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*date:* July 31, 1995

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*from:* Eric Haskin, UNM 


*subj:* FINAL REPORT FOR SUBCONTRACT 9-XU0-D1200-1, TASK 46

This memorandum along with the attached sheets will serve as the final report, which you requested for the closeout of Subcontract 9-XU0-D1200-1, Task 46.

The attached sheets are progress reports submitted to Mike Butner, LANL, for the months of October 1991 to May 1992. Also included are review comments sent to Kent Sasser, LANL, on SAIC-368-91-035 and a copy of Min Huang's Master's thesis, *Event Group Importance Measures for Top Event Frequency Analyses*.

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MASTER

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*date:* December 7, 1991  
*to:* Mike Butner, LANL  
  
*from:* Eric Haskin, UNM  
*subj:* Progress Report for November

The tasks undertaken in November included:

1. Participating in the K Reactor PRA review meeting in Rockville, MD on November 13, 14, and 15.
2. Reviewed and revised backend portion of review team report and provided input to Willard Thomas (SEA) on November 20 & 21.
3. Completed the extension of the distribution shift statistic described by Iman and Hora<sup>1</sup> to groups of events in TEMAC.
4. Began investigating the treatment of correlated event probabilities in TEMAC.

---

<sup>1</sup> R. L. Iman and S. C. Hora, "A Robust Measure of Uncertainty Importance for Use in Fault Tree System Analysis, Risk Analysis, Vol. 10, No. 3, 1990.

MASTER

date: January 6, 1991

to: Mike Butner, LANL

from: Eric Haskin, UNM

subj: Progress Report for December 1991

The tasks undertaken in December included:

1. Developed TEMAC version 2C, which applies the distribution shift statistic described by Iman and Hora<sup>1</sup> to groups of events in TEMAC and incorporates a correct treatment of totally correlated event probabilities.
2. Delivered an executable version to TEMAC 2B (the version Dale Talbott has been using at LANL) to Greg Wyss at Sandia. In turn, received an executable of Greg's interim version of the Latin Hypercube sampling code that is being upgraded by Sandia to include a wider variety of distributions.
3. Began testing the "robust" uncertainty importance measure recommended by Iman and Hora.<sup>1</sup> Investigating some peculiar results found when comparing TEMAC regression results to hand calculations.
4. Responded to questions from LANL and SEA regarding TEMAC and K reactor.

---

<sup>1</sup> R. L. Iman and S. C. Hora, "A Robust Measure of Uncertainty Importance for Use in Fault Tree System Analysis, Risk Analysis, Vol. 10, No. 3, 1990.



*date:* April 7, 1992

*to:* Mike Butner, LANL

*from:* Eric Haskin, UNM

*subj:* Progress Report for March 1992

The tasks undertaken in March included:

1. Received and began reviewing WSRC studies on Loss of River Water and Loss of Heat Sink.
2. Began preparing a paper on sensitivity/uncertainty results for next PSA conference to be co-authored by Dale Talbott and Kent Sasser.
3. Work continued on Min Huang's Master's thesis.

*copy:* Min Huang

*date:* April 7, 1992

*to:* Mike Butner, LANL

*from:* Eric Haskin, UNM

*subj:* Progress Report for March 1992

The tasks undertaken in March included:

1. Received and began reviewing WSRC studies on Loss of River Water and Loss of Heat Sink.
2. Began preparing a paper on sensitivity/uncertainty results for next PSA conference to be co-authored by Dale Talbott and Kent Sasser.
3. Work continued on Min Huang's Master's thesis.

*copy:* Min Huang

*date:* May 6, 1992

*to:* Mike Butner, LANL

*from:* Eric Haskin, UNM

*subj:* Progress Report for April 1992

The tasks undertaken in April included:

1. Completed reviewing WSRC studies on Loss of River Water and Loss of Heat Sink.
2. Completed summary of a paper on sensitivity/uncertainty results for next PSA conference to be co-authored by Kent Sasser and Desmond Stack.
3. Responded to LANL proposal for continued assistance with K Reactor risk and uncertainty analyses.
4. Work continued on Min Huang's Master's thesis.

*copy:* Min Huang

date: April 27, 1992

to: Kent Sasser, LANL

from: Eric Haskin, UNM

subj: REVIEW COMMENTS ON "LOSS OF RIVER WATER (LORW) INITIATOR FREQUENCY CALCULATIONS," SAIC-368-91-035

REVIEW COMMENTS ON "LOSS OF HEAT SINK (LOHS) INITIATOR FREQUENCY CALCULATIONS SAIC-368-91-0037 Rev. 0.

As you requested, I have reviewed the subject reports as well a Harry Martz's comments. I agree with Harry's comments; however, I believe we should stress to WSRC that most of the LORW comments apply as well to the LOHS analysis. I have the following additional comments on the LORW document.

1. Assumption 17 is that "loss of AC pump supply to both pumphouses is treated as a loss of grid power." This in essence dismisses the possibility of any combination of events that could cause loss of AC power to both pumphouses without causing loss of grid power. Substations 451-D and 504-1G supply both pumphouses. The line from 451-D to pumphouse 681-1G crosses the line from substation 504-1G to pumphouse 681-3G. It seems counter-intuitive that the chance of losing AC to both pumphouses without loss of grid would be less than the frequency of common cause failure of 16 pumps (the dominant cut set at  $3.2 \times 10^{-4}$  per year). This assumption should either be re-examined or better justified. Also, possible failures of the 15/20 MVA 115/4.16 kV pumphouse transformers should be addressed.
2. Assumption 11 says "the pumphouse operator will not have sufficient time to reopen the cone valves of all nine operating pumps." Why would the operator have to re-open more than three cone valves to meet the success criteria?
3. Assumption 2 states that "if less than 56,000 gpm is provided, further actions are required." Perhaps this refers to the Safety Evaluation Report restart criterion that after a LORW event the reactor can be cooled for a period of 72 hours without recovery of the RWS. Such actions would presumably be initiated given any of the multiple indications of LORW mentioned under Assumption 13. Do the setpoints for these indicators actually correspond to  $<56,000$  gpm RW flow, or would LORW actions actually be initiated given less severe RWS failures. In the latter case, less severe RWS failures would actually constitute initiating events (the 3/20 success criteria would not reflect actual operations).

4. Probably due to the nature of the dominant cut sets and the fact that the LORW analysis is an initiating event analysis, there is essentially no treatment of recovery (aside from re-opening cone valves) in the LORW analysis.

Yet, how would recovery of RWS factor into the analysis of accident sequences initiated by LORW? Should RWS recovery be treated in this document, elsewhere, or not at all?

5. The appearance of external flood as the lone external event in the LORW fault tree at first glance seems strange. Some discussion of why other external events are not important or are treated elsewhere would be appropriate.
6. The basis for assumption 7 should be stated in the report, since most would not have access to more informal notes (Ref. 3). Alternatively (and better) include such notes as an appendix.

EVENT GROUP IMPORTANCE MEASURES FOR  
TOP EVENT FREQUENCY ANALYSES

by

MIN HUANG

B.S. Chang Chun Institute of Optics and Fine Mechanics

THESIS

Submitted in Partial Fulfillment of the  
Requirements for the Degree of

Master of Science in Nuclear Engineering

The University of New Mexico  
Albuquerque, New Mexico

December, 1992

## ACKNOWLEDGEMENTS

The author wishes to gratefully acknowledge his advisor, Professor F. Eric Haskin, for his guidance, friendship, and many contributions to this work including his derivations of the general variance reduction equations and his conscientious editing of the manuscript.

The author also wishes to acknowledge the financial support and technical cooperation of Los Alamos National Laboratory. In particular, thanks are due to Mr. Desmond Stack, Mr. Kent Sasser, and Mr. Dale Talbott.

EVENT GROUP IMPORTANCE MEASURES FOR  
TOP EVENT FREQUENCY ANALYSES

by  
MIN HUANG

ABSTRACT OF THESIS

Submitted in Partial Fulfillment of the  
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The University of New Mexico  
Albuquerque, New Mexico

December, 1992



# EVENT GROUP IMPORTANCE MEASURES FOR TOP EVENT FREQUENCY ANALYSIS

Min Huang

B.S., Optical Physics, Chang Chun Institute of Optics and Fine Mechanics, 1984

M.S., Nuclear Engineering, University of New Mexico, 1992

Three traditional importance measures, risk reduction, partial derivative, and variance reduction, have been extended to permit analyses of the relative importance of groups of underlying failure rates to the frequencies of resulting top events. The partial derivative importance measure was extended by assessing the contribution of a group of events to the gradient of the top event frequency. Given the moments of the distributions that characterize the uncertainties in the underlying failure rates, the expectation values of the top event frequency, its variance, and all of the new group importance measures can be quantified exactly for two familiar cases: 1) when all underlying failure rates are presumed independent, and 2) when pairs of failure rates based on common data are treated as being equal (totally correlated). In these cases, the new importance measures, which can also be applied to assess the importance of individual events, obviate the need for Monte Carlo sampling. The event group importance measures are illustrated using a small example problem and demonstrated by applications made as part of a major reactor facility risk assessment. These illustrations and applications indicate both the utility and the versatility of the event group importance measures.

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## 1 INTRODUCTION

Probabilistic risk assessments of nuclear facilities have often used importance measures to compare the contributions of individual initiating and basic events to the overall fuel damage frequency and the frequencies of dominant accident sequences. Several measures indicate the sensitivity of such top event frequencies to changes in the value of an initiating event frequencies or a basic event probabilities. The more commonly applied sensitivity measures, which are discussed in Section 3 of this work, are the partial derivative, the logarithmic derivative, and the risk reduction measures. In addition, uncertainty importance measures have been developed to estimate the contribution an individual initiating or basic event makes to the uncertainty in top event frequencies.

A widely used computer code for calculating event importance measures is TEMAC (Top Event Matrix Analysis Code).<sup>1,2</sup> Given a single estimate of the initiating event frequencies and basic event probabilities, TEMAC calculates the top event frequency and various sensitivity measures for each of the initiating and basic events. Alternatively, as discussed in Chapter 2, probability distributions can be used to characterize the uncertainty in each of the initiating event frequencies and basic event probabilities. TEMAC is designed to accept multiple estimates of the initiating event frequencies and basic event probabilities generated by Monte Carlo sampling via the LHS (Latin Hypercube Sampling) code.<sup>3</sup> From LHS-generated input, TEMAC estimates the means, medians, percentiles and variances of the corresponding distributions for the top event frequency and the sensitivity measures for each event. Using such LHS-generated input, TEMAC can also be used to calculate an uncertainty importance measure developed by Iman and Hora, which assesses the contribution of each basic and initiating event to the uncertainty in the logarithm of the top event frequency.<sup>4</sup>

The Engineering and Safety Analysis Group (N-6) of the Los Alamos National Laboratory recently performed an independent probabilistic risk assessment of the K Production Reactor, which is located at the Department of Energy Savannah River Site.<sup>5</sup> For this assessment, the development of measures of the importance of groups of basic and initiating events was initiated at The University of New Mexico. Treating all initiating event frequencies and basic event failure rates as independent, the new event group importance measures were quantified for fourteen dominant accident sequences and for the overall fuel damage frequency using the following event groups: initiating events, electrical failures, instrumentation failures, common cause failures, human errors, and nonrecovery events. Additional analyses with other event groups were also performed to examine the importance of key plant systems, various types of electrical failures, and different types of initiating events. The results of these computations significantly enhanced the insights obtained from the K Reactor risk assessment.<sup>6</sup>

This work documents and furthers the develops event group importance measures. Chapter 3 discusses the extension of the traditional event-based risk reduction and partial derivative sensitivity measures to groups of events. The partial derivative importance measure is extended by assessing the contribution of a group of events to the gradient of the top event frequency. In Chapter 4, the variance reduction importance measure is developed to assess the contributions of both individual events and event groups to uncertainty in the top event frequency. Given the moments of the distributions that characterize the uncertainties in the underlying failure rates, the expectation values of all of three importance measures are quantified exactly for two significant cases: 1) when all underlying failure rates are presumed independent, and 2) when pairs of failure rates based on common data are treated as being equal (totally correlated). The ability to treat totally

correlated failure rates is a significant extension of the capability demonstrated in the K Reactor risk assessment. In both cases, the new importance measures, obviate the need for Monte Carlo sampling. The event group importance measures are illustrated in Chapters 2, 3, and 4 using an example problem. Demonstration applications to the K Reactor fuel damage equation are presented in Chapter 5. These illustrations and applications indicate both the utility and the versatility of the event group importance measures.

To facilitate computations of the new event group importance measures the TEMAC computer code was modified and adapted to run on a personal computer. Along with the addition of the new group importance measures, the modified version of TEMAC calculates logarithmic derivatives and the exact variance of the top event frequency for independent and totally correlated input cases. Appendix A describes the input requirements for the modified version of TEMAC. Other appendices contain illustrative input and output. Input compatibility with previous versions of TEMAC has been maintained.

## 2 TOP EVENTS AND THEIR FREQUENCIES

### 2.1 Boolean Top Event Expressions

Estimated rates of occurrence of specified damage states in complex facilities such as nuclear power plants are often evaluated through the use of event trees and fault trees. The top event refers to the occurrence of a specified damage state. Sets of events that lead to the top event (specified damage state) are called cut sets. A minimal cut set is a cut set that has no other cut set as a subset. If any event is removed from a minimal cut set, the resulting reduced set of events would not cause the top event. Boolean logic is used to express the top event,  $T$ , as the union of the minimal cut sets,  $S_k$ :

$$T = \bigcup_{k=1}^K S_k$$

Each minimal cut set may in turn be represented as the intersection (Boolean AND) of specific basic or initiating events. For a general top event expression, the preceding equation can be written in the form

$$T = \bigcup_{k=1}^K S_k = \bigcup_{k=1}^K \bigcap_{\ell=1}^L E_{k\ell}$$

where  $L$  is the number of basic and initiating events,  $E_\ell$ , and

$$\begin{aligned} E_{k\ell} &= E_\ell && \text{if event } E_\ell \text{ occurs in minimal cut set } k \\ &= /E_\ell && \text{if the complement event } /E_\ell \text{ occurs in minimal} \\ &&& \text{cut set } k \\ &= /E_\ell + E_\ell = \Omega, && \text{the universal set, if the occurrence or} \\ &&& \text{nonoccurrence of event } \ell \text{ is irrelevant to} \\ &&& \text{minimal cut set } k. \end{aligned}$$



A simple example of a top event expression is developed in the following paragraphs. This example is used in subsequent sections to illustrate various concepts. Chapter 5 is devoted to a far more complex demonstration application.

A fluid system is any system that is used to direct the flow of liquids or gases. Modular logic can be used to delineate the minimal cut sets corresponding to failure of a fluid system.<sup>7</sup> In the modular approach the system is divided into interconnected segments, with each segment permitting fluid flow between two nodes. Failures within a segment may result from loss of fluid flow due to the failure of an active component (such as a pump or a compressor), blockage of fluid flow due to a component or support system fault (such as valve failure locally or valve actuation system failure), loss of system function due to a component or support system fault (such as a heat exchanger failure or a cooling water system failure), misdirection or diversion of fluid flow due to a component or support system fault, or loss of fluid flow due to a pipe rupture.

Figure 1 depicts part of a fluid flow system, a simple cross tie arrangement composed of two input segments (1 and 2), two output segments (3 and 4), and a single crosstie segment (5). Either input segment has sufficient capacity to supply the required flow through either or both of the output segments. Thus, by itself, blockage in segment 1 would not cause insufficient output flow. Likewise, blockage in segment 3 alone would not cause insufficient output flow. Considering only blockage failures, output flow through segment 3 would fail due to blockage in segment 3, blockages in both input segments (1 and 2), or blockages in both the adjoining input segment (1) and the crosstie segment (5). Similar logic applies to output flow through segment 4.

