AN INVESTIGATION OF THE EFFECT OF VIOLATING THE ASSUMPTION OF HOMOGENEITY OF REGRESSION SLOPES IN THE ANALYSIS OF COVARIANCE MODEL UPON THE F-STATISTIC

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The study seeks to determine the effect upon the F-statistic of violating the assumption of homogeneity of regression slopes in the one-way, fixed-effects analysis of covariance model. The study employs a Monte Carlo simulation technique to vary the degree of heterogeneity of regression slopes with varied sample sizes within experiments to determine the effect of such conditions. One hundred and eighty-three simulations were used.

The simulation procedure involved setting the number of treatment populations ($k = 2, 3,$ or 5), sample sizes ($n = 20, 30, 40, 100,$ and 200), and regression coefficients. The regression coefficients were set at differing degrees of heterogeneity varying from equal slopes to sets of slopes with values as different as .1 and .9. For each sample size, a set of concomitant values, X_{ij}, was generated by the computer with a $N(0, 1)$. In a given simulation, the values of the criterion variable, Y_{ij}, were computer generated using the model

$$Y_{ij} = \beta_j X_{ij} + e_{ij}.$$

The e_{ij} terms were randomly generated by the computer using a random number generator. The F value was calculated by analysis of covariance. This F-producing procedure was replicated 2000 times for a given set of conditions. The 2000 F-statistics were used to produce an empirical distribution which was compared graphically and analytically with the appropriate F-distribution.

The data which constituted the basis for the findings were the empirically obtained significance levels, called "actual significance levels," for each simulation. These levels were compared to the nominal (theoretically expected) significance levels, .20, .10, .05, and .01. By using a 95 per cent confidence interval about the nominal level, these actual significance levels were determined to be significantly deviant from their corresponding nominal levels.

Several patterns are apparent in the data. First, for equal sample sizes within an experiment, the empirical F-distributions closely approximate the normal theory F-distributions for all but the most heterogeneous regression slopes. Also, the critical or tabled value of F associated with the nominal significance level is actually a conservative value of F when the assumption of homogeneity of regression slopes is violated. This phenomenon is observed from the pattern of empirically derived proportions which are consistently less than the expected proportions.

Another pattern emerges, however, when considering experiments in which there are unequal sample sizes. For simulations in which the smaller samples were paired with the smaller slopes within an experiment, the actual significance levels which differ significantly from the nominal levels yield a positive bias almost without exception. That is, the actual levels are greater than the nominal levels for these simulations. If, however, the smaller slopes are matched with the larger samples within an experiment, the bias is again negative.

The following conclusions appear to be appropriate:

1. For equal sample sizes within an experiment, the one-way, fixed-effects analysis of covariance model is robust to violation of the homogeneous regression slopes assumption under all but the most heterogeneous slopes. As the degree of heterogeneity of the slopes increases, the test becomes more conservative with respect to making a Type I error.

2. With unequal sample sizes within an experiment, no generalization can be made. The direction of bias in such experiments with heterogeneous regression coefficients cannot be predicted, and consequently the effect on Type I error cannot be determined.

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CHAPTER I

INTRODUCTION

The proper application of a statistical technique such as the analysis of covariance depends upon the satisfaction of certain assumptions. Exact satisfaction of these theoretical assumptions is never achieved in a practical situation. The question then arises as to the extent to which these assumptions may be violated without seriously affecting the results obtained through application of the statistical technique.

The analysis of covariance, one of the most widely used statistical techniques, has approximately five such assumptions, depending on the form in which they are stated. Among these assumptions is the assumption of homogeneity of regression slopes (10, p. 586). This assumption implies that the regression of the criterion variable Y on the covariate X is the same for all treatment populations. In most situations this assumption is seldom reported to be satisfied, and the actual meeting of this assumption is subject to question in many research studies (7, p. 34). Knowledge regarding the extent to which the covariance technique is robust with respect to this assumption would tend to validate many past studies and justify the use of this method in many future studies where precise satisfaction of the assumption is not possible. Knowledge to the contrary should have an opposite effect.

A review of the literature on this subject points out the remarkable degree to which the analysis of variance model can tolerate departure from the assumption of the model (1, 2, 5). This fact has led researchers to feel no longer constrained to transform data to achieve normal distributions and equal variances for all treatment groups. Winer (10, p. 219) states that such transformations are no longer considered important in the light of the robustness of the analysis of variance model.

The robustness of the analysis of variance model naturally leads to a question of the robustness of the covariance model. Should the results of this study support the robustness of the covariance model, then many studies, past and future, in which the assumption of homogeneity of regression slopes are not strictly satisfied will be validated. On the other hand, should the results lend support to a strict satisfaction of the assumption, the results of many research studies employing analysis of covariance, in the behavioral sciences as well as other disciplines, will be made suspect whenever the assumption is not met.

Problem

The problem of this study was the effect of violating the assumption of homogeneity of regression slopes upon the

F-statistic in the one-way fixed-effects analysis of covariance model.

Purpose

The purpose of the study was to employ a Monte Carlo simulation to vary the degree of heterogeneity of regression slopes with varied sample sizes within experiments to determine the effect of such conditions.

Rationale

Wilson and Carry contend that:

> With the availability of computer programs, there is considerable misuse of analysis of covariance in educational research. One of the most frequent and flagrant misuses is the failure to test for homogeneity of regression. . . . There is little evidence that the test of homogeneity of regression has been used in educational research except as a nuisance parameter in analysis of covariance. It is a nuisance (1) because the analysis of covariance may be invalidated unless the hypothesis of homogeneity of regression can be accepted, and (2) it is not the main hypothesis of interest in the research (9, p. 86).

Cochran states that:

> Although the effects of failures in these assumptions on the analysis of covariance as such do not appear to have been investigated, much of the related work on the analysis of variance carries over--for instance, that on the effects of non-normality or inhomogeneity of variance in e_{ij} (3, p. 277).

Winer, however, points out that:

> Evidence from the usual analysis of variance indicates that F-tests in the analysis of covariance are robust with respect to the violation of the two assumptions, normality and homogeneity of the

residual variance. The effect of nonhomogeneity of within-class regression. . . has not been studied (10, p. 586).

Kirk substantiates this view when he reports that:

> In general, tests of significance in the analysis of covariance are robust with respect to violation of the assumptions of normality and homogeneity of the residual variance. Little is known concerning the effect of violations of the assumption of homogeneity of within-group regression coefficients (4, p. 469).

The Model

A one-way fixed-effects analysis of covariance model requires that the following assumptions be made:

1. The criterion variable Y_{ij} is expressed as a linear combination of independent components: an overall mean, μ; a treatment effect, τ_j; a linear regression on X, $\beta(X_{ij} - \overline{X}_{..})$; and an error term, e_{ij} (6, p. 15).

2. The treatment effects are constants rather than random variables (6, p. 15).

3. The X's are fixed over all replications and are measured without error (8, p. 309).

4. The e_{ij}'s are normally distributed with a mean of zero and a variance, σ_e^2, which is constant across treatment groups (6, p. 15).

5. The regression of Y on X after removal of. . . treatment differences is linear and independent of treatments. . . (8, p. 309).

