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NUMERICAL SPIN TRACKING IN A SYNCHROTRON

COMPUTER CODE *SPINK* - EXAMPLES (RHIC)

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ALTERNATING GRADIENT SYNCHROTRON DEPARTMENT

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Numerical Spin Tracking in a Synchrotron. Computer Code *Spink*. Examples (RHIC)

A.Luccio, September 1995

1. Spin Tracking

In the course of acceleration of polarized protons in a synchrotron, many depolarizing resonances are encountered [1]. They are classified in two categories: intrinsic resonances that depend on the lattice structure of the ring and arise from the coupling of betatron oscillations with horizontal magnetic fields, and imperfection resonances caused by orbit distortions due to field errors.

In general, the spectrum of resonances vs. spin tune $G\gamma$ ($G = 1.7928$, the proton gyromagnetic anomaly, and γ the proton relativistic energy ratio) for a given lattice tune ν, or vs. ν for a given $G\gamma$, contains a multitude of lines with various amplitudes or resonance strengths. The depolarization due to the resonance lines can be studied by numerically tracking protons with spin in a model accelerator. Tracking will allow one to check the strength of resonances, to study the effects of devices like Siberian Snakes [2], to find safe lattice tune regions where to operate, and finally to study in detail the operation of special devices such as Spin Flippers [3].

A few computer codes exist that calculate resonance strengths ε_k and perform tracking, for proton and electron machines. Most relevant to our work for the AGS and RHIC machines are the programs *Depol* and *Snake*. *Depol*, originally written by E.D. Courant [4] calculates the ε_k's by Fourier analysis. The input to *Depol* is the output of a machine model code, such as *Synch* or *Mad*, containing all details of the lattice. *Snake*, written by J. Buon and modified by E.D. Courant, S.Y. Lee and others, does the tracking, starting from a synthetic machine, that contains a certain number of periods, of FODO cells, of Siberian snakes, etc.

We believed the complexities of machines like the AGS or RHIC could not be adequately represented by *Snake*. Then, we decided to write a new code, *Spink*, that combines some of the features of *Depol* and *Snake*. I.e. *Spink* reads a *Mad* output like *Depol* and tracks as *Snake* does. The structure of the code and examples for RHIC are described in the following.

2. Spink Formalism.

The general idea is to track a certain number of protons, randomly generated in a phase space volume, through the machine lattice. Each proton is characterized by four transverse coordinates, x, x', y, y', by two longitudinal coordinates $dp = p - p_S$, $d\phi$, and by three spin coordinates, S_x, S_y, S_z (where $S_x^2 + S_y^2 + S_z^2 = 1$). Matrices are used to transform orbit (transverse and longitudinal) and spin coordinates. Orbit matrices are built from a *Twiss* file, output of *Mad*. Since for spin motion only magnets and RF cavities ("active" elements) are relevant, everything else in the lattice description is lumped in a drift space. Typically, for RHIC the number of *Spink* matrices is 981, each active element being surrounded by two drifts, keyword (D) in the code. Presently, active elements are: Bends (B), Quadrupoles (Q), Snakes (S), RF Cavities (R), Spin Flippers (F).

The spin, treated as a three dimensional vector, is transformed by rotation, using matrices. In a Bend the rotation is around a vertical y-axis, in a Quad, around a radial axis, in a Snake, around an axis of orientation given as input, and in a Spin Flipper around an horizontal rotating or oscillating axis.

Tracking a fair number of particles, say 25, for many revolutions, say 100,000, through a thousand or so matrices takes a considerable computer time, of the order of many hours for a typical fast workstation. To insure a "reasonable" turnaround time, some price had to be paid. The results presented in this paper were obtained with the following limitations: (i) only the vertical motion was considered, (ii) the longitudinal synchrotron motion was decoupled from the other degrees of freedom, (iii) there was no attempt to consider in detail the spin precession within each element (i.e. for the purpose of representing spin motion the machine elements are thin). These limitations are not too bad, however to study fine details, like side bands of resonance lines due to synchrotron or horizontal motion, full six-dimensional matrices are needed.

2.a. Orbit

With these limitation, the orbit matrices are 2×2 and act only on (y,y'). They are computed from the *Twiss* file using twiss function values at the entrance and exit of each element, as follows

$$\begin{pmatrix} y \\ y' \end{pmatrix} := \mathbf{T}\begin{pmatrix} y \\ y' \end{pmatrix}, \quad \mathbf{T} \equiv \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

(2-1)
$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \equiv \begin{pmatrix} \sqrt{\frac{\beta_2}{\beta_1}}(\cos\Delta\phi + \alpha_1 \sin\Delta\phi) & \sqrt{\beta_1\beta_2}\sin\Delta\phi \\ \frac{AD-1}{C} & \sqrt{\frac{\beta_1}{\beta_2}}(\cos\Delta\phi - \alpha_2 \sin\Delta\phi) \end{pmatrix}$$

with

$$\Delta\phi = \phi_2 - \phi_1$$

Orbit matrices are unitary, as they should, except than in a RF (thin) cavity, where the D element is decreased by a quantity proportional to the momentum gain per turn dp_m, corresponding to an instantaneous change of vertical orbit angle (together with the change of momentum)

(2-2)
$$D = 1 - \frac{dp_m}{\beta\gamma}, \quad dp_m = V_{RF}\sin\phi, \quad G\gamma = G\sqrt{1+(\beta\gamma)^2}$$

Since an important study is to track polarized protons through resonances at constant energy (storage mode) as a function of lattice tune, we need a means to vary the tune . It would be very cumbersome to run again *Mad* and read its output for every tune in the range. Besides, one might want to study acceleration with a programmed tune change. To accomplish this, two thin lenses of strength δ are added to each quadrupole in the machine, immediately up and down stream. The new quadrupole matrix and the quadrupole gradient become (See Appendix A)

(2-3)
$$\begin{pmatrix} A + B\delta & B \\ C + 2A\delta + B\delta^2 & D + B\delta \end{pmatrix}, \quad \begin{cases} K_1 := K_1 - \frac{2A}{B}\delta - \delta^2 & \text{(focusing quad)} \\ K_1 := K_1 + \frac{2A}{B}\delta + \delta^2 & \text{(defocusing quad)} \end{cases}$$

Spink compute the lattice tune simply by counting zeroes of the orbit performing betatron oscillations. If $\delta = 0$, this tune coincides with the tune calculated in *Mad*.

