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Automation and Other Extensions of the SMAC Modal Parameter Extraction Package

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ABSTRACT

As model validation techniques gain more acceptance and increase in power, the demands on the modal parameter extractions increase. The estimation accuracy, the number of modes desired, and the data reduction efficiency are required features. An algorithm known as SMAC (Synthesize Modes And Correlate), based on principles of modal filtering, has been in development for a few years[1,2]. SMAC has now been extended in two main areas. First, it has now been automated. Second, it has been extended to fit complex modes as well as real modes. These extensions have enhanced the power of modal extraction so that, typically, the analyst needs to manually fit only 10 percent of the modes in the desired bandwidth, whereas the automated routines will fit 90 percent of the modes. SMAC could be successfully automated because it generally does not produce computational roots.

NOMENCLATURE

FRF:	frequency response function
SDOF:	single degree-of-freedom
ωr:	natural frequency of r^{th} mode
ω_j:	j^{th} frequency line of FRF
ζ_r:	damping coefficient of r^{th} mode
ψ:	weighting vector
$H_X(\omega)$:	experimental FRF
$H_A(\omega)$:	analytical generalized coordinate FRF
$H_P(\omega)$:	predicted generalized coordinate FRF
MIF:	Mode Indicator Function
CMIF:	Complex Mode Indicator Function
NMIF:	Normal Mode Indicator Function
$\{\omega\}$:	vector of all frequency components
A_r:	modal residue coefficient
j:	$\sqrt{-1}$

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MOTIVATION

To reduce costs, many organizations are demanding higher fidelity models to be used for design studies. The prudent approach is to validate these models so that one has confidence in the modeling approach and results. This puts high demands on the modal testing results. Organizations want accurate results delivered quickly, and they want results to higher frequency ranges than were previously obtained. When the frequency range is extended, typically the modal density is higher. To extract these modes, more force inputs are required to separate the modes in areas of high modal density. In addition to the number of modes required being higher, this creates more data to reduce. The ideal solution is an algorithm that automatically does the work with less user interaction than has been required in the past. Part of the problem with automated algorithms in the past has been the difficulty of separating and eliminating computational roots from the true modal parameters of the physical system. The SMAC (Synthesize Modes and Correlate) approach, in general, does not yield computational roots and, therefore, lends itself to a reliable automation process. When all the important roots are accurately extracted, then the mode shapes for a particular degree of freedom can be accurately estimated by fitting a summation of single degree of freedom systems to the corresponding experimental frequency response function (FRF). If there are no computational roots, the confidence in the modal parameters can be evaluated by comparing a synthesized FRF with the data or in a more global sense, comparing the synthesized mode indicator function (MIF) with the data.

THEORY OF THE EXTRACTION OF ROOTS

The SMAC algorithm is based upon the modal filtering approach rather than an assumed matrix polynomial form. In

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the strictest sense this means that there must be at least as many response measurements as there are active modes in the frequency band of interest. The sensors should be placed so that the associated experimental mode shape matrix is well conditioned for inversion. Since the algorithm is not based on a matrix polynomial, there are no computational roots, eliminating a major set of decisions the analyst must make in deciding on the true system roots. The theory for the SMAC approach has been presented previously[1,2]. A few equations and an intuitive explanation will be provided here regarding the SMAC theory.

Assume that a set of FRFs for one reference input are measured, and the functions are arranged in a matrix so that each column represents a different sensor, and each row contains the complex values at each frequency line. This matrix is called $[H_x]$. Then values for a natural frequency, ωr, and a damping ratio, ζ_r, of the system are arbitrarily selected for a number of spectral lines *(NS)*. $\{H_A\}$ is a vector calculated, using the selected frequency and damping values, for a SDOF FRF based on a real mode assumption with an arbitrary amplitude as shown in Equation (1).

$$\{\mathbf{H}_A\} = \begin{Bmatrix} \dfrac{-\mathbf{A}_r\omega_1^2}{\omega r^2 + j2\zeta_r\omega r\omega_1 - \omega_1^2} \\ \dfrac{-\mathbf{A}_r\omega_2^2}{\omega r^2 + j2\zeta_r\omega r\omega_2 - \omega_2^2} \\ \vdots \\ \dfrac{-\mathbf{A}_r\omega_{NS}^2}{\omega r^2 + j2\zeta_r\omega r\omega_{NS} - \omega_{NS}^2} \end{Bmatrix} \tag{1}$$