Using  $T$  to denote the top event, failure to achieve flow through either output segment on demand, it follows that

$$T = II * B1 * B2 + II * B3 * B4 + II * B1 * B4 * B5 + II * B2 * B3 * B5$$

Here, following the conventional practice, the summation symbol is used to denote the Boolean OR (union), and the product symbol is used to denote the Boolean AND (intersection). The events on the left hand side of the preceding equation are

$E_1 = II$  - initiating event, demand for flow through either of two output segments

$E_2 = B1$  - blockage in input segment 1

$E_3 = B2$  - blockage in input segment 2

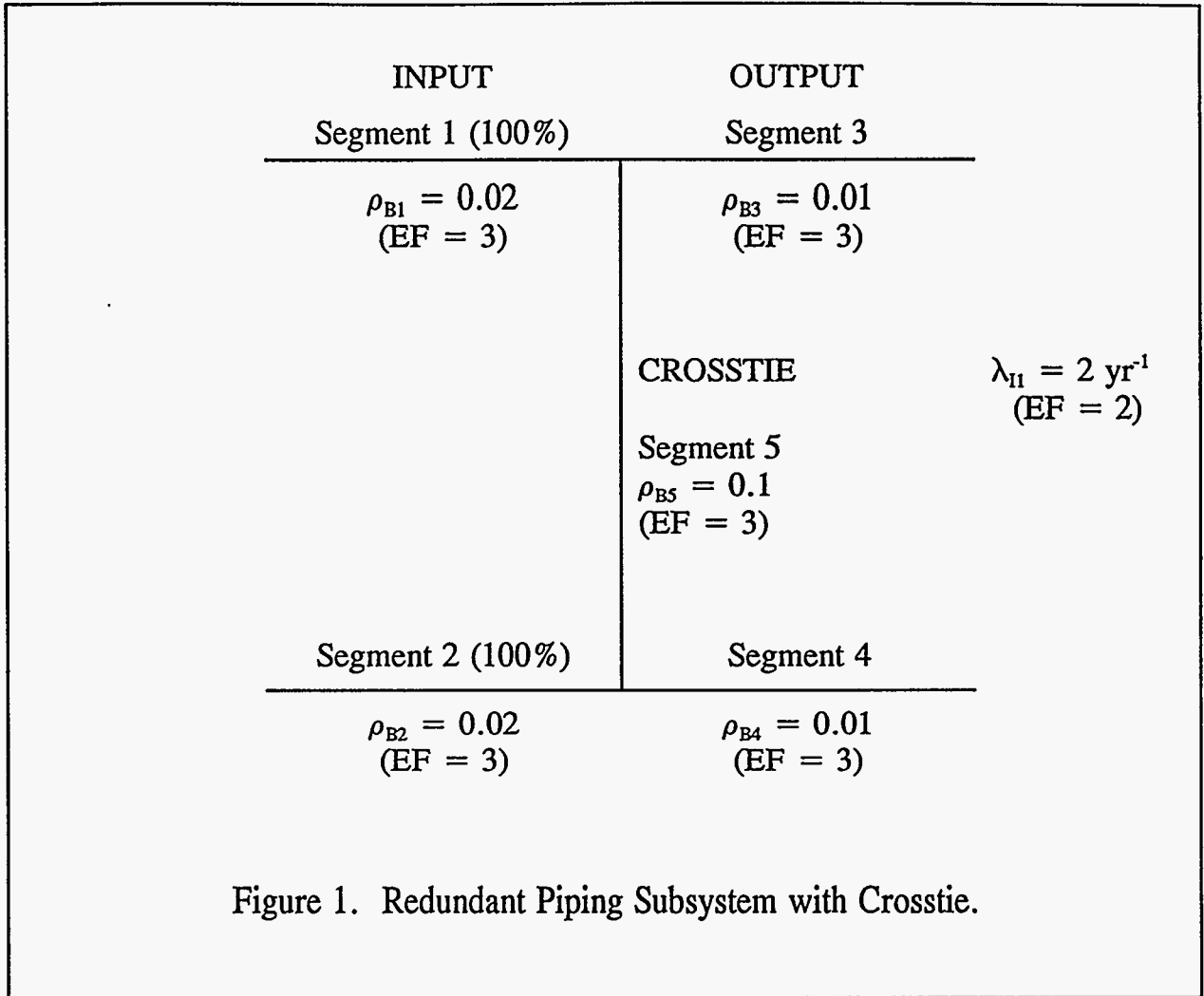
$E_4 = B3$  - blockage in output segment 3

$E_5 = B4$  - blockage in output segment 4

$E_6 = B5$  - blockage in crosstie segment 5

Each of the four minimal cut sets in the preceding Boolean expression is the intersection (Boolean AND) of three or four of these events. All other cut sets (*e.g.*,  $II * B3 * B4 * B5$ ) include one of the four minimal cut sets as a subset. In this example, each minimal cut set contains the single initiating event  $II$ . In general, either each minimal cut set contains an initiating event, or there are no initiating events (*e.g.*, if the top event were the probability of system failure per demand).

Equation (2) is an example of a coherent top event expression because it contains no complement events, only initiating and basic events. Noncoherent top event expressions are not frequently encountered in practice; nevertheless, the methods developed in the following sections encompass this possibility.



## 2.2 Top Event Quantification

If  $E$  is a basic event, for example a pump failing to start, it is presumed to occur at random but with a constant probability  $\rho_E$  per trial. For example, if there are  $n$  recorded instances in which a certain pump has been signaled to start, and if the pump failed to start in  $n_E$  of these cases, the best estimate of  $\rho_E$  is simply the observed frequency of failure,  $n_E/n$ . Conceptually, the "true" value of  $\rho_E$  is the limit of this frequency as the number of observations becomes infinitely large; that is,  $\rho_E$  is the frequentist interpretation of probability:

$$\rho_E = Pr(E) = \lim_{n \rightarrow \infty} \frac{n_E}{n} .$$

Section 2.3 discusses the treatment of uncertainties in the values of such parameters.

If  $E$  is an initiating event, its likelihood is characterized by its average rate of occurrence per unit time  $\lambda_E$ , commonly called the event's frequency. The parameter  $\lambda_E$  may be time dependent; nevertheless, there is generally a functional relationship between  $\lambda_E$  and the probability  $\rho_E$  of occurrence over some finite time interval. In this work, it will suffice to consider an infinitesimal time interval  $\delta t$ . The probability that the initiating event will occur in  $\delta t$  is simply

$$\rho_E = Pr(E|\delta t) = \lambda_E \delta t .$$

The term frequency is reserved by some to distinguish parameters like  $\rho$  and  $\lambda$ , which in principal can be estimated from occurrence data, from Bayesian and/or subjective probabilities. The frequentist philosophy and its limitations are discussed elsewhere.<sup>8</sup>

In this work, Greek symbols subscripted by event names or indices are used to denote frequentist probabilities and occurrence rates. Frequentist probabilities are denoted by the symbol  $\rho$ . The term frequency is used to describe occurrence rates, which are denoted by the symbol  $\lambda$  for initiating events and by the symbol  $\Phi$  for top events and cut sets. In equations where the distinction between initiating event frequencies and basic event probabilities is irrelevant, the symbol  $\Theta$  is used to represent both types of parameters.

The frequencies (or probabilities) of the top event and its minimal cut sets can be expressed as functions of the basic event probabilities and the initiating event frequencies. The functional relationships follow from the elementary rules of probability. First, the probability of the intersection of two events is given by

$$Pr(A * B) = Pr(A) \cdot Pr(B|A) = Pr(B) \cdot Pr(A|B) .$$

If the two events are independent,  $Pr(B|A) = Pr(B)$  and  $Pr(A|B) = Pr(A)$  so that

$$Pr(A*B) = Pr(A) \cdot Pr(B) \quad \text{if } A \text{ and } B \text{ are independent events.}$$

Thus the frequency  $\Phi_k$  of the k-th cut set  $S_k$  is the product of the initiating event frequency and the probabilities of the basic events in the cut set. For example, the frequencies of the first two cut sets in the example problem are

$$\Phi_1 = \lambda_{II} \Theta_{B1} \Theta_{B2} = (2 \text{ yr}^{-1})(0.02)(0.02) = 0.0008 \text{ yr}^{-1} ,$$

$$\Phi_2 = \lambda_{II} \Theta_{B3} \Theta_{B4} = (2 \text{ yr}^{-1})(0.01)(0.01) = 0.0002 \text{ yr}^{-1} .$$

The general expression for the frequency of the k-th minimal cut set given independent initiating event frequencies and basic event probabilities is

$$\Phi_k = \prod_{\ell=1}^L \Theta_{\ell}^{a_{k\ell}} (1 - \Theta_{\ell})^{c_{k\ell}} .$$

where

$$\begin{aligned} \Theta_{\ell} &= \rho_{\ell} && \text{if } E_{\ell} \text{ is a basic event,} \\ &= \lambda_{\ell} && \text{if } E_{\ell} \text{ is an initiating event,} \\ a_{k\ell} &= 1 && \text{if cut set } k \text{ contains event } E_{\ell}, \\ &= 0 && \text{otherwise, and} \\ c_{k\ell} &= 1 && \text{if cut set } k \text{ contains compliment event } E_{\ell}, \\ &= 0 && \text{otherwise.} \end{aligned}$$

The probability of the union of two events,  $A$  and  $B$ , is given by

$$Pr(A+B) = Pr(A)+Pr(B)-Pr(A * B) .$$

If the intersection of the two events  $P(A*B)$  is the null set  $\Omega$  (not possible if  $A$  and  $B$  are independent events)  $A$  and  $B$  are said to be mutually exclusive or disjoint.

$$P(A+B) = P(A) + P(B), \quad \text{if } A \text{ and } B \text{ are disjoint (mutually exclusive).}$$

Since initiating events are selected to be mutually exclusive, the preceding equation applies when the events in question are minimal cut sets which have different initiating events. However, if the events in question are minimal cut sets with the same initiating event, the cross product term  $P(A*B)$  must be considered. For example, the union of the first two minimal cut sets in the example problem is

$$\begin{aligned} \Phi(M_1+M_2) &= \Phi_{11} Pr(B1 * B2 + B3 * B4) = \Phi_{11} [\Theta_1 \Theta_2 + \Theta_3 \Theta_4 - \Theta_1 \Theta_2 \Theta_3 \Theta_4] \\ &= 2yr^{-1} [(0.02)(0.02) + (0.01)(0.01) - (0.02)(0.02)(0.01)(0.01)] \\ &= 0.0008 + 0.0002 - 0.00000008yr^{-1} \approx 0.001yr^{-1} . \end{aligned}$$

Clearly, the cross product term is negligible; that is, if two non-mutually-exclusive events have small probabilities, then  $P(A+B)$  is approximately equal to  $P(A) + P(B)$ . This so-called rare event approximation provides an upper bound on the actual top event frequency:

$$\Phi_T \leq \Phi_{\hat{T}} = \sum_{k=1}^K \prod_{\ell=1}^L \Theta_{\ell}^{a_{k\ell}} (1 - \Theta_{\ell})^{c_{k\ell}}$$

Here the upper bound  $\Phi_{\hat{T}}$  is the indicated function of the input vector

$$\bar{\Theta} = \Theta_1 \bar{u}_1 + \Theta_2 \bar{u}_2 + \dots + \Theta_L \bar{u}_L ,$$

and  $\bar{u}_\ell$  denotes a unit vector in the direction assigned to  $\Theta_\ell$ .

Unfortunately, if the probabilities of basic events are not small, the rare event approximation can result in unreasonably large estimates. For example, if  $Pr(A) = Pr(B) = 0.9$ , then  $Pr(A) + Pr(B) = 1.8$ ; whereas,  $Pr(A+B) = 0.9 + 0.9 - (0.9) \cdot (0.9) = 0.99$ . Nevertheless, for most practical problems, the rare event approximation provides a satisfactory estimate of the frequency of the top event.

### 2.3 Uncertainties

In a Bayesian, or subjectivist, framework, uncertainty in the values of the inputs of the top event frequency model is characterized by a joint probability density function  $p(\Theta_1, \Theta_2, \dots, \Theta_L)$ . This function represents the analyst's degree of belief about the possible values of the inputs and reflects the state of his knowledge about these parameters. The marginal probability density function (probability distribution) for a particular input  $\Theta_e$  is the integral of the joint probability density function over the other inputs:

$$p_e(\Theta_e) = \int_0^\infty \int_0^\infty \dots \int_0^\infty p(\Theta_1, \Theta_2, \dots, \Theta_n) d\Theta_1 d\Theta_2 \dots d\Theta_{e-1} d\Theta_{e+1} \dots d\Theta_L .$$

The cumulative distribution function is simply the integral of the marginal density function:

$$P_e(\theta_e) = Pr(\Theta_e \leq \theta_e) = \int_0^{\theta_e} p_e(\Theta_e) d\Theta_e .$$

The  $\alpha$  quantile of  $\Theta_e$  is the value of  $\theta_e$  that satisfies  $P_e(\theta_e) = \alpha$ . The median is the 0.5 quantile. The expectation value of the n-th power of  $\Theta_e$  is the n-th moment of the marginal probability density function  $p_e(\Theta_e)$

$$\langle (\Theta_e)^n \rangle = \int_0^{\infty} (\Theta_e)^n p_e(\Theta_e) d\Theta_e .$$

The first moment  $\langle \Theta_e \rangle$  is, of course, the mean.

One measure of the uncertainty in  $\Theta_e$  is its variance,

$$Var(\Theta_e) = \langle (\Theta_e)^2 \rangle - \langle \Theta_e \rangle^2 .$$

An alternative measure, the standard deviation, is simply the square root of the variance. The uncertainty in  $\Theta_e$  can also be measured by a probability interval (Bayesian confidence interval). A commonly used interval is the 0.90 interval, which is the range from the 0.05 quantile to the 0.95 quantile. A related measure that is often used to characterize input uncertainties is the error factor. The error factor is the ratio of the 0.95 quantile to the 0.05 quantile.

### 2.3.1 Lognormal Distributions

A commonly used distribution for characterizing the uncertainties in failure rates is the lognormal distribution, which is derived from a normal distribution on  $\ln(\Theta_e)$ . The lognormal distribution is usually written in the form



$$p_e(\Theta_e) = \frac{1}{\Theta_e \sigma_e \sqrt{2\pi}} \exp\left[-\frac{(\ln \Theta_e - \mu_e)^2}{2\sigma_e^2}\right]$$

where the parameters  $\mu_e \equiv \langle \ln \Theta_e \rangle$  and  $\sigma_e^2 \equiv \text{Var}(\ln \Theta_e)$  of the normal distribution for  $\ln \Theta_e$  are related to the mean  $\alpha = \langle \Theta_e \rangle$  and the error factor  $EF_e$  of the lognormal distribution for  $\Theta_e$  by the relations

$$\sigma_e^2 \equiv \text{Var}(\ln \Theta_e) = \left[ \frac{\ln(EF_e)}{1.645} \right]^2$$

$$\mu_e \equiv \langle \ln \Theta_e \rangle = \ln \alpha_e - \sigma_e^2/2$$

Given  $\sigma_e$  and  $\mu_e$ , the variance of the lognormal distribution is

$$\text{Var}(\Theta_e) = \alpha_e^2 \left[ \exp(\sigma_e^2) - 1 \right]$$

The moments of the lognormal distribution are

$$\langle \Theta_e^n \rangle = \exp(n\mu_e + n^2\sigma_e^2/2)$$

### 2.3.2 Uncertainty in the Top Event Frequency

The top event frequency has its own probability density function whose parameters (mean, percentiles, variance, etc.) have traditionally been determined by Monte Carlo sampling. This involves evaluating the top event frequency for many combinations of the input values, which are obtained by random sampling from the joint probability density function assigned to the inputs. Monte Carlo simulation thus constructs an approximation to the cumulative distribution of the top event frequency. To overcome the cost disadvantage of ordinary Monte Carlo, stratified Monte Carlo schemes based on Latin hypercube sampling have been developed to reduce the number of input combinations that have to be evaluated.<sup>3</sup>

In this work, rather than rely on Monte Carlo sampling, analytic methods are developed that permit key parameters of the probability density function for the top event frequency to be calculated as functions of the known parameters of the joint probability density functions for the inputs. The parameters calculated in this manner include the mean and the variance of the top event frequency and various importance measures described in Sections 3 and 4. Equations are developed herein to permit such calculations for two important situations: 1) when all of the inputs are independent and 2) when some of the inputs are totally correlated.

### 2.3.3 Independent Inputs

In the simplest case, the various inputs are independent (*i.e.*, they are treated as independent random variables) so that the joint probability density function is the product of the marginal probability density functions:

$$p(\vec{\Theta}) = p(\Theta_1, \Theta_2, \dots, \Theta_L) = p_1(\Theta_1)p_2(\Theta_2) \dots p_L(\Theta_L) .$$

In this case, the determination of certain parameters of the top event frequency distribution is greatly simplified because of each of the  $\Theta_e$  appears in the top event frequency equation to the first order only. In particular, the mean (expectation value) of the top event frequency is the top event frequency evaluated at the means of the inputs:

$$\langle \Phi_T \rangle = \Phi_T(\langle \vec{\Theta} \rangle)$$

To illustrate, for the example problem,

$$\Phi_T = \lambda_{II} \rho_{B1} \rho_{B2} + \lambda_{II} \rho_{B3} \rho_{B4} + \lambda_{II} \rho_{B1} \rho_{B4} \rho_{B5} + \lambda_{II} \rho_{B2} \rho_{B3} \rho_{B5}$$

and, if the inputs are all independent,

$$\begin{aligned} \langle \Phi_T \rangle &= \langle \lambda_{II} \rangle \langle \rho_{B1} \rangle \langle \rho_{B2} \rangle + \langle \lambda_{II} \rangle \langle \rho_{B3} \rangle \langle \rho_{B4} \rangle \\ &+ \langle \lambda_{II} \rangle \langle \rho_{B1} \rangle \langle \rho_{B4} \rangle \langle \rho_{B5} \rangle + \langle \lambda_{II} \rangle \langle \rho_{B2} \rangle \langle \rho_{B3} \rangle \langle \rho_{B5} \rangle . \end{aligned}$$

The variance of the top event frequency is given by

$$Var(\Phi_T) = \langle \Phi_T^2 \rangle - \langle \Phi_T \rangle^2 .$$

Expanding the rare event approximation in the cut set frequencies gives

$$\begin{aligned} Var(\Phi_T) &= \sum_{k=1}^K [\langle \Phi_k^2 \rangle - \langle \Phi_k \rangle^2] \\ &+ 2 \sum_{j=1}^{K-1} \sum_{k=j+1}^K [\langle \Phi_j \Phi_k \rangle - \langle \Phi_j \rangle \langle \Phi_k \rangle] . \end{aligned}$$

Here, the expectation values of the cut set cross products is

$$\langle \Phi_j \Phi_k \rangle = \prod_{\ell=1}^L \langle (\Theta_{\ell})^{a_{j\ell} + a_{k\ell}} \rangle \langle (1 - \Theta_{\ell})^{c_{j\ell} + c_{k\ell}} \rangle .$$

Applying the preceding equations to the example problem, yields the equation at the top of the next page, which illustrates that the variance expression, although fairly complex even for the simple example problem, can be quantified without Monte Carlo sampling provided that the second moments of the input distributions are known. The second moments can always be determined if the means and variances of the inputs are known since  $\langle \Theta^2 \rangle = Var(\Theta) + \langle \Theta \rangle^2$ .

