UCRL-JC-133757
PREPRINT

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This paper was prepared for submittal to the
American Society of Mechanical Engineers International
Mechanical Engineering Congress and Exposition
Nashville, TN
November 14-19, 1999

March 1999

Lawrence Livermore National Laboratory

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Abstract

Development of methods to spray form materials by precisely controlled deposition of droplets can result in new manufacturing processes which offer improved metallurgical performance and reduced production costs. These processes require a more detailed knowledge of the fluid mechanics, heat transfer and solidification that occur during droplet spreading. Previous work using computer simulations of this process have been difficult to implement and have required long running times. This paper examines the use of an alternative, simplified, method developed by Madjeski for solving for the problem of droplet spreading and solidification. These simplifications reduce the overall splat spreading and solidification problem to a closed-form differential equation. This differential equation is then solved under various conditions as reported from recent publications of experimental and numerical results of drop analysis. The results from the model are compared in terms of maximum splat diameter, minimum splat thickness, and time for the droplet spreading to reach 95% of the maximum diameter. The results indicate that the accuracy of the model can be improved by accounting for energy losses in the initial rate of droplet spreading. The model results show that the predictions of experimental results are improved to within 30% over a wide range of conditions.

Nomenclature

a	solid phase thermal diffusivity
b	thickness of the liquid layer
d	splat diameter
d^*	non-dimensional splat diameter, d/D
D	initial droplet diameter
e	ratio of droplet energy after impact to energy before impact
k	freezing parameter, Equation (2)
Pe	Peclet number, wD/a
R	cylinder radius
R_o	initial cylinder ratio
Re	Reynolds number, $wD\rho/\mu$
t	time
t^*	non-dimensional time, tw/D
U	constant in Equation (3)
w	droplet speed before impact
We	Weber number, $\rho D w^2/\sigma$
y	solid layer thickness
z	splat height
z^*	non-dimensional splat height, z/D
ε	ratio R_o/D
ρ_l	liquid phase density
ρ_s	solid phase density
σ	surface tension

Introduction

The problem of liquid droplets impinging on a solid surface has practical applications to multiple processes such as spray cooling, spray forming, microfabrication, microcasting, ink jet technology, and precision solder droplet dispensing. An improved control of these processes requires a more detailed knowledge of the fluid mechanics, heat transfer and solidification that occur during droplet spreading.

The process of droplet spreading and solidification is difficult to analyze due to the effect of multiple factors, including inertia, viscosity, gravity, surface tension, wetting, heat transfer (mainly by conduction and radiation), solidification (possibly under non-equilibrium conditions, Kang et al., 1995), phase transitions in the solid phase (Amon et al., 1996), and interactions with other droplets (Kang et al., 1994).

Some numerical analyses of the process of droplet spreading have been presented to date (Trapaga and Szekely, 1991; Trapaga et al., 1992; Fukai et al., 1993; Fukai et al., 1995; Waldvogel et al., 1996; Zhao et al., 1996; Waldvogel and Poulikakos, 1997; Pasandideh-Fard et al., 1998). These solve for the continuity equation, for the Navier-Stokes equations, and for the energy equation in the liquid and solid phases (and ideally in the substrate too). The solution of these equations is complicated by the presence of phenomena such as a free surface, conduction heat transfer through a contact resistance, a rapidly deforming geometry with a big aspect ratio, and liquid-solid and solid-solid phase transitions. As a result, computer simulations have been difficult to implement and have required long running times.

An alternative, simplified, method for solving for the problem of droplet spreading and solidification has been presented by Madejski (1976). This method uses an overall energy balance for the droplet. The initial kinetic energy of the droplet is converted into potential (surface) energy, or is dissipated due to viscous effects as the droplet spreads. The balance between kinetic, potential and dissipated energy determines the rate of droplet spreading. Solidification may result due to contact with a cold substrate.

Madejski's method uses a series of simplifications to reduce the overall splat spreading and solidification problem to a closed-form differential equation. The main assumptions are:

1. The droplet transforms instantly from a sphere into a cylinder when it impacts the substrate. No energy is dissipated during this initial transformation of the droplet.
2. The Navier-Stokes equations are not solved. Instead, a velocity field is assumed. The velocity profile selected is a simple velocity field that satisfies the continuity equation.
3. The droplet superheat disappears instantly when the droplet touches the substrate. Contact between the droplet and the substrate is assumed to be perfect (infinite contact heat transfer coefficient).
4. For cases that consider freezing, it is assumed that the solidification starts immediately after impingement. The solid layer thickness at any radial position is proportional to the square root of the time elapsed since the moment in which the droplet touched that radial position. The proportionality constant is calculated from a solution to the Stefan problem for a semi-infinite medium (Carslaw and Jaeger, 1990).
5. The liquid (unfrozen) phase has a thickness that is a function of time, but not a function of position along the splat.

6. The effect of surface wetting and contact angles is neglected.
7. Droplets do not break up when they collide with the substrate.

Madejski's method takes into account three of the most important effects in droplet spreading: viscous dissipation, surface tension, and freezing. This results in a very complete method, considering the simplicity of the analysis. However, many important effects are not considered. These include droplet superheat, thermal contact resistance (Liu et al., 1995), substrate melting, droplet recoil (Fukai et al., 1993), and wetting effects (Fukai et al., 1995).

