# Mathematical Representation of Viscosity of lonic Liquid + Molecular Solvent Mixtures at Various Temperatures Using the Jouyban-Acree Model 

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#### Abstract

The viscosity of room temperature ionic liquid + molecular solvents is correlated using the Jouyban-Acree model, and the accuracy of the model is evaluated using average relative deviations (ARD) of the calculated and experimental values. The overall ARD of $6.9 \%$ was obtained for the proposed model where the overall ARD for a similar model from the literature was $22.4 \%$. The mean difference between ARDs of the models was statistically significant. Combined forms of the Jouyban-Acree model and the Abraham parameters were used for predicting the viscosity of the mixtures at various temperatures. The overall ARD for the investigated mixtures was 15.0 \% when the experimental viscosity of the pure solvents was included. The ARD of the in silico version of the model was $20.7 \%$.




## INTRODUCTION

Room temperature ionic liquids (RTILs) are a new class of solvents attracting considerable attention in the pharmaceutical/ chemical industries. RTILs are used in manufacturing nanomaterials, high temperature lubricants, excipients for new drug delivery systems, novel stationary phases for gas chromatographic systems, and extraction solvents for separation of analytes. Very low vapor pressure, multiple solvation interactions with organic and inorganic compounds, very good chemical and thermal stability, high ionic conductivity, and wide melting point temperature ranges are the main advantages of RTILs over common molecular solvents. In addition to the applications of RTILs, their mixtures with molecular solvents are used in the industry providing wider physicochemical properties. Viscosity data of the mixtures are required in some related computations and a number of attempts have been made to collect the experimental values of the viscosity and density of these mixtures. ${ }^{1-23}$ Despite these experimental efforts, a number of mathematical models were proposed to compute the properties. The aims of this work are to propose a mathematical model to fit the viscosity data of RTILs + solvent mixtures at various temperatures and also provide global versions of the model to predict the viscosity data.

## COMPUTATIONAL METHODS

Larriba et al. ${ }^{10}$ used Bringham mixing rule to estimate the viscosity of binary mixtures containing two ionic liquids. The equation is a reciprocal viscosity prediction as

$$
\begin{equation*}
\frac{1}{\left(\eta_{\text {mix }}\right)_{T}}=\frac{x_{1}}{\left(\eta_{1}\right)_{T}}+\frac{x_{2}}{\left(\eta_{2}\right)_{T}} \tag{1}
\end{equation*}
$$

where $\left(\eta_{\text {mix }}\right)_{T}$ is the viscosity of the binary mixture at a given temperature $(T),\left(\eta_{1}\right)_{T}$ and $\left(\eta_{2}\right)_{T}$ denote the viscosity of RTIL and molecular solvent, $x_{1}$ and $x_{2}$ are mole fractions of RTIL and molecular solvent, respectively. One could probably generalize the equation to include other mixture compositions, such as volume fractions, mass fractions, etc. The equation is a strictly empirical equation. It is an outright prediction of viscosity in that there are no curve-fit parameters. All that one needs is the viscosity of the two pure liquids at each temperature studied.

Domanska and co-workers ${ }^{15}$ described the temperature dependence of viscosity with the Vogel-Fulcher-Tamman (VFT) equation

$$
\begin{equation*}
\eta=A T^{0.5} \exp \left(\frac{B}{T-T_{\mathrm{o}}}\right) \tag{2}
\end{equation*}
$$

and Wang et al. ${ }^{1}$ used a slightly different variation

$$
\begin{equation*}
\eta=\eta_{\mathrm{o}} \exp \left(\frac{B}{T-T_{o}}\right) \tag{3}
\end{equation*}
$$

where $A, T_{o}, B$, and $\eta_{o}$, are curve-fitting parameters depending on which temperature dependence was used. In each study,