$$\begin{aligned}
\text{Var}(\Phi_{\hat{T}}) = & \langle \lambda_{II}^2 \rangle \langle \rho_{B1}^2 \rangle \langle \rho_{B2}^2 \rangle - \langle \lambda_{II} \rangle^2 \langle \rho_{B1} \rangle^2 \langle \rho_{B2} \rangle^2 \\
& + \langle \lambda_{II}^2 \rangle \langle \rho_{B3}^2 \rangle \langle \rho_{B4}^2 \rangle - \langle \lambda_{II} \rangle^2 \langle \rho_{B3} \rangle^2 \langle \rho_{B4} \rangle^2 \\
+ & \langle \lambda_{II}^2 \rangle \langle \rho_{B1}^2 \rangle \langle \rho_{B4}^2 \rangle \langle \rho_{B5}^2 \rangle - \langle \lambda_{II} \rangle^2 \langle \rho_{B3} \rangle^2 \langle \rho_{B4} \rangle^2 \langle \rho_{B5} \rangle^2 \\
+ & \langle \lambda_{II}^2 \rangle \langle \rho_{B2}^2 \rangle \langle \rho_{B3}^2 \rangle \langle \rho_{B5}^2 \rangle - \langle \lambda_{II} \rangle^2 \langle \rho_{B2} \rangle^2 \langle \rho_{B3} \rangle^2 \langle \rho_{B5} \rangle^2 \\
& + 2[\langle \lambda_{II}^2 \rangle - \langle \lambda_{II} \rangle^2] \langle \rho_{B1} \rangle \langle \rho_{B2} \rangle \langle \rho_{B3} \rangle \langle \rho_{B4} \rangle \\
& + 2[\langle \lambda_{II}^2 \rangle \langle \rho_{B1}^2 \rangle - \langle \lambda_{II} \rangle^2 \langle \rho_{B1} \rangle^2] \langle \rho_{B2} \rangle \langle \rho_{B4} \rangle \langle \rho_{B5} \rangle \\
& + 2[\langle \lambda_{II}^2 \rangle \langle \rho_{B2}^2 \rangle - \langle \lambda_{II} \rangle^2 \langle \rho_{B2} \rangle^2] \langle \rho_{B1} \rangle \langle \rho_{B3} \rangle \langle \rho_{B5} \rangle \\
& + 2[\langle \lambda_{II}^2 \rangle \langle \rho_{B4}^2 \rangle - \langle \lambda_{II} \rangle^2 \langle \rho_{B4} \rangle^2] \langle \rho_{B1} \rangle \langle \rho_{B3} \rangle \langle \rho_{B5} \rangle \\
& + 2[\langle \lambda_{II}^2 \rangle \langle \rho_{B3}^2 \rangle - \langle \lambda_{II} \rangle^2 \langle \rho_{B3} \rangle^2] \langle \rho_{B2} \rangle \langle \rho_{B4} \rangle \langle \rho_{B5} \rangle \\
+ & 2[\langle \lambda_{II}^2 \rangle \langle \rho_{B5}^2 \rangle - \langle \lambda_{II} \rangle^2 \langle \rho_{B5} \rangle^2] \langle \rho_{B1} \rangle \langle \rho_{B2} \rangle \langle \rho_{B3} \rangle \langle \rho_{B4} \rangle.
\end{aligned}$$

### 2.3.4 Totally Correlated Inputs

Two or more inputs are totally correlated if they are equal. The number of inputs is reduced by one for each such equality specified. The general form of the rare event approximation still applies; that is,

$$\Phi_T \leq \Phi_{\hat{T}} = \sum_{k=1}^K \prod_{\ell=1}^L \theta_{\ell}^{a_{k\ell}} (1 - \theta_{\ell})^{c_{k\ell}}$$

However, the interpretation of  $\theta_{\ell}$ ,  $a_{k\ell}$ ,  $c_{k\ell}$ , and  $L$  must be generalized as follows:

- $\theta_{\ell}$  = probability (or frequency) of  $\ell$ -th independent input,
- $a_{k\ell}$  = number of appearances of events having probability (or frequency)  $\theta_{\ell}$  in cut set  $k$ ,

- $c_{k\ell}$  = number of appearances of complements of events assigned probability  $\Theta_\ell$  in cut set  $k$ ,
- $L$  = number of independent inputs.

For the example problem, if  $\rho_{B1} = \rho_{B2}$  and  $\rho_{B3} = \rho_{B4}$ , the preceding equation yields

$$\Phi_{\hat{T}} = \lambda_{II}(\rho_{B1})^2 + \lambda_{II}(\rho_{B3})^2 + 2\lambda_{II}\rho_{B1}\rho_{B3}\rho_{B5}$$

When pairs of events are totally correlated, the mean of the top event frequency can not be obtained by evaluating the top event frequency at the input means. The moments of the input probability density functions can, however, be used to evaluate the mean of the top event frequency. The expectation value of the top event frequency for the example problem with the specified total correlations is

$$\langle \Phi_{\hat{T}} \rangle = \langle \lambda_{II} \rangle \langle \rho_{B1}^2 \rangle + \langle \lambda_{II} \rangle \langle \rho_{B3}^2 \rangle + 2\langle \lambda_{II} \rangle \langle \rho_{B1} \rangle \langle \rho_{B3} \rangle \langle \rho_{B5} \rangle$$

Adopting the generalized interpretation of  $\Theta_\ell$ ,  $a_{k\ell}$ ,  $c_{k\ell}$ , and  $L$ , the expressions developed in Section 2.3.4 for the variance of the top event frequency remain applicable to the case where there are totally correlated inputs. For the example problem with  $\rho_{B1} = \rho_{B2}$  and  $\rho_{B3} = \rho_{B4}$

$$\begin{aligned}
\text{Var}(\Phi_f) = & \langle \lambda_{II}^2 \rangle \langle \rho_{B1}^4 \rangle - \langle \lambda_{II} \rangle^2 \langle \rho_{B1}^2 \rangle^2 \\
& + \langle \lambda_{II}^2 \rangle \langle \rho_{B3}^4 \rangle - \langle \lambda_{II} \rangle^2 \langle \rho_{B3}^2 \rangle^2 \\
+ & 4 \langle \lambda_{II}^2 \rangle \langle \rho_{B1}^2 \rangle \langle \rho_{B3}^2 \rangle \langle \rho_{B5}^2 \rangle - 4 \langle \lambda_{II} \rangle^2 \langle \rho_{B1} \rangle^2 \langle \rho_{B3} \rangle^2 \langle \rho_{B5} \rangle^2 \\
& + 2[\langle \lambda_{II}^2 \rangle - \langle \lambda_{II} \rangle^2] \langle \rho_{B1}^2 \rangle \langle \rho_{B3}^2 \rangle \\
& + 4[\langle \lambda_{II}^2 \rangle \langle \rho_{B1}^3 \rangle - \langle \lambda_{II} \rangle^2 \langle \rho_{B1} \rangle \langle \rho_{B1}^2 \rangle] \langle \rho_{B3} \rangle \langle \rho_{B5} \rangle \\
& + 4[\langle \lambda_{II}^2 \rangle \langle \rho_{B3}^3 \rangle - \langle \lambda_{II} \rangle^2 \langle \rho_{B3} \rangle \langle \rho_{B3}^2 \rangle] \langle \rho_{B1} \rangle \langle \rho_{B5} \rangle.
\end{aligned}$$

If the top event expression is coherent, there are no complement events so that  $c_{kl} = 0$ , and, even if there are total correlations between some failure rates, the top event frequency estimate under the rare event approximation is a monotonic function of each of the remaining, independent inputs. In this case, the median of the top event frequency is equal to the top event frequency evaluated at the medians of the inputs. This is not necessarily true if there are both complement events and total correlations, because the local extrema may exist in the top event frequency function in such cases.

Tables 1 and 2 summarize the key parameters of the lognormal input distributions and the top event frequency distribution, respectively, for the example problem. Both the case in which all inputs are treated as independent and the case with the total correlations  $\rho_{B1} = \rho_{B2}$  and  $\rho_{B3} = \rho_{B4}$  are included.

Table 1. Input distribution parameters for example problem.

Input	Mean	Error Factor	Variance	Moments		
				Second	Third	Fourth
$\lambda_{I1}$	2	2	0.777	4.78	13.6	46.4
$\rho_{B1}$	0.02	3	2.25e-4	6.25e-4	3.05e-5	2.32e-6
$\rho_{B2}$	0.02	3	2.25e-4	6.25e-4	3.05e-5	2.32e-6
$\rho_{B3}$	0.01	3	5.62e-5	1.56e-4	3.81e-6	1.45e-7
$\rho_{B4}$	0.01	3	5.62e-5	1.56e-4	3.81e-6	1.45e-7
$\rho_{B5}$	0.1	3	5.62e-3	1.56e-2	3.81e-3	1.45e-3

Table 2. Parameters of top event frequency distribution for example problem.

$\Phi_T = \lambda_{I1} \rho_{B1} \rho_{B2} + \lambda_{I1} \rho_{B3} \rho_{B4} + \lambda_{I1} \rho_{B1} \rho_{B4} \rho_{B5} + \lambda_{I1} \rho_{B2} \rho_{B3} \rho_{B5}$		
	All Independent Inputs	Total Correlations $\rho_{B1} = \rho_{B2} \text{ \& } \rho_{B3} = \rho_{B4}$
Median	6.23e-4	6.23e-4
Mean	1.08e-3	1.64e-3
Variance	1.51e-6	1.08e-5

### 3 SENSITIVITY MEASURES

Given the uncertainties that typically exist in the values of the inputs ( $\rho$ 's and  $\lambda$ 's), it is appropriate to assess the sensitivity of the estimated top event frequency to changes in the input values. This is commonly done by changing one input at a time in a consistent manner and observing the resulting changes in the estimated top event frequency. The objective is to prioritize (rank order) the inputs according to the magnitude of the individual sensitivities obtained. A variety of ways exist in which individual inputs can be changed. A corresponding variety of sensitivity measures exist, each with a slightly different interpretation.

#### 3.1 Partial Derivative, $PD_e$

A partial derivative of the top event frequency  $\Phi_T$  indicates the change in  $\Phi_T$  per infinitesimal change in one of the inputs with all other inputs held constant. The partial derivative is sometimes called the Birnbaum Importance.<sup>9</sup> It is a widely used measure of the importance of basic and initiating events and is denoted here by the symbol  $PD_e$ .

$$PD_e = \frac{\partial \Phi_T}{\partial \theta_e} .$$

Using the small probability approximation, the partial derivative is given by

$$PD_e = \sum_{k=1} \prod_{\ell=1} [a_{k\ell} \theta_\ell^{a_{k\ell}-1} (1-\theta_\ell)^{c_{k\ell}} - c_{k\ell} \theta_{k\ell}^{a_{k\ell}} (1-\theta_{k\ell})^{c_{k\ell}-1}] .$$

To illustrate, for initiating event  $II$  of the example problem,

$$PD_{II} = \rho_1 \rho_2 + \rho_3 \rho_4 + \rho_1 \rho_4 \rho_5 + \rho_2 \rho_3 \rho_5 .$$



The right hand side of the preceding equation is the conditional probability of the top event given the occurrence of the initiating event  $II$ . This interpretation of the partial derivative holds for any initiating event; that is, the top event frequency can always be expressed as a sum of the products of initiating event frequencies and corresponding conditional probabilities of the top event.

If all of the inputs vary independently, the top event expression under the rare event approximation is first order in each input and may be expressed as

$$\Phi_{\hat{f}} = A_{0e} + PD_e \Theta_e .$$

Both  $A_{0e}$  and  $PD_e$  are composite variables that are independent of  $\Theta_e$  and are first order in each of the remaining inputs. It follows that the mean (expectation value) of a partial derivative is the partial derivative evaluated at the means of the inputs. For initiating event  $II$  of the example problem,

$$\begin{aligned} \langle PD_{II} \rangle &= \langle \rho_{B1} \rangle \langle \rho_{B2} \rangle + \langle \rho_{B3} \rangle \langle \rho_{B4} \rangle + \langle \rho_{B1} \rangle \langle \rho_{B4} \rangle \langle \rho_{B5} \rangle + \langle \rho_{B2} \rangle \langle \rho_{B3} \rangle \langle \rho_{B5} \rangle \\ &= 0.0004 + 0.0001 + 0.00002 + 0.00002 \\ &= 0.00054 \end{aligned}$$

In contrast, if two or more inputs are totally correlated, then as for the top event frequency, the means of some partial derivatives can not be obtained by evaluating these partial derivatives at the input means. If totally correlated inputs are involved, the moments of the input distributions can be used to evaluate the means of such partial derivatives. For example, given total correlations between  $\rho_1$  and  $\rho_2$  and between  $\rho_3$  and  $\rho_4$  in the example problem, the expectation value of the partial derivative of the top event frequency with respect to  $\lambda_{II}$  is

$$\langle PD_{II} \rangle = \langle \rho_{B1}^2 \rangle + \langle \rho_{B3}^2 \rangle + 2 \langle \rho_{B1} \rangle \langle \rho_{B4} \rangle \langle \rho_{B5} \rangle .$$

Using the fact that  $\langle \Theta^2 \rangle = \langle \Theta \rangle^2 + \text{Var}(\Theta)$ , the preceding equation yields

$$\langle PD_{II} \rangle = \langle \rho_{B1} \rangle^2 + \langle \rho_{B3} \rangle^2 + 2\langle \rho_{B1} \rangle \langle \rho_{B4} \rangle \langle \rho_{B5} \rangle + Var(\rho_{B1}) + Var(\rho_{B2}).$$

The first three terms on the right hand side are quantitatively the same as the result for the independent case,  $5.4 \times 10^{-4}$ . Since  $Var(\rho_1) = 2.25 \times 10^{-4}$  and  $Var(\rho_3) = 5.62 \times 10^{-5}$ , the preceding equation yields  $PD_{II} = 8.21 \times 10^{-4}$ . This result illustrates that total correlations between pairs of inputs tend to increase the magnitudes of the partial derivatives because nonlinear terms are introduced in the top event expression. The means of the partial derivatives with respect to each of the inputs in the example problem are listed in Table 3. Note that the only mean partial derivative that is not changed as a result of inducing the indicated total correlations is  $PD_{B5}$ . This partial derivative remains linear in the other independent variables.

Table 3. Mean partial derivatives for the example problem.

Event	All Inputs Independent	Total Correlations $\rho_{B1} = \rho_{B2} \ \& \ \rho_{B3} = \rho_{B4}$
II	5.40e-4	8.21e-4
B1	4.20e-2	8.40e-2
B2	4.20e-2	8.40e-2
B3	2.40e-2	4.80e-2
B4	2.40e-2	4.80e-2
B5	8.00e-4	8.00e-4

### 3.2 Normalized Contribution to Square of Mean Gradient, $NMPD_e$

The gradient of a function of two or more variables is a vector quantity whose direction is that in which the function changes most rapidly (has the largest directional derivative). The magnitude of the gradient is equal to the largest directional derivative at the point of interest in multidimensional space. The gradient of the top event frequency is the vector sum of components attributable to each of the independent inputs,

$$\vec{\nabla}\Phi_T = \sum_{\ell=1}^L \frac{\partial\Phi_T}{\partial\Theta_\ell} \vec{u}_\ell .$$

Here  $\vec{u}_\ell$  denotes a unit vector in the  $\Theta_\ell$  direction. Similarly, the square of the magnitude of the gradient is a scalar sum of components attributable to each of the independent inputs:

$$|\vec{\nabla}\phi_T|^2 = \sum_{\ell} \left[ \frac{\partial\phi_T}{\partial\Theta_\ell} \right]^2 .$$

The preceding equation suggests a way of normalizing the partial derivative to reflect the fractional contribution made by a single input to the gradient. Specifically, what can be estimated analytically is the normalized contribution to the square of the mean gradient:

$$NMPD_e = \frac{\langle PD_e \rangle^2}{\sum_{\ell=1}^L \langle PD_\ell \rangle^2} = \frac{1}{\langle \vec{\nabla}\phi_T \rangle^2} \left\langle \frac{\partial\phi_T}{\partial\theta_e} \right\rangle^2 .$$

This normalized sensitivity measure is the fractional (or percentage) contribution that a particular input  $\Theta_e$  makes to the square of the magnitude of the mean gradient.

Note that  $NMPD_e$  is normalized so that the sum over all independent inputs is unity; that is,

$$\sum_{\ell} NMPD_{\ell} = 1 .$$

This permits the sensitivity measure to be extended to assess the sensitivity of the gradient to groups of events. The normalized contribution to the square of the mean gradient for a group of events is simply

$$NMPD_g = \sum_{\ell \in g} NMPD_{\ell} ,$$

where the sum over  $\ell \in g$  indicates inclusion of all terms  $NMPD_{\ell}$  corresponding to the independent inputs  $\Theta_{\ell}$  that characterize failure rates of events in the group. The normalized contribution to the square of the mean gradient for an event group is the fraction (or percentage) of the square of the mean gradient attributable to events in the group. It indicates the relative rate of change in the top event frequency achievable by changing the probabilities (frequencies) of events in the group. If the event groups are mutually exclusive, the normalized contributions to the square of the mean gradient for all event groups sum to unity.

Table 4 lists the normalized contributions to the mean gradient for the example problem. The values of  $NMPD_e$  are listed for the case in which all failure rates are treated as independent inputs and for the case in which the total correlations  $\rho_{B1} = \rho_{B2}$  and  $\rho_{B3} = \rho_{B4}$  are induced. Notice that the rankings of these sensitivity measures are the same as the corresponding partial derivatives. This is always true because the normalization process does not alter the rank-ordering of the derivative-based sensitivity measures.

