ROBUSTNESS OF THE ONE-SAMPLE KOLMOGOROV TEST TO SAMPLING FROM A FINITE DISCRETE POPULATION

DISSERTATION

Presented to the Graduate Council of the
University of North Texas in Partial
Fulfillment of the Requirements

For the Degree of
DOCTOR OF PHILSOPHY

By

Joanne M. Tucker, B.B.A, M.B.A.
Denton, Texas
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One of the most useful and best known goodness of fit test is the Kolmogorov one-sample test. The assumptions for the Kolmogorov (one-sample test) test are:

1. A random sample
2. A continuous random variable
3. F(x) is a completely specified hypothesized cumulative distribution function

The Kolmogorov one-sample test has a wide range of applications. Knowing the effect from using the test when an assumption is not met is of practical importance. The purpose of this research is to analyze the robustness of the Kolmogorov one-sample test to sampling from a finite discrete distribution.

The standard tables for the Kolmogorov test are derived based on sampling from a theoretical continuous distribution. As such, the theoretical distribution is infinite. The standard tables do not include a method or adjustment factor to estimate the effect on table values for statistical experiments where the sample stems from a finite discrete distribution without replacement.

This research provides an extension of the Kolmogorov test when the hypothesized distribution function is finite and discrete, and the sampling distribution is based on sampling without replacement. An investigative study has been conducted to explore possible tendencies and relationships in the distribution of Dn when sampling with and without replacement for various parameter settings. In all, 96 sampling distributions were derived. Results show the standard Kolmogorov table values are conservative, particularly when the sample sizes are small or the sample represents 10% or more of the population.

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CHAPTER 1

INTRODUCTION

The One-sample Kolmogorov Test

Statisticians frequently need to test hypotheses related to the distribution of a population. The test concerned with agreement between the distribution of a set of sample values (empirical distribution) and an hypothesized distribution is called a "test of goodness-of-fit." This type of test is designed for a null hypothesis about the form of the cumulative distribution function, or probability function, of the population from which the sample is drawn. Generally, the alternative hypothesis is broad, including differences in location, dispersion, and form. Goodness-of-fit tests are typically used when the form of the population is in question and the analyst expects the null hypothesis to be supported.

The probability that a set of values is a sample from a known distribution can be tested simply by comparing the empirical cumulative distribution function of the sample with the hypothesized cumulative distribution of the null hypothesis. Several goodness-of-fit test statistics are functions of the deviations between the empirical cumulative distribution function and the hypothesized cumulative distribution function. The goodness-of-fit statistics include sum of squares, sum of absolute values, and the maximum deviation.

One of the most useful and best known goodness-of-fit tests is the Kolmogorov one-sample test introduced in 1933 by Andrei Nikalayevich Kolmogorov. The test focuses on the question of how well the empirical distribution and the hypothesized distribution resemble each other. Smirnov extended this one-sample test by developing a two-sample version of the same test in 1939 (Smirnov 1939a). Both Kolmogorov and Smirnov developed their tests using the maximum vertical distance between the empirical and hypothesized cumulative distributions, called Dn, as an indication of how well the functions resemble each other. Specifically, $Dn = \text{Max} \mid F(x) - S(x) \mid$ where $F(x)$ is a completely specified hypothesized cumulative distribution and $S(x)$ is an empirical cumulative distribution. The cumulative distribution is defined as the proportion of observations that are less than or equal to some specified value.

The Dn statistic, referred to as the Kolmogorov one-sample statistic, is particularly useful in nonparametric statistical inference because the probability distribution of Dn is dependent upon sample size, but does not depend on the hypothesized cumulative distribution, provided the variable is continuous.

The complete set of assumptions (Miller, 1956) for the Kolmogorov one-sample test are:

1. That the sample has been randomly taken from an infinite population, or a sample taken from a finite population with replacement;

2. That X is a continuous random variable and the precision of measurement will prohibit tied observations; and

3. That F(x) is a completely specified hypothesized cumulative distribution function.

Statistical inference techniques usually generate questions about the appropriateness of the technique to different situations, and how the estimator behaves under different sampling schemes. General issues concerning statistical inferential techniques are:

1. How is the test statistic (Dn) mathematically derived, and what is its sampling distribution?

2. How robust is the test to each of the assumptions?

3. How is the test statistic affected if an assumption is not met? Is there a function that describes how the test statistic is affected?

4. What is the relative efficiency of available alternative statistical tests when assumptions are not met?

The research presented in this paper focuses on how and in which way(s) the test statistic is affected when two of the assumptions are violated; that is when the hypothesized distribution is discrete and when sampling is performed without replacement. Distributions could naturally be discrete like the binomial or Poisson, or originally continuous data that has been grouped into categories. If samples are selected from finite discrete distributions without replacement, then the assumptions of test are violated.

Applications of the Kolmogorov Test

The Kolmogorov one-sample test has a wide range of applications. One use of the Kolmogorov one-sample test is to test the normality assumption required by other statistical tests. Residual analysis, used in regression analysis or ANOVA techniques, includes testing for conditional normality of the dependent variable. The traditional parametric test, based on Student's t distribution, is derived under the assumption of a normal population. The probabilities of Type I and Type II errors depend on the normality of the population.

Reliability is a topic linked with many quality and statistical concepts. Reliability is defined as the probability that a device will perform its intended function satisfactorily for a stated period of time under specified operation conditions. Reliability is in part a probability-related concept. One concern for reliability is the recognition of the configuration of the failure rate distribution. Common distributions used are the exponential distribution , normal distribution , and Weibull distribution. Knowledge of the failure rate curve is required if proper use is to be made of the several standardized acceptance sampling schemes. Two of the more widely employed life testing standards are H-108 and MIL-STD 690B. These two standards assume that failure rate follows an exponential distribution.

Simulation studies have also used the Kolmogorov one-sample test to identify whether certain probability distributions could be assumed to describe a variable. In this way, the Kolmogorov one-sample test helps to specify the probability distribution of variables of interest. Proper identification of variables and their distribution is a deciding factor in the success of a simulation analysis.

Researchers have used the Kolmogorov one-sample test to show support, or lack thereof, in Title VII age discrimination cases. Consider the following hypothetical example. Downsizing reduces the population work force. Discrimination against individuals released from their employment because of age requires that the distribution of ages for individuals released is different from those who are not. The variable of interest, age, is often grouped into ordinal categories. If no age discrimination occurred with a downsizing, the distribution of ages for individuals released would not significantly differ from the distribution of ages of the workforce before the downsizing. There would be a "good fit" between the two distributions. If there were discrimination against older employees, the distributions of ages would not fit well, (e.g., the proportion of employees released that were under say age 52 might be much less than the proportion of the workforce under age 52). In this situation, the population could be represented by the distribution of ages of the workforce while the sample could be represented by the distribution of ages of the employees released. Sampling, the firing or releasing of an employee would be without replacement since the company would not fire the same employee twice. The Kolmogorov one-sample test could be used to determine if the distribution of ages for released employees mirrors that of the population.

The following is another example of applying the Kolmogorov one-sample test. In accounts receivable management, the number of days past due, known as aging, form ordinal groupings for the individual accounts. An acceptable percentage of accounts more than 30 days past due may be deemed by a manager to indicate that the accounts are "in control". A random sampling of the accounts with their distribution of aging could be compared to the standard for control purposes.

The Kolmogorov one-sample test has many applications. Knowing the effect of using the test when an assumption is not met is of practical importance. From an academic viewpoint, it is of theoretical importance and practical interest to know the consequences of statistical conclusions based on assumptions that do not adhere to the assumptive foundations of the test.

Purpose, Problem, and Significance

The purpose of this research is to analyze the robustness of the Kolmogorov one-sample test statistic when sampling from a finite discrete distribution. The research in this study investigates the relationship between the critical values (percentiles) of the sampling distribution of Dn when the underlying population is discrete and the critical values taken from the standard Kolmogorov table. This study also explores the relationship between the sampling distribution of Dn when samples are taken from a discrete population without replacement, versus samples taken with replacement.

The Kolmogorov one-sample test assumes a random sample will be taken from an infinite continuous population. This assumption may be a problem in some applications of the test. Samples are many times taken without replacement from a finite discrete population. Although a few researchers (Schmid 1958, Maag 1973, Conover 1980) have addressed the issue of discrete populations and the Kolmogorov one-sample test, there has not yet been any published research regarding the robustness of Kolmogorov one-sample test to sampling without replacement, which actually exists in many practical applications. The standard tables of critical values for the Kolmogorov one-sample test are derived based on sampling from a theoretical continuous distribution. As such, the theoretical distribution is infinite. The standard tables do not include a method or adjustment factor to estimate the effect on tabled critical values for statistical experiments where the sample stems from a finite discrete distribution without replacement. The basic conclusion about the Kolmogorov one-sample test when used with discrete data is that the test is conservative (Neother 1967). Some researchers (Slakter 1966, Pettitt 1977) have suggested that the test is extremely conservative, and in some cases, useless.

When the data are continuous, all measurements must be made on a discrete scale. The extent of the deviation between a theoretical continuous value and the value measured is a function of the coarseness of the measurement device. The conflict between this theoretical assumption of continuity of the data and the empirical reality of data measured on a discrete scale is a consideration for users of any test. This research investigates the effect on the Kolmogorov one-sample test when a discrete population is hypothesized and samples are taken without replacement.

The Kolmogorov goodness-of-fit test is known to be conservative when the hypothesized distribution function is discrete. There is a need for a goodness-of-fit test that will provide accurate critical values when the hypothesized distribution is not continuous (Conover 1972). It is desirable to have a goodness-of-fit test which can be used when the hypothesized distribution is discrete (Gibbons 1992). One goal of this study is to find a usable adjustment, for the standard table values to use when sampling from discrete populations without replacement.

There has been research on the effect of the sampling distribution from sampling with and without replacement on other statistical analyses. Changing the sampling scheme produces a different sampling distribution in statistical experiments. In perhaps the most obvious example, when sampling from a dichotomous population consisting of an attribute such as success or failure, a change from sampling with replacement to sampling without replacement from a finite population changes the relevant probability distribution from the binomial to the hypergeometric. Research (Freund 1992) has shown that the mathematical function that relates the variances of these two distributions is the finite population correction factor. The research objectives of this study are: (1) to determine whether there is a correction factor that can relate the variances of the sampling distribution of Dn in a similar manner; (2) to investigate the differences between the two sampling distributions in central tendency and variation; and (3) to measure differences in percentiles. Research on the effects of sampling without replacement on the sampling distribution of Dn has not been published to date.

Previous studies have focused on the effects of sampling without replacement for other sample statistics. Wright (1991) constructed extensive tables providing exact confidence intervals for an attribute when sampling from small finite distributions based on the hypergeometric probability distribution. Approximations of a confidence interval for a parameter of an attribute are frequently based on the binomial, Poisson, or normal distributions and assume sampling with replacement or infinite populations. However, Buonaccorsi (1987) found that, for certain combinations of population size (N), sample size (n), and confidence level, these approximations are not suitable and can lead to incorrect inferences about the attribute. One of the features of the Kolmogorov one-sample test is that a confidence band may be constructed for the true unknown distribution function. Likewise, confidence bands that are constructed for the true unknown population distribution function based on the standard Kolmogorov table values are merely approximations if samples are selected without replacement from finite populations. Knowledge of the effect of sampling without replacement could result in more reliable inferences about the population distribution function.

There has been growing acceptance and use of statistical analysis in a diversity of decision-making contexts, including legal environments and quality control. Broader application of a statistical test can be achieved by studying the robustness of a statistical test to conditions different from those theoretically assumed by a statistical test. When theoretical conditions do not exist for a test, the consequences are inaccurate conclusions derived from the analysis, as well as an incorrect assessment of the probability for type I and type II errors. A research interest of this study is to address the likelihood of an inappropriate conclusion when the test is performed using data taken from a finite discrete population without replacement. This study fills a void where scant research has been conducted in the area of finite discrete distributions, sampling without replacement, and the Kolmogorov one-sample test. The earlier research has generally been in the form of simulation studies because general analytical solutions have not been available. This study also uses simulation, and will focus on values of Dn when finite discrete populations are hypothesized and samples are taken without replacement.

CHAPTER II

LITERATURE REVIEW

Previous Research on the One-sample Kolmogorov Test

The test statistic, Dn, is a random variable independent of the special form of F(x), if X is continuous and has been specified. That is, regardless of the specified form of the hypothesized distribution (e.g., normal, gamma, exponential), the same standard Kolmogorov table can be used to determine critical values. If the hypothesized distribution is not completely specified, then the standard Kolmogorov one-sample test is not applicable. The standard Kolmogorov table has been modified to be used in several situations where parameters of the hypothesized distribution are estimated from sample data because the standard table values are no longer valid for all distributions; they change from one hypothesized distribution to another. Lilliefors (1967) initially developed critical values for a test to fit unspecified normal distributions. In later research (Lilliefors, 1969; Lilliefors, 1973), he developed critical values for tests to fit exponential distributions and gamma distributions. The critical values obtained by Lilliefors were arrived at by simulating 1000 samples, calculating the test statistics, then forming the sampling distribution. In similar research, critical values have also been developed for the Weibull distributions (Chandra 1981). If the theoretical distribution is not completely specified, and the standard tables are used, then the test becomes conservative (Conover 1980). The size of the exact critical values for Dn were shown to be as much as two-thirds of the standard Kolmogorov table values (Lilliefors 1967). One of the research interests of this study is to determine how conservative the standard tables are when sampling without replacement from a discrete population.

Previous research has also focused on other areas of the Kolmogorov one-sample test. Beginning in 1939, for example, Kolmogorov derived the limiting distribution for a two-sided test. This was followed by Smirnov (1939b) who found the limiting distribution for the one-sided tests. Later, Feller (1949) and Doob (1949) simplified the limiting distributions originally derived by Kolmogorov and Smirnov. These limiting distributions assume a sample size that approaches infinity.

The standard Kolmogorov table typically provides critical values derived from exact distributions for small samples, and asymptotic approximation values for Dn when the sample size exceeds 35. Birnbaum (1952) provided a method of evaluating the limiting distribution for small finite samples, and Massey (1950) calculated a table of critical values for small samples. Comparisons were made between the exact distributions for small samples and the limiting distribution by Birnbaum (1952). Birnbaum showed that the limiting values are always greater than the exact one, and he stated that, for sample sizes greater than 80, he believed the limiting distribution to be quite good. The difference between exact critical values for small samples and the asymptotic approximate critical values are close enough for practical applications as long as the sample size exceeds 35, according to Gibbons (1992).

There have been only a limited number of studies to analyze the impact on the test statistic for the Kolmogorov one-sample test when a discrete random variable, rather than a continuous random variable, is involved. The Kolmogorov one-sample test is known to be conservative when the hypothesized distribution function is discrete (Neother 1967). Slakter (1965) used Monte Carlo techniques to illustrate that the standard table values are extremely conservative. Conover (1972) developed a method of obtaining the approximate critical value for samples of 30 or less from an infinite discrete population. According to his research, the critical values for discrete data can be about one-third as large as their counterparts from the standard table.

The sampling distribution of Dn is no longer distribution-free when the variable is not continuous. Consequently, one of the factors in Conover's approximation formula is the form of the hypothesized distribution. Attempts to obtain a general limiting distribution for discrete distributions have, until now, been unsuccessful. All prior attempts (Schmid 1958, Carnal 1962, Taha 1966) have produced functions that become degenerate or undefined when the hypothesized distribution is purely discrete (Damianou 1990).

Pettit and Stephens (1977) furnished critical levels that can be used for grouped data from a continuous distribution, and for discrete data when the expected frequencies for each group are equal. They determined critical values for ordinal groups ranging from three to ten, with samples sizes ranging from 3 to 30. In comparing the critical values to standard tabulated Kolmogorov values, they found the tabulated values to be conservative, as stated in Noether's research (1967). The greatest discrepancies between the corresponding critical values occurred when the number of ordinal categories was small.

Research Questions

There is an absence of research regarding the effects that sampling without replacement from a discrete finite population would have on the critical values for the Kolmogorov one-sample test. This gap in current knowledge generates the first research question.

Research Question 1

> What is the general relationship between different sampling distributions of Dn? Specifically, this study will compare distributions of Dn derived when sampling with replacement versus sampling without replacement when the underlying population is finite. Is there a correction factor, similar to the finite population correction factor used between the hypergeometric and binomial distributions, that can be used to adjust the variance of the Dn test statistic?

Another research issue involves the relationship between the critical values of the sampling distribution of Dn when sampling without replacement from a discrete population and the standard Kolmogorov table values. This interest addresses a general concern of many statistical tests which is the robustness of the Kolmogorov standard table values to conditions different than theoretically assumed. Missing from the literature is the robustness of the standard table values to sampling without replacement. This omission provides the foundation for the second research question.

Research Question 2

> Is there an adjustment factor that can be used on the standard Kolmogorov table of values that will provide a good approximation of the actual critical value when sampling without replacement from a finite population? The standard table includes an asymptotic function for large samples. Is there a function that will also include the finite population factor of sampling without replacement from a discrete finite population?

The discrepancy between the theoretical foundation of the Kolmogorov one-sample test and the reality of sampling without replacement from a discrete finite population is a consideration that has been neglected by researchers. Given the extent of prior research to other issues involving of the Kolmogorov one-sample test, a logical step forward is to focus research on this aspect of the Kolmogorov one-sample test. This research will provide an extension of the Kolmogorov one-sample test when the hypothesized distribution function is finite and discrete, and the sampling distribution is based on sampling without replacement.

The effect of sampling without replacement from a finite population on the one-sample Kolmogorov test is an aspect that previous research has neglected. The increased accuracy that the adjustment factor will provide in determining the probability of committing a type I and type II error will increase the usefulness of the test.

CHAPTER III

RESEARCH FRAMEWORK

Simulation Goodness-of-fit Studies

In the past, researchers have used simulation to generate approximate theoretical distributions. A study by Wood and Altavela (1978) used simulation techniques to develop critical values for large sample goodness-of-fit tests using grouped data from discrete distributions. Another study (Slakter 1965) used grouped data, and compared the efficiency of the Pearson Chi-Square to the Kolmogorov test by simulating the sampling distributions. A Monte Carlo study compared distribution assumptions with a data-generating process (Anderson 1994).

Simulated distributions have also been used in developing other goodness-of-fit tests. Standard notation of the Shapiro-Wilk test statistic is W. With a sample size of three, the probability distribution of W is known and is used to determine the significance level. When the sample size is greater than three, simulation results are used to determine the significance levels (SAS Procedure Guide 1988).

Lilliefors estimated critical values for the one-sample Kolmogorov test to fit unspecified normal distribution (Lilliefors 1967) using simulation techniques. The problem had been too difficult to solve analytically, therefore, Lilliefors used a computer simulation with random numbers to obtain an approximate solution for the sampling distribution. Using the same simulation procedure, Lilliefors (1969) also found the critical values for a test to fit exponential distributions with an unspecified parameter.

Research Simulation Design

This study will use simulation techniques to provide critical values for the test statistic when fitting one of three specified finite discrete distributions. The procedure is as follows: Taking a random sample of size n observations without replacement from a finite discrete population of size N, calculate Dn =Max $|F(x) - S(x)|$. This sampling process is repeated 7,500 times to calculate Dn. In the case when the uniform population is assumed, the process is repeated 15,000 times. The resulting distribution of Dn is the estimated sampling distribution when sampling under these conditions. Percentiles and summary measures are made for the sampling distribution. The same analysis will be repeated for various population and sample size combinations.

This simulation procedure will also provide values for the test statistic in fitting a specified finite discrete distribution when sampling with replacement. The procedure is identical to the sampling process described above with the exception of the random samples are selected with replacement. The resulting distribution of Dn is the estimated sampling distribution when sampling under these conditions. Percentiles and summary measures are made for the sampling distribution. The same analysis will be repeated for various population and sample size combinations. The resultant sampling distributions of

Dn when sampling with and without replacement will provide the data to address the research issues of this study.

For a given combination of population and sample size, the two sampling distributions --- one stemming from random samples taken with replacement, the other stemming from random samples taken without replacement --- will be analyzed. These sampling distributions will be compared to determine if there is a population correction factor that relates the corresponding variances. Other comparisons about the central tendency between the distributions will also be made.

The effect of sampling without replacement on the critical values of the test statistic, Dn, is another concern of this research. Critical values for the Dn statistic from sampling without replacement will be compared to the critical values in the standard Kolmogorov one-sample table to determine the relationship between them. An analysis of the critical values of the simulated sampling distribution and the standard Kolmogorov table values will provide information about their relationship. The analysis may lead to an adjustment factor for the table value that will allow more accurate conclusions when using the test.

The simulated sampling distributions will vary in the following parameters: (1) population size; (2) sample size; and (3) shape of finite population. Accordingly, this study will include finite population sizes of small, middle, and large, and samples of various sizes. Table 1 defines specific parameter values for sample size (n) and population size (N).

Table 1.--Parameters values of n and N

Sample/population ratio	n, N			
5%	6, 120	12, 240	24, 480	48, 960
10%	6, 60	12, 120	24, 240	48, 480
20%	6, 30	12, 60	24, 120	48, 240
30%	6, 20	12, 40	24, 80	48, 160

A finite discrete population frequency distribution is assumed for each of the combinations listed in the table above. The shape of the assumed population frequency distribution is either uniform, symmetrical normal-like, or positively skewed. Each assumed population frequency distribution and simulations of the combinations in Table 1 constitutes a phase of this study. The three phases will provide a total of 96 simulated sampling distributions of Dn. The assumed finite discrete population frequency distributions and the simulated sampling distributions of Dn are listed in Table 69 in the appendix.

Investigative Pilot Study

An investigative pilot study was conducted to explore possible tendencies and relationships in the data. Based on sampling with and without replacement, the sampling distributions were derived based on the following parameters: ratio of (n/N) equals 20%

(2, 10) selected from a discrete uniform and a ratio of (n/N) equals 30% (3,10) selected from a discrete uniform.

The four derived sampling distributions were exact, with all possible samples of the stated size being selected and analyzed. The parameters for this pilot study made it feasible to generate the entire sampling distribution. These sampling distributions are presented in table form, accompanied by some descriptive statistics in Tables 70 and 71 in the appendix. Because these distributions are accurately portrayed, the magnitude of differences between them are also accurately portrayed.

Comparing the variances of the sampling distributions with replacement to sampling distributions without replacement gives the ratios in Table 2.

Table 2.--Ratio of Variances from Pilot Study

Ratio of n/N	$\sigma^2_{\text{without}} / \sigma^2_{\text{with}}$ = ratio
20% (2,10)	.0204/.0253=.8063
30% (3,10)	.0131/.0189=.6931

The ratio of the variances is approximately equal to (1-n/N). Comparing the critical values from the exact distribution without replacement to the standard table values is more difficult given the small population sizes of this pilot study. Interpolation is necessary to compare the most commonly used alphas in statistical inferences. The standard table values are greater than the critical values from the exact sampling distribution. In this study, the relationship between actual and table critical values will be estimated using a regression function.

In summary, the results of this investigative study indicate that there are discrepancies between the exact critical value and standard Kolmogorov table values when the hypothesized distribution is not continuous, as well as between the variances of sampling distributions.

CHAPTER IV

PHASE ONE OF DESIGN

Discrete Uniform Population

In order to determine how sampling from a finite discrete population affects the sampling distribution of Dn, the simulation study takes four factors into consideration. One factor, size of the sample drawn, has levels of 6, 12, 24, and 48 units. In applications of the one-sample Kolmogorov test, the first three sample sizes are considered small, while the last one is usually considered a large sample. A second factor in the design is the ratio of the sample size to the population size. This factor also has four levels: 5%, 10%, 20%, and 30%. It is along these two dimensions that the study determines the effect of sampling, with or without replacement, from a finite discrete population on the sampling distribution of Dn.

The shape of the discrete population from which simulated samples are taken is assumed to be one of three forms, and comprises the third factor in the study. Each of the three forms serves as the distinguishing aspect for a phase in the simulation. In the first phase, the shape of the population is a discrete uniform distribution with 10 categories. The cumulative distribution is found for each of 15,000 samples, taken with replacement, from this uniform distribution. Each of the sample cumulative distributions is compared to the population cumulative distribution, and the statistic, Dn, is then calculated. These

15,000 Dn values represent the sampling distribution. Another 15,000 samples are then taken from the discrete uniform population; this time, however, without replacement. The cumulative distribution for each of these samples is computed, compared to the cumulative population distribution, and the statistic, Dn, determined. These 15,000 Dn values represent the sampling distribution. The resulting sampling distributions from the simulation represent an approximation to the theoretical distribution of Dn values which exist for the population and sample designs.

The comparison of the two simulated sampling distributions provides an estimate of the effect that sampling with or without replacement has on Dn. Also of interest in this study is the comparison of the simulated sampling distributions of Dn and the critical values of Dn found in standard one-sample Kolgmorov tables. The critical values found in standard tables are associated with specified alphas and sample sizes. The levels of significance, 10%, 5%, 2.5%, 1%, and 0.5%, represent the levels for the final factor of this study.

The simulated sampling distributions stem from samples taken from a uniform discrete population in the initial phase of the study. In the second phase, the form of the population sampled is changed from uniform to an unimodal, symmetrical distribution that has ten categories. The form is similar to the shape of a normal distribution, but is discrete like that of a binomial distribution. The same size samples, both with and without replacement, that were generated from the uniform population will be taken from this symmetric distribution. Similar comparisons will be made between simulated sampling distributions and the critical values found in standard Kolmogorov tables. In the last phase, the form of the assumed population is a distribution that is skewed to the right, similar to the shape of a Poisson distribution. The population values will also be spread over ten categories. The same sample designs and analyses, as in the other two phases, will be completed. Table 3 further depicts the relationships and arrangement of the factors in this simulation study.

Table 3.--Factors of Simulation

	Phase I	Phase II	Phase III
Sampled Population:	Uniform Discrete distributed over 10 categories	Unimodal Symmetrical discrete distributed over 10 categories	Unimodal Skewed distributed over 10 categories
Sample Size:	6, 12, 24, 48	6, 12, 24, 48	6, 12, 24, 48
Ratio of n/N:	5%, 10%, 20% 30%	5%, 10%, 20% 30%	5%, 10%, 20% 30%
Sampling with Replacement:	Yes, No	Yes, No	Yes, No

There are 32 combinations, or treatments, for each phase, and 96 treatments for the entire simulation study. The sampling distribution of Dn for each treatment is based on 15,000 simulated samples. This chapter has focused on the results obtained from comparing the sampling distributions that were generated from a finite uniform discrete population. Forthcoming chapters concentrate on results and analyses from sampling from a binomial shaped population and from a population that is skewed to the right.

The test statistic, Dn, is defined as the maximum absolute difference between the sample cumulative distribution, S(x), and the hypothesized population cumulative distribution, F(x). Figures 1, 2, and 3 illustrate how Dn is determined and show how extreme values for Dn can occur. The figures depict the value of Dn for three different samples taken from an hypothesized population of 25 observations uniformly distributed over 5 categories, (0, 1, 2, 3, 4). Samples used in this example are assumed to be selected with replacement, and taken exclusively from either the high, low, and middle values of the population, respectively.