Now a weighted sum of the FRFs will be calculated to attempt to create a vector that reproduces $\{H_A\}$. What is required is that the weighting vector must be found that suppresses all the other modes except a mode at the frequency of interest. Theoretically

$$\{H_A\} \cong [H_x]\{\Psi\}, \tag{2}$$

and the weight vector, $\{\Psi\}$, can be calculated in a least squares solution utilizing the pseudo-inverse of $[H_x]$. After the weights are calculated, the predicted FRF $\{H_P\}$ can be generated as shown in Equation 3 below

$$\{H_P\} = [H_x]\{\Psi\}. \tag{3}$$

Now $\{H_P\}$ can be compared with $\{H_A\}$ through calculating the correlation coefficient[3] between the vectors. The selected frequency and damping is evaluated utilizing the correlation coefficient. Intuitively a good guess of one of the roots would yield a good match and high correlation coefficient between $\{H_P\}$ and $\{H_A\}$, whereas a poor guess would yield a worse match and lower correlation coefficient. The computer is used to make many guesses for frequency and damping, and the local maxima are found for the correlation coefficient as a function of frequency and damping. The frequencies and damping ratios associated with these maxima are the estimated roots.

After *n* roots are found by SMAC, the mode shapes are determined by performing a least squares fit of the residues for each mode to the FRF for each sensor as shown in Equation (4).

$$\mathrm{Imag}\begin{Bmatrix} \mathbf{H}_x(\omega_1) \\ \mathbf{H}_x(\omega_2) \\ \vdots \\ \mathbf{H}_x(\omega_{4n}) \end{Bmatrix} = \mathrm{Imag}\begin{bmatrix} \mathbf{H}^1_{ker}(\omega_1) & \mathbf{H}^2_{ker}(\omega_1) & \cdots & \mathbf{H}^n_{ker}(\omega_1) \\ \mathbf{H}^1_{ker}(\omega_2) & \mathbf{H}^2_{ker}(\omega_2) & \cdots & \mathbf{H}^n_{ker}(\omega_2) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}^1_{ker}(\omega_{4n}) & \mathbf{H}^2_{ker}(\omega_{4n}) & \cdots & \mathbf{H}^n_{ker}(\omega_{4n}) \end{bmatrix} \begin{Bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \vdots \\ \mathbf{A}_n \end{Bmatrix} \tag{4}$$

To establish this matrix, four frequency lines are selected near each resonant frequency estimated by SMAC. The kernel FRF in Equation 4 is made up of all known quantities as shown in Equation 5

$$\mathbf{H}^r_{ker}(\omega_j) = \frac{-\omega_j^2}{\omega r^2 + j2\zeta_r\omega r\omega_j - \omega_j^2}. \tag{5}$$

The modal parameters are then used to synthesize FRFs. Using these parameters an examination of the quality of the fit can be made by comparing synthesized FRFs with the data. A more global quality of fit is obtained by comparing synthesized MIFs with the MIFs calculated on the actual data. If a mode has been missed using this automated procedure, then a manual version of the extraction process can be used.

IMPLEMENTATION OF THE EXTRACTION OF ROOTS

The algorithm was first implemented in MATLAB with an assumption of real modes, i.e., A_r in Equation (1) is a real number. The solution to Equation (2) was constrained to obtain real valued weights, which should be the case for a structure with real modes except where out-of-band modes are significantly influencing the response. Initially the analyst must choose the frequency range to investigate. There are no limits except the range of the data itself. Then a reasonable damping value is assumed, and the correlation coefficient between $\{H_A\}$ and $\{H_P\}$ is calculated and plotted at each frequency line in the bandwidth under consideration. The peaks in the correlation coefficient are indications of the modes in the bandwidth as shown in Figure 1.