Assumption one leads to the expression of the criterion variable Y as

$$Y_{ij} = \mu + \tau_j + \beta(X_{ij} - \bar{X}..) + e_{ij}$$

where X_{ij} is the concomitant variable (covariate) and $\bar{X}..$ is the grand mean of the X values.

Implicit in assumption five is the assumption of homogeneous regression slopes.

The over-riding question which sets the direction of this investigation is:

> What is the effect of violating the assumption of homogeneity of regression slopes (assumption five) upon the F-statistic of the analysis of covariance under certain experimental conditions?

This question becomes an inquiry as to, under the null condition, how the empirically derived distributions of F-ratios obtained under the conditions of this study compare with the theoretical F-distributions.

The null condition incorporates the assumption that treatment populations have identical distributions on the dependent (criterion) variable and identical distributions with respect to the concomitant variable. This simulates the condition in which the analysis of covariance is employed to increase precision in a "true" experiment where subjects are randomly assigned to treatments.

Definition of Terms

For the purposes of this study the following definitions were formulated:

Nominal significance level--The nominal significance level, α, is the percentage of F-ratios which exceed the tabled value of F associated with k-1 and N-k-1 degrees of freedom, $_{1-\alpha}F_{k-1,\ N-k-1}$. This tabled F associated with α is the point on the central F distribution above which 100α per cent of F-ratios will occur when all assumptions of the analysis of coveriance are met.

Actual significance level--The actual significance level, α', is the percentage of F-ratios which exceed the tabled value of F, $_{1-\alpha}F_{k-1,\ N-k-1}$, in an empirical distribution.

Robust--A statistical model is robust with respect to a violation of an assumption to the degree that the model can tolerate violation of the assumption without seriously affecting the results. Specifically, the degree to which the actual significance level fails to deviate significantly from the nominal significance level determines the robustness of the F-statistic with respect to violation of the assumption.

Significant difference between nominal and actual significance levels--An actual significance level is said to differ significantly from a nominal significance level when it fails to fall within a ninety-five per cent confidence interval about the nominal level.

Monte Carlo simulation--A Monte Carlo simulation is a procedure in which random samples are drawn from populations having certain parameters and then a particular statistic is computed. The process is repeated until an empirical sampling distribution of the statistic is obtained. When a certain number of repetitions have yielded the empirical sampling distribution, this distribution is then compared with the normal theory distribution to determine the degree to which it departs from the normal theory distribution (6, p. 19).

Limitations

This study was limited to experimental conditions simulated with the following conditions:

1. The treatment effects, τ_j, are such that $\sum \tau_j = 0$;
2. The number of treatment groups considered include only k = 2, 3, and 5;
3. Only selected sets of regression slopes with varying degrees of heterogeneity were used;
4. Only selected sample sizes were employed;
5. Selected combinations of slopes with sample sizes were used.

Basic Assumptions

The random number generator was tested, as described in the chapter on procedures, to verify the characteristics of uniform or normal distribution and independence of numbers in the sequence. However, since this procedure is not infallible, it is assumed that the generator produced numbers which are randomly sampled.

Hypotheses

To set the direction of this study and investigate the following question, the subsequent hypotheses were formulated:

Question: What is the effect of violating the assumption of homogeneity of regression slopes upon the F-test of the analysis of covariance under certain experimental conditions?

Hypothesis 1. Actual significance levels will not differ significantly from nominal significance levels for regression slopes as heterogeneous as:

(a) $\beta_1 = .3$, $\beta_2 = .7$, for k = 2, and nominal levels of .010 and .050,

(b) $\beta_1 = .4$, $\beta_2 = .6$, for k = 2, and nominal levels of .100 and .200.

(c) $\beta_1 = .3$, $\beta_2 = .5$, $\beta_3 = .7$, for k = 3, and nominal levels of .010, .050, .100, and .200,

(d) $\beta_1 = .3$, $\beta_2 = .4$, $\beta_3 = .5$, $\beta_4 = .6$, $\beta_5 = .7$, for k = 5, and nominal levels of .010 and .050,

(e) $\beta_1 = .4$, $\beta_2 = .4$, $\beta_3 = .5$, $\beta_4 = .6$, $\beta_5 = .6$, for k = 5, and nominal levels of .100 and .200.

Hypothesis 2. For a given set of regression slopes, for all sample sizes, there will be no significant difference between the actual significance levels and the nominal significance levels.

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CHAPTER II

SURVEY OF RELATED RESEARCH

Kendall (5, p. 245) admonishes researchers that the analysis of covariance "is not a mill which will grind out results automatically without care or forethought on the part of the operator." He depicts the covariance technique as "a rather delicate instrument which can be called into play when precision is needed," but "requires skill as well as enthusiasm to apply to the best advantage."

The question of the "delicacy" of statistical techniques such as covariance has intrigued researchers for decades. Pearson (7), in 1931, published his work on the effect of non-normal variation on the F-test of analysis of variance. In 1935, Bartlett (3) reported his work on the effect of non-normality of distributions on the $\underline{t}$ distribution.

The list of studies of the robustness of the general linear model in analysis of variance has continued to grow since the Pearson study.

Studies by Atiqullah (1, 2) and Norton (6) exemplify two primary directions robustness studies have taken. Atiqullah has utilized a mathematical analysis approach to his study of the robustness of the F-test. Norton, following the example of Pearson, has employed an empirical approach which is sometimes called a Monte Carlo simulation.

Norton investigated the effects of non-normality and of heterogeneity of variance upon the F-distribution. In his procedure he constructed "card populations" of 10,000 cases each from which he drew samples using electric tabulating equipment. In each phase of his work Norton selected 3000 or 3333 sets of k random samples of n cases each, computed F-ratios, and thus obtained an empirical sampling distribution of 3000 or 3333 F's. The values of k were 3 or 4 and the values of n used were 3, 5, 6, and 10. The empirical distributions thus obtained were each compared with the appropriate normal-theory F-distribution. Norton concluded that so long as the distribution of criterion measures is homogeneous in both form and variance for the various treatment populations, and as long as it is neither markedly peaked nor flat, the F-distribution seems insensitive to the violation of assumptions of distribution and variance.

Box (4) analytically studied the effect of group-to-group inequality of variance upon the F-statistic in the analysis of variance. He verified the conclusions of Norton's study, which was restricted to equal numbers within treatment groups. However, with unequal groups Box reported that inequality of variances across groups does seriously affect the F-distribution.

Atiqullah's mathematical analysis work on the "robustness of the covariance analysis of a one-way classification" led him to conclude that "unlike the analysis of variance F-test, the analysis of covariance F-test is found to be appreciably affected by non-normality even in balanced classifications" (1, p. 365). He states that the distribution of the concomitant variables determines the degree of sensitivity to non-normality. In addition he concludes that "the violation of the assumption of no treatment-slope interaction does not seem to have serious effect on the F-test in large samples, but a much more serious effect appears to be that of a quadratic component of regression on the F-test" (1, p. 365). Atiqullah's work employed asymptotic results requiring the number of treatment groups to approach infinity. As such, the results of his investigation are less specific and less useful. Peckham points out that "the assumptions required for this analysis are such that little light is shed on the practical effects of the bias in typical research applications" (8, p. 28).