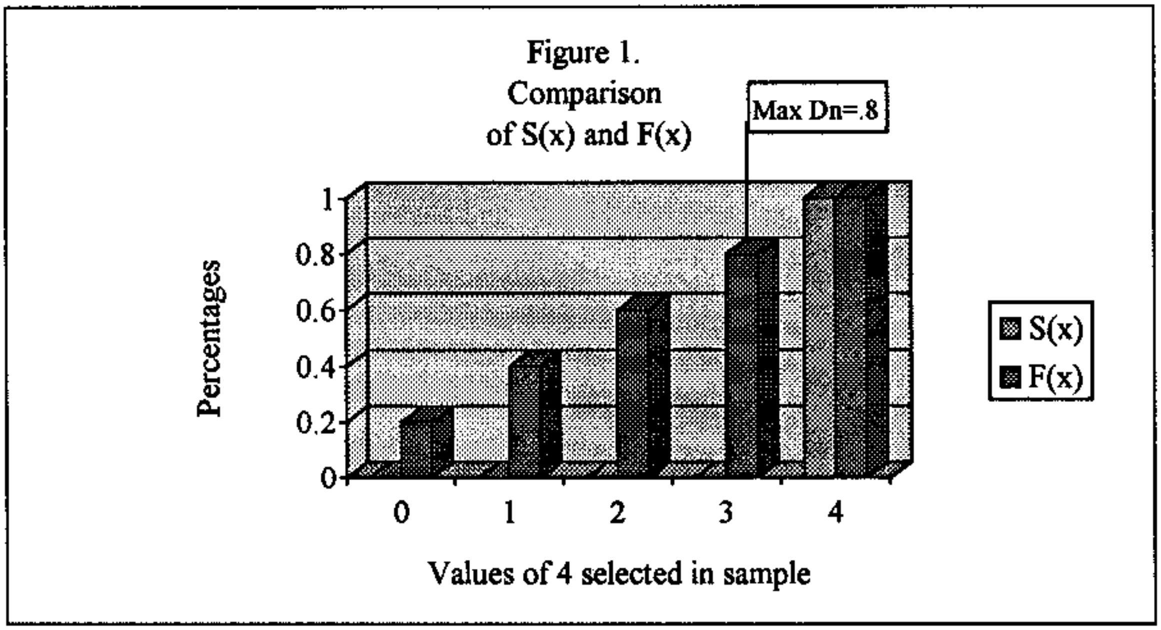


Figure 1.
Comparison
of S(x) and F(x)

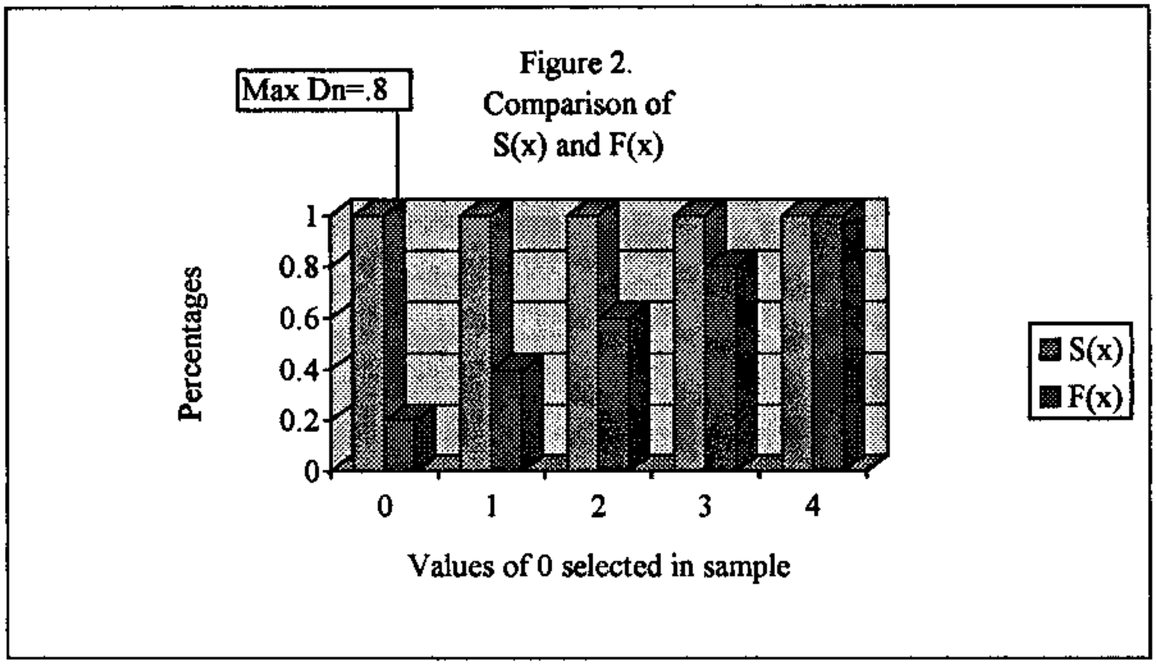


Figure 2.
Comparison of
S(x) and F(x)

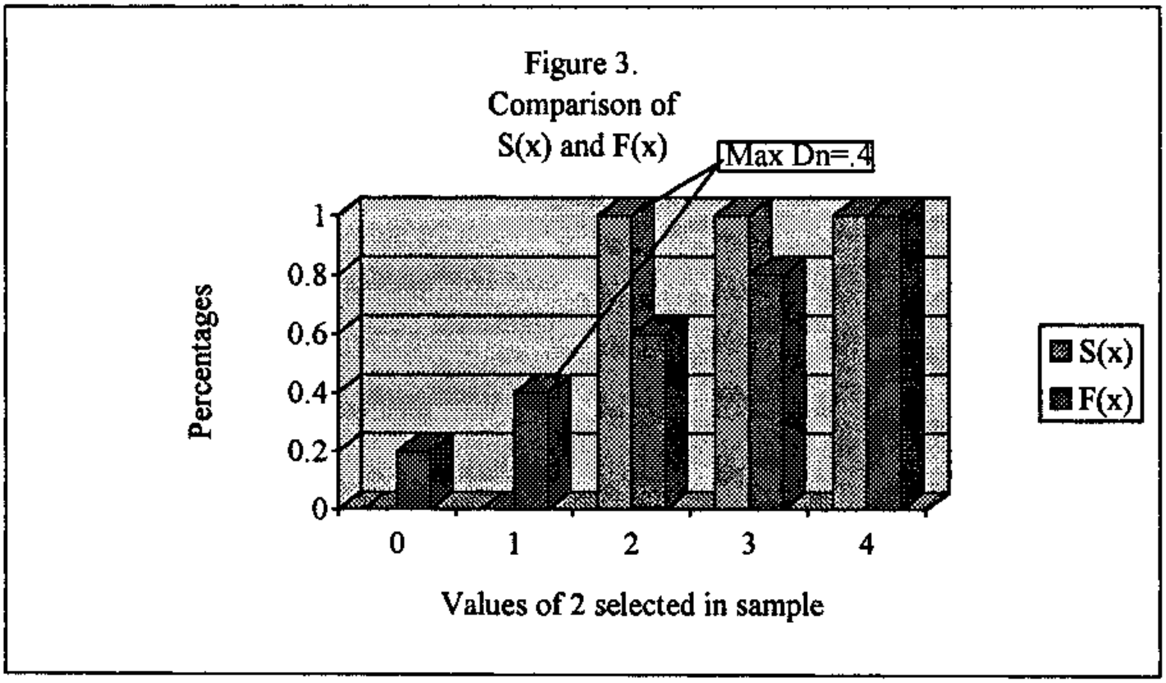


Figure 3.
Comparison of
S(x) and F(x)

The figures present differences between the sample and population cumulative distributions as the difference in height between two adjacent bars. Figure 1 presents the differences between cumulative distribution for a sample in which all values selected are fours and the population cumulative distribution. The maximum difference between adjacent bars for this situation is 0.8. The test statistic, Dn, therefore is 0.8. In figure 2, the sampled values are all assumed to be zeroes. The maximum Dn for this situation is also 0.8. The value of zero and four in the population are extreme values because they are far from the center of the distribution. In figure 3, the sample consists entirely of twos. In this case, the Dn is 0.4. The value of two in the population is close to the mean of the population values and, therefore, is not an extreme value. Using this example, the minimum value of Dn (not shown) would be 0.0, in which the sample consists of five different values; 0, 1, 2, 3, and 4. The cumulative S(x) and F(x) distributions would be of equal heights at all points. Unusually high values for Dn occur because extreme population values are selected in the sample. When sampling without replacement from a population, repeat selection of extreme values is not possible (or is not as likely) and, therefore, large Dn values are less frequent.

Sampling Distribution of Dn
Comparison of the Mean and Variance
When Sampling With and Without Replacement

One of the factors in this study is sample size. With 4 sample sizes and 4 ratios of n/N, there are sixteen combinations of n and N. The sampling distribution of Dn when sampling with replacement is compared to the sampling distribution of Dn when sampling without replacement in order to consider the effect of sampling without replacement on the value of Dn. This study shows that the mean Dn for the sampling distribution, when sampling without replacement, is lower in all sixteen comparisons. When a statistical test (a two-independent sample test) was performed to indicate significant differences between the two means for the sixteen comparisons, all comparisons were significant.

When comparing the magnitude of the difference across the ratio levels, it is greatest when the sample/population ratio is 30%, and it steadily decreases as the ratio decreases. This is true across all sample sizes in this study. Comparing the differences in means between different sample sizes, the greatest differences in means are from samples of size six, and it steadily decreases as the sample size increases. This is true across all ratios in this study.

Another interesting observation is that, for a given sample size, the mean of Dn when sampling with replacement is fairly constant regardless of the ratio. For a sample of size 6, the range for the mean is 0.28342 to 0.28587. For a sample of size 48, the range for the mean is 0.09839 to 0.09907. In contrast, the mean of Dn when sampling without replacement decreases steadily in value as the ratio increases. For a sample of size 6, the range for the mean is 0.27761 for a ratio of 5%, and it decreases to 0.24596 for a ratio of 30%. For a sample of size 48, the range for the mean is 0.09739 to 0.08281. The mean of the Dn values in both cases decreases as the sample size increases. According to the

Glivenko-Cantelli theorem (Gibbons 1992), this is because S(x) converges uniformly to F(x) as the sample size approaches infinity. These statistics are presented in Table 4.

In analyzing the variance of the sampling distribution of 15,000 Dn values when sampling with replacement, some trends are evident. First, as with the means of Dn when sampling with replacement, the variance of the Dn values is fairly constant for a given sample size. The constant value for the mean and variance for Dn suggest the distribution is stable for a given sample size. This appears to be true regardless of the sample/population ratio. The sample size is the dominant factor in the value of the variance for Dn. This is not the case, however, when sampling without replacement. In addition, the variances of the Dn values decrease as the sample size increases. The larger sample provides more consistent or reliable information about the value of Dn. This is similar to the relationship of the *Central Limit Theorem* and the standard error of sample means. As the sample size increases, the standard error decreases.

When sampling without replacement, the variance of Dn also decreases as the sample size increases, but its value is influenced by the sample/population ratio. As the sample/population ratio increases for a given sample size, the variance of Dn decreases in value. In comparing the two variances for the sixteen combinations of n and n/N, the variance from sampling without replacement is always smaller than the variance from sampling with replacement. Sampling without replacement yields a smaller variance value because of the covariance between any two observations in the sample (Freund 1992). The covariance of random variables drawn from a finite population without replacement is

Table 4.--Mean: Sampling Distribution of Dn when Sampling With and Without Replacement Uniform Population

Ratio	n, N	Mean of Dn With	Mean of Dn Without	Difference*
5%	6, 120	0.28342	0.27761	0.00581
10%	6, 60	0.28403	0.27258	0.01145
20%	6, 30	0.28448	0.25831	0.02617
30%	6, 20	0.28587	0.24596	0.03991
5%	12, 240	0.19835	0.19319	0.00516
10%	12, 120	0.19896	0.18909	0.00986
20%	12, 60	0.19849	0.17924	0.01925
30%	12, 40	0.19810	0.16842	0.02968
5%	24, 480	0.14023	0.13697	0.00326
10%	24, 240	0.14070	0.13325	0.00745
20%	24, 120	0.14023	0.12560	0.01462
30%	24, 80	0.14038	0.11836	0.02203
5%	48, 960	0.09839	0.09739	0.00100
10%	48, 480	0.09907	0.09372	0.00536
20%	48, 240	0.09850	0.08858	0.00992
30%	48, 160	0.09839	0.08281	0.01558

*All Significant at 0.05

$-\sigma^2/(N-1)$. The covariance of random variables drawn from a finite population with replacement is zero. Therefore, the variance of the Dn values will be smaller as a result of sampling without replacement.

In general, when sampling from a finite population without replacement and when extreme values in the population are selected and not replaced, subsequent items selected must be taken from values closer to the mean. Consequently, differences in the sample cumulative and the population cumulative distribution are smaller than if samples are taken with replacement. With replacement, extreme values can be selected repeatedly, thereby providing a cumulative sample distribution that differs more from the population cumulative distribution. The mean value of Dn when sampling with replacement would tend to be larger and the Dn values would have more variability. When sampling without replacement from a population, repeated extreme values are not possible, or not as likely, and large Dn values are less frequent, thereby giving a smaller value for the variance and the mean for the sampling distribution of Dn. The likelihood that extreme values are repeatedly selected decreases as the sample size increases. It is more likely to draw 6 zeroes in row than to draw 48 or 24 zeroes in a row. Some populations might not have "n" zeroes in which case it would be impossible to draw "n" zeroes in a row.

Analyzing the ratio of the variance when sampling without replacement to the variance when sampling with replacement displays two patterns. The first pattern shows that, for a given sample size, the variance ratio decreases in value as the sample/population ratio increases. This is a logical extension because of previous observations made about the value of the two variances. The second, and somewhat more fascinating pattern, is that the ratio seems to approximate the finite correction factor, (1 - n/N). These statistics are presented in Table 5.

This latter pattern is a remarkable, although not unique, relationship. In studying the relationship between the sampling distributions of sample means or sums, when sampling with and without replacement, the *Central Limit Theorem* identifies a finite correction factor to adjust the variances. In sampling from a population with a fixed amount of variability, the variance of the sampling distribution of means depends primarily on the size of the sample, and only to a lesser extent on the sample/population ratio. With respect to the sampling distributions of Dn, there appears to be a similar relationship between the variances of the sampling distributions.

Table 5.--Variance: Sampling Distribution of Dn when Sampling With and Without Replacement Uniform Population

Ratio	n, N	σ^2 of Dn With	σ^2 of Dn Without	Ratio of σ^2 (wo / w)
5%	6, 120	0.01029	0.00986	0.9572
10%	6, 60	0.01044	0.00954	0.9138
20%	6, 30	0.01054	0.00835	0.7922
30%	6, 20	0.01058	0.00741	0.7004
5%	12, 240	0.00547	0.00521	0.9525
10%	12, 120	0.00551	0.00502	0.9111
20%	12, 60	0.00540	0.00435	0.8056
30%	12, 40	0.00553	0.00381	0.6890
5%	24, 480	0.00277	0.00267	0.9639
10%	24, 240	0.00286	0.00254	0.8881
20%	24, 120	0.00277	0.00219	0.7906
30%	24, 80	0.00283	0.00197	0.6961
5%	48, 960	0.00139	0.00137	0.9856
10%	48, 480	0.00141	0.00127	0.9007
20%	48, 240	0.00141	0.00111	0.7872
30%	48, 160	0.00139	0.00100	0.7194

Sampling Distribution of Dn
Critical Values
When Sampling With and Without Replacement
vs. Tabulated Kolmogorov One-sample Values

Since the sampling distributions of Dn are used to determine critical values for specified alphas in the one-sample Kolmogorov test, the analysis now focuses on the differences in the critical values of Dn under these three circumstances: (1) when samples are taken with replacement from a finite discrete population; (2) when samples are taken without replacement from a finite discrete population; and (3) when samples are taken from an infinite continuous distribution assumed by the Kolmogorov one-sample tables. The alphas selected for comparison are commonly selected levels of significance in statistical tests: 10%, 5%, 2.5%, 1% and 0.5%. The analysis, therefore, shifts to a small but important portion of the cumulative distribution between the 90th and the 99.5th percentiles.

The sixteen combinations of n and n/N in this study produced sixteen sampling distributions of Dn in which the critical values for levels of significance are determined. The critical values are compared across sampling distributions to determine patterns. All the critical values associated when sampling without replacement are lower than the corresponding critical values when sampling with replacement. The smaller values for the mean and variance when sampling without replacement results in a cumulative sampling distribution that has shifted to the left, therefore the 90th and 95th percentiles are associated with smaller values of Dn.

The analysis also shows that, in all combinations, the critical values when sampling with and without replacement are smaller than the corresponding critical values from one-sample Kolmogorov tables. The table values of Dn give conservative critical values when samples are taken from discrete populations. The conservative values can be attributed to the approximation of the theoretical continuous distribution used by the standard Kolmogorov table by a discrete cumulative distribution. The stair-step configuration of a discrete cumulative distribution is attempting to approximate the smooth curve of a continuous cumulative distribution. For discrete ogives, maximum differences can only occur at stair-step points. For continuous ogives, maximum differences can occur at an infinite number of points. It is only possible for the maximum Dn of a discrete ogive and a continuous ogive to be the same if the maximum difference in the continuous ogive occurs at a stair-step point of the discrete ogive. Consequently, the discrete ogives usually result in smaller Dn values and the table critical values are conservative. A 5% critical table value might actually represent 4% alpha. As a result, if the test statistic, Dn, for a discrete population and sample distribution test is greater than the standard table value, then the researcher is safe in rejecting the null hypothesis because the actual probability of alpha is less than the stated table alpha. The test for population discrete data using tabulated values is said to be "conservative" and less powerful.

Table 6 provides three sets of critical values for five alphas given a sample and population size. The sets of critical values are the critical values from a one-sample Kolmogorov table and from the sampling distribution of Dn when sampling with and without replacement. In most comparisons, the critical values of the sampling distributions of Dn differ the most from the standard table values at either the 1% or 0.5% percentile. The minimum differences occurred at the 10% alpha. Because there is more area under the curve associated with the 90th percentile than is associated with the 99th percentile, bigger shifts in the critical values occur with the higher percentiles.

Table 6.--Comparison of Critical Values Sampling With and Without Replacement from Finite Discrete Uniform Population One-sample Kolmogorov Table Values

	Dn Standard Table	$Dn_{(1-\alpha)}$ With	$Dn_{(1-\alpha)}$ Without	Difference $Dn_{Table}-Dn_{Without}$
Alpha	*Ratio 5%, n=6, N=120*			
0.1	0.41037	0.40558	0.39749	0.01288
0.05	0.46799	0.45905	0.44409	0.02390
0.025	0.51926	0.50144	0.49336	0.02590
0.01	0.57741	0.56784	0.53169	0.04572
0.005	0.61661	0.59726	0.59048	0.02613
Alpha	*Ratio 10%, n=6, N=60*			
0.1	0.41037	0.40855	0.39236	0.01801
0.05	0.46799	0.46310	0.43281	0.03518
0.025	0.51926	0.50324	0.48680	0.03246
0.01	0.57741	0.56850	0.52545	0.05196
0.005	0.61661	0.59597	0.57917	0.03744
Alpha	*Ratio 20%, n=6, N=30*			
0.1	0.41037	0.41004	0.37076	0.03928
0.05	0.46799	0.46458	0.41544	0.05255
0.025	0.51926	0.50697	0.45809	0.06117
0.01	0.57741	0.56880	0.49683	0.08058
0.005	0.61661	0.59539	0.52276	0.12661
Alpha	*Ratio 30%, n=6, N=20*			
0.1	0.41037	0.40909	0.34687	0.06350
0.05	0.46799	0.46520	0.39270	0.07529
0.025	0.51926	0.50891	0.42364	0.09562
0.01	0.57741	0.57327	0.47525	0.10216
0.005	0.61661	0.59802	0.49647	0.12015

Table 6.--Continued.

	Dn Standard Table	$Dn_{(1-\alpha)}$ With	$Dn_{(1-\alpha)}$ Without	Difference Dn_{Table}-$Dn_{Without}$
Alpha	*Ratio 5%, n=12, N=240*			
0.1	0.29577	0.29335	0.28465	0.01112
0.05	0.33815	0.33005	0.32222	0.01593
0.025	0.37543	0.35861	0.34985	0.02559
0.01	0.41918	0.40780	0.39861	0.02057
0.005	0.44905	0.42987	0.42619	0.02286
Alpha	*Ratio 10%, n=12, N=120*			
0.1	0.29577	0.29390	0.27856	0.01722
0.05	0.33815	0.33094	0.31590	0.02225
0.025	0.37543	0.36023	0.34546	0.02997
0.01	0.41918	0.41559	0.38222	0.03696
0.005	0.44905	0.43488	0.42013	0.02892
Alpha	*Ratio 20%, n=12, N=60*			
0.1	0.29577	0.29254	0.26102	0.03475
0.05	0.33815	0.32962	0.29499	0.04316
0.025	0.37543	0.35819	0.32523	0.05020
0.01	0.41918	0.40758	0.35328	0.06590
0.005	0.44905	0.43021	0.38333	0.06572
Alpha	*Ratio 30%, n=12, N=40*			
0.1	0.29577	0.29347	0.24444	0.05133
0.05	0.33815	0.32895	0.27430	0.06385
0.025	0.37543	0.35850	0.30456	0.07087
0.01	0.41918	0.41022	0.33839	0.08079
0.005	0.44905	0.43212	0.35433	0.09472
Alpha	*Ratio 5%, n=24, N=480*			
0.1	0.21205	0.20833	0.20433	0.00772
0.05	0.24242	0.23464	0.23050	0.01193
0.025	0.26931	0.25990	0.25517	0.01414
0.01	0.30104	0.28605	0.28457	0.01647
0.005	0.32286	0.31167	0.29083	0.03203
Alpha	*Ratio 10%, n=24, N=240*			
0.1	0.21205	0.21097	0.19956	0.01249
0.05	0.24242	0.23691	0.22557	0.01685
0.025	0.26931	0.26329	0.24753	0.02178
0.01	0.30104	0.28937	0.27783	0.02321
0.005	0.32286	0.30611	0.29500	0.02786

Table 6.--Continued.

	Dn Standard Table	$Dn_{(1-\alpha)}$ With	$Dn_{(1-\alpha)}$ Without	Difference $Dn_{Table}-Dn_{Without}$
Alpha	*Ratio 20%, n=24, N=120*			
0.1	0.21205	0.20833	0.18587	0.02618
0.05	0.24242	0.23464	0.20899	0.03343
0.025	0.26931	0.25990	0.22925	0.04006
0.01	0.30104	0.28605	0.25833	0.04271
0.005	0.32286	0.31167	0.27284	0.05002
Alpha	*Ratio 30%, n=24, N=80*			
0.1	0.21205	0.20952	0.17527	0.03678
0.05	0.24242	0.23611	0.19815	0.04427
0.025	0.26931	0.26104	0.21787	0.05144
0.01	0.30104	0.28842	0.24250	0.05854
0.005	0.32286	0.31029	0.26081	0.06205
Alpha	*Ratio 5%, n=48, N=960*			
0.1	0.15139	0.14760	0.14651	0.00488
0.05	0.17302	0.16579	0.16396	0.00906
0.025	0.19221	0.18302	0.18169	0.01052
0.01	0.21493	0.20344	0.20197	0.01296
0.005	0.23059	0.21936	0.21667	0.01392
Alpha	*Ratio 10%, n=48, N=480*			
0.1	0.15139	0.14838	0.13985	0.01154
0.05	0.17302	0.16706	0.15813	0.01490
0.025	0.19221	0.18490	0.17474	0.01747
0.01	0.21493	0.20566	0.19631	0.01862
0.005	0.23059	0.22292	0.20795	0.02264
Alpha	*Ratio 20%, n=48, N=240*			
0.1	0.15139	0.14712	0.13254	0.01885
0.05	0.17302	0.16642	0.14899	0.02403
0.025	0.19221	0.18555	0.16427	0.02794
0.01	0.21493	0.20708	0.18186	0.03307
0.005	0.23059	0.21964	0.19142	0.03917
Alpha	*Ratio 30%, n=48, N=160*			
0.1	0.15139	0.14760	0.12358	0.02781
0.05	0.17302	0.16579	0.14026	0.03276
0.025	0.19221	0.18302	0.15684	0.03537
0.01	0.21493	0.20344	0.17222	0.04271
0.005	0.23059	0.21936	0.18472	0.04587

Previous research (Neother 1967) has determined that the standard table values are conservative when sampling with replacement from discrete populations. A research

Table 7.--One-sample Kolmogorov Tabulated Values and Critical Values When Sampling Without Replacement Uniform Population

	Alphas			
	0.10	0.05	0.025	0.01
Standard Table Values:	*0.410*	*0.468*	*0.519*	*0.577*
Ratio, n = 6				
5%	0.397	0.444	0.493	0.532
10%	0.392	0.433	0.487	0.525
20%	0.371	0.415	0.458	0.497
30%	0.347	0.393	0.424	0.475
Standard Table Values:	*0.296*	*0.338*	*0.375*	*0.419*
Ratio, n=12				
5%	0.285	0.322	0.350	0.398
10%	0.278	0.316	0.345	0.382
20%	0.261	0.295	0.325	0.353
30%	0.244	0.274	0.305	0.338
Standard Table Values:	*0.212*	*0.242*	*0.269*	*0.301*
Ratio, n=24				
5%	0.204	0.231	0.255	0.285
10%	0.200	0.226	0.247	0.278
20%	0.186	0.209	0.229	0.258
30%	0.175	0.198	0.218	0.243
Standard Table Values:	*0.151*	*0.173*	*0.192*	*0.215*
Ratio, n=48				
5%	0.147	0.164	0.182	0.202
10%	0.140	0.158	0.175	0.196
20%	0.133	0.149	0.164	0.182
30%	0.124	0.140	0.157	0.172

interest of this study is to explore the relationship between the table values of the one-sample Kolmogorov test and the critical values when sampling without replacement from a alpha of approximately discrete finite population. The differences between the tabulated table values and the critical values, when sampling without replacement, becomes the smallest when the sample size is large, or when the sample/population ratio is small. For example, the table critical value for an alpha of 10% and a sample of 48 is actually a critical value for an7.5% when the sample/population ratio is 10%, resulting in a 25% reduction in alpha. Conversely, when the sample size is small, or when the sample/population ratio is large, the differences are maximized. A table critical value for an alpha of 10% and a sample of 6 is actually a critical value for an alpha of approximately 3% when the sample/population ratio is 30%, resulting in a 70% reduction in alpha. One goal of this study is to produce an adjustment factor to be used with the standard Kolmogorov table values so that the desired alpha is maintained.

Table 7 provides a listing of the critical values from the sampling distribution when sampling without replacement, and the standard critical values of the one-sample Kolmogorov test. The relationship between these critical values is explored and the adjustment factor determined through regression analysis. The estimates of the adjustment to the standard critical value of Dn, given the sample/population ratio and the corresponding standard errors, are provided in Table 8.

The following two examples illustrate the application of the adjustment factor. First, if a sample of size 12 is taken from a uniform discrete population of size 48 (then n/N=.25), and an alpha of 10% is desired, then the adjustment to the table critical value of 0.296 is reduced by 0.043 (-0.173 times 0.25) to an adjusted value of 0.253 (0.296 minus 0.043). As a second illustration, if a sample of size 6 is taken from a uniform discrete population of size 24 (n/N=.25), and an alpha of 1% is desired, then the table critical value of 0.577 is reduced by 0.095 to an adjusted value of 0.482.

Table 8.--Adjustment Function for Tabled Kolmogorov Values Alphas Uniform Population

	Alphas			
	0.10	0.05	0.025	0.01
Standard Table Values:	*0.410*	*0 .468*	*0 .519*	*0 .577*
n = 6				
Estimated adjustment	-0.206r	-0.265r	-0.319r	-0.381r
Standard error	0.0025	0.0083	0.0059	0.0188
where r = sample/population ratio				
Standard Table Values:	*0.296*	*0 .338*	*0 .375*	*0 .419*
n =12				
Estimated adjustment	-0.173r	-0.216r	-0.250r	-0.296r
Standard error	0.0015	0.0030	0.0084	0.0181
where r = sample/population ratio				
Standard Table Values:	*0.212*	*0 .242*	*0 .269*	*0 .301*
n =24				
Estimated adjustment	-0.126r	-0.156r	-0.185r	-0.205r
Standard error	0.0012	0.0032	0.0045	0.0044
where r = sample/population ratio				
Standard Table Values:	*0.151*	*0 .173*	*0 .192*	*0 .215*
n =48				
Estimated adjustment	-0.095r	-0.116r	-0.130r	-0.154r
Standard error	0.0012	0.0029	0.0042	0.0043
where r = sample/population ratio				

In the second example, the alphas associated with the adjusted and unadjusted table value change from 1% (unadjusted = 0.577) to an alpha between 0.05 and 0.025 (adjusted = 0.482). These adjustments allow a practitioner to make a better estimate of the actual Dn associated with a desired alpha with more accuracy than provided in the current standard Kolmogorov tables. The equations provide a way of improving decision-making when using the one-sample Kolmogorov test.

This concludes the results of this study of the effects on the values of Dn when sampling from a uniform discrete population. The next phase considers an hypothesized population that is finite, discrete and similar in shape to a normal distribution.

