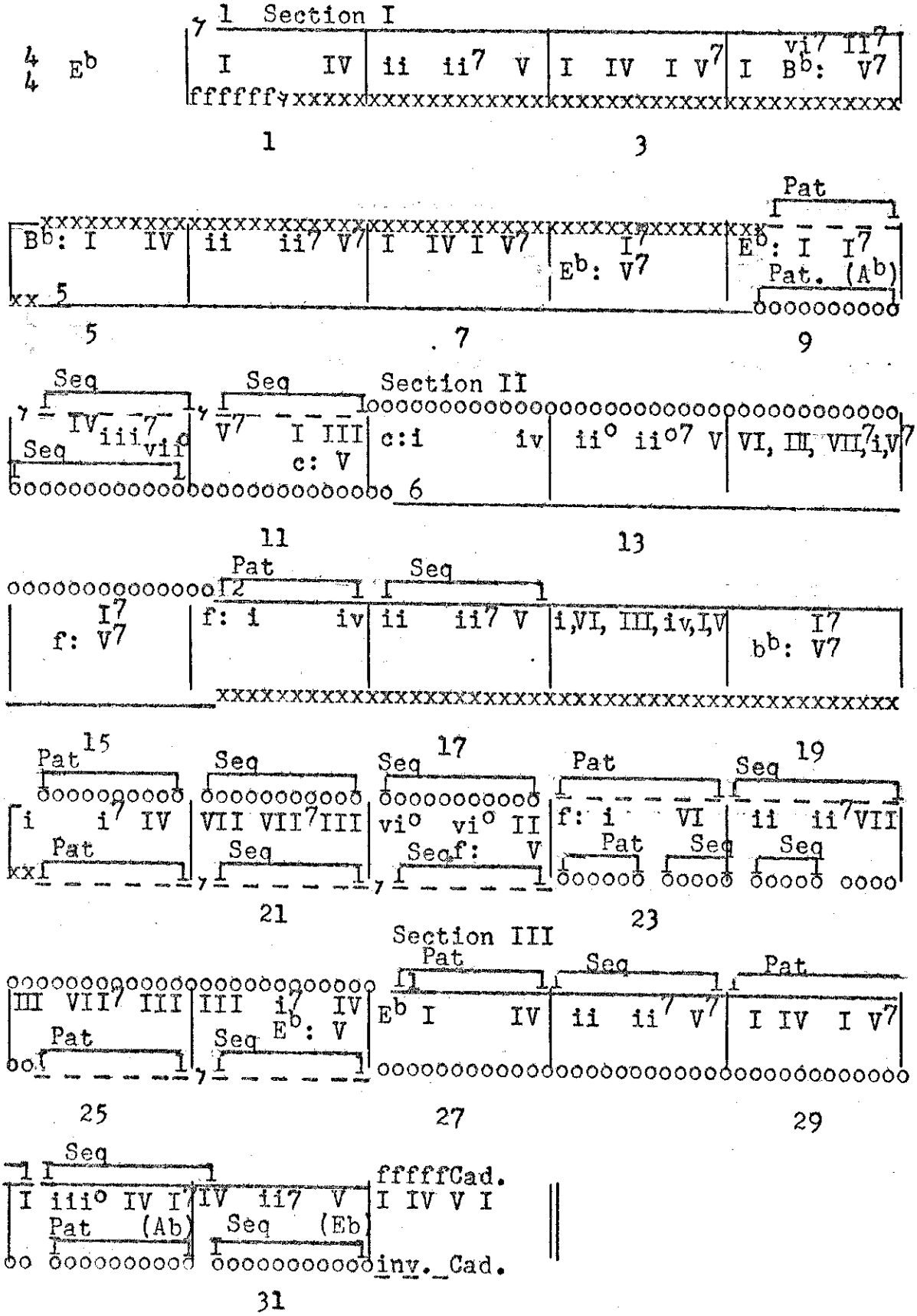

Fig. 25--Invention No. V, graphic analysis

material. A climax occurs on the highest note of the phrase (E^b in measure 4).

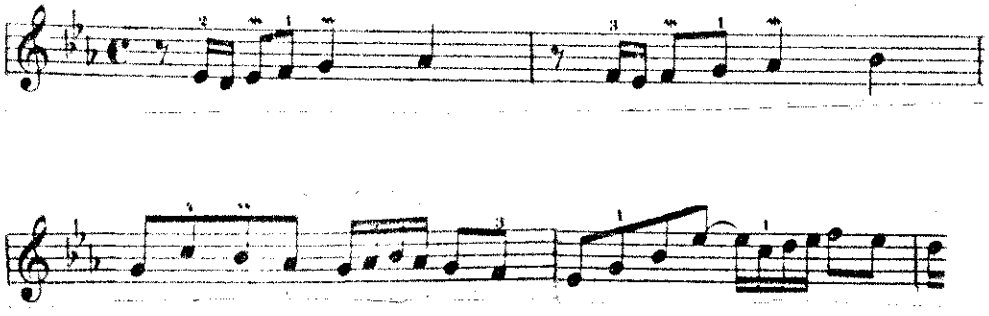

Fig. 26--Invention No. V, motive, measures 1-5 (8, p. 2)

Eleven intervals (39 per cent) change direction while nineteen (61 per cent) continue in the same direction. All of the intervals are a fifth or smaller.

The countermotive is fourteen and one-half beats long and appears consistently with the motive. It consists entirely of sixteenth notes except for the last, an eighth note and has a range of a tenth. Basically, it is a sequence with minor alterations of one figure.

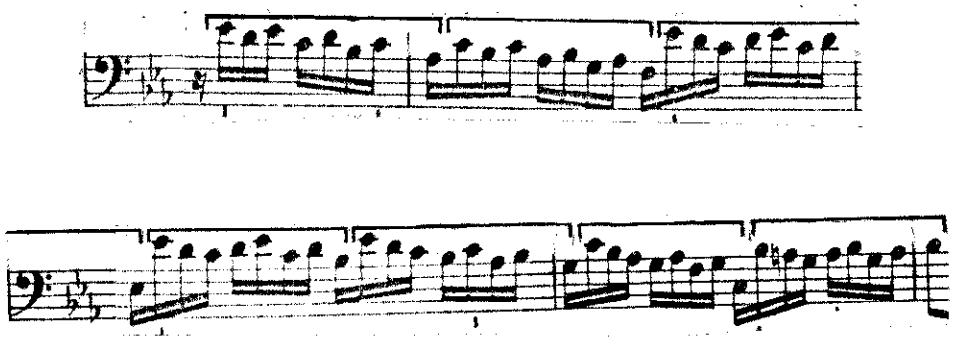

Fig. 27--Invention No. V, countermotive, measures 1-5

There are many changes in the directions of the line--forty-six in all--for a percentage of 84. Only nine intervals (16 per cent) continue in the same direction. Fifty-one intervals (93 per cent) are a fifth or smaller, and four (7 per cent) are larger than a fifth.

Leonard discusses the motive and countermotive in the following passage:

> In the Two Part Invention No. 5 (E^b Major), the "joy motive" appears as the beginning of the subject, which ends with the "step of confidence". . . . The counterpoint is, as often happens, a long motive of "simple joy." Here, however, is plainly one of the "conflicts" of direction, which Bach so often created, with the "upward striving" of one theme in combat with the downward strength of another (9, p. 573).

Episodes.--Only two beats (1 per cent) of the material is not derived from the motive, and this occurs at a cadence point. Two-hundred-fifty-four beats (99 per cent) is derived from the exposition.

Satterfield classifies 18.8 per cent of the bars in the upper voice "smooth" and 81.2 per cent "rough" in melodic accents. In the lower voice 9.4 per cent are "smooth" and 90.6 per cent are "rough" (14, p. 174). There is one ornament, a short trill.

Textural Characteristics

Rhythmic relationships.--The following table shows the number of beats and the percentage of each type of Species of counterpoint in Invention No. V.

TABLE IX

FREQUENCY OF COUNTERPOINT SPECIES IN INVENTION NO. V

Species	Number of Beats	Per Cent of Total Beats
First	20.25	16
Second	58	45
Third	41	32
Fourth	4	3
Other	4.75	4

Second Species is the most common and Fourth Species the least.

Within the First Species counterpoint forty-three progressions (57 per cent) are in parallel and similar motion and thirty-two progressions (43 per cent) are in contrary motion. None of the motion is oblique.

Two authors have made other textural studies. An analysis of the total Invention produces the following figures for the types of motion: parallel--10 per cent, similar--11 per cent, oblique--53 per cent, and contrary--26 per cent. The index of melodic independence is 1.9 (12, p. 291). Based on a

study of durational accents 6.4 per cent of the measures are "rough" and 93.6 per cent are "smooth" (14, p. 177).

Tonal relationships.--Of the 239 intervals having metric accents, 23.4 per cent (fifty-six in number) were found to be dissonant and 76.6 per cent (forty-four) consonant (13, p. 278). There are eight modulations to new keys and one transient modulation. An average of 2.9 harmonies are implied in each measure. There are thirty-nine altered tones in this composition.

Rhythmic Characteristics

The meter is quadruple simple. The following table shows the frequency of the note values appearing in the first voice.

TABLE X

FREQUENCY OF NOTE VALUES IN INVENTION NO. V

Note Values	Upper Voice Number of Beats	Per Cent of Total Beats
Quarter note (beat note)	23	19
Eighth note	34.5	27
Sixteenth note	63.75	50
Other note values and rests	6.75	4

Note Values	Lower Voice Number of Beats	Per Cent of Total Beats
Quarter note (beat note)	20	16
Eighth note	24	19
Sixteenth note	77.25	60
Other note values and rests	6.75	5

Half of the total time is in sixteenth notes. Since this Invention employs constant use of the motive and countermotive the rhythmic figures appearing therein are seen repeatedly throughout the piece.

Authorities disagree on the correct tempo for this piece. Keller suggests a tempo of ♩=69. Bodky recommends a tempo of ♩=±80 (1, p. 345).

Formal Design

Invention No. V may be considered a three-section composition. Cadences are not well-defined, however, and the division probably would not be detected from oral observation alone since the episodes do not present any new material and very little development of the motive and countermotive. Rather, the whole composition is a series of repetitions of the motive accompanied by the countermotive. According to Bodky this is the first Invention to use a countermotive throughout the entire piece (1, p. 267).

The first section begins in the key of E^b Major and contains modulations to B^b Major and back to E^b Major. The second section begins in c minor modulates to f minor, b^b minor and back to f minor. The final section begins in E^b Major, contains a transient modulation to A^b Major and closes in E^b Major. This Invention is described as being "full of grace and dignity" (17, p. 61).

Invention No. VI

Linear Characteristics

Motives.--Invention No. VI is based on two coordinate motives. Motive A appears in the lower voice and has a range of an octave. Basically, it is an ascending scale line.

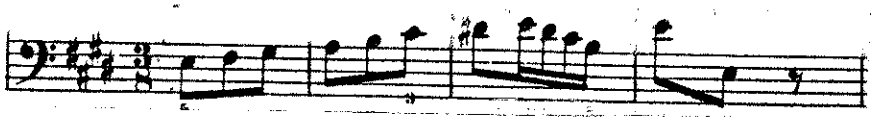

Fig. 28--Invention No. VI, motive A, measures 1-4 (8, p. 2)

There are only three changes (27 per cent) in the direction of the motive; there are nine continuances (73 per cent). Only one (9 per cent) of the twelve intervals is a fifth or larger. The remaining eleven (91 per cent) move by step.

The second motive is a mirror of the first. It begins on the second half of the first beat and continues in this syncopated rhythm until the fourth measure. Motive B is twelve-and-one-half beats in length and covers a range of a ninth.

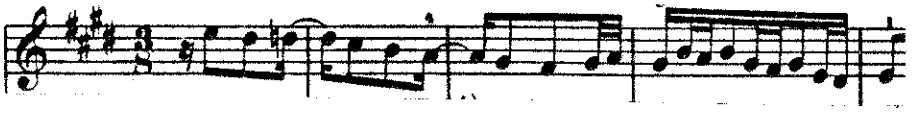

Fig. 29--Invention No. VI, motive B, measures 1-5 (8, p. 2)

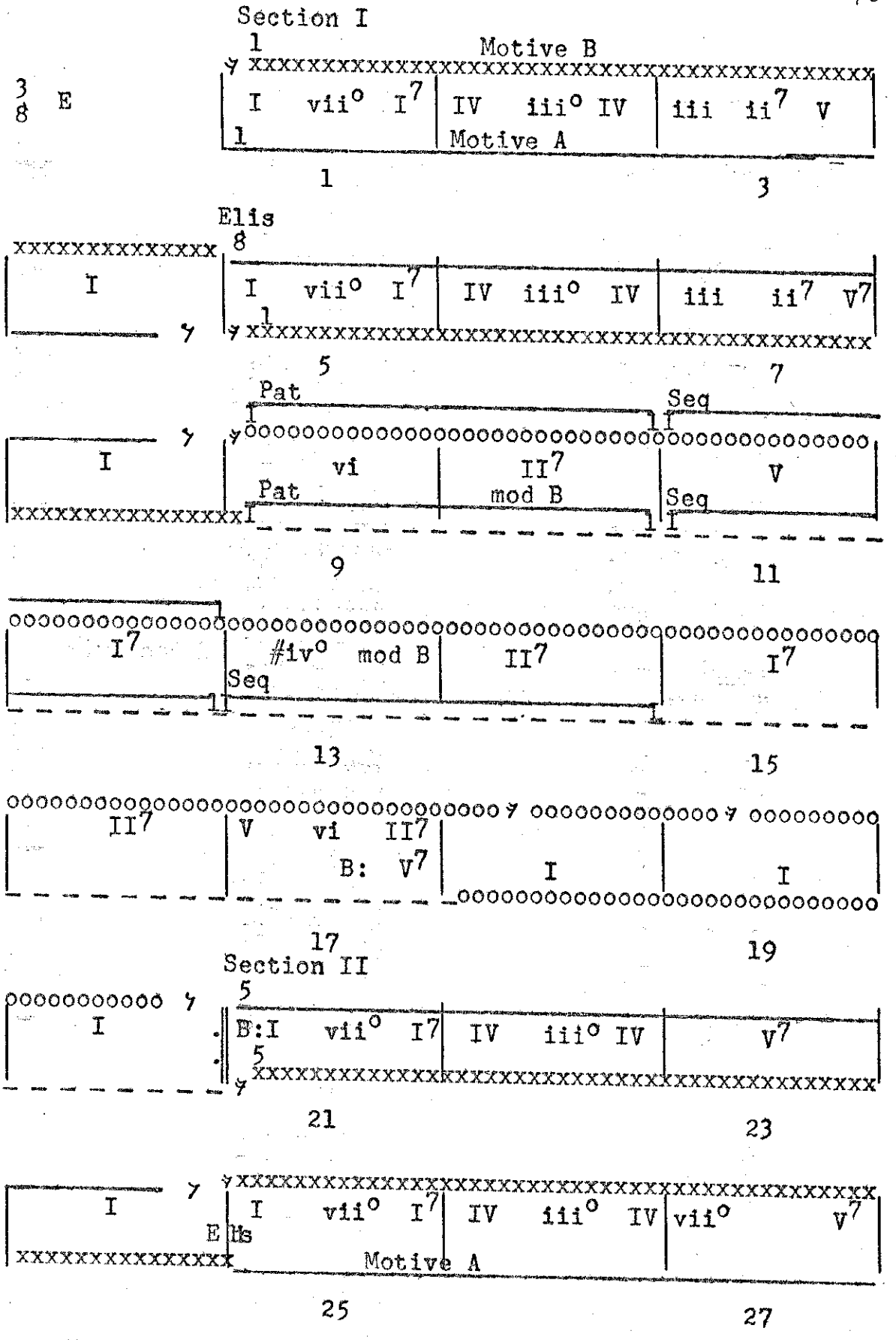

Fig. 30--Invention No. VI, graphic analysis (continued on next page)

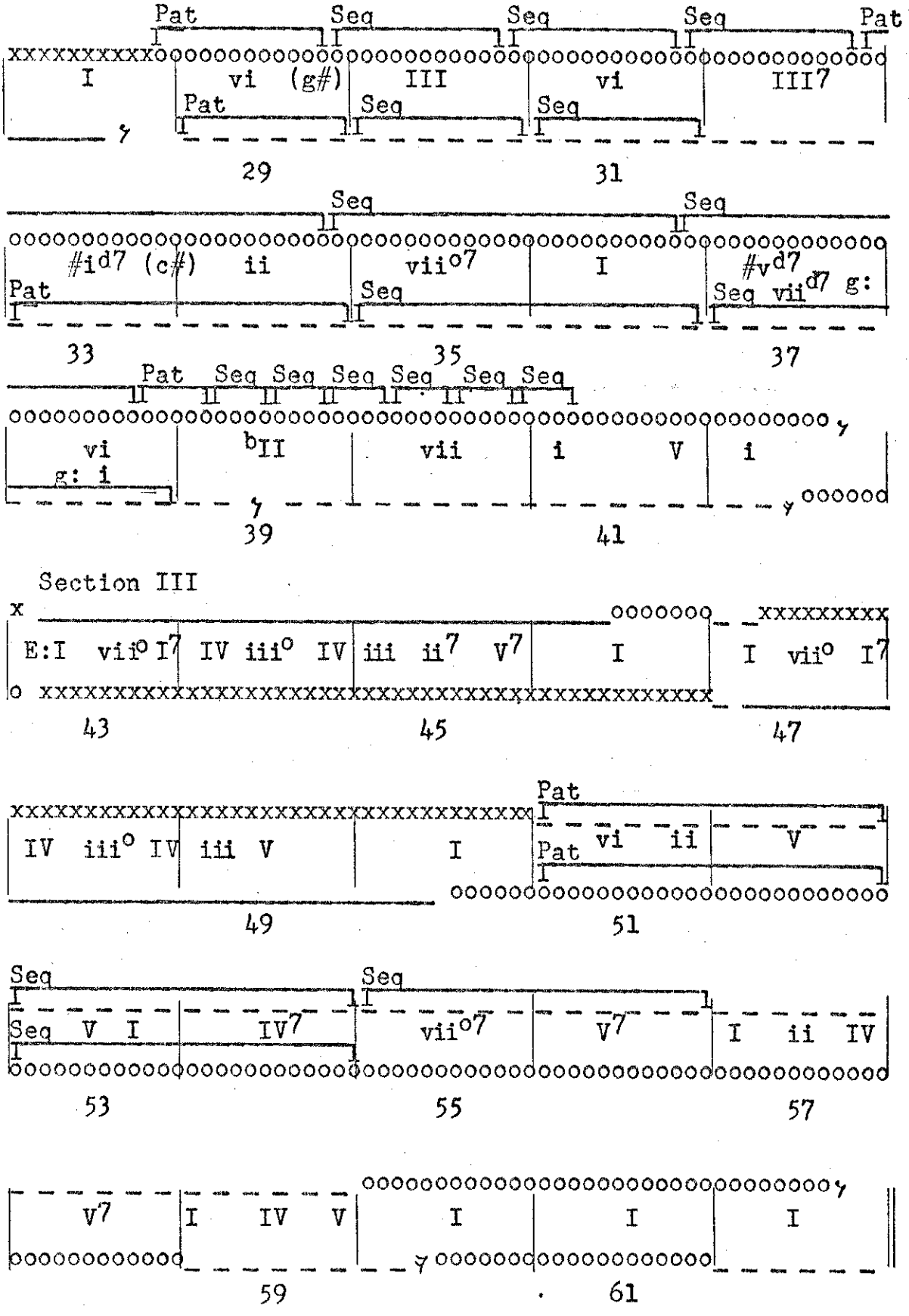

Fig. 30--Continued

There are nine changes and nine continuances in the direction of the melodic intervals. All of the intervals are a fifth or smaller. Motive B consists basically of the descending scale with an interruption of an embellished arpeggio before reaching tonic.

Leonard ascribes the following meaning to the two lines: "It [Invention No. VI] sets forth the 'torturing grief' of the chromatic line, and the 'faltering uncertainty' or syncopation . . . . Against these two stalks the confidence of the 'step motive,' in conflicting contrary direction; and the whole theme ends with the motive of 'joyous agitation'. . . . (9, p. 645).