Violation of the assumption of homogeneous regression slopes has been investigated under certain conditions in a computer simulation study by Robinson (9). Robinson generated normal bivariate populations with identical means and standard deviations but with different regression slopes. He restricted his investigation to three treatment populations within an experiment, samples of equal size chosen from each population, and five combinations of five regression combinations. He concluded that "as sample size increased, the deviations from expectancy increased in size, thus indicating a definite trend toward over-estimation of Type I error" (9, pp. 36-37).

Peckham claims that a Monte Carlo simulation done by him "could be considered a first step in a comprehensive investigation to determine the limits of conditions under which the fixed-effects analysis of covariance is robust with respect to violation of the assumption of homogeneous slopes and other violations" (8, p. 61). In his study Peckham simulated the combination of regression slopes with varying numbers of treatment groups (k = 2, 3, and 5) and varying sample sizes (n = 5, 10, and 20). For each experiment simulated the sample sizes were equal, while the regression slopes were allowed to take on increasing degrees of heterogeneity. His conclusion was that:

> . . . the one-way fixed-effects analysis of covariance is robust to violations of the assumption of homogeneous slopes in the randomized experiment except when those violations become extreme. As the degree of heterogeneity increases, the analysis becomes more conservative with respect to making a Type I error. The robustness is maintained when the number of treatment groups and the sample sizes are varied (8, p. 60).

As a result of his findings Peckham suggested that other conditions "should be studied singly or in combination with each other and with heterogeneous regression slopes. . ." (8, p. 61). Among these conditions he recommended that investigation should include unequal sample sizes within an experiment (8, p. 61).

The approach taken in this study is similar to Peckham's. Unlike Peckham's investigation, the present study utilizes unequal sample sizes within simulated experiments and includes larger sample sizes than those employed by Peckham. As such, this study is an extension of Peckham's work. Care was taken to follow many of Peckham's conventions pertaining to conditions set on parameters and interpretation of results so as to increase the comparability of the results. The study of Box (4) cited previously points up the danger of generalizing to unequal groups from results restricted to equal groups. Possible discrepancies between Peckham's study with small samples and the previously cited work of Atiqullah with large groups, together with the results of Box, seem to justify further investigation using unequal groups and larger samples.

The present study uses a Monte Carlo simulation approach to the investigation of the effects of violation of the regression slopes assumption in analysis of covariance. The sample sizes were varied, with samples of 20, 30, 40, 100, and 200 used to give simulated experiments with unequal groups. Regression slopes were systematically varied to yield differing degrees of heterogeneity within experiments. The number of groups was also varied, with experiments involving two, three, and five groups being simulated.

Summary

Some possibly crucial theoretical and methodological questions remain concerning the robustness of the analysis of covariance model. Very little investigation has been made of situations where more than one assumption is violated. The state of the art concerning interpretation of heterogeneous regressions needs to be extended to the point, according to Wilson and Carry, "where we can test hypotheses in true experiments when heterogeneity of regression is predicted and interpreted; where it spells success for the experiment rather than defeat" (10, p. 88).

The present study attempts to answer a question concerning the violation of one assumption. As such, it should be considered one step in a comprehensive investigation in which the ultimate goal is a fuller and surer application of covariance.

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CHAPTER III

PROCEDURES

A one-way fixed-effects analysis of covariance model requires that the following assumptions be made:

1. The criterion variable Y_{ij} is expressed as a linear combination of independent components; an overall mean, μ; a treatment effect, τ_j; a linear regression on X, $\beta(X_{ij} - \bar{X}_{..})$; and an error term, e_{ij} (4, p. 15).

2. The treatment effects are constants rather than random variables (4, p. 15).

3. The X's are fixed over all replications and are measured without error (5, p. 309).

4. The e_{ij}'s are normally distributed with a mean of zero and a variance, σ_e^2, which is constant across treatment groups (4, p. 15).

5. The regression of Y on X after removal of. . . treatment differences is linear and independent of treatments (5, p. 309).

Implicit in assumption five is the assumption of homogeneous regression slopes.

The over-riding question which set the direction of this investigation was:

> What is the effect of violating the assumption of homogeneity of regression slopes upon the F-statistic of the analysis of covariance, one-way fixed-effects with one concomitant variable, under certain specified experimental conditions?

This question became an inquiry as to, under the null condition, how the empirically derived distributions of F-ratios obtained under conditions described in the following two paragraphs compared with the central F-distributions. In particular, how did the actual significance levels compare with nominal significance levels?

The null condition incorporates the assumption that the treatment populations have identical distributions on the dependent (criterion) variable and identical distributions with respect to the concomitant variable. This simulates the condition in which the analysis of covariance is employed to increase precision in a "true" experiment where subjects are randomly assigned to treatments. For purposes of this study these conditions were simulated by setting $E(Y._j) + \overline{X}._j = 0$ and $\sigma_y^2 = MS_x = 1$, where

$$MS_{x_j} = \sum_{i=1}^{n_j} \frac{(X_{ij} - \overline{X}.j)^2}{n_j},$$

with the values of the concomitant variable chosen to conform as closely as possible to the normal distribution with variance one.

Specifically, the experimental conditions referred to in the original question can be further delineated as follows:

- equal means were set on the dependent variable Y and the concomitant variable X across treatment groups (this simulated random assignment to treatment groups);
- the number of treatment groups was varied;
- the number of observations was varied within each group within an experiment (i.e. equal and unequal sample size within an experiment);
- the regression slopes were varied within an experiment.

The conditions of equal sample sizes and unequal sample sizes within an experiment were studied in combination with heterogeneous regression slopes. Specifically, sample n's of 20, 30, and 40 were chosen as representative of situations encountered in educational research. Combinations of these sample sizes were taken with combinations of regression coefficients with a mean of .5. The conditions for each of the simulations are specified in Appendix A. The number of treatment groups included were k = 2, 3, and 5. Sample sizes of 100 and 200 were included with equal n's only within an experiment. Simulations occur in sets of five, the only difference between simulations of a given set being the regression slopes, which were set at differing degrees of heterogeneity, ranging from equal slopes of .5 to simulations involving slopes as different as .1 and .9.

The first simulation in each set of five has equal regression slopes. This provided a check on the simulation process since no assumptions were violated. Further evidence for validation of the simulation process was gathered and reported in Table VII in Appendix B. These data include the values of the statistics, obtained in each simulation, corresponding to population parameters over all replications of the experiment. While the values for the means, variances, and regression coefficients showed large random variations from replication to replication, the means of these statistics over all replications in a simulation yielded a close approximation to all parameters.

The specifications for each of the simulations are given in Table V, Appendix A. Simulations 1-5, 31-35, and 81-85 are replications of Peckham's work (4, pp. 49-50). Other simulations in this study extend Peckham's study both to larger sample sizes and to a consideration of unequal sample sizes within an experiment. Simulations 21-30, 61-70, and 121-130 represent extension of Peckham's study with equal n's to larger samples, n = 30 and n = 40.

Table V also describes simulations which extend sample sizes to large (n = 100 and n = 200) samples (simulations 136-165). As such, this set of simulations is also an extension of Peckham's study to larger samples.