[^0]Table 1. Details of the Investigated Experimental Viscosity Data Collected from the Literature and the Average Absolute Relative Deviations (ARDs) of Equations 5 and 6

| no. | RTIL | molecular solvent/RTIL | ref no. | $N$ | eq 5 | eq 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | [AP4443][ALA] | water | 1 | 30 | 37.6 | 7.8 |
| 2 | [AP4443][LEU] | water | 1 | 30 | 50.7 | 8.1 |
| 3 | [AP4443][VAL] | water | 1 | 30 | 32.3 | 7.2 |
| 4 | [BHEAP] | 1-propanol | 2 | 52 | 16.0 | 21.4 |
| 5 | [BHEAP] | ethanol | 2 | 52 | 17.8 | 25.3 |
| 6 | [BHEAP] | methanol | 2 | 52 | 15.9 | 31.9 |
| 7 | [BMIM][ALA] | benzyl alcohol | 3 | 44 | 4.0 | 1.6 |
| 8 | [BMIM][ALA] | methanol | 3 | 44 | 6.2 | 7.3 |
| 9 | [BMIM][BF4] | dimethyl sulfoxide | 4 | 77 | 8.6 | 1.2 |
| 10 | [BMIM][BF4] | ethylene glycol | 4 | 55 | 8.1 | 3.5 |
| 11 | [BMIM][BF4] | water | 5 | 88 | 7.1 | 4.0 |
| 12 | $\begin{aligned} & {[\mathrm{BMIM}]} \\ & {[\mathrm{CF} 3 \mathrm{SO} 3]} \end{aligned}$ | water | 6 | 77 | 9.5 | 6.8 |
| 13 | [BMIM][PF6] | dimethyl sulfoxide | 8 | 15 | 9.7 | 3.2 |
| 14 | [BMIM][PF6] | methanol | 8 | 15 | 25.6 | 1.7 |
| 15 | [BMIM][PF6] | methyl methacrylate | 7 | 143 | 11.8 | 4.7 |
| 16 | [BMIM][PF6] | tetrahydrofuran | 8 | 15 | 7.4 | 6.4 |
| 17 | [BMIM][SCN] | 1-butanol | 9 | 72 | 5.6 | 5.2 |
| 18 | [BMIM][SCN] | 1-hexanol | 9 | 72 | 3.9 | 4.4 |
| 19 | [BMIM][SCN] | 1-pentanol | 9 | 78 | 4.4 | 5.4 |
| 20 | [BPY][BF4] | [BPY][Tf2N] | 10 | 72 | 5.9 | 1.1 |
| 21 | [BUPY][BF4] | water | 11 | 130 | 15.0 | 8.1 |
| 22 | [C4MIM][PF6] | 2-butanone | 13 | 15 | 8.8 | 0.8 |
| 23 | [C4MIM][PF6] | acetone | 13 | 15 | 2.6 | 0.7 |
| 24 | [C4MIM][PF6] | acetonitrile | 8 | 15 | 35.3 | 2.0 |


|  |  |  |  |  |  |  |
| :---: | :--- | :--- | :---: | :--- | ---: | ---: |
| no. | RTIL |  | molecular <br> solvent/RTIL | ref <br> no. | $N$ | eq 5 | eq 6

the authors calculated the curve-fit coefficients for each of the binary solvent mixtures. There was no attempt to incorporate a compositional dependence into the coefficients. One might be able to reasonably put these two ideas together as

$$
\begin{equation*}
\frac{1}{\left(\eta_{\text {mix }}\right)_{T}}=\frac{x_{1}}{\eta_{0,1} \exp \left(\frac{B_{1}}{T-T_{0,1}}\right)}+\frac{x_{2}}{\eta_{0,2} \exp \left(\frac{B_{2}}{T-T_{0,2}}\right)} \tag{4}
\end{equation*}
$$

in which the three curve-fit parameters for each pure solvent could be computed by fitting the viscosity of each as a function of temperature. Predictions based on the above equations could be compared to eq 4 as it is a logical combination of two ideas being used in the recent literature.

The Eyring kinetic equation was used for correlating viscosity $\left(\eta_{m, T}\right)$ data of binary mixtures by Gong et al. ${ }^{24}$ The equation is

$$
\begin{equation*}
\eta_{m, T}=\left(A_{1}+A_{2} x_{1}\right) \exp \left[\frac{\left(A_{3}+\frac{A_{4}}{T}\right)+\left(A_{5}+\frac{A_{6}}{T}\right) x_{1}+\left(A_{7}+\frac{A_{8}}{T}\right) x_{1}^{2}}{R T}\right] \tag{5}
\end{equation*}
$$

where $A_{1}-A_{8}$ are the model constants calculated using a nonlinear least-squares analysis, $R$ is the gas constant, and $T$ is the absolute temperature.