Some improvements on Madejski's method have been published in the literature. Markworth and Saunders (1992) developed an improved velocity field for application to Madejski's method. The velocity field is improved because it not only satisfies the continuity equation. It also satisfies the appropriate no stress boundary condition at the free surface. The new velocity field can be used to derive new versions of Madejski's equations. Rangel and Bian (1997) have published the resulting equations. Rangel and Bian (1997) have also extended Madejski's method by using the full energy equation and an energy balance to track the solid-liquid interface. The extended method can therefore take into account droplet superheat and substrate melting. Thermal contact resistance could also be implemented in the model.

The main advantage of Madejski's method is its closed form, which is very simple to use. Only three non-dimensional parameters (the Reynolds number, Re; the Weber number, We; and a freezing parameter, k) are enough to determine droplet spreading with Madejski's method. In contrast, detailed numerical analyses of the droplet spreading and solidification require a major effort for implementation and long computer run times. Detailed numerical solutions may also be subjected to limitations in accuracy, due to an inappropriate knowledge of the wetting and heat transfer interactions between the droplet and the substrate. These factors are recognized to be very important in the process of droplet spreading, and obtaining real data for practical applications is a challenge (Pasandideh-Fard et al., 1998).

While a detailed analysis is necessary in many cases, it is important to determine the accuracy of Madejski's model as compared to existing experimental and numerical data. This comparison may help in determine under which regimes Madejski's model gives the best results, and under which regimes it may be used at least as a preliminary tool in determining droplet spreading parameters.

This report presents a comparison of the results obtained with Madejski's model and those obtained in recent publications, both experimental and numerical. Three magnitudes are compared: maximum splat diameter, minimum splat thickness, and time for droplet spreading to 95% of the maximum diameter.

Analysis

This work uses Madejski's (1976) method with the improved velocity profile presented by Markworth and Saunders (1992). The Schwarz solution (Carslaw and Jaeger, 1990) is used instead of the Neumann solution to predict the freezing rate, according to the recommendation of Rangel and Bian (1997). The full energy equations applied in the solution of Rangel and Bian are not used in this work.

An additional modification to Madejski's method is done beyond those discussed in the previous paragraph. Madejski (1976) defines the parameter ε as $\varepsilon \equiv R_o/D$, where D is the droplet diameter before impact and R_o is the initial radius of the cylinder immediately after impact (it is assumed that the spherical droplet becomes a cylinder immediately upon impact; see Assumption 1 in the Introduction). Madejski originally used $\varepsilon=0.5$. Rangel and Bian later used $\varepsilon=0.74$, which satisfies the equation of conservation of potential energy. However, using $\varepsilon=0.74$ results in considerable instantaneous spreading of the droplet upon impact. Conservation of mass indicates that, for $\varepsilon=0.74$, the height of the cylinder immediately after impact, b, is only 30% of the droplet diameter before impact. This results in a very short time for completion of droplet spreading under many conditions.

In this work, $\varepsilon=0.408$ is used. This results in an initial cylinder height b=D, and a much improved prediction of the time for droplet spreading. Potential energy is not conserved during the impact. However, total energy (kinetic+potential) can still be conserved if the equation for the initial speed of droplet spreading (Equation (16) in Rangel and Bian, 1997) is modified appropriately. The new version of this equation is:

$$\left[\frac{dR}{dt}\right]_{t=0} = w\sqrt{\frac{\frac{5}{3}+\frac{20}{We}\left(1-\varepsilon^2-\frac{1}{3\varepsilon}\right)}{1+\frac{11}{252\varepsilon^6}}} \qquad (1)$$

where We is the Weber number, defined as $We \equiv \rho_l D w^2/\sigma$, w is the droplet velocity before impact, ρ_l is the liquid density and σ is surface tension. The non-dimensional freezing parameter k is defined as (Madejski, 1976):

$$k \equiv 6\varepsilon^2 U \frac{\rho_s}{\rho_l}\sqrt{\frac{\varepsilon}{Pe}} \qquad (2)$$

where Pe is the Peclet number, defined as $Pe \equiv wD/a$, a is the thermal diffussivity of the solid layer, and U is the proportionality constant in the solid layer thickness equation, obtained from the solution to the Schwarz problem (Carslaw and Jaeger, 1990).

$$y = U\sqrt{at} \qquad (3)$$

Madejski's equations are solved with Euler's method for solving the differential equation. The integral equation that determines the thickness of the solid layer is solved with Simpson's rule. The program was tested for time step size sensitivity by halving the time step until a negligible change in the solution was obtained. Typical running time is of the order of 1 minute in a current engineering workstation.

Results

A survey of the recent literature on splat formation was conducted, and multiple experimental and numerical results were collected from the available papers. Results were collected for maximum splat diameter, minimum splat thickness, and time for reaching 95% of the maximum diameter. The time for 95% of the maximum diameter is used because this is much easier to determine than the time for reaching the maximum diameter (Fukai et al., 1995).

The conditions and the results found in the references are converted to non-dimensional parameters (Re, We, k, $d^*=d/D$, $z^*=z/D$, and $t^*=tw/D$). A summary of all the information collected from the references is included in Table 1. Much of the information was collected from figures, and it is therefore subjected to reading errors, especially in the case of the splat thickness, due to the small values frequently obtained for it. Some of the spaces are empty because these values were not reported in the references.