For each sample size, n, a set of concomitant variable values, X's, was chosen (See Table VI, Appendix A). Each set of X's was chosen so as to be approximately normally distributed with a mean of zero and a variance approximately one. These sets of concomitant variable values were generated by the IBM 360 Model 50 computer system at North Texas State University. The IBM subroutine GAUSS was utilized to generate, for each n, a set of random numbers with the mean and variance approximately zero and one, and the distribution approximately normal. (See Appendix D on the application of the random number generator for more detailed explanation). The values generated were then transformed linearly to produce values with mean exactly zero, the variances and distributions remaining the same. The sets of X's generated for each n were used for all replications involving the given n.

In a given simulated experiment, a set of conditions was set for the treatment populations involved. Within each treatment population the sample size, n_j, and a regression slope, β_j, was set.

The values of the criterion variable, Y, were then generated from the concomitant variable, X, using a random error term, e_{ij}. This general procedure is justified by the following comments.

The general linear model for a one-way analysis of covariance, fixed-effects model, is

$$Y_{ij} = \mu + \tau_j + \beta_j(X_{ij} - \bar{X}_{..}) + e_{ij} \quad (6, \text{ p. } 584)$$

where

Y_{ij} = criterion score for an individual;

μ = the general elevation of the criterion scores;

τ_j = the treatment effect for treatment j;

β = the [illegible]

X_{ij} = the concomitant variable score for an individual;

$\bar{X}..$ = the over-all mean of the concomitant variable;

e_{ij} = the error term for an individual.

Under the null hypothesis, which was assumed, the means of the treatment groups on the adjusted criterion, $\mu + \tau_j$, were assumed to be equal (1, p. 146). Without loss of generality these means were set equal to zero. Thus the model became

$$Y_{ij} = \beta_j(X_{ij} - \bar{X}..) + e_{ij}.$$

Further, the over-all mean of the concomitant variable, $\bar{X}..$, was set equal to zero when the X values were selected. This procedure was justified by considering the concomitant variable as a standard score with a mean of zero. For simplification without loss of generality, standard scores ($\mu = 0$, $\sigma = 1$) were used throughout the study for both dependent and independent variables. Therefore, the model finally took the form

$$Y_{ij} = \beta_j X_{ij} + e_{ij}.$$

This formula was used to generate an individual's Y score using his X score, a regression slope, and random error term. For a given sample from a treatment population, the β_j was set and set of X's constructed for the sample size was used. The random error terms were generated by the computer and the corresponding Y values were calculated.

The error terms were generated by the computer such that they had a mean of zero and normal distribution. One of the assumptions for analysis of covariance is that these error terms will have a constant variance across treatment populations. This was found, under violation of the homogeneous regression slopes assumption and the specified conditions of this study, to be impossible.

For each simulated experiment all assumptions were met, with the above noted exception, except for the assumption of homogeneous regression slopes. These slopes were set at differing degrees of heterogeneity.

The simulation process was accomplished using a FORTRAN computer program (Appendix E) in which the experimental conditions and parameters were set by the investigator.

The following conditions and parameters were set for each simulation:

1. The number of treatment populations, k;

2. The expectation of the dependent variable, Y (criterion), within each population, $E(Y_{ij})$, which was set equal to zero;

3. The variance of the dependent variable within each population, which was set equal to one, (i.e. $\sigma_y^2 = 1$);

4. The slope, β_j, of the regression of Y on X within each population;

5. The size of the samples to be taken from each population;

6. The fixed set of concomitant variable, X, values, sizes, for each observation within each population.

The computer program then performed the sampling from each population and computed the F-statistic. This process was repeated for a given set of conditions and parameters 2000 times, thus yielding 2000 F-statistics. These F-ratios were then used to obtain an empirical F-distribution which was compared to the theoretical F-distribution.

Consequently, for each simulation (each set of conditions and parameters) a frequency distribution of 2000 F-ratios was obtained. This empirical distribution was plotted by the computer (Appendix C), superimposed upon the theoretical F-distribution. The computer also output frequencies for computation of actual significance levels for comparison with nominal levels of .200, .100, .050, and .010. (See Table II, Chapter IV).

Actual significance levels were judged to be significantly different from their corresponding nominal levels if they did not fall within the 95 per cent confidence limits presented in Table I established originally by Peckham (4, p. 44).

The Computer Installation

The computer system of the North Texas State University Computing Center was utilized in this study. Primary equipment used was an IBM 360 Model 50 Computer System with 262, 144 bytes of memory, disk storage facility with six drives, four magnetic tape drives,and a Calcomp 663 drum plotter.

TABLE I

NINETY-FIVE PER CENT CONFIDENCE LIMITS FOR PROPORTIONS CORRESPONDING TO NOMINAL SIGNIFICANCE LEVELS

Number of Sample Points	Limits	Proportions			
		.20	.10	.05	.01
2000	Upper	.218	.114	.061	.016
	Lower	.183	.087	.041	.006

The Random Number Generator

A random number generator was employed by the computer to generate the error terms of the model. It was also used to select the values of the concomitant variable. The term "random number" used in the context of this study should be understood to be "pseudorandom number." Pseudorandom number sequences generated internally by the computer are not random in the true sense, because they are completely determined by the starting data and have limited precision. Thus, according to Naylor (3, p. 57), the pseudorandom numbers must be submitted to a number of statistical tests used to detect non-randomness. If they pass the tests, these pseudorandom numbers can be treated as "truly" random numbers even though they are not.

The IBM subroutine RANDU was used to generate random numbers assumed to be from a uniform distribution. Tests of uniformity, oscillatory nature, and independence of the pseudo-random numbers were applied with acceptable results. Detailed explanations of these tests are presented in Appendix D.

Normal random deviates with mean of zero and variance $1 - \beta_j^2$ were obtained from the uniformly distributed variates produced by RANDU using the trigonometric transformations

$$x_1 = (-2 \log_e r_1)^{\frac{1}{2}} \cos 2\pi r_2$$
$$x_2 = (-2 \log_e r_1)^{\frac{1}{2}} \sin 2\pi r_2.$$

The r_1 and r_2 are two uniformly distributed independent random variates defined on the interval (0, 1). The transformed values x_1 and x_2 are two random variates from a standard normal distribution. These values were multiplied by $1 - \beta_j^2$ to give a variance of $1 - \beta_j^2$. According to Muller, "this method produces exact results and the speed of calculation compares fairly well with the Central Limit approach subject to the efficiency of the special function subroutines" (2, p. 382).

Summary

Summarizing the procedures utilized in this Monte Carlo simulation, we have the following:

1. The model used was

$$Y_{ij} = \beta_j X_{ij} + e_{ij}.$$

2. Each simulated experiment involved setting the number of treatment populations, sizes of the samples to be selected from each population, and regression coefficients for each treatment population. The number of treatment populations were two, three, and five. The sample sizes were 20, 30, 40, 100, and 200. The regression slopes were set at differing degrees of heterogeneity from equality to sets of slopes with values as different as .1 and .9.

3. The simulations, totaling 183, occur in sets of five with the exception of the last three, differing within a set only with the regression slopes set at increasing degrees of heterogeneity.

4. For each sample size a set of concomitant variable values was generated by the computer with mean of zero, variance of approximately one, and an approximate normal distribution. These values were fixed for all replications involving a given n.