The purpose of this work is to present the capabilities of the Jouyban-Acree model ${ }^{25-27}$ and its combined version with the Abraham parameters for representing the experimental data. The basic model for describing the physicochemical properties $\left(P_{m, T}\right)$ of the binary mixtures with respect to solvent composition and temperature is

$$
\begin{align*}
\ln P_{m, T}= & x_{1} \ln P_{1, T}+x_{2} \ln P_{2, T}+J_{0}\left[\frac{x_{1} x_{2}}{T}\right] \\
& +J_{1}\left[\frac{x_{1} x_{2}\left(x_{1}-x_{2}\right)}{T}\right]+J_{2}\left[\frac{x_{1} x_{2}\left(x_{1}-x_{2}\right)^{2}}{T}\right] \tag{6}
\end{align*}
$$

where $P_{m, T}, P_{1, T}$, and $P_{2, T}$ are the numerical values of the property (viscosity, density, or molar volume) of the mixture and liquid 1 (RTIL in this work) and 2 (molecular solvent in this work) at temperature $T$, respectively, and $J_{i}$ represents the model constants. The $J_{i}$ terms could be calculated by regressing $\left(\ln P_{m, T}-x_{1} \ln P_{1, T}-x_{2} \ln P_{2, T}\right)$ against $\left[x_{1} x_{2} / T\right],\left[\left(x_{1} x_{2}\left(x_{1}-x_{2}\right)\right) / T\right]$, and $\left[\left(x_{1} x_{2}\left(x_{1}-x_{2}\right)^{2}\right) / T\right]$ by a no intercept least-squares analysis. ${ }^{28}$

The correlated data was compared with the corresponding experimental data by computing the average absolute relative deviation (ARD) using

$$
\begin{equation*}
\mathrm{ARD}=\frac{100}{N} \sum\left|\frac{\text { calculated }}{\text { experimental }}-1.0\right| \tag{7}
\end{equation*}
$$

in which $N$ is the number of data points in each set.

## RESULTS

The gathered viscosity data were correlated using the proposed model and the names of the components, their references, and the ARD values for eqs 5 and 6 are listed in Table 1. The proposed model, that is, eq 6, is able to represent the viscosity of binary mixtures with the overall ARD of $6.9 \pm 7.8 \%$ (number of data sets, NDS $=45$ ). The maximum ARD of $31.9 \%$ was observed for [BHEAP] + methanol mixtures and the minimum value ( $0.4 \%$ ) was observed for $[\mathrm{C} 4 \mathrm{MIM}][\mathrm{PF} 6]+$ pentanone. Excluding these two data sets reduces the