2.b.Spin

Spin rotation matrices are less familiar

$$\begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} := \mathbf{R} \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}$$

In a Bend, the spin vector precesses around the y-axis by an angle ψ proportional to the bend angle θ

$$\psi = G\gamma\theta$$

The spin matrix of a Bend is

(2-4)
$$\mathbf{R}_B = \begin{pmatrix} \cos\psi & 0 & -\sin\psi \\ 0 & 1 & 0 \\ \sin\psi & 0 & \cos\psi \end{pmatrix}$$

In a Quadrupole, the spin vector precesses around a radial axis. The angle of precession is proportional to the quadrupole integrated gradient K_1L and to the orbit (vertical) displacement, y_β due to the betatron motion, and y_e due to lattice errors

$$\psi = -K_1L(1+G\gamma)(y_\beta + y_e)$$

The spin rotation matrix for a quadrupole is

(2-5)
$$\mathbf{R}_Q = \begin{pmatrix} 0 & 1 & 0 \\ -\cos\psi & 0 & \sin\psi \\ \sin\psi & 0 & \cos\psi \end{pmatrix}$$

In a Siberian snake, two angles are needed: the angle ϕ_A of the precession axis with the radial x-axis (we assume that the precession axis is in the horizontal plane) and the snake rotation ψ. Both angles are given as input to the program. The spin rotation matrix for a snake is (see Appendix B)

$$\mathbf{R}_S = \begin{pmatrix} 1-2\sin^2\phi_A \sin^2\frac{1}{2}\psi & \sin 2\phi_A \sin^2\frac{1}{2}\psi & \sin\phi_A \sin\psi \\ -\sin\phi_A \sin\psi & 1-2\sin^2\frac{1}{2}\psi & \cos\phi_A \sin\psi \\ \sin\phi_A \sin\psi & -\cos\phi_A \sin\psi & 1-2\cos^2\phi_A \sin^2\frac{1}{2}\psi \end{pmatrix} \tag{2-6}$$

In a RF Spin Flipper, the spin precesses around an axis that rotates (or oscillates) in a horizontal plane with a frequency ω. This frequency varies. When ω equals half of the revolution frequency of the protons in the machine (with two snakes on), the spin flips. There are variation of this scheme that will be described later. The spin flipping angle is proportional to the strength of the SF

$$\psi = \frac{\int B d\ell}{B\rho} G\gamma$$

The spin rotation matrix for a spin flipper is

$$\mathbf{R}_F = \begin{pmatrix} 1-2\sin^2\omega t \sin^2\frac{1}{2}\psi & \sin\omega t \sin\psi & \sin 2\omega t \sin^2\frac{1}{2}\psi \\ -\sin\omega t \sin\psi & 1-2\sin^2\frac{1}{2}\psi & \cos\omega t \sin\psi \\ \sin 2\omega t \sin^2\frac{1}{2}\psi & -\cos\omega t \sin\psi & 1-2\cos^2\omega t \sin^2\frac{1}{2}\psi \end{pmatrix} \tag{2-7}$$

2.c. Population

Montecarlo population is extracted at the beginning of a tracking run in the transverse and longitudinal phase spaces. There are two basic ways to do the random extraction. One is to extract more particles per unit area where the density function is higher. The other is to extract particles with an even distribution, but to attribute weights to each one, proportional to the local density value. If the particles are not too many, the second method is probably better. In *Spink* this second method is used.

2.d. Froissart-Stora

Spink can be used to check the strength of resonances calculated by *Depol* using the Froissart-Stora formula [5]. Conditions are that the resonances being studied are reasonably separated (say, ten times their width) and are not too strong. The first condition insures that the polarization, at the beginning of the run is stable around the value of one. The second should be met in order than Froissart-Stora would not saturate. It can always be used (at least for intrinsic resonances) by tracking particles close to the ring horizontal plane, thus reducing the effective strength.

Assume that after crossing an isolated resonance the polarization (no snakes on) is reduced to some final value $<P>$. The resonance strength is then calculated with

(2-8) $$\varepsilon_k = \sqrt{-4\alpha \ln \frac{1+<P>}{2}}, \quad \alpha = \frac{\Delta\gamma_m}{2\pi}$$

with α the rate of resonance crossing ($\Delta\gamma_m$, the change of γ per turn). Since after the crossing the polarization performs some oscillations, $<P>$ is calculated as the average over a few turns, starting after a value of $G\gamma$ given as input .

4. Examples

The following examples are for RHIC. RHIC contains two full Siberian snakes (ψ = 180°) with axis at ±45° respectively. $G\gamma$ ranges from 45 to 500 [6].

4.a. Example: Strength of an isolated Intrinsic Resonance.

Intrinsic resonances are located where

$$G\gamma = \text{Integer} \pm \nu \ ,$$

ν being the (vertical) tune of the lattice. Integers that are multiple of some characteristic periodicity of the lattice are particularly important.

A spectrum of intrinsic resonances for RHIC, calculated by *Depol* is shown in Fig. 1. The spectrum has been calculated for a particle on the contour of a vertical emittance of 10π mm-mrad for a tune of RHIC $\nu = 29.18000$. One of the strongest resonance is

$$\varepsilon_k = 0.41288, \quad G\gamma = 381.82 = 5 \times 81 - (\nu - 6).$$

With *Spink*, one particle was tracked through this resonance. The Froissart-Stora formula was used to calculate the strength and compare with *Depol*. In the run, the emittance was reduced by a factor of 100, since the strength of the resonance would have saturated Froissart-Stora. With this reduced emittance, a predicted value for the strength is 10 times smaller, i.e. 0.0413. A series of acceleration rates were used. The results are shown in Fig. 2 and in the following table

dp [(Gev/c)/turn]	<*P*>	ε_k
0.03	-0.865	0.0395
0.04	-0.752	0.0400
0.05	-0.646	0.0408

We consider the agreement with *Depol* very good, also recalling that only one test particle was used.

Tracking through the same resonance was repeated, with both snakes on, and emittance back to 10π. The results are in Fig. 3, that shows how snakes are an effective means to avoid depolarization.

4.b. Example: Scanning the tune.

When the lattice tune is close to a fraction with small denominator, resonances can become so strong that snakes are not capable of avoiding depolarization of the beam. In this example we have investigated the crossing of the same intrinsic resonance of the previous example, i.e. $G\gamma = 381.82$, with a variable fractional tune around 1/6 = 1.66667.