CHAPTER V

PHASE TWO OF SIMULATION

Unimodal, Symmetrical Population

In the second phase of the simulation, the population from which samples are taken is assumed to be a unimodal, symmetrical, and discrete distribution with ten categories. The population shape is similar to the shape of a normal distribution. In analyzing the data, 7,500 samples are taken with replacement from this population. Each of the cumulative distributions is then compared to the cumulative population distribution and the statistic, Dn, is then calculated. These 7,500 Dn values represent the sampling distribution when sampling with replacement. In turn, another 7,500 samples are taken from the unimodal, symmetrical population; this time, however, without replacement. The cumulative distribution for each of these samples is computed, compared to the cumulative population distribution, and the statistic, Dn, is then determined. These 7,500 Dn values represent the sampling distribution when sampling without replacement. The resulting sampling distributions from the simulation represent an approximation to the theoretical distribution of Dn values which exist for the population and sample designs.

The comparison of the two simulated sampling distributions provides an estimate of the effect that sampling with or sampling without replacement has on Dn. The comparisons of the critical values from the simulated sampling distributions of Dn and the tabulated critical values of Dn found in standard one-sample Kolgmorov tables also provide an estimate of the differences in the distributions. The levels of significance selected for comparison purposes are: 10%, 5%, 2.5%, 1%, and 0.5%. Table 9 summarizes the factors involved.

Table 9.--Factors for Phase II of Simulation

Sampled Population	Unimodal Symmetrical discrete distributed over 10 categories
Sample Size:	6, 12, 24, 48
Ratio of n/N	5%, 10%, 20%, 30%
Sampling with Replacement:	Yes, No

The possible values for sample/population ratio and sample sizes, together with sampling with and without replacement, produce 32 treatments. For each treatment the sampling distribution of Dn is based on 7,500 simulated samples. The assumed populations of the three phases of this study have the same number of categories and means. In this phase the population assumed is unimodal, symmetric instead of uniform. The distinguishing factor between this phase and the previous phase is the shape of the assumed population.

Sampling Distribution of Dn
Comparison of the Mean and Variance
When Sampling With and Without Replacement

One of the interests of this study is to consider the effect of sampling with and without replacement on the sampling distribution of Dn. With 4 sample sizes and 4 ratios of sample/population size, there are sixteen combinations of n and N. For each combination of n and N, samples were taken with and without replacement to produce sampling distributions. This study shows that the mean Dn for the sampling distribution from sampling without replacement is smaller in all sixteen combinations. This indicates a shift to the left of the sampling distribution when sampling without replacement. A statistical test (a two-independent sample test) indicates all the differences are significant.

The magnitude of the difference across the ratio levels is maximized when the sample/population ratio is 30%, and steadily decreases as the ratio decreases. This pattern is present for all sample sizes in this study. Comparing the differences in means between different sample sizes, the largest differences in means are from small sample sizes, decreasing as sample size increases. This relationship is consistent for all sample/population ratios.

The mean of Dn when sampling with replacement is fairly constant for a given sample size regardless of the sample/population ratio. For a sample of size 6, the range for the mean is 0.24295 to 0.24562. For a sample of size 48, the range for the mean is 0.08618 to 0.08714. The mean of Dn when sampling without replacement, however, shifts to the left for a given sample size as the sample/population ratio increases. For a sample of size 6, the mean shifts from 0.23790 to 0.21165. For a sample of size 48, the means changes from 0.08441 to 0.07176. These statistics are presented in Table 10.

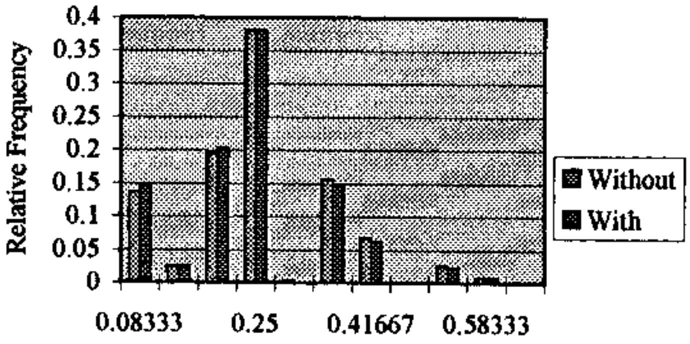
Table 10.--Mean: Sampling Distribution of Dn when Sampling With and Sampling Without Replacement Unimodal Symmetrical Population

Ratio	n, N	Mean of Dn With	Mean of Dn Without	Difference*
5%	6, 120	0.24295	0.23790	0.00505
10%	6, 60	0.24471	0.23294	0.01176
20%	6, 30	0.24644	0.22501	0.02143
30%	6, 20	0.24562	0.21165	0.03397
5%	12, 240	0.17116	0.16805	0.00311
10%	12, 120	0.16723	0.15938	0.00785
20%	12, 60	0.16749	0.14938	0.01811
30%	12, 40	0.17125	0.14438	0.02687
5%	24, 480	0.12077	0.11869	0.00208
10%	24, 240	0.12283	0.11587	0.00697
20%	24, 120	0.12116	0.10779	0.01337
30%	24, 80	0.12083	0.10141	0.01943
5%	48, 960	0.08681	0.08441	0.00240
10%	48, 480	0.08714	0.08263	0.00451
20%	48, 240	0.08714	0.07767	0.00947
30%	48, 160	0.08618	0 .07176	0.01442

*All Significant at 0.05

An interesting aspect in analyzing the means of Dn in this phase, in contrast to those of the previous phase, is that the differences between means are approximately the same magnitude for both phases, (i.e., differences in means when sampling with and without replacement are the same if an uniform or if unimodal, symmetrical population is assumed). Additionally, the relationships between the means with the sample size and sample/population ratio are the same as those found in the previous phase. This study shows that, regardless of the assumed population, sampling with replacement or sampling without replacement has a similar effect on the differences between the means of the sampling distribution. There appears to be a consistent effect on the values of Dn when sampling with and without replacement. Figure 4 illustrates the comparison of the sampling distributions of Dn when sampling with and without replacement from the unimodal symmetrical population for one combination.

Figure 4
Sampling Distribution of Dn
n=6, N=120
Unimodal symmetrical population



Another interesting aspect about the corresponding means of the two phases is the further shift to the left when the assumed population is changed from discrete uniform to unimodal, symmetrical discrete. Distributions based on small sample sizes shift more, while the shift decreases as sample size increases. For example, when the assumed population changes, the corresponding means of Dn shifted by approximately 0.04 for samples of size 6. When samples of size 48 are drawn , the corresponding means shift by approximately 0.01. The shift to the left is because of the differences in variances between the two assumed populations. The unimodal symmetrical population has a smaller variance and, therefore, a greater percentage of its values clustered close to the mean. There are fewer values in the tails of the distribution and, consequently, extreme values for Dn are not as likely.

In analyzing the variance of the Dn values when sampling with replacement, trends that were observed in the previous phase are present in this phase of the study. Both the mean and variance of the Dn values when sampling with replacement are fairly stable for a given sample size. The constant value for the mean and variance for Dn suggests a stable distribution given a sample size. This is true regardless of the sample/population ratio. The sample size, not sample/population ratio, is the dominant factor in the value of the variance for Dn. As the sample size increases, the variance of Dn decreases. This is similar to the relationship of n and the size of the standard error provided by the *Central Limit Theorem*.

In comparing the two variances for each of the sixteen combinations, the variance from sampling without replacement is always smaller than the variance from sampling with replacement. When sampling without replacement, the variance of the Dn values is not constant but decreases as the sample size increases, --- its value is influenced by the sample/population ratio. The value of the variance also decreases as the sample/population ratio increases for a given sample size.

In analyzing the ratio of the variance when sampling without replacement to the variance when sampling with replacement, two patterns appear. First, for a given sample size, the variance ratio decreases in value as the sample/population ratio increases.

Table 11.--Variance: Sampling Distribution of Dn when Sampling With and Without
Unimodal Symmetrical Population

Ratio	n, N	σ^2 of Dn With	σ^2 of Dn Without	Ratio of σ^2 (wo / w)
5%	6, 120	0.01105	0.01095	0.9908
10%	6, 60	0.01102	0.01000	0.9075
20%	6, 30	0.01030	0.00845	0.8201
30%	6, 20	0.01030	0.00749	0.7272
5%	12, 240	0.00564	0.00534	0.9478
10%	12, 120	0.00564	0.00509	0.9031
20%	12, 60	0.00574	0.00458	0.7986
30%	12, 40	0.00527	0.00380	0.7216
5%	24, 480	0.00267	0.00255	0.9554
10%	24, 240	0.00280	0.00246	0.8786
20%	24, 120	0.00280	0.00228	0.8140
30%	24, 80	0.00268	0.00193	0.7194
5%	48, 960	0.00133	0.00129	0.9707
10%	48, 480	0.00140	0.00120	0.8551
20%	48, 240	0.00135	0.00109	0.8110
30%	48, 160	0.00140	0.00100	0.6824

Secondly, the ratio seems to approximate the finite correction factor (1 - n/N). Table 11 presents these statistics.

One of the goals of this study is to explore the relationship between the variability of the two sampling distributions. The finite correction factor is the difference between the variances of Dn, as was the case when a uniform population was assumed in the first phase. The *Central Limit Theorem* applies to the summing of sample results, specifically sample means and sample sums. The cumulative distribution is also the result of summing relative frequencies from a sample.

Sampling Distribution of Dn Percentiles and Critical Values When Sampling With and Without Replacement vs. Tabulated Kolmogorov One-sample Values

One purpose of this research is to investigate the differences between the simulated sampling distributions and the critical values of Dn found in standard one-sample Kolmogorov tables. The standard table values that are based on an hypothesized continuous distribution are known to be conservative when a discrete distribution is hypothesized. The alphas selected for comparison are the same used in the previous phase: 10%, 5%, 2.5%, 1% and 0.5%. Now the focus on the sampling distributions is centered on the upper percentiles of the cumulative distributions. Table 12 provide specifics about the critical values.

Table 12.--Comparison of Critical Values Sampling With and Without Replacement from Finite Discrete Unimodal Symmetric Population with One-sample Kolmogorov Table Values

	Dn Standard Table	$Dn_{(1-\alpha)}$ With	$Dn_{(1-\alpha)}$ Without	Difference $Dn_{Table}-Dn_{Without}$
Alpha	*Ratio 5%, n=6, N=120*			
0.1	0.41037	0.33805	0.32690	0.08348
0.05	0.46799	0.39909	0.39396	0.07403
0.025	0.51926	0.48968	0.44452	0.07474
0.01	0.57741	0.49901	0.49632	0.08109
0.005	0.61661	0.53664	0.53965	0.07696
Alpha	*Ratio 10%, n=6, N=60*			
0.1	0.41037	0.34281	0.32239	0.08798
0.05	0.46799	0.40409	0.38163	0.08636
0.025	0.51926	0.45705	0.42568	0.09358
0.01	0.57741	0.49817	0.48051	0.09690
0.005	0.61661	0.54167	0.49878	0.11783
Alpha	*Ratio 20%, n=6, N=30*			
0.1	0.41037	0.34876	0.31692	0.09345
0.05	0.46799	0.38921	0.36039	0.10760
0.025	0.51926	0.48259	0.39400	0.12526
0.01	0.57741	0.51439	0.46320	0.11421
0.005	0.61661	0.54280	0.49320	0.12341
Alpha	*Ratio 30%, n=6, N=20*			
0.1	0.41037	0.34914	0.28881	0.12156
0.05	0.46799	0.41267	0.32449	0.14350
0.025	0.51926	0.47129	0.37372	0.14554
0.01	0.57741	0.54445	0.42949	0.14792
0.005	0.61661	0.57567	0.46955	0.14706
Alpha	*Ratio 5%, n=12, N=240*			
0.1	0.29577	0.25117	0.25028	0.04549
0.05	0.33815	0.32780	0.32653	0.01162
0.025	0.37543	0.33151	0.33115	0.04428
0.01	0.41918	0.40938	0.33993	0.07925
0.005	0.44905	0.41418	0.41305	0.03600
Alpha	*Ratio 10%, n=12, N=120*			
0.1	0.29577	0.23235	0.22205	0.07372
0.05	0.33815	0.26366	0.24604	0.09212
0.025	0.37543	0.30706	0.29022	0.08521
0.01	0.41918	0.33310	0.32628	0.09290
0.005	0.44905	0.38086	0.36414	0.08764

Table 12.--Continued.

	Dn Standard Table	$Dn_{(1-\alpha)}$ With	$Dn_{(1-\alpha)}$ Without	Difference Dn_{Table}-$Dn_{Without}$
Alpha	*Ratio 20%, n=12, N=60*			
0.1	0.29577	0.23352	0.20936	0.08641
0.05	0.33815	0.26915	0.23884	0.09931
0.025	0.37543	0.30994	0.26806	0.10738
0.01	0.41918	0.33920	0.31270	0.10648
0.005	0.44905	0.38322	0.32758	0.12147
Alpha	*Ratio 30%, n=12, N=40*			
0.1	0.29577	0.24640	0.23849	0.05728
0.05	0.33815	0.31836	0.24569	0.09247
0.025	0.37543	0.32828	0.24928	0.12615
0.01	0.41918	0.40232	0.29596	0.12322
0.005	0.44905	0.41023	0.31893	0.13012
Alpha	*Ratio 5%, n=24, N=480*			
0.1	0.21205	0.17929	0.17050	0.04155
0.05	0.24242	0.20802	0.20732	0.03511
0.025	0.26931	0.24625	0.24515	0.02416
0.01	0.30104	0.28594	0.24938	0.05167
0.005	0.32286	0.38965	0.28799	0.03487
Alpha	*Ratio 10%, n=24, N=240*			
0.1	0.21205	0.20245	0.16959	0.03286
0.05	0.24242	0.20966	0.20633	0.03609
0.025	0.26931	0.24702	0.24183	0.02749
0.01	0.30104	0.28607	0.24925	0.05179
0.005	0.32286	0.29026	0.28629	0.03657
Alpha	*Ratio 20%, n=24, N=1200*			
0.1	0.21205	0.17294	0.15496	0.05709
0.05	0.24242	0.20161	0.17939	0.06303
0.025	0.26931	0.22836	0.20127	0.06804
0.01	0.30104	0.25306	0.23114	0.06990
0.005	0.32286	0.27604	0.24760	0.07526
Alpha	*Ratio 30%, n=24, N=80*			
0.1	0.21205	0.17004	0.14453	0.06752
0.05	0.24242	0.19882	0.16302	0.07940
0.025	0.26931	0.22287	0.20611	0.06320
0.01	0.30104	0.24863	0.20809	0.09295
0.005	0.32286	0.28974	0.22716	0.09570

Table 12.--Continued.

	Dn Standard Table	$Dn_{(1-\alpha)}$ With	$Dn_{(1-\alpha)}$ Without	Difference $Dn_{Table}-Dn_{Without}$
Alpha	*Ratio 5%, n=48, N=960*			
0.1	0.15139	0.13999	0.12774	0.02365
0.05	0.17302	0.14955	0.14563	0.02739
0.025	0.19221	0.16634	0.16556	0.02665
0.01	0.21493	0.18715	0.18648	0.02845
0.005	0.23059	0.20649	0.20566	0.02494
Alpha	*Ratio 10%, n=48, N=480*			
0.1	0.15139	0.14042	0.12473	0.02666
0.05	0.17302	0.16086	0.14440	0.02862
0.025	0.19221	0.16936	0.16402	0.02819
0.01	0.21493	0.19167	0.18462	0.03032
0.005	0.23059	0.20752	0.18854	0.04205
Alpha	*Ratio 20%, n=48, N=240*			
0.1	.15139	0.13889	0.12174	0.02965
0.05	.17302	0.14956	0.13996	0.03306
0.025	.19221	0.16781	0.14842	0.04379
0.01	.21493	0.18982	0.16693	0.04800
0.005	.23059	0.20632	0.18562	0.04498
Alpha	*Ratio 30%, n=48, N=160*			
0.1	.15139	0.14189	0.10409	0.04730
0.05	.17302	0.16277	0.12421	0.04882
0.025	.19221	0.16644	0.13627	0.05594
0.01	.21493	0.18726	0.15268	0.06225
0.005	.23059	0.20146	0.16384	0.06675

All the critical values associated with sampling without replacement are lower than the corresponding critical values associated without sampling with replacement. As shown in first phase, the smaller values for the mean and variance of Dn when sampling without replacement results in a cumulative sampling distribution that has shifted to the left. Therefore, the 90th and 95th percentiles are associated with smaller values of Dn.

The analysis also shows that, in all combinations, the critical values when sampling with and without replacement are smaller than the corresponding standard values from one-sample Kolmogorov tables. The tabulated critical values of Dn are conservative.

The differences between the standard table values and the critical values, when sampling without replacement, are minimized when the sample size is large, or when the sample/population ratio is small. For samples of size 48, differences range 0.02 to 0.07 for n/N ratios ranging from 5% to 30% respectively; for samples of size 6, differences range from 0.08 to 0.15 for n/N ratios ranging from 5% to 30% respectively. The relationships between the critical values and differences are consistent with those found in the previous phase with one exception: the magnitude of the differences is significantly larger, especially when sample sizes are small. The increased shift of the means to the left, resulting from the change of the assumed population, increases the differences between the table and actual critical values. Table 13 provides a listing of the standard table values and critical values when sampling without replacement.

The relationship between these critical values and the adjustment factor is estimated through regression analysis. The estimates of the adjustment to the table value of Dn given the sample/population ratio and the corresponding standard errors are provided in Table 14.

Table 13.--One-sample Kolmogorov Tabulated Values and Critical Values When Sampling Without Replacement Unimodal Symmetrical Population

	Alphas			
	0.10	0.05	0.025	0.01
Standard Table Values:	*0.410*	*0.468*	*0.519*	*0.577*
Ratio, n = 6				
5%	0.327	0.394	0.445	0.496
10%	0.322	0.382	0.426	0.481
20%	0.317	0.360	0.394	0.463
30%	0.289	0.324	0.374	0.429
Standard Table Values:	*0.296*	*0.338*	*0.375*	*0.419*
Ratio, n=12				
5%	0.250	0.327	0.331	0.340
10%	0.222	0.246	0.290	0.326
20%	0.209	0.239	0.268	0.313
30%	0.238	0.246	0.249	0.296
Standard Table Values:	*0.212*	*0.242*	*0.269*	*0.301*
Ratio, n=24				
5%	0.171	0.207	0.245	0.249
10%	0.170	0.206	0.241	0.249
20%	0.155	0.179	0.201	0.231
30%	0.144	0.163	0.206	0.208
Standard Table Values:	*0.151*	*0.173*	*0.192*	*0.215*
Ratio, n=48				
5%	0.128	0.146	0.166	0.186
10%	0.125	0.144	0.164	0.185
20%	0.122	0.140	0.148	0.167
30%	0.104	0.124	0.136	0.153

Table 14.--Adjustment Function for Tabled Kolmogorov Values Unimodal Symmetrical Population

	Alphas			
	0.10	0.05	0.025	0.01
Standard Table Values:	*0 .410*	*0 .468*	*0 .519*	*0 .577*
n = 6				
Estimated adjustment	-0.06-0.204r	-0.06-0.266r	-0.06-0.300r	-0.06-0.294r
Standard error	0.0098	0.0039	0.0045	0.0063
where r = sample/population ratio				
Standard Table Values:	*0 .296*	*0 .338*	*0 .375*	*0 .419*
n =12				
Estimated adjustment	-0.04-0.190r	-0.04-0.220r	-0.04-0.309r	-0.04-.319r
Standard error	0 .0115	0.0309	0.0116	0.0196
where r = sample/population ratio				
Standard Table Values:	*0.212*	*0 .242*	*0 .269*	*0 .301*
n =24				
Estimated adjustment	-0.02-0.175r	-0.02-0.202r	-0.02-0.165r	-0.02-0.257r
Standard error	0.0085	0.0041	0.0110	0 .0117
where r = sample/population ratio				
Standard Table Values:	*0 .151*	*0 .173*	*0 .192*	*0 .215*
n =48				
Estimated adjustment	-0.02-0.077r	-0.02-0.088r	-0.02-0.117r	-0.02-0.138r
Standard error	0 .0042	0.0034	0.0021	0.0022
where r = sample/population ratio				

As in the previous phase, two examples are used to illustrate the application of the adjustment factor. First, if a sample of size 12 is taken from the hypothesized unimodal, symmetric discrete population of size 48 (then n/N=0.25), and an alpha of 10% is desired, then the adjustment to the tabulated critical value of 0.296 is reduced it by 0.0875 (0.190 times 0.25 plus 0.04) to an adjusted value of 0.209 (0.296 minus 0.0875). In the second illustration, if a sample of size 6 is taken from an hypothesized discrete population of size

24 (n/N=0.25), and an alpha of 1% is desired, then the table critical value of 0.577 is reduced by 0.1306 to an adjusted value of 0.446. The alphas associated with the adjusted and unadjusted tabled value of this second example change from 1% (unadjusted = 0.577) to an alpha between 0.10 and 0.05 (adjusted = 0.446). These adjustments are approximations, but allow a practitioner to make a better estimate of the actual Dn associated with a desired alpha with more accuracy than provided in the current standard Kolmogorov tables.

Two points can be made in comparing the adjustment factor of the two phases,. First, the adjustment factor for this phase included a fixed and a variable term. The addition of the fixed term is attributed to the change in the assumed population. The shift of the sampling distributions of Dn to the left are a result of the change in assumed populations. The fixed portion decreases as the sample size increases. Second, the variable terms of corresponding adjustment factor of the two phases are most alike when a large sample is selected. The equations provide a way of improving decision-making when using the one-sample Kolmogorov test.

This concludes the results of this study of the effects on the values of Dn when sampling from a unimodal, symmetric discrete population. The next phase considers an hypothesized population that is a finite, discrete, and similar in shape to a Poisson distribution.

CHAPTER VI

PHASE THREE OF SIMULATION

Skewed Positive Discrete Population

In the final phase of the simulation, the population from which samples are taken is assumed to be a positive skewed discrete distribution with ten categories. The population shape is similar to the shape of a Poisson distribution. From this assumed distribution, 7,500 samples are taken with replacement. Each of the cumulative distributions is compared to the cumulative population distribution and the statistic, Dn, is then calculated. These 7,500 Dn values represent the sampling distribution of Dn when sampling with replacement. In turn, another 7,500 samples are taken from the skewed distribution; this time, however, without replacement. The cumulative distribution for each of these samples is computed, compared to the cumulative population distribution, and the statistic, Dn, is then calculated. These 7,500 Dn values represent the sampling distribution of Dn when sampling without replacement. The resultant sampling distributions from the simulation are an approximation of the theoretical distribution of Dn values which exist given the population and sample designs.

Comparisons made in previous phases are made again using the sampling distributions of this phase. The comparison of the two simulated sampling distributions provides an estimate of the effect that sampling with, or sampling without, replacement has on Dn. Comparisons are also made between percentiles of the simulated sampling distributions and the critical values of Dn found in standard one-sample Kolmogorov tables. The factors are similar to those used in the previous phases. Table 15 below summarizes the factors involved.

Table 15.--Factors for Phase III of Simulation

Sampled Population:	Skewed positive discrete distributed over 10 categories
Sample Size:	6, 12, 24, 48
Ratio of n/N:	5%, 10%, 20%, 30%
Sampling with Replacement:	Yes, No

There are 32 treatments. For each treatment, the sampling distribution of Dn is approximated based on 7,500 simulated samples. In each phase of this study, the assumed populations all have the same number of categories and means. They do have different variances and shapes. In this phase, the assumed population is skewed positive.

Sampling Distribution of Dn
Comparison of the Mean and Variance
When Sampling With and Without Replacement

One of the purposes of this study is to assess the effect that sampling with and without replacement has on the sampling distribution of Dn. As in the previous phases, sampling distributions of the sixteen combinations of n and N are simulated.

Relationships between the means and variances of the Dn values, when sampling with and without replacement, are examined based on these sampling distributions. In comparing the corresponding mean values of Dn, the following relationships are evident: 1) the mean of the Dn values shifts to the left when sampling without replacement; 2) the differences in the means of Dn when sampling with and without replacement are statistically significant; 3) when the sample size is fixed and sample/population ratio increases, the differences in means increases; and 4) when the sample/population ratio is fixed and sample size increases, the differences in means decrease. These relationships are more or less evident in all three phases.

The mean of the Dn values is stable for a given sample size when sampling with replacement, whereas the mean value of Dn varies depending on both sample size and sample/population ratio when sampling without replacement. The differences between the means of the sampling distribution of Dn when sampling with and without replacement are fairly constant regardless of the assumed population. These relationships have been true in each phase of this study. These statistics on the mean values of Dn are listed in Table 16.

A comparison of corresponding mean Dn values when 30% of the population is sampled highlights the movement of the sampling distribution. The means of the Dn values when the assumed population changes from discrete uniform population, symmetric population, and positively skewed population, respectively, show a shift to the left when samples are selected without replacement. The movement in the values of the means of Dn for a given sample size is brought about by the change in assumed population. The movement becomes smaller as the sample size increases. Maximum differences between the values of the means of Dn occur when the sample size is small.

Table 16.--Mean: Sampling Distribution of Dn when Sampling With and Without Replacement Skewed Positive Population

Ratio n, N	Mean of Dn With	Mean of Dn Without	Difference*
5% 6, 120	0.25336	0.25574	0.00263
10% 6, 60	0.25805	0.24801	0.01004
20% 6, 30	0.25430	0.22988	0.02452
30% 6, 20	0.25556	0.22001	0.03555
5% 12, 240	0.18494	0.17882	0.00613
10% 12, 120	0.18510	0.17573	0.00937
20% 12, 60	0.18324	0.16658	0.01667
30% 12, 40	0.18326	0.15507	0.02819
5% 24, 480	0.12898	0.12695	0.00203
10% 24, 240	0.13019	0.12455	0.00564
20% 24, 120	0.13097	0.11618	0.01479
30% 24, 80	0.13099	0.10955	0.02145
5% 48, 960	0.09211	0.09069	0.00142
10% 48, 480	0.09257	0.08706	0.00551
20% 48, 240	0.09246	0.08258	0.00988
30% 48, 160	0.09221	0.07739	0.01484

*All Significant at 0.05

In comparing the variances of the Dn values, relationships that appeared in other phases are also evident in this phase. They are: 1) when the sample size is fixed, the variance of the Dn values is stable when sampling with replacement; 2) the variance of the

Dn values when sampling without replacement depends upon sample size and sample/population ratio; 3) the variance of the Dn values when sampling without replacement is smaller than when sampling with replacement; and 4) the finite correction factor, (1 - n/N), approximates the relationship between the variances of Dn when sampling with and without replacement. The consistency of this result in all three phases provides empirical evidence regarding the first research hypothesis. Table 17 provides specific values for the variances of Dn.

Table 17.--Variance: Sampling Distribution of Dn when Sampling With and Without Replacement Skewed Positive Population

Ratio	n, N	σ^2 of Dn With	σ^2 of Dn Without	Ratio of σ^2 (wo / w)
5%	6, 120	0.01046	0.00984	0.9406
10%	6, 60	0.01106	0.00964	0.8720
20%	6, 30	0.01078	0.00822	0.7828
30%	6, 20	0.01042	0.00687	0.6594
5%	12, 240	0.00532	0.00510	0.9580
10%	12, 120	0.00528	0.00493	0.9350
20%	12, 60	0.00538	0.00450	0.8342
30%	12, 40	0.00528	0.00362	0.6858
5%	24, 480	0.00263	0.00260	0.9883
10%	24, 240	0.00273	0.00247	0.9059
20%	24, 120	0.00264	0.00206	0.7882
30%	24, 80	0.00274	0.00019	0.6766
5%	48, 960	0.00135	0.00130	0.9676
10%	48, 480	0.00139	0.00126	0.9045
20%	48, 240	0.00140	0.00111	0.7901
30%	48, 160	0.00134	0.00097	0.7234

Sampling Distribution of Dn
Percentiles and Critical Values
When Sampling With and Without Replacement
Tabulated Kolmogorov One-sample Values

The second research question of this study is to investigate the differences between the critical values of the simulated sampling distributions of Dn and the critical values of Dn found in standard one-sample Kolmogorov tables. The alphas selected for comparison are 10%, 5%, 2.5%, 1% and 0.5%. The analysis of the sampling distribution now shifts to the upper percentiles of the cumulative distributions instead of the mean or variance.