Detailed analyses and experiments often show droplet recoil due to surface tension effects, especially for low Weber numbers. Madejski's model does not predict droplet recoil. For this reason, the splat properties listed in Table 1 do not represent final, equilibrium splat conditions. Instead, they represent values at the end of the first splat expansion cycle, before recoil starts. No comparison between Madejski's method and detailed models is possible after the recoil process starts.

Most of the data found in the literature are obtained for isothermal droplet spreading, at conditions for which no freezing occurs. A few cases with freezing are also reported. The literature survey used for this report is by no means exhaustive, but the data cover a wide range of Reynolds and Weber numbers, and should give a good indication of the accuracy of Madejski's method under multiple regimes.

Figure 1 shows non-dimensional maximum droplet diameter (d^*) as a function of Reynolds number and Weber number. The figure shows all the points listed in Table 1, with the original reference identified by symbols. The numbers located next to the symbols indicate the maximum splat diameter obtained from the references. A range of values is given for points that originate from a single reference and are located very close to each other in the figure (data from Fukai et al., 1995, and from Pasandideh-Fard et al., 1998). The figure shows the wide ranges of Reynolds (10^2-10^6) and Weber (1-10^4) numbers studied in the literature.

The lines in the figure show contours of constant maximum splat diameter, calculated by Madejski's model, assuming no freezing conditions (k=0). Both the lines and the symbols show that the maximum splat diameter tends to grow as the Reynolds and the Weber numbers increase. Some anomalous results are easily identified in the figure. These include the values reported by Zhao et al., 1996, which appear high compared to the values for nearby points reported in other references. This may be due to effects, such as surface wetting, which cannot be described in terms of the Reynolds and the Weber numbers alone. Another effect not properly described by the Reynolds and the Weber numbers is freezing, and the points with freezing (open symbols in the figure) have maximum diameters that are lower than the neighboring points with no freezing.

Figure 1 indicates that there is good qualitative agreement between Madejski's method and the existing literature. Figure 2 is included to facilitate a quantitative comparison between the references and Madejski's model. Figure 2 shows the ratio of the maximum droplet diameter calculated from Madejski's model and the maximum droplet diameter reported in the references. The figure shows that Madejski's model tends to overestimate the maximum splat diameter. Figure 2 shows that the best agreement is obtained for the data of Trapaga and Szekely (1992) and for the data of Fukai et al. (1995). The agreement improves as the Reynolds and Weber numbers increase. The results for the freezing cases are also overestimated by 30-60%.

Figure 3 shows the ratio of the minimum splat thickness calculated by Madejski's model and the minimum splat thickness as reported in the references. Some points in the figure do not include a number because the minimum splat thickness is not reported in these references. The figure indicates that Madejski's model underestimates the minimum splat thickness in almost all cases. This is consistent with the overestimation of the splat diameter shown in Figure 2, since a bigger splat is necessarily thinner. The relative error in using Madejski's model for predicting splat thickness is greater than the relative error in predicting the splat diameter (Figure 2). This is due in part to the difficulty of reading the small values of the splat thickness from plots presented in the references.

Figure 4 shows the ratio of the time for reaching 95% of the maximum splat diameter calculated by Madejski's model and the time for reaching 95% of the maximum splat diameter reported in the literature. Unlike figures 2 and 3, this figure does not show a definite trend. Madejski's model underpredicts the time for some points and overpredicts the time for some other points. The trend seems to depend more on the original reference than on the location on the Reynolds-Weber plane. Spreading time is underestimated when compared with the results of Trapaga and Szekely (1991), but it is overestimated when compared to those of Fukai et al., (1993). Very good agreement is once more obtained with the results of Fukai et al. (1995).

Figures 5 through 7 show a comparison of the splat spreading as a function of time between Madejski's model and some of the references. Figure 5 shows the spreading process for splats at the conditions studied by Trapaga and Szekely (1991). The figure shows the results of Trapaga and Szekely with solid lines and the results of Madejski's method with dotted lines. Madejski's method underpredicts the initial slope of the lines, but the slope then remains fairly constant for a long period of time, resulting in final diameters that closely predict those of Trapaga and Szekely. The lines from Trapaga and Szekely have slopes that drop rapidly. This is due to substantial viscous dissipation in the early stages of the spreading process. In contrast, Madejski's method assumes that no energy losses occur in the initial spreading of the droplet.

Figure 6 shows a comparison of the results of Madejski's model with those of Waldvogel et al. (1996). The figure shows the results of Waldvogel et al. with solid lines and the results of Madejski's method with dotted lines. Figure 2 shows that the conditions used by Waldvogel et al. (low Reynolds and Weber number) result in a poor agreement with Madejski's model. Figure 6 shows that Waldvogel et al. report substantial droplet recoil, down to diameters lower than the original droplet diameter in some cases. As previously discussed, Madejski's model does not predict droplet recoil in its current form. Droplet solidification should reduce the amount of droplet recoil and improve the accuracy of Madejski's model for low Reynolds and Weber number cases. Figure 6 shows once more that Madejski's model underpredicts the initial rate of viscous energy dissipation. As a consequence, the slopes of the lines remain constant for a long time compared to those of Waldvogel et al.