5. In a given simulation, k, n_j, and β_j (j = 1, ... k) having been set, the values of the criterion variable, Y_{ij}, were generated by the computer using the formula for the model,

$$Y_{ij} = \beta_j X_{ij} + e_{ij}.$$

The e_{ij} terms were randomly generated by the computer using a random number generator with mean set equal to zero, variance equal to $1 - \beta_j^2$, and distribution normal.

6. The computer calculated the value of F using analysis of covariance. This process (steps 5 and 6) was replicated

2000 times for a given set of conditions and parameters. Thus the only changes from replication to replication within a given simulation were the values of the e_{ij} which were randomly generated, and the values of the criterion, Y_{ij}, which was a function of e_{ij}.

7. The 2000 F-statistics were used to produce an empirical F-distribution which was compared graphically and analytically with the appropriate theoretical F-distribution.

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CHAPTER IV

FINDINGS

A comparison of the actual significance levels with the nominal significance levels of .200, .100, .050, and .010 obtained on the 183 simulations of this study are presented in Table II. The number of groups and their sample sizes are also presented along with the regression coefficients for each simulated experiment. Graphic comparisons of the empirically derived distributions with the theoretical distributions are contained in Appendix C. A complete summary of simulation results is found in Table V of Appendix B.

The data in Table II are arranged in sets of five with varying degrees of heterogeneity of the regression coefficients. The first simulation in each set gives the results of the simulations with all assumptions satisfied including equality of regression slopes. These simulations serve as a verification of the simulation process as the discrepancies between the nominal and actual significance levels are well within sampling error limits with only three exceptions. This indicates that the simulation process performed accurately. Additional data confirming the accuracy of the simulation process can be found in Appendix D. The only set of simulations not containing five simulations is simulations 181-183, which include two simulations with non-symmetric slopes.

TABLE II

COMPARISON OF ACTUAL SIGNIFICANCE LEVEL WITH NOMINAL SIGNIFICANCE LEVEL IN SIMULATED EXPERIMENTS

Simulation Number	Number of Groups, K	Sample Sizes, n_j	Regression Coefficients, β_j	Actual Significance Levels Corresponding to Nominal Significance Levels of			
				.200	.100	.050	.010
1	2	20,20	.5,.5	.2040	.1045	.0465	.0120
2	2	20,20	.4,.6	.1870	.0990	.0490	.0090
3	2	20,20	.3,.7	.1895	.0895	.0490	.0105
4	2	20,20	.2,.8	.1620*	.0735*	.0365*	.0050*
5	2	20,20	.1,.9	.1495*	.0595*	.0220*	.0040*
6	2	20,20	.5,.5	.1915	.0920	.0460	.0085
7	2	20,30	.4,.6	.2160	.1065	.0565	.0115
8	2	20,30	.3,.7	.2255*	.1195*	.0595	.0140
9	2	20,30	.2,.8	.2210*	.1085	.0560	.0085
10	2	20,30	.1,.9	.2240*	.1190*	.0630*	.0110
11	2	20,40	.5,.5	.1990	.1000	.0525	.0075
12	2	20,40	.4,.6	.2140	.1050	.0480	.0110
13	2	20,40	.3,.7	.2465*	.1270*	.0735*	.0235*
14	2	20,40	.2,.8	.2500*	.1350*	.0785*	.0160
15	2	20,40	.1,.9	.2345*	.1295*	.0720*	.0200*
16	2	30,40	.5,.5	.2025	.0965	.0445	.0150
17	2	30,40	.4,.6	.1955	.0940	.0445	.0090
18	2	30,40	.3,.7	.1895	.1005	.0495	.0115
19	2	30,40	.2,.8	.1870	.0980	.0485	.0100
20	2	30,40	.1,.9	.1880	.0920	.0420	.0070
21	2	30,30	.5,.5	.2020	.0940	.0410	.0115
22	2	30,30	.4,.6	.2000	.1040	.0530	.0085
23	2	30,30	.3,.7	.2050	.0990	.0450	.0065
24	2	30,30	.2,.8	.1855	.0850*	.0375*	.0080
25	2	30,30	.1,.9	.1510	.0665	.0290	.0040

TABLE II--Continued

Simulation Number	Number of Groups, K	Sample Sizes, n_j	Regression Coefficients, β_j	Actual Significance Levels Corresponding to Nominal Significance Levels of .200	.100	.050	.010
26	2	40,40	.5,.5	.2070	.0910	.0440	.0115
27	2	40,40	.4,.6	.2075	.1055	.0445	.0100
28	2	40,40	.3,.7	.1910	.0990	.0510	.0070
29	2	40,40	.2,.8	.1755*	.0785*	.0365*	.0060
30	2	40,40	.1,.9	.1600*	.0705*	.0300*	.0040*
31	3	20,20,20	.5,.5,.5	.2000	.0990	.0510	.0105
32	3	20,20,20	.4,.5,.6	.1925	.0920	.0445	.0095
33	3	20,20,20	.3,.5,.7	.1600*	.0755*	.0360*	.0100
34	3	20,20,20	.2,.5,.8	.1670*	.0845*	.0385*	.0080
35	3	20,20,20	.1,.5,.9	.1475*	.0720*	.0345*	.0075
36	3	20,20,30	.5,.5,.5	.1980	.1010	.0555	.0155
37	3	20,20,30	.4,.5,.6	.2250*	.1175*	.0575	.0110
38	3	20,20,30	.3,.5,.7	.1940	.0990	.0490	.0085
39	3	20,20,30	.2,.5,.8	.2210*	.1150*	.0635*	.0145
40	3	20,20,30	.1,.5,.9	.2020	.1060	.0575	.0110
41	3	20,20,40	.5,.5,.5	.1960	.1010	.0510	.0150
42	3	20,20,40	.4,.5,.6	.2195*	.1165*	.0600	.0140
43	3	20,20,40	.3,.5,.7	.2385*	.1190*	.0635*	.0130
44	3	20,20,40	.2,.5,.8	.2270*	.1150*	.0635*	.0165*
45	3	20,20,40	.1,.5,.9	.2125	.1160*	.0660*	.0185*
46	3	20,30,30	.5,.5,.5	.1910	.0940	.0450	.0085
47	3	20,30,30	.4,.5,.6	.2070	.1050	.0555	.0145
48	3	20,30,30	.3,.5,.7	.2070	.1055	.0525	.0110
49	3	20,30,30	.2,.5,.8	.1805*	.0895	.0495	.0090
50	3	20,30,30	.1,.5,.9	.1920	.1035	.0475	.0150
51	3	30,40,40	.5,.5,.5	.1985	.1015	.0470	.0115
52	3	30,40,40	.4,.5,.6	.1965	.0965	.0520	.0145
53	3	30,40,40	.3,.5,.7	.2005	.1020	.0495	.0075
54	3	30,40,40	.2,.5,.8	.1965	.0950	.0455	.0090
55	3	30,40,40	.1,.5,.9	.1735*	.0880	.0415	.0040*