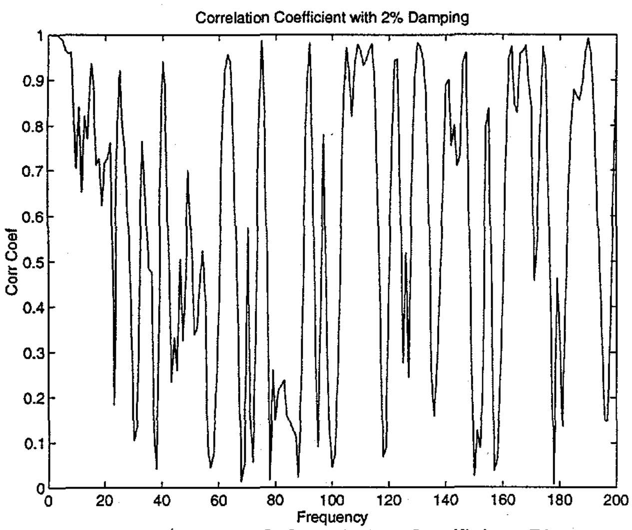

Figure 1 - SMAC Correlation Coefficient Plot

The frequency of each peak is saved in a table, and these become the starting points for the automated algorithm described below. The analyst may choose thresholds below which no peaks are considered. In our experience only correlation coefficient peaks above 0.9 are worthy of investigation; however, this probably varies considerably from test to test. The correlation coefficient plot could be made with any frequency resolution, but the authors have found the frequency resolution of the FRF to be sufficient.

Next the automated algorithms are executed. The algorithm starts with one frequency from the table of peaks and the initial guess of the damping ratio. Two routines, which operate in the same manner, are used to converge on the root. In the first routine a narrow frequency band around the suspected root is selected. The user specifies the frequency band; the authors have used a bandwidth of 1 to 3 percent of the root frequency with success. The correlation coefficient is calculated at five equally spaced frequency points using the assumed damping. Then a parabola is fit to the five points, and the frequency of the maximum point of the parabola is calculated. This becomes the first estimate of the natural frequency as shown in Figure 2.

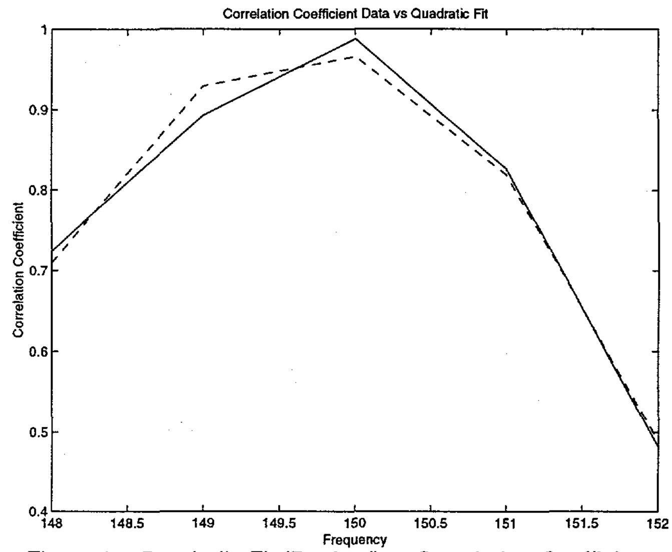

Figure 2 – Parabolic Fit (Dashed) to Correlation Coefficient (Solid) versus Frequency (Estimated 2 Percent Damping)

Then the second routine is executed, which is exactly like the first except the abscissa is now a range of damping ratios, chosen by the user, and the frequency is assumed to be the estimate just calculated above. A best estimate of damping (2.8 percent damping) is then obtained from the parabolic fit of the data shown in Figure 3.

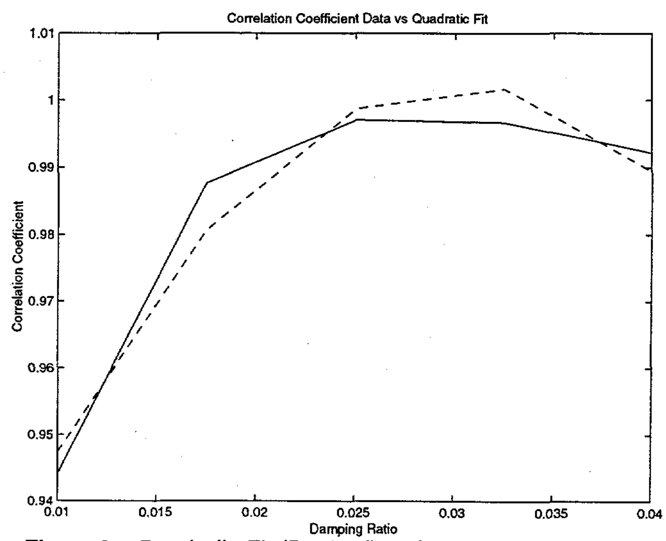

Figure 3 – Parabolic Fit (Dashed) to Correlation Coefficient (Solid) versus Damping (Estimated Frequency 149.7 Hz)

The program oscillates back and forth between these two routines. With each oscillation the range of frequency and/or damping is reduced. When the damping ceases to change more than 0.5 percent of the damping value, the root is considered successfully converged and is saved. If the optimization process attempts to extend beyond the original frequency range, the root is rejected. Sometimes the same root is converged upon from two different starting points, so there is a built-in check to eliminate duplicate roots. Then the program repeats the process for the second frequency in the peak table and continues until all candidate roots have been converged upon or eliminated. Typically, 90 percent of the roots, which can be found utilizing the SMAC process, are calculated by the automated algorithms. A manual version of these algorithms is used to converge on any roots that are missed. It has been found that the automated process sometimes misses roots when two roots are within the initial 1 to 3 percent frequency band, or the damping ratio is very low or out of the range of the damping ratios that the analyst considers reasonable. These omissions can usually be discovered in the quality comparisons, and the roots can be extracted through the manual process.