Episodes.--All of the material is related to the motives. There are no ornaments indicated by sign. However, the mordent figure, first seen in the second motive, appears frequently.

In the upper voice of this Invention 27.5 per cent of the bars are "smooth" and 72.5 per cent "rough" in melodic accents. In the lower voice 17.7 per cent of the bars are "smooth" and 82.3 per cent "rough" (14, p. 174).

Textural Characteristics

Rhythmic relationships.--The following table shows the number of beats and the percentage of each type of Species of counterpoint in this Invention.

TABLE XI

FREQUENCY OF COUNTERPOINT SPECIES IN INVENTION NO. VI

Species	Number of Beats	Per Cent of Total Beats
First	5	3
Second	24	13
Third	4	2
Fourth	95	51
Other	58	31

Atypically, most of the counterpoint is in Fourth Species with the least amount in Third Species.

Within the First Species counterpoint there were found to be nineteen progressions. Of these, thirteen (68 per cent) are in similar and parallel motion, six (32 per cent) in contrary motion, and none in oblique motion.

Two authors offer other approaches. Based on an analysis of the whole Invention there is 5 per cent parallel motion, 6 per cent similar motion, 76 per cent oblique motion, and 13 per cent contrary motion. The index of melodic independence is 2.0 (12, p. 291). Based on a study of the relationship of durational accents and metric accents 3.2 per cent of the measures are classified as "rough" and 96.8 per cent as "smooth" (14, p. 177).

Tonal relationships.--A study of the 237 intervals having relative accents reveals that eighty-five (35.9 per cent) are

dissonant and 152 (64.1 per cent) are consonant (13, p. 278). There are two major modulations, to the dominant and returning to the tonic, and four transient modulations. An average of 1.7 identifiable chordal structures occur in each measure. There are forty-six altered tones in Invention No. VI.

Rhythmic Characteristics

The meter is triple simple and the signature is $\frac{3}{8}$. The following table shows the frequency of note values in this Invention.

TABLE XII

FREQUENCY OF NOTE VALUES IN INVENTION NO. VI

Upper Voice		
Note Values	Number of Beats	Per Cent of Total Beats
Eighth note (beat note)	101	54
Sixteenth note	50	27
Thirty-second note	26	14
Other note values and rests	9	5
Lower Voice		
Note Values	Number of Beats	Per Cent of Total Beats
Eighth note (beat note)	118	63
Sixteenth note	24.5	13
Thirty-second note	12	6
Other note values and rests	31.5	18

Most of the movement is in eighth notes and a relatively large amount in thirty-second notes, which appear 208 times. The rhythmic patterns of syncopation and embellishment occurring in the second motive and the eighth note pattern from the first motive are all extremely important throughout the Invention.

There is considerable disagreement among editors, performers and pedagogues concerning the correct tempo. Keller suggests a tempo of ♩=96 and Bodky recommends a tempo of ♩=±120 (1, p. 366).

Formal Design

Mason classifies Invention No. VI as a three part song form although the return to the beginning in the last twenty measures has the voices interchanged (11, p. 80). It is the only Invention divided into two repeating sections (3, p. 339). Schroder compares the section from measure twenty-nine to measure forty-three, largely modulatory, to a "miniature development section of a sonata form" (15, p. 30). Leonard describes this passage in the following statements: "After the double bar the first theme returns (measure 21), but the joyous motive takes possession in measure 29 and is continually interrupting grievous thoughts till measure 43. There the

original conflict recommences and ends with a double assertion of the joyous agitation" (9, p. 645). Both the second and third sections are marked by strong cadences. The final three measures constitute a cadence rhyme to measures eighteen to twenty (the last measure in the first section) (15, p. 30). Says Schroder: "This device of cadence rhyme was one of the distinguishing features of eighteenth century binary form, of which this Invention is a good example. The formal effect is to tie the composition together as a whole and reveal the essential relatedness of each of the two sections to each other" (15, p. 30).

Spitta points to the prominence of double counterpoint in Invention No. VI and characterizes it as being "full of roguish fun" (17, p. 61). Similarly, Landowska calls it "jocose and spirited" (7, p. 452). Schroder terms this piece a "highly imaginative little scherzo" the opening measures of which "remind one of two dancing partners coming together, after which both change their respective positions and move apart once again" (15, p. 27).

Invention No. VII

Linear Characteristics

Motive and countermotive.--The motive, beginning on the second half of the beat, is two beats in length. Like

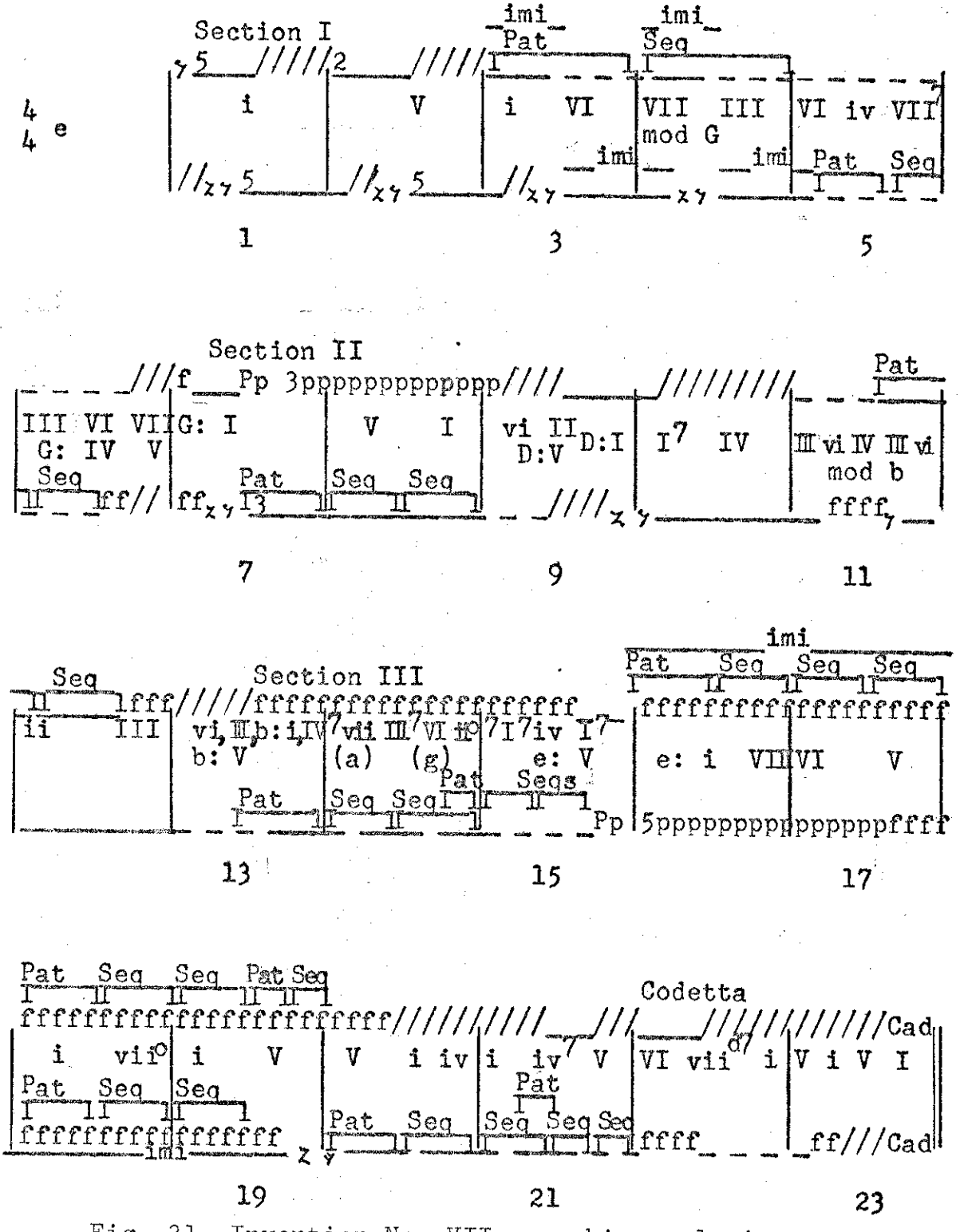

Fig. 31--Invention No. VII, graphic analysis

Inventions I and II the range is only a fifth and the movement largely is in sixteenth notes. The striking feature is the descent, mainly by step, from the dominant note to the tonic followed by the leap back to the dominant.

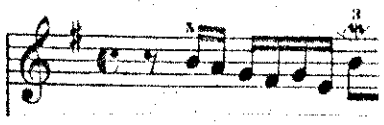

Fig. 32--Invention No. VII, motive, measure 1 (8, p. 3)

There are three changes in direction and two continuations, 60 per cent and 40 per cent, respectively. All of the intervals are a fifth or smaller.

There is no countermotive. In the first measure, the motive is accompanied by a pattern of an octave leap, which reappears several times.

Episodes.--There are thirty-eight beats of free material (20 per cent) in Invention No. VII. This is a relatively high figure. One-hundred-and-forty-six beats (80 per cent) are either derived from the motive or, if not, have gained thematic importance through development and use.

Satterfield classifies 30.4 per cent of the bars in the upper voice "smooth" and 69.6 "rough" in melodic accents. In the lower voice he finds 8.7 per cent "smooth" and 91.3 per cent "rough" (14, p. 174). A relatively large number of

ornaments are indicated: nine pralltrillers, one short trill, one extended trill, and seven mordents.

Textural Characteristics

Rhythmic relationships.--The following table shows the number of beats and the percentages of each type of Species of counterpoint in Invention No. VII.

TABLE XIII

FREQUENCY OF COUNTERPOINT SPECIES IN INVENTION NO. VII

Species	Number of Beats	Per Cent of Total Beats
First	28.5	31
Second	20.75	23
Third	0	0
Fourth	19.5	21
Other	23.25	25

Unlike most of the Inventions, Invention VII contains more First Species than Second Species counterpoint. The presence of the two pedal points and the several rests is reflected in the percentage for other rhythmic relationships.

Three other statistical analyses have been made. Within the First Species counterpoint of fifty-six progressions, twenty-five (45 per cent) were found to be in parallel and similar motion, twenty-four (43 per cent) in contrary motion,

and seven (12 per cent) in oblique motion. In his analysis of the total Invention Miller reports the following figures for the types of motion: parallel--five per cent, similar-- nine per cent, oblique--seventy per cent, and contrary--sixteen per cent. The index of melodic independence is 2.0 (12, p. 291). In his analysis of rhythmic relationships Satterfield classifies 6.4 per cent of the measures "rough" and 93.6 per cent "smooth" (14, p. 177).

Tonal relationships.--An analysis of the 159 intervals having relative metric accents revealed that thirty one (19.5 per cent) are dissonant. Thus, slightly over four-fifths of the intervals under study were found to be consonant (13, p. 278). There is an average of 2.5 implied tonalities per measure. There are four well defined modulations and two transient modulations. Thirty-four altered tones occur in this piece.

Rhythmic Characteristics

The meter signature is $\frac{4}{4}$. The following table shows the frequency of the note values appearing in the first voice of this Invention.

TABLE XIV

FREQUENCY OF NOTE VALUES IN INVENTION NO. VII

Upper Voice Note Values	Number of Beats	Per Cent of Total Beats
Quarter note (beat note)	0	0
Eighth note	21	23
Sixteenth note	42.5	46
Other note values and rests	28.5	31
Lower Voice **Note Values**	**Number of Beats**	**Per Cent of Total Beats**
Quarter Note	2	2
Eighth note	14.5	16
Sixteenth note	48	52
Other note values and rests	27.5	30

Most of the movement is in sixteenth notes. Contributing to the large percentage of "other note values and rests" is a pedal point and a large number of dotted eighth notes. The rhythmic figure of the dotted eighth note followed by a sixteenth note appears in the first measure and plays a prominent role thenceforth. Leonard describes this pattern as symbolic of "grievous passion" but claims that Invention No. VII expresses "heaviness of spirit" rather than "extreme sorrow" (9, p. 645).

Authorities disagree on the correct tempo (see Appendix G). Bodky assigns this piece a tempo of ♩=±60, explaining

that the complicated ornaments necessitate a rather slow tempo (1, p. 268). Keller suggests a tempo of ♩=72 (1, p. 348).

Formal Design

Invention No. VII in e minor is in three sections. The beginnings of both the second and third sections are marked by strong cadences. However, the cadence into the third section is not well-defined. The initial tonal centers for the sections are e minor (i), G Major (III) and b minor (v). Spitta recognizes in this Invention an "affinity" in form with No. I but sees no similarity in expression. "It is suppliant and mournful; still, in spite of its disturbed character, has extraordinary melodic beauty" (17, p. 62).

Invention No. VIII

Linear Characteristics

Motive and countermotive.--The motive is three beats in length and, like Inventions I, II, V, VI and VII begins off the beat. The notes of the motive outline the tonic chord. The range is an octave.

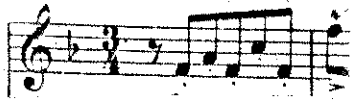

Fig. 33--Invention No. VIII, motive, measures 1-2 (8, p. 4)

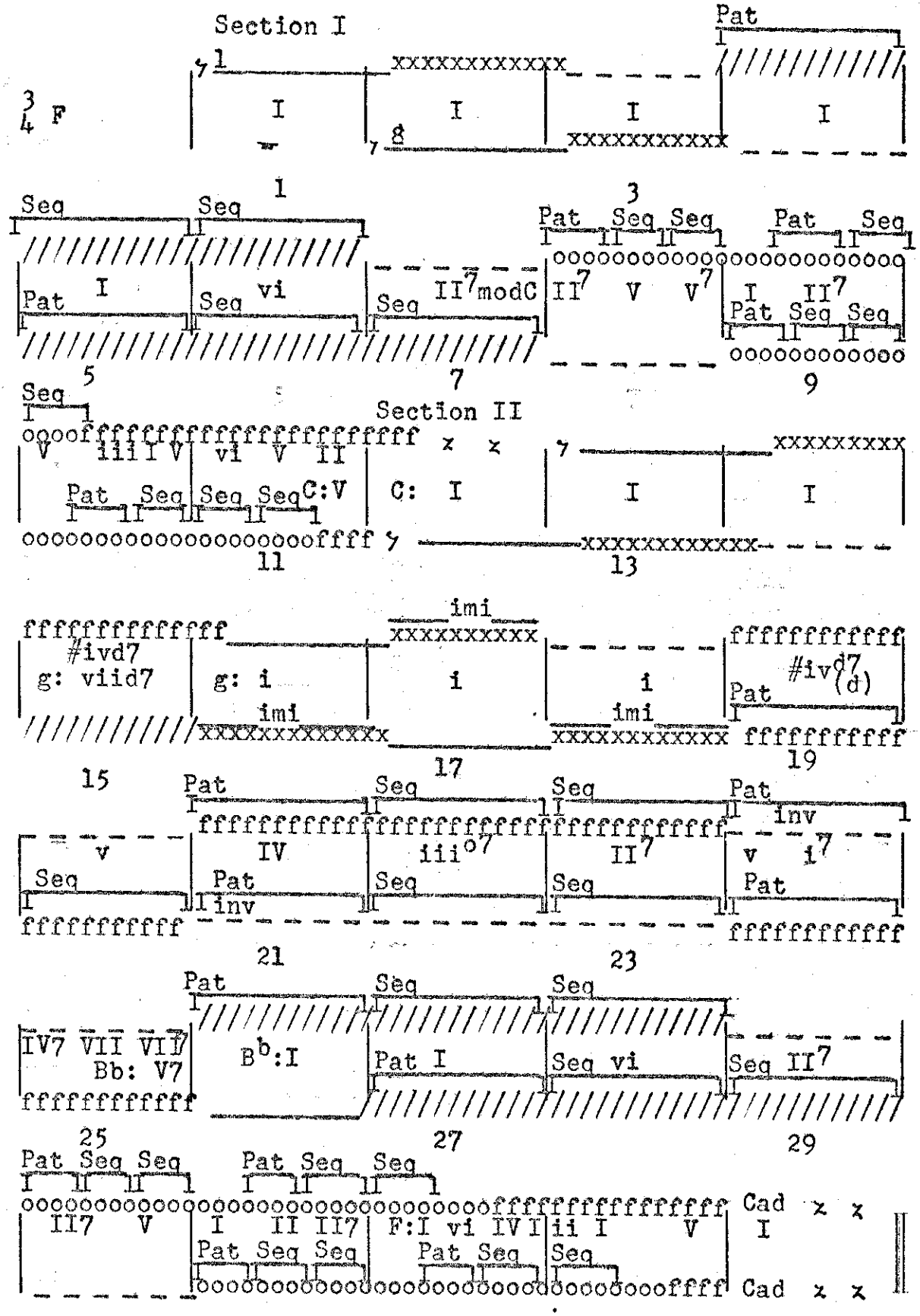

Fig. 34--Invention No. VIII, graphic analysis

All of the intervals change the direction of the line. These tonal and directional patterns are the distinguishing characteristics of the motive. Classed as a "tumult" idea (9, p. 645) the motive contains four intervals (80 per cent) a fifth or smaller and one (20 per cent) larger than a fifth.

Like the motive the countermotive is three beats long and has a range of an octave.

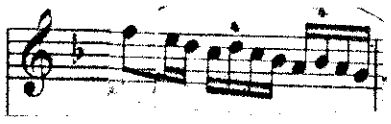

Fig. 35--Invention No. VIII, countermotive, measures 2-3 (8, p. 4)

Three of the intervals, or one-third, change the direction of the line while six, or two-thirds, continue in the same direction. All of the intervals are seconds. The countermotive basically is simply a descending scale line in sixteenth notes, thus offering a marked contrast to the ascending eighth note arpeggio of the motive.

Episodes.--There are thirty-one beats of free material, representing 15 per cent. The remaining 85 per cent consists of material either derived from the exposition, or thematically important. The following pattern first appears in measure four and reappears in measures five through seven and twenty-six through twenty-nine.

Fig. 36--Invention No. VIII, episodic pattern, measure 4

Another pattern with a quasi-pedal point is first seen in measure fifteen and is found in measures nineteen to twenty-three. In a somewhat altered form it also appears in measures twenty-four and twenty-three.

Fig. 37--Invention No. VIII, episodic pattern, measure 15

In the first voice 20.7 per cent of the bars are "smooth" and 79.3 per cent are "rough" in melodic accents. In the second voice 27.3 per cent of the measures are "smooth" and 72.7 "rough" (14, p. 174). There are no ornaments.

Textural Characteristics

Rhythmic relationships.--Invention No. VIII is characteristic in its large percentage of Second Species counterpoint. The following table shows the number of beats and the percentage of each type of species.

TABLE XV

FREQUENCY OF SPECIES OF COUNTERPOINT
IN INVENTION NO. VIII

Species	Number of Beats	Per Cent of Total Beats
First	23.5	23
Second	68	67
Third	0	0
Fourth	0	0
Other	10.5	10

Within the twenty-three beats of First Species counterpoint, seventy-five progressions (96 per cent) are in similar and parallel motion and three (4 per cent) are in contrary motion. None of the movement is oblique.

Two authors have made other textural studies. An analysis of all the progressions in the piece produced the following figures for the types of movement: parallel--22 per cent, similar--14 per cent, oblique--38 per cent, and contrary--26 per cent. The index of melodic independence is 1.7 (12, p. 291). From an analysis of the relationship of durational accents and metric accents 6.1 per cent of the measures were termed "rough" and 93.9 per cent "smooth" (14, p. 177).

Tonal relationships.--Satterfield finds 183 intervals having relative metric accents. Of these, thirty-two (17.5

per cent) are dissonant, and 151 intervals (82.5 per cent) consonant (13, p. 278).

There is an average of 1.5 identifiable chordal structures implied in each measure. Five significant modulations occur. There are twenty-five altered tones in this composition.

Rhythmic Characteristics

The meter is triple simple and the time signature is $\frac{3}{4}$. The following table shows the frequencies of the note values in Invention No. VIII.