Table 4 also presents normalized contributions to the mean gradient of the example problem by event groups. Events *B1* and *B2* are grouped together as are events *B3*, *B4*, and *B5*. It is important to note that two totally correlated failure rates must be assigned to the same event group. This is not a practical limitation because such correlations are only induced when failure rates are estimated from data for similar components.

Table 4. Mean partial derivatives and normalized contributions to the square of the mean gradient for the example problem.

Event	All Inputs Independent			Total Correlations $\rho_{B1} = \rho_{B2}$ and $\rho_{B3} = \rho_{B4}$		
	$\langle PD_e \rangle$	$NMPD_e$	Rank	$\langle PD_e \rangle$	$NMPD_e$	Rank
I1	5.40e-4	0.0062%	6	8.21e-4	0.0072%	5
B1	4.20e-2	37.7%	1.5	8.40e-2	75.4%	1.5
B2	4.20e-2	37.7%	1.5			
B3	2.40e-2	12.3%	3.5	4.80e-2	24.6%	3.5
B4	2.40e-2	12.3%	3.5			
B5	8.00e-4	0.0137%	5	8.00e-4	0.0068%	6
Event Group		$NMPD_e$	Rank		$NMPD_e$	Rank
I1	--	0.0062%	4	--	0.0072%	4
B1, B2	--	75.4%	1	--	75.4%	1
B3, B4	--	24.6%	2	--	24.6%	2
B5	--	0.0137%	3	--	0.0068%	3

### 3.3 Logarithmic Derivatives, $LD_e$ and $NMLD_g$

The total derivative of the top event frequency may be expressed in terms of the partial derivatives as

$$d\Phi_T = \nabla\Phi_T \cdot d\bar{\Theta} = \frac{\partial\Phi_T}{\partial\Theta_1}d\Theta_1 + \frac{\partial\Phi_T}{\partial\Theta_2}d\Theta_2 + \dots + \frac{\partial\Phi_T}{\partial\Theta_L}d\Theta_L$$

From this equation, it is clear that the inputs having the largest partial derivatives produce the largest change in the total derivative. That is, the partial derivative measures the change induced in the top event frequency per unit change in an input. One drawback of the partial derivative as a sensitivity measure is that a given change in one variable  $d\theta_1$  may be a large fractional change if the variable is small but a small fractional change if the variable is large. This drawback can be addressed by rewriting the preceding equation in terms of fractional changes:

$$\frac{d\Phi_T}{\Phi_T} = \left[ \frac{\partial\ln\Phi_T}{\partial\ln\Theta_1} \right] \frac{d\Theta_1}{\Theta_1} + \left[ \frac{\partial\ln\Phi_T}{\partial\ln\Theta_2} \right] \frac{d\Theta_2}{\Theta_2} + \dots + \left[ \frac{\partial\ln\Phi_T}{\partial\ln\Theta_n} \right] \frac{d\Theta_n}{\Theta_n} .$$

The quantities in parenthesis multiplying the fractional change in the inputs represent the fractional change in the output  $\phi_T$  resulting from a fractional change in the indicated input. The logarithmic derivative sensitivity measure,  $LD_e$ , which is sometimes called the *Vessely-Fussell Importance*,<sup>9</sup> is defined as

$$LD_e \equiv \frac{\partial\ln\Phi_T}{\partial\ln\Theta_e} = \frac{d\Phi_T/\Phi_T}{d\Theta_e/\Theta_e} = \frac{\Theta_e}{\Phi_T} PD_e .$$

The mean of the logarithmic derivative can only be determined analytically for trivial cases. For example, if the top event is comprised of a single minimal cut set, the logarithmic derivative is equal to one for all base and initiating events in the cut set. The capability to estimate the parameters of the logarithmic

derivative probability distribution by Monte Carlo sampling was added to the TEMAC code as part of this work.

The mean logarithmic derivative can be normalized in a manner analogous to that used for the partial derivative:

$$NMLD_e = \frac{\langle LD_e \rangle^2}{\sum_l \langle LD_l \rangle^2} .$$

This permits the logarithmic derivative sensitivity measure to be applied to groups of events according to the relation

$$NMLD_g = \sum_{l \in g} NMLD_l .$$

Because the mean logarithmic derivative cannot be evaluated analytically, however, this normalized measure has not been extensively investigated as part of this work. Values of the logarithmic derivatives for the example problem based on a Latin Hypercube sample of size 50 are provided in Table 5.

Table 5. Mean logarithmic derivatives and normalized contributions to the square of the mean logarithmic gradient for the example problem.

Event	All Inputs Independent			Total Correlations $\rho_{B1} = \rho_{B2}$ and $\rho_{B3} = \rho_{B4}$		
	$\langle LD_e \rangle$	$NMLD_e$	Rank	$\langle LD_e \rangle$	$NMLD_e$	Rank
I1	1	43.5%	1	1	27.6%	3
B1	7.76e-1	26.2%	2	1.56	67.0%	1.5
B2	7.68e-1	25.6%	3			
B3	2.24e-1	2.18%	5	4.41e-1	5.35%	4.5
B4	2.32e-1	2.35%	4			
B5	7.69e-2	0.26%	6	6.20e-2	0.11%	6
Event Group	--	$NMLD_g$	Rank	--	$NMLD_g$	Rank
I1	--	43.5%	2	--	27.6%	2
B1, B2	--	51.8%	1	--	67.0%	1
B3, B4	--	4.53%	3	--	5.35%	3
B5	--	0.26%	4	--	0.11%	4

### 3.4 Risk Reduction, $RR_g$ and $NMRR_g$

For a particular input  $\Theta_e$ , the risk reduction sensitivity measure is the decrease in the top event frequency obtained when the value of  $\Theta_e$  is changed from its point estimate to zero. Mathematically,

$$RR_e = \phi_T(\bar{\Theta}) - \phi_T(\bar{\Theta} - \bar{\Theta}_e)$$

where the vector  $\bar{\Theta}_e$  is the component of the vector  $\bar{\Theta}$  in the direction  $\bar{u}_e$ ,



$$\bar{\theta}_e = \theta_e \bar{u}_e$$

The risk reduction is sometimes called the risk reduction worth increment.

If the compliment of the (group) event  $E_e$  does not appear in the top event expression, the risk reduction for this event is simply the sum of the frequencies of the minimal cut sets containing event  $E_e$ ,

$$RR_e = \sum_{E_e \in S_k} \Phi_k .$$

If all of the  $\theta_e$  vary independently, so that  $\Phi_{\hat{T}} = A_{0e} + PD_e \theta_e$

$$RR_e = PD_e \theta_e$$

As indicated, in this case, the risk reduction is directly related to the partial derivative. The product  $PD_e \theta_e$ , which in this common case is equal to the risk reduction  $RR_e$ , is sometimes called the inspection importance.<sup>9</sup> The mean (expectation value) of  $RR_e$  is equal to the risk reduction evaluated at the means of the independent inputs.

To illustrate, if all of the failure rates for the for the example problem are independent,

$$\Phi_{\hat{T}} = \lambda_{11} \rho_{B1} \rho_{B2} + \lambda_{11} \rho_{B3} \rho_{B4} + \lambda_{11} \rho_{B1} \rho_{B4} \rho_{B5} + \lambda_{11} \rho_{B2} \rho_{B3} \rho_{B5}$$

and the risk reduction achieved by eliminating base event B1 is

$$RR_{B1} = \lambda_{11} \rho_{B1} \rho_{B2} + \lambda_{11} \rho_{B1} \rho_{B4} \rho_{B5}$$

The mean of this risk reduction is simply

$$\begin{aligned}\langle RR_{B1} \rangle &= \langle \lambda_{II} \rangle \langle \rho_{B1} \rangle \langle \rho_{B2} \rangle + \langle \lambda_{II} \rangle \langle \rho_{B1} \rangle \langle \rho_{B4} \rangle \langle \rho_{B5} \rangle \\ &= (2)(0.02)(0.02) + (2)(0.02)(0.01)(0.1) = 8.4 \times 10^{-4}\end{aligned}$$

In the case where some of the inputs are totally correlated,

$$\Phi_T = A_{0e} + A_{1e} \Theta_e + A_{2e} \Theta_e^2 + \dots$$

and

$$RR_e = A_{1e} \Theta_e + A_{2e} \Theta_e^2 + \dots$$

In this case,  $RR_e$  is not directly related to  $PD_e$ , and the mean (expectation value) of  $RR_e$  can only be determined if the moments of the underlying distributions are known. To illustrate, if the total correlations  $\rho_{B1} = \rho_{B2}$  and  $\rho_{B3} = \rho_{B4}$  are imposed in the example problem,

$$\Phi_T = \lambda_{II} \rho_{B1}^2 + \lambda_{II} \rho_{B3}^2 + 2\lambda_{II} \rho_{B1} \rho_{B3} \rho_{B5}$$

and the risk reduction achieved by forcing the probability  $\rho_{B1}$  to be zero is now

$$RR_{B1} = \lambda_{II} \rho_{B1}^2 + 2\lambda_{II} \rho_{B1} \rho_{B4} \rho_{B5}$$

Notice that eliminating event B1 also eliminates event B2 in this case because the probability  $\rho_{B1}$  is equal to the probability  $\rho_{B2}$ . The mean risk reduction associated with either event is then

$$\begin{aligned}\langle RR_{B1} \rangle &= \langle RR_{B2} \rangle = \langle \lambda_{II} \rangle \langle \rho_{B1}^2 \rangle + 2\langle \lambda_{II} \rangle \langle \rho_{B1} \rangle \langle \rho_{B3} \rangle \langle \rho_{B5} \rangle \\ &= (2)(6.25 \times 10^{-4}) + 2(2)(0.02)(0.01)(0.1) = 1.34 \times 10^{-3}\end{aligned}$$

This illustrates that total correlations tend to increase the magnitudes of risk reduction sensitivity measures.

As for the partial derivative and the logarithmic partial derivative it is convenient to normalize the mean risk reduction. In this work the normalized mean risk reduction is simply the fraction (or percentage) reduction in the mean top event frequency that would be achieved if event  $E_e$  could be eliminated,

$$NMRR_e = \frac{\langle RR_e \rangle}{\langle \Phi_T \rangle} .$$

The sum of  $NMRR_e$  over all base and initiating events is not unity except in the trivial case when each minimal cut set contains only one event.

Both the unnormalized and normalized risk reduction measures can be extended to groups of events in a straight forward manner. The risk reduction achievable by eliminating a group of events is simply

$$RR_g = \Phi_T(\bar{\Theta}) - \Phi_T(\bar{\Theta} - \bar{\Theta}_g)$$

where the scalar elements of the modified vector  $\bar{\Theta}_g$  are

$$\begin{aligned} \Theta_{g,l} &= \Theta_l \quad \text{for } l \in g , \\ \Theta_{g,l} &= 0 \quad \text{for } l \notin g . \end{aligned}$$

If complement events do not appear in the top event expression, then under the rare event approximation, the group risk reduction  $RR_g$  is equal to the sum of the frequencies of cut sets that contain events in the g-th group:

$$RR_g = \sum_{G_k \cap S_k \neq \emptyset} \Phi_k$$

The normalized mean risk reduction for a group of events is simply

$$NMRR_g = \frac{\langle RR_g \rangle}{\langle \Phi_T \rangle} .$$

Normalized and unnormalized risk reductions for the example problem are listed in Table 6 both by event and by event group. Notice that the normalization process does not affect the rank ordering of the results.

Table 6. Mean risk reductions for the example problem.

Event	All Inputs Independent			Total Correlations $\rho_{B1} = \rho_{B2}$ and $\rho_{B3} = \rho_{B4}$		
	$\langle RR_e \rangle$	$NMRR_e$	Rank	$\langle RR_e \rangle$	$NMRR_e$	Rank
I1	1.08e-3	100%	1	1.64e-3	100%	1
B1	8.40e-4	77.8%	2.5	1.33e-3	81.0%	2
B2	8.40e-4	77.8%	2.5			
B3	2.40e-4	22.2%	4.5	3.92e-4	23.9%	3
B4	2.40e-4	22.2%	4.5			
B5	8.00e-5	7.41%	6	8.00e-5	4.87%	4
Event Group	$\langle RR_g \rangle$	$NMRR_g$	Rank	$\langle RR_g \rangle$	$NMRR_g$	Rank
I1	1.08e-3	100%	1	1.64e-3	100%	1
B1, B2	8.80e-4	81.5%	2	1.33e-3	81.0%	2
B3, B4	2.80e-4	25.9%	3	3.92e-4	23.9%	3
B5	8.00e-5	7.41%	4	8.00e-5	4.87%	4

## 4 UNCERTAINTY IMPORTANCE MEASURES

### 4.1 Variance Reduction by Event, $VR_e$

Since the top event frequency  $\Phi_T$  is a function of several uncertain inputs, it is reasonable to ask how much each input contributes to the uncertainty in  $\Phi_T$ . A measure of this contribution is the reduction in the variance of  $\Phi_T$  achieved by fixing the value of one of the inputs  $\Theta_e$ . The top event conditional on a specific value of  $\Theta_e$  is denoted  $\Phi_T | \Theta_e$ . The variance of  $\Phi_T$  conditional on a specific value of  $\Theta_e$  is denoted by  $Var(\Phi_T | \Theta_e)$  and given by

$$Var(\Phi_T | \Theta_e) = \langle (\Phi_T | \Theta_e)^2 \rangle - \langle \Phi_T | \Theta_e \rangle^2$$

The variance reduction achieved by fixing the value of  $\Theta_e$  is

$$VR_e = Var(\Phi_T) - Var(\Phi_T | \Theta_e) .$$

The mean (expectation value) of the variance reduction over all possible values of  $\Theta_e$  is

$$\langle VR_e \rangle = Var(\Phi_T) - \langle Var(\Phi_T | \Theta_e) \rangle$$

where

$$\langle Var(\Phi_T | \Theta_e) \rangle \equiv \int_0^{\infty} p_e(\Theta_e) Var(\Phi_T | \Theta_e) d\Theta_e .$$

The unconditional variance of a random variable can be expressed in terms of the conditional variances as follows:<sup>10</sup>

$$Var(\Phi_T) = \langle Var(\Phi_T | \Theta_e) \rangle + Var(\langle \Phi_T | \Theta_e \rangle)$$

In words, the unconditional variance is equal to the mean of the conditional

variance plus the variance of the conditional mean. This implies that the mean variance reduction is equal to the variance of the conditional mean; that is,

$$\langle VR_e \rangle = Var(\langle \Phi_T | \Theta_e \rangle) .$$

This importance measure can be normalized by expressing the mean variance reduction as a fraction (or percentage) of the unconditional variance:

$$NMVR_e = \frac{\langle VR_e \rangle}{Var(\Phi_T)} .$$

#### 4.1.1 Independent Inputs

If all failure rates are treated as independent inputs, the top event frequency under the rare event approximation can be written in the form

$$\Phi_{\hat{T}} = A_{0e} + PD_e \Theta_e$$

where  $A_{0e}$  and  $PD_e$  are composite variables that are independent of  $\Theta_e$ . It follows that

$$\langle \Phi_{\hat{T}} | \Theta_e \rangle = \langle A_{0e} \rangle + \langle PD_e \rangle \Theta_e$$

and the mean variance reduction associated with fixing the failure rate  $\Theta_e$  is

$$\langle VR_e \rangle = Var(\langle \Phi_{\hat{T}} | \Theta_e \rangle) = \langle PD_e \rangle^2 Var(\Theta_e)$$

To illustrate, note that for the example problem,  $\langle PD_{B1} \rangle = 4.2 \times 10^{-2}$  and  $Var(\rho_{B1}) = 2.25 \times 10^{-4}$ , so that  $\langle VR_{B1} \rangle = 4.0 \times 10^{-7}$ . Since the unconditional variance for this case from Table 2 is  $Var(\Phi_T) = 1.53 \times 10^{-6}$ ,  $NMVR_{B1} = 0.26$  (26%).