As reported in a previous paper[2], there are an optimum number of frequency lines over which the correlation coefficient should be calculated, and this is chosen by the analyst. From all the data that the authors have analyzed, 10 to 30 lines on each side of the estimated resonance appears to be adequate. The optimum number of lines will result in a high correlation coefficient at the true root and a rapid monotonic roll-off of the correlation coefficient at damping values away from the true root, Figure 4.

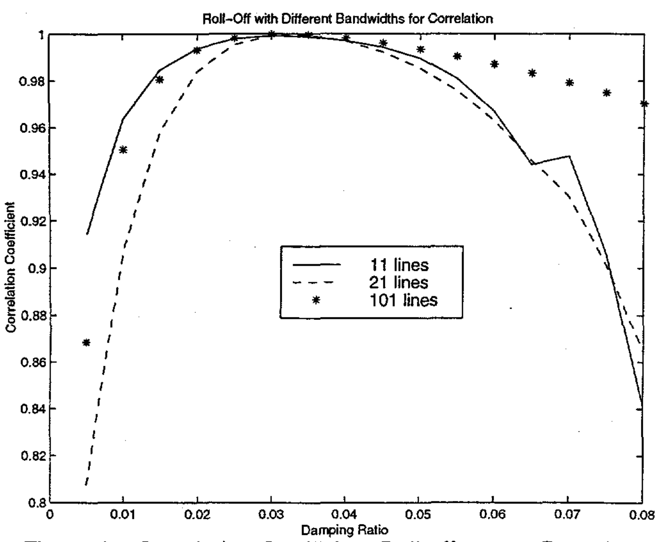

Figure 4 – Correlation Coefficient Roll-off versus Damping

Too few lines can result in nonmonotonic roll-off as seen in the 11-line plot. Once this optimum is found for a mode near the center of the frequency bandwidth, it is generally good for the entire data analysis. Adding a few extra lines is good insurance for monotonic roll-off and does not significantly impact the roll-off rate.

The modal density can become so high that the optimization, based on the correlation coefficient surface, is not effective. This is characterized by a flat plot with very high correlation coefficients, and SMAC will sometimes fail in such a bandwidth. However, the best approach is to let the automated algorithms attempt to find the roots, even in the regions of high modal density, and then examine the results using the quality evaluations (synthesized versus actual MIFs). Sometimes the routines work well, even in difficult regimes.

It has been found that these algorithms will do an amazing job at estimating the roots for very weakly excited modes. Sometimes these modes are so weakly excited that their estimated mode shapes are extremely noisy. Consequently, the authors recommend publishing the shapes for only the well-excited modes. Fitting these weaker modes can improve the fit of the mode shapes of nearby well-excited modes. Sometimes even weakly excited modes have satisfactory shape estimates. The analyst's experience is required for these judgments.

DATA QUALITY CHECKS USING THE MODE INDICATOR FUNCTION

After the process of automatically extracting modal parameters using SMAC, it is important to assess the quality of the extraction. Comparisons of the synthesized and actual FRF data are a typical method of checking whether all the modes of interest in a particular frequency band have been properly identified. This time-consuming approach can require viewing all or most of the response channels from an experiment. Thus, the method of verifying the quality of the extraction by comparing synthesized and actual mode indicator functions (MIFs) has been explored.

The MIF includes effects of the data from all the FRFs in a single curve to indicate the modes. If the synthesized MIF depicts modes in the same way as the MIF using the experimental data, then there is high confidence in the modal parameters. The Complex Mode Indicator Function (CMIF) [5] is a good tool for comparing the strength of the modes excited by a particular reference. However, the CMIF can obscure the effects of the weakly excited modes in the data. Because the SMAC algorithms have been shown to be extremely robust in extracting not only well excited but also weakly excited modes, the Normal Mode Indicator (NMIF) [6] is, typically, selected because it tends to be a more sensitive comparison for all the modes.