The sampling distributions of Dn for the sixteen combinations of n and N provide the critical values for comparison purposes. Table 18 provides a listing of the critical values.

Table 18.--Comparison of Critical Values Sampling With and Without Replacement from Finite Discrete Population Skewed Positive One-sample Kolmogorov Table Values

	Dn Standard Table	$Dn_{(1-\alpha)}$ With	$Dn_{(1-\alpha)}$ Without	Difference $Dn_{Table}-Dn_{Without}$
Alpha	*Ratio 5%, n=6, N=120*			
0.1	0.41037	0.36154	0.36000	0.05037
0.05	0.46799	0.47318	0.47030	-0.00231
0.025	0.51926	0.49877	0.49749	0.02177
0.01	0.57741	0.42701	0.52614	0.05127
0.005	0.61661	0.53240	0.53087	0.08574

Table 18.--Continued.

	Dn Standard Table	$Dn_{(1-\alpha)}$ With	$Dn_{(1-\alpha)}$ Without	Difference $Dn_{Table}-Dn_{Without}$
Alpha	*Ratio 10%, n=6, N=60*			
0.1	0.41037	0.34659	0.34177	0.06860
0.05	0.46799	0.48294	0.45705	0.01094
0.025	0.51926	0.50130	0.49486	0.02440
0.01	0.57741	0.51302	0.50920	0.06821
0.005	0.61661	0.58572	0.51420	0.10241
Alpha	*Ratio 20%, n=6, N=30*			
0.1	0.41037	0.33504	0.32852	0.08185
0.05	0.46799	0.44139	0.35357	0.11442
0.025	0.51926	0.48597	0.36610	0.15316
0.01	0.57741	0.51633	0.48000	0.09741
0.005	0.61661	0.52645	0.50778	0.10883
Alpha	*Ratio 30%, n=6, N=20*			
0.1	0.41037	0.35958	0.33305	0.07732
0.05	0.46799	0.38925	0.35768	0.11031
0.025	0.51926	0.50858	0.36383	0.15543
0.01	0.57741	0.52587	0.41416	0.16326
0.005	0.61661	0.53025	0 .48265	0.13396
Alpha	*Ratio 5%, n=12, N=240*			
0.1	0.29577	0.28034	0.26699	0.02878
0.05	0.33815	0.32942	0.30778	0.03037
0.025	0.37543	0.34067	0.33301	0.04242
0.01	0.41918	0.38477	0.36979	0.05121
0.005	0.44905	0.41499	0.41487	0.03418
Alpha	*Ratio 10%, n=12, N=120*			
0.1	0.29577	0.27725	0.25737	0.03840
0.05	0.33815	0.32530	0.30584	0.03231
0.025	0.37543	0.34208	0.33226	0.04317
0.01	0.41918	0.38833	0.36631	0.05287
0.005	0.44905	0.41404	0.40991	0.03914
Alpha	*Ratio 20%, n=12, N=60*			
0.1	0.29577	0.26119	0.24751	0.04826
0.05	0.33815	0.32007	0.26502	0.07314
0.025	0.37543	0.33877	0.32287	0.05256
0.01	0.41918	0.38974	0.34370	0.07548
0.005	0.44905	0.41875	0.36912	0.07993

Table 18.--Continued.

	Dn Standard Table	$Dn_{(1-\alpha)}$ With	$Dn_{(1-\alpha)}$ Without	Difference Dn_{Table}-$Dn_{Without}$
Alpha	*Ratio 30%, n=12, N=40*			
0.1	0.29577	0.27578	0.22726	0.06851
0.05	0.33815	0.30912	0.26284	0.07531
0.025	0.37543	0.35095	0.29052	0.08491
0.01	0.41918	0.38132	0.32308	0.09610
0.005	0.44905	0.42391	0.34683	0.10222
Alpha	*Ratio 5%, n=24, N=480*			
0.1	0.21205	0.19781	0.19101	0.02104
0.05	0.24242	0.24941	0.22060	0.02182
0.025	0.26931	0.24755	0.24721	0.02210
0.01	0.30104	0.28102	0.27560	0.02545
0.005	0.32286	0.29193	0.29340	0.02946
Alpha	*Ratio 10%, n=24, N=240*			
0.1	0.21205	0.20185	0.18761	0.02444
0.05	0.24242	0.22423	0.21583	0.02659
0.025	0.26931	0.24860	0 .24053	0.02878
0.01	0.30104	0.27794	0.26667	0.03437
0.005	0.32286	0.30298	0.28862	0.03424
Alpha	*Ratio 20%, n=24, N=120*			
0.1	0.21205	0.19877	0.17173	0.04032
0.05	0.24242	0.22461	0.19286	0.04956
0.025	0.26931	0.24566	0.21963	0.04968
0.01	0.30104	0.27500	0.24451	0.05653
0.005	0.32286	0.30096	0.26162	0.06124
Alpha	*Ratio 30%, n=24, N=80*			
0.1	0.21205	0.19805	0.16319	0.04886
0.05	0.24242	0.22591	0.19063	0.05179
0.025	0.26931	0.24844	0.20551	0.06380
0.01	0.30104	0.28171	0.23266	0.06838
0.005	0.32286	0.30271	0.24110	0.08176
Alpha	*Ratio 5%, n=48, N=960*			
0.1	0.15139	0.13861	0.13663	0.01476
0.05	0.17302	0.15750	0.15608	0.01694
0.025	0.19221	0.17643	0.17517	0.01704
0.01	0.21493	0.19757	0.19552	0.01941
0.005	0.23059	0.21521	0.20498	0.02561

Table 18.--Continued.

	Dn Standard Table	$Dn_{(1-\alpha)}$ With	$Dn_{(1-\alpha)}$ Without	Difference $Dn_{Table}-Dn_{Without}$
Alpha	*Ratio 10%, n=48, N=480*			
0.1	0.15139	0.14459	0.13271	0.01686
0.05	0.17302	0.16255	0.15251	0.02051
0.025	0.19221	0.17858	0.16970	0.02251
0.01	0.21493	0.19821	0.19006	0.02487
0.005	0.23059	0.21052	0.20563	0.04497
Alpha	*Ratio 20%, n=48, N=240*			
0.1	0.15139	0.14240	0.12429	0.02711
0.05	0.17302	0.16254	0.14355	0.02947
0.025	0.19221	0.18021	0.16082	0.03139
0.01	0.21493	0.20313	0.17865	0.03628
0.005	0.23059	0.22161	0.18912	0.04147
Alpha	*Ratio 30%, n=48, N=160*			
0.1	0.15139	0.14084	0.11853	0.03286
0.05	0.17302	0.16048	0.13269	0.04033
0.025	0.19221	0.17823	0.14883	0.04338
0.01	0.21493	0.19840	0.16389	0.05104
0.005	0.23059	0.20841	0.18042	0.05017

All of the critical values associated with sampling without replacement are lower than the corresponding critical values when sampling with replacement. As shown in the first phase, the smaller values for the mean and variance of Dn when sampling without replacement results in a cumulative sampling distribution that has shifted to the left. Therefore, the 90th and 95th percentiles are associated with smaller values of Dn. The analysis also shows that, in most combinations, the critical values when sampling with and without replacement are smaller than the corresponding tabulated values from one-sample Kolmogorov tables.

Table 19.--One-sample Kolmogorov Tabulated Values and Critical Values When Sampling Without Replacement Skewed Positive Population

	Alphas			
	0.10	0.05	0.025	0.01
Standard Table Values:	*0.410*	*0.468*	*0.519*	*0.577*
Ratio, n = 6				
5%	0.360	0.470	0.497	0.526
10%	0.342	0.457	0.495	0.509
20%	0.329	0.354	0.366	0.480
30%	0.333	0.357	0.364	0.414
Standard Table Values:	*0.296*	*0.338*	*0.375*	*0.419*
Ratio, n=12				
5%	0.267	0.308	0.333	0.370
10%	0.257	0.306	0.332	0.366
20%	0.248	0.265	0.323	0.344
30%	0.227	0.263	0.291	0.323
Standard Table Values:	*0.212*	*0.242*	*0.269*	*0.301*
Ratio, n=24				
5%	0.191	0.221	0.247	0.276
10%	0.188	0.216	0.241	0.267
20%	0.172	0.193	0.220	0.245
30%	0.163	0.191	0.206	0.233
Standard Table Values:	*0.151*	*0.173*	*0.192*	*0.215*
Ratio, n=48				
5%	0.137	0.156	0.175	0.196
10%	0.133	0.153	0.170	0.190
20%	0.124	0.144	0.161	0.179
30%	0.119	0.133	0.149	0.164

The differences between the table values and the critical values when sampling without replacement are minimized in two situations: 1) when the sample size is large; and 2) given a sample size that is fixed, when the sample/population ratio is small.

The relationships between the critical values and differences are consistent with those found in the previous phase. The differences are larger than corresponding differences found in the first phases because of the shift of the sampling distribution resulting from the change in assumed population.

The increased shift of the means to the left resulting from the change of the assumed population increases the differences between the table and actual critical values. An interest of this research is to study the relationship of the differences to develop an adjustment factor for the one-sample Kolmogorov table. Table 18 provides a listing of the table values and critical values when sampling without replacement.

The relationship between these critical values and the adjustment factor is estimated through regression analysis. The adjustment factor corrects the standard critical value to a value closer to the critical value that more accurately reflects the alpha desired in testing an hypothesized population. The estimates of the adjustment to the table value of Dn, given the sample/population ratio and the corresponding standard errors, are provided in Table 20.

As in the previous phases, two examples are used to illustrate the application of the adjustment factor. First, if a sample of size 12 is taken from the hypothesized unimodal, symmetric discrete population of size 48 (then n/N=0.25), and an alpha of 10% is desired, then the adjustment to the standard critical value of 0.296 is reduced it by 0.060 (.158 times .25 plus .02) to an adjusted value of .236 (.296 minus .060). As a second illustration, if a sample of size 6 is taken from an hypothesized discrete population of size 24 (n/N=0.25), and an alpha of 1% is desired, then the table critical value of

0.577 is reduced by .131 to an adjusted value of .446. In the second example, the alphas associated with the adjusted and unadjusted table value change from 1%

Table 20.--Adjustment Function for Tabled Kolmogorov Values Skewed Positive Population

	Alphas			
	0.10	0.05	0.025	0.01
Standard Table Values:	*0.410*	*0 .468*	*0.519*	*0.577*
n = 6				
Estimated adjustment	-0.04-0.173r	-0.04-0.217r	-.04-0.384r	-0.04-0.364r
Standard error	0.0123	0.0461	0.0433	0.0135
where r = sample/population ratio				
Standard Table Values:	*0 .296*	*0 .338*	*0 .375*	*0 .419*
n =12				
Estimated adjustment	-0.02-0.158r	-0.02-0.203r	-0.02-0.206r	-.02-.271r
Standard error	0.0025	0.0092	0.0088	0.0102
where r = sample/population ratio				
Standard Table Values:	*0 .212*	*0 .242*	*0.269*	*0.301*
n =24				
Estimated adjustment	-0.01-0.139r	-0.01-0.159r	-0.01-0.186r	-0.01-0.211r
Standard error	0.0032	0.0061	0.0024	0.0051
where r = sample/population ratio				
Standard Table Values:	*0 .151*	*0 .173*	*0 .192*	*0 .215*
n =48				
Estimated adjustment	-0.01-0.080r	-.01-0.101r	-.01-0.112r	-.01-0.137r
Standard error	0.0011	0 .0012	0.0013	0.0017
where r = sample/population ratio				

(unadjusted =0.577) to an alpha between 0.10 and 0.05 (adjusted = 0.446). These adjustments are estimations, but allow a practitioner to make a better estimate of the actual Dn associated with a desired alpha, and with more accuracy than provided in the current standard Kolmogorov tables.

The adjustment factor for this phase has a fixed component and a variable component. The fixed component is smaller than that of the previous phase as a result of the distance between the means of respective sampling distributions of Dn are not as great as in the previous phase. The fixed portion decreases as the sample size increases. The variable term of the adjustment factor is fairly constant for all three phases when the sample size is large. The standard error is the smallest for the large sample.

The population shape affects the sampling distribution of Dn especially when small samples are selected. The standard error is a measure of the usefulness of the adjustment factor. When the error is small, the adjustment factor provides a way of improving decision- making when using the one-sample Kolmogorov test. In some of the cases, the error is large and the resultant correction is less efficient. Populations with a larger number of extreme values appear to have an effect of increasing the mean value of the Dn for the sampling distribution. The uniform discrete population had more values in the tails of the distribution than the unimodal symmetrical or the skewed positive discrete populations. The means of the corresponding Dn values for samples taken from an assumed uniform population consistently were higher. The effect of the population shape is a factor not formally addressed in this study, but in the future this factor could be investigated.

This concludes the results of this study of the effects on the values of Dn when sampling from a positively skewed discrete population. The following chapter summarizes the results of the simulation and conclusions based on the analysis.

CHAPTER VII

CONCLUSIONS

Research Questions Answered

This study has addressed two research questions regarding the effects of sampling with and without replacement from a finite population on the sampling distribution of the test statistic, Dn, the one-sample Kolmogorov test statistic. Through the simulation of this study it was found that the effects can be dramatic. The study shows that these research interests raise relevant questions, particularly when small samples are selected from a finite population.

The first research question focuses on the relationships between the sampling distributions of Dn when sampling with and without replacement.

Research Question 1

> What is the general relationship between the sampling distributions of Dn? Specifically, this study will compare distributions derived when sampling with replacement from a finite population to another distribution derived when sampling without replacement. Is there a correction factor similar to the finite correction factor used between the hypergeometric and binomial distributions?

The study shows that the finite correction factor can be used when the sampled finite population is uniformly distributed. The finite correction factor serves as the connection between the values of the two variances when the population is uniformly distributed. As the sampled population deviates from the uniform, the finite correction factor fits, although, not with the same degree of accuracy as with the uniform. When the sampled population changes from the uniform, this study shows the ratio of variances are to within 3% accuracy of the percentage that the finite correction factor would give as the ratio.

The mean for the sampling distribution of Dn values when sampling without replacement is smaller than the mean of the Dn values from sampling with replacement. This was true in every phase, regardless of assumed population, sample/population ratio, or sample size. This indicates a shift in the sampling distribution to the left when sampling without replacement. The size of the difference between corresponding Dn means is maximized when the sample/population ratio is 30%, and minimized when the sample/population ratio is 5%. This pattern is present given a fixed sample size, and is evident in each of the phases.

Comparing the differences in Dn means, between different sample sizes, the largest differences occur from small sample sizes, and the smallest differences are from large sample sizes. This relationship is present for all sample/population ratios and in all phases of the study. The mean of the Dn values when sampling with replacement is constant for a given sample size. The mean, when sampling without replacement, however, shifts to the left for a given sample size as the sample/population ratio increases. The research shows that the differences between the means of the sampling of Dn, when sampling with replacement or sampling without replacement, is the same given a sample size and sample/population ratio independent of the assumed population. There appears to be a consistent effect on the values of Dn when sampling with and without replacement no matter what the assumed population is.

The mean and variance of the Dn values when sampling with replacement are stable for a given sample size. This suggests a stable distribution given a sample size that is independent of sample/population ratio. The sample size, not sample/population ratio, is the dominant factor in the value of the mean and variance for Dn. As the sample size increases, the mean and variance of the Dn values decrease in size. A larger sample size provides more information about the hypothesized population because of a smaller variance.

When sampling without replacement, the variance of Dn also decreases as the sample size increases, however its value is influenced by the sample/population ratio. The value of the variance decreases as the sample/population ratio increases for a given sample size. The reduction in the size of the variance is a result of the effect the finite correction factor has on its value. The variance from sampling without replacement is always smaller than the variance from sampling with replacement. The *Central Limit Theorem* applies to the summation of sample results, specifically sample means and sample sums. The cumulative distribution is a summing of relative frequencies.

The second research question is concerned about relating the critical value of Dn when sampling without replacement from a finite population to the tabled Dn value found in standard Kolmogorov tables.

Research Question 2

> Is there an adjustment factor that can be used on the standard Kolmogorov table of values that will provide a good approximation of the actual critical value when sampling without replacement from a finite population? The standard table includes an asymptotic function for large samples. Is there a function that will also tie-in the factor of sampling without replacement from a discrete finite population?

The analysis also shows that, in most comparisons, the critical values when sampling with and without replacement are smaller than the corresponding table values from one-sample Kolmogorov tables. The exceptions occurred in the last phase when a skewed distribution was assumed and the sample size was six. The table values of Dn are conservative compared to the actual values. The differences between the table values and the critical values when sampling without replacement are minimized when the sample size is large and when the sample/population ratio is small.

The relationships between the critical values and differences are consistent throughout the phases with respect to the factors of this study with one exception: the magnitude of the differences is significantly larger when the assumed population shifts from the uniform. The shift of the means to the left, which results from changing the shape of the assumed population, increases the differences between the table and actual critical values. The study shows how improvement can be made to the table values of Dn through an adjustment factor derived by regression analysis.

Limitations of the Study

Limiting factors of this study are as follows:

1. Only three discrete populations were used in this study (uniform population, unimodal and symmetric population, and skewed positive population --- each having 10 categories and common mean). Each was included as a factor to determine, in general, how the population shape would effect the values of Dn. The study showed how the inclusion of this factor had an effect, however the effect diminishes as the sample size increases. Other populations are encountered aside from those included in this study, and future research may focus on determining a more precise effect from this factor.

2. This research considered three populations distributed over 10 categories with a common mean. Of course, other populations of interest may be distributed over more or fewer categories.

3. The populations in this study have equal mean values. The sampled populations have different variances and skewness and, consequently, the effect on the Dn values when the assumed population changes from the uniform can not be attributed singularly to either of these moments.

4. The sample sizes used in this study were 6, 12, 24, and 48. There is some inference that can be made about the effect on Dn from the size of the sample selected; however, small sample sizes (ie:between 6 and 24) are levels for this factor for future studies.

5. The sample/population ratios considered by this research are 5%, 10%, 20%, and 30%. Inferences can be made about the effect on Dn by this factor. The ratios are the basis of the linear function between the tabulated Dn values and simulated Dn when sampling without replacement. Additional ratios would provide additional data points that perhaps would give a better equation for estimating the Dn value.

6. The study was empirical in nature, and not mathematical.

Future Directions for Research

1. Although this study has showed that the shape of the population has an effect on the values of Dn, particularly when small samples are selected, a more extensive study in which the variance and skewness are systematically changed to explore the effect in more detail.

2. The number of sample/population ratios could be increased to fill-in between those used in this study. Additional ratio values would provide more information about the relationship between ratio and the value of Dn. This might increase the accuracy in predicting the Dn value.

3. Sample sizes other than the ones used in this research can be explored in simulation studies. A table of Dn values, similar to Table 4, Table 5, and Table 6 produced in this paper, could provide corresponding Dn values for other sample sizes.

Major Contribution of the Research

As a popular goodness-of-test, the one-sample Kolmogorov test is not always applied in situations that are assumed by the test. The information provided in this paper allows the use of the one-sample Kolmogorov test to be expanded to situations when samples are selected from finite populations without replacement. The linear function developed from this study allows the Kolmogorov test to be extended to finite population sampling. The difference in the Dn value when sampling without replacement can be significant in terms of stated and effective alpha levels. The function provides a Dn value, thereby reducing the error in actual and perceived levels of significance involved in using the table Dn value. These equations allow a practitioner to make a more accurate estimate of the actual Dn associated with a desired alpha than is provided in the current standard Kolmogorov tables. The equations provide a way of improving decision-making when using the one-sample Kolmogorov test.

The table of critical values for Dn created by Lilliefor apply to unspecified normal populations. The normal population is a commonly assumed distribution of statistical analysis. Likewise, sampling without replacement, is a common sampling occurrence. The values of Dn provided by this research are appropriate when this occurs. The differences that exist between the standard table Dn values and Lilliefor's values are comparable to the differences that exist when sampling without replacement. The adjustments to the table Dn values provided by this research are important because of the size of the error incurred when the adjustment is not used, and because of the frequency with which samples are taken without replacement from finite populations.

Another contribution of this research is the application of the finite correction factor. The finite population correction factor is an interesting result of the research in that the correction factor applies to the variance of the sampling distribution of Dn, as well as the well known sample statistics: sample sum and sample mean.

APPENDIX

TABLES

Table 21.--Simulated Sampling Distribution of Dn (n=6, N=120)
Sampling from Uniform Population

Sampling With replacement			Sampling Without replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.06667	8	.05%	0.06667	6	.04%
0.10000	171	1.19%	0.10000	207	1.42%
0.13333	914	7.29%	0.13333	1002	8.10%
0.16667	818	12.74%	0.16667	888	14.02%
0.20000	2383	28.63%	0.20000	2473	30.51%
0.23333	2356	44.33%	0.23333	2378	46.36%
0.26667	1352	53.35%	0.26667	1362	55.44%
0.30000	2409	69.41%	0.30000	2381	71.31%
0.33333	1176	77.25%	0.33333	1155	79.01%
0.36667	836	82.82%	0.36667	826	84.52%
0.40000	971	89.29%	0.40000	889	90.45%
0.43333	633	93.51%	0.43333	602	94.46%
0.46667	289	95.44%	0.46667	251	96.13%
0.50000	300	97.44%	0.50000	256	97.84%
0.53333	209	98.83%	0.53333	183	99.06%
0.56667	22	98.98%	0.56667	16	99.17%
0.60000	85	99.55%	0.60000	70	99.63%
0.63333	43	99.83%	0.63333	33	99.85%
0.70000	23	99.99%	0.70000	22	100.00%
0.73333	1	99.99%			
0.80000	1	100.00%			
Average	0.283		Average	0.27761	
Variance	0.010		Variance	0.00986	
Skewness	0.774		Skewness	0.77387	
Kurtosis	0.619		Kurtosis	0.65652	

Table 22.--Simulated Sampling Distribution of Dn (n=6, N=60)
Sampling from Uniform Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.06667	12	.08%	0.06667	12	.08%
0.10000	157	1.13%	0.10000	227	1.59%
0.13333	904	7.15%	0.13333	1080	8.79%
0.16667	877	13.00%	0.16667	1022	15.61%
0.20000	2347	28.65%	0.20000	2460	32.01%
0.23333	2371	44.45%	0.23333	2487	48.59%
0.26667	1324	53.28%	0.26667	1357	57.63%
0.30000	2318	68.73%	0.30000	2239	72.56%
0.33333	1236	76.97%	0.33333	1173	80.38%
0.36667	886	82.88%	0.36667	828	85.90%
0.40000	910	88.95%	0.40000	798	91.22%
0.43333	616	93.05%	0.43333	576	95.06%
0.46667	327	95.23%	0.46667	215	96.49%
0.50000	319	97.36%	0.50000	250	98.16%
0.53333	216	98.80%	0.53333	165	99.26%
0.56667	25	98.97%	0.56667	12	99.34%
0.60000	91	99.57%	0.60000	64	99.77%
0.63333	40	99.84%	0.63333	23	99.92%
0.70000	21	99.98%	0.70000	10	99.99%
0.73333	2	99.99%	0.73333	1	99.99%
0.80000	1	100.00%	0.80000	1	100.00%

Average	0.28403	Average	0.27258
Variance	0.01044	Variance	0.00954
Skewness	0.77011	Skewness	0.76113
Kurtosis	0.56235	Kurtosis	0.59926

Table 23.--Simulated Sampling Distribution of Dn (n=6, N=30)
Sampling from Uniform Population

Sampling With Replacement			Sampling Without replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.06667	9	.06%	0.06667	17	.11%
0.10000	165	1.16%	0.10000	290	2.05%
0.13333	926	7.33%	0.13333	1287	10.63%
0.16667	840	12.93%	0.16667	1183	18.51%
0.20000	2318	28.39%	0.20000	2833	37.40%
0.23333	2407	44.43%	0.23333	2560	54.47%
0.26667	1318	53.22%	0.26667	1381	63.67%
0.30000	2350	68.89%	0.30000	2073	77.49%
0.33333	1168	76.67%	0.33333	1109	84.89%
0.36667	897	82.65%	0.36667	692	89.50%
0.40000	889	88.58%	0.40000	611	93.57%
0.43333	649	92.91%	0.43333	462	96.65%
0.46667	335	95.14%	0.46667	171	97.79%
0.50000	304	97.17%	0.50000	200	99.13%
0.53333	239	98.76%	0.53333	82	99.67%
0.56667	30	98.96%	0.60000	40	99.94%
0.60000	94	99.59%	0.63333	5	99.97%
0.63333	44	99.88%	0.70000	3	99.99%
0.83333	18	100.00%	0.80000	1	100.00%

Average	0.28448	Average	0.25831
Variance	0.01054	Variance	0.00835
Skewness	0.76072	Skewness	0.76714
Kurtosis	0.49084	Kurtosis	0.60913

Table 24.--Simulated Sampling Distribution of Dn (n=6, N=20)
Sampling from Uniform Population

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.06667	10	.07%
0.10000	143	1.02%
0.13333	886	6.93%
0.16667	875	12.76%
0.20000	2276	27.93%
0.23333	2359	43.66%
0.26667	1371	52.80%
0.30000	2274	67.96%
0.33333	1225	76.13%
0.36667	951	82.47%
0.40000	955	88.83%
0.43333	642	93.11%
0.46667	296	95.09%
0.50000	296	97.06%
0.53333	247	98.71%
0.56667	24	98.87%
0.60000	101	99.54%
0.63333	36	99.78%
0.70000	25	99.95%
0.73333	5	99.98%
0.80000	3	100.00%

Average	0.285869
Variance	0.010584
Skewneww	0.788553
Kurtosis	0.674279

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.06667	36	.24%
0.10000	410	2.97%
0.13333	1609	13.70%
0.16667	1286	22.27%
0.20000	2968	42.06%
0.23333	2506	58.77%
0.26667	1387	68.01%
0.30000	1985	81.25%
0.33333	1049	88.24%
0.36667	650	92.57%
0.40000	466	95.68%
0.43333	385	98.25%
0.46667	70	98.71%
0.50000	132	99.59%
0.53333	47	99.91%
0.60000	13	99.99%
0.70000	1	100.00%

Average	0.245962
Variance	0.00741
Skewneww	0.668041
Kurtosis	0.379305

Table 25.--Simulated Sampling Distribution of Dn (n=12, N=240)
Sampling from Uniform Population

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.05000	18	.12%
0.06667	164	1.21%
0.08333	213	2.63%
0.10000	628	6.82%
0.11667	1254	15.18%
0.13333	1610	25.91%
0.15000	1660	36.98%
0.16667	987	43.56%
0.18333	922	49.71%
0.20000	1393	58.99%
0.21667	1384	68.22%
0.23333	1130	75.75%
0.25000	761	80.83%
0.26667	643	85.11%
0.28333	448	88.10%
0.30000	474	91.26%
0.31667	345	93.56%
0.33333	269	95.35%
0.35000	229	96.88%
0.36667	180	98.08%
0.38333	65	98.51%
0.40000	44	98.81%
0.41667	62	99.22%
0.43333	53	99.57%
0.45000	38	99.83%
0.46667	10	99.89%
0.50000	6	99.93%
0.51667	7	99.98%
0.53333	2	99.99%
0.55000	1	100.00%
Average	0.19835	
Variance	0.00548	
Skewness	0.78011	
Kurtosis	0.52392	

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.03333	3	.02%
0.05000	27	.20%
0.06667	199	1.53%
0.08333	243	3.15%
0.10000	697	7.79%
0.11667	1255	16.16%
0.13333	1695	27.46%
0.15000	1828	39.65%
0.16667	1052	46.66%
0.18333	886	52.57%
0.20000	1374	61.73%
0.21667	1416	71.17%
0.23333	1025	78.00%
0.25000	717	82.78%
0.26667	614	86.87%
0.28333	436	89.78%
0.30000	419	92.57%
0.31667	283	94.46%
0.33333	243	96.08%
0.35000	215	97.51%
0.36667	143	98.47%
0.38333	47	98.78%
0.40000	36	99.02%
0.41667	44	99.31%
0.43333	49	99.64%
0.45000	26	99.81%
0.46667	13	99.90%
0.50000	4	99.93%
0.51667	9	99.97%
0.55000	2	100.00%
Average	0.19319	
Variance	0.00521	
Skewness	0.82096	
Kurtosis	0.71231	

Table 26.--Simulated Sampling Distribution of Dn (n=12, N=120)
Sampling from Uniform Population

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.03333	3	.02%
0.05000	26	.19%
0.06667	161	1.27%
0.08333	231	2.81%
0.10000	578	6.66%
0.11667	1144	14.29%
0.13333	1555	24.65%
0.15000	1797	36.63%
0.16667	1004	43.33%
0.18333	881	49.20%
0.20000	1484	59.09%
0.21667	1454	68.79%
0.23333	1102	76.13%
0.25000	702	80.81%
0.26667	592	84.76%
0.28333	490	88.03%
0.30000	467	91.14%
0.31667	352	93.49%
0.33333	265	95.25%
0.35000	229	96.78%
0.36667	176	97.95%
0.38333	72	98.43%
0.40000	42	98.71%
0.41667	62	99.13%
0.43333	52	99.47%
0.45000	43	99.76%
0.46667	13	99.85%
0.48333	3	99.87%
0.50000	3	99.89%
0.51667	7	99.93%
0.53333	6	99.97%
0.55000	3	99.99%
0.61667	1	100.00%

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.03333	4	.03%
0.05000	30	.23%
0.06667	228	1.75%
0.08333	319	3.87%
0.10000	708	8.59%
0.11667	1339	17.52%
0.13333	1794	29.48%
0.15000	1849	41.81%
0.16667	1025	48.64%
0.18333	942	54.92%
0.20000	1426	64.43%
0.21667	1319	73.22%
0.23333	972	79.70%
0.25000	647	84.01%
0.26667	582	87.89%
0.28333	443	90.85%
0.30000	352	93.19%
0.31667	284	95.09%
0.33333	223	96.57%
0.35000	191	97.85%
0.36667	118	98.63%
0.38333	55	99.00%
0.40000	30	99.20%
0.41667	34	99.43%
0.43333	53	99.78%
0.45000	17	99.89%
0.46667	9	99.95%
0.50000	2	99.97%
0.51667	5	100.00%

Table 26.--Continued.