#### 4.1.2 Totally Correlated Inputs

If total correlations exist between pairs of failure rates, the top event frequency under the rare event approximation can be written in the form

$$\langle \Phi_{\hat{T}} | \Theta_e \rangle = \sum_{m=0}^M \langle A_{me} \rangle \Theta_e^m .$$

It follows that the mean variance reduction associated with fixing the failure rate  $\Theta_e$  is

$$\begin{aligned} \langle VR_e \rangle &= \sum_{m=1}^M \langle A_{me} \rangle^2 \text{Var}(\Theta^{2m}) \\ &+ 2 \sum_{m=1}^{M-1} \sum_{n=m+1}^M \langle A_{me} \rangle \langle A_{ne} \rangle [ \langle \Theta^{m+n} \rangle - \langle \Theta^m \rangle \langle \Theta^n \rangle ] . \end{aligned}$$

Notice that if  $M=1$  this result reduces to the first order case presented in the previous section for independent failure rates. In essence, if total correlations exist between pairs of failure rates, higher order moments of the probability distributions of the correlated inputs are required in order to evaluate the mean variance reductions. Consider the example problem with the total correlations  $\rho_{B1} = \rho_{B2}$  and  $\rho_{B3} = \rho_{B4}$ . The expectation value of the top event frequency conditional on the failure rate  $\rho_{B1}$  is

$$\langle \Phi_{\hat{T}} | \rho_{B1} \rangle = \langle \lambda_{II} \rangle \langle \rho_{B3}^2 \rangle + 2 \langle \lambda_{II} \rangle \langle \rho_{B3} \rangle \langle \rho_{B5} \rangle \rho_{B1} + \langle \lambda_{II} \rangle \rho_{B1}^2 .$$

That is, with respect to  $\rho_{B1}$ ,

$$\begin{aligned} \langle A_0 \rangle &= \langle \lambda_{II} \rangle \langle \rho_{B3}^2 \rangle , \\ \langle A_1 \rangle &= 2 \langle \lambda_{II} \rangle \langle \rho_{B3} \rangle \langle \rho_{B5} \rangle , \\ \langle A_2 \rangle &= \langle \lambda_{II} \rangle , \end{aligned}$$

and, according to the preceding result for  $\langle VR_e \rangle$ ,

$$\begin{aligned} \langle VR_{B1} \rangle &= 4\langle \lambda_{II} \rangle^2 \langle \rho_{B3} \rangle^2 \langle \rho_{B5} \rangle^2 \text{Var}(\rho_{B1}) + \langle \lambda_{II} \rangle^2 \text{Var}(\rho_{B2}^2) \\ &\quad + 4\langle \lambda_{II} \rangle^2 \langle \rho_{B3} \rangle \langle \rho_{B5} \rangle [\langle \rho_{B3}^3 \rangle - \langle \rho_{B2}^2 \rangle \langle \rho_{B2} \rangle] . \end{aligned}$$

Quantitatively, this yields  $\langle VR_{B1} \rangle = 8.03 \times 10^{-6}$ . Since the unconditional variance for this case from Section 2.3 is  $\text{Var}(\Phi_T) = 1.08 \times 10^{-5}$ , the normalized mean variance reduction  $NMVR_{B1}$  is 74.5%.

#### 4.2 Variance Reduction by Event Group, $VR_g$

In the more general case, the variance reduction associated with fixing the failure rates associated with a group of events is sought. Based on the same logic presented in Section 4.1, it follows that

$$\langle VR_g \rangle = \text{Var}(\langle \Phi_T | \Theta_g \rangle) .$$

As in section 3.4, the elements of the vector  $\vec{\Theta}_g$  are

$$\begin{aligned} \Theta_{g,\ell} &= \Theta_\ell \quad \text{for } \ell \in g \\ \Theta_{g,\ell} &= 0 \quad \text{for } \ell \notin g \end{aligned}$$

That is, fixing this vector fixes the failure rates of all events in group  $g$ . To evaluate the mean group variance reduction analytically, the conditional mean can be written in the form

$$\langle \Phi_T | \theta_g \rangle = \sum_{k=1}^K \langle B_k \rangle \Gamma_k$$

Here each cut set  $\Phi_k$  has been factored into two terms.  $B_k$  contains those failure rates that are not associated with event group  $g$ , and  $\Gamma_k$  contains those failure rates that are associated with events in group  $g$ . Specifically,



$$B_{kg} = \prod_{\ell \notin g} \theta_{\ell}^{a_{\ell}} (1 - \theta_{\ell})^{c_{\ell}} ,$$

and

$$\Gamma_{kg} = \prod_{\ell \in g} \theta_{\ell}^{a_{\ell}} (1 - \theta_{\ell})^{c_{\ell}} .$$

The variance reduction associated with fixing the failure rates associated with events in group  $g$  can then be expressed as

$$\begin{aligned} \langle VR_g \rangle &= \sum_{k=1}^K \langle B_{kg} \rangle^2 \left[ \langle \Gamma_{kg}^2 \rangle - \langle \Gamma_{kg} \rangle^2 \right] \\ &+ 2 \sum_{k=1}^{K-1} \sum_{k=j+1}^K \langle B_{jg} \rangle \langle B_{kg} \rangle \left[ \langle \Gamma_{jg} \Gamma_{kg} \rangle - \langle \Gamma_{jg} \rangle \langle \Gamma_{kg} \rangle \right] \end{aligned}$$

where

$$\langle \Gamma_{jg} \Gamma_{kg} \rangle = \prod_{\ell \in g} \langle \theta_{\ell}^{a_{\ell} + a_{\ell}} \rangle \langle (1 - \theta_{\ell})^{c_{\ell} + c_{\ell}} \rangle$$

All of the expectation values on the right hand side of the preceding equation for  $\langle VR_g \rangle$  can be evaluated provided that the moments of all of the failure rate distributions are known.

Table 7 lists the mean variance reductions for the example problem with and without total correlations between some pairs of failure rates. It should be noted that the variance reductions do not sum to the total variance (the  $NMVR_{\ell}$ 's do not sum to one). This would only be the case were the top event frequency linear (as opposed to first order) in all of the uncertain failure rates; that is, if each cut set contained only one uncertain failure rate.

Table 7. Mean variance reductions for the example problem.

Event	All Inputs Independent			Total Correlations $\rho_{B1} = \rho_{B2}$ and $\rho_{B3} = \rho_{B4}$		
	$\langle VR_e \rangle$	$NMVR_e$	Rank	$\langle VR_e \rangle$	$NMVR_e$	Rank
I1	2.26e-7	15.0%	3	5.23e-7	4.85%	3
B1	3.97e-7	26.3%	1.5	8.03e-6	74.5%	1
B2	3.97e-7	26.3%	1.5			
B3	3.24e-8	2.15%	4.5	5.60e-7	5.19%	2
B4	3.24e-8	2.15%	4.5			
B5	3.8e-15	nil	6	4.0e-15	nil	4
Event Group	$\langle VR_g \rangle$	$NMVR_e$	Rank	$\langle VR_g \rangle$	$NMVR_e$	Rank
I1	2.26e-7	15.0%	2	5.23e-7	4.85%	3
B1, B2	9.96e-7	66.0%	1	8.03e-6	74.5%	1
B3, B4	7.74e-8	5.13%	3	5.60e-7	5.19	2
B5	3.8e-15	nil	4	4.0e-15	nil	4

#### 4.3 Reduction in Variance of $\ln(\Phi_T)$ , $UI_e$

Iman and Hora observe that the normalized variance reduction as defined by

$$\langle VR_e \rangle = \text{Var}(\langle \Phi_T | \Theta_E \rangle)$$

can be estimated by the coefficient of regression ( $R^2$  statistic) calculated from a linear regression of values of  $\Phi_T$  obtained from a Monte Carlo sample versus  $\Theta_e$ .<sup>4</sup>

When long-tailed distributions such as the lognormal distribution are used to characterize the input uncertainties, however, sample based estimates of  $\text{Var}(\Phi_T)$  and  $\text{VR}_e$  are not robust for samples of manageable size; they vary significantly from sample to sample.

To circumvent this lack of robustness, Iman and Hora, recommend a sample based estimate of the variance reduction in the logarithm of the top event frequency.

$$UI_e = \frac{\text{Var}(\langle \ln \Phi_T | \Theta_e \rangle)}{\text{Var}(\ln \Phi_T)} .$$

In TEMAC, this uncertainty importance measure is denoted by the symbol  $UI_e$  and calculated as the coefficient of correlation ( $R^2$  statistic) of a fit of sample values of  $\Phi_T$  versus sample values of  $\Theta_e$ .

It is important to observe that a major reason for the apparent "robustness" of Iman and Hora's uncertainty importance measure is due to the fact that it reflect the reduction in the variance of the logarithm of  $\Phi_T$  rather than  $\Phi_T$  itself. If the regression is performed on  $\Phi_T$  rather than  $\ln(\Phi_T)$  large sample to sample variations occur. Further, the Iman and Hora uncertainty importance measure is not readily extended to groups of events. Table 8 compares the Iman-Hora uncertainty importance results for the example problem to the exact results obtained by the moments method described in Section 4.1.1. Because the Iman-Hora uncertainty importance measures the reduction in the variance in  $\ln(\Phi_T)$  rather than  $\Phi_T$  itself, it should not be surprising that the rankings obtained by the two methods differ even for this simple example.

Table 8. Mean variance reductions and Iman-Hora uncertainty importances for the example problem with all inputs independent.

Event	$NMVR_e$	Rank	$UI_e$	Rank
I1	15.0%	3	33.4%	1
B1	26.3%	1.5	31.2%	2
B2	26.3%	1.5	29.2%	3
B3	2.15%	4.5	9.71%	4
B4	2.15%	4.5	4.67%	5
B5	nil	6	nil	6

## 5. DEMONSTRATION APPLICATION

The event group importance measures describe in the preceding section have been applied in a variety of different ways as part of a complete accident sequence analysis for a major reactor facility. The results presented here illustrate the types of analyses permitted by the group importance measures. For details regarding the system models and the implications of the group analysis results, the interested reader should refer to the referenced report.<sup>5</sup>

A principal objective of the accident sequence analysis was to estimate the contribution of electrical and nonelectrical failures to each of the dominant accident sequences and to the overall fuel damage frequency. For this purpose, the basic and initiating events were divided into seven groups: initiating events, electrical failures, mechanical failures, instrumentation failures, common cause failures, human errors, and nonrecovery events. The assignment of events to these groups is fairly unambiguous; however, it does not account for common cause or initiating events that are predominately electrical or mechanical. In particular, initiating events could have been subdivided into the other groups (e.g., electrical, mechanical, etc.).

The results presented in the following subsections were obtained by modifying a version of the TEMAC computer code to incorporate the new group importance measures. All of the new importance measures are calculated as functions of the means and moments of the basic event probabilities and initiating event frequencies. None of the new importance measures require Monte Carlo sampling for their quantification. Appendix A provides an input guide for the modified version of the TEMAC code.

To aid in understanding the results from the various group sensitivity and uncertainty analyses, equivalent rankings are included in the tables used to display the analysis results. The event or group yielding the largest value of a given importance measure is assigned the rank 1.0, indicating that it is the most important event group by that measure. Similarly, the group yielding the second largest value is assigned the rank 2.0, the group yielding the third largest value is assigned the rank 3.0, and so on. In the case of a tie, the rank assigned is the average of the rank that would have been assigned had there been no tie. For example, there are several sequences for which two groups had 100% risk reduction. In these cases, the two groups are assigned the rank of 1.5.

To assess sensitivities and uncertainties associated with the overall fuel damage frequency, a fuel damage equation (FDE) was formed by taking the Boolean sum of the 14 dominant accident sequences which account for ~ 98% of the fuel damage frequency. Only cut sets with point estimate (mean) frequencies in excess of  $10^{-8}$  per reactor year were retained in this top event expression, which has 113 cut sets, 7 initiating events, and 75 basic events. The complete TEMAC input for the fuel damage equation is provided as Appendix B.

### 5.1. Event Importance Measure Results

Tables 9 presents the fuel damage frequency results. Tables 10, 11, and 12 present the risk reduction, partial derivative, and variance reduction measures for the fuel damage equation. Results are presented both for the case where all failure rates are treated as independent and for the case in which failure rates based on the same data for similar components are totally correlated. Such correlations were specified by Los Alamos National Laboratory. Only the top ten events by each measure are included in Tables 10 through 12.

Table 9. Parameters of fuel damage frequency distribution.

	Independent Inputs	Total Correlations
Median	5.92e-5	6.1e-5
Mean	1.40	1.72
Variance	3.60e-9	9.63e-9

Table 10. Mean risk reductions by event for fuel damage equation with and without correlated failure rates.

Event	With Independent Failure Rates			With Total Correlations		
	$\langle RR_e \rangle$	$NMRR_e$ %	Rank	$\langle RR_e \rangle$	$NMRR_e$ %	Rank
ICW20002X-PIP-BR	2.05e-5	34.63	1.5	2.05e-5	33.55	1.5
RFAIL-ISOCW-OE	2.05e-5	34.63	1.5	2.05e-5	33.55	1.5
MIA-INIT-FL-OELP	1.73e-5	29.22	3	1.73e-5	28.31	3
MEDIUM-LOCA	1.12e-5	18.92	4	1.29e-5	21.11	4
IOAC15112-BUS-FL	1.03e-5	17.40	5	1.04e-5	17.02	5
RFAIL-RRIVERWTR	8.08e-6	13.65	6	8.08e-6	13.22	7
RFAIL-ISORW-OEM	8.05e-6	13.60	7	9.79e-6	16.02	6
SMALL-LOCA	7.49e-6	12.65	8	7.54e-6	12.34	8
LORWNPLOSTK	5.74e-6	9.70	9	5.74e-6	9.39	11
CC-CW200-2OF2FL	3.64e-6	6.15	14	7.28e-6	11.91	9.5

The events with the largest mean risk reduction measures in Table 10 are for the most part initiating events or operator errors. For example, eliminating all accidents initiated by MEDIUM-LOCAs would eliminate roughly 20% of the mean fuel damage frequency. The impact of correlations on the mean risk-reduction results is weak.

Table 11. Mean partial derivatives by event for fuel damage equation with and without correlated failure rates.

Event	With Independent Failure Rates			With Total Correlations		
	$\langle PD_e \rangle$	$NMPD_e$	Rank	$\langle PD_e \rangle$	$NMPD_e$	Rank
CC-CW200-2OF2FL	7.00e-1	24.96%	2.5	1.40	66.6%	1.5
CC-CW247-2OF2FL	7.00e-1	24.96%	2.5			
CC-CWBRE-2OF2FL	7.00e-1	24.96%	2.5	7.00e-1	16.6%	3.5
CC-CWBPV-2OF2FL	7.00e-1	24.96%	2.5	7.00e-1	16.6%	3.5
LARGE-LOCA	4.29e-2	0.09%	5	4.29e-2	0.06%	5
ICW20002X-PIP-BR	3.41e-2	0.06%	6	3.41e-2	0.04%	6
LORWNPLOSTK	1.43e-2	0.01%	7	1.43e-2	0.01%	7
MEDIUM-LOCA	1.02e-2	0.01%	8	1.17e-2	0.00%	8
LORWNPAVALK	5.56e-3	0.00%	9	5.56e-3	0.00%	11
9-MISALIGN	4.50e-3	0.00%	11.5	9.00e-3	0.00%	9.5
9CLUTCH-FAIL	4.50e-3	0.00%	11.5			

In Table 11, the sum of the normalized mean partial derivatives is unity. Notice that with or without total correlations, four common cause events account for nearly all of the mean gradient. This implies that the fuel damage frequency is most sensitive to changes in the values of these four common cause events. As for the risk reduction results in Table 10, the results partial derivative results presented in Table 11 are relatively insensitive to the induced total correlations.



Table 12. Mean variance reduction by event for fuel damage equation with and without correlated failure rates.

Event	With Independent Failure Rates			With Total Correlations		
	$\langle VR_e \rangle$	$NMVR_e$ %	Rank	$\langle VR_e \rangle$	$NMVR_e$ %	Rank
MEDIUM-LOCA	7.86e-9	21.80	1	1.05e-8	10.9	1
SMALL-LOCA	3.35e-9	9.31	2	3.40e-9	3.53	2
ICW20002X-PIP-BR	2.35e-9	6.53	3	2.36e-9	2.45	3
MIA-INIT-FL-OELP	1.68e-9	4.66	4	1.68e-9	1.74	10
RFAIL-ISOCW-OE	1.11e-9	3.07	5	1.11e-9	1.15	11
ECCV381X-COO-MF	1.07e-9	2.97	6	1.69e-9	1.76	8.5
WD1206AX-SUC-PG	1.08e-11	0.03	34.5	1.77e-9	1.84	5.5
WD1206BX-SUC-PG	1.08e-11	0.03	34.5			
WD1206A1-RVS-FL	1.08e-11	0.03	34.5	1.77e-9	1.84	5.5
WD1206B4-RVS-FL	1.08e-11	0.03	34.5			

Table 12 shows that the largest impact on uncertainty in the fuel damage frequency is associated with three pipe break initiating events MEDIUM-LOCA, SMALL-LOCA, and ICW20002X-PIP-BR. The top four events in Table 12 also appear in Table 11. This should not be surprising because, for the independent case  $\langle VR_e \rangle = \langle PD_e \rangle^2 \text{Var}(\Theta_e)$ ; that is, events with large partial derivatives and large variances have large variance reductions. Note that inducing correlations tends to make the last four events in Table 12 contribute proportionately more to the overall variance.

## 5.2 Primary Event Group Results

Tables 13 and 14 present the normalized mean risk reduction, gradient, and variance reduction measures for the fuel damage equation by event group for the independent and totally correlated cases respectively. Uncertainties in initiating-event frequencies typically dominate the uncertainties in top event frequencies because every cut set contains an initiating event and the magnitudes of the uncertainties in initiating event frequencies generally exceed those for basic event probabilities. To clarify the uncertainty importance associated with basic event group, conditional uncertainty importance calculations were performed in which the initiating-event frequencies were fixed at their point estimate (mean) values.

As shown in the risk-reduction columns in Tables 13 and 14, the elimination of nonrecovery events or human errors would reduce the point estimate frequency by more than 70% and 60%, respectively. In addition, the elimination of either common cause or mechanical failures would reduce the point estimate FDE frequency by approximately 17%. Because an initiating event appears in every cut set, the initiating event group has a risk-reduction measure of 100%.

Tables 13 and 14 also indicates that the point-estimate sequence frequencies would be reduced by slightly over 15% if all the electrical failures were eliminated from this sequence. However, this electrical event risk-reduction estimate does not include initiating events related to the loss of power supplied by the Savannah River Site grid. In particular, if the initiating event for loss of normal power to the reactor is included in the electrical group, the electrical risk-reduction measure increases to 32%. If the initiating events associated with loss of electrical power to the river

water pumps are also included in the electrical group, electrical risk-reduction measure increases further to almost 41%.

Electrical failures are a relatively minor contributor to the top event gradients. The gradient of the fuel damage frequency is dominated by common cause and human-error event groups.