The following figures display CMIF (Figure 5) and NMIF (Figure 6) comparisons for a set of experimental data and a SMAC modal extraction synthesis. The CMIF data show three dominant modes in the frequency band (868 Hz, 976 Hz and 1072 Hz). The mode at 868 Hz is the most strongly excited mode for this excitation location. It is also apparent from this curve that there are some weakly excited modes between 900-950 Hz that have been obscured by the CMIF analysis. However, by viewing the NMIF, we can see that those weakly excited modes between 900-950 Hz stand out more clearly and can be compared more easily using this function.

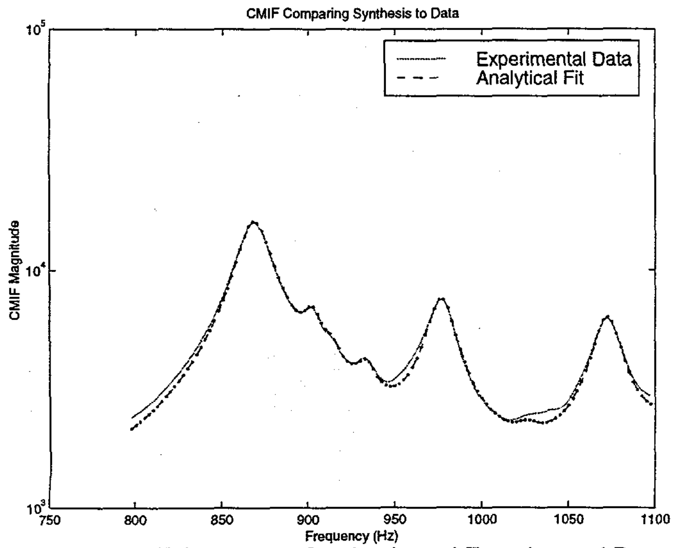

Figure 5. CMIF Comparing Synthesis and Experimental Data

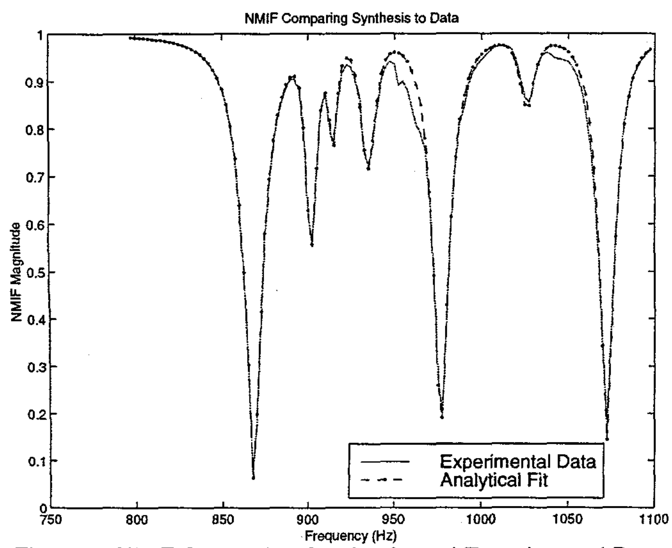

Figure 6. NMIF Comparing Synthesis and Experimental Data

COMPLEX MODES APPROACH AND ADDITIONAL EXTENSIONS

Another extension made to the SMAC Modal Extraction Package is the ability to fit complex modes. The real mode approach tends to work well when the system response is lightly damped or has proportional damping distributions. However, when the system is characterized by non-proportional damping -e.g., concentrated damping sources- the assumption of real modes is not appropriate. As a result, the modal extraction can produce multiple real roots when a single complex root is actually a more accurate model of the data.

The following figures (Figures 7 and 8) depict the results of a SMAC extraction where real modes were initially used to fit some experimental data. Comparisons of mode indicator functions (both the NMIF and CMIF) show that the mode near 800 Hz is not fit very well using the real mode approximation. However, by reanalyzing the data using a complex mode extraction, the comparisons indicate a much better fit of the 800 Hz mode. This mode is dominated by the mass of an internal component exercising a viscoelastic spring, resulting in localized damping.