Sampling With Replacement		Sampling Without Replacement	
Average	0.19896	Average	0.18909
Variance	0.00551	Variance	0.00502
Skewness	0.82515	Skewness	0.81504
Kurtosis	0.75724	Kurtosis	0.63120

Table 27.--Simulated Sampling Distribution of Dn (n=12, N=60)
Sampling from Uniform Population

Dn	*Frequency*	*Cum. %*
0.03333	4	.03%
0.05000	18	.15%
0.06667	168	1.27%
0.08333	227	2.78%
0.10000	615	6.88%
0.11667	1132	14.43%
0.13333	1537	24.67%
0.15000	1765	36.44%
0.16667	973	42.93%
0.18333	976	49.43%
0.20000	1464	59.19%
0.21667	1457	68.91%
0.23333	1113	76.33%
0.25000	737	81.24%
0.26667	591	85.18%
0.28333	471	88.32%
0.30000	456	91.36%
0.31667	347	93.67%
0.33333	256	95.38%
0.35000	235	96.95%
0.36667	169	98.07%
0.38333	62	98.49%
0.40000	47	98.80%
0.41667	66	99.24%
0.43333	48	99.56%
0.45000	34	99.79%
0.46667	11	99.86%
0.50000	11	99.93%
0.51667	8	99.98%
0.55000	1	99.99%
0.61667	1	100.00%
Average	0.19849	
Variance	0.00540	
Skewness	0.79981	
Kurtosis	0.67816	

Dn	*Frequency*	*Cum. %*
0.03333	2	.01%
0.05000	52	.36%
0.06667	299	2.35%
0.08333	343	4.64%
0.10000	885	10.54%
0.11667	1511	20.61%
0.13333	2011	34.02%
0.15000	1925	46.85%
0.16667	1093	54.14%
0.18333	946	60.45%
0.20000	1302	69.13%
0.21667	1301	77.80%
0.23333	883	83.69%
0.25000	615	87.79%
0.26667	502	91.13%
0.28333	359	93.53%
0.30000	316	95.63%
0.31667	186	96.87%
0.33333	183	98.09%
0.35000	123	98.91%
0.36667	66	99.35%
0.38333	22	99.50%
0.40000	13	99.59%
0.41667	32	99.80%
0.43333	15	99.90%
0.45000	10	99.95%
0.51667	5	100.00%
Average	0.17924	
Variance	0.00434	
Skewness	0.79620	
Kurtosis	0.64167	

Table 28.--Simulated Sampling Distributuion of Dn (n=12, N=40)
Sampling from Uniform Population

Dn	*Frequency*	*Cum. %*
0.03333	1	.01%
0.05000	21	.15%
0.06667	157	1.19%
0.08333	236	2.77%
0.10000	646	7.07%
0.11667	1181	14.95%
0.13333	1598	25.60%
0.15000	1719	37.06%
0.16667	1046	44.03%
0.18333	931	50.24%
0.20000	1391	59.51%
0.21667	1449	69.17%
0.23333	1070	76.31%
0.25000	617	80.42%
0.26667	619	84.55%
0.28333	515	87.98%
0.30000	498	91.30%
0.31667	342	93.58%
0.33333	289	95.51%
0.35000	223	96.99%
0.36667	149	97.99%
0.38333	68	98.44%
0.40000	46	98.75%
0.41667	62	99.16%
0.43333	55	99.53%
0.45000	35	99.76%
0.46667	10	99.83%
0.48333	3	99.85%
0.50000	5	99.88%
0.51667	7	99.93%
0.53333	6	99.97%
0.55000	2	99.98%
0.56667	1	99.99%
0.60000	2	100.00%

Dn	*Frequency*	*Cum. %*
0.03333	7	.05%
0.05000	65	.48%
0.06667	444	3.44%
0.08333	472	6.59%
0.10000	1069	13.71%
0.11667	1837	25.96%
0.13333	2113	40.05%
0.15000	1888	52.63%
0.16667	1115	60.07%
0.18333	997	66.71%
0.20000	1297	75.36%
0.21667	1102	82.71%
0.23333	723	87.53%
0.25000	558	91.25%
0.26667	437	94.16%
0.28333	275	95.99%
0.30000	197	97.31%
0.31667	106	98.01%
0.33333	121	98.82%
0.35000	89	99.41%
0.36667	50	99.75%
0.40000	14	99.84%
0.41667	11	99.91%
0.43333	8	99.97%
0.45000	5	100.00%

Table 28.--Continued.

Sampling With Replacement		Sampling Without Replacement	
Average	0.19810	Average	0.168421
Variance	0.00553	Variance	0.003814
Skewness	0.83040	Skewness	0.783736
Kurtosis	0.75102	Kurtosis	0.597675

Table 29.--Simulated Sampling Distribution of Dn (n=24, N=480)
Sampling from Uniform Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.02500	1	.01%	0.02500	2	.01%
0.03333	10	.07%	0.03333	14	.11%
0.04167	26	.25%	0.04167	31	.31%
0.05000	104	.94%	0.05000	143	1.27%
0.05833	271	2.75%	0.05833	306	3.31%
0.06667	447	5.73%	0.06667	461	6.38%
0.07500	740	10.66%	0.07500	811	11.79%
0.08333	441	13.60%	0.08333	461	14.86%
0.09167	1003	20.29%	0.09167	1090	22.13%
0.10000	1001	26.96%	0.10000	990	28.73%
0.10833	1084	34.19%	0.10833	1092	36.01%
0.11667	1243	42.47%	0.11667	1352	45.02%
0.12500	671	46.95%	0.12500	715	49.79%
0.13333	1212	55.03%	0.13333	1240	58.05%
0.14167	637	59.27%	0.14167	623	62.21%
0.15000	864	65.03%	0.15000	809	67.60%
0.15833	860	70.77%	0.15833	815	73.03%
0.16667	548	74.42%	0.16667	570	76.83%
0.17500	794	79.71%	0.17500	683	81.39%
0.18333	432	82.59%	0.18333	397	84.03%
0.19167	442	85.54%	0.19167	429	86.89%
0.20000	361	87.95%	0.20000	323	89.05%
0.20833	308	90.00%	0.20833	275	90.88%
0.21667	290	91.93%	0.21667	275	92.71%
0.22500	248	93.59%	0.22500	221	94.19%
0.23333	190	94.85%	0.23333	185	95.42%
0.24167	140	95.79%	0.24167	124	96.25%
0.25000	145	96.75%	0.25000	139	97.17%
0.25833	90	97.35%	0.25833	79	97.70%
0.26667	117	98.13%	0.26667	86	98.27%
0.27500	58	98.52%	0.27500	49	98.60%
0.28333	57	98.90%	0.28333	52	98.95%
0.29167	46	99.21%	0.29167	54	99.31%
0.30000	14	99.30%	0.30000	18	99.43%

Table 29.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.30833	28	99.49%
0.31667	5	99.52%
0.32500	17	99.63%
0.33333	18	99.75%
0.34167	4	99.78%
0.35000	16	99.89%
0.35833	2	99.90%
0.36667	6	99.94%
0.37500	4	99.97%
0.38333	1	99.97%
0.39167	1	99.98%
0.40833	3	100.00%

Average	0.14023
Variance	0.00277
Skewness	0.84531
Kurtosis	0.85352

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.30833	32	99.64%
0.31667	10	99.71%
0.32500	16	99.81%
0.33333	10	99.88%
0.34167	2	99.89%
0.35000	7	99.94%
0.36667	3	99.96%
0.37500	3	99.98%
0.39167	2	99.99%
0.41667	1	100.00%

Average	0.13697
Variance	0.00267
Skewness	0.85317
Kurtosis	0.80312

Table 30.--Simulated Sampling Distribution of Dn (n=24, N=240)
Sampling from Uniform Population

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.02500	5	.03%
0.03333	11	.11%
0.04167	26	.28%
0.05000	127	1.13%
0.05833	260	2.86%
0.06667	479	6.05%
0.07500	759	11.11%
0.08333	433	14.00%
0.09167	1021	20.81%
0.10000	904	26.83%
0.10833	1094	34.13%
0.11667	1272	42.61%
0.12500	608	46.66%
0.13333	1210	54.73%
0.14167	649	59.05%
0.15000	859	64.78%
0.15833	849	70.44%
0.16667	533	73.99%
0.17500	751	79.00%
0.18333	426	81.84%
0.19167	448	84.83%
0.20000	368	87.28%
0.20833	306	89.32%
0.21667	322	91.47%
0.22500	249	93.13%
0.23333	224	94.62%
0.24167	133	95.51%
0.25000	128	96.36%
0.25833	105	97.06%
0.26667	111	97.80%
0.27500	68	98.25%
0.28333	62	98.67%
0.29167	69	99.13%
0.30000	23	99.28%

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.03333	31	.21%
0.04167	29	.40%
0.05000	166	1.51%
0.05833	338	3.76%
0.06667	537	7.34%
0.07500	907	13.39%
0.08333	487	16.63%
0.09167	1174	24.46%
0.10000	979	30.99%
0.10833	1166	38.76%
0.11667	1343	47.71%
0.12500	737	52.63%
0.13333	1245	60.93%
0.14167	660	65.33%
0.15000	795	70.63%
0.15833	783	75.85%
0.16667	461	78.92%
0.17500	652	83.27%
0.18333	382	85.81%
0.19167	343	88.10%
0.20000	301	90.11%
0.21667	534	93.67%
0.22500	189	94.93%
0.23333	160	95.99%
0.24167	138	96.91%
0.25000	125	97.75%
0.25833	60	98.15%
0.26667	73	98.63%
0.27500	38	98.89%
0.28333	50	99.22%
0.29167	34	99.45%
0.30000	20	99.58%
0.30833	30	99.78%
0.31667	3	99.80%

Table 30.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.30833	45	99.58%
0.31667	12	99.66%
0.32500	15	99.76%
0.33333	13	99.85%
0.34167	2	99.86%
0.35000	9	99.92%
0.35833	3	99.94%
0.36667	5	99.97%
0.39167	3	99.99%
0.42500	1	100.00%

Average	0.14070
Variance	0.00286
Skewness	0.80589
Kurtosis	0.61150

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.32500	7	99.85%
0.33333	9	99.91%
0.34167	5	99.94%
0.35000	6	99.98%
0.37500	1	99.99%
0.39167	2	100.00%

Average	0.13325
Variance	0.00254
Skewness	0.85844
Kurtosis	0.78609

Table 31.--Simulated Sampling Distribution of Dn (n=24, N=120)
Sampling from Uniform Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.02500	1	.01%	0.02500	5	.03%
0.03333	10	.07%	0.03333	41	.31%
0.04167	26	.25%	0.04167	65	.74%
0.05000	104	.94%	0.05000	202	2.09%
0.05833	271	2.75%	0.05833	442	5.03%
0.06667	447	5.73%	0.06667	718	9.82%
0.07500	740	10.66%	0.07500	1028	16.67%
0.08333	441	13.60%	0.08333	629	20.87%
0.09167	1003	20.29%	0.09167	1279	29.39%
0.10000	1001	26.96%	0.10000	1053	36.41%
0.10833	1084	34.19%	0.10833	1160	44.15%
0.11667	1243	42.47%	0.11667	1361	53.22%
0.12500	671	46.95%	0.12500	748	58.21%
0.13333	1212	55.03%	0.13333	1192	66.15%
0.14167	637	59.27%	0.14167	637	70.40%
0.15000	864	65.03%	0.15000	724	75.23%
0.15833	860	70.77%	0.15833	682	79.77%
0.16667	548	74.42%	0.16667	484	83.00%
0.17500	794	79.71%	0.17500	559	86.73%
0.18333	432	82.59%	0.18333	382	89.27%
0.19167	442	85.54%	0.19167	358	91.66%
0.20000	361	87.95%	0.20000	281	93.53%
0.20833	308	90.00%	0.20833	206	94.91%
0.21667	290	91.93%	0.21667	177	96.09%
0.22500	248	93.59%	0.22500	162	97.17%
0.23333	190	94.85%	0.23333	98	97.82%
0.24167	140	95.79%	0.24167	75	98.32%
0.25000	145	96.75%	0.25000	78	98.84%
0.25833	90	97.35%	0.25833	24	99.00%
0.26667	117	98.13%	0.26667	55	99.37%
0.27500	58	98.52%	0.27500	27	99.55%
0.28333	57	98.90%	0.28333	22	99.69%
0.29167	46	99.21%	0.29167	15	99.79%
0.30000	14	99.30%	0.30000	3	99.81%

Table 31.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.30833	28	99.49%
0.31667	5	99.52%
0.32500	17	99.63%
0.33333	18	99.75%
0.34167	4	99.78%
0.35000	16	99.89%
0.35833	2	99.90%
0.36667	6	99.94%
0.37500	4	99.97%
0.38333	1	99.97%
0.39167	1	99.98%
0.40833	3	100.00%

Average	0.14023
Variance	0.00277
Skewness	0.84531
Kurtosis	0.85352

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.30833	14	99.91%
0.32500	8	99.96%
0.33333	2	99.97%
0.35000	2	99.99%
0.35833	1	99.99%
0.36667	1	100.00%

Average	0.12560
Variance	0.00219
Skewness	0.78616
Kurtosis	0.64062

Table 32.--Simulated Sampling Distribution of Dn (n=24, N=80)
Sampling from Uniform Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.01667	1	.01%	0.02500	6	.04%
0.02500	5	.04%	0.03333	54	.40%
0.03333	20	.17%	0.04167	87	.98%
0.04167	29	.37%	0.05000	302	2.99%
0.05000	105	1.07%	0.05833	558	6.71%
0.05833	275	2.90%	0.06667	888	12.63%
0.06667	494	6.19%	0.07500	1213	20.72%
0.07500	702	10.87%	0.08333	715	25.49%
0.08333	451	13.88%	0.09167	1448	35.14%
0.09167	1056	20.92%	0.10000	1153	42.83%
0.10000	922	27.07%	0.10833	1156	50.53%
0.10833	1018	33.85%	0.11667	1342	59.48%
0.11667	1324	42.68%	0.12500	731	64.35%
0.12500	658	47.07%	0.13333	1115	71.79%
0.13333	1188	54.99%	0.14167	593	75.74%
0.14167	629	59.18%	0.15000	646	80.05%
0.15000	883	65.07%	0.15833	605	84.08%
0.15833	808	70.45%	0.16667	437	86.99%
0.16667	568	74.24%	0.17500	440	89.93%
0.17500	728	79.09%	0.18333	340	92.19%
0.18333	418	81.88%	0.19167	274	94.02%
0.19167	476	85.05%	0.20000	189	95.28%
0.20000	395	87.69%	0.20833	194	96.57%
0.20833	304	89.71%	0.21667	125	97.41%
0.21667	301	91.72%	0.22500	97	98.05%
0.22500	234	93.28%	0.23333	85	98.62%
0.23333	213	94.70%	0.24167	52	98.97%
0.24167	135	95.60%	0.25000	50	99.30%
0.25000	161	96.67%	0.25833	19	99.43%
0.25833	86	97.25%	0.26667	37	99.67%
0.26667	117	98.03%	0.27500	16	99.78%
0.27500	69	98.49%	0.28333	7	99.83%
0.28333	41	98.76%	0.29167	12	99.91%
0.29167	59	99.15%	0.30000	1	99.91%

Table 32.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.30000	12	99.23%
0.30833	36	99.47%
0.31667	17	99.59%
0.32500	18	99.71%
0.33333	14	99.80%
0.34167	4	99.83%
0.35000	11	99.90%
0.36667	7	99.95%
0.37500	2	99.96%
0.38333	2	99.97%
0.39167	2	99.99%
0.40000	1	99.99%
0.40833	1	100.00%

Average	0.14038
Variance	0.00283
Skewness	0.81686
Kurtosis	0.72277

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.30833	5	99.95%
0.31667	4	99.97%
0.32500	1	99.98%
0.33333	2	99.99%
0.35000	1	100.00%

Average	0.11836
Variance	0.00197
Skewness	0.82123
Kurtosis	0.70906

Table 33.--Simulated Sampling Distribution of Dn (n=48, N=960)
Sampling from Uniform Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.01667	3	.02%	0.01667	2	.01%
0.02083	2	.03%	0.02083	4	.04%
0.02500	16	.14%	0.02500	18	.16%
0.02917	38	.39%	0.02917	41	.43%
0.03333	70	.86%	0.03333	86	1.01%
0.03750	175	2.03%	0.03750	162	2.09%
0.04167	106	2.73%	0.04167	120	2.89%
0.04583	277	4.58%	0.04583	294	4.85%
0.05000	440	7.51%	0.05000	434	7.74%
0.05417	537	11.09%	0.05417	542	11.35%
0.05833	563	14.85%	0.05833	611	15.43%
0.06250	372	17.33%	0.06250	364	17.85%
0.06667	676	21.83%	0.06667	699	22.51%
0.07083	808	27.22%	0.07083	823	28.00%
0.07500	919	33.35%	0.07500	938	34.25%
0.07917	696	37.99%	0.07917	686	38.83%
0.08333	445	40.95%	0.08333	493	42.11%
0.08750	694	45.58%	0.08750	726	46.95%
0.09167	858	51.30%	0.09167	856	52.66%
0.09583	815	56.73%	0.09583	816	58.10%
0.10000	526	60.24%	0.10000	499	61.43%
0.10417	462	63.32%	0.10417	452	64.44%
0.10833	529	66.85%	0.10833	531	67.98%
0.11250	631	71.05%	0.11250	612	72.06%
0.11667	549	74.71%	0.11667	495	75.36%
0.12083	335	76.95%	0.12083	347	77.67%
0.12500	320	79.08%	0.12500	335	79.91%
0.12917	355	81.45%	0.12917	341	82.18%
0.13333	392	84.06%	0.13333	367	84.63%
0.13750	300	86.06%	0.13750	308	86.68%
0.14167	255	87.76%	0.14167	236	88.25%
0.14583	243	89.38%	0.14583	230	89.79%
0.15000	219	90.84%	0.15000	198	91.11%
0.15417	201	92.18%	0.15417	205	92.47%

Table 33.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.15833	160	93.25%
0.16250	154	94.27%
0.16667	138	95.19%
0.17083	96	95.83%
0.17500	86	96.41%
0.17917	89	97.00%
0.18333	81	97.54%
0.18750	69	98.00%
0.19167	50	98.33%
0.19583	38	98.59%
0.20000	24	98.75%
0.20417	46	99.05%
0.20833	29	99.25%
0.21250	15	99.35%
0.21667	12	99.43%
0.22083	17	99.54%
0.22500	8	99.59%
0.22917	27	99.77%
0.23333	4	99.80%
0.23750	2	99.81%
0.24167	5	99.85%
0.24583	8	99.90%
0.25000	6	99.94%
0.25833	1	99.95%
0.26250	2	99.96%
0.26667	2	99.97%
0.27083	2	99.99%
0.29167	1	99.99%
0.29583	1	100.00%

Average	0.09839
Variance	0.00139
Skewness	0.81260
Kurtosis	0.71029

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.15833	172	93.62%
0.16250	153	94.64%
0.16667	154	95.67%
0.17083	95	96.30%
0.17500	75	96.80%
0.17917	56	97.17%
0.18333	81	97.71%
0.18750	56	98.09%
0.19167	53	98.44%
0.19583	34	98.67%
0.20000	32	98.88%
0.20417	38	99.13%
0.20833	31	99.34%
0.21250	19	99.47%
0.21667	5	99.50%
0.22083	11	99.57%
0.22500	13	99.66%
0.22917	15	99.76%
0.23333	8	99.81%
0.23750	3	99.83%
0.24167	3	99.85%
0.24583	6	99.89%
0.25417	3	99.91%
0.26250	2	99.93%
0.26667	3	99.95%
0.27083	5	99.98%
0.28333	2	99.99%
0.29167	1	100.00%

Average	0.09739
Variance	0.00137
Skewness	0.82995
Kurtosis	0.78941

Table 34.--Simulated Sampling Distribution of Dn (n=48, N=480)
Sampling from Uniform Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.01667	1	.01%	0.01250	1	.01%
0.02500	6	.05%	0.01667	1	.01%
0.02917	41	.32%	0.02917	79	.54%
0.03333	75	.82%	0.03333	84	1.10%
0.03750	169	1.95%	0.03750	225	2.60%
0.04167	106	2.65%	0.04167	143	3.55%
0.04583	260	4.39%	0.04583	317	5.67%
0.05000	419	7.18%	0.05000	548	9.32%
0.05417	489	10.44%	0.05417	627	13.50%
0.05833	578	14.29%	0.05833	681	18.04%
0.06250	373	16.78%	0.06250	388	20.63%
0.06667	663	21.20%	0.06667	728	25.48%
0.07083	846	26.84%	0.07083	897	31.46%
0.07500	937	33.09%	0.07500	967	37.91%
0.07917	690	37.69%	0.07917	710	42.64%
0.08333	447	40.67%	0.08333	514	46.07%
0.08750	685	45.23%	0.08750	692	50.68%
0.09167	819	50.69%	0.09167	812	56.09%
0.09583	821	56.17%	0.09583	799	61.42%
0.10000	492	59.45%	0.10000	496	64.73%
0.10417	421	62.25%	0.10417	460	67.79%
0.10833	534	65.81%	0.10833	514	71.22%
0.11250	639	70.07%	0.11250	581	75.09%
0.11667	558	73.79%	0.11667	529	78.62%
0.12083	380	76.33%	0.12083	328	80.81%
0.12500	350	78.66%	0.12500	344	83.10%
0.12917	356	81.03%	0.12917	342	85.38%
0.13333	403	83.72%	0.13333	352	87.73%
0.13750	334	85.95%	0.13750	216	89.17%
0.14167	234	87.51%	0.14167	222	90.65%
0.14583	230	89.04%	0.14583	207	92.03%
0.15000	236	90.61%	0.15000	170	93.16%
0.15417	233	92.17%	0.15417	162	94.24%
0.15833	150	93.17%	0.15833	120	95.04%

Table 34.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.16250	152	94.18%
0.17083	225	95.68%
0.17500	90	96.28%
0.17917	72	96.76%
0.18333	85	97.33%
0.18750	69	97.79%
0.19167	60	98.19%
0.19583	41	98.46%
0.20000	25	98.63%
0.20417	42	98.91%
0.20833	39	99.17%
0.21250	22	99.31%
0.21667	10	99.38%
0.22083	14	99.47%
0.22500	8	99.53%
0.22917	25	99.69%
0.23333	4	99.72%
0.23750	2	99.73%
0.24167	8	99.79%
0.24583	9	99.85%
0.25000	10	99.91%
0.25417	2	99.93%
0.25833	2	99.94%
0.26667	2	99.95%
0.27083	2	99.97%
0.28333	2	99.98%
0.30833	2	99.99%
0.32500	1	100.00%

Average	0.09907
Variance	0.00141
Skewness	0.84552
Kurtosis	0.88598

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.16250	115	95.81%
0.16667	110	96.54%
0.17083	83	97.09%
0.17500	65	97.53%
0.17917	46	97.83%
0.18333	63	98.25%
0.18750	48	98.57%
0.19167	36	98.81%
0.19583	25	98.98%
0.20000	26	99.15%
0.20417	21	99.29%
0.20833	32	99.51%
0.21250	11	99.58%
0.21667	8	99.63%
0.22083	7	99.68%
0.22500	15	99.78%
0.22917	9	99.84%
0.23333	5	99.87%
0.23750	1	99.88%
0.24167	2	99.89%
0.24583	2	99.91%
0.25000	5	99.94%
0.25417	1	99.95%
0.26667	3	99.97%
0.27083	2	99.98%
0.27917	1	99.99%
0.28750	1	99.99%
0.29167	1	100.00%

Average	0.09372
Variance	0.00127
Skewness	0.85758
Kurtosis	0.94398

Table 35.--Simulated Sampling Distribution of Dn (n=48, N=240)
Sampling from Uniform Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.01667	1	.01%	0.01250	1	.01%
0.02083	1	.01%	0.01667	2	.02%
0.02500	16	.12%	0.02083	8	.07%
0.02917	44	.41%	0.02500	28	.26%
0.03333	66	.85%	0.02917	92	.87%
0.03750	176	2.03%	0.03333	141	1.81%
0.04167	101	2.70%	0.03750	263	3.57%
0.04583	282	4.58%	0.04167	181	4.77%
0.05000	392	7.19%	0.04583	423	7.59%
0.05417	507	10.57%	0.05000	635	11.83%
0.05833	658	14.96%	0.05417	737	16.74%
0.06250	398	17.61%	0.05833	771	21.88%
0.06667	674	22.11%	0.06250	431	24.75%
0.07083	799	27.43%	0.06667	724	29.58%
0.07500	863	33.19%	0.07083	989	36.17%
0.07917	706	37.89%	0.07500	1041	43.11%
0.08333	464	40.99%	0.07917	725	47.95%
0.08750	700	45.65%	0.08333	472	51.09%
0.09167	873	51.47%	0.08750	678	55.61%
0.09583	820	56.94%	0.09167	879	61.47%
0.10000	474	60.10%	0.09583	841	67.08%
0.10417	408	62.82%	0.10000	478	70.27%
0.10833	543	66.44%	0.10417	426	73.11%
0.11250	652	70.79%	0.10833	468	76.23%
0.11667	544	74.41%	0.11250	563	79.98%
0.12083	371	76.89%	0.11667	452	82.99%
0.12500	355	79.25%	0.12083	323	85.15%
0.12917	334	81.48%	0.12500	266	86.92%
0.13333	430	84.35%	0.12917	263	88.67%
0.13750	318	86.47%	0.13333	246	90.31%
0.14167	243	88.09%	0.13750	223	91.80%
0.14583	226	89.59%	0.14167	196	93.11%
0.15000	197	90.91%	0.14583	184	94.33%
0.15417	200	92.24%	0.15000	132	95.21%

Table 35.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.15833	148	93.23%
0.16250	141	94.17%
0.17500	78	96.35%
0.17917	68	96.80%
0.18333	71	97.27%
0.18750	64	97.70%
0.19167	54	98.06%
0.19583	29	98.25%
0.20000	34	98.48%
0.20417	50	98.81%
0.20833	40	99.08%
0.21250	34	99.31%
0.21667	14	99.40%
0.22083	21	99.54%
0.22500	10	99.61%
0.22917	12	99.69%
0.23333	14	99.78%
0.23750	2	99.79%
0.24167	8	99.85%
0.24583	4	99.87%
0.25000	5	99.91%
0.25417	4	99.93%
0.25833	2	99.95%
0.26667	3	99.97%
0.27500	2	99.98%
0.27917	1	99.99%
0.30833	2	100.00%