Table 13. Group importance measures for fuel damage equation with no correlated failure rates.

Event Group	Risk Reduction		Contribution to Gradient		Contribution to Variance		Conditional <sup>a</sup> Contribution to Variance	
	$NMRR_g$ % (rank)		$NMPD_g$ % (rank)		$NMVR_g$ % (rank)		$NMVR_g$ % (rank)	
Initiating Events	100.0	1	0.2	3	39.9	1	--	--
Recovery Actions	70.1	2	0.0	4	6.25	3	35.2	2
Operator Errors	61.7	3	25.0	2	6.58	2	35.9	1
Common Cause	17.1	4	74.9	1	1.28	5	7.2	4
Mechanical	16.9	5	0.0	4	3.52	4	19.6	3
Electrical	15.2 (32.4) <sup>b</sup> (41.0) <sup>c</sup>	6	0.0	4	0.35	6	1.9	5
Instrumentation	0.8	7	0.0	4	0.06	7	0.2	6

<sup>a</sup> Initiating event frequencies fixed at mean values.

<sup>b</sup> Loss of off site power initiator included with electrical group

<sup>c</sup> Loss of off site power and loss of river water initiators included with electrical group.

Table 14. Group importance measures for fuel damage equation with correlated failure rates.

Event Group	Risk Reduction $NMRR_g$ % (rank)		Contribution to Gradient $NMPD_g$ % (rank)		Contribution to Variance $NMVR_g$ % (rank)		Conditional <sup>a</sup> Contribution to Variance $NMVR_g$ % (rank)	
	Initiating Events	100.00	1	0.07	3	17.7	1	--
Recovery Actions	70.9	2	0.00	7	2.52	4	14.8	3
Operator Errors	60.2	3	9.99	2	2.56	3	15.0	2
Common Cause	16.6	5	89.9	1	0.92	6	5.41	5
Mechanical	18.7	4	0.00	4	5.02	2	29.4	1
Electrical	15.2	6	0.00	5	1.97	5	11.6	4
Instrumentation	0.74	7	0.00	6	0.02	7	0.13	6

<sup>a</sup> Initiating event frequencies fixed at mean values.

The initiating-event group accounts for the majority of the unconditional FDE variance. Human errors and nonrecovery events have the largest conditional variance reductions. That is, eliminating uncertainties in the quantification of human error events and nonrecovery events would yield the most reduction in the uncertainty in fuel damage frequency that remains when the initiating-event frequencies are fixed at their point-estimate (mean) values. Mechanical failures are ranked third by their contributions to the conditional variance, followed by common-cause failures. Electrical failures are relatively small contributors to the conditional variance.

### 5.3. Alternative Event Group Results

Grouping of basic events can be accomplished in any arrangement desired by the analyst. This is particularly useful for analyzing the importance of various groupings of components of interest, e.g. plant safety systems or individual components within a system. Note that system failure probabilities are often initiating event dependent and that no single probability estimate can describe the system's importance to safety.

Table 15 shows the results in which all of the noninitiating events were grouped by system. The systems used in the analysis are the process water system (PWS), the emergency cooling system (ECS), the cooling water system (CWS),- the electrical systems; the water disposal system (WDS), the Moderator Recovery System (MRS), the River Water System, and the instrument air (IA) system. Based on the risk-reduction measure, the most important system is the ECS. Elimination of ECS failures would reduce the FDE frequency by almost 49%. The analyst can review the dominant sequences and cutsets to determine the most important ECS events (components or human errors), or can conduct an importance analysis of each basic event assigned to a system.

Table 15. System importance measures for the fuel damage equation without correlated failure rates.

System	Risk Reduction $NMRR_g$ % (rank)		Contribution to Gradient $NMPD_g$ % (rank)		Contribution to Variance $NMVR_g$ % (rank)		Conditional <sup>a</sup> Contribution to Variance $NMVR_g$ % (rank)	
	Initiating Events	100.0	1	0.17	2	39.9	1	
Emergency Cooling	48.9	2	0.00	4	8.38	2	40.2	1
Cooling Water	29.3	3	99.8	1	2.12	3	10.2	2
Electrical	14.1	4	0.00	5	0.30	5	1.43	4
Water Disposal	10.3	5	0.00	6	0.45	4	2.16	3
Moderator Recovery	5.51	6	0.00	8	0.16	6	0.79	5
River Water	1.73	7	0.00	7	0.05	8	0.22	7
Scram	1.69	8	0.00	3	0.07	7	0.33	6
Instrument Air	0.21	9	0.00	9	0.00	9	0.01	8

<sup>a</sup> Initiating event frequencies fixed at mean values.

Table 16. System importance measures for the fuel damage equation with correlated failure rates.

System	Risk Reduction $NMRR_g$ % (rank)		Contribution to Gradient $NMPD_g$ % (rank)		Contribution to Variance $NMVR_g$ % (rank)		Conditional <sup>a</sup> Contribution to Variance $NMVR_g$ % (rank)	
Initiating Events	100.0	1	0.07	2	17.7	1		
Emergency Cooling	47.5	2	0.00	4	3.71	2	21.8	1
Cooling Water	28.5	3	99.9	1	1.30	4	7.61	3
Electrical	13.8	4	0.00	5	0.14	5	0.80	4
Water Disposal	12.5	5	0.00	6	3.33	3	19.5	2
Moderator Recovery	5.38	6	0.00	8	0.06	6	0.36	5
River Water	1.68	7	0.00	7	0.02	8	0.10	7
Scram	1.64	8	0.00	3	0.03	7	0.17	6
Instrument Air	0.20	9	0.00	9	0.00	9	0.00	8

<sup>a</sup> Initiating event frequencies fixed at mean values.

## 6 CONCLUSIONS

Representative quantitative results illustrate the utility of the event-group importance measures. Events may be grouped by event type (initiating, mechanical, electrical, etc.), by subtypes within a type, by system, or by component type (pumps, valves, switches, etc.). The three group importance measures can be used to rank groups of events according to different attributes:

1. the potential to reduce the top event frequency (risk reduction),
2. the potential for inducing change in the point estimate of the top event frequency (gradient importance), and
3. the potential for reducing the uncertainty in the top event frequency (variance reduction).

The group importance frequently provide insights that refute preconceived expectations. For example, in the reactor facility analyzed in Chapter 5, both operator errors and recovery actions proved much more important than expected by the Los Alamos National Laboratory team that modeled the facility. Mechanical failures proved less important than expected.

Given the moments of the distributions that characterize the uncertainties in the underlying failure rates, the expectation values of the top event frequency, its variance, and all of the new group importance measures can be quantified exactly for two familiar cases:



1. when all underlying failure rates are presumed independent,  
and
2. when pairs of failure rates based on common data are treated  
as being equal (totally correlated).

In these cases, the new importance measures can also be applied to assess the importance of either individual events or, for the first time, groups of events. In addition, the new importance measures can be evaluated analytically. They obviate the need for Monte Carlo sampling. They are free from the statistical errors associated with importance measures based on Monte Carlo sampling. In summary, the methods developed for this thesis provide very precise and robust measures of the importance of individual events and groups of events.

## APPENDIX A

### TEMAC INPUT FILES

A modified Top Event Matrix Analysis Code, TEMAC PC2, has been developed to perform the uncertainty and sensitivity analysis calculations illustrated in the previous sections, and to summarize and provide graphical displays for many of the results. The code TEMAC requires up to six input files. A detailed explanation of these input files is given in this section. This discussion is augmented with example input files.

TEMAC requires up to six files as input. INPUT FILE 1 contains the filenames for INPUT FILES 2 and 3, the TEMAC keywords, and their associated options including the name of INPUT FILE 6 if this file is required. INPUT FILE 2 contains the top event expression in disjunctive normal form. INPUT FILE 3 contains event names, their associated nominal or mean values, and, optionally, their standard deviations and the event groups to which they are assigned. INPUT FILES 1, 2, and 3 are always required. Three additional files may also be required depending on the form of the analysis desired. If Monte Carlo sampling is used for the uncertainty analysis INPUT FILES 4 and 5 are required. INPUT FILE 4 contains the name of INPUT FILE 5 and specifies the locations of basic event frequencies in INPUT FILE 5, which contains the Monte Carlo Sample. INPUT FILE 6 defines groups of the cut sets in the top event expression. These groups may correspond to accident sequences or plant damage states for which TEMAC is asked to compute split fractions. The structure and the content of each of the input files will now be discussed.

#### A.1 INPUT FILE 1

The first two records of INPUT FILE 1 must contain the names of INPUT FILE 2 and INPUT FILE 3. These filenames must be consistent with the file naming convention on the computer used to execute the program. Up to 18 keywords with their associated

options can be used in INPUT FILE 1 to designate the desired calculations and output. The left hand side of Table A1 provides an example of INPUT FILE 1 with all keywords and options listed. The keywords numbers on the left hand side of Table A1 are not included in the input file. They are used to indicate the five categories into which keyword fall as indicated in Table A2.

**Table A1. INPUT FILE 1 With All Keyword and Options**

1	INPUT2.XH1						
2	INPUT3.XH1						
3	TITLE	TEMAC EXAMPLE PROBLEM H1					
4	TOP EVENT	TEQN	EVCS	EVPO			
5	NOMINAL EVALUATIONS	EVDP	EVL	EVRR	EVRI	CSFQ	CSNF
6	EVENT CUTOFF	10					
7	CUT SET CUTOFF	1.E-8					
8	DELETE CUT SETS	10.					
9	SAMPLE SIZE	25					
10	UNCERTAINTY INTERVALS	EVDP	EVL	EVRR	EVRI	CSFQ	CSNF
11	UNCERTAINTY GRAPHS	EVDP	EVL	EVRR	EVRI	CSFQ	CSNF
12	QUANTILE RATIO CUTOFF	5					
13	UNCERTAINTY IMPORTANCE	EVUI	EVVR	QUAN			
14	RANK SUMMARY	EVUI					
15	EVENT GROUPS						
16	INITIATING EVENTS	I					
17	CUT SET GROUPS	INPUT6.XH1					
18	TOTAL CORRELATION						

**Table A2. Keyword Categories**

Record in Table A1	Keyword Category
3, 4, 5, 6, 7, 12	Control the form of the output.
8	Delete selected cut sets from the top event.
5, 14	Designate the calculations to be performed and output with respect to nominal values.
9, 10, 11, 13	Designate the calculations to be performed and output with respect to the uncertainty calculations.
16	Define the initiating event prefix.
15, 17	Define whether calculations are to be performed for groups of events or cut sets.
18	Together with the input file 4, require the total dependent case calculation.

The following rules apply to all keywords:

1. All keywords must be spelled out completely as in Table A1;
2. A keyword record can have no leading blanks, but at least one blank must follow the keyword on the record;
3. All keywords are optional;
4. The keywords may appear in any order;
5. Options on keywords with multiple associated options (keywords 2, 3, 6, 8, and 9) may appear in any order, but must appear on the same record as the keyword. The multiple options on the keyword DELETE CUT SETS may continue to the next record(s) and must end with a period; and
6. Omitting all options on any keyword with associated options will result in that keyword being ignored excepting keywords 13 & 15. When keyword 13 appears alone, it is equivalent to the keyword plus the EVUI option. When keyword 16 appears, it must be followed by the initiating event prefix.

There are a number of internal checks built into TEMAC to ensure the correct specification of the keywords and their associated options. Improper keyword specification causes an appropriate error message to be printed and the execution of the program to be terminated. The function of each keyword is now considered in detail.

#### TITLE

This keyword may be used to provide a title of up to 74 characters on each page header. Record 3 in Table A1 illustrates the use of this keyword.

#### TOP EVENT

This keyword may be used to request any of three possible output options with respect to the top event expression being analyzed. Record 2 in Table A1 illustrates the use of this keyword with all three options requested. The function of each of the options is as follows:

TEQN - Used to request the output of the top event (in disjunctive normal form);

EVCS - Used to request a version of the Top Event Indicator Matrix T as output. The user should be aware that this option will generate a lot of output on large scale problems. The number of pages of output generated by this option can be found as the product of the smallest integer greater than or equal to the number of cut sets divided by 20, and the smallest integer greater than or equal to the number of events divided by 50; and

EVPO - Used to request the product of the Top Event Indicator Matrix and its transpose as output. The user should be aware that this option will generate a lot of output on large scale problems. The number of pages of output generated by this option can be found as the product of the smallest integer greater than or equal to the number of events divided by 20, and the smallest integer greater than or equal to the number of events divided by 50.

#### NOMINAL EVALUATIONS

This keyword may be used to request any of six evaluations described below. These evaluations are all based on user supplied nominal values. Thus, these evaluations

should be regarded as estimates in the statistical sense only to the extent that the nominal values are estimates in the statistical sense. Otherwise, these evaluations are more accurately referred to as evaluations made as nominal values. An example of the use of this keyword appears on record 5 of Table A1 where all six options are required. The function of each of the six options is as follows:

- EVDP - Used to request the partial derivatives with respect to basic events and initiating events. The basic event results are listed separately from the initiating event results in the output;
- EVLDP - Used to request the log-log scale partial derivatives with respect to basic events and initiating events. The basic event results are listed separately from the initiating event results in the output;
- EVRR - Used to request the risk reductions with respect to basic events and initiating events. The basic event results are listed separately from the initiating event results in the output;
- EVRI - Used to request the risk increases with respect to basic events;
- CSFQ - Used to request the output of the individual cut set frequencies. These frequencies are output in decreasing order; and
- CSNF - Used to request the output of the individual normalized cut set frequencies (cut set frequencies expressed as a percentage of the top event frequency). These frequencies are output in decreasing order.

If the options EVDP, EVRR, and EVRI are all requested the results will be printed out in a single table and sorted with respect to EVRR. If the option EVRR is not requested the results are sorted on the basis of EVLD. If the options EVRR and EVLD are not requested the results are sorted on the basis of EVRI. If the options EVRR, EVLD, and EVRI are not requested the results are sorted on the basis of EVDP. These defaults can be changed by placing an asterisk immediately following the option that the user wishes to designate as the sort key.

## EVENT CUTOFF

This keyword is followed by an integer that is used to truncate the output with respect to both basic events and initiating events. Keyword 4 of Table A1 illustrates the use of this keyword where the integer 10 has been used to signify that the output will be shown only for the first 10 basic events and the first 10 initiating events. This cutoff applies to all output involving events except that requested for the rank summary.

## CUT SET CUTOFF

This keyword is followed by a real number that is used to truncate all output with respect to cut sets. Keyword 5 of Table A1 illustrates the use of this keyword where the real number 1.E-8 has been used to signify that the output should show only those cut sets whose frequency is bigger than or equal to 1.E-8.

## DELETE CUT SETS

This keyword is followed by integers that identify cut sets to be deleted from the top event expression. Keyword 6 of Table A1 illustrates the use of this keyword where cut sets 17, 25 through 30 and 10 are to be deleted from the top event expression. The cut set numbers may appear in any order and may be continued for as many records as needed, except that ranges cannot be broken up across records. Spaces are used to delimit cut set number and hyphens are used to designate a range of cut sets. No space are allowed on either side of the hyphen when designating a range of cut sets. A period must follow the number of the last cut set to be deleted.

## SAMPLE SIZE

This keyword is used in conjunction with a Monte Carlo Sample simulation. The keyword is followed by a positive integer greater than 1, corresponding to the number of observations in the Monte Carlo Sample. An example of the use of this keyword appears on record 9 of Table A1 where a sample size of 100 has been indicated.

## UNCERTAINTY INTERVALS

This keyword is used in conjunction with a Monte Carlo Sample simulation. It allows the user to request the output of approximate 90% uncertainty intervals for any of the six options described under the keyword NOMINAL EVALUATIONS. These intervals are derived from the sampling distributions generated by the Monte Carlo Sample simulation for each event. The six options associated with the keyword NOMINAL EVALUATIONS. The selection of the options CSFO or CSNF with this keyword overrides the corresponding selection on the keyword NOMINAL EVALUATIONS. An example of the use of this keyword is shown on record 10 of Table A1 where uncertainty intervals are requested for all six possible options.

## UNCERTAINTY GRAPHS

This keyword is used in conjunction with a Monte Carlo Sample simulation. It allows the user to request the output of line printer graphs of 90% uncertainty intervals for any of the options described under the keyword NOMINAL EVALUATIONS. These graphs are based on the uncertainty intervals calculated with the use of the keyword UNCERTAINTY INTERVALS. The options selected with this keyword are, however, independent of the options selected with the keyword UNCERTAINTY INTERVALS. The six options associated with this keyword are identical to those associated with the keyword NOMINAL EVALUATIONS. An example of the use of this keyword is shown on record 11 of Table A1 where graphs of 90% uncertainty intervals are requested for all six possible options.

## QUANTILE RATIO CUTOFF

This keyword is used in conjunction with the QUAN in UNCERTAINTY IMPORTANCE to request the quantile ratio measurements. This keyword is followed by a number between 0 and 100 that is used to control the calculation of quantile ratio for those event whose uncertainty is above the number. For example, QUANTILE RATIO CUTOFF 5 requests quantile ratio calculations only for those input variables



whose uncertainty importance measure (UI) is at least 5%. The default cutoff value for the quantile ratio cutoff is 1%.

#### UNCERTAINTY IMPORTANCE

This keyword may be used to request any of three evaluations described below. Two of the evaluations are used in conjunction with a Monte Carlo sampling. One is used to invoke analytic variance reduction calculations. The function of the options are as follows:

EVVR - Used to request the analytic normalized variance reduction by using parameters of the input distributions;

EVUI - Used in conjunction with a Monte Carlo Sample to calculate the logarithmic Iman-Hora uncertainty importance by regression method; and

QUAN - Used to calculate the Iman-Hora quantile ratios obtained from the Monte Carlo.