Figure 7 shows a comparison of the results of Madejski's model with those of Pasandideh-Fard et al. (1998). The figures show the results of Pasandideh-Fard et al. with solid lines and the results of Madejski's method with dotted lines. The results of Pasandideh-Fard et al. presented in the figure include the effect of freezing. Some droplet recoil is observed in the results of Pasandideh-Fard et al., although the magnitude of the recoil is much reduced compared to the results of Waldvogel et al. (1996) shown in Figure 6, partly due to the higher Reynolds number and partly due to freezing. Figure 7 shows that Madejski's model predicts the freezing time reasonably well. This is due in part to the use of near zero initial droplet superheat by Pasandideh-Fard et al. As previously discussed, Madejski's freezing model neglects superheat. The effect of droplet superheat could be estimated and incorporated in Madejski's model, as previously shown by Rangel and Bian (1997).

The previous figures indicate that Madejski's model overpredicts the splat diameter, and as a consequence it underpredicts the splat thickness. This is due in part to the assumption that the droplet turns into a cylinder immediately upon impact with no energy losses in the process. More detailed analyses (Trapaga and Szekely, 1991) indicate that there is substantial energy dissipation in the early stages of splat spreading. Including some energy losses in Madejski's model is certain to result in better predictions. Energy losses in the initial stage of splat spreading can be included in Madejski's model by modifying the initial rate of droplet spreading (Equation 1). The new form of this equation is:

$$\left[\frac{dR}{dt}\right]_{t=0} = w\sqrt{\frac{\frac{5e}{3}+\frac{20}{We}\left(e-\varepsilon^2-\frac{1}{3\varepsilon}\right)}{1+\frac{11}{252\varepsilon^6}}} \tag{4}$$

The rest of the formulation is not changed. In Equation (4), e is defined as the ratio of the cylinder energy immediately after impact to the droplet energy before impact. The parameter e was varied in the range $0.1<e<1.0$, and the best agreement with published results was obtained with $e=0.6$. This value is in reasonably good agreement with the viscous energy dissipation calculated by Trapaga and Szekely (1991) during the initial impact process.

Low values of e combined with low values of We may result in negative values inside the radical in Equation (4), which results in a non-physical, imaginary, number for the initial rate of droplet expansion. For $e=0.6$, the radical turns negative for points with $We<7.7$, which includes 4 points in Table 1 and in Figures 2, 3 and 4. For these points the method cannot be applied, and a higher value of e (0.9) has been used for comparison with the results from the references.

The results of the analysis for e=0.6 are shown in Figure 8. Figure 8 shows the same information as Figure 2, with the only difference that the Madejski calculations are done with e=0.6 (e=0.9 for We<7.7). Figure 8 shows the ratio of the maximum droplet diameter calculated from Madejski's model with e=0.6 and the maximum droplet diameter reported in the references. The figure shows a much-improved agreement between Madejski's model and the original references, compared with Figure 2. Almost all the results, including the results for the cases that include freezing, fall within 30% of the values given in the references, and in most cases the agreement is much better throughout the ranges of Reynolds and Weber numbers. This figure indicates that it is necessary to incorporate the initial diffusion of energy in Madejski's model to obtain results that are broadly applicable.

Conclusions

This paper uses a modified Madejski's model for calculating droplet spreading and solidification upon impingement on a solid surface. Madejski's model is modified by including an improved velocity profile, the Schwarz solution to predict the freezing rate, a modified aspect ratio for the cylinder formed upon impingement, and a different initial speed of droplet spreading. The results are compared to data published in the recent literature over a Reynolds number range of 10^2 to 10^6 and a Weber number range of 1 to

10^4. Three magnitudes are compared: maximum splat diameter, minimum splat thickness, and time for droplet spreading to 95% of the maximum diameter.

The results show that Madejski's model overpredicts the maximum droplet diameter and underpredicts the minimum splat thickness. No clear trend was found for the predictions of spreading time. Examination of the results shows that the overprediction of the maximum diameter occurs partly because Madejski's model neglects energy losses in the initial stages of the spreading process. These energy losses result from viscous dissipation. Madejski's model is then modified to take into account these losses by changing the initial rate of droplet spreading. The new results show improved agreement with the maximum splat diameters (within 30% for almost all cases). These results indicate that the modified Madejski's model of droplet spreading presented in this paper can be used to give reasonable predictions of maximum droplet diameter and minimum splat thickness.

Acknowledgments

This work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract W-7405-ENG-48.

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Table 1. Non-dimensional parameters obtained from the literature for the problem of droplet spreading, without and with freezing. The test results are the maximum splat diameter, the minimum splat thickness, and the time to reach 95% of the maximum splat diameter.