Table 2. Abraham Solvent Parameters for RTIL/Solvents ${ }^{a}$

| RTIL/molecular solvent | c | $e$ | $s$ | $a$ | $b$ | $v$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| [BMIM][BF4] | -0.206 | 0.355 | 2.278 | -0.172 | -4.415 | 3.635 |
| [BMIM][CF3SO3] | -0.191 | 0.272 | 2.362 | -0.060 | -4.273 | 3.603 |
| [BMIM][PF6] | -0.113 | 0.150 | 2.521 | -1.382 | -4.572 | 3.641 |
| [BMIM][SCN] | -0.481 | -1.152 | 4.939 | 0.749 | -5.310 | 3.768 |
| [BPY][BF4] | -0.423 | 0.168 | 1.860 | -0.257 | -4.469 | 4.001 |
| [BPY][Tf2N] | -0.267 | 0.082 | 1.610 | -1.467 | -4.174 | 3.785 |
| [BUPY][BF4] | -0.423 | 0.168 | 1.860 | -0.257 | -4.469 | 4.001 |
| [C4MIM][PF6] | -0.113 | 0.150 | 2.521 | -1.382 | -4.572 | 3.641 |
| [C8IQUIN][NTf2] | 0.149 | -0.058 | 2.422 | -2.861 | -3.920 | 6.099 |
| [EMIM][BF4] | -0.061 | 0.425 | 2.353 | -0.483 | -3.584 | 4.024 |
| [EMIM][EtSO4] | -0.124 | -0.013 | 2.252 | 1.524 | -4.941 | 3.718 |
| [EMISE] | -0.124 | -0.013 | 2.252 | 1.524 | -4.941 | 3.718 |
| [EPY][SO4] | -0.341 | -0.200 | 1.834 | 1.439 | -4.995 | 4.084 |
| [OCPY][BF4] | -0.275 | 0.028 | 2.040 | -0.441 | -4.581 | 4.197 |
| [OMIM][BF4] | 0.035 | 0.010 | 2.701 | -1.566 | -4.684 | 3.837 |
| [PDMIM][BF4] | -0.192 | 0.577 | 2.572 | -0.327 | -4.150 | 4.169 |
| 1-butanol | 0.376 | 0.434 | -0.718 | -0.097 | -2.350 | 2.682 |
| 1-hexanol | -0.006 | 0.460 | -0.940 | 0.142 | -3.284 | 3.792 |
| 1-pentanol | 0.185 | 0.367 | -0.732 | 0.105 | -3.100 | 3.395 |
| 1-propanol | 0.148 | 0.436 | -1.098 | 0.389 | -3.893 | 4.036 |
| 2 -propanol | 0.102 | 0.315 | -1.020 | 0.532 | -3.865 | 4.023 |
| acetone | 0.313 | 0.312 | -0.121 | -0.608 | -4.753 | 3.942 |
| acetonitrile | 0.413 | 0.077 | 0.326 | -1.566 | -4.391 | 3.364 |
| dimethyl formamide | -0.305 | -0.058 | 0.343 | 0.358 | -4.865 | 4.486 |
| dimethyl sulfoxide | -0.194 | 0.327 | 0.791 | 1.260 | -4.540 | 3.361 |
| ethanol | 0.208 | 0.409 | -0.959 | 0.186 | -3.645 | 3.928 |
| ethyl acetate | 0.441 | 0.591 | -0.699 | -0.325 | -4.261 | 3.666 |
| ethylene glycol | -0.243 | 0.695 | -0.670 | 0.726 | -2.399 | 2.670 |
| methanol | 0.329 | 0.299 | -0.671 | 0.080 | -3.389 | 3.512 |
| tetrahydrofuran | 0.207 | 0.372 | -0.392 | -0.236 | -4.934 | 4.447 |
| water | -0.994 | 0.577 | 2.549 | 3.813 | 4.841 | -0.869 |

${ }^{a}$ Numerical values were either taken the literature ${ }^{29}$ or calculated using a group contribution method. ${ }^{38}$
overall ARD to $6.4 \pm 6.9$. The overall ARDs of eq 6 for RTIL + water mixtures was $7.1 \pm 2.5(\mathrm{NDS}=12)$ and that of RTIL + organic solvents (or RTIL) was $6.8 \pm 9.0 \%$ (NDS $=33$ ), in which there was no statistically significant difference among overall ARD values revealing good capability of the model to fit both aqueous and nonaqueous binary mixtures. Equation 5 reproduced the viscosity data of the binary mixtures with the overall ARD ( $\pm$ SD) of $22.4 \pm$ 43.8 \% which was significantly ( $p<0.017$ ) more than the $6.9 \%$ of eq 6.

The main drawback of eq 6 is that it requires a number of viscosity data in the binary mixtures for computing the numerical values of the model constants, and also the viscosity of the pure solvents. To provide a global model to predict the viscosity of the mixtures, it is possible to include some physicochemical parameters representing the interactions within the binary mixtures responsible for deviation from the linear behavior. One of the best parameters to represent the interaction within the mixtures are the Abraham parameters which are developed for nonionized chemicals. ${ }^{29,30}$ In recent works, the parameters were also presented for ionized forms and were used in a number of applications. ${ }^{31-36}$

The $J_{i}$ terms of eq 6 which stand for the various solventsolvent interactions in the mixture, ${ }^{37}$ may be represented by Abraham parameters and hence eq 6 may be rewritten as eq 8 for viscosity data,