Average	0.09850
Variance	0.00141
Skewness	0.85245
Kurtosis	0.85777

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.15417	119	96.01%
0.15833	84	96.57%
0.17083	68	98.31%
0.17500	39	98.57%
0.17917	42	98.85%
0.18333	34	99.08%
0.18750	47	99.39%
0.19167	17	99.51%
0.19583	13	99.59%
0.20000	9	99.65%
0.20417	15	99.75%
0.20833	13	99.84%
0.21250	5	99.87%
0.21667	2	99.89%
0.22083	3	99.91%
0.22500	4	99.93%
0.22917	4	99.96%
0.23333	3	99.98%
0.24167	2	99.99%
0.25833	1	100.00%

Average	0.08858
Variance	0.00111
Skewness	0.77357
Kurtosis	0.58979

Table 36.--Simulated Sampling Distribution of Dn (n=48, N=160)
Sampling from Uniform Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.01667	3	.02%	0.01250	1	.01%
0.02083	2	.03%	0.01667	4	.05%
0.02500	16	.14%	0.02083	7	.12%
0.02917	38	.39%	0.02500	27	.39%
0.03333	70	.86%	0.02917	68	1.07%
0.03750	175	2.03%	0.03333	152	2.59%
0.04167	106	2.73%	0.03750	252	5.11%
0.04583	277	4.58%	0.04167	173	6.84%
0.05000	440	7.51%	0.04583	351	10.35%
0.05417	537	11.09%	0.05000	484	15.19%
0.05833	563	14.85%	0.05417	600	21.19%
0.06250	372	17.33%	0.05833	611	27.30%
0.06667	676	21.83%	0.06250	356	30.86%
0.07083	808	27.22%	0.06667	593	36.79%
0.07500	919	33.35%	0.07083	709	43.88%
0.07917	696	37.99%	0.07500	707	50.95%
0.08333	445	40.95%	0.07917	480	55.75%
0.08750	694	45.58%	0.08333	348	59.23%
0.09167	858	51.30%	0.08750	400	63.23%
0.09583	815	56.73%	0.09167	479	68.02%
0.10000	526	60.24%	0.09583	477	72.79%
0.10417	462	63.32%	0.10000	295	75.74%
0.10833	529	66.85%	0.10417	266	78.40%
0.11250	631	71.05%	0.10833	290	81.30%
0.11667	549	74.71%	0.11250	303	84.33%
0.12083	335	76.95%	0.11667	279	87.12%
0.12500	320	79.08%	0.12083	174	88.86%
0.12917	355	81.45%	0.12500	173	90.59%
0.13333	392	84.06%	0.12917	138	91.97%
0.13750	300	86.06%	0.13333	129	93.26%
0.14167	255	87.76%	0.13750	107	94.33%
0.14583	243	89.38%	0.14167	101	95.34%
0.15000	219	90.84%	0.14583	74	96.08%
0.15417	201	92.18%	0.15000	65	96.73%

Table 36.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.15833	160	93.25%
0.16250	154	94.27%
0.17500	86	96.41%
0.17917	89	97.00%
0.18333	81	97.54%
0.18750	69	98.00%
0.19167	50	98.33%
0.19583	38	98.59%
0.20000	24	98.75%
0.20417	46	99.05%
0.20833	29	99.25%
0.21250	15	99.35%
0.21667	12	99.43%
0.22083	17	99.54%
0.22500	8	99.59%
0.22917	27	99.77%
0.23333	4	99.80%
0.24167	7	99.85%
0.24583	8	99.90%
0.25000	6	99.94%
0.25833	1	99.95%
0.26250	2	99.96%
0.26667	2	99.97%
0.27083	2	99.99%
0.29167	1	99.99%
0.29583	1	100.00%

Average	0.09839
Variance	0.00139
Skewness	0.81260
Kurtosis	0.71029

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.15417	52	97.25%
0.15833	39	97.64%
0.17083	30	98.93%
0.17500	21	99.14%
0.17917	17	99.31%
0.18333	14	99.45%
0.18750	15	99.60%
0.19167	9	99.69%
0.19583	3	99.72%
0.20000	6	99.78%
0.20417	6	99.84%
0.20833	7	99.91%
0.21250	4	99.95%
0.22083	2	99.97%
0.22500	1	99.98%
0.22917	1	99.99%
0.25000	1	100.00%

Average	0.08281
Variance	0.00100
Skewness	0.84829
Kurtosis	0.77673

Table 37.--Simulated Sampling Distribution of Dn (n=6, N=120) Sampling from Unimodal Symmetrical Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.08333	1022	13.63%	0.08333	1101	14.68%
0.15000	183	16.07%	0.15000	196	17.29%
0.16667	1462	35.56%	0.16667	1520	37.56%
0.25000	2855	73.63%	0.25000	2852	75.59%
0.31667	28	74.00%	0.33333	1117	90.48%
0.33333	1171	89.61%	0.41667	466	96.69%
0.41667	512	96.44%	0.50000	181	99.11%
0.48333	3	96.48%	0.58333	62	99.93%
0.50000	201	99.16%	0.75000	5	100.00%
0.58333	58	99.93%			
0.75000	5	100.00%			
Average	0.24295		Average	0.23790	
Variance	0.01105		Variance	0.01095	
Skewness	0.57432		Skewness	0.62527	
Kurtosis	0.56838		Kurtosis	0.70211	

Table 38.--Simulated Sampling Distribution of Dn (n=6, N=60)
Sampling from Unimodal Symmetrical
Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.08333	928	12.37%	0.08333	1091	14.55%
0.15000	210	15.17%	0.15000	200	17.21%
0.16667	1491	35.05%	0.16667	1653	39.25%
0.25000	2927	74.08%	0.25000	2867	77.48%
0.31667	31	74.49%	0.33333	1081	91.89%
0.33333	1105	89.23%	0.41667	402	97.25%
0.41667	510	96.03%	0.50000	171	99.53%
0.50000	228	99.07%	0.58333	34	99.99%
0.58333	65	99.93%	0.75000	1	100.00%
0.65000	1	99.95%			
0.75000	4	100.00%			
Average	0.24471		Average	0.23294	
Variance	0.01102		Variance	0.01000	
Skewness	0.63284		Skewness	0.56148	
Kurtosis	0.65605		Kurtosis	0.48153	

Table 39.--Simulated Sampling Distribution of Dn (n=6, N=30)
Sampling from Unimodal Symmetrical
Population

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.06667	34	.45%
0.10000	799	11.11%
0.13333	326	15.45%
0.16667	1430	34.52%
0.23333	1595	55.79%
0.26667	1265	72.65%
0.30000	92	73.88%
0.33333	1066	88.09%
0.40000	618	96.33%
0.46667	2	96.36%
0.50000	179	98.75%
0.56667	88	99.92%
0.73333	6	100.00%

Average	0.24644
Variance	0.01030
Skewness	0.71155
Kurtosis	0.68132

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.06667	62	.83%
0.10000	980	13.89%
0.13333	439	19.75%
0.16667	1684	42.20%
0.23333	1737	65.36%
0.26667	1162	80.85%
0.33333	910	92.99%
0.40000	372	97.95%
0.50000	125	99.61%
0.56667	29	100.00%

Average	0.22501
Variance	0.00845
Skewness	0.71452
Kurtosis	0.58434

Table 40.--Simulated Sampling Distribution of Dn (n=6, N=20) Sampling from Unimodal Symmetrical Population

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.08333	65	.87%
0.10000	635	9.33%
0.11667	457	15.43%
0.16667	1439	34.61%
0.23333	305	38.68%
0.25000	2470	71.61%
0.28333	226	74.63%
0.33333	1130	89.69%
0.40000	97	90.99%
0.41667	396	96.27%
0.45000	15	96.47%
0.50000	182	98.89%
0.56667	12	99.05%
0.58333	62	99.88%
0.61667	4	99.93%
0.73333	1	99.95%
0.75000	4	100.00%

Average	0.24562
Variance	0.01030
Skewness	0.75606
Kurtosis	0.88003

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.08333	136	1.81%
0.10000	841	13.03%
0.11667	807	23.79%
0.16667	1829	48.17%
0.23333	406	53.59%
0.25000	2323	84.56%
0.33333	876	96.24%
0.41667	195	98.84%
0.50000	78	99.88%
0.58333	9	100.00%

Average	0.21165
Variance	0.00750
Skewness	0.70086
Kurtosis	0.54632

Table41.--Simulated Sampling Distribution of Dn (n=12, N=240)
Sampling from Unimodal Symmetrical,
Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.02083	22	.29%	0.02083	34	.45%
0.06250	21	.57%	0.06250	12	.61%
0.07500	39	1.09%	0.07500	45	1.21%
0.07917	156	3.17%	0.07917	194	3.80%
0.08333	385	8.31%	0.08333	401	9.15%
0.08750	541	15.52%	0.08750	525	16.15%
0.09167	1119	30.44%	0.09167	1120	31.08%
0.14583	61	31.25%	0.14583	51	31.76%
0.15833	261	34.73%	0.15833	281	35.51%
0.16250	733	44.51%	0.16250	701	44.85%
0.16667	1000	57.84%	0.16667	1042	58.75%
0.17083	1142	73.07%	0.17083	1227	75.11%
0.24167	126	74.75%	0.22917	3	75.15%
0.24583	415	80.28%	0.24167	124	76.80%
0.25000	638	88.79%	0.24583	398	82.11%
0.25417	325	93.12%	0.25000	570	89.71%
0.32500	31	93.53%	0.25417	323	94.01%
0.32917	164	95.72%	0.32500	23	94.32%
0.33333	237	98.88%	0.32917	139	96.17%
0.40833	2	98.91%	0.33333	209	98.96%
0.41250	28	99.28%	0.41250	36	99.44%
0.41667	41	99.83%	0.41667	34	99.89%
0.49583	5	99.89%	0.49583	5	99.96%
0.50000	7	99.99%	0.50000	2	99.99%
0.57917	1	100.00%	0.57917	1	100.00%

Average	0.17116	Average	0.16805
Variance	0.00564	Variance	0.00534
Skewness	0.81965	Skewness	0.82726
Kurtosis	0.68539	Kurtosis	0.77183

Table 42.--Simulated Sampling Distribution of Dn (n=12, N=120)
Sampling from Unimodal Symmetrical,
Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.01667	22	.29%	0.01667	28	.37%
0.06667	13	.47%	0.06667	10	.51%
0.08333	2329	31.52%	0.08333	2578	34.88%
0.15000	38	32.03%	0.15000	24	35.20%
0.16667	3172	74.32%	0.16667	3244	78.45%
0.25000	1492	94.21%	0.25000	1303	95.83%
0.33333	360	99.01%	0.33333	260	99.29%
0.41667	64	99.87%	0.41667	46	99.91%
0.50000	10	100.00%	0.50000	7	100.00%
Average	0.16723		Average	0.15938	
Variance	0.00563		Variance	0.00509	
Skewness	0.75389		Skewness	0.79978	
Kurtosis	0.53789		Kurtosis	0.71057	

Table 43.--Simulated Sampling Distribution of Dn (n=12, N=60)
Sampling from Unimodal Symmetrical,
Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.01667	29	.39%	0.01667	37	.49%
0.06667	11	.53%	0.06667	16	.71%
0.08333	2317	31.43%	0.08333	3019	40.96%
0.15000	45	32.03%	0.16667	3135	82.76%
0.16667	3184	74.48%	0.25000	1060	96.89%
0.25000	1451	93.83%	0.33333	210	99.69%
0.33333	383	98.93%	0.41667	23	100.00%
0.41667	71	99.88%			
0.50000	9	100.00%			
Average	0.16749		Average	0.14938	
Variance	0.00574		Variance	0.00458	
Skewness	0.76736		Skewness	0.80767	
Kurtosis	0.54096		Kurtosis	0.41128	

Table 44.--Simulated Sampling Distribution of Dn (n=12, N=40)
Sampling from Unimodal Symmetrical,
Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.02500	14	.19%	0.02500	45	.60%
0.05833	16	.40%	0.05833	29	.99%
0.06667	52	1.09%	0.06667	106	2.40%
0.08333	1151	16.44%	0.08333	1981	28.81%
0.10000	954	29.16%	0.10000	1169	44.40%
0.14167	78	30.20%	0.15000	314	48.59%
0.15000	328	34.57%	0.16667	2799	85.91%
0.16667	2888	73.08%	0.23333	38	86.41%
0.22500	19	73.33%	0.25000	869	98.00%
0.23333	164	75.52%	0.33333	136	99.81%
0.25000	1385	93.99%	0.41667	13	99.99%
0.31667	44	94.57%	0.50000	1	100.00%
0.33333	315	98.77%			
0.40000	6	98.85%			
0.41667	79	99.91%			
0.50000	5	99.97%			
0.58333	2	100.00%			

Average	0.17125	Average	0.14438
Variance	0.00527	Variance	0.00380
Skewness	0.86931	Skewness	0.91501
Kurtosis	0.94214	Kurtosis	0.86131

Table 45.--Simulated Sampling Distribution of Dn (n=24, N=480)
Sampling from Unimodal Symmetrical, Population

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.01875	3	.04%
0.02292	1	.05%
0.03542	8	.16%
0.03750	35	.63%
0.03958	4	.68%
0.04167	96	1.96%
0.04583	173	4.27%
0.04792	361	9.08%
0.06458	101	10.43%
0.07708	196	13.04%
0.07917	471	19.32%
0.08333	617	27.55%
0.08750	695	36.81%
0.08958	572	44.44%
0.11875	231	47.52%
0.12083	528	54.56%
0.12500	726	64.24%
0.12917	639	72.76%
0.16042	93	74.00%
0.16250	365	78.87%
0.16667	464	85.05%
0.17083	320	89.32%
0.20417	201	92.00%
0.20833	243	95.24%
0.21250	80	96.31%
0.24583	79	97.36%
0.25000	105	98.76%
0.25417	8	98.87%
0.28542	3	98.91%
0.28750	28	99.28%
0.29167	32	99.71%
0.32917	12	99.87%
0.33333	8	99.97%
0.37083	2	100.00%

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.01875	4	.05%
0.02292	5	.12%
0.03542	10	.25%
0.03750	44	.84%
0.03958	7	.93%
0.04167	113	2.44%
0.04583	197	5.07%
0.04792	387	10.23%
0.06458	81	11.31%
0.07708	205	14.04%
0.07917	457	20.13%
0.08333	657	28.89%
0.08750	678	37.93%
0.08958	551	45.28%
0.10625	23	45.59%
0.11875	211	48.40%
0.12083	550	55.73%
0.12500	696	65.01%
0.12917	675	74.01%
0.14792	4	74.07%
0.16250	440	79.93%
0.16667	478	86.31%
0.17083	301	90.32%
0.20208	18	90.56%
0.20417	163	92.73%
0.20833	225	95.73%
0.21250	72	96.69%
0.24375	4	96.75%
0.24583	84	97.87%
0.25000	100	99.20%
0.25417	6	99.28%
0.28750	13	99.45%
0.29167	30	99.85%
0.32917	1	99.87%

Table 45.--Continued.

Sampling With Replacement		Sampling Without Replacement		
		Dn	*Frequency*	*Cum. %*
		0.33333	6	99.95%
		0.37500	3	99.99%
		0.41250	1	100.00%
Average	0.12077	Average	0.11869	
Variance	0.00267	Variance	0.00255	
Skewness	0.91056	Skewness	0.85369	
Kurtosis	0.96019	Kurtosis	0.89912	

Table 46.--Simulated Sampling Distribution of Dn (n=24, N=240)
Sampling from Unimodal Symmetrical, Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.02083	4	.05%	0.02083	4	.05%
0.03333	11	.20%	0.03750	83	1.16%
0.03750	46	.81%	0.04167	130	2.89%
0.04167	111	2.29%	0.04583	233	6.00%
0.04583	164	4.48%	0.05000	421	11.61%
0.05000	348	9.12%	0.06250	89	12.80%
0.06250	88	10.29%	0.07500	215	15.67%
0.07500	219	13.21%	0.07917	507	22.43%
0.07917	426	18.89%	0.08333	674	31.41%
0.08333	553	26.27%	0.08750	704	40.80%
0.08750	698	35.57%	0.09167	484	47.25%
0.09167	531	42.65%	0.10417	25	47.59%
0.10417	43	43.23%	0.11667	230	50.65%
0.11667	226	46.24%	0.12083	533	57.76%
0.12083	520	53.17%	0.12500	724	67.41%
0.12500	697	62.47%	0.12917	640	75.95%
0.12917	664	71.32%	0.15833	84	77.07%
0.15833	114	72.84%	0.16250	292	80.96%
0.16250	360	77.64%	0.16667	492	87.52%
0.16667	485	84.11%	0.17083	265	91.05%
0.17083	301	88.12%	0.20000	20	91.32%
0.18750	1	88.13%	0.20417	152	93.35%
0.20000	26	88.48%	0.20833	239	96.53%
0.20417	194	91.07%	0.21250	69	97.45%
0.20833	266	94.61%	0.24167	2	97.48%
0.21250	91	95.83%	0.24583	41	98.03%
0.24167	4	95.88%	0.25000	89	99.21%
0.24583	88	97.05%	0.25417	6	99.29%
0.25000	118	98.63%	0.28333	2	99.32%
0.25417	6	98.71%	0.28750	19	99.57%
0.28333	1	98.72%	0.29167	21	99.85%
0.28750	32	99.15%	0.33333	8	99.96%
0.29167	40	99.68%	0.37500	2	99.99%
0.32500	1	99.69%	0.41250	1	100.00%

Table 46.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.37500	6	99.99%
0.41667	1	100.00%

Average	0.12283	Average	0.11587
Variance	0.00280	Variance	0.00246
Skewness	0.91477	Skewness	0.87600
Kurtosis	1.03042	Kurtosis	0.98850

Table 47.--Simulated Sampling Distribution of Dn (n=24, N=120)
Sampling from Unimodal Symmetrical,
Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.01667	4	.05%	0.01667	8	.11%
0.02500	2	.08%	0.02500	8	.21%
0.04167	710	9.55%	0.04167	1047	14.17%
0.06667	58	10.32%	0.08333	3021	54.45%
0.08333	2464	43.17%	0.12500	1985	80.92%
0.10833	34	43.63%	0.16667	947	93.55%
0.12500	2135	72.09%	0.20833	357	98.31%
0.16667	1261	88.91%	0.25000	95	99.57%
0.20833	545	96.17%	0.29167	30	99.97%
0.25000	207	98.93%	0.33333	1	99.99%
0.29167	68	99.84%	0.37500	1	100.00%
0.33333	11	99.99%			
0.37500	1	100.00%			

Average	0.12116	Average	0.10779
Variance	0.00280	Variance	0.00228
Skewness	0.78751	Skewness	0.85316
Kurtosis	0.61240	Kurtosis	0.87583

Table 48.--Simulated Sampling Distribution of Dn (n=24, N=80)
Sampling from Unimodal Symmetrical,
Population

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.01250	4	.05%
0.02917	3	.09%
0.03750	9	.21%
0.04167	339	4.73%
0.04583	406	10.15%
0.07083	38	10.65%
0.07917	191	13.20%
0.08333	1683	35.64%
0.08750	550	42.97%
0.11250	14	43.16%
0.12083	194	45.75%
0.12500	1992	72.31%
0.16250	82	73.40%
0.16667	1201	89.41%
0.20833	543	96.65%
0.25000	182	99.08%
0.28750	2	99.11%
0.29167	55	99.84%
0.33333	8	99.95%
0.37500	3	99.99%
0.41667	1	100.00%

Average	0.12083
Variance	0.00268
Skewness	0.79681
Kurtosis	0.79210

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.01250	7	.09%
0.02917	8	.20%
0.03750	18	.44%
0.04167	688	9.61%
0.04583	643	18.19%
0.07917	256	21.60%
0.08333	2337	52.76%
0.08750	512	59.59%
0.12083	119	61.17%
0.12500	1766	84.72%
0.16667	845	95.99%
0.20417	3	96.03%
0.20833	237	99.19%
0.25000	52	99.88%
0.29167	6	99.96%
0.33333	3	100.00%

Average	0.10141
Variance	0.00193
Skewness	0.80528
Kurtosis	0.76196

Table 49.--Simulated Sampling Distribution of Dn (n=48, N=960)
Sampling from Unimodal Symmetrical, Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.01458	1	.01%	0.01667	5	.07%
0.01667	4	.07%	0.01979	7	.16%
0.01875	1	.08%	0.02083	16	.37%
0.01979	3	.12%	0.02188	22	.67%
0.02083	12	.28%	0.02500	55	1.40%
0.02500	66	1.16%	0.02708	114	2.92%
0.02708	100	2.49%	0.03542	68	3.83%
0.03542	79	3.55%	0.03750	145	5.76%
0.03750	143	5.45%	0.04167	250	9.09%
0.04167	203	8.16%	0.04271	71	10.04%
0.04271	63	9.00%	0.04583	300	14.04%
0.04583	266	12.55%	0.04792	338	18.55%
0.04792	312	16.71%	0.05625	221	21.49%
0.05625	202	19.40%	0.05833	402	26.85%
0.05833	364	24.25%	0.05833	0	26.85%
0.06250	433	30.03%	0.06250	484	33.31%
0.06667	515	36.89%	0.06354	39	33.83%
0.06875	258	40.33%	0.06667	467	40.05%
0.07708	192	42.89%	0.06875	272	43.68%
0.07917	440	48.76%	0.07708	158	45.79%
0.08333	544	56.01%	0.07917	440	51.65%
0.08750	502	62.71%	0.08333	525	58.65%
0.09792	178	65.08%	0.08750	433	64.43%
0.10000	344	69.67%	0.08958	73	65.40%
0.10417	464	75.85%	0.09792	128	67.11%
0.10833	264	79.37%	0.10000	351	71.79%
0.11875	50	80.04%	0.10417	417	77.35%
0.12083	257	83.47%	0.10521	3	77.39%
0.12500	301	87.48%	0.10833	312	81.55%
0.12917	147	89.44%	0.11875	44	82.13%
0.13958	15	89.64%	0.12083	214	84.99%
0.14167	140	91.51%	0.12500	272	88.61%
0.14583	196	94.12%	0.12917	158	90.72%
0.15000	74	95.11%	0.13958	15	90.92%

Table 49.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.16250	68	96.01%
0.16667	121	97.63%
0.18750	47	99.05%
0.19167	4	99.11%
0.20417	10	99.24%
0.20833	35	99.71%
0.22500	5	99.77%
0.22917	11	99.92%
0.24583	1	99.93%
0.25000	2	99.96%
0.26667	3	100.00%

Average	0.08681
Variance	0.00133
Skewness	0.86739
Kurtosis	0.90529

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.14167	129	92.64%
0.14583	186	95.12%
0.16250	51	96.67%
0.16667	85	97.80%
0.17083	28	98.17%
0.18125	1	98.19%
0.18333	27	98.55%
0.18750	45	99.15%
0.19167	4	99.20%
0.20417	15	99.40%
0.20833	21	99.68%
0.22500	9	99.80%
0.22917	8	99.91%
0.24583	1	99.92%
0.25000	2	99.95%
0.27083	2	99.97%
0.29167	2	100.00%

Average	0.08441
Variance	0.00129
Skewness	0.93022
Kurtosis	1.24215

Table 50.--Simulated Sampling Distribution of Dn (n=48, N=480)
Sampling from Unimodal Symmetrical, Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.01667	2	.03%	0.00625	1	.01%
0.01875	9	.15%	0.01458	1	.03%
0.02083	13	.32%	0.01667	2	.05%
0.02500	63	1.16%	0.01875	13	.23%
0.02708	93	2.40%	0.02083	26	.57%
0.03542	58	3.17%	0.02292	21	.85%
0.03750	144	5.09%	0.02500	61	1.67%
0.04167	232	8.19%	0.02708	122	3.29%
0.04375	64	9.04%	0.03542	93	4.53%
0.04583	290	12.91%	0.03750	167	6.76%
0.04792	316	17.12%	0.04167	251	10.11%
0.05625	197	19.75%	0.04375	56	10.85%
0.05833	397	25.04%	0.04583	301	14.87%
0.06250	445	30.97%	0.04792	317	19.09%
0.06458	44	31.56%	0.05625	246	22.37%
0.06667	488	38.07%	0.05833	405	27.77%
0.06875	279	41.79%	0.06250	516	34.65%
0.07708	169	44.04%	0.06667	527	41.68%
0.07917	409	49.49%	0.06875	250	45.01%
0.08333	472	55.79%	0.07708	159	47.13%
0.08750	458	61.89%	0.07917	438	52.97%
0.08958	76	62.91%	0.08333	543	60.21%
0.10000	475	69.24%	0.08750	484	66.67%
0.10417	409	74.69%	0.08958	57	67.43%
0.10833	307	78.79%	0.09792	92	68.65%
0.11875	46	79.40%	0.10000	344	73.24%
0.12083	239	82.59%	0.10417	445	79.17%
0.12500	298	86.56%	0.10833	286	82.99%
0.12917	181	88.97%	0.11875	29	83.37%
0.13958	19	89.23%	0.12083	220	86.31%
0.14167	145	91.16%	0.12500	296	90.25%
0.14583	194	93.75%	0.12500	0	90.25%
0.15000	72	94.71%	0.14167	242	93.48%
0.16042	6	94.79%	0.14583	174	95.80%

Table 50.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.16250	75	95.79%
0.16667	113	97.29%
0.18333	35	98.09%
0.18750	60	98.89%
0.19167	8	99.00%
0.20417	15	99.20%
0.20833	28	99.57%
0.21250	1	99.59%
0.22500	6	99.67%
0.22917	11	99.81%
0.23333	1	99.83%
0.24583	6	99.91%
0.25000	3	99.95%
0.27083	3	99.99%
0.31250	1	100.00%

Average	0.08714
Variance	0.00140
Skewness	0.93008
Kurtosis	1.11136

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.15000	55	96.53%
0.16042	4	96.59%
0.17083	20	98.45%
0.18333	25	98.79%
0.18750	52	99.48%
0.19167	6	99.56%
0.20417	8	99.67%
0.20833	18	99.91%
0.22500	1	99.92%
0.22500	0	99.92%
0.22917	6	100.00%

Average	0.08263
Variance	0.00120
Skewness	0.81953
Kurtosis	0.71506

Table 51.--Simulated Sampling Distribution of Dn (n=48, N=240)
Sampling from Unimodal Symmetrical, Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.02083	35	.47%	0.00833	1	.01%
0.02500	44	1.05%	0.01250	2	.04%
0.02917	83	2.16%	0.01667	8	.15%
0.03333	85	3.29%	0.02083	68	1.05%
0.03750	166	5.51%	0.02500	82	2.15%
0.04167	272	9.13%	0.02917	142	4.04%
0.04583	282	12.89%	0.03333	137	5.87%
0.05000	273	16.53%	0.03750	214	8.72%
0.05417	219	19.45%	0.04167	390	13.92%
0.05833	351	24.13%	0.04583	421	19.53%
0.06250	515	31.00%	0.05000	326	23.88%
0.06667	492	37.56%	0.05417	250	27.21%
0.07083	213	40.40%	0.05833	462	33.37%
0.07500	225	43.40%	0.06250	561	40.85%
0.07917	415	48.93%	0.06667	497	47.48%
0.08333	513	55.77%	0.07083	203	50.19%
0.08750	454	61.83%	0.07500	197	52.81%
0.09167	78	62.87%	0.07917	458	58.92%
0.09583	149	64.85%	0.08333	524	65.91%
0.10000	334	69.31%	0.08750	434	71.69%
0.10417	419	74.89%	0.09583	132	73.45%
0.10833	299	78.88%	0.10000	307	77.55%
0.11667	60	79.68%	0.10417	416	83.09%
0.12083	217	82.57%	0.10833	263	86.60%
0.12500	308	86.68%	0.11667	33	87.04%
0.12917	172	88.97%	0.12083	168	89.28%
0.13750	29	89.36%	0.12500	249	92.60%
0.14167	144	91.28%	0.12917	119	94.19%
0.14583	220	94.21%	0.13750	9	94.31%
0.15000	66	95.09%	0.14167	88	95.48%
0.15833	11	95.24%	0.14583	131	97.23%
0.16250	66	96.12%	0.15000	33	97.67%
0.16667	98	97.43%	0.15833	2	97.69%
0.17083	20	97.69%	0.16250	39	98.21%

Table 51.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.18333	33	98.13%
0.18750	60	98.93%
0.20417	18	99.31%
0.20833	28	99.68%
0.22500	7	99.77%
0.22917	11	99.92%
0.23333	1	99.93%
0.25000	4	99.99%
0.26667	1	100.00%

Average	0.08714
Variance	0.00135
Skewness	0.86770
Kurtosis	0.81587

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.16667	58	98.99%
0.17083	16	99.20%
0.20417	6	99.71%
0.20833	9	99.83%
0.21250	1	99.84%
0.22500	4	99.89%
0.22917	3	99.93%
0.22917	0	99.93%
0.27083	4	99.99%
0.31250	1	100.00%

Average	0.07767
Variance	0.00109
Skewness	0.92957
Kurtosis	1.43466

Table 52.--Simulated Sampling Distribution of Dn (n=48, N=160)
Sampling from Unimodal Symmetrical, Population

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.01667	1	.01%
0.01875	1	.03%
0.02083	53	.73%
0.02292	41	1.28%
0.02500	88	2.45%
0.03750	72	3.41%
0.04167	660	12.21%
0.04375	77	13.24%
0.04583	330	17.64%
0.05833	200	20.31%
0.06250	1313	37.81%
0.06458	38	38.32%
0.06667	283	42.09%
0.07917	167	44.32%
0.08333	1426	63.33%
0.08542	8	63.44%
0.08750	90	64.64%
0.10000	87	65.80%
0.10417	1048	79.77%
0.10625	2	79.80%
0.12083	43	80.37%
0.12500	682	89.47%
0.14167	20	89.73%
0.14583	376	94.75%
0.16250	5	94.81%
0.16667	213	97.65%
0.18333	1	97.67%
0.18750	106	99.08%
0.20833	47	99.71%
0.22917	14	99.89%
0.25000	2	99.92%
0.27083	6	100.00%

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.00417	1	.01%
0.01667	1	.03%
0.01875	5	.09%
0.02083	129	1.81%
0.02292	74	2.80%
0.02500	209	5.59%
0.03750	135	7.39%
0.04167	1118	22.29%
0.04375	74	23.28%
0.04583	451	29.29%
0.05833	244	32.55%
0.06250	1698	55.19%
0.06667	243	58.43%
0.07917	132	60.19%
0.08333	1368	78.43%
0.08750	40	78.96%
0.10000	38	79.47%
0.10417	805	90.20%
0.12083	12	90.36%
0.12500	430	96.09%
0.14583	195	98.69%
0.16667	70	99.63%
0.18750	20	99.89%
0.20833	6	99.97%
0.22917	2	100.00%

Table 52.--Continued.