Results without freezing	Test conditions			Test results		
Reference	Reynolds	Weber	k	$d^* \equiv d/D$	$z^* \equiv z/D$	$t^* \equiv tw/D$
Trapaga and Szekely, 1991	585	6,800	0	3.60		2.40
	600	1,290	0	3.65	0.05	2.20
	6,000	1,290	0	5.80	0.02	3.20
	16,000	4,500	0	7.20		4.50
	60,000	1,290	0	9.30	0.01	5.95
	63,570	1,304	0	9.40		6.40
	600,000	1,290	0	16.00	0.01	13.00
Fukai et al., 1993	100	1.4	0	1.30	0.50	0.40
	120	80	0	2.35	0.08	1.10
	500	10	0	1.65	0.15	0.60
	600	8	0	1.65		0.55
	1,200	80	0	2.90	0.05	1.25
	1,200	100	0	3.05	0.08	2.00
	1,200	500	0	2.63		1.10
	1,200	1,000	0	3.05		1.45
	1,200	5,000	0	3.80		2.07
	1,944	31.6	0	2.37		1.10
	6,000	80	0	3.18	0.05	1.50
	12,000	80	0		0.04	
Fukai et al., 1995	3,010	58.4	0	4.04		1.90
	3,130	64.1	0	3.61	0.07	1.71
	3,320	57.5	0	3.94		2.08
	3,490	56.8	0	4.23	0.11	2.01
	4,130	111	0	4.48	0.06	2.29
	7,390	364	0	5.42	0.04	2.58
	8,800	359	0	6.03	0.03	2.86
Zhao et al., 1996	485	20	0	2.05		0.95
	1,650	235	0	3.62		1.90
Waldvogel et al., 1996	156	2.38	0	1.26	0.40	0.30
	313	4.76	0	1.36	0.18	0.41
	627	19	0	1.66	0.07	0.57
Results with freezing						
Reference	Reynolds	Weber	k	d/D	z/D	tw/D
Trapaga et al., 1992	23,537	185	0.07	2.73	0.13	1.53
	27,700	223	0.06	2.83	0.12	2.10
Waldvogel and Poulikakos, 1997	235	2.68	0.23	1.29	0.48	0.42
Pasandideh-Fard et al., 1998	12,412	67.5	0.01	3.10		4.19
	12,412	67.5	0.03	2.95		2.13

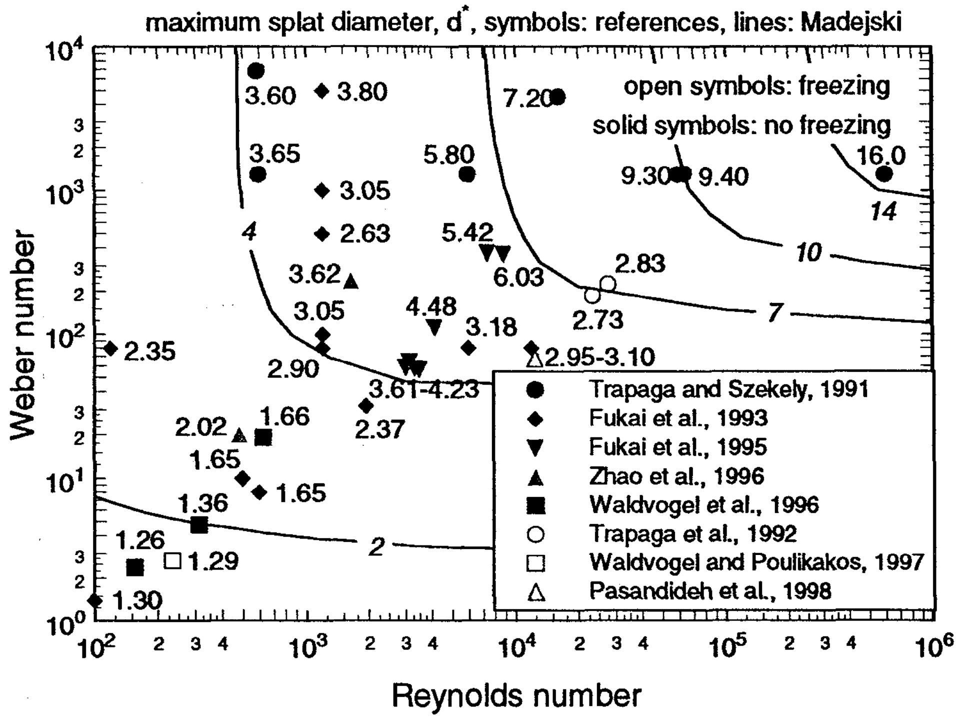

Figure 1. Non-dimensional maximum droplet diameter (d*) as a function of Reynolds number and Weber number. The figure shows all the points listed in Table 1, with the original reference identified by symbols. The numbers located next to the symbols indicate the maximum splat diameter obtained from the references. The lines in the figure show contours of constant maximum splat diameter, calculated by Madejski's model assuming no freezing conditions (k=0).