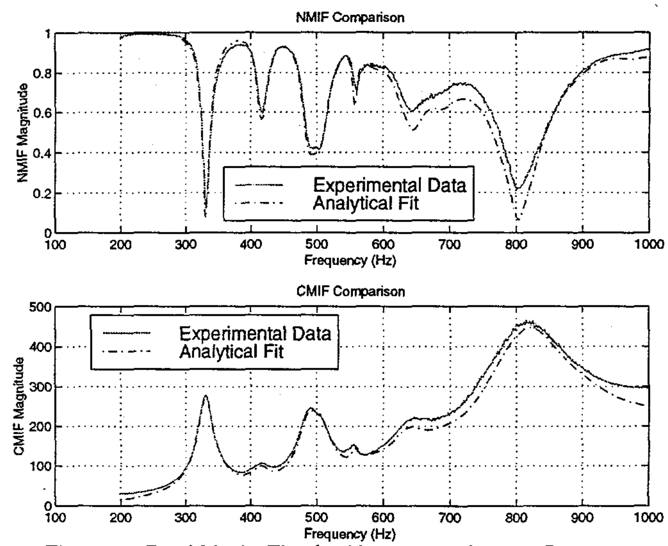

Figure 7. Real Mode Fit of a Nonproportionally Damped Mode (800 Hz)

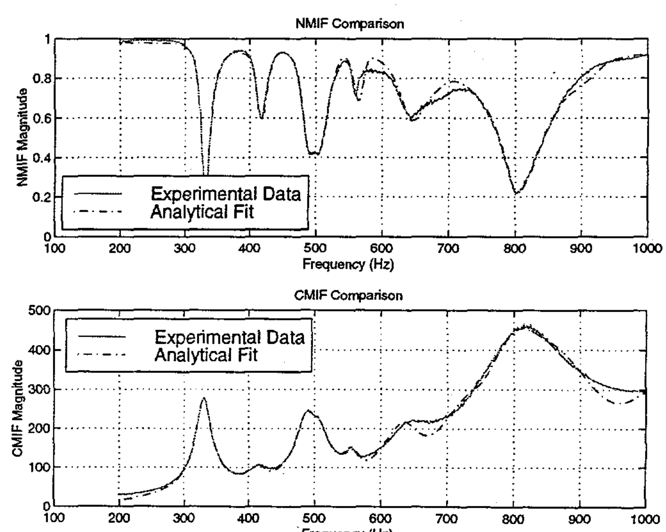

Figure 8. Complex Mode Fit of a Nonproportionally Damped Mode (800 Hz)

The complex mode approach for SMAC root extraction is carried out in a very similar manner to that described above in Equations (1-3). The extractions are still based on the modal filtering concept; however, in this case the residue coefficients of the SDOF FRF $\{H_A\}$ are complex values as are the weights of the reciprocal modal vector, $\{\Psi\}$. These changes do increase the overall computational time of the algorithms.

The SMAC package includes algorithms for calculating the residue amplitudes and mode shape coefficients for the complex modes identified in an extraction. As before these algorithms are based on a least squares fit of the FRF data. To establish the matrix for estimating the residues, a user-specified number of frequency lines are selected near each resonant frequency identified by SMAC. Eliminating all the frequency lines except those closest to each estimated complex root minimizes errors in the residue calculations. The residue amplitudes are solved using the following $2NL*2n$ equation for the n unknown problem:

$$\begin{Bmatrix} \mathbf{H}_x(\omega_1) \\ \mathbf{H}_x(\omega_2) \\ \vdots \\ \mathbf{H}_x(\omega_{NL*n}) \\ \hline \mathbf{H}^*_x(\omega_1) \\ \mathbf{H}^*_x(\omega_2) \\ \vdots \\ \mathbf{H}^*_x(\omega_{NL*n}) \end{Bmatrix} = \begin{bmatrix} \left[\mathbf{H}^n(\omega_j)\right] & \left[\mathbf{H}^{n*}(\omega_j)\right] \\ \left[\mathbf{H}^n(-\omega_j)\right] & \left[\mathbf{H}^{n*}(-\omega_j)\right] \end{bmatrix} \begin{Bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \vdots \\ \mathbf{A}_n \\ \hline \mathbf{A}^*_1 \\ \mathbf{A}^*_2 \\ \vdots \\ \mathbf{A}^*_n \end{Bmatrix}. \quad (6)$$