In the tracking, 25 particles were used, extracted at random with a gaussian distribution in a vertical phase-space ellipse containing 99.9% of the particles in 10π mm-mrad, and a gaussian energy distribution with $3\sigma = 0.1\%$. Twiss parameters for the yy' ellipse were given in a typical *Mad* run. The rate of acceleration was 4.10^{-5} (GeV/c)/turn. Synchrotron oscillations were included.

The results are shown in Fig. 4. Each plot corresponding to about 100,000 turns shows the polarization averaged over the 25 particles and the average plus/minus the statistical dispersion of the polarization. As a measure of the final depolarization, the difference between these maximum and minimum curves was taken, showing how the polarization cone of all the particles in the distribution remain open around the value of 1/6 of the fractional tune. Fig. 5 shows the profile of this "tune" line, that appeared two-peaked, with an amplitude $\Delta\nu = 0.0012$.

4.c. Example: Spin Flipper

For experiments with polarized protons in RHIC, the spin must be reversed periodically in the machine. A way to achieve this goal is to use a RF Spin Flipper. In an idealization of such a device, a magnetic field is established in some position around the machine, that rotates in the horizontal plane, or simply oscillates along the radial or longitudinal direction. The frequency of rotation of the field is varied. It can be shown that, with two Siberian snakes in action (with axis at $\pm 45^\circ$), when the frequency of the SF field becomes equal to half the frequency of revolution of the protons in the ring, and the field is strong enough, the polarization flips over.

We have simulated a rotating field spin flipper by tracking with *Spink*. That was done at constant proton energy (storage mode) $G\gamma = 378$, at various SF field strengths and frequency sweep speeds. Since at that energy the frequency of revolution in RHIC is 78.25042 KHz, the SF frequency range was taken between 38 and 41 KHz. The results are shown in Fig. 6. A good flipping was found at the right frequency of 39.125 KHz with an integrated FP field of 0.006 Tesla-m and a sweep rate of 0.05 Hz/turn.

It has been shown that an oscillating field SF will produce the desired results when the snakes precession axes are somewhat detuned from their standard values ±45°. The required field of a transverse SF is about $\sqrt{2}$ times larger than in a rotating field SF, however the former is much easier to build. We studied with *Spink* such transverse oscillating field SF with the Snake axes detuned by ±5°. The results are given in Fig. 7, that shows how a transverse field SF flips the polarization back and forth at two frequencies.

5. Program Notes (User's Manual?)

Spink is a straightforward Fortran program in double precision with few or no extensions. It should compile and run on any platform, with static storage for all the variables. Needed files are:

File name	Type	
Code		
Spink.f	ASCII	Source code -Fortran 77
Spink.dimension Spink.common Spink.format	ASCII	Insert files -Fortran 77 Somewhat redundant, but convenient
Input		
Spink.dat	ASCII	Namelist and Data
"twiss_file"	ASCII	*Mad* standard "twiss" output file
Output		
terminal		
Spink.matr	ASCII	Contains the 2×2 orbit matrices
Spink.bmatr	binary	Contains the matrices in binary form for fast access
Spink.track		Contains results of the convolution over many particles. Values of the vertical spin are averaged, then the average and the values of $\langle S_z \rangle \pm \sigma$ are recorded.
Spink.pop	ASCII	Contains the starting Montecarlo population of protons

Spink.dat is used in the following way.

Namelist $Set		
twissf	character*80	Name of twiss_file
read_mad	logical	If .true. twissf is read, the orbit matrices are built and Spink.matr and Spink.bmatr written. You do this only once if you don't change the *Mad* run
no_spin	logical	In some case you may just want the orbits
outf	logical	If .false. some output files are not created
intrinsic imperf	logical	Intrinsic or imperfection resonances/ or both
term	logical	If .false. only a minimal output appears on terminal
Namelist $Dyna		
n_turn	integer	Maximum allowed number of turns
dn_pr	integer	Turn print step (... print every dn_pr turns)
emity	real*8	Vertical emittance containing 99% of protons, in units of π mm-mrad
phi	real*8	Angle of a particle on an emity contour. Used with iseed=0, see Namelist $Cycle
Ggamso Ggamsf	real*8	Initial and final values of $G\gamma$ for a synchronous particle
dp	real*8	Acceleration with no synchrotron motion (obsolete)
dq	real*8	Detune parameter. Changes the lattice tune
volt	real*8	RF Voltage, in GV
harm	integer	RF harmonic No.
deltaph deltap	real*8	Initial longitudinal phase space coordinates of a proton. Used with iseed=0, see Namelist $Cycle
dps	real*8	Synch. proton momentum change per turn, in GeV/c
Namelist $Snakes		
snk_on	logical	If .true. snakes are "on"
rot_sn()	vector real*8	Snake rotation in degrees
ax_sn()	vector real*8	Snake precession axis angle with z-axis, in degrees

Namelist $Froissart-Stora		
ff_ss	logical	If .true. F-S evaluation of resonance strength is on
ggam_fs	real*8	Value of $G\gamma$ where to start F-S evaluation average
Namelist $Spin_flippers		
sf_on	logical	If .true. Spin Flippers are on
bdl_sf	real*8	SF strength, in Tesla-m
rot_sf	character*1	'r', rotating field SF, 'x' or 'z' transverse field SF
f_sfo	real*8	Start frequency of SF field, in Hz
df_sf	real*8	Frequency variation of SF field per turn, in Hz
nt_sf	integer	Turn No. where to turn SF on
Namelist $Cycle		
cycle_on	character*4	'none' means: don't cycle 'dq' means: cycle on dq values listed at end of namelist 'pop' means: cycle on y,y' values listed ......to be augmented
npart	integer	Together with cycle='none'. No. of Particles for random extraction (gaussian in y,y' and energy, parabolic for ϕ)
iseed	integer	Seed for random extraction. If iseed = 0, the extraction is not random, and a particle is used on the emity contour with the angle phi given in $Dyna

Appendix A. Modified Quadrupole [7]

Add two thin quadrupoles at both ends of a regular quadrupole

$$\mathbf{M}=\begin{pmatrix}1 & 0\\ \delta & 1\end{pmatrix}\begin{pmatrix}A & B\\ C & A\end{pmatrix}\begin{pmatrix}1 & 0\\ \delta & 1\end{pmatrix}=\begin{pmatrix}A+B\delta & B\\ C+2A\delta+B\delta^2 & A+B\delta\end{pmatrix}$$