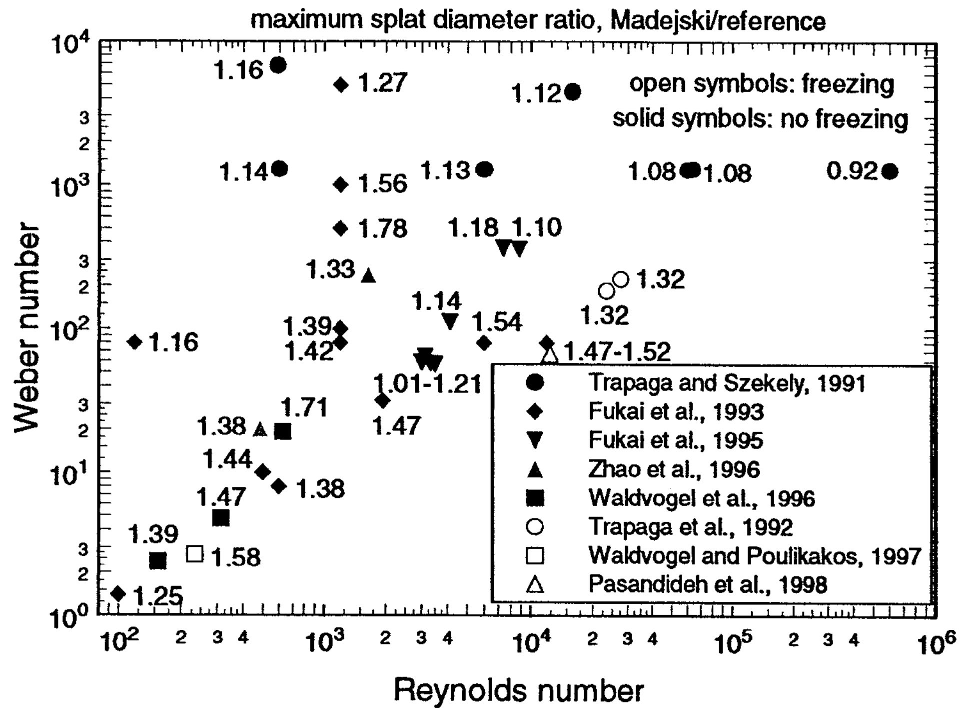

Figure 2. Ratio of the maximum droplet diameter calculated from Madejski's model and the maximum droplet diameter reported in the references as a function of Reynolds number and Weber number. The figure shows all the points listed in Table 1, with the original reference identified by symbols.

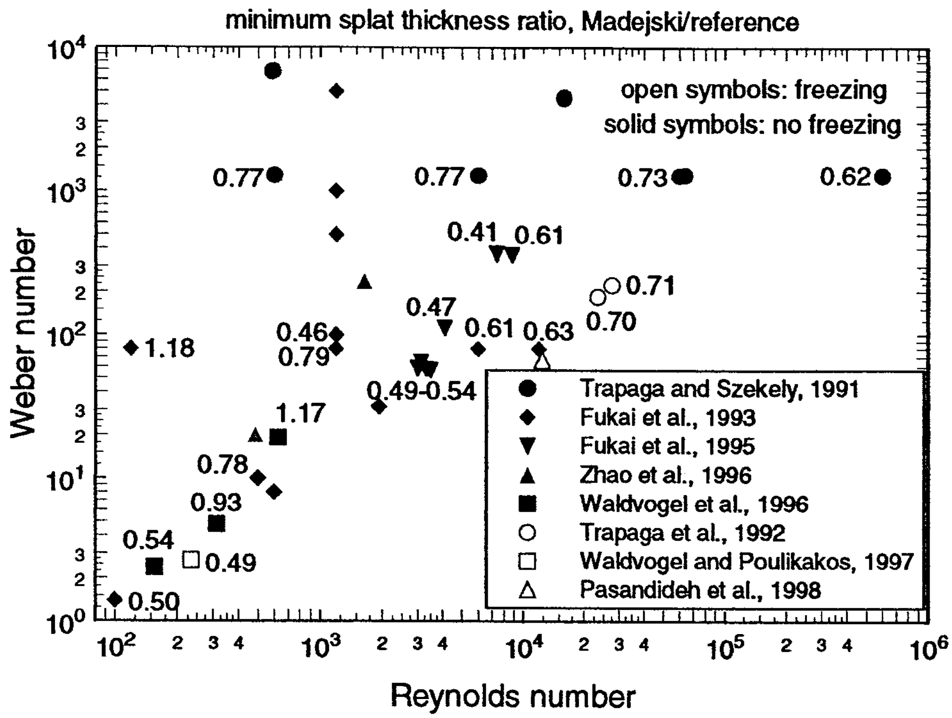

Figure 3. Ratio of the minimum splat thickness calculated from Madejski's model and the minimum splat thickness reported in the references as a function of Reynolds number and Weber number. The figure shows all the points listed in Table 1, with the original reference identified by symbols.