The variable, NL, is the number of frequency lines selected near resonance; n is the number of modes; and ω_i represents the chosen frequency lines of the FRF. The kernel matrix, **H**, is of the following form

$$\left[\mathbf{H}^n\{\omega_j\}\right] = \begin{bmatrix} \mathbf{H}^1_{\mathrm{ker}}(\omega_1) & \mathbf{H}^2_{\mathrm{ker}}(\omega_1) & \cdots & \mathbf{H}^n_{\mathrm{ker}}(\omega_1) \\ \mathbf{H}^1_{\mathrm{ker}}(\omega_2) & \mathbf{H}^2_{\mathrm{ker}}(\omega_2) & \cdots & \mathbf{H}^n_{\mathrm{ker}}(\omega_2) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}^1_{\mathrm{ker}}(\omega_{NL*n}) & \mathbf{H}^2_{\mathrm{ker}}(\omega_{NL*n}) & \cdots & \mathbf{H}^n_{\mathrm{ker}}(\omega_{NL*n}) \end{bmatrix}, \quad (7)$$

and the kernel function is given in Equation (5). Note that Equation (6) contains both the positive and negative frequencies, forcing the residues to be conjugate pairs. Each FRF is then synthesized from the calculated residue coefficients and the kernel matrices over a prescribed frequency band

$$\{\mathbf{H}_{\mathrm{syn}}\} = \left[\left[\mathbf{H}^n(\omega_j)\right] \; \left[\mathbf{H}^{n*}(\omega_j)\right]\right] \begin{Bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \vdots \\ \mathbf{A}_n \\ \hline \mathbf{A}^*_1 \\ \mathbf{A}^*_2 \\ \vdots \\ \mathbf{A}^*_n \end{Bmatrix}. \quad (8)$$

An additional feature that has been coded into the SMAC extraction package is the ability to include residual terms when comparing synthesized and actual FRF data. These residual terms allow the user to account appropriately for out-of-band modes in the synthesis. This is more important when dealing with complex modes because of the effects of the real parts of the FRFs. When SMAC computes real modes using a quadrature response[7] (imaginary part of FRF only), the residual terms are negligible because the imaginary part of the FRF approaches zero away from a resonance. In the coding the residual term is calculated by adding two columns to the kernel matrix before solving the least squares problem in Equation (6). The inertance and compliance residuals are solved for in Equation (9) along with the residue coefficients.

$$\begin{Bmatrix} \mathbf{H}_x(\omega_1) \\ \mathbf{H}_x(\omega_2) \\ \vdots \\ \mathbf{H}_x(\omega_{NL*n}) \\ \hline \mathbf{H}^*_x(\omega_1) \\ \mathbf{H}^*_x(\omega_2) \\ \vdots \\ \mathbf{H}^*_x(\omega_{NL*n}) \end{Bmatrix} = \begin{bmatrix} \left[\mathbf{H}^n(\omega_j)\right] & \left[\mathbf{H}^{n*}(\omega_j)\right] & \begin{matrix} 1 \\ \vdots \\ 1 \end{matrix} & \begin{matrix} -\omega_j^2 \\ \vdots \\ -\omega_{NL*n}^2 \end{matrix} \\ \left[\mathbf{H}^n(-\omega_j)\right] & \left[\mathbf{H}^{n*}(-\omega_j)\right] & \begin{matrix} 1 \\ \vdots \\ 1 \end{matrix} & \begin{matrix} -\omega_j^2 \\ \vdots \\ -\omega_{NL*n}^2 \end{matrix} \end{bmatrix} \begin{Bmatrix} \mathbf{A}_1 \\ \vdots \\ \mathbf{A}_n \\ \hline \mathbf{A}^*_1 \\ \vdots \\ \mathbf{A}^*_n \\ \hline \mathbf{R}_I \\ \mathbf{R}_C \end{Bmatrix} \quad (9)$$

The following examples show experimental data that have been fit using complex mode extraction with and without residual terms included in the synthesis (Figures 9 and 10). Note particularly the lower and higher frequency bands where the comparison between analytical fit and experimental data shows much better agreement with the residual terms included.

APPLICATIONS

A companion paper [4] provides examples and practical issues associated with the application of the package.