TABLE XVI

FREQUENCY OF NOTE VALUES IN INVENTION NO. VIII

Upper Voice Note Values	Number of Beats	Per Cent of Total Beats
Quarter note (beat note)	1	1
Eighth note	43.5	43
Sixteenth note	51.5	50
Other note values and rests	4	6
Lower Voice **Note Values**	**Number of Beats**	**Per Cent of Total Beats**
Quarter note (beat note)	1	1
Eighth note	40	39
Sixteenth note	55	54
Other note values and rests	6	6

The great number of sixteenth notes produce a driving rhythm in this piece. There are very few beats which are not divided into sixteenth notes in one or both voices.

There is considerable disagreement among authorities concerning the correct tempo. Keller recommends a tempo of ♩=116-126 and Bodky suggests a tempo of ♩=±100 (1, p. 358).

Formal Design

Invention No. VIII in F Major is in two-sections. (11, p. 79) written in canonic imitation (4, p. 30). In the twelfth bar the second voice takes the lead and the canon becomes less strict (17, p. 62). The second section begins in the dominant key and contains modulations to g minor, d minor, and B^b Major before returning to the tonic key.

Probably the most popular of all the Inventions (4, p. 339), this piece is described as being "delightful" (6, p. 621), "full of happy and innocent contentment" (17, p. 62), as possessing a "vivacity replete with the very joy of live" (18, p. 782) and as displaying "some concertizing elements en miniature" (1, p. 233).

Invention No. IX

Linear Characteristics

Motives.--The motive is twelve beats in length and has a range of a twelfth.

Fig. 38--Invention No. IX, motive A, measures 1-4 (8, p. 4)

Ten of the intervals (32 per cent) change direction while the remaining twenty-one (68 per cent) continue in the same direction. Thirty of the intervals (94 per cent) are a fifth or smaller and two (6 per cent) larger than a fifth.

The motive is accompanied by a second motive which is also twelve beats long and also has a tonal range of a twelfth.

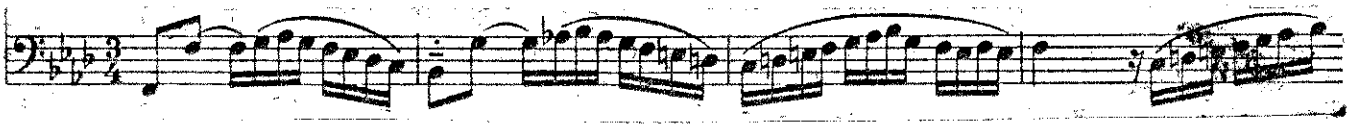

Fig. 39--Invention No. IX, motive B, measures 1-4 (8, p. 4)

Eight note progressions (27 per cent) change direction and, twenty-one note progressions (73 per cent) do not. Twenty-eight of the intervals (93 per cent) are a fifth or smaller and two (7 per cent) larger than a fifth.

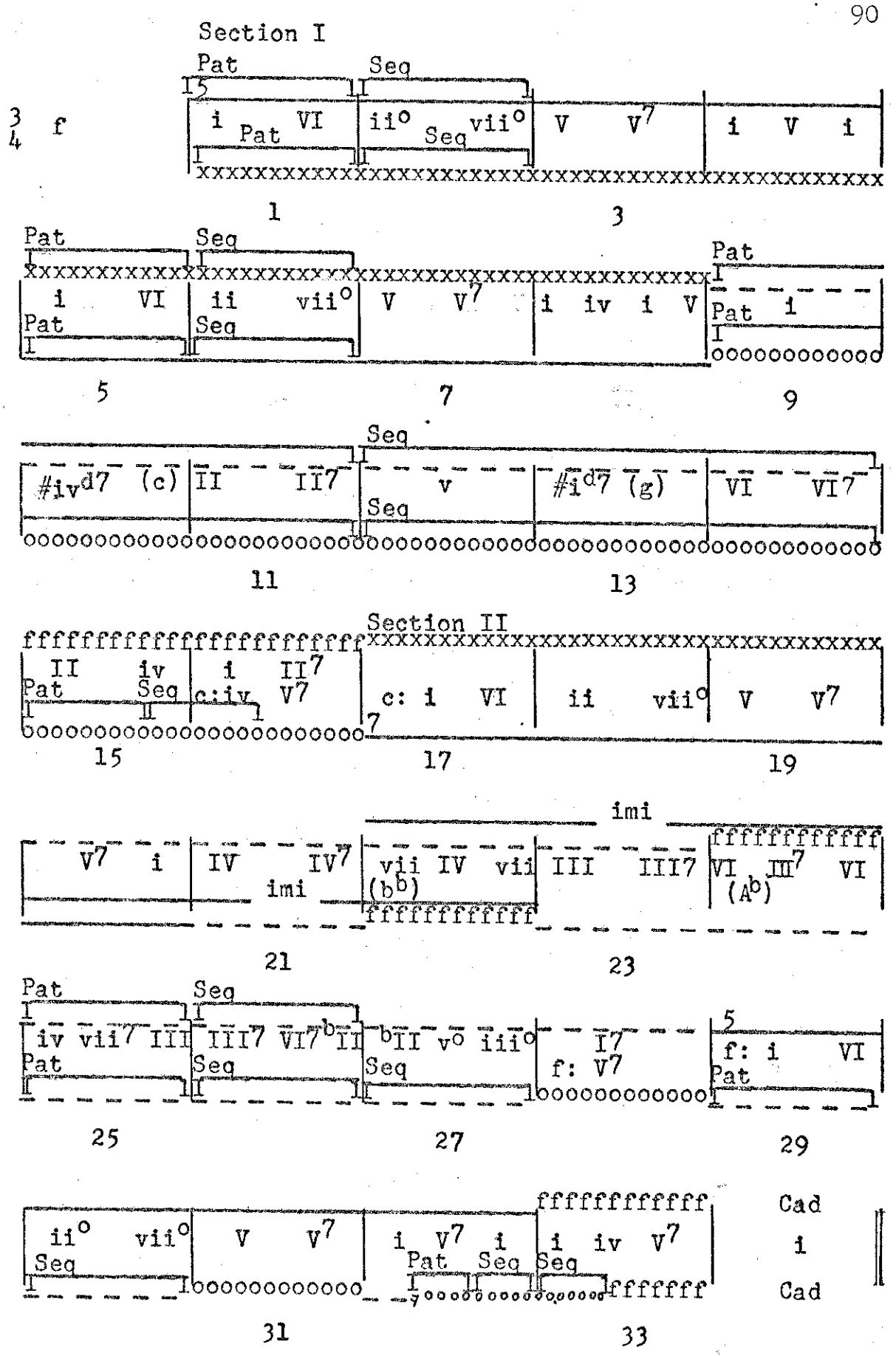

Fig. 40--Invention No. IX, graphic analysis

Episodes.--There are seventeen beats of free material (8 per cent) and 187 beats of material (92 per cent) derived from the exposition. In the upper voice 17.6 per cent of the bars are "smooth" and 82.4 per cent "rough" in melodic accents. In the lower voice 5.9 per cent are "smooth" and 94.1 per cent "rough" (14, p. 174). There are two ornaments: one short trill and one turn.

Textural Characteristics

Rhythmic relationships.--The following table shows the number of beats and the percentage of each type of Species of counterpoint in this Invention.

TABLE XVII

FREQUENCY OF COUNTERPOINT SPECIES IN INVENTION NO. IX

Species	Number of Beats	Per Cent of Total Beats
First	19	19
Second	22	22
Third	1	1
Fourth	49.75	50
Other	9.75	8

Half of the motion is in Fourth Species; the least amount is in Third Species. Of the seventy-two progressions in the First Species counterpoint fifty were found to be in parallel

and similar motion (sixty-nine per cent), twenty in contrary motion (twenty-eight per cent), and two in oblique motion (two per cent).

Two authors have conducted other studies of rhythmic relationships in the Inventions. An analysis of all progressions in the Invention produced the following figures on the types of motion: parallel--16 per cent, similar--4 per cent, oblique--59 per cent, and contrary--21 per cent. The index of melodic independency is 1.9 (12, p. 291). Based on a textural analysis of the durational accents 2.9 per cent of the measures are classified as "rough" and 97.1 per cent as "smooth" (14, p. 177).

Tonal relationships.--Of the 195 intervals having relative metric accents, thirty (15.4 per cent) were found to be dissonant. The remaining 165 intervals (84.6 per cent) are consonant (13, p. 278). There is an average of 2.2 identifiable chordal structures implied in each measure. There are three major modulations and four transient modulations. Ninety-one altered tones occur.

Rhythmic Characteristics

The meter is triple simple. The following table shows the frequency of note values appearing in this Invention.

TABLE XVIII

FREQUENCY OF NOTE VALUES IN INVENTION NO. IX

Upper Voice Note Values	Number of Beats	Per Cent of Total Beats
Quarter note (beat note)	0	0
Eighth note	22.25	21
Sixteenth note	45	42
Other note values and rests	40.75	37
Lower Voice **Note Values**	**Number of Beats**	**Per Cent of Total Beats**
Quarter note (beat note)	2	1
Eighth note	17	13
Sixteenth note	61.75	45
Other note values and rests	55.25	41

The sixteenth note is the most common note value. A rhythmic figure involving a tied note appears two times in the motive and two times in the countermotive.

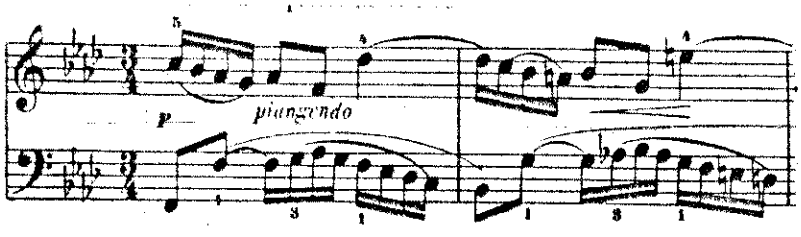

Fig. 41--Invention No. IX, rhythmic pattern (measures 1-2)

This tied note pattern appears frequently throughout the Invention and accounts for the large percentage for "other

note values and rests."

Keller suggests a tempo of ♩=46 (1, p. 361). Bodky assigns to this Invention a tempo of ♩=±60 and adds: "This relatively slow speed is necessary to bring out the richness of Bach's original articulation" (1, p. 268).

Formal Design

A two-section Invention, it has been likened in form to Invention No. V (17, p. 62). Geiringer calls this Invention a "kind of double fugue in which theme and counter melody usually appear together. . . ." (2, p. 275). The first section passes through two transient modulations before the cadence into c minor, the beginning tonality of section two. At this point a counterexposition appears. Section two contains two transient modulations and ends in the tonic key of f minor. The third section states the two motives in accompaniment before beginning the final cadence. This Invention is one of two Inventions provided with slurs by Bach (1, pp. 202-203).

Invention No. IX is described as "mild and composed" (1, p. 228), as "tender" (6, p. 621), as being "full of impassioned strains of sadness, which rise to a great intensity of effect in bars 21 to 26" (17, p. 62). Bodky comments: "It is this [pathos and declamation] which must have caused Bach to distinguish the Invention in f minor with so many slurs for articulation" (1, p. 131).

Invention No. X

Linear Characteristics

Motive and countermotive.--The length of the motive is twelve beats (four compound beats). Basically, the motive is an arpeggiated chordal outline moving primarily in eighth notes. "Arpeggio forms," says Leonard, "especially rising ones are of cheerful mood, intensified always if the intervals become wider" (9, p. 645).

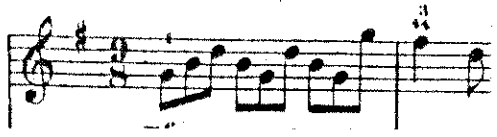

Fig. 42--Invention No. X, motive, measures 1-2 (8, p. 4)

There are five changes and four continuances in the direction of the motivic line, 56 per cent and 44 per cent, respectively. Nine of the intervals (90 per cent) are a fifth or smaller, one (10 per cent) is larger than a fifth.

The countermotive is nine beats (three compound) in length and has a range of a seventh. Like the motive, it is basically a chordal outline--in this case the outlines of two chords, the dominant and the tonic. The movement is in eighth notes.

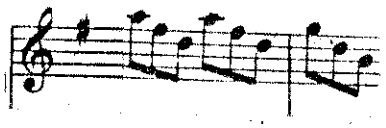

Fig. 43--Invention No. X, countermotive, measures 2-3

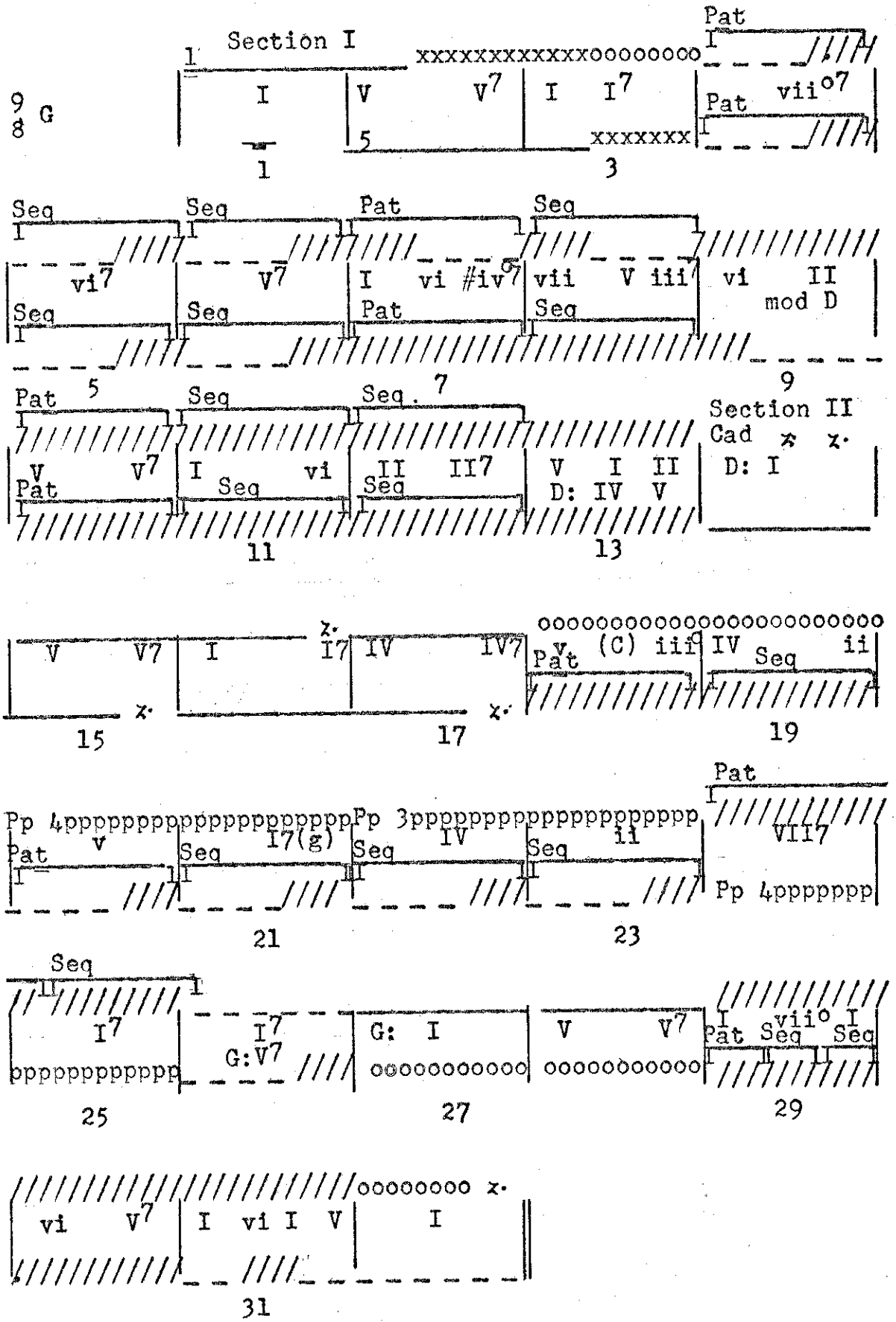

Fig. 44--Invention No. X, graphic analysis

Four of the intervals (59 per cent) change direction while three (41 per cent) continue in the same direction. All of the eight intervals are a fifth or smaller.

Episodes.--All of the material in the Invention is either derived from the motive or countermotive or is thematically important. The following three note patterns appear very frequently in the piece.

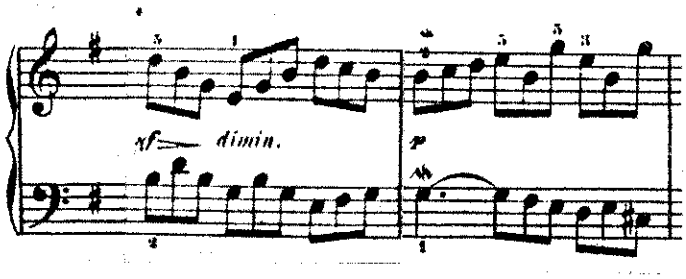

Fig. 45--Episodic patterns, (measures 5 and 7, respectively)

These patterns appear to be equally as important as those present in the countermotive and motive.

In his study of the relative congruence of melodic and metric accents within each bar Satterfield classifies 37.5 per cent of the bars in the upper voice "smooth" and 62.5 per cent of the bars "rough." In the lower voice 29.0 per cent of the bars are "smooth" and 71.0 per cent "rough" (14, p. 174).

There are numerous ornaments. Invention No. X contains five pralltrillers, three extended trills, and three mordents.

Textural Characteristics

Rhythmic relationships.--The following table shows the number of beats and the percentage of each type of counterpoint Species in this Invention.

TABLE XIX

FREQUENCY OF COUNTERPOINT SPECIES IN INVENTION NO. X

Species	Number of Beats	Per Cent of Total Beats
First	127	44
Second	15	6
Third	0	0
Fourth	64	22
Other	82	28

A relatively high percentage of beats are in First Species Counterpoint, whereas none of the material is in Third Species and a very small percentage in Second Species. Of the 127 progressions within the First Species Counterpoint, fifty-five, (forty-three per cent) are in similar and parallel motion; seventy-two, (fifty-seven per cent) are in contrary motion; and none are in oblique motion.

Two authors offer other approaches. In his analysis of the whole Invention Miller found seventeen per cent of the progressions in parallel motion, eight per cent in similar motion, forty-three per cent in oblique motion, and thirty-two per cent in contrary motion. The index of melodic

independence is 1.9 (12, p. 291). Based on his study of the relationships of durational and metric accents, Satterfield classifies 3.1 measures "rough" and 96.9 measures "smooth" (14, p. 177).

Tonal relationships.--There are 87 intervals having relative metric accents in this Invention. Of these, eighteen, (20.7 per cent) are dissonant and sixty-nine, (79.3 per cent) consonant (13, p. 278). There is an average of 1.8 identifiable chordal structures in each measure. Two major modulations--to the dominant and back to the tonic--and one transient modulation occur in this piece. There are eleven altered tones in this piece.

Rhythmic Characteristics

The meter is triple compound; the time signature is $\frac{9}{8}$. The following table shows the frequency of note values appearing in the Invention.

TABLE XX

FREQUENCY OF NOTE VALUES IN INVENTION NO. X

Note Values	Upper Voice Number of Beats	Per Cent of Total Beats
Eighth note (beat note)	84	75
Sixteenth note	0	0
Thirty-second note	0	0
Other note values and rests	73	25

(Continued on next page)

TABLE XX--Continued

Note Values	Lower Voice Number of Beats	Per Cent of Total Beats
Eighth note (beat note)	188	65
Sixteenth note	0	0
Thirty-second note	0	0
Other note values and rests	100	35

The great majority of the movement is in the basic eighth note pulse.

Authorities disagree concerning the correct tempo. Bodky assigns to this Invention a tempo of ♩.=±100 whereas Keller suggests ♩.=108 (1, p. 370).

Formal Design

Described as a "fugue whose main subject consists of broken chords, providing the Invention with a basically harmonic character" (2, p. 275), Invention No. X is a two-section composition containing a counterexposition. There is a brief recapitulation section starting in measure twenty-seven and a deceptive cadence in measure thirty before the final cadence in measure thirty-two.