$$
\begin{align*}
& \ln \eta_{m, T}=x_{1} \ln \eta_{1, T} \\
& \quad+x_{2} \ln \eta_{2, T}+\frac{x_{1} x_{2}}{T}\left[\begin{array}{l}
W_{0}+W_{1}\left(c_{1}-c_{2}\right)^{2}+W_{2}\left(e_{1}-e_{2}\right)^{2} \\
+W_{3}\left(s_{1}-s_{2}\right)^{2}+W_{4}\left(a_{1}-a_{2}\right)^{2} \\
+W_{5}\left(b_{1}-b_{2}\right)^{2}+W_{6}\left(v_{1}-v_{2}\right)^{2}
\end{array}\right] \\
& \quad+\frac{x_{1} x_{2}\left(x_{1}-x_{2}\right)}{T}\left[\begin{array}{l}
W_{0}^{\prime}+W_{1}^{\prime}\left(c_{1}-c_{2}\right)^{2}+W_{2}^{\prime}\left(e_{1}-e_{2}\right)^{2} \\
+W_{3}^{\prime}\left(s_{1}-s_{2}\right)^{2}+W_{4}^{\prime}\left(a_{1}-a_{2}\right)^{2} \\
+W_{5}^{\prime}\left(b_{1}-b_{2}\right)^{2}+W_{6}^{\prime}\left(v_{1}-v_{2}\right)^{2}
\end{array}\right] \\
& \quad+\frac{x_{1} x_{2}\left(x_{1}-x_{2}\right)^{2}}{T}\left[\begin{array}{l}
W_{0}^{\prime \prime}+W_{1}^{\prime \prime}\left(c_{1}-c_{2}\right)^{2}+W_{2}^{\prime \prime}\left(e_{1}-e_{2}\right)^{2} \\
+W_{3}^{\prime \prime}\left(s_{1}-s_{2}\right)^{2}+W_{4}^{\prime \prime}\left(a_{1}-a_{2}\right)^{2} \\
+W_{5}^{\prime \prime}\left(b_{1}-b_{2}\right)^{2}+W_{6}^{\prime \prime}\left(v_{1}-v_{2}\right)^{2}
\end{array}\right] \tag{8}
\end{align*}
$$

where subscripts 1 and 2 are the properties of solvents 1 and 2 , and $W$ terms are the model constants representing the two and three body interactions of the components. From a strictly mathematical point-of-view, they are adjustable curve-fit parameters determined from regressing $\left(\ln \eta_{\mathrm{m}, \mathrm{T}}-x_{1} \ln \eta_{1, \mathrm{~T}}-\right.$ $\left.x_{2} \ln \eta_{2, \mathrm{~T}}\right)$ against $x_{1} x_{2} / T,\left(x_{1} x_{2}\left(c_{1}-c_{2}\right)^{2}\right) / T,\left(x_{1} x_{2}\left(e_{1}-e_{2}\right)^{2}\right) /$ $T,\left(x_{1} x_{2}\left(s_{1}-s_{2}\right)^{2}\right) / T,\left(x_{1} x_{2}\left(a_{1}-a_{2}\right)^{2}\right) / T,\left(x_{1} x_{2}\left(b_{1}-b_{2}\right)^{2}\right) / T$, $\left(x_{1} x_{2}\left(v_{1}-v_{2}\right)^{2}\right) / T,\left(x_{1} x_{2}\left(x_{1}-x_{2}\right)\right) / T,\left(x_{1} x_{2}\left(x_{1}-x_{2}\right)\left(c_{1}-\right.\right.$ $\left.\left.c_{2}\right)^{2}\right) / T,\left(x_{1} x_{2}\left(x_{1}-x_{2}\right)\left(e_{1}-e_{2}\right)^{2}\right) / T,\left(x_{1} x_{2}\left(x_{1}-x_{2}\right)\left(s_{1}-s_{2}\right)^{2}\right) /$ $T,\left(x_{1} x_{2}\left(x_{1}-x_{2}\right)\left(a_{1}-a_{2}\right)^{2}\right) / T,\left(x_{1} x_{2}\left(x_{1}-x_{2}\right)\left(b_{1}-b_{2}\right)^{2}\right) / T$,
$\left(x_{1} x_{2}\left(x_{1}-x_{2}\right)\left(v_{1}-v_{2}\right)^{2}\right) / T,\left(x_{1} x_{2}\left(x_{1}-x_{2}\right)^{2}\right) / T,\left(x_{1} x_{2}\left(x_{1}-\right.\right.$ $\left.\left.x_{2}\right)^{2}\left(c_{1}-c_{2}\right)^{2}\right) / T,\left(x_{1} x_{2}\left(x_{1}-x_{2}\right)^{2}\left(e_{1}-e_{2}\right)^{2}\right) / T,\left(x_{1} x_{2}\left(x_{1}-\right.\right.$ $\left.\left.x_{2}\right)^{2}\left(s_{1}-s_{2}\right)^{2}\right) / T,\left(x_{1} x_{2}\left(x_{1}-x_{2}\right)^{2}\left(a_{1}-a_{2}\right)^{2}\right) / T,\left(x_{1} x_{2}\left(x_{1}-\right.\right.$ $\left.\left.x_{2}\right)^{2}\left(b_{1}-b_{2}\right)^{2}\right) / T$ and $\left(x_{1} x_{2}\left(x_{1}-x_{2}\right)^{2}\left(v_{1}-v_{2}\right)^{2}\right) / T$, using a no intercept least-squares analysis. The collected data with available Abraham parameters (see Table 2) were fitted to eq 8 and after excluding the nonsignificant coefficients ( $p>0.05$ ), the obtained trained model was