If the keyword appears alone, it is equivalent to the keyword plus the option EVUI.

#### RANK SUMMARY

If this keyword is specified a table of ranks of certain measures with respect to basic events is output. These measures are: partial derivatives, log-log scale partial derivatives, risk reductions, risk increases, and uncertainty importance. If EVPD, EVLD, EVRR, and/or EVRI are specified with any of the NOMINAL EVALUATIONS, UNCERTAINTY INTERVALS or UNCERTAINTY GRAPHS keywords then the corresponding ranks of those measures will appear in table summary on the basis of the nominal evaluations of each of the measures. The ranks of the uncertainty importance measure are included in the summary if this measure has been requested. In addition to the rank summary, a matrix of top-down correlation coefficients<sup>11</sup> is output. This keyword cannot be used unless two or more of the measures above have been requested. An example of the use of this keyword appears on record 13 of Table A1.

The rank summary table is output with the event results sorted by one of the following measures: risk reduction, risk increase, partial derivatives, log-log scale partial derivatives, or uncertainty importance. The user may specify the measure on which results are sorted by adding one of the EVRR, EVRI, EVPD, EVLD, or EVUI options respectively, with the keyword RANK SUMMARY. The default hierarchy for sorting is EVRR, EVRI, and EVPD, depending on which calculations have been performed. The default sorting hierarchy is used in two cases: (1) the user does not specify any option and (2) the user has asked for a sorting option for which the corresponding calculations have not been requested.

### EVENT GROUPS

This keyword is used to require the sensitivities for grouped events. When the keyword is used, the name of group of each event belongs to must be presented at the last column of INPUT FILE 3.

### INITIATING EVENTS

This keyword is used to let user have the ability to define the initiating event names by defining the beginning letters of events' names. The beginning letters must be three or less. The default of the beginning letters are 'IE-'. There should be some initiating event definition followed the keyword, provided the keyword was presented.

### CUT SETS GROUPS

This keyword is used require the analysis of cut set groups. If this keyword appears in INPUT FILE 1, a file name for INPUT FILE 6 must be specified following the keyword. INPUT FILE 6 is discussed separately in Section A.6.

### TOTAL CORRELATION

This keyword together with appropriate position number in input file 4 will give out the calculation for total correlated events.

## A.2 INPUT FILE 2

The first record in INPUT FILE 1 contains the name of INPUT FILE 2. The top event expression is stored in INPUT FILE 2 in disjunctive normal form. The top event name must be followed by an equals sign. The cut sets follow the equals sign and are each separated by plus signs. Note that due to the individual cut set analysis within TEMAC, the program allows only low order terms in the cut sets. That is, no higher order terms involving intersections of cut sets with their corresponding possible minus sign on the cut set are permitted.

The events within cut sets are separated by asterisks. Event names (including the top event name) must be 16 characters or less. Initiating events must be designated by using the prefix defined by user in INPUT FILE 1. The default prefix for initiating event names is IE. Complements of basic events can be used in the top event expression, and if present are denoted by a '/' immediately before the basic event name. The slash is not included in the 16-character name limit. The top event expression may be continued from one record (line) to the next except that event names may not be broken up over records. Blanks and blank lines may be used freely except within event names and complements. The top event expression must be terminated by a period. Table A3 shows the content of INPUT FILE 2 as it could be used in the analysis of the problem in the previous section.

---

**Table A3. Content of INPUT FILE 2 for Example Problem**

---

$$T = I1 * B1 * B2 + I1 * B3 * B4 \\ + I1 * B1 * B4 * B5 + I1 * B2 * B3 * B5.$$

---

### A.3 INPUT FILE 3

The second record in INPUT FILE 1 contains the name of INPUT FILE 3. Each record of INPUT FILE 3 contains an event name, its associated nominal value, and may also contain its associated standard deviation and the name of the event group to which the event belongs. If an event is used in the top event expression then it must have a nominal value in this file. But an event can appear in INPUT FILE 3 and not be present in INPUT FILE 1. If EVVR is requested the standard deviation must be present in this file after the nominal value. If the EVENT GROUPS keyword appears in INPUT FILE 1, then the group name of which the event must be appeared in the last column. The standard deviation and group names can be appear in INPUT FILE 3 even though the corresponding keyword or option (flag) are not appeared in INPUT FILE 1. If the event is a basic event then the nominal value is assumed to be probability and it must have a value between zero and one. If the event is an initiating event, however, the nominal value is assumed to be a frequency and the value must be nonnegative. Table A4 shows the content of INPUT FILE 3 as it could be used in the analysis of the problem in the previous section.

**Table A4. Contents of INPUT FILE 3 for a Example Problem**

I1	2	0.88156	IE
B1	0.02	1.4994E-2	G1
B2	0.02	1.4994E-2	G1
B3	0.01	7.4972E-3	G2
B4	0.01	7.4972E-3	G2
B5	0.10	7.4972E-2	B5

#### A.4 INPUT FILE 4

The first record of this file contains the name of INPUT FILE 5 as shown in Table A5. The following records will contain in order: the event name and an integer identifying the position number in a sample vector where the sampled value for the event in question is found. These records must contain an event name/ integer pair for each sampled event in the top event expression. If an event is not sampled the user has two choices with respect to the corresponding record in INPUT FILE 4. First a record for a nonsampled event does not need to appear in INPUT FILE 4. Second, if the event does appear in INPUT FILE 4, the position number associated with it should be a zero (not blank).

Completely dependent events, that is, events having exactly the same sample values, are identified in INPUT FILE 4 by assigning the same position number within each group of dependent events. In the case of dependent events, the user must take care to assign identical nominal values in INPUT FILE 3.

Additionally, the uncertainty importance calculation is based on the individual means of the sampled events. For this calculation it is assumed that the nominal value of the event is a mean. If the nominal value is not a mean the user should place an asterisk immediately following the position number in INPUT FILE 4, such as 7\* for position 7. This will cause TEMAC to calculate a sample mean from the Monte Carlo Sample and substitute it into the uncertainty importance calculation rather than using the nominal value.

Table A5. Content of INPUT FILE 4 for the sample problem

---

INPUT5.XH1	
I1	1
B1	2
B2	2
B3	3
B4	3
B5	4

---

#### A.5 INPUT FILE 5

When uncertainty calculations are requested, a Monte Carlo sample must be supplied to TEMAC in INPUT FILE 5. The sample must be stored in LHS format.<sup>3</sup> Table A6 shows the LHS of size 25 used in the Monte Carlo simulation. Each line in Table A6 starts with the sample vector number, followed by the number of events and the actual sampled valued for each of the six events.

Table A6. The Latin Hypercube Sample Contained in INPUT FILE 5

---

1	4	1.10568	0.154474E-01	0.489035E-02	0.162340
2	4	1.48431	0.692179E-02	0.598766E-02	0.499842E-01
3	4	2.89050	0.400007E-01	0.113455E-01	0.525915E-01
4	4	1.83547	0.499559E-01	0.638315E-02	0.178912
5	4	2.37111	0.783144E-02	0.707873E-02	0.155595
6	4	4.12632	0.199994E-01	0.125699E-01	0.235312E-01
7	4	1.97703	0.135249E-01	0.162993E-01	0.105357
8	4	1.19871	0.311473E-02	0.196053E-01	0.115273
9	4	2.03925	0.341511E-01	0.155508E-01	0.396435
10	4	1.38208	0.180411E-01	0.429503E-02	0.129277
11	4	2.15701	0.158887E-01	0.369916E-02	0.625047E-01
12	4	1.73722	0.215302E-01	0.219969E-02	0.419493E-01
13	4	0.49800	0.260571E-01	0.545242E-02	0.747974E-01
14	4	1.51770	0.921915E-02	0.749740E-02	0.339495E-01
15	4	2.62026	0.294796E-01	0.272688E-01	0.123440
16	4	1.21975	0.172809E-01	0.240203E-01	0.811808E-01
17	4	3.57150	0.245154E-01	0.353932E-02	0.676037E-01
18	4	2.50637	0.131234E-01	0.866606E-02	0.843134E-01
19	4	2.29125	0.889073E-02	0.982116E-02	0.395214E-01
20	4	3.00597	0.110629E-01	0.280498E-02	0.232091
21	4	1.67184	0.529924E-02	0.108197E-01	0.986942E-01
22	4	1.91896	0.125496E-01	0.540771E-02	0.568833E-01
23	4	1.57872	0.106002E-01	0.135570E-01	0.907034E-01
24	4	0.94492	0.903703E-01	0.911969E-02	0.272143E-01
25	4	1.31054	0.234545E-01	0.806144E-02	0.681443E-01

## A.6 INPUT FILE 6

This keyword is used for plant damage state and split fractions. Risk assessments frequently proceed by evaluation subgroups of the cut sets in the top event expression. These subgroups are referred to as plant damage states. INPUT FILE 6 allow the user do define the plant damage states and split fractions and to output the results as part of TEMAC analysis. As an example, consider a top event having 1000 cut sets. Suppose we wish to define two plant damage states where the first plant damage state consists of cut sets 1 to 100 and 275 to 325, while the second plant damage state consists of cut sets 450 to 840. Further, we need to define a split fraction consisting of the ratio of the frequency of cut sets 1 to 100 to the frequency of the first plant damage state. Table A7 shows the way to construct INPUT FILE 6 to obtain the desired results. Comments have been added within parentheses in Table A7, but are not a part of the file. Note that a period is required at the end of the cut sets defining each plant damage state or split fraction.

Table A7. Content of INPUT FILE 6 for a hypothetical example.

INPUT	EXPLANATION
2	Two plant damage states are to be evaluated.
1-100 275-325.	Cut sets 1-100 and 275-325 are in the first plant damage state. The period is necessary.
450-840.	Cut sets 450-840 in the second plant damage state.
1	One split fraction is to be evaluated.
1	Cut sets for the denominator of the split fraction are found in plant damage state 1.
1-100.	Cut sets 1-100 are in the numerator of the split fraction.



INPUT FILE 6 is optional. If the user doesn't want the calculation about the damage state or split fraction, the key word of CUT SETS GROUPS should not be in the INPUT FILE 1.

In the event the user does desire to make calculations based on INPUT FILE 6 then UNCERTAINTY INTERVALS must be requested in INPUT FILE 1. The output from an analysis involving INPUT FILE 6 will stored in a file named as TEMAC4.OUT.

# APPENDIX B

## DEMONSTRATION APPLICATION FILES

INPUT FILE 1

INPUT2.FD  
 INPUT3.FD  
 TITLE MS K- PROBLEM 16  
 NOMINAL EVALUATIONS EVRI EVRR EVPD EVLD  
 UNCERTAINTY IMPORTANCE EVVR  
 EVENT GROUPS  
 TOTAL CORRELATIONS

INPUT FILE 2

FUEL-DAMAGE =  
 ICW20002X-PIP-BR \* MIA-INIT-FL-OELP \* RFAIL-ISOCW-OE +  
 IOAC15112-BUS-FL \* CC-CWBPV-2OF2FL +  
 IOAC15112-BUS-FL \* CC-CW247-2OF2FL +  
 IOAC15112-BUS-FL \* CC-CW200-2OF2FL +  
 OAC15112-BUS-FL \* ACOODGXX-SEQ-FL \* SMALL-LOCA \* RFAIL-ISOPW-OES +  
 MRSXXXX-ACT-OE \* SMALL-LOCA \* MIA-INIT-FL-OES5 +  
 MEDIUM-LOCA \* ECCV381X-COO-MF +  
 IOAC15112-BUS-FL \* CC-CWBRE-2OF2FL +  
 LARGE-LOCA \* FLOOD-OEL \* RFAIL-ISOPW-OEL +  
 ICW20002X-PIP-BR \* ECCV380X-COO-MF \* RFAIL-ISOCW-OE +  
 ECCV381X-COO-MF \* ICW20002X-PIP-BR \* RFAIL-ISOCW-OE +  
 MEDIUM-LOCA \* CC-WD126-2OF2FL \* RFAIL-ISOPW-OEM +  
 THROTTLE-FL-OELM \* MEDIUM-LOCA \* RFAIL-ISOPW-OEM +  
 LORWNPVALK \* LORWISL1-CWH-OE \* RFAIL-RRIVERWTR +  
 ACOODG2X-DGN-ST \* RFAIL-RRIVERWTR \* LORWNPLOSTK \* LORWISL2-CWH-OE +  
 ACOODG1X-DGN-ST \* RFAIL-RRIVERWTR \* LORWNPLOSTK \* LORWISL2-CWH-OE +  
 RFAIL-RRIVERWTR \* LORWNPLOSTK \* LORW1072-PMP-OE +  
 LORWISL1-CWH-OE \* RFAIL-RRIVERWTR \* LORWNPLOSTK +  
 MEDIUM-LOCA \* OAC15112-BUS-FL \* ACOODGXX-SEQ-FL \* RFAIL-ISOPW-OEM +  
 LORWNPVALK \* RFAIL-RRIVERWTR \* LORWDIAG-ACC-OE +  
 ACOODG2X-DGN-RN \* RFAIL-RRIVERWTR \* LORWNPLOSTK \* LORWISL2-CWH-OE +  
 ACOODG1X-DGN-RN \* RFAIL-RRIVERWTR \* LORWNPLOSTK \* LORWISL2-CWH-OE +  
 RFAIL-RRIVERWTR \* LORWDIAG-ACC-OE \* LORWNPLOSTK +  
 SC-SCRAM-FS \* NI-SCRAM-FS \* SMALL-LOCA +  
 LORWNPVALK \* RFAIL-RRIVERWTR \* LORW190-PMP-OE +  
 LORWNPVALK \* RFAIL-RRIVERWTR \* LORW1071-PMP-OE +  
 ECCV381X-COO-MF \* MRSXXXX-ACT-OE \* SMALL-LOCA +  
 MEDIUM-LOCA \* CC-EC380-3OF3FL +  
 MEDIUM-LOCA \* WD1206A1-PMP-ST \* WD1206B4-PMP-ST \* RFAIL-ISOPW-OEM +  
 MEDIUM-LOCA \* WD126A2X-CCC-MF \* WD1206B4-PMP-ST \* RFAIL-ISOPW-OEM +  
 MEDIUM-LOCA \* WD1206A1-PMP-ST \* WD126B2X-CCC-MF \* RFAIL-ISOPW-OEM +  
 MEDIUM-LOCA \* WD126A2X-CCC-MF \* WD126B2X-CCC-MF \* RFAIL-ISOPW-OEM +  
 CC-ACDST-2OF2FL \* RFAIL-RRIVERWTR \* LORWNPLOSTK \* LORWISL2-CWH-OE +  
 MEDIUM-LOCA \* WD1206A1-PMP-RE \* WD1206B4-PMP-ST \* RFAIL-ISOPW-OEM +  
 MEDIUM-LOCA \* WD1206A1-PMP-RE \* WD126B2X-CCC-MF \* RFAIL-ISOPW-OEM +  
 MEDIUM-LOCA \* WD1206A1-PMP-ST \* WD1206B4-PMP-RE \* RFAIL-ISOPW-OEM +  
 MEDIUM-LOCA \* WD126A2X-CCC-MF \* WD1206B4-PMP-RE \* RFAIL-ISOPW-OEM +  
 ACOOB4X-SWT-CO \* ICW20002X-PIP-BR \* RFAIL-ISOCW-OE +  
 ACOATS4X-ATS-CO \* ICW20002X-PIP-BR \* RFAIL-ISOCW-OE +  
 MIAHS723-SWT-FL \* ICW20002X-PIP-BR \* RFAIL-ISOCW-OE +  
 MEDIUM-LOCA \* WD1206A1-PMP-RE \* WD1206B4-PMP-RE \* RFAIL-ISOPW-OEM +  
 IOAC15112-BUS-FL \* CC-CW004-2OF2FL \* CC-ACDST-2OF2FL \* RFAIL-OPENBYP-OE +  
 9-MISALIGN \* SMALL-LOCA +  
 9CLUTCH-FAIL \* SMALL-LOCA +  
 CC-EC380-3OF3FL \* ICW20002X-PIP-BR \* RFAIL-ISOCW-OE +  
 MEDIUM-LOCA \* WD1206AX-SUC-PG \* WD126B2X-CCC-MF \* RFAIL-ISOPW-OEM +  
 MEDIUM-LOCA \* WD1206AX-SUC-PG \* WD1206B4-PMP-ST \* RFAIL-ISOPW-OEM +  
 MEDIUM-LOCA \* WD1206A1-RVS-FL \* WD126B2X-CCC-MF \* RFAIL-ISOPW-OEM +  
 MEDIUM-LOCA \* WD1206A1-PMP-ST \* WD1206BX-SUC-PG \* RFAIL-ISOPW-OEM +  
 MEDIUM-LOCA \* WD1206A1-PMP-ST \* WD1206B4-RVS-FL \* RFAIL-ISOPW-OEM +