Bodky states: "The Invention in G Major (X) is the only example of allegro assai in $\frac{9}{8}$ time, among Bach's keyboard

works and is an early brother to the gigantic Gigue that Bach wrote in $\frac{9}{16}$ time for the Fourth Partita" (1, p. 138). Another author Kinscella says "Number X is like nothing so much as an old style Gigue" (6, p. 621).

Spitta describes this piece in the following statement:

> No. 10, in G major, begins like a fugue, but without any of the fetters of that form; it flits to and fro--now in imitation, and now in episodic extension. There is a piece of quite wanton fun at the reprise, when the upper part takes upon itself the double duty of a theme and response (dux and comes) (17, p. 62).

In his discussion of symbolism in Bach's music Leonard states:

> The gigue form, whether it has the rhythm of dotted notes or of even triplets, designates, according to its tempo, as moderato or headlong, the idea of placid or ecstatic happiness, or the gaiety of "animal spirits." It occurs . . . in Two Part Invention, No. 10 (9, p. 645).

Invention No. XI

Linear Characteristics

Motives A and B.--Invention No. XI is based on two motives (11, p. 82). The motive appearing in the upper voice, Motive A, is eight beats long and has a range of a ninth. It moves entirely in sixteenth notes and contains two sequences. The leaps of a seventh and the sequences are the distinguishing characteristics of Motive A.

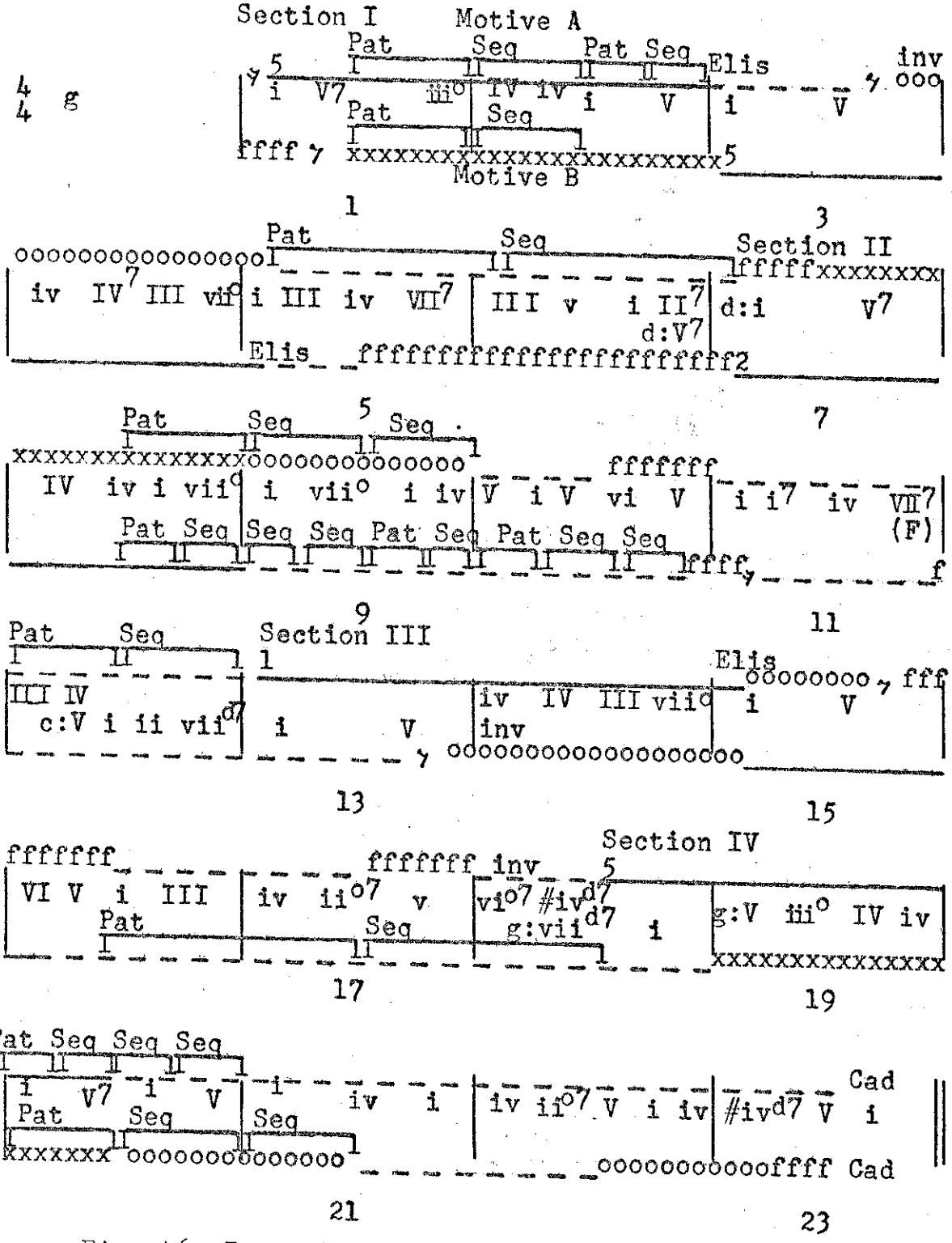

Fig. 46--Invention No. XI, graphic analysis

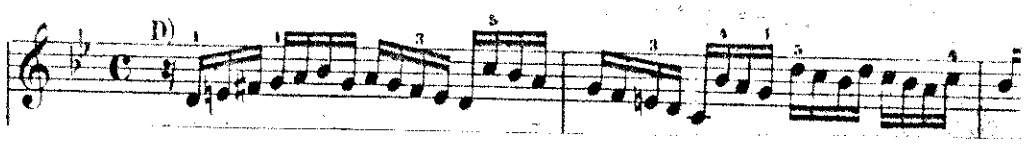

Fig. 47--Invention No. XI, motive A, measures 1-3 (8, p. 4)

There are twelve changes in direction (40 per cent) and eighteen continuances (60 per cent) among the intervals. Twenty-nine intervals are a fifth or smaller and two are larger than a fifth (93 per cent and 7 per cent respectively).

The second motive is six-and-three-fourths beats long and has a range of a fourth. The distinguishing features are the sequence appearing near the beginning and the syncopated rhythmic pattern near the end.

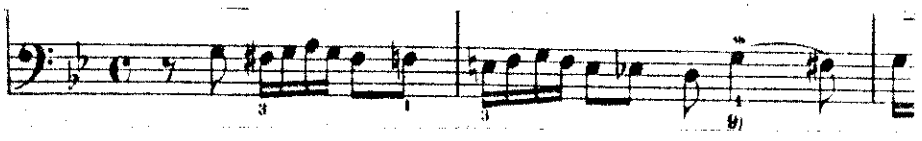

Fig. 48--Invention No. XI, motive B, measures 1-3

There are seven directional changes (47 per cent) and eight continuances (53 per cent) among the intervals. All of the sixteen intervals are a fifth or smaller.

Episodes.--There are seventeen-and-one-half beats of free (9 per cent) material in Invention No. XI. The remaining 167 beats (88 per cent) are material derived from the motives.

In the upper voice 13.0 per cent of the bars are "smooth" and 87.0 per cent "rough." In the lower voice 0.0 per cent are "smooth" and 100 per cent "rough" (14, p. 174). There is one ornament, a pralltriller.

Textural Characteristics

Rhythmic relationships.--The following table shows the number of beats and the percentage of each type of Species of counterpoint in this Invention.

TABLE XXI

FREQUENCY OF COUNTERPOINT SPECIES IN INVENTION NO. XI

Species	Number of Beats	Per Cent of Total Beats
First	37.25	41
Second	33.50	36
Third	0	0
Fourth	14.50	16
Other	6.75	7

Atypically, most of the counterpoint is in First Species. None is in Third Species. Within the First Species counterpoint sixty-seven progressions (56 per cent) are in similar and parallel movement, fifty-two progressions (43 per cent) are in contrary movement, and one progression (1 per cent) is in oblique motion.

Two authors have made other studies of rhythmic relationships in the Inventions. An analysis of all progressions in the Invention produced the following figures for the types of motion: parallel--21 per cent, similar--13 per cent, oblique--41 per cent, and contrary--25 per cent. The index of melodic independence is 1.7 (12, p. 291). Based on a textural analysis 4.3 of the measures are "rough" and 95.7 per cent are "smooth" (14, p. 177).

Tonal relationships.--Of the 175 intervals having relative metric accents, thirty-three (18.9 per cent) are dissonant and 142 (81.1 per cent) are consonant (13, p. 278). There are an average of 3.6 identifiable chordal structures in each measure. There are three major modulations--from tonic to minor dominant, to minor subdominant, back to tonic--and two transient modulations. Sixty-one altered tones occur in this Invention.

Rhythmic Characteristics

The meter is quadruple simple. The following table shows the frequency of note values appearing in this Invention.

TABLE XXII

FREQUENCY OF NOTE VALUES IN INVENTION NO. XI

Upper Voice Note Values	Number of Beats	Per Cent of Total Beats
Quarter note (beat note)	3	3
Eighth note	12.5	14
Sixteenth note	62.25	68
Other note values and rests	14.25	15
Lower Voice Note Values	**Number of Beats**	**Per Cent of Total Beats**
Quarter note (beat note)	7	8
Eighth note	22.5	24
Sixteenth note	56	61
Other note values and rests	6.5	7

The two rhythmic figures appearing in Motive B are used frequently in the piece. The continuous motion in sixteenth notes in Motive A is evident throughout the piece and is reflected in the high percentage of sixteenth notes seen in Table XX.

Authorities disagree on the correct tempo. Bodky recommends a tempo of ♩ =±80; Keller suggests a tempo of ♩=58 (1, p. 345).

Formal Design

This Invention may be divided into four sections with the following tonal scheme; g minor, d minor, c minor, and g minor.

Geiringer describes it as "a kind of double fugue, in which theme and countermelody usually appear together" (2, p. 275).

Spitta makes the following comments about Invention No. XI:

> The piece which follows [Invention XI] has a character of tormented restlessness. A chromatic counter-subject of two bars long, attached from the outset to the chief theme, evolves, by means of inversion in the fourth bar, an episode of the most painful and insistent kind, which reappears in alternation with the original counterpoint at bar 14. The phrases consist of six bars, but at the last two recurrences it is only five bars and a half long, and these two are merged directly in one another without any cadences to give a moment's rest (see bars 12, 13, and 18); each beat of the bar is more restless than the last (17, p. 62).

Invention No. XII

Linear Characteristics

Motives A and B.--Like Invention No. XI, this one is based on two motives. Motive A, that motive appearing in the upper voice, is eighteen-and-one-half beats long (or six-and-one-eighth compound beats) and has a range of an octave.

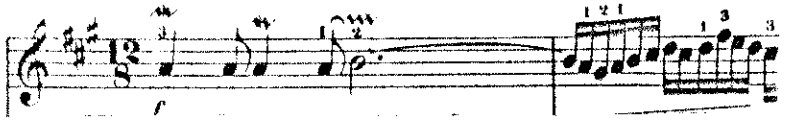

Fig. 49--Invention No. XII, motive A, measures 1-2 (8, p. 5)

Five of the intervals (37 per cent) change direction, eight (50 per cent) continue in the same direction, and three

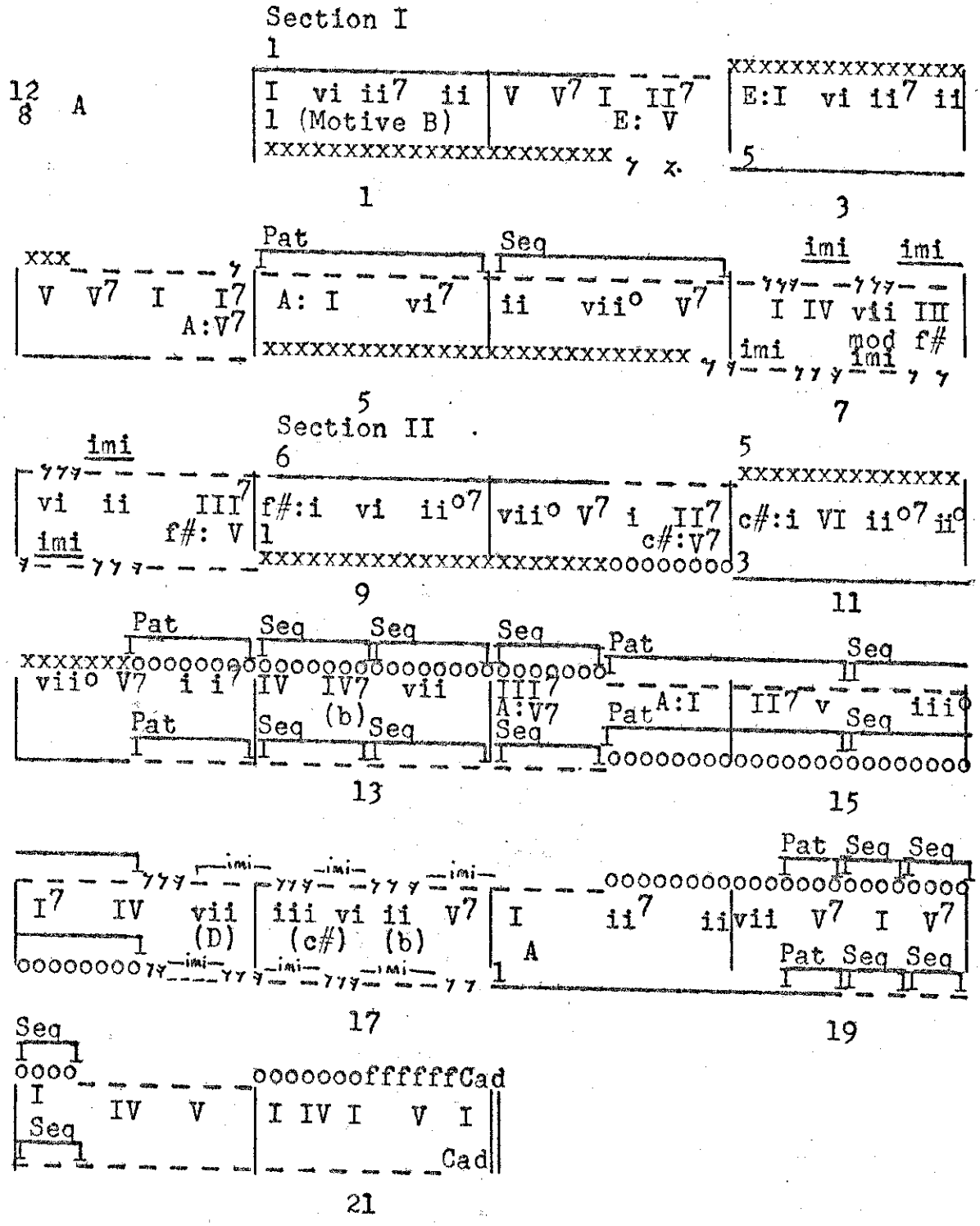

Fig. 50--Invention No. XII, graphic analysis

(13 per cent) involve note repetition. All of the sixteen intervals are smaller than a fifth. The outstanding characteristics of the motive are the repeated notes, the embellishments, and the two sequence figures.

Motive B is twenty beats (or six-and-two-thirds compound beats) in length and has a range of a thirteenth, an unusually large range. Fourteen note progressions (52 per cent) change direction while fifteen note progressions (48 per cent) continue in the same direction.

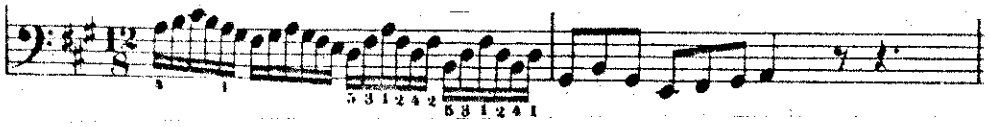

Fig. 51--Invention No. XII, motive B, measures 1-2 (8, p. 5)

All of the thirty intervals are smaller than a fifth.

The distinguishing feature is the winding nature of the notes in the first measure, which is quite similar to the second measure of motive A. The second measure of motive B is basically a harmonic outline of the dominant chord ending on the tonic note.

Episodes.--Like Invention No. X all the material in the Invention is derived from the two motives. The following sequential passage appears first in measure seven and reappears later in measure sixteen.

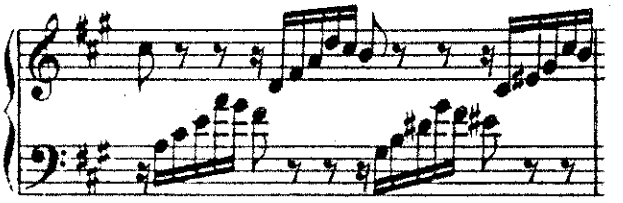

Fig. 52--Invention No. XII, episodic pattern, measure 7

This pattern could be considered a derivation of either of the motives.

In his study of the relative congruence of melodic and metric accents within each bar Satterfield classifies 9.5 per cent of the bars in the upper voice "smooth" and 90.5 per cent "rough." In the lower voice 14.3 per cent of the bars were found to be "smooth" and 85.7 per cent "rough" (14, p. 174).

Invention No. XII contains a relatively high number of ornaments. There are six extended trills and seventeen mordents. (These figures include several ornaments printed in small type. They appear originally in the motive in ordinary type, however. According to the performance practice of the day the embellishments appearing in the motive as it is first presented must be executed each time the motive occurs.

Textural Characteristics

Rhythmic relationships.--The following table shows the

number of beats and the percentage of each type of counterpoint species in this Invention.

TABLE XXIII

FREQUENCY OF COUNTERPOINT SPECIES IN INVENTION NO. XII

Species	Number of Beats	Per Cent of Total Beats
First	18.5	7
Second	105	42
Third	41	16
Fourth	2	1
Other	39	34

A high percentage of the counterpoint is in Second Species, and as the table indicates, much of the texture involves rests and pedal points (inherent in motive A).

Of the seventy-seven progressions within the First Species counterpoint, forty-one, representing fifty-three per cent, are in similar and parallel motion and thirty-six, representing forty-seven per cent, are in contrary motion. None are in oblique motion.

Two authors offer other approaches. In his analysis of the whole Invention Miller found six per cent of the progressions in parallel motion, eleven per cent in similar motion, sixty-one per cent in oblique motion, and twenty-two per cent in contrary motion. The index of melodic independence is

2.0 (12, p. 291). Based on his study of the relationships of durational and metric accents, Satterfield classifies 4.7 per cent of the bars "rough" and 95.3 per cent "smooth" (14, p. 177).

Tonal relationships.--There are 206 intervals having relative metric accents in Invention No. XII. Of these, thirty-six (17.4 per cent) are dissonant and the remaining 170 (82.6 per cent) are consonant (13, p. 278). There are an average of 3.5 identifiable chordal structures in each measure. Five major modulations and four transient modulations occur in this piece. Thirty-three altered tones occur.

Rhythmic Characteristics

The meter is quadruple compound, a meter unique in the Two Part Inventions. The following table shows the frequency of note values appearing in the Invention.

TABLE XXIV

FREQUENCY OF NOTE VALUES IN INVENTION NO. XII

Note Values	Upper Voice Number of Beats	Per Cent of Total Beats
Eighth note (beat note)	50	20
Sixteenth note	150.5	60
Thirty-second note	0	0
Other note values and rests	51.5	20

(Continued on next page)

TABLE XXIV--Continued

Note Value	Lower Voice Number of Beats	Per Cent of Total Beats
Eighth note (beat note)	49	20
Sixteenth note	130	51
Thirty-second note	0	0
Other	73	29

Most of the motion is in sixteenth notes. The rhythmic figures appearing in the two motives appear throughout the Invention.

There is considerable disagreement among authorities concerning the correct tempo. Bodky assigns to this Invention a tempo of ♩.=±60 (1, p. 268). Keller suggests a tempo of ♩.=72 (1, p. 370).