$$
\begin{align*}
\ln \eta_{m, T}= & x_{1} \ln \eta_{1, T}+x_{2} \ln \eta_{2, T}+\frac{x_{1} x_{2}}{T}[181.555 \\
& +135.913\left(c_{1}-c_{2}\right)^{2}-48.327\left(e_{1}-e_{2}\right)^{2} \\
& \left.-6.473\left(a_{1}-a_{2}\right)^{2}+2.643\left(b_{1}-b_{2}\right)^{2}\right] \\
& +\frac{x_{1} x_{2}\left(x_{1}-x_{2}\right)}{T}\left[-132.339+182.404\left(c_{1}-c_{2}\right)^{2}\right. \\
& -124.917\left(e_{1}-e_{2}\right)^{2}+10.572\left(s_{1}-s_{2}\right)^{2} \\
& \left.-7.140\left(b_{1}-b_{2}\right)^{2}\right]+\frac{x_{1} x_{2}\left(x_{1}-x_{2}\right)^{2}}{T} \\
& \times\left[6.893\left(b_{1}-b_{2}\right)^{2}\right] \tag{9}
\end{align*}
$$

which correlates the viscosity data with the correlation coefficient of $0.866, F$ value of 449 and significance level of $<0.0005$ ( $N=1652$ ). A wide viscosity range ( 0.000316 to 1.036980 $\mathrm{kPa} \cdot \mathrm{s}$ ) was correlated by the model. When the viscosity data were back-calculated by eq 9, the minimum ARD ( $4.2 \%$ ) was observed for [C4MIM][PF6] + acetone mixtures and the maximum ARD (67.1 \%) for [BMIM][BF4] + ethylene glycol mixtures. The overall ARD was $15.0 \pm 12.6 \%(\mathrm{NDS}=30)$ (for details of ARDs see Table 3). Figure 1 shows a plot of the calculated versus experimental viscosity data along with the linear equation and correlation coefficient in which excellent correlation was observed $(R=0.991)$. Equation 9 requires the experimental viscosity data of the pure solvents at each temperature as input data to predict the viscosity of binary mixtures.

Further reduction could be achieved by using another version of the model which is

Table 3. Average Relative Deviation of eqs 9 and 11

| RTIL | molecular solvent/RTIL | $N$ | eq 9 | eq 11 |
| :---: | :---: | :---: | :---: | :---: |
| [BMIM][BF4] | dimethyl sulfoxide | 77 | 5.6 | 4.4 |
| [BMIM][BF4] | ethylene glycol | 55 | 67.1 | 40.9 |
| [BMIM][BF4] | water | 88 | 17.0 | 18.4 |
| [BMIM][CF3SO3] | water | 77 | 15.0 | 22.0 |
| [BMIM][PF6] | dimethyl sulfoxide | 15 | 13.1 | 22.2 |
| [BMIM][PF6] | methanol | 15 | 15.8 | 16.5 |
| [BMIM][PF6] | tetrahydrofuran | 15 | 17.2 | 37.4 |
| [BMIM][SCN] | 1-butanol | 72 | 6.5 | 13.3 |
| [BMIM][SCN] | 1-hexanol | 72 | 4.6 | 14.8 |
| [BMIM][SCN] | 1-pentanol | 78 | 5.9 | 17.3 |
| [BPY][BF4] | [BPY][Tf2N] | 72 | 22.5 | 15.7 |
| [BUPY][BF4] | water | 130 | 30.5 | 26.7 |
| [C4MIM][PF6] | acetone | 15 | 4.2 | 25.4 |
| [C4MIM][PF6] | acetonitrile | 15 | 7.2 | 13.9 |
| [C4MIM][PF6] | dimethyl formamide | 66 | 7.1 | 23.9 |
| [C4MIM][PF6] | ethyl acetate | 15 | 8.0 | 24.3 |
| [C4MIM][PF6] | methanol | 15 | 15.7 | 16.4 |
| [C8IQUIN][NTf2] | 1-butanol | 55 | 33.5 | 10.8 |
| [EMIM][BF4] | water | 77 | 23.0 | 18.9 |
| [EMIM][EtSO4] | water | 56 | 13.0 | 14.9 |
| [EMISE] | 1-propanol | 33 | 4.3 | 26.8 |
| [EMISE] | 2-propanol | 33 | 7.7 | 35.6 |
| [EMISE] | ethanol | 36 | 9.4 | 15.1 |
| [EMISE] | methanol | 39 | 21.6 | 18.3 |
| [EMISE] | water | 30 | 8.4 | 12.5 |
| [EPY][SO4] | ethanol | 33 | 6.8 | 34.4 |
| [EPY][SO4] | propan-1-ol | 36 | 6.9 | 25.3 |
| [OCPY][BF4] | water | 140 | 25.7 | 23.8 |
| [OMIM][BF4] | ethanol | 104 | 13.1 | 15.3 |
| [PDMIM][BF4] | water | 88 | 13.4 | 15.6 |
|  | overall |  | 15.0 | 20.7 |