TABLE II--Continued

Simulation Number	Number of Groups, K	Sample Sizes, n_j	Regression Coefficients, β_j	Actual Significance Levels Corresponding to Nominal Significance Levels of .200	.100	.050	.010
56	3	30,30,40	.5,.5,.5	.2055	.1030	.0480	.0080
57	3	30,30,40	.4,.5,.6	.1955	.0975	.0550	.0125
58	3	30,30,40	.3,.5,.7	.2080	.1075	.0530	.0110
59	3	30,30,40	.2,.5,.8	.1965	.1080	.0560	.0085
60	3	30,30,40	.1,.5,.9	.1960	.0930	.0445	.0075
61	3	30,30,30	.5,.5,.5	.2040	.1005	.0465	.0115
62	3	30,30,30	.4,.5,.6	.2110	.1060	.0520	.0090
63	3	30,30,30	.3,.5,.7	.2040	.1080	.0505	.0065
64	3	30,30,30	.2,.5,.8	.1705*	.0865*	.0405*	.0085
65	3	30,30,30	.1,.5,.9	.1590*	.0805*	.0445	.0105
66	3	40,40,40	.5,.5,.5	.2020	.1035	.0530	.0120
67	3	40,40,40	.4,.5,.6	.1890	.0890	.0420	.0065
68	3	40,40,40	.3,.5,.7	.1895	.0885	.0400*	.0115
69	3	40,40,40	.2,.5,.8	.1625*	.0830*	.0400*	.0085
70	3	40,40,40	.1,.5,.9	.1665*	.0775*	.0360*	.0080
71	3	20,40,40	.5,.5,.5	.2215*	.1080	.0490	.0095
72	3	20,40,40	.4,.5,.6	.2020	.0975	.0455	.0070
73	3	20,40,40	.3,.5,.7	.2120	.1135	.0620*	.0125
74	3	20,40,40	.2,.5,.8	.2055	.1120	.0605	.0150
75	3	20,40,40	.1,.5,.9	.1990	.1100	.0570	.0160
76	3	20,30,40	.5,.5,.5	.1930	.0885	.0455	.0085
77	3	20,30,40	.4,.5,.6	.2070	.1050	.0540	.0140
78	3	20,30,40	.3,.5,.7	.2235*	.1145*	.0700*	.0165*
79	3	20,30,40	.2,.5,.8	.2210*	.1210*	.0595	.0155
80	3	20,30,40	.1,.5,.9	.2210*	.1215*	.0590	.0150
81	5	20,20,20,20,20	.5,.5,.5,.5,.5	.2025	.1010	.0555	.0090
82	5	20,20,20,20,20	.4,.4,.5,.6,.6	.1795*	.0950	.0500	.0115
83	5	20,20,20,20,20	.3,.4,.5,.6,.7	.1975	.0975	.0510	.0105
84	5	20,20,20,20,20,	.2,.4,.5,.6,.8	.1730*	.0965	.0455	.0085
85	5	20,20,20,20,20	.1,.3,.5,.7,.9	.1685*	.0815*	.0445	.0100

TABLE II--Continued

Simulation Number	Number of Groups k	Sample Sizes, n_j	Regression Coefficients, β_j	Actual Significance Levels Corresponding to Nominal Significance Levels of .200	.100	.050	.010
86	5	20,20,20,30,30	.5,.5,.5,.5,.5	.2070	.1065	.0560	.0120
87	5	20,20,20,30,30	.4,.4,.5,.6,.6	.2165	.1210*	.0635*	.0165*
88	5	20,20,20,30,30	.3,.4,15,.6,.7	.2200*	.1105	.0575	.0115
89	5	20,20,20,30,30	.2,.4,.5,.6,.8	.1980	.1125	.0620*	.0125
90	5	20,20,20,30,30	.1,.3,.5,.7,.9	.1885	.0985	.0505	.0110
91	5	20,20,20,30,40	.5,.5,.5,.5,.5	.1965	.1060	.0565	.0070
92	5	20,20,20,30,40	.4,.4,.5,.6,.6	.2230*	.1175*	.0590	.0145
93	5	20,20,20,30,40	.3,.4,.5,.6,.7	.2230*	.1145*	.0615*	.0110
94	5	20,20,20,30,40	.2,.4,.5,.6,.8	.2225*	.1210*	.0605	.0135
95	5	20,20,20,30,40	.1,.3,.5,.7,.9	.2145	.1105	.0620*	.0140
96	5	20,20,30,30,30	.5,.5,.5,.5,.5	.1975	.0960	.0460	.0115
97	5	20,20,30,30,30	.4,.4,.5,.6,.6	.2015	.0965	.0470	.0130
98	5	20,20,30,30,30	.3,.4,.5,.6,.7	.2100	.1135	.0610	.0130
99	5	20,20,30,30,30	.2,.4,.5,.6,.8	.1910	.0955	.0440	.0115
100	5	20,20,30,30,30	.1,.3,.5,.7,.9	.1945	.1075	.0570	.0130
101	5	20,20,30,40,40	.5,.5,.5,.5,.5	.2135	.1060	.0540	.0065
102	5	20,20,30,40,40	.4,.4,.5,.6,.6	.2185*	.1135	.0615*	.0135
103	5	20,20,30,40,40	.3,.4,.5,.6,.7	.2305*	.1260*	.0615*	.0145
104	5	20,20,30,40,40	.2,.4,.5,16,.8	.2285*	.1215*	.0695*	.0150
105	5	20,20,30,40,40	.1,.3,.5,.7,.9	.2165	.1175*	.0685*	.0185*
106	5	20,20,40,40,40	.5,.5,.5,.5,.5	.1960	.0980	.0530	.0075
107	5	20,20,40,40,40	.4,.4,.5,.6,.6	.2130	.1160*	.0595	.0130
108	5	20,20,40,40,40	.3,.4,.5,.6,.7	.2045	.1090	.0630	.0110
109	5	20,20,40,40,40	.2,.4,.5,.6,.8	.2335*	.1240*	.0645*	.0135
110	5	20,20,40,40,40	.1,.3,.5,.7,.9	.2090	.1095	.0605	.0215*
111	5	20,30,30,30,40	.5,.5,.5,.5,.5	.1800*	.0940	.0510	.0095
112	5	20,30,30,30,40	.4,.4,.5,.6,.6	.2240*	.1130	.0565	.0130
113	5	20,30,30,30,40	.3,.4,.5,.6,.7	.2150	.1075	.0535	.0095
114	5	20,30,30,30,40	.2,.4,.5,.6,.8	.2095	.1030	.0530	.0135
115	5	20,30,30,30,40	.1,.3,.5,.7,.9	.1995	.1115	.0615*	.0120

