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# Medical Imaging with Coded Apertures

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Abstract

We investigated new algorithms for image reconstruction in emission tomography which could incorporate complex instrumental effects such as might be obtained with a coded aperture system. Our investigation focused on possible uses of the wavelet transform to handle non-stationary instrumental effects and analytic continuation of the Radon transform to handle self-absorption. Neither investigation was completed during the funding period and whether such algorithms will be useful remains an open question.

## Introduction

Emission tomography involves the reconstruction of an object from its own internally emitted radiation In medicine, emission tomography is used to determine the volume density of an injected radioactive pharmaceutical by measuring the flux of gamma rays emitted in all angles around the patient. Such studies measure biochemical function or metabolism in specific organs such as the heart. Emission tomography is distinguished from transmission tomography which uses an external source of penetrating radiation usually x-rays primarily to determine structure.

Tomographic reconstructions, both emission and transmission, are performed with the Radon transform, an invertible transform which allows the reconstruction of a function from its projections or line integrals. However, in real tomographic systems, the collected data are only approximations of line integrals in two significant ways. First, no instrument has a perfect response so that a realistic forward transform would include a convolution of the projection with the response function of the instrument. Second, in emission tomography, the object both emits and absorbs radiation so that the correct integral in the transform is not a simple projection, but the radiative transfer integral. These two difficulties place significant constraints on the design of imaging systems and the interpretation of data. Understanding the Radon transform in greater generality would allow a significant improvement in imaging capability. The mathematical techniques developed would be equally useful in the 3D imaging of any volume emitter of radiation, for example, a medical patient, a plasma, or radioactive material.

## Results

In this project we identified two promising techniques for addressing these two problems. The first uses wavelet transforms which are appropriate for deconvolution in precisely such cases as tomography where the instrument response varies across the image (Beylkin, Coifman, and Rokhlin, 1991). Our investigation found that because the Radon transform is not diagonally dominant, the standard technique of wavelet based compression could not be applied. However, there are a number of approaches that have appeared in the literature from investigations at other institutions contemporaneous with ours (Bhatia, Karl, and Willsky 1994, Donoho 1994). None of these seems to offer significant advantages over standard inversion techniques. The central difficulty appears to be that the back-projection part of the inverse Radon transform is an expansive operator. Still this area of research seems promising.

As for our second problem we showed that the Radon transform formalism can be maintained even when considering the full radiative transfer equation in place of the line integral if the standard Radon transform is analytically continued to complex arguments. Inversion of this transform is then an exercise in complex integration. The integration may be performed along certain paths, some of which have been derived in the literature by other methods (Bellini et al. 1979). A general formula for the complex integration is difficult to derive because of essential singularities arising from the geometry of the Radon transform itself. For example, define two coordinate systems, one rotated by angle $\phi$,

$$\xi = x\cos\phi + y\sin\phi$$

$$\eta = x\sin\phi + y\cos\phi$$

Then an attenuated projection is defined as

$$p(\xi,\eta) = \int_{-\infty}^{+\infty} f(\xi,\eta)e^{-\alpha\eta}d\eta$$

where $\alpha$ is a constant attenuation coefficient. Taking a one dimensonal Fourier transform of the projection,

$$P(k,\phi) = \iint f(\xi,\eta)e^{-2\pi i(k\xi - i\alpha\eta)}d\xi d\eta$$

define complex numbers,

$$\lambda_x = k\cos\phi + \frac{i\alpha}{2\pi}\sin\phi$$

$$\lambda_y = k\sin\phi - \frac{i\alpha}{2\pi}\cos\phi$$

Formally, f may be recovered as an inverse complex Fourier transform, that is a path integral along some specified path. If we choose the path as the real axes of, $\lambda_x$ $\lambda_y$ then solving for k and $\phi$, we recover the inversion formula from Bellini et al (1979). In this case, $\phi$, is a complex number and a further analytic continuation is necessary. If k and $\phi$ are to be real numbers, then a possible path is,

$$\lambda_x = t_1 + \frac{i\alpha t_2}{2\pi t}$$

$$\lambda_y = t_1 - \frac{i\alpha t_1}{2\pi t}$$

where $t = \sqrt{t_1^2 + t_2^2}$. But at the origin this path is not analyticby virtue of an essential singularity. Perhaps an extension to higher dimensions would prove fruitful.

Further work is needed on these techniques and other fast algorithms to fully take advantage of the advanced capabilities of the next generation of medical imaging systems.

## REFERENCES


Bellini, S., Piacentini, M., Cafforio, C., Rocca, F. 1979, Compensation of tissue absorption in emission tomography, IEEE Trans. ASSP-27(3), 213-218

Bhatia, M., Karl, W.C. and Willsky, A.S. 1994, A Wavelet based method for multiscale tomographic reconstruction, MIT Tech. Rept. LIDS-P-2182, subm. IEEE Trans. Medical Imaging

G. Beylkin, R. Coifman, V. Rokhlin, 1991, Fast wavelet transforms and numerical algorithms, Comm. Pure and Applied Math, 44, pg. 141

Brooks and deChiro, 1976, Physics in Medicine and Biology, 21, pg. 689

Donoho, D 1994 Non-linear solution of linear inverse problems by Wavelet-Vaguelette Decomposition, Stanford University