Formal Design

As he does Invention No. V, IX, and XI, Geiringer likens this to a "kind of double fugue" (2, p. 275). It can be considered in two sections with the beginning tonalities of A Major and f# minor (returning to A Major in a codetta). The second section contains a counterexposition in which the voices have not been transplanted, as is usually the case. The coda, however, does begin with the inverted counterpoint--motive A appears in the lower voice and motive B in the upper voice. After stating the motives and developing them briefly the section closes.

Two other authors comment on Invention No. XII. Says Spitta, "A feeling of honest German fun is given by No. 12, in A Major, which corresponds in form to Nos. 5 and 9" (17, p. 62). Leonard similarly states that the Invention is based on motives of "typical joy" (9, p. 645).

Invention No. XIII

Linear Characteristics

Motives and countermotives A and B.--Invention No. XIII again is based on two motives, each having its own exposition (11, p. 82). Kennan concurs with Mason in recognizing two "coordinate" motives (5, p. 68). Motive A, that motive appearing first, is two-and-one-fourth beats long and has a range of a seventh. Three intervals (50 per cent) change direction and three (50 per cent) continue in the same direction. All of the seven intervals are smaller than a fifth. Except for the last note, all the movement is in sixteenth notes.

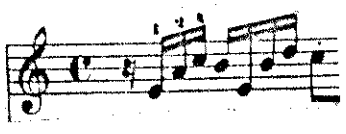

Fig. 53--Invention No. XIII, motive A, measure 1

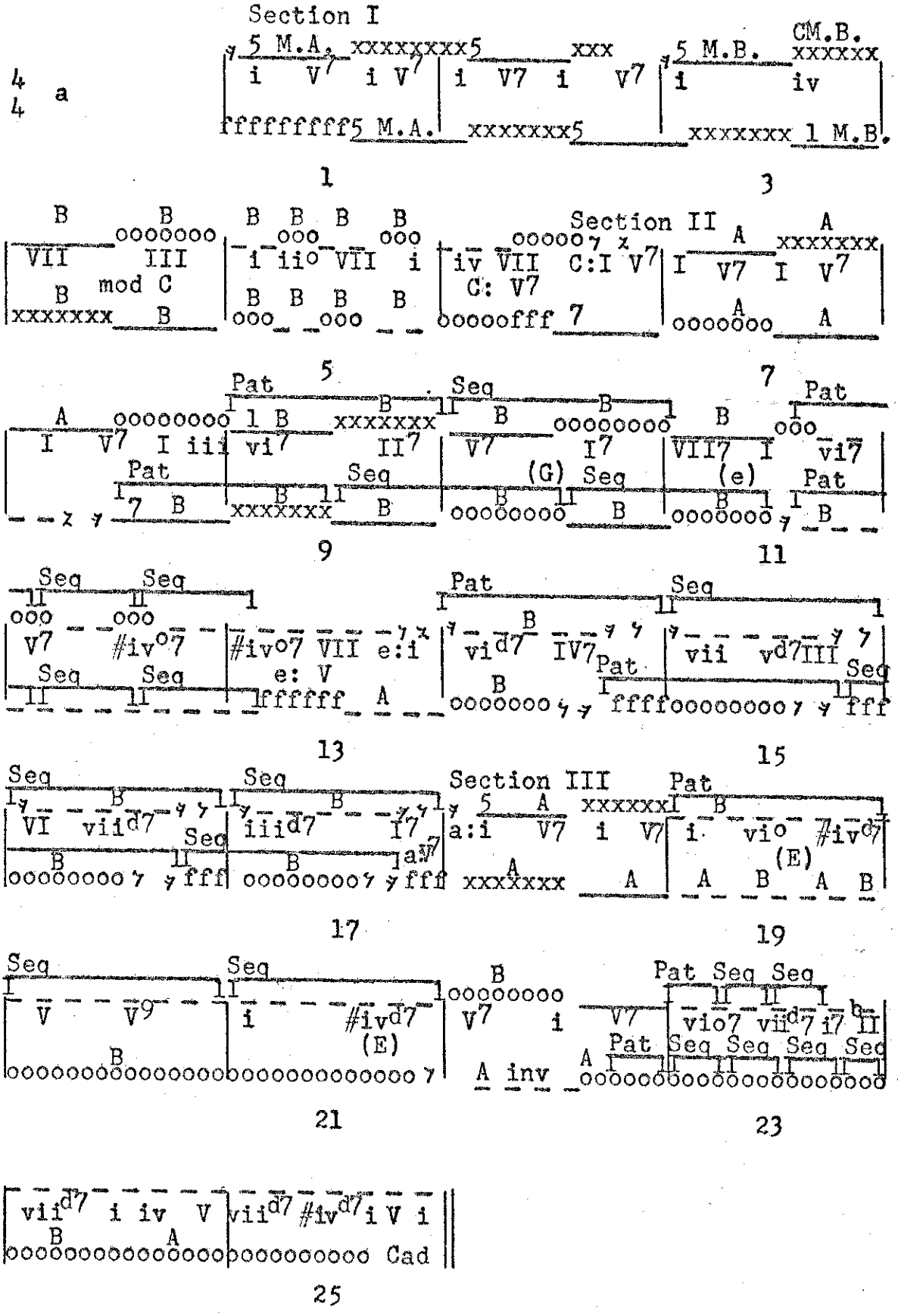

Fig. 54--Invention No. XIII, graphic analysis

The distinguishing feature is the "somersaulting" directions of the motive. Leonard classes this motive as a "typical joy" idea. (This type motive is characterized by movement in sixteenth notes and constant return to one or two notes) (9, p. 645).

The countermotive accompanying motive A is one-and-three-fourths beats long. The range is a sixth.

Fig. 55--Invention No. XIII, countermotive A, measure 1

All of the intervals change direction. One of the three intervals is a fifth or smaller and the other two are larger than a fifth.

Motive B, first appearing in measure three in the upper voice, is two-and-three-fourths beats long and has a range of an octave. The distinguishing characteristic is the number of changes in the direction of the intervals: all of the motivic intervals change direction. One interval (29 per cent) is larger than a fifth while the remaining six intervals (84 per cent) are smaller than a fourth.

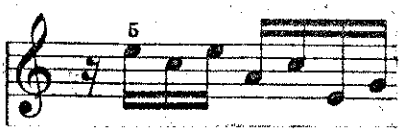

Fig. 56--Invention No. XIII, motive B, measure 3

Like countermotive A countermotive B is one-and-three-fourths beats long and has a range of an octave. All of the intervals continue in the same direction.

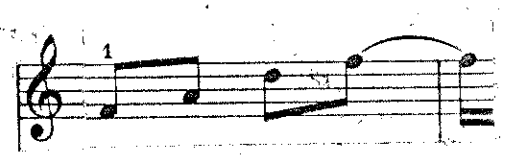

Fig. 57--Invention No. XIII, countermotive B, measure 3

One interval is larger than a third and the other three a third or smaller.

Episodes.--Three per cent (six beats) of the material in the Invention is free. The remaining 97 per cent (194 beats) is derived from the motives or countermotives. There are no ornaments.

In his study of the relative congruence of melodic and metric accents within each bar Satterfield classifies 20.0 per cent of the bars in the upper voice "smooth" and 80.0 per cent of the bars "rough." In the lower voice he finds 8.0 per cent "smooth" and 92.0 per cent "rough" (14, p. 174).

Textural Characteristics

Rhythmic relationships.--The following table shows the number of beats and the percentage of each type of counterpoint Species in this Invention.

TABLE XXV

FREQUENCY OF COUNTERPOINT SPECIES IN INVENTION NO. XIII

Species	Number of Beats	Per Cent of Total Beats
First	6.5	7
Second	66	66
Third	1	1
Fourth	6	6
Other	20.5	20

The greatest amount of counterpoint is Second Species and the least is in Third Species.

An analysis of the twenty-five progressions in First Species counterpoint produced the following percentages for the types of motion: similar and parallel--sixty per cent, and contrary--forty per cent. None of the progressions were in oblique motion.

Two authors have made studies of rhythmic relationships in the Inventions. An analysis of all progressions in the Invention produced the following figures for the types of motion: parallel--six per cent; similar--thirteen per cent; oblique--forty-nine per cent, and contrary--thirty-two per cent. The index of melodic independence is 2.1 (12, p. 291). Based on a textural analysis four per cent of the bars are "rough" and ninety-six per cent "smooth" in durational accents (14, p. 177).

Tonal relationships.--Of the 164 intervals having relative metric accents, thirty-six, or twenty-two per cent, are dissonant. The remaining seventy-eight per cent are consonant (13, p. 278). An average of three identifiable chordal structures are implied in each measure. There are three major modulations--from minor tonic to relative major, to minor dominant, back to tonic--and seven transient modulations. Fifty-four altered tones occur.

Rhythmic Characteristics

The meter is quadruple simple. The following table shows the frequency of note values appearing in this Invention.

TABLE XXVI

FREQUENCY OF NOTE VALUES IN INVENTION NO. XIII

Upper Voice Note Values	Number of Beats	Per Cent of Total Beats
Quarter note (beat note)	1	1
Eighth note	22.5	23
Sixteenth note	62	62
Other note values and rests	14.5	14
Lower Voice **Note Values**	**Number of Beats**	**Per Cent of Total Beats**
Quarter note	1	1
Eighth note	50	50
Sixteenth	38.25	38
Other note values and rests	10.75	11

Most of the movement in the upper voice appears in sixteenth notes, whereas most of the movement in the lower voice is in eighth notes. Thus, the sixteenth note movement of the motives and the eighth note movement of the countermotives appear to be nearly equally reflected in the Invention.

Authorities differ in their opinions of the correct tempo (see appendix). Keller recommends a tempo of ♩=69 and Bodky suggests a tempo of ♩=±80 (1, p. 345).

Formal Design

Invention XIII is in three-sections. The exposition based on motive A contains four entrances of the motive; the exposition based on motive B contains two entrances of the motive. Section two presents two counterexpositions whereas section three states only motive A. The key scheme for the sections is a minor, C Major and a minor. Section two contains a modulation to e minor, perhaps suggesting a subsection.

Two authors offer comments concerning this Invention. Kinscella says ". . . Number XIII is another example of Bach's frequent use of the simple triad as subject matter" (6, p. 621). Spitta attributes to this piece the character of a prelude, "because the subjects and the workings-out alike move almost exclusively in harmonic passages" (17, p. 62).

Invention No. XIV

Linear Characteristics

Motive and countermotive.--The motive is twelve-and-one-fourth beats long and consists of sixty notes. The tonal range is two octaves and one step, an unusually wide range. The motive consists of one idea, or figure, and the following two sequences of that idea. The distinguishing features of this motivic idea are the strong rhythmic patterns involving thirty-second notes, the movement by arpeggio up the chord and then down (beats three and four are a melodic inversion of beats one and two) (4, p. 340), and the two sequences of this motive figure.

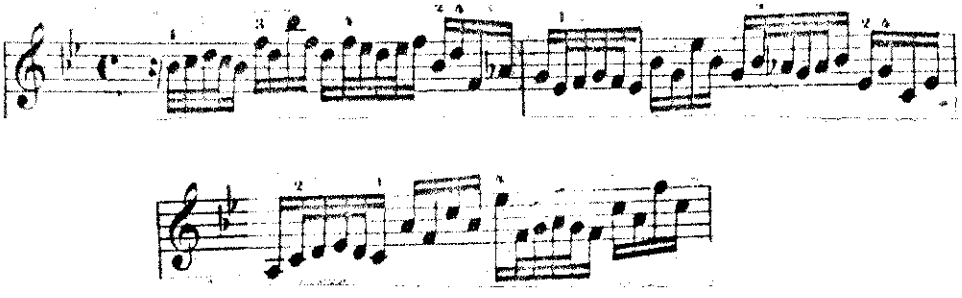

Fig. 58--Invention No. XIV, motive, measures 1-3

There are forty-one changes in the direction of the intervals (71 per cent) and seventeen continuances (29 per cent). Fifty-three intervals (90 per cent) are a fifth or

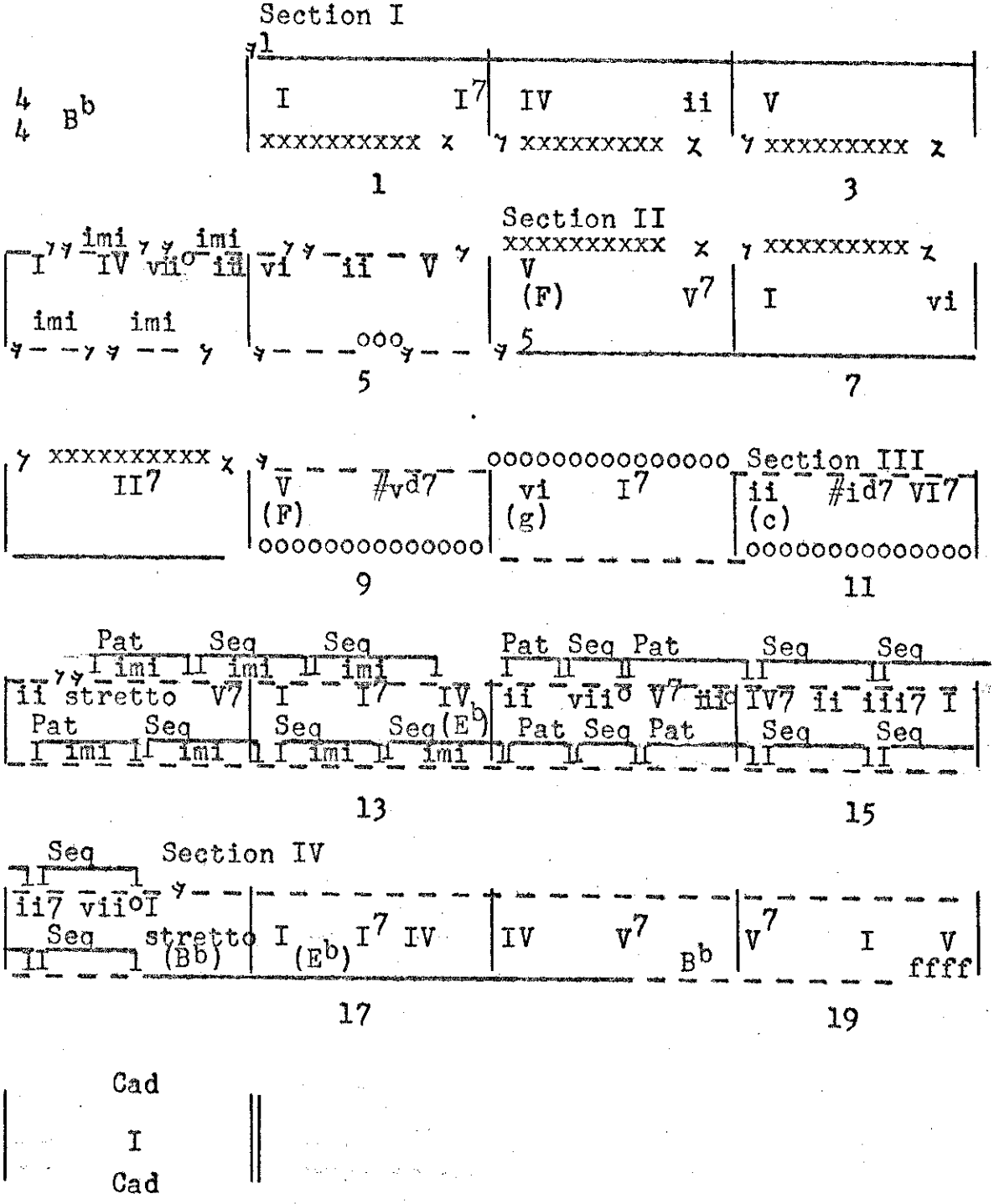

Fig. 59--Invention No. XIV, graphic analysis

smaller and six intervals (10 per cent) are larger than a fifth. This motive is symbolic of "typical joy" (9, p. 645).

The countermotive is a simple arpeggio figure moving primarily in eighth notes. This figure appears in a sequence as does that of the motive. These figures are separated by rests.

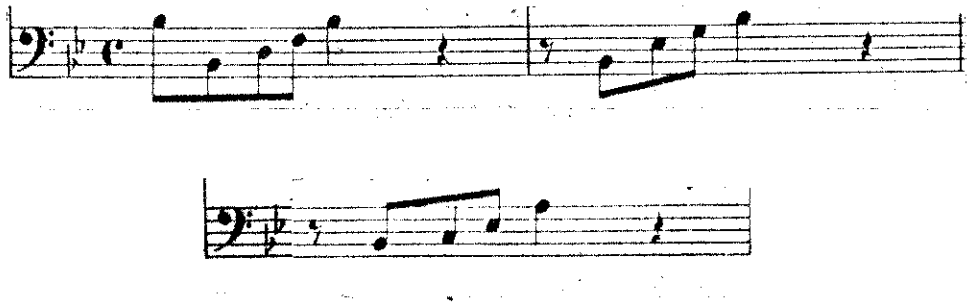

Fig. 60--Invention No. XIV, countermotive, measures 1-3

Eleven beats in length, the countermotive contains only one change (11 per cent) in the direction of the intervals (notes separated by rests were not considered in this analysis) and eight continuations (89 per cent). Nine intervals (90 per cent) are a fifth or smaller and one interval (10 per cent) is larger than a fifth. The countermotive (with an octave range) functions primarily as an accompaniment figure.

Episodes.--All of the material is derived from the motive and countermotive. There are no ornaments.

In his study of the relative congruence of melodic and metric accents within each bar Satterfield classifies forty per cent of the bars of the upper voice "smooth" and sixty per cent "rough." In the lower voice twenty-five per cent of the bars were found to be "smooth" and seventy-five per cent "rough" (14, p. 174).

Textural Characteristics

Rhythmic relationships.--The following table shows the number of beats and the percentage of each type of counterpoint Species in this piece.

TABLE XXVII

FREQUENCY OF COUNTERPOINT SPECIES IN INVENTION NO. XIV

Species	Number of Beats	Per Cent of Total Beats
First	16.75	21
Second	17.5	22
Third	0	0
Fourth	6	7
Other	39.75	50

Most of the counterpoint in this Invention could not be classified as First, Second, Third, or Fourth Species. Within the First Species counterpoint there are eighty-seven progressions. Of these, sixty-eight progressions (78 per cent) were

found to be in similar and parallel motion, and nineteen (22 per cent) in contrary motion. None of the progressions were found to be in oblique motion.

Two other authors offer statistical approaches to this Invention. In his analysis of the whole Invention Miller found twenty-one per cent of the motion to be parallel, thirteen per cent to be similar, forty-nine per cent to be oblique, and twenty-one per cent to be contrary. The index of melodic independence is 1.6 (12, p. 291). Based on his study of the relationships of durational and metric accents, Satterfield classifies all of the measures "smooth" and none "rough" (14, p. 177).

Tonal relationships.--In another study Satterfield found 215 intervals having relative accents in Invention No. XIV. Of these, thirty-four, or 15.8 per cent were classified dissonant and the remaining 181, or 84.2 per cent, consonant (13, p. 278). There are an average of 2.5 identifiable chordal structures in each measure. Because of the continually shifting tonal center all of this Invention was analyzed in terms of the tonic key. The tonality changes six times before finally returning to the tonic key. Some of these tonalities last for several measures, others for only a few beats. There are only eighteen altered tones in this piece.

Rhythmic Characteristics

The meter is quadruple simple. The following table shows the frequency of note values appearing in the Invention.

TABLE XXVIII

FREQUENCY OF NOTE VALUES IN INVENTION NO. XIV

Upper Voice Note Values	Number of Beats	Per Cent of Total Beats
Quarter note (beat note)	4	5
Eighth note	8.5	11
Sixteenth note	34	43
Thirty-second note	17.5	22
Other note values and rests	16	19
Lower Voice **Note Values**	**Number of Beats**	**Per Cent of Total Beats**
Quarter note (beat note)	13	4
Eighth note	16.5	20
Sixteenth note	30.5	38
Thirty-second note	16.5	20
Other note values and rests	13.5	18

The greatest amount of motion is in sixteenth notes. The rhythmic pattern from the motive involving the thirty-second note is very important thematically and, therefore, there is an unusually high percentage of motion using that note value.