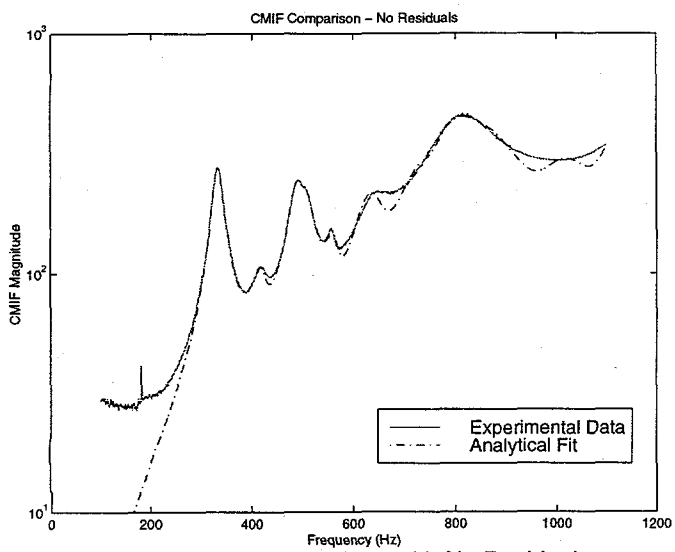

Figure 9. CMIF Comparison with No Residuals

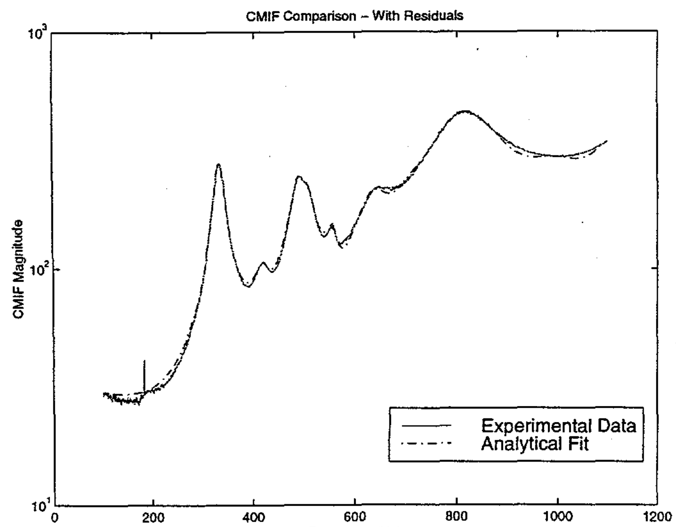

Figure 10. CMIF Comparison with Residuals

UNCERTAINTY

The quality evaluation utilizing the normal and complex MIF provides the most objective quantification of uncertainty for this work. If there is a poor match of the synthesized MIF to the data, then there is high uncertainty in the estimated modal parameters. Measures, such as the correlation coefficient and comparisons of synthesized and actual FRFs, provide additional information regarding the uncertainty of the modal extraction. The authors have found that the observation of the mode shape is still necessary to determine a particular mode's value for model validation.

FUTURE WORK

The SMAC algorithms are limited to analyzing data from one reference at a time. In the future a multireference version might be of value. This is somewhat contingent on the test structure being very linear. Many structures of interest are slightly nonlinear, which results in the frequencies and damping being amplitude dependent. Inputs applied at two locations will, therefore, result in two different frequencies and damping for the same mode. This will provide difficulties for any multi-input extraction algorithm. A more valuable addition to the current technology would be some sifting technique that could take all the results from the SMAC analyses from several inputs and consolidate the results down to a single set with some confidence levels.

CONCLUSIONS

The SMAC algorithms have now been automated, allowing the analyst to spend more time understanding the dynamics and significantly less time in extraction. In general, the real mode approach has had excellent results for providing modal data for model validation. A few cases have required complex modes, so this capability has been developed. The effects of residual terms have also been included. The algorithms are limited to analyzing data from one reference at a time.

REFERENCES

[1] Mayes, Randall L. and Johansen, David D., "A Modal Parameter Extraction Algorithm Using Best–Fit Reciprocal Vectors", *Proceedings of the 16th International Modal Analysis Conference,* pp.517-521, February 1998.

[2] Mayes, Randall L. and Klenke, Scott E., "The SMAC Modal Extraction Package", *Proceedings of the 17th International Modal Analysis Conference,* pp.812-818, February 1999.

[3] Thomas, L.F. and Young, J.I., *An Introduction to Educational Statistics: The Essential Elements, Third Edition*, pp77, 1993.

[4] Mayes, Randall L., Dorrell, Larry R. and Klenke, Scott E., "Applications of the Automated SMAC Modal Parameter Extraction Package", *Proceedings of the 18th International Modal Analysis Conference,* February 2000.

[5] Shih, C.Y. and Brown, D.L., *"The Complex Mode Indicator Function Approach to Modal Analysis", Pre-IMAC 8 Symposium,* UC-Irvine, pp0-10, 1989.

[6] *User Manual for Modal Analysis 9.0*, General Electric CAE International, 1985, pp6-85.

[7] Mayes, Randall L., "A Multi-Degree-of-Freedom Mode Shape Estimation Algorithm Using Quadrature Response", *Proceedings of the 11th International Modal Analysis Conference,* pp.1026-1034, 1993.