For a focusing quad it is

$$A=\cos\phi_0,\quad B=\frac{1}{\sqrt{k_0}}\sin\phi_0,\quad C=-\sqrt{k_0}\sin\phi_0$$

Write the identities

$$\begin{cases}A+B\delta=\cos\phi_0+\dfrac{\delta}{\sqrt{k_0}}\sin\phi_0=\cos\phi\\[2ex] B=\dfrac{1}{\sqrt{k_0}}\sin\phi_0=\dfrac{1}{\sqrt{k}}\sin\phi \quad\Rightarrow\quad k=k_0\dfrac{\sin^2\phi}{\sin^2\phi_0}\\[2ex] C+2\delta A+B\delta^2=\dfrac{k_0-\delta^2}{\sqrt{k_0}}\sin\phi_0+2\delta\cos\phi_0=-\sqrt{k}\sin\phi\end{cases}$$

From the above obtain

$$k=k_0-2\delta\frac{A}{B}-\delta^2$$

The corresponding result for a diverging quadrupole is

$$k=k_0+2\delta\frac{A}{B}+\delta^2$$

Appendix B. Spin Rotation in a Snake.

X = (**x**,**y**,**z**) are the Lab coordinates, **U** = (**u**,**v**,**w**) are the proton coordinates. **R** is a rotation of **U** with respect to **X**.

$\mathbf{S}_u$ are the spin coordinates in **U**. Assume that in **U** the spin precesses around the axis **û** by an angle θ

$$\mathbf{S}'_u = \mathbf{P}\mathbf{S}_u, \quad \mathbf{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix}, \quad \begin{cases} c = \cos\theta \\ s = \sin\theta \end{cases}$$

If $\mathbf{S}_x$ are the spin coordinates in the Lab, it is

$$\mathbf{S}'_x = \mathbf{R}^{-1}\mathbf{P}\mathbf{R}\mathbf{S}_x$$

with

$$\mathbf{R} = \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ \mu_1 & \mu_2 & \mu_3 \\ \nu_1 & \nu_2 & \nu_3 \end{pmatrix}, \quad \mathbf{R}^{-1} = \begin{pmatrix} \lambda_1 & \mu_1 & \nu_1 \\ \lambda_2 & \mu_2 & \nu_2 \\ \lambda_3 & \mu_3 & \nu_3 \end{pmatrix}$$

and

$$\lambda, \mu, \nu \quad \text{direction cosines of } \hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}} \quad \text{in } \mathbf{U}$$

We obtain

$$\text{(B-1)} \quad \mathbf{R}^{-1}\mathbf{P}\mathbf{R} = \begin{pmatrix} \lambda_1^2 + A_{11}c & \lambda_1 \cdot \lambda_2 + A_{12}c + B_{12}s & \lambda_1 \cdot \lambda_3 + A_{13}c + B_{13}s \\ \lambda_1 \cdot \lambda_2 + A_{12}c - B_{12}s & \lambda_2^2 + A_{22}c & \lambda_2\lambda_3 + A_{23}c + B_{23}s \\ \lambda_1 \cdot \lambda_3 + A_{13}c - B_{13}s & \lambda_2\lambda_3 + A_{23}c - B_{23}s & \lambda_3^2 + A_{33}c \end{pmatrix}$$

with

$$\begin{cases} A_{ij} = \mu_i\mu_j + \nu_i\nu_j \\ B_{ij} = \mu_i\nu_j - \nu_i\mu_j \end{cases}$$

Note that all spin rotation matrices in Sec. 2.b are a special case of Eq. (B-1). We obtain:

- a spin rotation around **y** (Bend), with **u** = **y**, and **v** = **x** (arbitrary);
- around **x** (Quadrupole) is obtained by putting **u** = **x**;
- around **z**, with **u** = **z**, and **v** = **y** (arbitrary);
- around an axis in the *xz* plane (Snake and Spin Flipper), with **w** = **y**.

The last case as an example: precession axis in the xz plane (fig. 8)

$$\text{(B-7)} \qquad \mathbf{R}^{-1}\mathbf{PR} = \begin{pmatrix} \cos^2\phi + \sin^2\phi\cos\theta & \sin\phi\cos\phi(1-\cos\theta) & \sin\phi\sin\theta \\ \sin\phi\cos\phi(1-\cos\theta) & \sin^2\phi + \cos^2\phi\cos\theta & -\cos\phi\sin\theta \\ -\sin\phi\sin\theta & -\cos\phi\sin\theta & \cos\theta \end{pmatrix}$$

6. References

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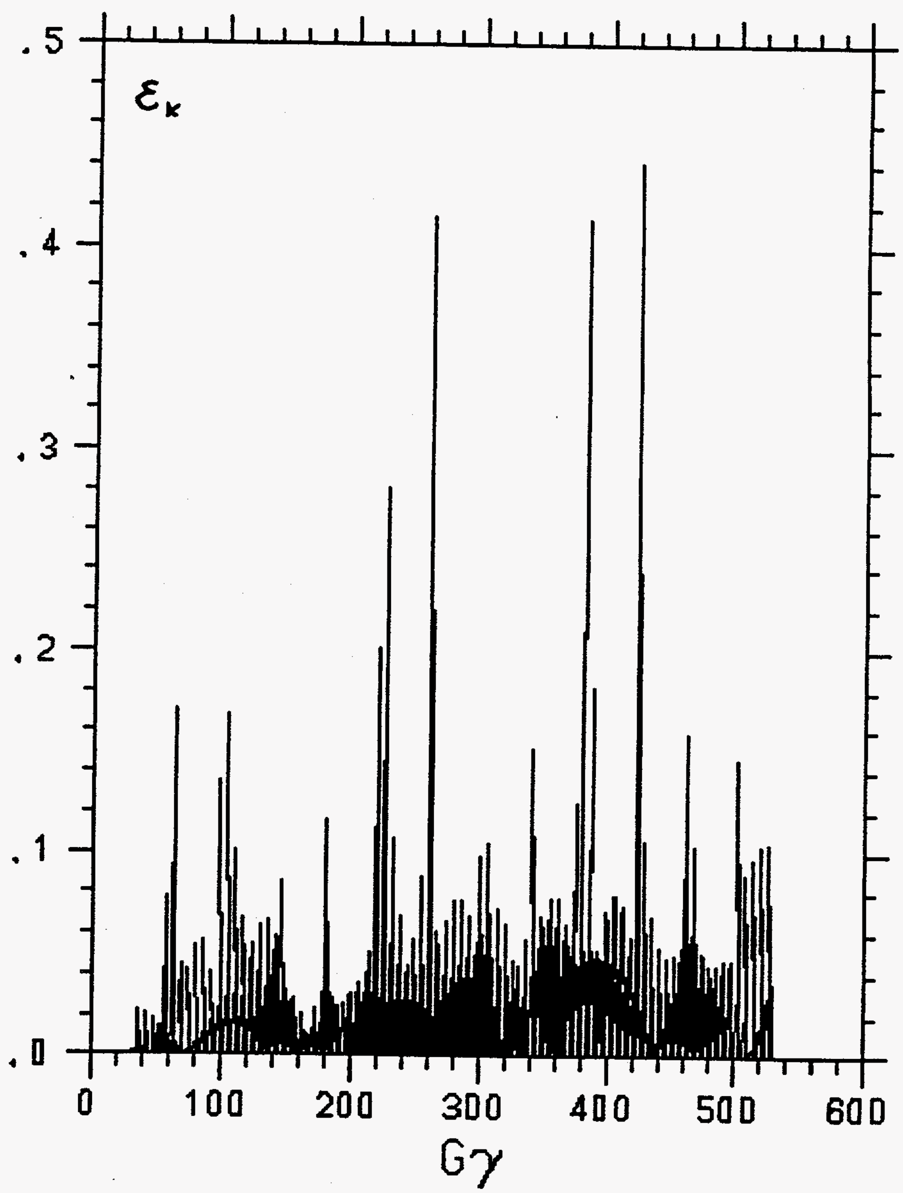