There is considerable disagreement among editors, performers, and pedagogues concerning the correct tempo. Keller

suggests a tempo of ♪=88. Bodky assigns to this Invention a tempo of ♩=60 (1, p. 268).

Formal Design

Invention No. XIV can be considered in four sections with the following key scheme: B^b Major, F Major, c minor, and B^b Major. A highly sequential and modulatory passage occurs in measures twelve through sixteen, at which point (Section four) a stretto occurs.

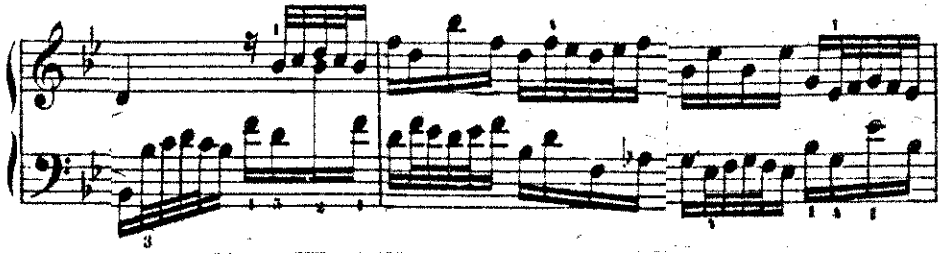

Fig. 61--Invention No. XIV, stretto, measures 16-17

Spitta discusses this Invention in the following excerpt:

> The two next inventions, in A minor and B Flat major, both have somewhat the character of preludes, because the subject and the workings-out alike move almost exclusively in harmonic passages; the second betrays a close relationship to the prelude of the B flat partita in the first portion of the Clavierübung. The three-fold division is, however, preserved in this case. In the B flat invention the first subject comes in in canon at its repetition (bar 16, in the middle), and in this place it has a bold, soaring character, while in the former it was dreamy and rather melancholy. (17, pp. 62-63).

Invention No. XV

Linear Characteristics

Motive and countermotive.--The motive is eight beats in length and has a range of a seventh. The outstanding features are the continual recurrence of a few notes and the appearance of the melodic interval of a sixth.

Thirteen intervals (61 per cent) change direction, five intervals (28 per cent) continue in the same direction, and two intervals (11 per cent) involve note repetition. Seventeen intervals (89 per cent) are a fifth or smaller and two (11 per cnet) are larger than a fifth.

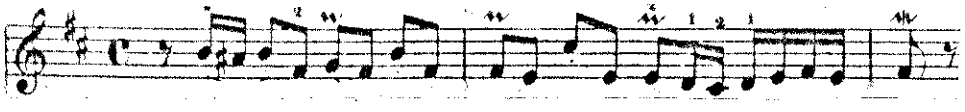

Fig. 62--Invention No. XV, motive, measures 1-3

One author suggests symbolic meaning in this motive. Leonard sees a combination of joy (a) and sadness, the "sighing" motive of grief (b) (9, p. 645).

The countermotive first appears opposite the motive in the first Imitation. It is five-and-one-fourth beats in length and has a range of a sixth.

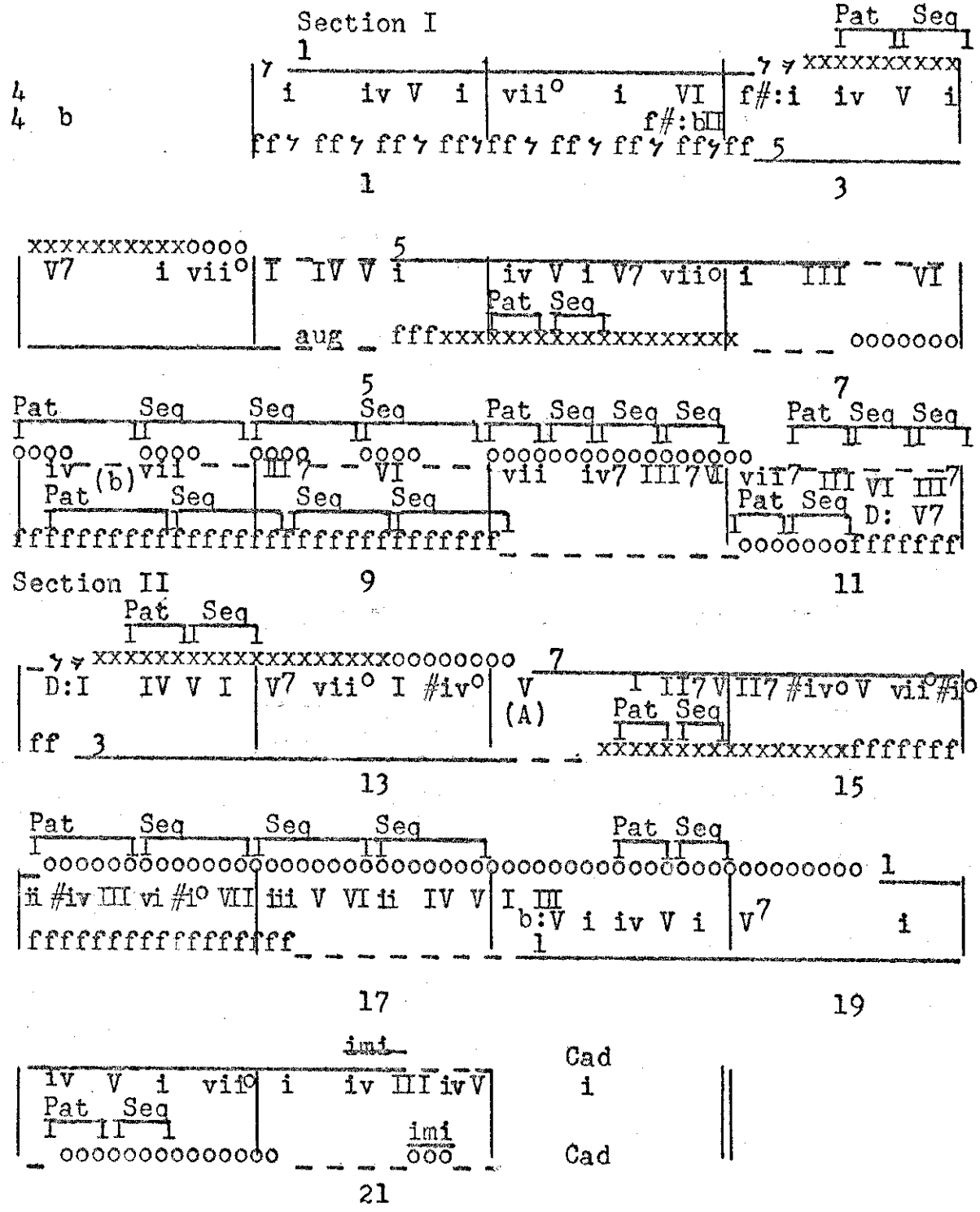

Fig. 63--Invention No. XV, graphic analysis

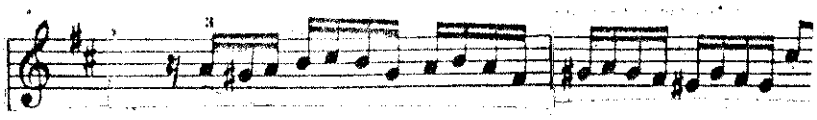

Fig. 64--Invention No. XV, countermotive, measures 3-4

Half of the intervals change direction and half continue in the same direction. Only one interval (6 per cent) is larger than a fifth; the remaining eighteen (94 per cent) are a fifth or smaller.

Episodes.--Twenty-two beats of the material (13 per cent) are classified as free. One-hundred-fifty-four beats (87 per cent) are derived from the motive or countermotive. Leonard points to several symbolic figures in the episodic material related to the motive or countermotive. The sequenced figure (appearing in the upper voice) in measures eight to ten is designated a "tumult motive."

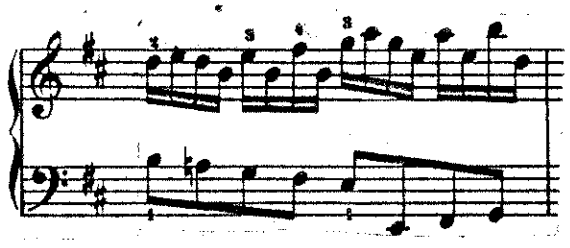

Fig. 65--Invention No. XV, episodic pattern (measure 8)

In the lower voice accompanying the "tumult motive" is the "step motive." The passage appearing in the lower voice in

measures sixteen and seventeen is also symbolic, according to Leonard.

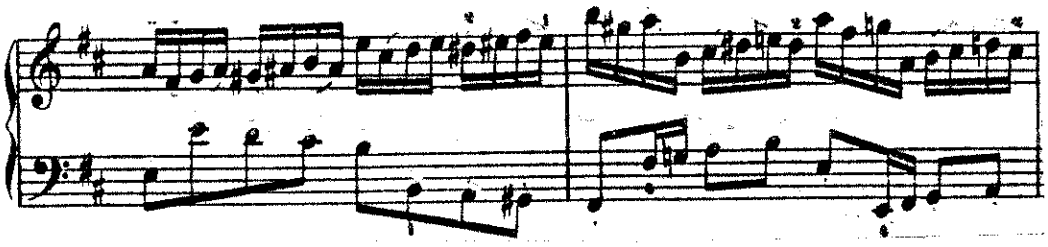

Fig. 66--Invention No. XV, episodic pattern, (measures 16-17)

At this point, the Invention continues with "wide skips and turns of sadness" (9, p. 645).

In his analysis of the relative congruence of melodic and metric accents within the measure Satterfield found 27.3 per cent "rough." In the lower voice he found 22.7 per cent "smooth" and 77.3 per cent "rough" (14, p. 174).

There are a great number of ornaments. Twenty-four pralltrillers and one mordent occur in this composition.

Textural Characteristics

The following table shows the number of beats and the percentage of each type of Species of counterpoint in this Invention.

TABLE XXIX

FREQUENCY OF COUNTERPOINT SPECIES IN INVENTION NO. XV

Species	Number of Beats	Per Cent of Total Beats
First	12	13
Second	61	69
Third	0	0
Fourth	3.75	4
Other	11.25	14

Most of the counterpoint is in Second Species. Within the First Species counterpoint two-thirds of the progressions are in similar and parallel motion with one-third appearing in contrary motion. None of the progressions are in oblique motion.

Two other authors have made statistical studies of texture. In his analysis of all the progressions in the Invention Miller reports the following figures for the types of motion: parallel--14 per cent, similar--15 per cent, and contrary--21 per cent. The index of melodic independence is 1.9 (12, p. 291). Satterfield in his study classifies 13.6 per cent of the measures "rough" and 86.4 per cent of the measures "smooth" (14, p. 177).

Tonal relationships.--There are 156 intervals having relative metric accents in Invention No. XV. Forty-eight

(30.8 per cent) are dissonant and 108 intervals (69.2 per cent) are consonant (13, p. 278). An average of 3.8 identifiable chordal structures is implied in each measure. There are three major modulations and two transient modulations. Thirty-five altered tones occur.

Rhythmic Characteristics

The meter is quadruple simple. The following table shows the frequency of note values appearing in this Invention.

TABLE XXX

FREQUENCY OF NOTE VALUES IN INVENTION NO. XV

Note Values	Upper Voice Number of Beats	Per Cent of Total Beats
Quarter note (beat note)	0	0
Eighth note	30.5	35
Sixteenth note	49.25	56
Other note values and rests	8.25	9

Note Values	Lower Voice Number of Beats	Per Cent of Total Beats
Quarter note (beat note)	0	0
Eighth note	51	58
Sixteenth note	27.5	31
Other note values and rests	9.5	11

In the upper voice most of the motion is in sixteenth notes whereas in the lower voice most of the motion is in eighth notes.

Authorities disagree on the correct tempo. Keller suggests a tempo of ♩=92. Bodky recommends a tempo of ♩=±80 (1, p. 345).

Formal Design

A two-section Invention, this piece, according to Geiringer "has the character of a two-part fugue, although, contrary to tradition, the first entrance of the subject is supported by brief notes in the bass" (2, p. 275). The first section modulates to the minor dominant as the lower voice enters in imitation of the motive. A strong cadence confirms this tonality at the beginning of the third entrance of the motive.

A cadence to D Major marks the beginning of the second section. In measures sixteen to seventeen a series of altered chords appear, largely as a result of melodic patterns in the upper voice. The motive appears in the tonic in measures eighteen and is imitated in measure nineteen before the Invention comes to a cadence in measure twenty-two

Geiringer describes Invention No. XV as "gay and high spirited" (2, p. 275). Another author, Kinscella calls it "an elaborate dancelike movement of great brilliancy" (6, p. 261) and, lastly, Spitta says:

> The last invention comes in gravely, yet not without a certain dignified grace; in its fugal working-out it alternates with episodic interludes, founded on the counterpoint to the theme. It is remarkable that the theme does not come in alone, but is supported by short notes in harmony in the bass. In no other of the inventions is this the case; but it happens so in the sinfonias (or inventions in three parts), and I believe that these had a reactive influence on this invention . . . (17, p. 63).

Summary

The purpose of this chapter has been to present an analysis of line, texture, rhythm, and formal design in the Two Part Inventions. The study of linear characteristics included examination of the motive (s), countermotive, and the episodes. It was found that the motives of nine Inventions contain 50 per cent or more changes in the direction of the melodic intervals and those of six Inventions contain less than 50 per cent. The motives of nine Inventions have ranges of an octave or greater. Motives of six pieces contain 10 per cent or more intervals of a sixth or larger; nine pieces have less than 10 per cent of intervals a sixth or larger. The countermotives of nine Inventions contain 50 per cent or more melodic intervals that change direction and those of six Inventions contain less than 50 per cent. The countermotives of nine Inventions have ranges of an octave or greater; ranges of countermotives of six pieces are smaller than an octave. The

countermotives of only two Inventions contain 10 per cent or more intervals of a sixth or larger.

In studying the episodes, the derivation of material, the amount of "rough" and "smooth" measures according to melodic accents, and the presence and amount of ornaments were discussed. It was found that five Inventions contain 10 per cent or more free material. Ten pieces have less than 10 per cent free material. All of the Inventions contain 62 per cent or more "rough" measures according to melodic accents in the upper voice; all of the pieces contain 64 per cent or more "rough" measures in the lower voice. Four Inventions have no ornaments. Eleven compositions contain from one to twenty-five ornaments. Seven pieces contain six or more embellishments.

The study of texture included an examination of the following rhythmic relationships: the counterpoint species, the types of motion, the index of melodic independence, and the relationship of durational accents and metric accents. It was found that most of the counterpoint occurs in Second Species. Nine Inventions contain 36 per cent or more Second Species. Only two pieces have 36 per cent or more First Species counterpoint. None of the pieces contain as much as 36 per cent of Third Species counterpoint, but two Inventions contain 36 per cent or more Fourth Species.

Two analyses of the types of contrapuntal motion were made. In the first analysis it was found that in the First Species counterpoint twelve Inventions contain 50 per cent or more similar motion; most of the motion in the other three pieces occurs in oblique and contrary motion. An analysis of all the contrapuntal progressions revealed that most of the motion is oblique. The average frequencies of the types of motion is as follows: oblique--54 per cent; contrary--23 per cent; parallel--13 per cent; and similar--10 per cent. The average index of melodic independence is 1.9. Eleven Inventions received an index of 1.9 or greater. Four pieces received an index of less than 1.9.

The last textural study of rhythm concerned "rough" and "smooth" measures according to durational accents. It was found that all the Inventions contain 86.4 per cent or more measures classified as "smooth."

In addition to a study of rhythmic relationships the following aspects of tonal relationships were examined: the amount of dissonance and consonance, the number of identifiable chordal structures, the number of altered tones, and the frequency of modulations. All of the Inventions contain at least 15.8 per cent of relatively accented dissonance. Three compositions contain over 25 per cent of relatively accented dissonance. The majority of the relatively accented

intervals in each Invention are consonant. Ten of the compositions have an average of 2.0 or more implied tonalities per measure; five pieces have an average of less than 2.0. Thirteen Inventions contain twenty-five or more altered tones; two contain fewer. Three pieces have over fifty altered tones. At least three modulations occur in every Invention; eight pieces contain six or more modulations.

The study of the characteristics of rhythm included the following: meter, the frequency and amount of the note values, and suggested tempos. Eight pieces have a time signature of C; two have a time signature of $\frac{3}{4}$; three have a time signature of $\frac{3}{8}$; one has a time signature of $\frac{9}{8}$; and one has a time signature of $\frac{12}{8}$. The most frequent note value is the sixteenth note. Eight Inventions have 50 per cent or more of the beats in sixteenth notes. Two pieces contain 50 per cent or more of the beats in eighth notes. Five Inventions have less than 50 per cent of the beats in any one note value. According to Keller the most rapid Inventions are Nos. IV, VIII, X, XII, and XV. Bodky's most rapid tempo suggestion is for No. X. He also suggests relatively rapid tempos for Nos. III, IV, X, and XII.

Study of the formal design produced several observations. Eight of the Inventions can be divided into three sections;

six can be divided into two sections; and one can be divided into four sections. Seven of the pieces are based on two coordinate motives. One is based on only one motive and no countermotive. Others contain a motive and countermotive. Two of the Inventions are canons.

Comments of writers concerning the Inventions have been included. It was found that these authors often hold opposing views concerning the mood and character of the pieces.

Selected findings related to the performance problems in the Inventions are discussed in Chapter III of this report. In addition a ranking based on the factors relating to difficulty is presented.

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CHAPTER III

IDENTIFICATION OF PERFORMANCE PROBLEMS AND RANKING OF THE TWO PART INVENTIONS

After having examined the stylistic characteristics of the Inventions certain factors were identified which would present problems to the intermediate pianist. These factors will be identified and discussed in relation to the Inventions. It should be emphasized that these factors do not constitute all the difficulties in these pieces.

Performance Problems Related to the Linear Characteristics

Five of the linear characteristics investigated were considered to contribute to the difficulty of an Invention. They are as follows: (a) a high percentage of intervals of the motive or countermotive which change direction, (b) wide tonal ranges of the motive or countermotive, (c) a high percentage of intervals of the motive or countermotive which are larger than a fifth, (d) a relatively large percentage of free material and (e) the presence and amount of ornaments.

The characteristics of the motive and countermotive were believed to be representative of the entire Invention

since the piece is based structurally on these two elements. (Should an Invention contain a second motive, or a second motive and a countermotive, only the characteristics of the first and second motives will be considered in this section). A high percentage of changes in the direction of the melodic intervals was felt to increase the technical difficulty of a piece. Therefore, the direction of the intervals in the motive and countermotive was examined. There are twelve Inventions containing 50 per cent or more changes in direction of melodic intervals in the motive or countermotive. They are Inventions Nos. I, II, III, V, VI, VII, VIII, X, XII, and XV.

Composite scores representing the percentage of changes in the direction of the motive and countermotive produced the following figures. Invention No. XIII contains the largest percentage (75) and Invention No. VII the lowest (30). Other Inventions with scores of fifty or more are Nos. I, V, VIII, X, and XV. Other Inventions with scores of less than fifty are Nos. II, III, IV, IX, XI, XII, and XIV.

Changes in hand position were believed to be technical problems. Two measurable factors felt to be related to this problem are wide tonal ranges of the motive or countermotive and a relatively high percentage of intervals in the motive or countermotive which are larger than a fifth.

The first of the two factors examined was the size of the range. All of the Inventions except Nos. I and VII have motives or countermotives with a range of at least a sixth. Invention No. XIV has the motive with the largest range--a sixteenth.

The examination of the size of the intervals within the motive and countermotive revealed the following facts. A majority of the intervals are no larger than a fifth. To produce a composite figure for each Invention the percentages of intervals larger than a fifth in the motive and countermotive were added together. The highest figure was 20 per cent (Nos. VIII and XIV). Inventions I, VII, and XII had no intervals larger than a fifth. Those Inventions in which intervals larger than a fifth constitute from eleven to twenty percent are Nos. IV, VIII, IX, XIII, XIV, and XV. Those Inventions with percentages of 10 per cent or less are Nos. I, II, III, V, VI, VII, X, XI, and XII.