TABLE II--Continued

Simulation Number	Number of Groups, k	Sample Sizes, n_j	Regression Coefficients, β_j	Actual Significance Levels Corresponding to Nominal Significance Levels of .200	.100	.050	.010
116	5	20,30,40,40,40	.5,.5,.5,.5,.5	.2060	.1035	.0485	.0075
117	5	20,30,40,40,40	.4,.4,.5,.6,.6	.1910	.0970	.0455	.0080
118	5	20,30,40,40,40	.3,.4,.5,.6,.7	.2220*	.1070	.0590	.0110
119	5	20,30,40,40,40	.2,.4,.5,.6,.8	.2060	.1085	.0595	.0145
120	5	20,30,40,40,40	.1,.3,.5,.7,.9	.2080	.1120	.0525	.0130
121	5	30,30,30,30,30	.5,.5,.5,.5,.5	.1935	.0850*	.0415	.0115
122	5	30,30,30,30,30	.4,.4,.5,.6,.6	.1980	.0960	.0490	.0160
123	5	30,30,30,30,30	.3,.4,.5,.6,.7	.1815*	.0935	.0445	.0065
124	5	30,30,30,30,30	.2,.4,.5,.6,.8	.1780*	.0855*	.0440	.0095
125	5	30,30,30,30,30	.1,.3,.5,.7,.9	.1595*	.0755*	.0395*	.0090
126	5	40,40,40,40,40	.5,.5,.5,.5,.5	.2005	.1070	.0530	.0140
127	5	40,40,40,40,40	.4,.4,.5,.6,.6	.1950	.0980	.0455	.0085
128	5	40,40,40,40,40	.3,.4,.5,.6,.7	.1885	.0985	.0435	.0100
129	5	40,40,40,40,40	.2,.4,.5,.6,.8	.1745*	.0945	.0480	.0095
130	5	40,40,40,40,40	.1,.3,.5,.7,.9	.1615*	.0780*	.0390*	.0070
131	5	30,30,40,40,40	.5,.5,.5,.5,.5	.1820*	.0855*	.0425*	.0080
132	5	30,30,40,40,40	.4,.4,.5,.6,.6	.1950	.0995	.0515	.0100
133	5	30,30,40,40,40	.3,.4,.5,.6,.7	.1945	.0935	.0425	.0095
134	5	30,30,40,40,40	.2,.4,.5,.6,.8	.1955	.0970	.0490	.0110
135	5	30,30,40,40,40	.1,.3,.5,.7,.9	.1685*	.0830*	.0375*	.0065
136	2	100,100	.5,.5	.2000	.1045	.0480	.0095
137	2	100,100	.4,.6	.1945	.0920	.0440	.0105
138	2	100,100	.3,.7	.2025	.0945	.0455	.0095
139	2	100,100	.2,.8	.1780*	.0885	.0395*	.0075
140	2	100,100	.1,.9	.1545*	.0660*	.0285*	.0025*
141	2	200,200	.5,.5	.2135	.1100	.0525	.0060
142	2	200,200	.4,.6	.1865	.0965	.0430	.0090
143	2	200,200	.3,.7	.1945	.0975	.0455	.0075
144	2	200,200	.2,.8	.1725*	.0880	.0435	.0080
145	2	200,200	.1,.9	.1390*	.0585*	.0260*	.0040*

TABLE II--Continued

Simulation Number	Number of Groups, K	Sample Sizes, n_j	Regression Coefficients, β_j	Actual Significance Levels Corresponding to Nominal Significance Levels of .200	.100	.050	.010
146	3	100,100,100	.5,.5,.5	.2000	.0975	.0470	.0100
147	3	100,100,100	.4,.5,.6	.1885	.0995	.0485	.0095
148	3	100,100,100	.3,.5,.7	.1800*	.0880	.0390*	.0045*
149	3	100,100,100	.2,.5,.8	.1820*	.0855	.0455	.0105
150	3	100,100,100	.1,.5,.9	.1415*	.0750*	.0330*	.0080
151	3	200,200,200	.5,.5,.5	.2085	.0965	.0445	.0110
152	3	200,200,200	.4,.5,.6	.1805*	.0860*	.0445	.0110
153	3	200,200,200	.3,.5,.7	.1820*	.0954	.0515	.0085
154	3	200,200,200	.2,.5,.8	.1780*	.0830*	.0425	.0130
155	3	200,200,200	.1,.5,.9	.1530*	.0670*	.0285*	.0095
156	5	100,100,100,100,100	.5,.5,.5,.5,.5	.2090	.1140	.0545	.0120
157	5	100,100,100,100,100	.4,.4,.5,.6,.6	.1905	.0985	.0500	.0125
158	5	100,100,100,100,100	.3,.4,.5,.6,.7	.1805*	.0915	.0405	.0095
159	5	100,100,100,100,100	.2,.4,.5,.6,.8	.1660*	.0805*	.0375*	.0065
160	5	100,100,100,100,100	.1,.3,.5,.7,.9	.1615*	.0870	.0425	.0095
161	5	200,200,200,200,200	.5,.5,.5,.5,.5	.1990	.1020	.0435	.0075
162	5	200,200,200,200,200	.4,.4,.5,.6,.6	.2030	.0990	.0455	.0085
163	5	200,200,200,200,200	.3,.4,.5,.6,.7	.1755*	.0835*	.0460	.0090
164	5	200,200,200,200,200	.2,.4,.5,.6,.8	.1595*	.0810*	.0435	.0100
165	5	200,200,200,200,200	.1,.3,.5,.7,.9	.1640*	.0805*	.0360*	.0065
166	2	30,40	.5,.5	.2025	.0965	.0445	.0105
167	2	30,40	.6,.4	.1810*	.0805*	.0325*	.0075
168	2	30,40	.7,.3	.1570*	.0735*	.0335*	.0035*
169	2	30,40	.8,.2	.1500*	.0625*	.0300*	.0040*
170	2	30,40	.9,.1	.1235*	.0450*	.0190*	.0015*
171	2	20,40	.5,.5	.1990	.1000	.0525	.0075
172	2	20,40	.6,.4	.1790*	.0750*	.0305*	.0075
173	2	20,40	.7,.3	.1670*	.0710*	.0350*	.0075
174	2	20,40	.8,.2	.1190*	.0475*	.0160*	.0005*
175	2	20,40	.9,.1	.0705*	.0255*	.0075*	.0000*

TABLE II--Continued

Simulation Number	Number of Groups, K	Sample Sizes, n_j	Regression Coefficients, β_j	Actual Significance Levels Corresponding to Nominal Significance Levels of .200	.100	.050	.010
176	3	20,20,40	.5,.5,.5	.1960	.1010	.0510	.0105
177	3	20,20,40	.6,.5,.4	.1880	.0930	.0425	.0075
178	3	20,20,40	.7,.5,.3	.1710*	.0750*	.0385*	.0080
179	3	20,20,40	.8,.5,.2	.1275*	.0600*	.0220*	.0050*
180	3	20,20,40	.9,.5,.1	.1145*	.0455*	.0200*	.0030*
181	3	20,20,20	-.4, 0,.4	.1635*	.0755*	.0305*	.0035*
182	3	20,20,20	.1,.1,.9	.1420*	.0630*	.0345*	.0070
183	3	20,20,20	.1,.2,.7	.1720*	.0820*	.0420	.0100

*Actual significance levels which differ significantly from their corresponding nominal significance level.

The actual significance levels, proportions of F-ratios which empirically exceeded the tabled F values, may be compared with the nominal significance levels by observing the discrepancy between the observed proportion and the theoretically expected proportion, the nominal values of .200, .100, .050, and .010. These deviations of the actual levels from the nominal levels are to be interpreted within the framework of the 95 per cent confidence intervals presented in Table I of Chapter III. The 95 per cent confidence intervals for nominal significance levels of .200, .100, .050, and .010 are from .183 to .218, .087 to .114, .041 to .061, and .006 to .016, respectively. These confidence intervals provide a means of determining whether the actual significance level deviations from the nominal level can be attributed, within certain probability limits, to sampling error or to the experimental conditions.

Several patterns are apparent from the data given in Table II. First, for equal sample sizes within an experiment, the empirical F-distributions closely approximate the normal theory F-distributions for all but the most heterogeneous regression slopes. The number of significant discrepancies is greater as the nominal significance level increases from .010 to .200. No attempt is herein made to explain this phenomena.