For the other data sets, the Abraham parameters were not available.

$$
\begin{align*}
& \ln \eta_{m, T}=\alpha_{0} x_{1}+\frac{x_{1}}{T}\left\{\begin{array}{l}
\alpha_{1}+\alpha_{2} c_{1}+\alpha_{3} e_{1}+\alpha_{4} s_{1}+\alpha_{5} a_{1}+\alpha_{6} b_{1}+\alpha_{7} v_{1} \\
+\alpha_{8} a_{1} \cdot b_{1}+\alpha_{9} \frac{a_{1}}{v_{1}}+\alpha_{10} \frac{b_{1}}{v_{1}}+\alpha_{11} \frac{a_{1} \cdot b_{1}}{v_{1}}
\end{array}\right\}+\beta_{0} x_{2} \\
& \quad+\frac{x_{2}}{T}\left\{\begin{array}{l}
\beta_{1}+\beta_{2} c_{2}+\beta_{3} e_{2}+\beta_{4} s_{2}+\beta_{5} a_{2}+\beta_{6} b_{2}+\beta_{7} v_{2} \\
+\beta_{8} a_{2} \cdot b_{2}+\beta_{9} \frac{a_{2}}{v_{2}}+\beta_{10} \frac{b_{2}}{v_{2}}+\beta_{11} \frac{a_{2} \cdot b_{2}}{v_{2}}
\end{array}\right\}+\frac{x_{1} x_{2}}{T}\left[\begin{array}{l}
W_{0}+W_{1}\left(c_{1}-c_{2}\right)^{2}+W_{2}\left(e_{1}-e_{2}\right)^{2} \\
+W_{3}\left(s_{1}-s_{2}\right)^{2}+W_{4}\left(a_{1}-a_{2}\right)^{2} \\
+W_{5}\left(b_{1}-b_{2}\right)^{2}+W_{6}\left(v_{1}-v_{2}\right)^{2}
\end{array}\right] \\
& \quad+\frac{x_{1} x_{2}\left(x_{1}-x_{2}\right)}{T}\left[\begin{array}{l}
W_{0}^{\prime}+W_{1}^{\prime}\left(c_{1}-c_{2}\right)^{2}+W_{2}^{\prime}\left(e_{1}-e_{2}\right)^{2} \\
+W_{3}^{\prime}\left(s_{1}-s_{2}\right)^{2}+W_{4}^{\prime}\left(a_{1}-a_{2}\right)^{2} \\
+W_{5}^{\prime}\left(b_{1}-b_{2}\right)^{2}+W_{6}^{\prime}\left(v_{1}-v_{2}\right)^{2}
\end{array}\right]+\frac{x_{1} x_{2}\left(x_{1}-x_{2}\right)^{2}}{T}\left[\begin{array}{l}
W_{0}^{\prime \prime}+W_{1}^{\prime \prime}\left(c_{1}-c_{2}\right)^{2}+W_{2}^{\prime \prime}\left(e_{1}-e_{2}\right)^{2} \\
+W_{3}^{\prime \prime}\left(s_{1}-s_{2}\right)^{2}+W_{4}^{\prime \prime}\left(a_{1}-a_{2}\right)^{2} \\
+W_{5}^{\prime \prime}\left(b_{1}-b_{2}\right)^{2}+W_{6}^{\prime \prime}\left(v_{1}-v_{2}\right)^{2}
\end{array}\right] \tag{10}
\end{align*}
$$