Fig.1. Intrinsic Resonances in RHIC, calculated by *Depol*. Proton on the contour of a vertical phase-space ellipse of 10π mm-mrad.

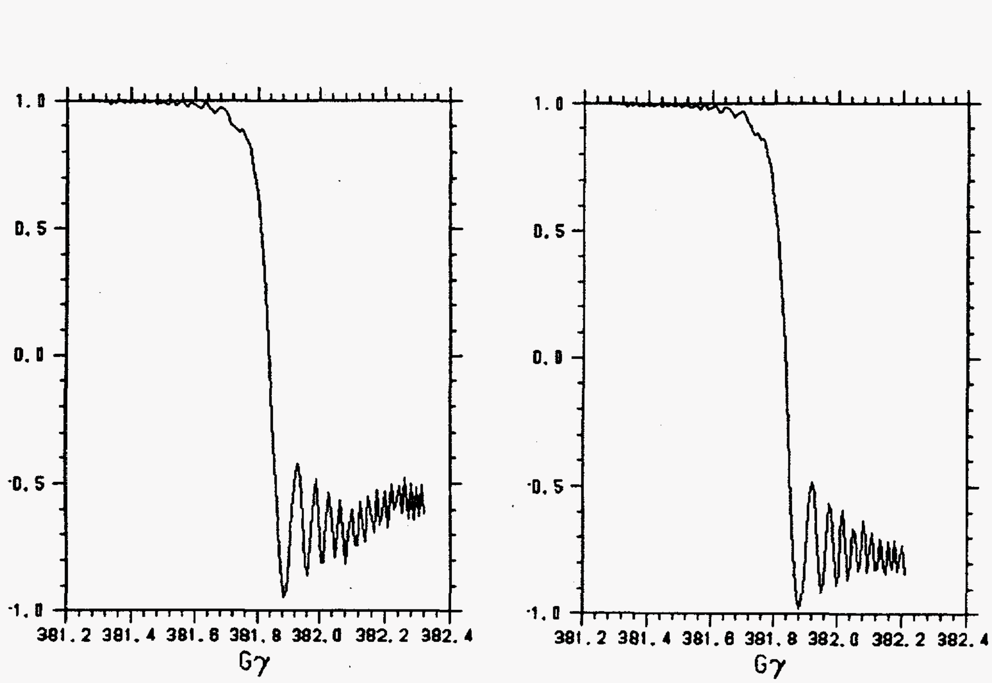

Fig. 2. Tracking with *Spink* through the intrinsic resonance at $G\gamma$=381.82. Proton on the contour of a vertical phase-space ellipse of 0.1π mm-mrad. Snakes off.

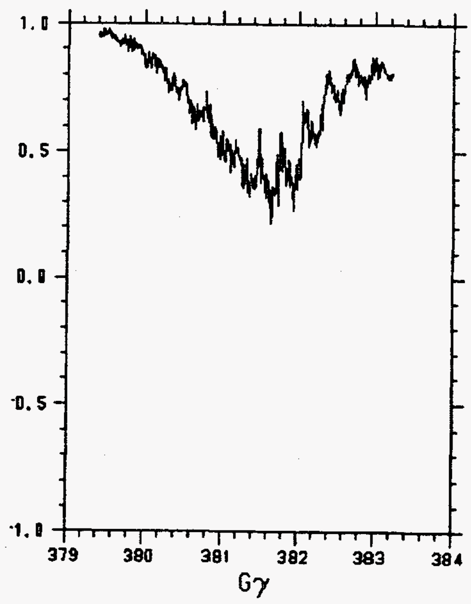

Fig. 3. Same tracking as in Fig. 2. Snakes on. 10 π mm-mrad.