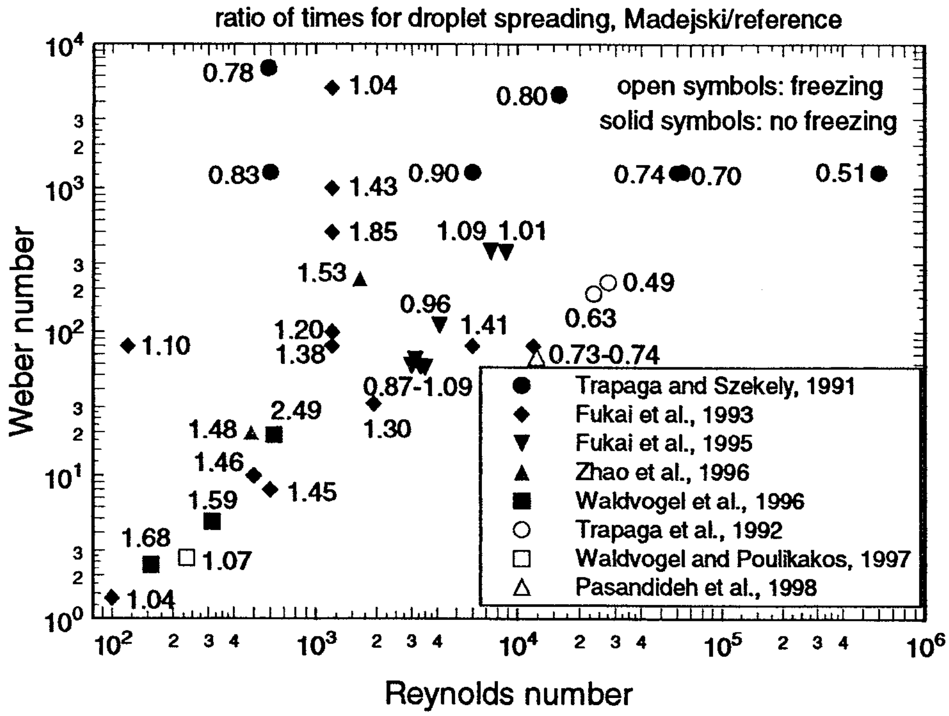

Figure 4. Ratio of the time for reaching 95% of the maximum splat diameter calculated by Madejski's model and the time for reaching 95% of the maximum splat diameter reported in the literature as a function of Reynolds number and Weber number. The figure shows all the points listed in Table 1, with the original reference identified by symbols.

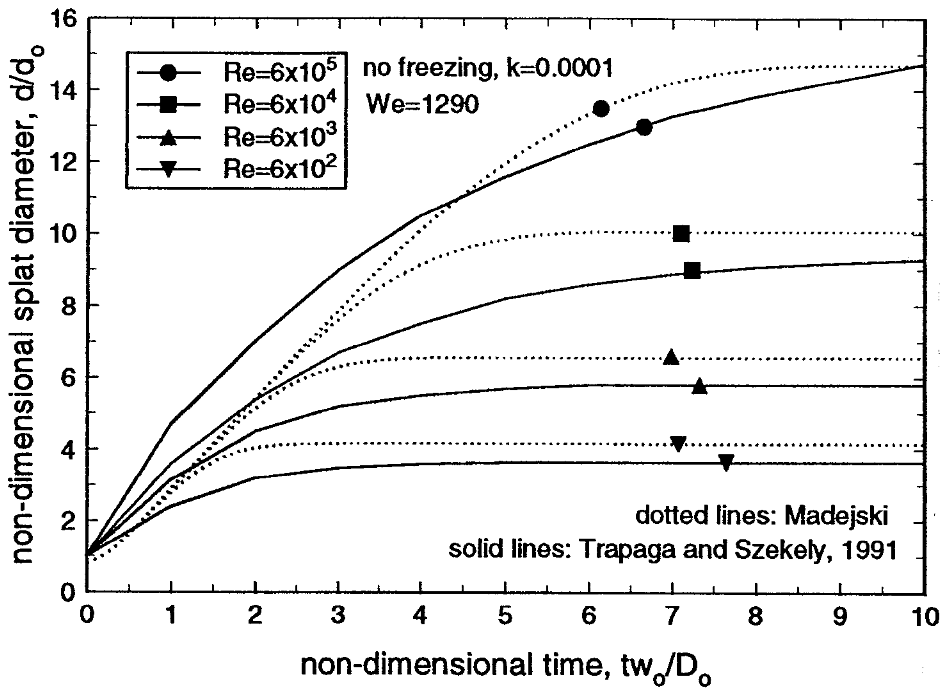

Figure 5. Comparison of the results of Madejski's model and the results of Trapaga and Szekely (1991). The figure shows splat diameter during the spreading process as a function of time. The results of Trapaga and Szekely are shown with solid lines and the results of Madejski's method with dotted lines.

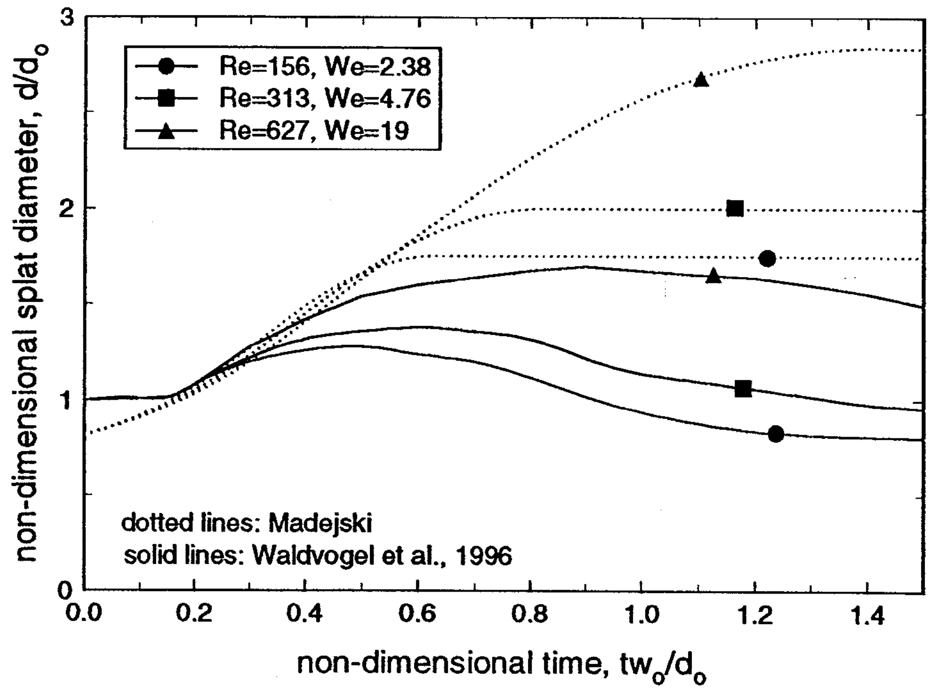

Figure 6. Comparison of the results of Madejski's model and the results of Waldvogel et al. (1996). The figure shows splat diameter during the spreading process as a function of time. The results of Waldvogel et al. are shown with solid lines and the results of Madejski's method with dotted lines.