This version does not require any experimental viscosity data. When available data were fitted to eq 10 , the trained model after excluding nonsignificant variables was


Figure 1. Plot of the calculated viscosities using eq 9 versus experimental values.

$$
\begin{align*}
& \ln \eta_{m, T}=-7.085 x_{1}+\frac{x_{1}}{T}\left\{-1381.472+317.451 c_{1}\right. \\
& \quad+128.713 e_{1}+29.338 s_{1}+427.664 a_{1}+576.548 v_{1} \\
& \left.\quad+3430.864 \frac{a_{1}}{v_{1}}-883.984 \frac{b_{1}}{v_{1}}+1083.890 \frac{a_{1} \cdot b_{1}}{v_{1}}\right\} \\
& \quad-5.137 x_{2}+\frac{x_{2}}{T}\left\{649.173-230.029 c_{2}+517.417 e_{2}\right. \\
& \quad+187.305 s_{2}-256.863 a_{2}+215.281 b_{2}+218.199 v_{2} \\
& \left.\quad-49.271 a_{2} \cdot b_{2}\right\}+\frac{x_{1} x_{2}}{T}\left[274.842+130.361\left(e_{1}-e_{2}\right)^{2}\right. \\
& \quad-14.032\left(s_{1}-s_{2}\right)^{2}-7.967\left(a_{1}-a_{2}\right)^{2} \\
& \left.\quad+9.154\left(v_{1}-v_{2}\right)^{2}\right]+\frac{x_{1} x_{2}\left(x_{1}-x_{2}\right)}{T}[-176.488 \\
& \quad+259.610\left(c_{1}-c_{2}\right)^{2}-309.306\left(e_{1}-e_{2}\right)^{2} \\
& \left.\quad+22.617\left(s_{1}-s_{2}\right)^{2}-22.911\left(b_{1}-b_{2}\right)^{2}\right] \\
& \quad+\frac{x_{1} x_{2}\left(x_{1}-x_{2}\right)^{2}}{T}\left[154.378-52.663\left(a_{1}-a_{2}\right)^{2}\right. \\
& \left.+52.629\left(v_{1}-v_{2}\right)^{2}\right] \tag{11}
\end{align*}
$$

Equation 11 correlates the viscosity data of RTIL + molecular solvents/RTIL with the correlation coefficient of 0.998 , F value of 14874 and significance level of $<0.0005(N=1652)$. When the viscosity data are back-calculated by eq 11 , the minimum $\operatorname{ARD}(4.4 \%)$ was observed for $[B M I M][B F 4]+$ dimethyl sulfoxide mixtures and the maximum ARD (40.9 \%) for [BMIM][BF4] + ethylene glycol mixtures where the overall ARD was $20.7 \pm 8.4 \%(N D S=30)$ (for details of ARDs see Table 3). Figure 2 illustrates the linear plot of the predicted viscosity data by eq 11 against the corresponding experimental values along with the equation and the correlation coefficient. In comparison with eq 9 , lower correlation and more scattered plot is obtained; however, eq 11 is an in silico model and requires the Abraham parameters of the pure solvents which usually are available from the literature or could be calculated using group contribution methods.

In conclusion, the proposed models provided reasonable accurate results to calculate the viscosity of RTILs + solvent mixtures with respect to their composition and temperature.


Figure 2. Plot of the calculated viscosities using eq 11 versus experimental values.

The overall ARD of 6.4 \% was obtained for correlated data, 15.0 \% for predicted data employing experimental data of $\eta_{1, T}$ and $\eta_{2, T}$ and $20.7 \%$ for in silico prediction of the data revealing that the Jouyban-Acree model is able to accurately represent the viscosity data of the mixtures and Abraham parameters are good predictors of the interactions occurring in the mixtures. The overall ARD of a similar model from the literature, that is, eq 5, was 22.4 which is significantly higher than the corresponding value for the proposed model.

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## Notes

The authors declare no competing financial interest.

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