Sampling With Replacement		Sampling Without Replacement	
Average	0.08618	Average	0.07176
Variance	0.00140	Variance	0.00096
Skewness	0.92724	Skewness	0.87712
Kurtosis	1.03509	Kurtosis	0.88749

Table 53.--Simulated Sampling Distribution of Dn (n=6, N=120)
Sampling from Skewed Positive Population

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.08333	62	.83%
0.10833	25	1.16%
0.13333	90	2.36%
0.14167	135	4.16%
0.15833	109	5.61%
0.16667	1396	24.23%
0.17500	771	34.51%
0.19167	613	42.68%
0.20000	755	52.75%
0.25000	207	55.51%
0.27500	94	56.76%
0.30000	375	61.76%
0.30833	431	67.51%
0.32500	169	69.76%
0.33333	1033	83.53%
0.35833	358	88.31%
0.36667	330	92.71%
0.41667	48	93.35%
0.47500	128	95.05%
0.49167	42	95.61%
0.50000	166	97.83%
0.52500	74	98.81%
0.53333	58	99.59%
0.58333	5	99.65%
0.60833	2	99.68%
0.65833	1	99.69%
0.66667	18	99.93%
0.70000	5	100.00%

Average	0.25836
Variance	0.01046
Skewness	0.92814
Kurtosis	0.52959

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.08333	63	.84%
0.10833	25	1.17%
0.13333	82	2.27%
0.14167	140	4.13%
0.15833	105	5.53%
0.16667	1439	24.72%
0.17500	807	35.48%
0.19167	557	42.91%
0.20000	759	53.03%
0.25000	231	56.11%
0.27500	102	57.47%
0.30000	349	62.12%
0.30833	456	68.20%
0.32500	179	70.59%
0.33333	1026	84.27%
0.35833	357	89.03%
0.36667	365	93.89%
0.41667	25	94.23%
0.44167	12	94.39%
0.46667	2	94.41%
0.47500	101	95.76%
0.49167	32	96.19%
0.50000	141	98.07%
0.52500	61	98.88%
0.53333	66	99.76%
0.58333	1	99.77%
0.65833	3	99.81%
0.66667	12	99.97%
0.70000	2	100.00%

Average	0.25574
Variance	0.00984
Skewness	0.89157
Kurtosis	0.43712

Table 54.--Simulated Sampling Distribution of Dn (n=6, N=60)
Sampling from Skewed Positive Population

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.08333	84	1.12%
0.11667	32	1.55%
0.13333	17	1.77%
0.15000	339	6.29%
0.16667	1373	24.60%
0.18333	2125	52.93%
0.25000	200	55.60%
0.28333	76	56.61%
0.30000	40	57.15%
0.31667	919	69.40%
0.33333	954	82.12%
0.35000	743	92.03%
0.41667	49	92.68%
0.45000	4	92.73%
0.46667	3	92.77%
0.48333	171	95.05%
0.50000	171	97.33%
0.51667	160	99.47%
0.58333	8	99.57%
0.61667	1	99.59%
0.65000	7	99.68%
0.66667	18	99.92%
0.68333	5	99.99%
0.83333	1	100.00%

Average	0.25805
Variance	0.01106
Skewness	0.93854
Kurtosis	0.52245

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.08333	80	1.07%
0.11667	40	1.60%
0.13333	15	1.80%
0.15000	316	6.01%
0.16667	1497	25.97%
0.18333	2322	56.93%
0.25000	191	59.48%
0.28333	75	60.48%
0.30000	25	60.81%
0.31667	907	72.91%
0.33333	947	85.53%
0.35000	662	94.36%
0.41667	18	94.60%
0.45000	5	94.67%
0.48333	115	96.20%
0.50000	141	98.08%
0.51667	125	99.75%
0.65000	5	99.81%
0.66667	10	99.95%
0.68333	4	100.00%

Average	0.24801
Variance	0.00964
Skewness	0.99792
Kurtosis	0.62706

Table 55.--Simulated Sampling Distribution of Dn (n=6, N=30)
Sampling from Skewed Positive Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.13333	1603	21.92%	0.13333	1938	26.95%
0.20000	2405	53.99%	0.20000	2835	64.75%
0.26667	128	55.69%	0.26667	144	66.67%
0.30000	1574	76.68%	0.30000	1323	84.31%
0.36667	1320	94.28%	0.36667	998	97.61%
0.43333	17	94.51%	0.46667	86	98.76%
0.46667	153	96.55%	0.53333	90	99.96%
0.53333	247	99.84%	0.70000	3	100.00%
0.70000	12	100.00%			
Average	0.25430		Average	0.22978	
Variance	0.01078		Variance	0.00822	
Skewness	0.78280		Skewness	0.87391	
Kurtosis	0.35321		Kurtosis	0.51305	

Table 56.--Simulated Sampling Distribution of Dn (n=6, N=20)
Sampling from Skewed Positive Population

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.13333	213	4.27%
0.15000	585	12.07%
0.16667	1123	27.04%
0.18333	624	35.36%
0.20000	1484	55.15%
0.28333	205	57.88%
0.30000	745	67.81%
0.33333	863	79.32%
0.35000	336	83.80%
0.36667	809	94.59%
0.50000	183	97.03%
0.51667	69	97.95%
0.53333	143	99.85%
0.68333	1	99.87%
0.70000	10	100.00%

Average	0.25556
Variance	0.01042
Skewness	0.83471
Kurtosis	0.37034

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.13333	309	7.31%
0.15000	822	18.27%
0.16667	1317	35.83%
0.18333	796	46.44%
0.20000	1910	71.91%
0.30000	647	80.53%
0.33333	716	90.08%
0.35000	135	91.88%
0.36667	508	98.65%
0.50000	73	99.63%
0.53333	28	100.00%

Average	0.22001
Variance	0.00687
Skewness	1.00838
Kurtosis	0.65540

Table 57.--Simulated Sampling Distribution of Dn (n=12, N=240)
Sampling from Skewed Positive Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.05417	13	.25%	0.05417	13	.28%
0.05833	18	.49%	0.05833	19	.53%
0.07500	4	.55%	0.07500	3	.57%
0.07917	24	.87%	0.07917	16	.79%
0.08333	574	8.52%	0.08333	681	9.87%
0.08750	277	12.21%	0.08750	316	14.08%
0.10833	266	15.76%	0.10833	296	18.03%
0.11250	146	17.71%	0.11250	151	20.04%
0.11667	354	22.43%	0.11667	384	25.16%
0.13333	294	26.35%	0.13333	278	28.87%
0.13750	52	27.04%	0.13750	46	29.48%
0.14167	358	31.81%	0.14167	419	35.07%
0.15833	6	31.89%	0.15833	4	35.12%
0.16250	151	33.91%	0.16250	158	37.23%
0.16667	1441	53.12%	0.16667	1481	56.97%
0.17083	321	57.40%	0.17083	303	61.01%
0.19167	374	62.39%	0.19167	359	65.80%
0.19583	72	63.35%	0.19583	69	66.72%
0.20000	342	67.91%	0.20000	351	71.40%
0.21667	276	71.59%	0.21667	208	74.17%
0.22500	355	76.32%	0.22500	357	78.93%
0.24583	119	77.91%	0.24583	112	80.43%
0.25000	641	86.45%	0.25000	595	88.36%
0.27500	208	89.23%	0.27500	181	90.77%
0.27917	12	89.39%	0.28333	165	92.97%
0.28333	163	91.56%	0.30000	40	93.51%
0.30000	64	92.41%	0.30833	120	95.11%
0.30833	144	94.33%	0.32917	32	95.53%
0.32917	39	94.85%	0.33333	160	97.67%
0.33333	180	97.25%	0.35833	61	98.48%
0.35833	63	98.09%	0.36250	1	98.49%
0.36250	2	98.12%	0.36667	36	98.97%
0.36667	45	98.72%	0.39167	16	99.19%
0.39167	29	99.11%	0.41250	3	99.23%

Table 57.--Continued.

Sampling With Replacement				Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*		*Dn*	*Frequency*	*Cum. %*
0.41250	11	99.25%		0.41667	36	99.71%
0.41667	31	99.67%		0.44167	9	99.83%
0.45000	16	99.88%		0.45000	7	99.92%
0.47500	5	99.95%		0.47500	1	99.93%
0.49583	1	99.96%		0.50000	2	99.96%
0.50000	2	99.99%		0.52500	3	100.00%
0.61667	1	100.00%				
Average	0.184942			Average	0.178816	
Variance	0.005318			Variance	0.005095	
Skewness	0.76866			Skewness	0.82345	
Kurtosis	0.63904			Kurtosis	0.76571	

Table 58.--Simulated Sampling Distribution of Dn (n=12, N=120)
Sampling from Skewed Positive Population

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.05833	28	.51%
0.07500	24	.83%
0.08333	581	8.57%
0.09167	255	11.97%
0.10833	405	17.37%
0.11667	322	21.67%
0.13333	362	26.49%
0.14167	402	31.85%
0.15833	183	34.29%
0.16667	1450	53.63%
0.17500	298	57.60%
0.19167	427	63.29%
0.20000	359	68.08%
0.21667	268	71.65%
0.22500	334	76.11%
0.24167	122	77.73%
0.25000	654	86.45%
0.27500	222	89.41%
0.28333	163	91.59%
0.30000	63	92.43%
0.30833	141	94.31%
0.32500	46	94.92%
0.33333	169	97.17%
0.35833	70	98.11%
0.36667	51	98.79%
0.38333	1	98.80%
0.39167	25	99.13%
0.40833	9	99.25%
0.41667	27	99.61%
0.44167	6	99.69%
0.45000	11	99.84%
0.47500	4	99.89%
0.49167	1	99.91%
0.50000	6	99.99%

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.05833	38	.60%
0.07500	30	1.00%
0.08333	732	10.76%
0.09167	299	14.75%
0.10833	457	20.84%
0.11667	434	26.63%
0.13333	342	31.19%
0.14167	426	36.87%
0.15833	194	39.45%
0.16667	1479	59.17%
0.17500	275	62.84%
0.19167	400	68.17%
0.20000	335	72.64%
0.21667	262	76.13%
0.22500	325	80.47%
0.24167	110	81.93%
0.25000	554	89.32%
0.27500	173	91.63%
0.28333	144	93.55%
0.30000	34	94.00%
0.30833	107	95.43%
0.32500	31	95.84%
0.33333	143	97.75%
0.35833	49	98.40%
0.36667	47	99.03%
0.39167	21	99.31%
0.40833	9	99.43%
0.41667	29	99.81%
0.45000	6	99.89%
0.47500	4	99.95%
0.50000	4	100.00%

Table 58.--Continued.

Sampling With Replacement				Sampling Without Replacement	
Dn	*Frequency*	*Cum. %*			
0.53333	1	100.00%			
Average	0.18510			Average	0.17572
Variance	0.00528			Variance	0.00493
Skewness	0.78547			Skewness	0.85900
Kurtosis	0.66771			Kurtosis	0.81318

Table 59.--Simulated Sampling Distribution of Dn (n=12, N=60)
Sampling from Skewed Positive Population

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.06667	53	.81%
0.08333	625	9.15%
0.10000	918	21.39%
0.11667	167	23.61%
0.13333	56	24.36%
0.15000	798	35.00%
0.16667	1422	53.96%
0.18333	1082	68.39%
0.20000	68	69.29%
0.21667	14	69.48%
0.23333	639	78.00%
0.25000	620	86.27%
0.26667	417	91.83%
0.31667	200	94.49%
0.33333	186	96.97%
0.35000	121	98.59%
0.40000	39	99.11%
0.41667	26	99.45%
0.43333	28	99.83%
0.48333	4	99.88%
0.50000	4	99.93%
0.51667	4	99.99%
0.60000	1	100.00%

Average	0.18324
Variance	0.00538
Skewness	0.82213
Kurtosis	0.84559

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.06667	81	1.32%
0.08333	885	13.12%
0.10000	1171	28.73%
0.11667	189	31.25%
0.13333	52	31.95%
0.15000	929	44.33%
0.16667	1422	63.29%
0.18333	1028	77.00%
0.20000	30	77.40%
0.23333	518	84.31%
0.25000	502	91.00%
0.26667	333	95.44%
0.31667	118	97.01%
0.33333	98	98.32%
0.35000	82	99.41%
0.40000	17	99.64%
0.41667	11	99.79%
0.43333	12	99.95%
0.48333	2	99.97%
0.50000	2	100.00%

Average	0.18324
Variance	0.00538
Skewness	0.82213
Kurtosis	0.84559

Table 60.--Simulated Sampling Distribution of Dn (n=12, N=40)
Sampling from Skewed Positive Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.05833	29	.57%	0.05833	66	1.43%
0.06667	46	1.19%	0.06667	70	2.36%
0.07500	241	4.40%	0.07500	458	8.47%
0.09167	460	10.53%	0.09167	753	18.51%
0.10000	197	13.16%	0.10000	345	23.11%
0.10833	281	16.91%	0.10833	390	28.31%
0.11667	499	23.56%	0.11667	654	37.03%
0.13333	271	27.17%	0.13333	347	41.65%
0.14167	438	33.01%	0.14167	481	48.07%
0.15000	260	36.48%	0.15000	267	51.63%
0.15833	675	45.48%	0.15833	710	61.09%
0.17500	736	55.29%	0.17500	674	70.08%
0.18333	263	58.80%	0.18333	276	73.76%
0.19167	363	63.64%	0.19167	362	78.59%
0.20000	417	69.20%	0.20000	345	83.19%
0.21667	263	72.71%	0.21667	188	85.69%
0.22500	355	77.44%	0.22500	293	89.60%
0.24167	378	82.48%	0.24167	221	92.55%
0.25833	254	85.87%	0.25833	148	94.52%
0.26667	113	87.37%	0.27500	133	96.29%
0.27500	180	89.77%	0.28333	78	97.33%
0.28333	182	92.20%	0.30000	29	97.72%
0.30000	59	92.99%	0.30833	73	98.69%
0.30833	147	94.95%	0.32500	26	99.04%
0.32500	85	96.08%	0.34167	28	99.41%
0.34167	76	97.09%	0.35833	21	99.69%
0.35000	24	97.41%	0.36667	8	99.80%
0.35833	57	98.17%	0.39167	11	99.95%
0.36667	45	98.77%	0.40833	1	99.96%
0.39167	29	99.16%	0.42500	2	99.99%
0.40833	4	99.21%			
0.42500	23	99.52%			
0.43333	5	99.59%			
0.44167	15	99.79%			

Table 60.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.45000	6	99.87%
0.47500	3	99.91%
0.49167	1	99.92%
0.50833	1	99.93%
0.52500	2	99.96%
0.53333	2	99.99%
0.59167	1	100.00%

Average	0.18326
Variance	0.00528
Skewness	0.83339
Kurtosis	0.87098

Sampling Without Replacement

Average	0.15507
Variance	0.00362
Skewness	0.79815
Kurtosis	0.65733

Table 61.--Simulated Sampling Distribution of Dn (n=24, N=480)
Sampling from Skewed Positive Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.02500	1	.01%	0.03333	1	.01%
0.03333	3	.05%	0.03958	3	.05%
0.03750	2	.08%	0.04167	91	1.27%
0.03958	6	.16%	0.04375	45	1.87%
0.04167	84	1.28%	0.04583	38	2.37%
0.04375	46	1.89%	0.05000	71	3.32%
0.04583	41	2.44%	0.05625	125	4.99%
0.05000	71	3.39%	0.05833	151	7.00%
0.05625	103	4.76%	0.06667	135	8.80%
0.05833	127	6.45%	0.06875	63	9.64%
0.06667	122	8.08%	0.07292	9	9.76%
0.06875	58	8.85%	0.07500	109	11.21%
0.07292	7	8.95%	0.07917	63	12.05%
0.07500	121	10.56%	0.08125	154	14.11%
0.07917	83	11.67%	0.08333	724	23.76%
0.08125	154	13.72%	0.08542	287	27.59%
0.08333	638	22.23%	0.08750	149	29.57%
0.08542	273	25.87%	0.09167	281	33.32%
0.08750	139	27.72%	0.09792	27	33.68%
0.09167	302	31.75%	0.10000	347	38.31%
0.10000	351	36.43%	0.10833	236	41.45%
0.10833	261	39.91%	0.11042	71	42.40%
0.11042	84	41.03%	0.11458	2	42.43%
0.11667	211	43.84%	0.11667	228	45.47%
0.12292	235	46.97%	0.12292	231	48.55%
0.12500	661	55.79%	0.12500	686	57.69%
0.12708	311	59.93%	0.12708	303	61.73%
0.12917	80	61.00%	0.12917	87	62.89%
0.13333	267	64.56%	0.13333	234	66.01%
0.14167	287	68.39%	0.13958	5	66.08%
0.15000	227	71.41%	0.14167	313	70.25%
0.15208	23	71.72%	0.15000	199	72.91%
0.15833	181	74.13%	0.15208	22	73.20%
0.16458	149	76.12%	0.15833	165	75.40%

Table 61.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.16667	297	80.08%
0.16875	162	82.24%
0.17083	33	82.68%
0.17500	138	84.52%
0.18333	177	86.88%
0.19167	150	88.88%
0.20000	114	90.40%
0.20625	108	91.84%
0.20833	83	92.95%
0.21042	89	94.13%
0.21250	8	94.24%
0.21250	0	94.24%
0.21667	30	94.64%
0.22500	82	95.73%
0.23333	47	96.36%
0.24167	46	96.97%
0.24792	42	97.53%
0.25000	16	97.75%
0.25208	17	97.97%
0.25417	3	98.01%
0.25833	14	98.20%
0.26667	24	98.52%
0.27500	23	98.83%
0.28333	18	99.07%
0.28958	16	99.28%
0.29167	16	99.49%
0.29375	4	99.55%
0.30833	8	99.65%
0.31667	6	99.73%
0.31875	1	99.75%
0.32500	7	99.84%
0.33125	6	99.92%
0.33333	3	99.96%
0.35000	1	99.97%
0.37292	1	99.99%

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.16458	158	77.51%
0.16667	280	81.24%
0.16875	206	83.99%
0.17083	24	84.31%
0.17500	113	85.81%
0.18333	198	88.45%
0.19167	126	90.13%
0.19375	3	90.17%
0.20000	80	91.24%
0.20625	81	92.32%
0.20833	63	93.16%
0.21042	57	93.92%
0.21250	7	94.01%
0.21667	32	94.44%
0.22500	89	95.63%
0.23333	62	96.45%
0.24167	51	97.13%
0.24792	31	97.55%
0.25000	25	97.88%
0.25208	24	98.20%
0.25833	6	98.28%
0.26667	31	98.69%
0.27500	22	98.99%
0.28333	14	99.17%
0.28958	19	99.43%
0.29375	6	99.51%
0.29583	1	99.52%
0.30833	9	99.64%
0.31667	4	99.69%
0.32500	9	99.81%
0.33125	4	99.87%
0.35000	2	99.89%
0.35833	4	99.95%
0.36667	1	99.96%
0.37292	2	99.99%

Table 61.--Continued.

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.40833	1	100.00%	0.43333	1	100.00%

Sampling With Replacement		Sampling Without Replacement	
Average	0.12898	Average	0.12695
Variance	0.00263	Variance	0.00260
Skewness	0.84921	Skewness	0.93434
Kurtosis	0.86013	Kurtosis	1.20570

Table 62.--Simulated Sampling Distribution of Dn (n=24, N=240)
Sampling from Skewed Positive Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.02917	2	.03%	0.01667	1	.01%
0.03333	1	.04%	0.02917	2	.04%
0.03750	4	.09%	0.03333	7	.13%
0.04167	98	1.40%	0.03750	3	.17%
0.04583	57	2.16%	0.04167	140	2.04%
0.05000	61	2.97%	0.04583	42	2.60%
0.05417	119	4.56%	0.05000	74	3.59%
0.05833	130	6.29%	0.05417	129	5.31%
0.06667	106	7.71%	0.05833	153	7.35%
0.07083	60	8.51%	0.06667	117	8.91%
0.07500	124	10.16%	0.07083	55	9.64%
0.08333	1115	25.03%	0.07500	148	11.61%
0.08750	209	27.81%	0.07917	138	13.45%
0.09167	278	31.52%	0.08333	1096	28.07%
0.09583	43	32.09%	0.08750	218	30.97%
0.10000	308	36.20%	0.09167	310	35.11%
0.10833	241	39.41%	0.09583	42	35.67%
0.11250	66	40.29%	0.10000	344	40.25%
0.11667	215	43.16%	0.10833	255	43.65%
0.12083	160	45.29%	0.11250	56	44.40%
0.12500	950	57.96%	0.11667	219	47.32%
0.12917	135	59.76%	0.12083	176	49.67%
0.13333	253	63.13%	0.12500	947	62.29%
0.14167	282	66.89%	0.12917	117	63.85%
0.15000	206	69.64%	0.13333	249	67.17%
0.15417	22	69.93%	0.14167	311	71.32%
0.15833	172	72.23%	0.15000	215	74.19%
0.16250	112	73.72%	0.15833	203	76.89%
0.16667	603	81.76%	0.16250	95	78.16%
0.17083	47	82.39%	0.16667	528	85.20%
0.17500	153	84.43%	0.17083	20	85.47%
0.18333	177	86.79%	0.17500	105	86.87%
0.19167	118	88.36%	0.18333	177	89.23%
0.20000	103	89.73%	0.19167	113	90.73%

Table 62.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.20417	45	90.33%
0.20833	226	93.35%
0.21667	45	93.95%
0.22500	87	95.11%
0.23333	64	95.96%
0.24167	37	96.45%
0.24583	18	96.69%
0.25000	91	97.91%
0.26667	45	98.51%
0.27500	31	98.92%
0.28333	17	99.15%
0.28750	2	99.17%
0.29167	21	99.45%
0.30000	1	99.47%
0.30833	7	99.56%
0.31667	11	99.71%
0.32500	6	99.79%
0.33333	8	99.89%
0.35833	2	99.92%
0.36667	1	99.93%
0.37083	1	99.95%
0.37500	2	99.97%
0.40833	2	100.00%

Average	0.13019
Variance	0.00273
Skewness	0.87038
Kurtosis	0.95604

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.19583	2	90.76%
0.20000	69	91.68%
0.20833	213	94.52%
0.21667	40	95.05%
0.22500	96	96.33%
0.23333	59	97.12%
0.24167	33	97.56%
0.24583	16	97.77%
0.25000	56	98.52%
0.25833	2	98.55%
0.26667	34	99.00%
0.27500	26	99.35%
0.28333	7	99.44%
0.28750	1	99.45%
0.29167	13	99.63%
0.30000	1	99.64%
0.30833	7	99.73%
0.31667	7	99.83%
0.32500	3	99.87%
0.32917	1	99.88%
0.33333	4	99.93%
0.35000	3	99.97%
0.36667	2	100.00%

Average	0.12455
Variance	0.00247
Skewness	0.87754
Kurtosis	0.92895

Table 63.--Simulated Sampling Distribution of Dn (n=24, N=120)
Sampling from Skewed Positive Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.02500	1	.01%	0.02500	3	.04%
0.03333	4	.07%	0.03333	12	.20%
0.03333	0	.07%	0.04167	188	2.71%
0.04167	73	1.04%	0.05000	225	5.71%
0.05000	142	2.93%	0.05833	291	9.59%
0.05833	183	5.37%	0.06667	255	12.99%
0.06667	163	7.55%	0.07500	313	17.16%
0.07500	230	10.61%	0.08333	1168	32.73%
0.08333	980	23.68%	0.09167	593	40.64%
0.09167	509	30.47%	0.10000	404	46.03%
0.10000	348	35.11%	0.10833	320	50.29%
0.10833	334	39.56%	0.11667	425	55.96%
0.11667	384	44.68%	0.12500	960	68.76%
0.12500	975	57.68%	0.13333	358	73.53%
0.13333	405	63.08%	0.14167	294	77.45%
0.14167	265	66.61%	0.15000	225	80.45%
0.15000	260	70.08%	0.15833	237	83.61%
0.15833	291	73.96%	0.16667	394	88.87%
0.16667	556	81.37%	0.17500	140	90.73%
0.17500	182	83.80%	0.18333	124	92.39%
0.18333	173	86.11%	0.19167	100	93.72%
0.19167	171	88.39%	0.20000	105	95.12%
0.20000	142	90.28%	0.20833	140	96.99%
0.20833	223	93.25%	0.21667	20	97.25%
0.21667	50	93.92%	0.22500	52	97.95%
0.22500	85	95.05%	0.23333	35	98.41%
0.23333	64	95.91%	0.24167	30	98.81%
0.24167	73	96.88%	0.25000	41	99.36%
0.25000	97	98.17%	0.25833	3	99.40%
0.25833	7	98.27%	0.26667	19	99.65%
0.26667	27	98.63%	0.27500	11	99.80%
0.27500	28	99.00%	0.28333	3	99.84%
0.28333	18	99.24%	0.29167	3	99.88%
0.29167	17	99.47%	0.30833	5	99.95%

Table 63.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.30000	1	99.48%
0.30833	13	99.65%
0.31667	7	99.75%
0.32500	9	99.87%
0.33333	3	99.91%
0.35000	2	99.93%
0.35833	1	99.95%
0.37500	1	99.96%
0.40000	1	99.97%
0.45000	1	99.99%
0.45833	1	100.00%

Average	0.13097
Variance	0.00264
Skewness	0.88780
Kurtosis	1.07774

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.31667	1	99.96%
0.32500	1	99.97%
0.33333	2	100.00%