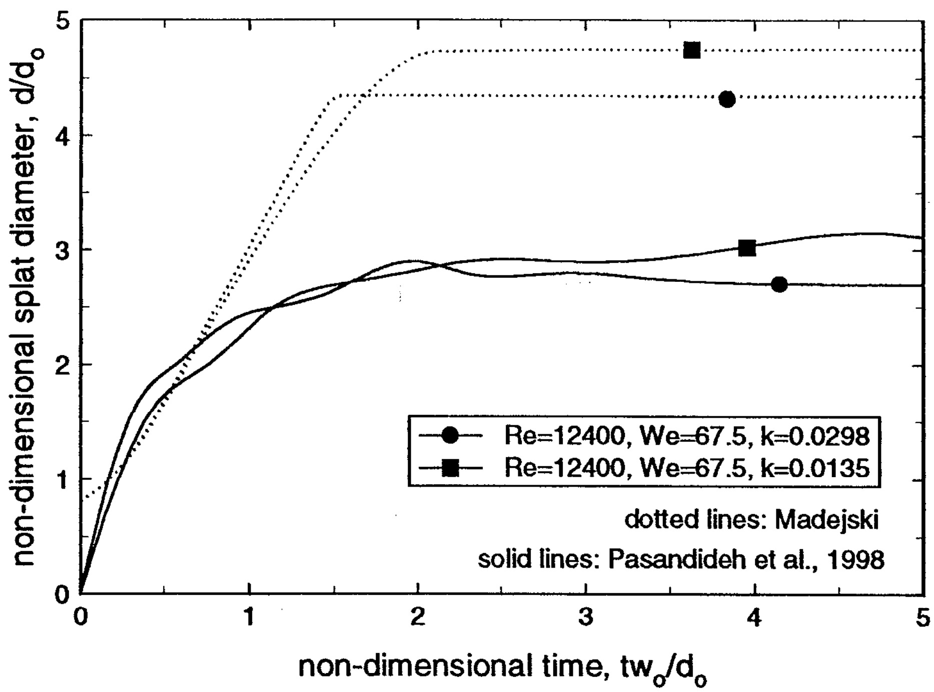

Figure 7. Comparison of the results of Madejski's model and the results of Pasandideh-Fard et al. (1998). The figure shows splat diameter during the spreading process as a function of time. The results of Pasandideh-Fard et al. are shown with solid lines and the results of Madejski's method with dotted lines.

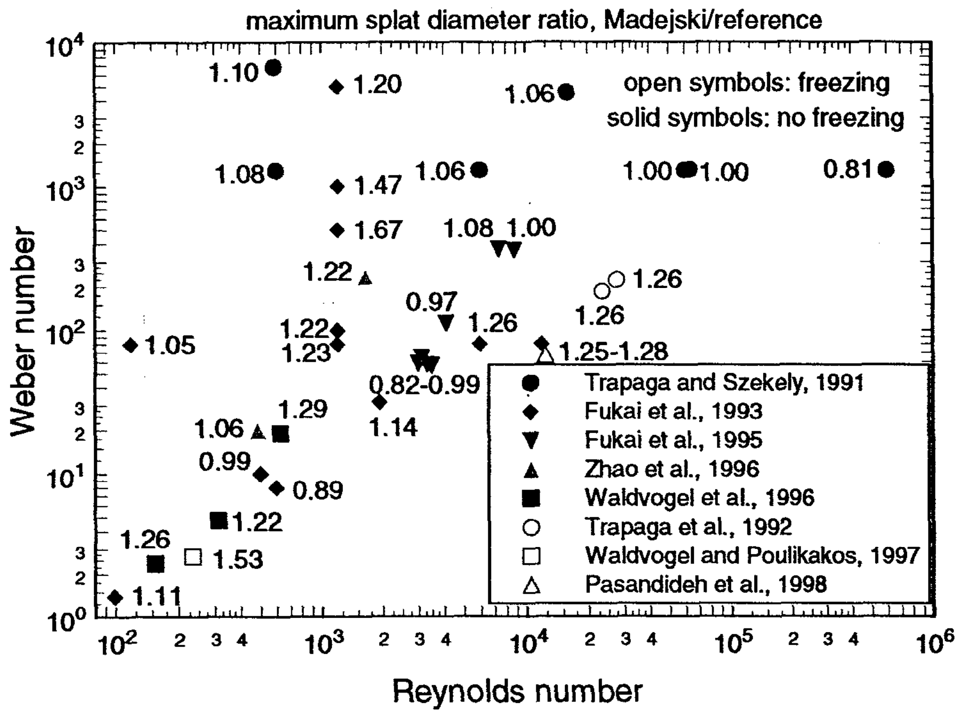

Figure 8. Ratio of the maximum droplet diameter calculated from Madejski's model and the maximum droplet diameter reported in the references as a function of Reynolds number and Weber number. Madejski's model uses an energy ratio e=0.6, except for points with We<7.7, for which e=0.9 is used. The figure shows all the points listed in Table 1, with the original reference identified by symbols.