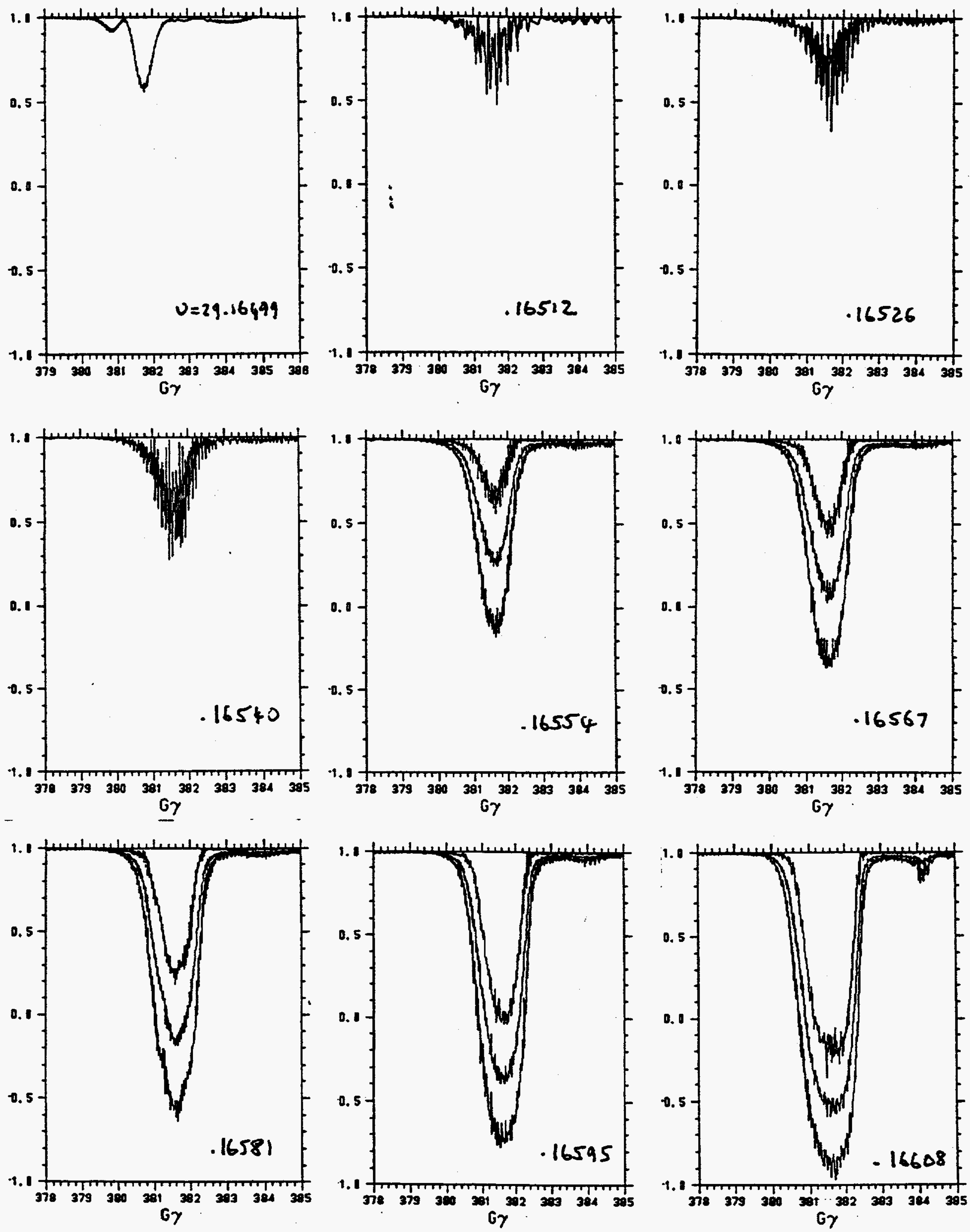

Fig. 4. Crossing the $G\gamma$=381.82 resonance with snakes on and varying the tune in proximity of 1/6.

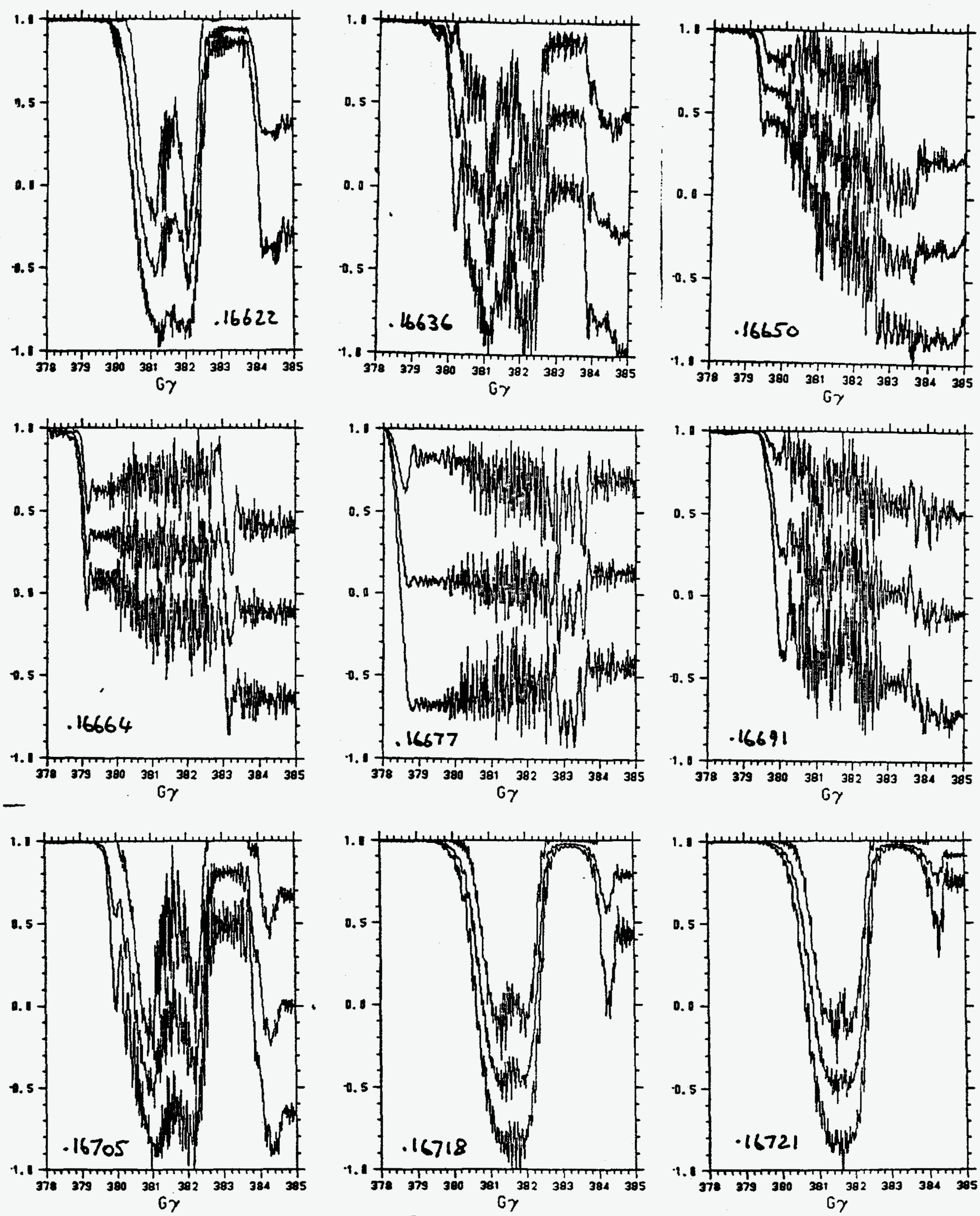

Fig. 4. (Continuation). Crossing the $G\gamma$=381.82 resonance with snakes on and varying the tune in proximity of 1/6.