It was felt that a large percentage of material not derived from the motive and thematically unimportant would tend to complicate the formal design of an Invention. This decreased distinction could affect negatively the student's understanding and memorization of the piece. Therefore, the amount of free material was determined in each Invention.

The Invention containing the greatest amount of free material is No. VII (20 per cent). Inventions Nos. VI, X, XII, and XIV contain no free material. Inventions with 15 to 19 per cent free material are Nos. III (17 per cent), IX (15 per cent), and XI (19 per cent). Amounts of free material in the remaining Inventions vary between 1 per cent and 10 per cent.

Teachers generally agree that ornaments contribute to the technical difficulty of a piece. Ornaments were divided into three categories. The first category include mordents, turns, and pralltrillers. Invention No. XV contains the greatest number of ornaments in this classification with a total of twenty-five. In order of decreasing number of mordents, turns, and pralltrillers are Invention Nos. XII (seventeen), VII (sixteen), II (twelve), X (eight), III (six), I (six), XI (one), and IX (one). All other Inventions contain no ornaments of these types.

The second category of ornaments consists of short trills. Those Inventions containing short trills (trills other than the pralltriller which are less than two beats) are Nos. I (eight) and Nos. V, VII, and IX with one short trill each.

The third category consists of extended trills. Inventions containing the extended trill (a trill of two beats or longer) are Nos. XII (six trills), X (three), IV (two), and VII (one).

Invention No. XV contains the greatest number of total ornaments with twenty-five; Invention No. XII contains the second highest number with twenty-three ornaments. Invention Nos. VI, VIII, XIII, and XIV contain no ornaments.

Performance Problems Related to the Textural Characteristics

Four textural characteristics were felt to be factors in the difficulty of an Invention. They are as follows: (a) a relatively high percentage of First Species counterpoint; (b) a relatively high percentage of parallel and similar motion within the First Species counterpoint; (c) a great number of altered tones; and (d) contrasting articulation in the voices.

A high percentage of First Species counterpoint (note against note) was regarded as a factor which might increase the technical difficulty of an Invention. Those Inventions in which First Species counterpoint constitutes at least 25 per cent of the total counterpoint are Inventions Nos. II (32 per cent), VII (31 per cent), X (44 per cent), and XI (41 per cent).

The type of motion within the First Species counterpoint was deemed important. Parallel and similar motion were thought to pose the most difficulty. Only those Inventions

in which First Species counterpoint represented at least 15 per cent of the material were considered. Of those, the following contain at least 50 per cent similar and parallel motion: Inventions Nos. II, III, IV, V, VII, VIII, IX, XI, and XIV. Especially noted were Invention No. VIII with 96 per cent similar and parallel motion and Invention No. XIV with 78 per cent of these types of motion.

Altered notes were thought to increase the difficulty of reading the Invention and, possibly, the difficulty of memorization. The Invention containing the greatest number of altered tones is No. IX with ninety-one. Next is No. XI with sixty-one. Other Inventions containing at least thirty-five altered tones are Nos. II, IV, V, VI, XIII, and XV.

Although articulation is an aspect of interpretation and therefore varies among performers, technical difficulties related to certain common performance practices will be mentioned here. If a great deal of the eighth notes are detached and the sixteenth notes played legato, the resulting contrast in articulation between the voices could pose technical difficulty in Invention Nos. IV, VIII, and XIII. This factor could also cause problems to a lesser degree in Nos. XI, XIV, and XV. Only two Inventions were amply edited by Bach. Invention Nos. III and IX are "well provided with

slurs" (2, pp. 202-203). The observance of these slurs could well cause difficulty for the intermediate student.

Performance Problems Related to the Rhythmic Characteristics

Two measurable factors related to rhythmic characteristics were felt to contribute to the performance problems of the Inventions. They are the tempo (as suggested by Keller) and the percentage of sixteenth notes.

Those Inventions in simple time with the most rapid tempos are Nos. VIII (♩=116-126) and XV (♩=92). Those Inventions in compound time with the most rapid tempos are Nos. IV (♩.=60); XII (♩.=72); and X (♩.=108). The percentages of sixteenth note movement in these Inventions are as follows: No. VIII--52 per cent, No. XV--43 per cent, No. IV--57 per cent, No. X--0 per cent, and No. XII--65 per cent. Other Inventions with high percentages of sixteenth note movement are Nos. XI (65 per cent), V (55 per cent), VII (53 per cent), and XIII (50 per cent).

The views concerning the correct tempos vary greatly among editors, teachers, and performers (see Appendix G). Maier states, however, that one of the virtues of these pieces is their capacity for being musically performed at varying tempos (9, pp. 2-3).

Performance Problems Related to the Formal Design

The complexity of the formal design was thought to affect the performer's intellectual grasp of the piece and his ability to memorize. Three measurable factors were felt to relate to the formal design of an Invention. They are as follows: (a) a relatively high number of implied chordal structures in each measure; (b) a relatively high number of modulations; and (c) obscurity of sectional divisions. In addition, the opinions of one pedagogue concerning the difficulty of analysis of each Invention will be given.

The first factor in the complexity of the formal design is a relatively high number of implied chordal structures in each measure. Those Inventions containing an average of at least 2.5 implied chordal structures in each measure are Nos. I, II, V, VII, XI, XII, XIII, XIV and XV. Of those, Invention Nos. XI, XII, and XV contained 3.5 or more implied chordal structures per measure.

The second factor in the complexity of the formal design is the number of modulations present in an Invention. All of the Inventions contained at least three modulations. Those Inventions containing at least six modulations are Nos. III, V, VI, VII, IX, XII, XIII and XIV. The greatest number of modulations occur in Invention Nos. V (nine), XII (nine) and XIII (ten).

The third factor related to the complexity of the formal design concerns the sectional division of the Invention. Those pieces in which the sections were well-defined were felt to be more easily grasped than those in which section divisions were somewhat obscure. Inventions Nos. V, XI, and, to lesser extents, XII and IX were felt to be somewhat less clear in sectional design than the other pieces. The cadences marking the sections were not well-defined.

One pedagogue (Terhune), in addition to evaluating the difficulties of technique, memorization, and interpretation of the Inventions, gave his opinion of the difficulty of analysis for each piece. He specified the analysis in terms of the performer, not the composer ("how difficult is the Invention to analyze in such a way as to make it intelligible to the student?"). Invention No. X received the highest score (8). (The rating scale was 1--easy and 9--difficult). The next in order of difficulty was No. VI with a score of 7. The Inventions of medium of analysis difficulty were Nos. IX (6), XV (6), XI (5), III (4), IV (4), VIII (4), and X (4). Those Inventions judged to be simple in regard to analysis were Nos. I (3), II (2), V (2), XII (2), and XIV (2).

Ranking of the Two Part Inventions

Two overall rankings of the Inventions were established. The first was based on the factors of difficulty determined through the analysis of the compositions. The second order was based on the evaluations of six pedagogues.

The first ranking was based on sixteen factors relating to difficulty (see Table XXXI). It was necessary to assign numerical values to each of the factors related to difficulty in order to produce an overall indication of the difficulty of the piece. The composite figures of the Inventions could then be compared and a ranking established.

It was decided that each factor would be measured on a scale from zero to ten. Those factors which had been measured in percentages were easily converted to this scale by dividing by ten. Thus, a factor which had been measured as 68 per cent received a value of 6.8. However, it was necessary to devise a procedure for assigning numbers to those factors not indicated by percentages. The following system was used: the greatest amount of a factor found in any of the Inventions was given a value of 10.0. All other amounts of the factor occurring in the remaining fourteen Inventions were assigned proportionate numbers. Relative tempos were based on the speed of the fastest note in each composition.

After values had been determined for all the factors, a composite average figure was established for each Invention. The following table shows the numerical values for the factors relating to difficulty and the composite averages for the Two Part Inventions.

TABLE XXXI

NUMERICAL VALUES OF THE FACTORS RELATING TO DIFFICULTY IN THE TWO PART INVENTIONS

Inventions	Percentage of Intervals that Change Direction		Range		Percentage of Intervals Larger than a Fifth		Percentage of Free Material
	M.*	CM.**	M.	CM.	M.	CM.	
I	6.7	5.0	3.1	2.3	0	0	.6
II	4.6	5.0	6.3	6.9	1.0	0	.4
III	5.8	4.0	3.1	6.2	0	.9	1.7
IV	2.7	4.0	4.4	5.4	1.7	0	1.0
V	3.9	8.4	6.3	7.7	0	.7	.1
VI	5.0	2.7	5.0	6.9	0	.9	0
VII	6.0	. .	3.1	. .	0	. .	2.0
VIII	10.0	3.3	5.0	6.2	2.0	0	1.5
IX	3.2	2.7	7.5	9.2	.6	.7	.8
X	5.6	5.9	5.0	5.4	1.0	0	0
XI	4.0	4.7	5.6	3.1	.7	0	.9
XII	3.7	5.2	5.0	10.0	0	0	0
XIII	5.0	10.0	4.4	6.2	0	1.4	.3
XIV	7.1	1.1	10.0	6.2	1.0	1.0	0
XV	6.1	4.4	4.4	4.6	1.1	.6	1.3

*M. means motive. **CM. means countermotive.
Rating scale: 1--easy; 9--difficult.

TABLE XXXI--Continued

Ornaments	Percentage of First Species Counterpoint	Percentage of Similar and Parallel Motion	Altered Tones	Tempos	Percentage of Sixteenth Notes	Number of Chordal Structures Per Measure	Modulations	Sections Not Well Defined	Composite Scores
2.4	.8	4.8	2.6	5.2	4.8	2.6	5.0	0	2.19
8.0	3.2	5.6	4.0	4.3	6.2	2.6	5.0	0	3.94
2.4	1.6	5.5	2.3	5.7	5.3	1.5	7.0	0	3.31
1.3	2.2	5.1	5.4	7.6	5.7	1.0	5.0	0	3.28
.4	1.6	5.7	4.3	6.0	5.5	2.9	9.0	6.7	4.33
0	3.0	6.8	5.1	8.0	3.0*	1.7	6.0	0	3.38
7.5	3.1	4.5	3.7	6.0	4.9	2.5	6.0	0	3.79
0	2.3	9.6	2.7	10.0	5.2	1.5	5.0	0	3.46
1.3	1.9	6.9	10.0	3.8	4.4	2.2	7.0	3.3	4.09
4.4	4.4	4.3	1.2	6.7	7.0**	1.8	3.0	0	3.48
.4	4.1	5.6	6.7	4.8	6.5	3.6	5.0	10.0	4.11
9.2	.7	5.3	3.6	8.9	5.6	3.5	9.0	3.3	4.56
0	.7	6.0	5.9	5.7	5.0	3.0	10.0	0	3.35
0	2.1	7.8	2.0	7.3	6.2*	2.5	6.0	0	3.77
10.0	1.3	6.7	3.8	7.6	4.4	3.8	5.0	0	4.11

*The percentages of sixteenth and thirty-second notes were combined.

**The percentage of eighth note movement was used as there were no sixteenth or thirty-second notes in the piece.

The figures in the last column, "composite scores," were arranged in order of decreasing value. The following table shows the Inventions and the scores beginning with the most difficult.

TABLE XXXII

RANKING OF THE TWO PART INVENTIONS IN ORDER OF DECREASING DIFFICULTY ACCORDING TO COMPOSITE SCORES OF FACTORS RELATING TO DIFFICULTY

Inventions	Composite Scores
XII	4.56
V	4.33
XI	4.11
XV	4.11
IX	4.09
II	3.94
VII	3.79
XIV	3.77
X	3.48
VIII	3.46
VI	3.38
XIII	3.35
III	3.31
IV	3.28
I	2.19

Only 2.37 points separates the most difficult Invention, No. XII from the least difficult, No. I. Four of the pieces have scores of 4.00 or above and one piece has a score of less than 3.00. The majority of the Inventions received scores between 3.00 and 4.00.

To establish the second ranking, six pedagogues evaluated the difficulties of technique, memorization and interpretation in each Invention. Scores of one, two, and three indicated least difficulty; scores of four, five and six indicated medium difficulty; and scores of seven, eight, and nine indicated the most difficulty. The following table shows the scores and the averages for each category of difficulty.

TABLE XXXIII

LEVELS OF DIFFICULTY OF PERFORMANCE OF THE INVENTIONS AS RATED BY SIX COLLEGE PIANO INSTRUCTORS*

Invention Nos.	Technique		Memorization
	Individual Piano Teachers' Ratings	Average	Individual Piano Teachers' Ratings
I	1 2 2 4 4 4	2.83	1 1 3 4 4 5
II	3 5 5 5 6 6	5.00	4 4 5 5 6 7
III	4 4 5 5 6 6	5.00	3 3 4 5 5 7
IV	3 4 4 4 4 7	4.33	2 2 3 4 4 5
V	3 5 5 5 6 7	5.17	5 5 5 6 6 6
VI	2 5 5 5 5 6	4.67	2 4 4 5 6 7
VII	4 5 5 5 6 6	5.17	2 5 5 6 6 **
VIII	2 4 4 5 7 8	5.00	1 2 4 4 4 6
IX	3 5 5 5 6 6	5.00	5 5 5 6 6 8
X	2 4 5 5 6 8	5.00	1 3 4 5 5 6
XI	2 5 6 6 7 7	5.50	4 5 6 6 6 7
XII	6 7 7 8 8 9	7.50	5 5 7 7 7 8
XIII	4 4 4 6 6 6	5.00	1 3 4 4 5 5
XIV	2 4 5 6 6 6	4.83	2 3 4 4 5 5
XV	2 4 5 6 6 7	5.00	2 5 5 6 7 7

*Rating scale: 1--easy; 9--difficult
**No rating given by one teacher

TABLE XXXIII--Continued

	Interpretation		Composite Average
Average	Individual Piano Teachers' Ratings	Average	
3.00	1 1 2 3 4 4	2.50	2.78
5.17	2 5 6 6 6 6	5.17	5.11
4.50	3 4 4 5 5 5	4.33	4.61
3.33	2 2 4 4 4 6	3.67	3.78
5.50	4 5 6 7 7 9	6.33	5.67
4.67	4 4 5 6 6 7	5.33	4.89
4.80	4 5 5 5 6 7	5.33	5.10
3.50	1 2 3 4 4 5	3.17	3.89
5.83	3 4 5 6 6 7	5.17	5.33
4.00	2 2 4 5 5 7	4.17	4.39
5.67	4 5 6 6 7 7	5.83	5.67
6.50	5 5 7 7 8 **	6.40	6.80
3.67	1 4 4 4 5 5	3.83	4.17
3.83	4 4 4 5 6 6	4.83	4.50
5.33	5 5 5 6 6 8	5.83	5.39

Invention No. I received the greatest number of "one's." Only two Inventions received a score of nine in any category. They are No. V, in the difficulty of interpretation, and No. XII, in technical difficulty.

There was a wide variance in the opinions of the pedagogues. The greatest range in figures occurred in the scores for technical difficulty in Invention Nos. VIII and X. In either case the lowest figure was "2" and the highest "8."

Table XXXIV below lists the Inventions in order of decreasing difficulty in technique, memorization, and interpretation. In addition, the last column presents the Inventions in order of overall difficulty. In each ranking Invention No. XII was judged to be most difficult and No. I to be the least difficult. Based on the rating scale of one to nine, however, the range of difficulty of all the Inventions was only 4.02 (from 2.78 to 6.80). Thus, all the Inventions were classified as either simple (1-3) or medium difficult (3-6).

TABLE XXXIV

RANKING OF THE TWO PART INVENTIONS ACCORDING TO TECHNIQUE, MEMORIZATION, INTERPRETATION AND COMPOSITE SCORES IN ORDER OF DECREASING DIFFICULTY*

Order According to Technique Scores	Order According to Memorization Scores	Order According to Interpretation Scores	Order According to Composite Scores
XII (7.50)	XII (6.50)	XII (6.40)	XII (6.80)
XI (5.50)	IX (5.83)	V (6.33)	V (5.67)
V (5.17)	XI (5.67)	XI (5.83)	XI (5.67)
VII (5.17)	V (5.50)	XV (5.83)	XV (5.39)
II (5.00)	XV (5.33)	VI (5.33)	IX (5.33)
III (5.00)	II (5.17)	VII (5.33)	II (5.11)
VIII (5.00)	VII (4.80)	II (5.17)	VII (5.10)
IX (5.00)	VI (4.67)	IX (5.17)	VI (4.89)
X (5.00)	III (4.50)	XIV (4.83)	III (4.61)
XIII (5.00)	X (4.00)	III (4.33)	XIV (4.50)
XIV (5.00)	XIV (3.83)	X (4.17)	X (4.39)
XV (5.00)	XIII (3.67)	XIII (3.83)	XIII (4.17)
VI (4.67)	VIII (3.50)	IV (3.67)	VIII (3.89)
IV (4.33)	IV (3.33)	VIII (3.17)	IV (3.78)
I (2.83)	I (3.00)	I (2.50)	I (2.78)

*Rating scale: 1--easy; 9--difficult

The next table presents a comparison of the two overall rankings of the Inventions.

TABLE XXXV

A COMPARISON OF THE RANKING OF THE TWO PART INVENTIONS BASED ON THE EVALUATIONS OF SIX PEDAGOGUES AND THE RANKING BASED ON THE FACTORS RELATING TO DIFFICULTY

Ranking Based on Evaluations of Six Pedagogues		Ranking Based on the Factors Relating to Difficulty	
Inventions	Scores	Inventions	Scores
XII	6.80	XII	4.56
V	5.67	V	4.33
XI	5.67	XI	4.11
XV	5.39	XV	4.11
IX	5.33	IX	4.09
II	5.11	II	3.94
VII	5.10	VII	3.79
VI	4.89	XIV	3.77
III	4.61	X	3.48
XIV	4.50	VIII	3.46
X	4.39	VI	3.38
XIII	4.17	XIII	3.35
VIII	3.89	III	3.31
IV	3.78	IV	3.28
I	2.78	I	2.19

Several observations can be made concerning the two orders shown in Table XXXV. First, ten of the Inventions (67 per cent) received the same positions in both rankings. This is a high correlation. Second, the orders of the seven most difficult Inventions and the two least difficult correspond exactly. Third, the five pieces whose orders differed vary no more than five places. The numerical difference in the widest variance is .46. Inventions Nos. XIV and X differ two places in the rankings; No. VI differs three positions; No. VIII varies four places; and No. III varies five positions.

Properly qualified, the following proposition appears to be valid: there is a large possibility that the factors identified and measured in this study are related to the difficulty as determined here by the evaluations of six pedagogues. Within limits, the difficulty of an Invention is proportionate to the amount of each of the factors relating to difficulty contained in it.

Requirements for Performance of Bach

Several authors offer opinions on the requirements of performance ability before studying Bach. Bryant states That "the performer must develop the technique to maintain a singable line . . . artistic performance will be in proportion to the relative absence of strain and excess physical effort"

(3, p. 53). Maier states similarly that the Two Part Inventions "should only be studied by players who possess well developed technique" (9, pp. 2-3).

Lanning discusses further the necessary technique for performance of Bach.

> Although this music may be played fairly well without proper attention to a well grounded finger technic, experience has shown that all the famous specialists and the best student performers of this music, all have fine finger action.
>
> Weak finger tips, especially when playing the black keys, will cause uneveness of tone and insecurity.
>
> Special attention should be given to the "up-stroke," or the exactness with which each key is released, for the blurring of any tone or tones by improper overlapping, is about the worst defect in almost all Bach playing (7, p. 5).

Three authors emphasize other musical requirements before playing Bach. Balogh states: "To play Bach well, one has to have a good legato and a crisp leggiero touch. To do justice to the polyphonic part-writing one must have independent and well-trained fingers, and one needs good ears to hear what is going on in the music" (1, p. 51). Schroder writes, "A good sense of musical phrasing and tonal control is absolutely imperative if one is to perform these Inventions in such a way as to do them justice" (11, p. 27). Tureck stresses "special mental preparation" before playing Bach.