Average	0.11618
Variance	0.00206
Skewness	0.82073
Kurtosis	0.76093

Table 64.--Simulated Sampling Distribution of Dn (n=24, N=80)
Sampling from Skewed Positive Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.02500	2	.03%	0.02083	1	.01%
0.02917	1	.04%	0.02500	2	.04%
0.03333	22	.33%	0.02917	6	.12%
0.03750	22	.63%	0.03333	51	.80%
0.04583	16	.84%	0.03750	40	1.33%
0.05000	162	3.00%	0.04583	30	1.73%
0.05417	78	4.04%	0.05000	383	6.84%
0.05833	20	4.31%	0.05417	183	9.28%
0.06250	146	6.25%	0.05833	44	9.87%
0.07083	148	8.23%	0.06250	268	13.44%
0.07500	578	15.93%	0.07083	224	16.43%
0.07917	130	17.67%	0.07500	852	27.79%
0.08750	84	18.79%	0.07917	220	30.72%
0.09167	893	30.69%	0.08750	83	31.83%
0.09583	267	34.25%	0.09167	1118	46.73%
0.10000	32	34.68%	0.09583	356	51.48%
0.10417	82	35.77%	0.10417	62	52.31%
0.11250	257	39.20%	0.11250	321	56.59%
0.11667	843	50.44%	0.11667	837	67.75%
0.12083	96	51.72%	0.12083	74	68.73%
0.13333	810	62.52%	0.13333	694	77.99%
0.13750	279	66.24%	0.13750	262	81.48%
0.14583	43	66.81%	0.15417	212	84.31%
0.15417	237	69.97%	0.15833	402	89.67%
0.15833	598	77.95%	0.16250	14	89.85%
0.16250	59	78.73%	0.17500	200	92.52%
0.17500	356	83.48%	0.17917	120	94.12%
0.17917	185	85.95%	0.19583	96	95.40%
0.18750	11	86.09%	0.20000	136	97.21%
0.19583	154	88.15%	0.21667	65	98.08%
0.20000	261	91.63%	0.22083	47	98.71%
0.21667	148	93.60%	0.23750	31	99.12%
0.22083	84	94.72%	0.24167	33	99.56%
0.23750	69	95.64%	0.25833	11	99.71%

Table 64.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.24167	121	97.25%
0.24583	6	97.33%
0.26250	80	98.40%
0.27917	23	98.71%
0.28333	36	99.19%
0.30000	17	99.41%
0.30417	10	99.55%
0.32083	6	99.63%
0.32500	9	99.75%
0.34167	6	99.83%
0.34167	0	99.83%
0.36250	8	99.93%
0.36667	1	99.95%
0.38750	3	99.99%
0.42500	1	100.00%

Average	0.13100
Variance	0.00274
Skewness	0.91452
Kurtosis	1.10250

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.26250	11	99.85%
0.27917	9	99.97%
0.28333	1	99.99%
0.30417	1	100.00%

Average	0.10955
Variance	0.00185
Skewness	0.83665
Kurtosis	0.63872

Table 65.--Simulated Sampling Distribution of Dn (n=48, N=960)
Sampling from Skewed Positive Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.01563	1	.01%	0.01458	1	.01%
0.01771	2	.04%	0.01771	1	.03%
0.01875	1	.05%	0.02292	2	.05%
0.02292	1	.07%	0.02396	7	.15%
0.02396	9	.19%	0.02604	8	.25%
0.02604	8	.29%	0.02708	9	.37%
0.02708	17	.52%	0.02813	18	.61%
0.02813	13	.69%	0.03125	45	1.21%
0.03125	38	1.20%	0.03438	27	1.57%
0.03438	15	1.40%	0.03542	41	2.12%
0.03542	32	1.83%	0.03646	40	2.65%
0.03646	30	2.23%	0.03854	92	3.88%
0.03854	98	3.53%	0.03958	63	4.72%
0.03958	63	4.37%	0.04375	46	5.33%
0.04375	37	4.87%	0.04479	141	7.21%
0.04479	125	6.53%	0.04688	75	8.21%
0.04688	70	7.47%	0.04792	57	8.97%
0.04792	63	8.31%	0.04896	117	10.53%
0.04896	111	9.79%	0.05208	274	14.19%
0.05208	250	13.12%	0.05521	124	15.84%
0.05521	116	14.67%	0.05625	93	17.08%
0.05625	90	15.87%	0.05729	128	18.79%
0.05729	119	17.45%	0.05938	283	22.56%
0.05938	273	21.09%	0.06042	65	23.43%
0.06042	58	21.87%	0.06458	67	24.32%
0.06458	66	22.75%	0.06563	291	28.20%
0.06563	284	26.53%	0.06771	143	30.11%
0.06771	148	28.51%	0.06875	89	31.29%
0.06875	79	29.56%	0.06979	187	33.79%
0.06979	183	32.00%	0.07292	355	38.52%
0.07292	424	37.65%	0.07604	195	41.12%
0.07604	191	40.20%	0.07708	65	41.99%
0.07708	50	40.87%	0.07813	149	43.97%
0.07813	153	42.91%	0.08021	296	47.92%

Table 65.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.08021	330	47.31%
0.08542	25	47.64%
0.08646	285	51.44%
0.08854	117	53.00%
0.08958	57	53.76%
0.09063	181	56.17%
0.09375	337	60.67%
0.09688	173	62.97%
0.09792	4	63.03%
0.09896	107	64.45%
0.10104	245	67.72%
0.10625	17	67.95%
0.10729	204	70.67%
0.10938	55	71.40%
0.11042	13	71.57%
0.11146	127	73.27%
0.11458	302	77.29%
0.11771	134	79.08%
0.11979	81	80.16%
0.12188	144	82.08%
0.12708	4	82.13%
0.12813	148	84.11%
0.13021	27	84.47%
0.13125	10	84.60%
0.13229	84	85.72%
0.13542	220	88.65%
0.13854	100	89.99%
0.14063	29	90.37%
0.14271	83	91.48%
0.14792	1	91.49%
0.14896	48	92.13%
0.15104	2	92.16%
0.15208	4	92.21%
0.15313	53	92.92%

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.08542	37	48.41%
0.08646	305	52.48%
0.08854	95	53.75%
0.08958	56	54.49%
0.09063	196	57.11%
0.09375	358	61.88%
0.09688	181	64.29%
0.09896	112	65.79%
0.10104	212	68.61%
0.10625	23	68.92%
0.10729	230	71.99%
0.10938	50	72.65%
0.11042	25	72.99%
0.11146	152	75.01%
0.11458	289	78.87%
0.11771	134	80.65%
0.11979	76	81.67%
0.12188	138	83.51%
0.13125	151	85.52%
0.13229	75	86.52%
0.13542	225	89.52%
0.13854	93	90.76%
0.14063	29	91.15%
0.14271	72	92.11%
0.14896	56	92.85%
0.15104	4	92.91%
0.15208	1	92.92%
0.15313	50	93.59%
0.15625	112	95.08%
0.15938	62	95.91%
0.16146	18	96.15%
0.16354	33	96.59%
0.16979	23	96.89%
0.17292	1	96.91%

Table 65.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.15625	138	94.76%
0.15938	45	95.36%
0.16146	16	95.57%
0.16354	47	96.20%
0.16979	25	96.53%
0.17188	2	96.56%
0.17396	19	96.81%
0.17708	65	97.68%
0.18021	29	98.07%
0.18229	7	98.16%
0.18438	18	98.40%
0.19063	5	98.47%
0.19479	8	98.57%
0.19792	36	99.05%
0.20104	9	99.17%
0.20313	6	99.25%
0.20521	8	99.36%
0.21146	6	99.44%
0.21563	5	99.51%
0.21875	14	99.69%
0.22188	3	99.73%
0.22396	1	99.75%
0.22396	0	99.75%
0.22604	3	99.79%
0.23646	2	99.81%
0.23958	7	99.91%
0.24271	1	99.92%
0.24479	1	99.93%
0.24688	1	99.95%
0.26042	2	99.97%
0.26354	1	99.99%
0.28646	1	100.00%

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.17396	20	97.17%
0.17708	63	98.01%
0.18021	25	98.35%
0.18229	6	98.43%
0.18438	16	98.64%
0.19063	11	98.79%
0.19375	1	98.80%
0.19479	8	98.91%
0.19792	30	99.31%
0.20104	6	99.39%
0.20521	9	99.51%
0.21146	2	99.53%
0.21563	4	99.59%
0.21875	12	99.75%
0.22188	2	99.77%
0.22604	1	99.79%
0.23958	6	99.87%
0.24271	3	99.91%
0.24688	2	99.93%
0.26354	4	99.99%
0.28438	1	100.00%

Table 65.--Continued.

Sampling With Replacement		Sampling Without Replacement	
Average	0.09210	Average	0.09069
Variance	0.00135	Variance	0.00130
Skewness	0.86343	Skewness	0.86100
Kurtosis	0.83326	Kurtosis	0.88796

Table 66.--Simulated Sampling Distribution of Dn (n=48, N=48)
Sampling from Skewed Positive Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.01458	1	.01%	0.01250	1	.01%
0.01875	1	.03%	0.01667	1	.03%
0.02083	8	.13%	0.01875	2	.05%
0.02292	5	.20%	0.02083	14	.24%
0.02500	10	.33%	0.02292	5	.31%
0.02708	20	.60%	0.02500	18	.55%
0.02917	12	.76%	0.02708	17	.77%
0.03125	6	.84%	0.02917	26	1.12%
0.03333	20	1.11%	0.03333	29	1.51%
0.03542	34	1.56%	0.03542	45	2.11%
0.03750	57	2.32%	0.03750	67	3.00%
0.03958	51	3.00%	0.03958	51	3.68%
0.04167	223	5.97%	0.04167	298	7.65%
0.04375	73	6.95%	0.04375	126	9.33%
0.04583	154	9.00%	0.04583	218	12.24%
0.04792	51	9.68%	0.04792	69	13.16%
0.05000	125	11.35%	0.05000	135	14.96%
0.05417	125	13.01%	0.05208	10	15.09%
0.05625	82	14.11%	0.05417	124	16.75%
0.05833	176	16.45%	0.05625	85	17.88%
0.06042	144	18.37%	0.05833	217	20.77%
0.06250	506	25.12%	0.06042	179	23.16%
0.06458	162	27.28%	0.06250	566	30.71%
0.06667	299	31.27%	0.06458	199	33.36%
0.06875	69	32.19%	0.06667	314	37.55%
0.07083	187	34.68%	0.06875	58	38.32%
0.07500	165	36.88%	0.07083	188	40.83%
0.07708	7	36.97%	0.07500	166	43.04%
0.07917	213	39.81%	0.07917	197	45.67%
0.08125	178	42.19%	0.08125	175	48.00%
0.08333	557	49.61%	0.08333	517	54.89%
0.08542	179	52.00%	0.08542	198	57.53%
0.08750	299	55.99%	0.08750	252	60.89%
0.08958	36	56.47%	0.08958	26	61.24%

Table 66.--Continued.

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.09167	180	58.87%	0.09167	192	63.80%
0.09583	163	61.04%	0.09583	166	66.01%
0.09792	4	61.09%	0.09792	3	66.05%
0.10000	167	63.32%	0.10000	160	68.19%
0.10208	153	65.36%	0.10208	155	70.25%
0.10417	323	69.67%	0.10417	314	74.44%
0.10625	138	71.51%	0.10625	155	76.51%
0.10833	206	74.25%	0.10833	170	78.77%
0.11250	125	75.92%	0.11042	8	78.88%
0.11667	115	77.45%	0.11250	114	80.40%
0.12083	132	79.21%	0.11667	93	81.64%
0.12292	116	80.76%	0.12083	119	83.23%
0.12500	136	82.57%	0.12292	122	84.85%
0.12708	90	83.77%	0.12500	127	86.55%
0.12917	119	85.36%	0.12708	80	87.61%
0.13125	3	85.40%	0.12917	128	89.32%
0.13333	69	86.32%	0.13125	4	89.37%
0.13750	76	87.33%	0.13333	67	90.27%
0.14167	93	88.57%	0.13750	68	91.17%
0.14375	84	89.69%	0.14167	68	92.08%
0.14583	57	90.45%	0.14375	62	92.91%
0.14792	71	91.40%	0.14583	46	93.52%
0.15000	96	92.68%	0.14792	43	94.09%
0.15417	49	93.33%	0.15000	60	94.89%
0.15833	56	94.08%	0.15208	1	94.91%
0.16250	68	94.99%	0.15417	34	95.36%
0.16458	44	95.57%	0.15833	31	95.77%
0.16667	36	96.05%	0.16250	51	96.45%
0.16875	23	96.36%	0.16458	30	96.85%
0.17083	48	97.00%	0.16667	13	97.03%
0.17500	16	97.21%	0.16875	14	97.21%
0.17917	25	97.55%	0.17083	47	97.84%
0.18333	22	97.84%	0.17500	12	98.00%
0.18542	19	98.09%	0.17917	19	98.25%
0.20417	12	99.28%	0.19583	4	99.19%

Table 66.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.20625	13	99.45%
0.18750	15	98.29%
0.18958	9	98.41%
0.19167	27	98.77%
0.19583	5	98.84%
0.20000	21	99.12%
0.20833	1	99.47%
0.21042	2	99.49%
0.21250	10	99.63%
0.22083	2	99.65%
0.22500	3	99.69%
0.22708	4	99.75%
0.23333	7	99.84%
0.24167	1	99.85%
0.24792	1	99.87%
0.25000	1	99.88%
0.25417	2	99.91%
0.26250	1	99.92%
0.26875	1	99.93%
0.27500	2	99.96%
0.28333	1	99.97%
0.28750	2	100.00%

Average	0.09257
Variance	0.00139
Skewness	0.88964
Kurtosis	0.92645

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.20000	9	99.31%
0.18333	20	98.52%
0.18542	19	98.77%
0.18750	2	98.80%
0.18958	12	98.96%
0.19167	13	99.13%
0.20417	11	99.45%
0.20625	5	99.52%
0.20833	1	99.53%
0.21042	3	99.57%
0.21250	6	99.65%
0.21667	1	99.67%
0.22083	7	99.76%
0.22500	1	99.77%
0.22708	2	99.80%
0.23333	6	99.88%
0.23333	0	99.88%
0.24167	1	99.89%
0.24583	2	99.92%
0.24792	1	99.93%
0.25417	4	99.99%
0.26250	1	100.00%

Average	0.08706
Variance	0.00126
Skewness	0.93185
Kurtosis	1.08065

Table 67.--Simulated Sampling Distribution of Dn (n=48, N=240)
Sampling from Skewed Positive Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.01667	1	.01%	0.01667	1	.01%
0.02083	9	.13%	0.02083	30	.41%
0.02500	16	.35%	0.02500	27	.77%
0.02917	35	.81%	0.02917	76	1.79%
0.03333	59	1.60%	0.03333	83	2.89%
0.03750	63	2.44%	0.03750	106	4.31%
0.04167	305	6.51%	0.04167	517	11.20%
0.04583	208	9.28%	0.04583	277	14.89%
0.05000	168	11.52%	0.05000	226	17.91%
0.05417	185	13.99%	0.05417	220	20.84%
0.05833	201	16.67%	0.05833	297	24.80%
0.06250	783	27.11%	0.06250	899	36.79%
0.06667	381	32.19%	0.06667	377	41.81%
0.07083	246	35.47%	0.07083	261	45.29%
0.07500	171	37.75%	0.07500	166	47.51%
0.07917	299	41.73%	0.07917	298	51.48%
0.08333	731	51.48%	0.08333	739	61.33%
0.08750	301	55.49%	0.08750	302	65.36%
0.09167	220	58.43%	0.09167	197	67.99%
0.09583	138	60.27%	0.09583	166	70.20%
0.10000	266	63.81%	0.10000	239	73.39%
0.10417	542	71.04%	0.10417	480	79.79%
0.10833	210	73.84%	0.10833	202	82.48%
0.11250	139	75.69%	0.11250	110	83.95%
0.11667	133	77.47%	0.11667	113	85.45%
0.12083	187	79.96%	0.12083	148	87.43%
0.12500	292	83.85%	0.12500	233	90.53%
0.12917	142	85.75%	0.12917	119	92.12%
0.13333	84	86.87%	0.13333	44	92.71%
0.13750	81	87.95%	0.13750	43	93.28%
0.14167	124	89.60%	0.14167	82	94.37%
0.14583	170	91.87%	0.14583	104	95.76%
0.15000	69	92.79%	0.15000	60	96.56%
0.15417	45	93.39%	0.15417	26	96.91%

Table 67.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.15833	50	94.05%
0.16250	70	94.99%
0.16667	105	96.39%
0.17083	45	96.99%
0.17500	11	97.13%
0.17917	21	97.41%
0.18333	26	97.76%
0.18750	43	98.33%
0.19167	16	98.55%
0.19583	7	98.64%
0.20000	12	98.80%
0.20417	20	99.07%
0.20833	18	99.31%
0.21250	6	99.39%
0.21667	3	99.43%
0.22083	4	99.48%
0.22500	8	99.59%
0.22917	8	99.69%
0.23333	6	99.77%
0.24167	2	99.80%
0.24583	4	99.85%
0.25000	4	99.91%
0.25417	3	99.95%
0.26250	3	99.99%
0.27500	1	100.00%

Average	0.09246
Variance	0.00140
Skewness	0.92044
Kurtosis	1.01144

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.15833	23	97.21%
0.16250	36	97.69%
0.16667	54	98.41%
0.17083	22	98.71%
0.17500	8	98.81%
0.17917	16	99.03%
0.18333	19	99.28%
0.18750	13	99.45%
0.19167	9	99.57%
0.20000	2	99.60%
0.20417	8	99.71%
0.20833	5	99.77%
0.21250	6	99.85%
0.21667	1	99.87%
0.22500	2	99.89%
0.22917	2	99.92%
0.23333	1	99.93%
0.24167	1	99.95%
0.24167	0	99.95%
0.25000	3	99.99%
0.29583	1	100.00%

Average	0.08258
Variance	0.00111
Skewness	0.89916
Kurtosis	1.12043

Table 68.--Simulated Sampling Distribution of Dn (n=48, N=48)
Sampling from Skewed Positive Population

Sampling With Replacement			Sampling Without Replacement		
Dn	*Frequency*	*Cum. %*	*Dn*	*Frequency*	*Cum. %*
0.01458	1	.01%	0.01667	3	.04%
0.01875	1	.03%	0.01875	12	.20%
0.02292	9	.15%	0.02292	21	.48%
0.02500	10	.28%	0.02500	41	1.03%
0.02708	7	.37%	0.02708	20	1.29%
0.02917	11	.52%	0.02917	53	2.00%
0.03125	18	.76%	0.03125	53	2.71%
0.03333	22	1.05%	0.03333	58	3.48%
0.03542	25	1.39%	0.03542	32	3.91%
0.03750	64	2.24%	0.03750	148	5.88%
0.03958	124	3.89%	0.03958	279	9.60%
0.04375	176	6.24%	0.04375	300	13.60%
0.04583	203	8.95%	0.04583	371	18.55%
0.04792	59	9.73%	0.04792	82	19.64%
0.05000	126	11.41%	0.05000	184	22.09%
0.05417	122	13.04%	0.05208	22	22.39%
0.05625	52	13.73%	0.05417	187	24.88%
0.05833	273	17.37%	0.05625	49	25.53%
0.06042	338	21.88%	0.05833	403	30.91%
0.06458	328	26.25%	0.06042	415	36.44%
0.06667	379	31.31%	0.06458	348	41.08%
0.06875	67	32.20%	0.06667	485	47.55%
0.07083	202	34.89%	0.06875	51	48.23%
0.07292	22	35.19%	0.07083	207	50.99%
0.07500	163	37.36%	0.07292	6	51.07%
0.07917	349	42.01%	0.07500	177	53.43%
0.08125	315	46.21%	0.07917	419	59.01%
0.08542	311	50.36%	0.08125	304	63.07%
0.08750	372	55.32%	0.08542	236	66.21%
0.08958	31	55.73%	0.08750	405	71.61%
0.09167	182	58.16%	0.08958	13	71.79%
0.09583	148	60.13%	0.09167	155	73.85%
0.10000	377	65.16%	0.09583	155	75.92%
0.10208	208	67.93%	0.10000	293	79.83%

Table 68.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.10625	170	70.20%
0.10833	305	74.27%
0.11042	10	74.40%
0.11250	116	75.95%
0.11667	131	77.69%
0.12083	254	81.08%
0.12292	113	82.59%
0.12708	66	83.47%
0.12917	207	86.23%
0.13125	5	86.29%
0.13333	71	87.24%
0.13750	70	88.17%
0.14167	171	90.45%
0.14375	60	91.25%
0.14792	13	91.43%
0.15000	139	93.28%
0.15208	2	93.31%
0.15417	34	93.76%
0.15833	39	94.28%
0.16250	105	95.68%
0.16458	30	96.08%
0.16875	5	96.15%
0.17083	72	97.11%
0.17500	14	97.29%
0.17917	20	97.56%
0.18333	53	98.27%
0.18542	7	98.36%
0.19167	34	98.81%
0.19583	6	98.89%
0.20000	13	99.07%
0.20417	23	99.37%
0.20625	5	99.44%
0.21250	13	99.61%
0.21667	1	99.63%
0.22083	1	99.64%

Sampling Without Replacement

Dn	*Frequency*	*Cum. %*
0.10208	136	81.64%
0.10625	85	82.77%
0.10833	259	86.23%
0.11250	98	87.53%
0.11667	99	88.85%
0.12083	192	91.41%
0.12292	61	92.23%
0.12708	19	92.48%
0.12917	156	94.56%
0.13333	39	95.08%
0.13750	41	95.63%
0.14167	82	96.72%
0.14375	29	97.11%
0.14792	2	97.13%
0.15000	63	97.97%
0.15417	16	98.19%
0.15833	18	98.43%
0.16250	41	98.97%
0.16458	3	99.01%
0.17083	22	99.31%
0.17500	5	99.37%
0.17917	5	99.44%
0.18333	15	99.64%
0.19167	4	99.69%
0.20000	1	99.71%
0.20417	8	99.81%
0.21250	7	99.91%
0.21667	2	99.93%
0.22500	3	99.97%
0.25417	1	99.99%
0.27500	1	100.00%

Table 68.--Continued.

Sampling With Replacement

Dn	*Frequency*	*Cum. %*
0.22500	10	99.77%
0.22708	1	99.79%
0.23333	5	99.85%
0.24167	2	99.88%
0.24583	6	99.96%
0.25417	3	100.00%

Sampling Without Replacement

Sampling With Replacement		Sampling Without Replacement	
Average	0.09222	Average	0.07739
Variance	0.00134	Variance	0.00097
Skewness	0.85836	Skewness	0.91625
Kurtosis	0.75673	Kurtosis	1.21086

Table 69.--Assumed Populations of Simulation

	Populations of 20 units			Populations of 30 units		
	Uniform	Unimodal Symmetrical	Skewed Positive	Uniform	Unimodal Symmetrical	Skewed Positive
Categories	Frequency	Frequency	Frequency	Frequency	Frequency	Frequency
0	2	0	0	3	0	0
1	2	1	2	3	1	1
2	2	1	2	3	2	3
3	2	3	3	3	5	5
4	2	5	4	3	7	7
5	2	5	4	3	7	5
6	2	3	3	3	5	5
7	2	1	2	3	2	2
8	2	1	0	3	1	1
9	2	0	0	3	0	1

	Populations of 40 units			Populations of 60 units		
	Uniform	Unimodal Symmetrical	Skewed Positive	Uniform	Unimodal Symmetrical	Skewed Positive
Categories	Frequency	Frequency	Frequency	Frequency	Frequency	Frequency
0	4	0	0	6	0	0
1	4	1	2	6	1	3
2	4	3	4	6	4	7
3	4	6	7	6	10	10
4	4	10	8	6	15	11
5	4	10	7	6	15	10
6	4	6	5	6	10	8
7	4	3	4	6	4	6
8	4	1	2	6	1	3
9	4	0	1	6	0	2

Table 69.--Continued.

	Populations of 80 units			Populations of 120 units		
	Uniform	Unimodal Symmetrical	Skewed Positive	Uniform	Unimodal Symmetrical	Skewed Positive
Categories	Frequency	Frequency	Frequency	Frequency	Frequency	Frequency
0	8	0	1	12	0	1
1	8	1	4	12	2	6
2	8	6	9	12	8	13
3	8	13	12	12	20	20
4	8	20	15	12	30	23
5	8	20	15	12	30	21
6	8	13	10	12	20	15
7	8	6	7	12	8	11
8	8	1	5	12	2	6
9	8	0	2	12	0	4

	Populations of 160 units			Populations of 240 units		
	Uniform	Unimodal Symmetrical	Skewed Positive	Uniform	Unimodal Symmetrical	Skewed Positive
Categories	Frequency	Frequency	Frequency	Frequency	Frequency	Frequency
0	16	0	2	24	1	2
1	16	3	7	24	4	11
2	16	11	18	24	17	27
3	16	26	27	24	39	40
4	16	40	30	24	59	46
5	16	40	28	24	59	42
6	16	26	21	24	39	31
7	16	11	14	24	17	21
8	16	3	8	24	4	13
9	16	0	5	24	1	7

Table 69.--Continued.

	Populations of 480 units			Populations of 960 units		
	Uniform	Unimodal Symmetrical	Skewed Positive	Uniform	Unimodal Symmetrical	Skewed Positive
Categories	Frequency	Frequency	Frequency	Frequency	Frequency	Frequency
0	48	1	5	96	2	10
1	48	8	22	96	17	48
2	48	34	53	96	67	107
3	48	79	81	96	158	162
4	48	118	91	96	236	183
5	48	118	84	96	236	167
6	48	79	64	96	158	126
7	48	34	42	96	67	83
8	48	8	25	96	17	48
9	48	1	13	96	2	26

Table 70.--Exact Sampling Distribution of Dn (n=2, N=10)
Sampling from Uniform Population

Samples selected without replacement

	Dn	*Frequency*	*S(x)*
	0.2	2	2.22%
	0.3	16	20.00%
	0.4	32	55.56%
	0.5	16	73.33%
	0.6	12	86.67%
	0.7	8	95.56%
	0.8	4	100.00%
	0.9	0	100.00%
Total		90	

Descriptive Statistics	
Min	0.2000
Max	0.8000
Average	0.4667
Variance	0.0204
Skewness	0.6082
Kurtosis	-0.2917

Exact Sampling Distribution of Dn (n=2, N=10)
Sampling from Uniform Population

Samples selected with replacement

	Dn	*Frequency*	S(x)
	0.2	2	2.00%
	0.3	16	18.00%
	0.4	32	50.00%
	0.5	18	68.00%
	0.6	14	82.00%
	0.7	10	92.00%
	0.8	6	98.00%
	0.9	2	100.00%
Total		100	

Descriptive Statistics	
Min	0.2
Max	0.9
Average	0.49
Variance	0.0253
Skewness	0.6206
Kurtosis	-0.2909

Table 71.--Exact Sampling Distribution of Dn (n=3, N=10)
Sampling from Uniform Population Sampling Without Replacement

Dn	*Frequency*	*S(x)*
0.167	12	1.67%
0.200	36	6.67%
0.233	60	15.00%
0.267	108	30.00%
0.300	120	46.67%
0.367	96	60.00%
0.400	107	74.86%
0.467	60	83.19%
0.500	73	93.33%
0.567	0	93.33%
0.600	36	98.33%
0.700	12	100.00%
0.800	0	100.00%
0.900	0	100.00%
Total	720	

Descriptive Statistics	
Min	0.1667
Max	0.7000
Average	0.3618
Variance	0.0131
Skewness	0.6738
Kurtosis	0.0381

Exact Sampling Distribution of Dn (n=3, N=10)
Sampling from Uniform Population Sampling With Replacement

Dn	Frequency	S(x)
0.167	12	1.20%
0.2	36	4.80%
0.233	60	10.80%
0.267	108	21.60%
0.3	144	36.00%
0.367	120	48.00%
0.4	150	63.00%
0.467	90	72.00%
0.5	116	83.60%
0.567	36	87.20%
0.6	74	94.60%
0.7	38	98.40%
0.8	14	99.80%
0.9	2	100.00%
Total	1000	

Descriptive Statistics	
Min	0.1667
Max	0.9
Average	0.4036
Variance	0.0189
Skewness	0.6720
Kurtosis	0.1137

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