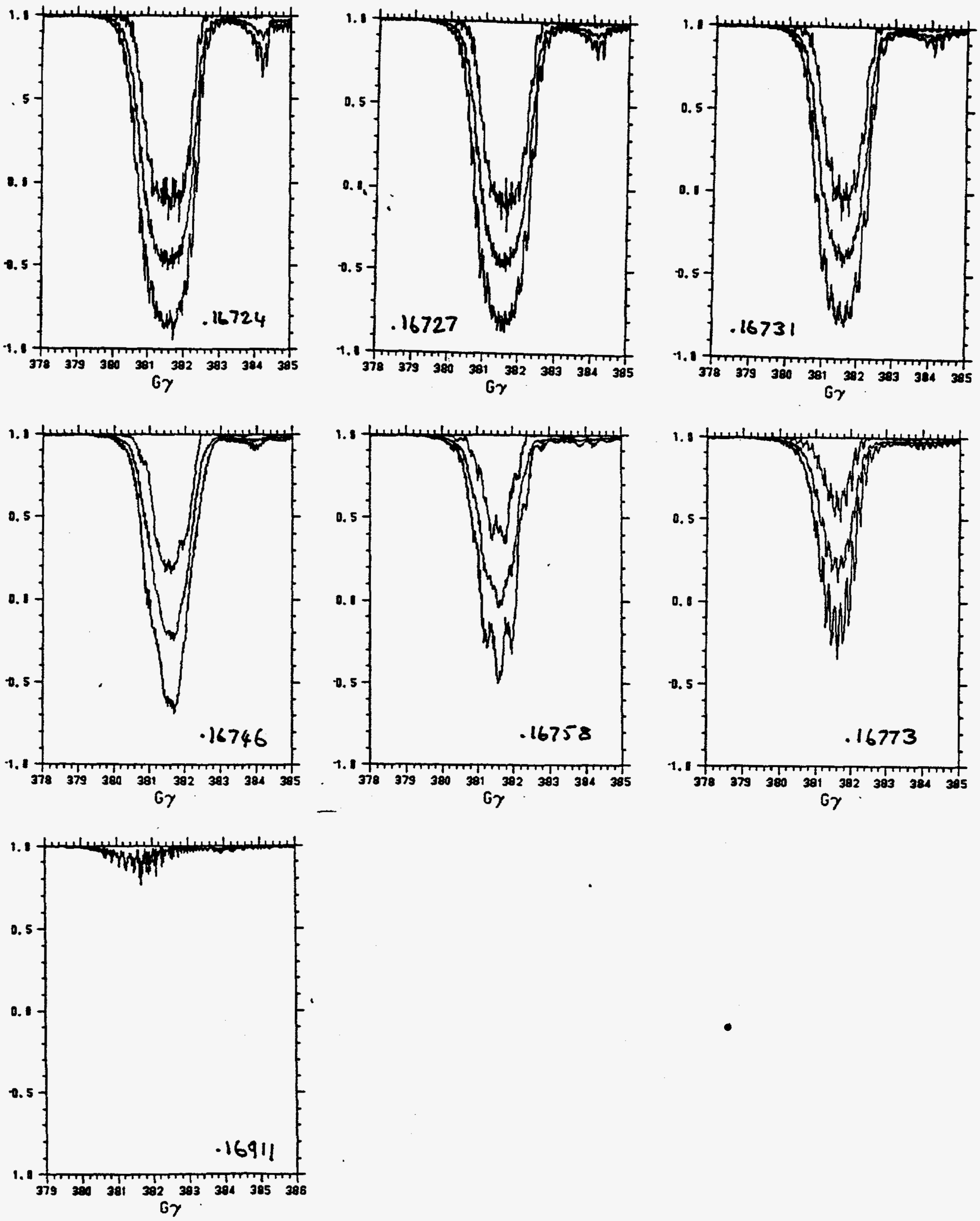

Fig. 4.(Continuation). Crossing the $G\gamma$=381.82 resonance with snakes on and varying the tune in proximity of 1/6.

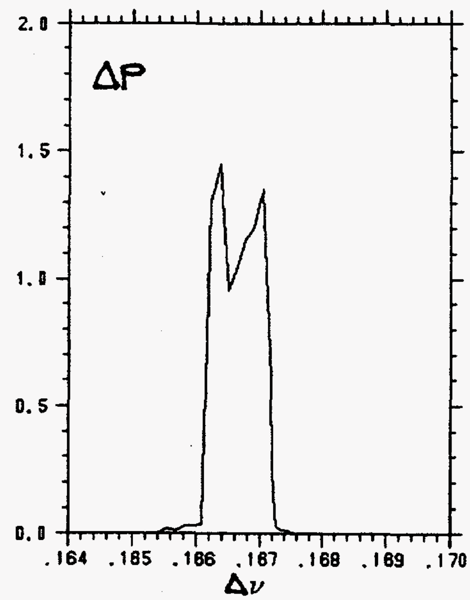

Fig. 5. Profile of the resonance tune line of Fig. 4.

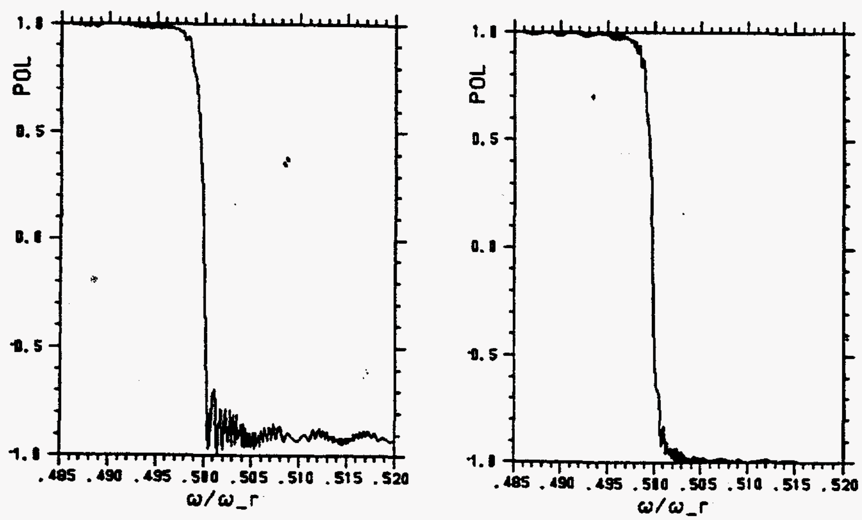

Fig. 6. Rotating Field Spin Flipper. Polarization vs. the ratio of SF frequency to revolution frequency in RHIC.