MEDIUM-LOCA \* WD126A2X-CCC-MF \* WD1206BX-SUC-PG \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD126A2X-CCC-MF \* WD1206B4-RVS-FL \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1206A1-RVS-FL \* WD1206B4-PMP-ST \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD126A2X-CCC-MF \* WD2142XX-XOC-RE \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1206A1-PMP-ST \* WD2142XX-XOC-RE \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD2141XX-XOC-RE \* WD126B2X-CCC-MF \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD2141XX-XOC-RE \* WD1206B4-PMP-ST \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1206A1-PMP-RE \* WD1206BX-SUC-PG \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1206A1-PMP-RE \* WD1206B4-RVS-FL \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1206AX-SUC-PG \* WD1206B4-PMP-RE \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1206A1-RVS-FL \* WD1206B4-PMP-RE \* RFAIL-ISOPW-OEM +  
SCTBPSWX-SOC-OE \* SMALL-LOCA +  
MEDIUM-LOCA \* WD2141XX-XOC-RE \* WD1206B4-PMP-RE \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1206A1-PMP-RE \* WD2142XX-XOC-RE \* RFAIL-ISOPW-OEM +  
ACOODG1X-DGN-ST \* ACOODG2X-DGN-ST \* IOAC15112-BUS-FL \* CC-CW004-20F2FL \* RFAIL-OPENBYP-OE +  
CC-SCTCZCZA-DE \* SMALL-LOCA +  
MEDIUM-LOCA \* IA00000X-DRY-OE \* SIA0000X-SYS-RE \* RFAIL-ISOPW-OEM +  
OAC15112-BUS-FL \* ACOODG1X-DGN-ST \* ACOODG2X-DGN-ST \* SMALL-LOCA \* RFAIL-ISOPW-OES +  
MEDIUM-LOCA \* WD1206AX-SUC-PG \* WD1206BX-SUC-PG \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1206AX-SUC-PG \* WD1206B4-RVS-FL \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1206A1-RVS-FL \* WD1206BX-SUC-PG \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1206A1-RVS-FL \* WD1206B4-RVS-FL \* RFAIL-ISOPW-OEM +  
SMALL-LOCA \* SCTOATDX-REL-CB \* SCTNGPGX-BUS-SH +  
SMALL-LOCA \* SCTOATDX-REL-CB \* SCTO22PX-BUS-SH +  
ACOODG1X-DGN-RN \* ACOODG2X-DGN-ST \* IOAC15112-BUS-FL \* CC-CW004-20F2FL \* RFAIL-OPENBYP-OE +  
ACOODG1X-DGN-ST \* ACOODG2X-DGN-RN \* IOAC15112-BUS-FL \* CC-CW004-20F2FL \* RFAIL-OPENBYP-OE +  
MEDIUM-LOCA \* WD2141XX-XOC-RE \* WD1206B4-RVS-FL \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD2141XX-XOC-RE \* WD1206BX-SUC-PG \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1206A1-RVS-FL \* WD2142XX-XOC-RE \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1206AX-SUC-PG \* WD2142XX-XOC-RE \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD2141XX-XOC-RE \* WD2142XX-XOC-RE \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* IALINESX-PIP-PG \* SIA0000X-SYS-RE \* RFAIL-ISOPW-OEM +  
MRSMR4X2-MCC-MF \* MRSMR3X4-MCC-MF \* SMALL-LOCA \* MIA-INIT-FL-OES5 +  
OAC15112-BUS-FL \* ACOODG1X-DGN-RN \* ACOODG2X-DGN-ST \* SMALL-LOCA \* RFAIL-ISOPW-OES +  
OAC15112-BUS-FL \* ACOODG1X-DGN-ST \* ACOODG2X-DGN-RN \* SMALL-LOCA \* RFAIL-ISOPW-OES +  
MEDIUM-LOCA \* IA00000X-DRY-OE \* SIARLFVX-RCO-MF \* RFAIL-ISOPW-OEM +  
ACOODG1X-DGN-RN \* ACOODG2X-DGN-RN \* IOAC15112-BUS-FL \* CC-CW004-20F2FL \* RFAIL-OPENBYP-OE +  
LORWNPVALK \* RFAIL-RRIVERWTR \* LORWSCRW-RSD-OE +  
MEDIUM-LOCA \* IARLFVX-RCO-MF \* SIA0000X-SYS-RE \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* IALINESX-PIP-PG \* SIARLFVX-RCO-MF \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* ACATSCS1-TCO-SP \* EC382XXM-MCC-MF +  
MEDIUM-LOCA \* WD1207AX-CCC-MF \* WD1207BX-CCC-MF \* WD1206B4-PMP-ST \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1207AX-CCC-MF \* WD126B2X-CCC-MF \* WD1207BX-CCC-MF \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1207AX-CCC-MF \* WD1206A1-PMP-ST \* WD1207BX-CCC-MF \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD126A2X-CCC-MF \* WD1207AX-CCC-MF \* WD1207BX-CCC-MF \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WDLEV7AX-SWT-FL \* WD126B2X-CCC-MF \* WD1207BX-CCC-MF \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1207AX-CCC-MF \* WD126B2X-CCC-MF \* WDLEV7BX-SWT-FL \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1207AX-CCC-MF \* WD1206B4-PMP-ST \* WDLEV7BX-SWT-FL \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1206A1-PMP-ST \* WDLEV7AX-SWT-FL \* WD1207BX-CCC-MF \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1207AX-CCC-MF \* WD1206A1-PMP-ST \* WDLEV7BX-SWT-FL \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WDLEV7AX-SWT-FL \* WD1207BX-CCC-MF \* WD1206B4-PMP-ST \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD126A2X-CCC-MF \* WDLEV7AX-SWT-FL \* WD1207BX-CCC-MF \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD126A2X-CCC-MF \* WD1207AX-CCC-MF \* WDLEV7BX-SWT-FL \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1206A1-PMP-ST \* WDLEV7AX-SWT-FL \* WDLEV7BX-SWT-FL \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD126A2X-CCC-MF \* WDLEV7AX-SWT-FL \* WDLEV7BX-SWT-FL \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WDLEV7AX-SWT-FL \* WD126B2X-CCC-MF \* WDLEV7BX-SWT-FL \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WDLEV7AX-SWT-FL \* WD1206B4-PMP-ST \* WDLEV7BX-SWT-FL \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD126A2X-CCC-MF \* WD6BL1X4-REL-EN \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD1206A1-PMP-ST \* WD6BL1X4-REL-EN \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD6AL1X1-REL-EN \* WD126B2X-CCC-MF \* RFAIL-ISOPW-OEM +  
MEDIUM-LOCA \* WD6AL1X1-REL-EN \* WD1206B4-PMP-ST \* RFAIL-ISOPW-OEM +  
ACPNLIX-TFM-FL \* ICW20002X-PIP-BR \* RFAIL-ISOCW-OE +  
MEDIUM-LOCA \* IA00000X-DRY-OE \* SIALINEX-PIP-PG \* RFAIL-ISOPW-OEM.

INPUT FILE 3

MEDIUM-LOCA 1.10E-03 2.76E-03 INIT  
ICW20002X-PIP-BR 6.00E-04 4.50E-04 INIT  
MIA-INIT-FL-OELP 4.00E-02 3.00E-02 OP-ERROR  
EC89CXX-AUTO-FL 3.40E-02 2.55E-02 MECH

SMALL-LOCA	4.50E-03	1.10E-02	INIT
CC-WD126-20F2FL	3.74E-03	4.74E-03	CC
ECCV381X-COO-MF	2.50E-03	6.17E-03	MECH
WD126B2X-CCC-MF	3.40E-02	3.46E-02	MECH
WD126A2X-CCC-MF	3.40E-02	3.46E-02	MECH
WD1206B4-PMP-ST	3.40E-02	4.31E-02	MECH
WD1206A1-PMP-ST	3.40E-02	4.31E-02	MECH
WD1206B4-PMP-RE	3.00E-02	2.25E-02	OP-ERROR
WD1206A1-PMP-RE	3.00E-02	2.25E-02	OP-ERROR
MIAHS723-SWT-FL	6.70E-04	5.02E-04	ELEC
ACOATS4X-ATS-CO	6.70E-04	5.02E-04	ELEC
ACOOOB4X-SWT-CO	6.70E-04	5.02E-04	ELEC
IOAC15112-BUS-FL	7.00E-01	9.15E-02	INIT
CC-CWBRE-20F2FL	3.00E-06	3.80E-06	OP-ERROR
MRSXXXXX-ACT-OE	3.20E-02	2.40E-02	OP-ERROR
MIA-INIT-SS-OE	2.00E-02	1.50E-02	OP-ERROR
MRSPMRA1-PMP-RE	3.00E-04	2.25E-04	OP-ERROR
MRSPMRB4-PMP-RE	3.00E-04	2.25E-04	OP-ERROR
CC-CWBPV-20F2FL	8.64E-07	1.14E-06	CC
OAC15112-BUS-FL	2.80E-03	2.04E-03	ELEC
ACOODGXX-SEQ-FL	1.00E+00	0.00E+00	ELEC
WD1206A1-RVS-FL	1.10E-02	2.72E-02	ELEC
WD1206AX-SUC-PG	1.10E-02	2.72E-02	MECH
WD1206B4-RVS-FL	1.10E-02	2.72E-02	ELEC
WD1206BX-SUC-PG	1.10E-02	2.72E-02	MECH
WD2141XX-XOC-RE	1.00E-02	7.50E-03	OP-ERROR
WD2142XX-XOC-RE	1.00E-02	7.50E-03	OP-ERROR
CC-EC380-30F3FL	3.26E-04	4.13E-04	CC
MRSMVNO1-XOC-RE	1.00E-02	7.50E-03	OP-ERROR
MRSMVNO2-XOC-RE	1.00E-02	7.50E-03	OP-ERROR
MRSMVNO3-XOC-RE	1.00E-02	7.50E-03	OP-ERROR
MRSMVNO4-XOC-RE	1.00E-02	7.50E-03	OP-ERROR
SIAOOOXX-SYS-RE	1.00E-02	7.50E-03	OP-ERROR
IAOOOXX-DRY-OE	1.50E-02	3.70E-02	OP-ERROR
MRSPMRA1-PMP-ST	6.20E-03	0.00E+00	MECH
MRSPMRB4-PMP-ST	6.20E-03	0.00E+00	MECH
CC-CW004-20F2FL	1.54E-04	1.95E-04	CC
CC-ACDST-20F2FL	3.60E-02	4.56E-02	CC
MIA-INIT-LS-OE	1.10E-01	8.25E-02	OP-ERROR
CC-CW200-20F2FL	5.20E-06	6.59E-06	CC
CC-CW247-20F2FL	5.20E-06	6.59E-06	CC
SC-SCRAM-FS	1.00E-02	7.50E-03	INSTRUM
NI-SCRAM-FS	1.00E-02	2.47E-02	INSTRUM
IALINESX-PIP-PG	9.84E-03	4.34E-03	MECH
SIARLFVX-RCO-MF	4.32E-03	3.24E-03	MECH
AC-PUMP-TRIP-OES	5.00E-02	3.75E-02	OP-ERROR
MRSPMRAX-CCC-MF	2.50E-03	6.17E-03	MECH
MRSPMRBX-CCC-MF	2.50E-03	6.17E-03	MECH
IARLFVVX-RCO-MF	4.32E-03	3.24E-03	MECH
ACOODG1X-DGN-ST	1.10E-01	8.25E-02	ELEC
ACOODG2X-DGN-ST	1.10E-01	8.25E-02	ELEC
WD1207BX-CCC-MF	3.40E-02	3.46E-02	MECH
WD1207AX-CCC-MF	3.40E-02	3.46E-02	MECH
WDLEV7BX-SWT-FL	3.39E-02	3.45E-02	MECH
WDLEV7AX-SWT-FL	3.39E-02	3.45E-02	MECH
9CLUTCH-FAIL	3.80E-05	9.38E-05	MECH
9-MISALIGN	3.80E-05	9.38E-05	MECH
WD6BL1X4-REL-EN	1.10E-03	4.85E-04	ELEC
WD6AL1X1-REL-EN	1.10E-03	4.85E-04	ELEC
SIALINEX-PIP-PG	2.40E-03	1.06E-03	MECH
WD1207B4-PMP-RE	3.00E-02	2.25E-02	OP-ERROR
WD1207A1-PMP-RE	3.00E-02	2.25E-02	OP-ERROR
WD1206B4-PMP-RN	7.92E-04	5.94E-04	MECH
WD1206A1-PMP-RN	7.92E-04	5.94E-04	MECH
ACPNLINX-TFM-FL	2.40E-05	1.06E-05	ELEC
AC1206A1-DCO-FL	6.70E-04	5.02E-04	ELEC
AC1206B4-DCO-FL	6.70E-04	5.02E-04	ELEC
OACOOO11-TCO-FL	6.70E-04	5.02E-04	ELEC
OACOOO44-TCO-FL	6.70E-04	5.02E-04	ELEC
ACOODG2X-DGN-RN	5.75E-02	2.53E-02	ELEC
ACOODG1X-DGN-RN	5.75E-02	2.53E-02	ELEC

SCTBPSWX-SOC-OE	2.00E-05	1.50E-05	OP-ERROR
MRSMPRB4-PMP-RN	9.60E-04	7.20E-04	MECH
MRSMPRA1-PMP-RN	9.60E-04	7.20E-04	MECH
EC382XXM-MCC-MF	1.60E-02	1.63E-02	MECH
ECCOOL3SX-REL-EN	1.10E-03	4.85E-04	ELEC
WD2145AX-SCC-MF	2.10E-02	1.57E-02	MECH
WD2146AX-SCC-MF	2.10E-02	1.57E-02	MECH
WD2145BX-SCC-MF	2.10E-02	1.57E-02	MECH
WD2146BX-SCC-MF	2.10E-02	1.57E-02	MECH
ACPMPMRA-NXX-SP	6.70E-04	5.02E-04	ELEC
ACOATS4X-TXX-TR	2.00E-02	3.02E-02	ELEC
ACO16A4X-DCO-FL	6.70E-04	5.02E-04	ELEC
ACOATS1X-TCO-SP	6.70E-04	5.02E-04	ELEC
ACOB16AX-SCO-SP	6.70E-04	5.02E-04	ELEC
ACPMPMRB-NXX-SP	6.70E-04	5.02E-04	ELEC
WD1206A1-CYC-OE	3.60E-04	2.70E-04	OP-ERROR
WD1206B4-CYC-OE	3.60E-04	2.70E-04	OP-ERROR
ACATSCS1-TCO-SP	6.70E-04	5.02E-04	ELEC
CC-SCTCZCZA-DE	1.10E-05	1.39E-05	ELEC
ECCV380X-COO-MF	2.50E-03	6.17E-03	MECH
FLOOD-OEL	8.10E-02	1.03E-01	OP-ERROR
LARGE-LOCA	4.00E-05	9.87E-05	INIT
LORW1071-PMP-OE	3.80E-03	4.80E-03	OP-ERROR
LORW1072-PMP-OE	9.50E-03	1.20E-02	OP-ERROR
LORWDIAG-ACC-OE	5.00E-03	6.33E-03	OP-ERROR
LORWISL1-CWH-OE	9.50E-03	1.20E-02	OP-ERROR
LORWISL2-CWH-OE	9.00E-02	1.14E-01	OP-ERROR
LORWNPVALK	4.20E-04	5.32E-04	INIT
LORWNPLOSTK	4.00E-04	5.10E-04	INIT
LORW0190-PMP-OE	3.80E-03	4.80E-03	OP-ERROR
LORWSCRMSD-OE	1.50E-04	1.90E-04	OP-ERROR
MIA-INIT-FL-OESS	2.00E-02	2.50E-02	OP-ERROR
MRSMR3X4-MCC-MF	1.60E-02	1.63E-02	MECH
MRSMR4X2-MCC-MF	1.60E-02	1.63E-02	MECH
RFAIL-ISOCW-OE	7.20E-01	3.70E-01	RECOV
RFAIL-ISOPW-OEL	5.30E-01	7.20E-02	RECOV
RFAIL-ISOPW-OEM	2.60E-01	1.95E-01	RECOV
RFAIL-ISOPW-OES	2.50E-01	2.54E-01	RECOV
RFAIL-OPENBYP-OE	5.00E-02	6.30E-02	OP-ERROR
RFAIL-RRIVERWTR	2.50E-01	2.54E-01	RECOV
SCTNGPGX-BUS-SH	7.68E-04	1.90E-03	ELEC
SCTO22PX-BUS-SH	7.68E-04	1.90E-03	ELEC
SCTOATDX-REL-CB	1.00E-02	1.27E-02	OP-ERROR
THROTTLE-FL-OELM	3.50E-03	4.30E-03	OP-ERROR

INPUT FILE 4

9-MISALIGN	1
9CLUTCH-FAIL	1
ACATSCS1-TCO-SP	2
ACOATS4X-ATS-CO	2
ACOODG1X-DGN-RN	3
ACOODG1X-DGN-ST	4
ACOODG2X-DGN-RN	3
ACOODG2X-DGN-ST	4
CC-CW247-20F2FL	6
CC-CW200-20F2FL	6
ECCV381X-COO-MF	5
ECCV380X-COO-MF	5
LORW1072-PMP-OE	7
LORWISL1-CWH-OE	7
MRSMR3X4-MCC-MF	8
MRSMR4X2-MCC-MF	8
SCTNGPGX-BUS-SH	18
SCTO22PX-BUS-SH	18
WD126A2X-CCC-MF	13
WD126B2X-CCC-MF	13
WD1206A1-PMP-RE	17
WD1206A1-PMP-ST	16
WD1206A1-RVS-FL	15

WD1206AX-SUC-PG	14
WD1206B4-PMP-RE	17
WD1206B4-PMP-ST	16
WD1206B4-RVS-FL	15
WD1206BX-SUC-PG	14
WD1207AX-CCC-MF	13
WD1207BX-CCC-MF	13
WD2141XX-XOC-RE	12
WD2142XX-XOC-RE	12
WD6AL1X1-REL-EN	11
WD6BL1X4-REL-EN	11
WDLEV7AX-SWT-FL	10
WDLEV7BX-SWT-FL	10

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