> You don't just sit down and learn him--you learn how to learn him. This intermediary step of learning how to learn is essential. It roots in habits of thought which grew from musical sources which are different from the sources of the 19th century. The purely pianistic aspects of Bach playing are also highly important. The student needs to develop (1) a finger technique which is much more complete than that which is generally acquired today. One of the most important factors is the strength and true independence of each finger: (2) a good, dependable legato: (3) a technique for changing fingers--sometimes on the same note, sometimes in sequences which demand a kind of inverted fingering, such as fourth to third, third to second, etc.: (4) a wide variety of staccato: (5) a swift foot for pedalling. All these are necessary: none is the least helpful musically unless the playing is bulwarked by certain ways of thinking (5, p. 13).

When to Begin the Study of the Inventions

Several authors emphasize the great importance of not beginning the study of the Two Part Inventions too soon. Spry says, "It is my belief that the Inventions are frequently given too soon in the young pupil's study, and thereby a distaste for Bach's music is acquired. If properly administered, the Inventions are a splendid preparation for the Partitas, and then the French and English Suites" (13, p. 640). Spencer states:

> . . . most piano teachers seem to think that there are only two collections of Bach's works that are available for teaching purposes: the "Inventions" and the "Well Tempered Clavichord." There is no greater pedagogical mistake than to begin the study of Bach with the "Inventions." To the untrained musical mind, they are stern and forbidding; and they should be left until the student has written advanced counterpoint for at least

> a year. When he has reached this stage of advancement, he not only will begin to comprehend the almost uncanny cleverness of these "Inventions," but he also will be able to grasp, in some degree, the deep musical feeling which pervades so many of them, particularly those in three parts (12, p. 62).

Spencer recommends an approach to Bach through the "almost limitless number of Gavottes, Bourees, Menuets and Gigues that are scattered throughout his many suites" (12, p. 62). Campbell recommends that the study of Bach begin, for the average student, in the sixth or seventh grade (a technical classification) (4, pp. 77-78).

Editions

Several writers offer recommendations concerning editions of the Inventions. Lanning recommends the Bischoff edition (7, p. 3). Piersel recommends the Landshoff edition (10, p. 6). Hutcheson recommends the Busoni edition (for "students needing a guide") and the Bischoff edition (6, p. 59-60). Lockwood suggests the Busoni edition "for those who need an intelligent guide. . . ." (8, p. 20). Stone suggests that the student study as many editions as possible (he mentions Bischoff, Czerny, Busoni and Landshoff). "But," he continues, "one should trust nothing but a conscientious Urtext edition, and draw one's interpretative conclusions exclusively from what Bach himself wrote" (14, pp. 12-14). He recommends the

Landshoff edition as "an excellent example of an Urtext edition" (14, p. 13).

Summary

The following characteristics were felt to be factors related to the performance problems in the Inventions: (1) a high percentage of intervals of the motive or countermotive which change direction, (2) wide tonal ranges of the motive or countermotive, (3) a high percentage of intervals of the motive or countermotive which are larger than a fifth, (4) a relatively large percentage of free material, (5) the presence and amount of ornaments, (6) a relatively high percentage of First Species counterpoint, (7) a relatively high percentage of parallel and similar motion within the First Species counterpoint, (8) a great number of altered tones, (9) contrasting articulation in the voices, (10) tempo, (11) a high percentage of sixteenth notes, (12) a relatively high number of implied chordal structures in each measure, (13) a relatively high number of modulations, and (14) obscurity of sectional divisions. Based on these factors the following ranking of the Inventions in order of decreasing difficulty was prepared: Nos. XII, V, XI, XV, IX, II, VII, XIV, X, VIII, VI, XIII, III, IV, I.

Six pedagogues were asked to evaluate the Two Part Inventions according to the difficulty of technique, memorization, and interpretation. The most technically difficult pieces are Nos. XII, XI, V, and VII; the easiest pieces are Nos. I, IV, and VI. The remaining Inventions were judged to be the same in technical difficulty. The following order shows the compositions according to difficulty of memorization, beginning with the most difficult: Nos. XII, IX, XI, V, XV, II, VII, VI, III, X, XIV, XII, VIII, IV, I. The next list presents the Inventions in order of decreasing difficulty of interpretation: Nos. XII; V; XI and XV (tied); VI and VII (tied); II and IX (tied); XIV; III; X; XIII; IV; VIII; I.

The last ranking represents the composite difficulty of the compositions. In order of decreasing difficulty they are Nos. XII, V, XI, XV, IX, II, VII, VI, III, XIV, X, XIII, VIII, IV, I.

A comparison of the ranking based on the evaluations of the six pedagogues and the ranking based on the factors related to difficulty revealed a high correlation. Ten Inventions hold identical positions in both rankings: the most difficult seven and the two least difficult were among the ten. This high agreement between the rankings suggested that, to some degree those factors identified and measured are related

to the difficulty as here determined. Within limits, the difficulty of an Invention is proportionate to the amount of each of the factors.

Several writers have offered comments concerning the requirements for performance of Bach. The essential musical abilities discussed are as follows: (a) a well-developed technique, (b) strong fingers, (c) a good legato and a crisp leggiero touch, (d) independent fingers, and (e) a good sense of musical phrasing. Tureck, in addition to pianistic requirements, emphasizes the need for mental preparation.

Three authors--Spry, Spencer, and Campbell--emphasize the importance of not beginning the study of the Inventions too soon. Spencer suggests that the student begin the study of Bach through selected Gavottes, Bourees, Menuets, and Gigues.

Three editions of the Two Part Inventions--Bischoff, Landshoff, and Busoni--are recommended by authors. Stone suggests that the student study as many editions as possible but trust only an Urtext. He recommends the Landshoff edition.

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CHAPTER IV

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary and Conclusions

The purpose of this report was to analyze the Bach Two Part Inventions and to prepare a graded list for pedagogical purposes. The analysis of the Inventions included an examination of the characteristics of line, texture, rhythm, and formal design. From the analyses, performance problems were determined for each composition and a ranking of the Inventions according to difficulty was established.

To determine the relative difficulty of the pieces six pedagogues were polled. These teachers were asked to evaluate the difficulty of technique, memorization, and interpretation in each of the Two Part Inventions. From the results of this survey a graded list of the compositions was prepared.

Chapter I of this report presents the purpose of the study, the sub-problems involved, definitions of terms, delimitations, the basic hypothesis of the report, the basic assumptions of the report, background for the study, methodology, and the plan of the report.

Chapter II contains the analyses of the Inventions and related comments of writers. The discussion of each piece is accompanied by a graphic analysis showing the form, the modulations, the implied chordal structures, and the contrapuntal devices used.

Chapter III presents the identification of performance problems in the Inventions and a graded list. In addition, a ranking based on the factors as measured in this study was established. A discussion of the musical and pianistic requirements for performing the Two Part Inventions, the suitable level for beginning the study, and the editions of the Inventions was also included.

Appendices A through G present the following: (a) the frequency of parallel, similar, oblique and contrary motion in the Inventions; (b) the frequency of relatively accented dissonant intervals in the Inventions; (c) the frequencies of "rough" and "smooth" measures according to melodic accents; (d) the frequencies of "rough" and "smooth" measures according to durational accents; (e) the list of piano pedagogues from whose surveys a graded list of the Inventions was prepared; (f) a copy of the rating sheet sent to the pedagogues; and (g) tempos for the Inventions suggested by five authorities.

The Two and Three Part Inventions were written about 1720 when Bach was conductor of the orchestra at Cöthen. They were intended as exercises for his son Wilhelm Friedmann Bach, then nine years old. According to Geiringer the first two and the last eight were written by the father and Nos. III through VII by Friedmann. The pieces are intended as studies for performing and for composing, both of which Bach considered necessary. Bodky concludes that the Inventions were intended to be performed on the clavichord.

Three studies related to the Two Part Inventions have been made by three authors--Platt, Miller and Satterfield. Platt points out the common use of melodic patterns in Bach's counterpoint, a practice which offers a clue to the less easily explained intricacies of his counterpoint. Miller examines the motion in the progressions. Oblique motion constitutes more than half of the progressions with other types of motion in order of decreasing frequency being contrary, parallel, and similar. The average index of melodic independence was found to be 1.9, an indication of a high degree of melodic independence. Satterfield finds the emotional content of the Inventions is related neither to the dissonance content nor to roughness in melodic accents. He concludes that poignancy is related to roughness in durational accents.

The following characteristics of the motive and countermotive are found in a majority of the Inventions: (a) 50 per cent or more changes in the direction of the melodic intervals; (b) less than 10 per cent of intervals a sixth or larger; and (c) a range of an octave or more.

A study of the episodes produced several facts. (a) Most of the material in the Inventions is derived from the motive and countermotive. (b) A majority of the measures are rough in melodic accents. (c) Most of the pieces contain ornaments, ranging in number from one to twenty-five.

An investigation of the texture revealed the following facts. Most of the counterpoint is in Second Species. A majority of the progressions within First Species counterpoint are in similar and parallel motion. Most of the relatively accented intervals are consonant. Two-thirds of the pieces have an average of two or more implied chordal structures per measure. Over three-fourths of the Inventions contain twenty-five or more altered tones. The majority of the pieces contain six or more modulations.

A study of the rhythmic characteristics produced the following observations. A majority of the pieces are in simple meter. The most frequent note value is the sixteenth note. Keller and Bodky agree on rapid tempos for Nos. VIII, X, and XII.

An analysis of the formal design revealed that most of the Inventions are in three sections. Two pieces are canons.

Two rankings of overall difficulty were established. Based on the factors identified in relation to the difficulty of the pieces the following ranking of the Inventions in order of decreasing difficulty was prepared: Nos. XII, V, XI, XV, IX, II, VII, XIV, X, VIII, VI, XIII, III, IV, I. According to the evaluations of six pedagogues the most technically difficult pieces are Nos. XII, XI, V, and VII; the easiest pieces are Nos. I, IV, and VI. The remaining Inventions were thought to be the same in technical difficulty. The following order shows the pieces according to difficulty of memorization beginning with the most difficult: Nos. XII, IX, XI, XV, II, VII, VI, III, X, XIV, XII, VIII, IV, I. The third list based on the opinions of the pedagogues presents the pieces in order of decreasing difficulty of interpretation: Nos. XII; V; XI and XV (tied); VI and VII (tied); II and IX (tied); XIV, III, X, XIII, IV, VIII, I. The second ranking indicating the composite difficulty of the compositions is as follows, beginning with the most difficult: Nos. XII, V, XI, XV, IX, II, VII, VI, III, XIV, X, XIII, VIII, IV, I. There is a high correlation between the two rankings of overall difficulty. Ten of the Inventions have identical positions in both orders.

Several authors make recommendations concerning the study of the compositions. First, the student should have a well-developed technique and a good sense of musical phrasing. Second, the student should not begin his study of polyphonic music with the Inventions. Third, the student should study as many editions as possible but should trust only an Urtext. The Landshoff edition is highly recommended.

Based on the results of the investigations of this study the following conclusions are drawn:

1. The ranking of the Inventions based on the evaluations of pedagogues is as follows, beginning with the most difficult: Nos. XII, V, XI, XV, IX, II, VII, VI, III, XIV, X, XIII, VIII, IV, I.

2. The ranking of the Inventions based on the identifiable factors related to difficulty is as follows, beginning with the most difficult: Nos. XII, V, XI, XV, IX, II, VII, XIV, X, VIII, VI, XIII, III, IV, I.

3. There is a high correlation between these two lists. This suggests a large possibility that the factors identified and measured in this study are related to the difficulty as determined by the evaluations of the six pedagogues.

4. A wide range of opinions concerning the difficulty of these pieces exists among pedagogues.

5. The majority of Inventions received scores indicating similar difficulty in each of the aspects of performance listed--technique, memorization and interpretation.

6. It is possible to determine and measure the factors related to difficulty in a group of compositions and to compare their overall difficulties.

Recommendations

Based on this study of Bach's Two Part Inventions the following recommendations are submitted to teachers, students, and researchers.

1. Teachers should not begin the intermediate student's study of polyphonic music with the Two Part Inventions.

2. Teachers should give serious consideration to the performance problems in each Invention before assigning it for study.

3. Teachers should give serious consideration to the particular strengths and weaknesses of the student in assigning the Inventions.

4. It is recommended that the student study as many editions of the Inventions as possible, but form his interpretative ideas on the basis of the Landshoff edition.

5. Further study should be made concerning the other smaller keyboard works of J. S. Bach.

6. Further study should be made to establish valid criteria for judging the difficulty of piano music.

APPENDIX

APPENDIX A

FREQUENCIES OF PARALLEL, SIMILAR, OBLIQUE, AND CONTRARY MOTION, AND MELODIC INDICES IN THE TWO PART INVENTIONS

TABLE III*

CONTRAPUNTAL PROGRESSION IN BACH'S TWO-PART INVENTIONS

Invention	Parallel	Similar	Oblique	Contrary	Index
No. 1, C major	7%	10%	58%	25%	2·0
No. 2, C minor	21·5	6	47	25·5	1·8
No. 3, D major	11	10	57	22	1·9
No. 4, D minor	12	10	53	25	1·9
No. 5, E♭ major	10	11	53	26	2·0
No. 6, E major	5	6	76	13	2·0
No. 7, E minor	5	9	70	16	2·0
No. 8, F major	22	14	38	26	1·7
No. 9, F minor	16	4	59	21	1·9
No. 10, G major	17	8	43	32	1·9
No. 11, G minor	21	13	41	25	1·7
No. 12, A major	6	11	61	22	2·0
No. 13, A minor	6	13	49	32	2·1
No. 14, B♭ major	21	13	49	17	1·6
No. 15, B minor	14	15	50	21	1·8
Average	13	10	54	23	1·9

*Complete table reproduced from 2, p. 291.

APPENDIX B

FREQUENCIES OF RELATIVELY ACCENTED DISSONANT INTERVALS IN THE TWO PART INVENTIONS

TABLE I *

Invention	Number of Intervals having relative Metric Accents	Number of relatively accented Dissonant Intervals	Percentage of relatively accented Dissonance
I	142	36	25·3%
II	185	44	23·8%
III	155	38	24·5%
IV	140	36	25·7%
V	239	56	23·4%
VI	237	85	35·9%
VII	159	31	19·5%
VIII	183	32	17·5%
IX	195	30	15·4%
X	87	18	20·7%
XI	175	33	18·9%
XII	206	36	17·4%
XIII	164	36	22·0%
XIV	215	34	15·8%
XV	156	48	30·8%

*Complete table reproduced from 3, p. 278.

APPENDIX C

PER CENT OF ROUGH AND SMOOTH MEASURES ACCORDING TO MELODIC ACCENTS

TABLE I*

Invention	Upper Voice		Lower Voice	
	Smooth	Rough	Smooth	Rough
	%	%	%	%
I	13·6	86·4	18·2	81·8
II	29·6	70·4	28·0	72·0
III	37·3	62·7	35·1	64·9
IV	28·8	71·2	24·0	76·0
V	18·8	81·2	9·4	90·6
VI	27·5	72·5	17·7	82·3
VII	30·4	69·6	8·7	91·3
VIII	20·7	79·3	27·3	72·7
IX	17·6	82·4	5·9	94·1
X	37·5	62·5	29·0	71·0
XI	13·0	87·0	0·0	100·0
XII	9·5	90·5	14·3	85·7
XIII	20·0	80·0	8·0	92·0
XIV	40·0	60·0	25·0	75·0
XV	27·3	72·7	22·7	77·3

*Complete table reproduced from 4, p. 174.

APPENDIX D

PER CENT OF ROUGH AND SMOOTH MEASURES ACCORDING TO DURATIONAL ACCENTS

TABLE II*

Invention	Time Signature	Key	Upper		Lower		Macro-	
			% R	% S	% R	% S	% R	% S
I	C	C	13·6	86·4	13·6	86·4	9·0	91·0
II	C	c	77·1	22·9	77·1	22·9	11·1	88·9
III	3/8	D	8·4	91·6	11·8	88·2	8·4	91·6
IV	3/8	d	11·5	88·5	5·7	94·3	9·6	90·4
V	C	E♭	59·4	40·6	40·6	59·4	6·4	93·6
VI	3/8	E	27·4	72·6	22·6	77·4	3·2	96·8
VII	C	e	34·8	65·2	13·0	87·0	13·0	87·0
VIII	3/4	F	6·1	93·9	6·1	93·9	6·1	93·9
IX	3/4	f	76·5	23·5	61·7	38·3	2·9	97·1
X	9/8	G	3·1	96·9	9·3	90·7	3·1	96·9
XI	C	g	34·8	65·2	43·4	56·6	4·3	95·7
XII	12/8	A	9·5	90·5	38·1	61·9	4·7	95·3
XIII	C	a	20·0	80·0	16·0	86·0	4·0	96·0
XIV	C	B♭	15·0	85·0	20·0	80·0	0·0	100·0
XV	C	b	22·7	77·3	31·8	68·2	13·6	86·4

*Complete table reproduced from 4, p. 177.

APPENDIX E

PIANO PEDAGOGUES

The rating sheet in Appendix F was sent to the following piano pedagogues:

Bardas, Stefan, School of Music, North Texas State University, Denton, Texas, 76201

Cass, Richard, School of Music, North Texas State University, Denton, Texas, 76201

Drath, Jan, School of Music, North Texas State University, Denton, Texas, 76201

Elliott, Denson, School of Music, Murray State University, Murray, Kentucky, 42071

Rogers, Robert, School of Music, North Texas State University, Denton, Texas, 76201

Terhune, Russell, School of Music, Murray State University, Murray, Kentucky, 42071

Winter, John, School of Music, Murray State University, Murray, Kentucky, 42071

APPENDIX F

RATING SHEET*

Please indicate the degree of difficulty of each of the Two Part Inventions on the scales below by circling the appropriate number.

Invention No. I

	Simple			Medium			Difficult		
a. Technical difficulty	1	2	3	4	5	6	7	8	9
b. Memorization difficulty	1	2	3	4	5	6	7	8	9
c. Interpretative difficulty	1	2	3	4	5	6	7	8	9
d. Other _____	1	2	3	4	5	6	7	8	9

Invention No. II

	Simple			Medium			Difficult		
a. Technical difficulty	1	2	3	4	5	6	7	8	9
b. Memorization difficulty	1	2	3	4	5	6	7	8	9
c. Interpretative difficulty	1	2	3	4	5	6	7	8	9
d. Other _____	1	2	3	4	5	6	7	8	9

Invention No. III

	Simple			Medium			Difficult		
a. Technical difficulty	1	2	3	4	5	6	7	8	9
b. Memorization difficulty	1	2	3	4	5	6	7	8	9
c. Interpretative difficulty	1	2	3	4	5	6	7	8	9
d. Other _____	1	2	3	4	5	6	7	8	9

Invention No. IV

	Simple			Medium			Difficult		
a. Technical difficulty	1	2	3	4	5	6	7	8	9
b. Memorization difficulty	1	2	3	4	5	6	7	8	9
c. Interpretative difficulty	1	2	3	4	5	6	7	8	9
d. Other _____	1	2	3	4	5	6	7	8	9

*The rating sheets for the remaining eleven Inventions followed the same form.

APPENDIX G

TEMPOS FOR THE TWO PART INVENTIONS SUGGESTED BY FIVE AUTHORITIES*

Inventions		Suggested Tempos				
Nos.	Beat Note	Bodky	Bischoff	Czerny	Keller	Landshoff
I	♩ =	±80	96	120	63	84
II	♩ =	±80	69	108	52	63
III	♩. =	±60	60	80	46	51
IV	♩. =	±60	76	72	60	48
V	♩ =	±80	72	108	72	69
VI	𝅘𝅥𝅯 =	±120	138	144	96	104
VII	♩ =	±60	69	112	72	60
VIII	♩ =	±100	126	144	116-126	116
IX	♩ =	±60	60	116	46	58
X	♩. =	±100	100	152	108	112
XI	♩ =	±80	80	108	58	69
XII	♩. =	±60	76	84	72	56
XIII	♩ =	±80	116	104	69	80
XIV	♩ =	±60	69	88	44	69
XV	♩ =	±80	96	104	92	84

*Tempos reproduced from 1, pp. 345-370.

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