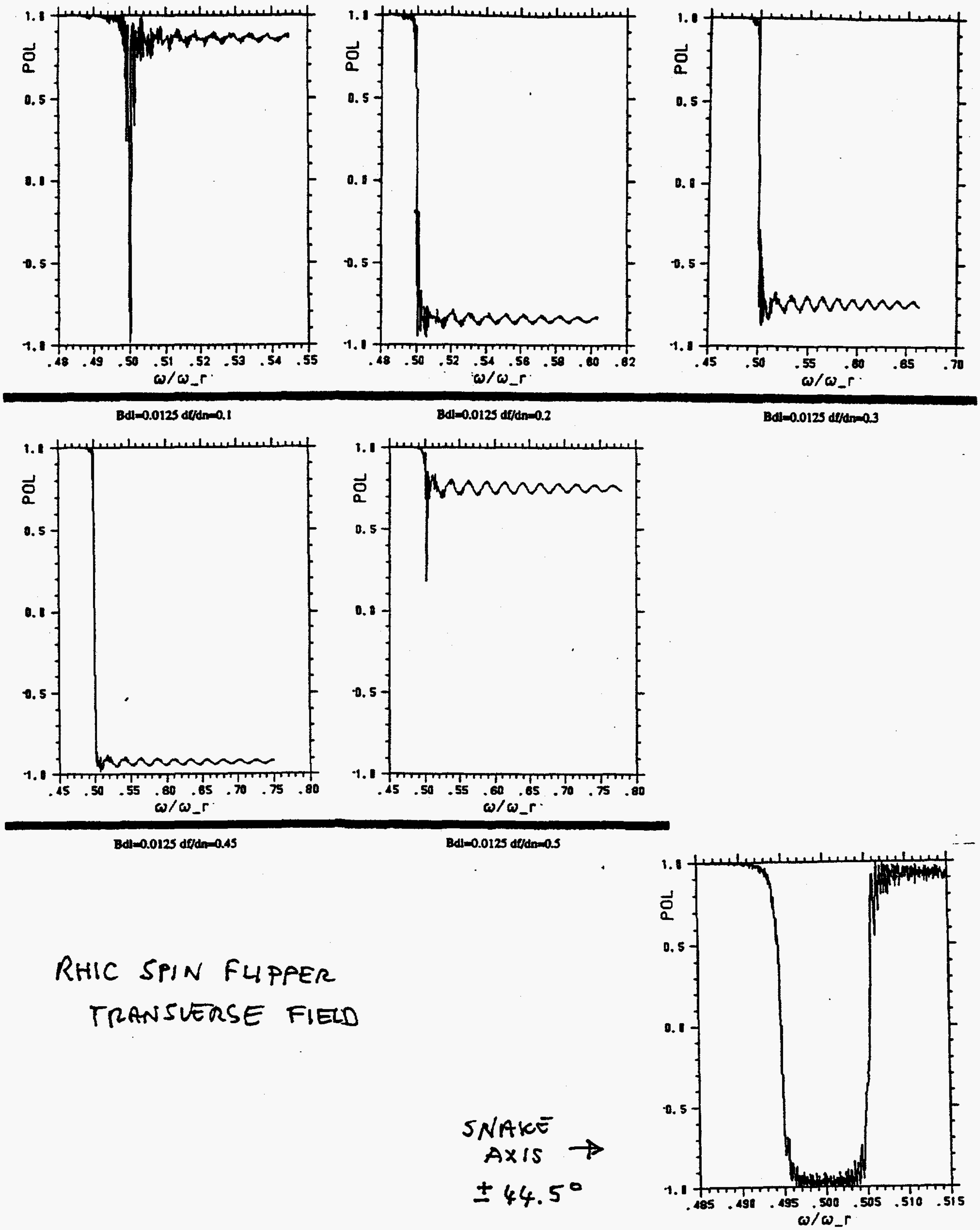

Fig. 7. Oscillating Transverse Field Spin Flipper. Polarization vs. the ratio of SF frequency to revolution frequency in RHIC. Snake axis detuned by ±5°.

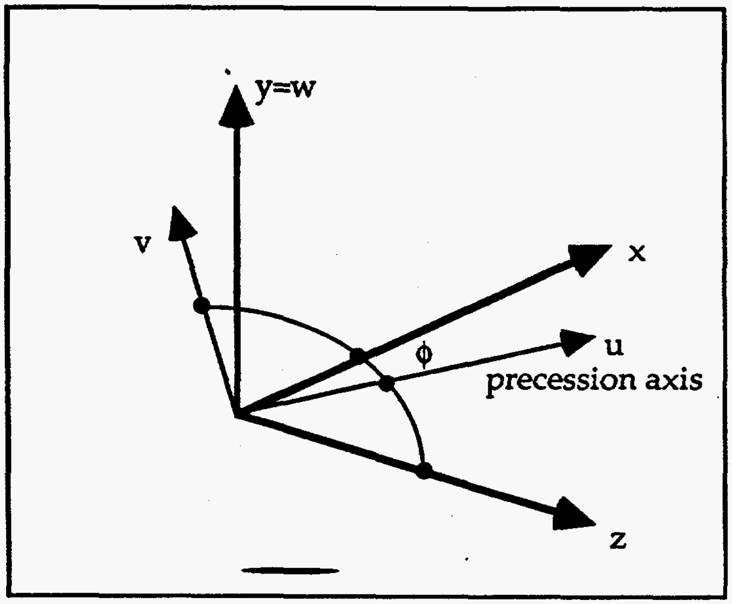

Fig. 8. Reference Coordinates for Spin Rotation Matrices.
Precession axis in the *xz* plane.