Second, for equal sample sizes within an experiment, the critical or tabled value of F associated with the nominal significance level α is actually a conservative value of F when the assumption of homogeneity of regression slopes is violated. This is observed from the pattern of negative bias of the empirically derived proportions which are consistently less than the expected proportions.

A third pattern emerges, however, when considering experiments in which there are unequal sample sizes. Simulations involving unequal sample sizes do not lead to the same conclusions as with equal sample sizes. All simulations involving unequal n, with the exception of simulations 166 through 183, were set in such a way so as to match the smaller samples within an experiment with the smaller regression slopes. In these cases the actual significance levels which differ significantly from the nominal levels almost without exception yield a positive bias in contrast to the negative bias obtained with equal n. That is, the actual levels generally are greater than the nominal levels for these simulations involving unequal n. Results from simulations 1-165, with the smaller and larger samples matched with the smaller and larger slopes respectively, indicated that as the samples within an experiment vary from very unequal n to slightly unequal n to equal n, the bias varies from positive to zero to negative bias. If, however, the smaller slopes are paired with the larger samples as in simulations 166-180, the bias is negative. The fact that the direction of bias with unequal n cannot be predicted with the same degree of certainty as with equal n leads to caution in interpreting results of experiments with unequal sample sizes.

In simulations where the sample sizes are unequal but only slightly unequal in absolute terms, such as simulations 16-20, 46-60, 71-75, 96-100, and 111-120, the violation of the assumption of homogeneous regression slopes resulted in only a few significant deviations from the nominal levels. In simulations 17-20, where no significant differences between nominal and actual significance levels were found, reversing the slopes to yield simulations 167-170 caused all but one of the actual significance levels to be significantly different from the nominal significance levels.

In simulations 181-183, which involved one simulation with slopes symmetric about zero and two simulations with non-symmetric slopes, it was found that violation of the assumption of equal regression slopes led to substantial deviations, some significant with positive bias and some with negative bias, from the nominal significance levels.

Two hypotheses were made to provide direction for this investigation. The first hypothesis is made up of five sub-hypotheses concerning the degree of heterogeneity of regression slopes possible (with different numbers of groups) without significantly effecting the results at the different nominal significance levels. The hypotheses were originally formulated with respect to simulations 1 through 165.

Hypothesis 1(a), that actual significance levels do not differ significantly from nominal significance levels for regression slopes as heterogeneous as $\beta_1=.3, \beta_2=.7$ at any of the four levels considered. For unequal n, however, significant differences were found in simulations 11 through 15 ($n_1 = 20$ and $n_2 = 40$) at the .05 and .01 levels for $\beta_1 = .3$, $\beta_2 = .7$. Thus the hypothesis must be rejected in the general case.

The (b) part of the first hypothesis, for k = 2, states that there are no significant differences between actual and nominal significance levels for slopes as heterogeneous as $\beta_1 = .4$ and $\beta_2 = .6$ at the .20 and .10 levels. This sub-hypothesis was retained since no significant deviation from the nominal level was found as the hypothesis stated.

The third sub-hypothesis, 1 (c), was rejected. This hypothesis stated that, for k = 3, there is no significant difference between nominal and actual significance levels for slopes as different as $\beta_1 = .3$, $\beta_2 = .5$, and $\beta_3 = .7$ for nominal levels .20, .10, .05, and .01. However, for equal n's of 20, 30, and 40 four out of twelve actual significance levels were significant for this set of slopes. For n's of 100 and 200, six out of sixteen actual significance levels for this set of regression slopes and the less heterogeneous set of $\beta_1 = .4$, $\beta_2 = .5$, and $\beta_3 = .6$ were found to be significantly different from the nominal levels. For equal n's of 20, 30, and 40 a less heterogeneous set of slopes then hypothesized, $\beta_1 = .4$, $\beta_2 = .5$, and $\beta_3 = .6$, yielded no significant deviations from the nominal levels. For unequal n's eight out of twenty-eight actual significance levels for the $\beta_1 = .3$, $\beta_2 = .5$, and $\beta_3 = .7$ hypothesized were significantly different from the nominal levels.

The (d) part of the first hypothesis stated that there is no significant difference between actual and nominal levels of significance for regression slopes as heterogeneous as $\beta_1 = .3$, $\beta_2 = .4$, $\beta_3 = .5$, $\beta_4 = .6$, and $\beta_5 = .7$ with k = 5 at the .01 and .05 nominal significance levels. This sub-hypothesis was rejected. However, for equal n no significant differences were found for the hypothesized slopes although one was found for $\beta_1 = .2$, $\beta_2 = .4$, $\beta_3 = .5$, $\beta_4 = .6$, and $\beta_5 = .8$. For unequal n five out of sixteen actual significance levels were found to be significantly different from the nominal levels at the .05 level. For equal and unequal n the .01 level yielded only three significant deviations for any set of regression slopes and two of these three occurred with the extremely heterogeneous slopes $\beta_1 = .1$, $\beta_2 = .3$, $\beta_3 = .5$, $\beta_4 = .7$, and $\beta_5 = .9$.

For the last part of the first hypothesis it was stated that there is no significant difference between actual and nominal significance levels for k = 5 and regression slopes as heterogeneous as $\beta_1 = .4$, $\beta_2 = .4$, $\beta_3 = .5$, $\beta_4 = .6$, and $\beta_5 = .6$ at nominal levels of .20 and .10. For equal n the hypothesis can be retained since only one significant actual value was found. However, six of sixteen actual significance levels for unequal n differed significantly from the nominal levels for the hypothesized slopes. Thus the sub-hypothesis was rejected.

Hypothesis two stated that for a given set of regression slopes, for all sample sizes, there will be no significant difference between the actual significance levels and the nominal significance levels. Observation of Table VII of Appendix B led to this hypothesis being rejected. It can be seen that for k = 2 and eight simulations for regression coefficients less heterogeneous than $\beta_1 = .3$ and $\beta_2 = .7$ no actual significance levels are significantly different from corresponding nominal levels. The number of actual significant levels differing significantly from nominal levels increases from six to sixteen to twenty-seven as the slopes vary from $\beta_1 = .3$, $\beta_2 = .7$ to $\beta_1 = .2$, $\beta_2 = .8$ to $\beta_1 = .1$, $\beta_2 = .9$.

For k = 3 and twelve simulations the number of actual significance levels which differ significantly from nominal levels follow a similar pattern. The frequency is 1, 6, 16, 22, and 21 as the slopes vary from $\beta_1 = .5$, $\beta_2 = .5$, $\beta_3 = .5$ to $\beta_1 = .1$, $\beta_2 = .5$, $\beta_3 = .9$. For k = 5, involving thirteen simulations, the frequency of significant differences are 4, 10, 13, 18, and 18 as the slopes vary from equal slopes of .5 to $\beta_1 = .1$, $\beta_2 = .3$, $\beta_3 = .5$, $\beta_4 = .7$, and $\beta_5 = .9$.

Table III presents data for chi-square test of homogeneity of the probability distributions for populations of F-ratios

TABLE III

CHI-SQUARE TESTS FOR HOMOGENEOUS DISTRIBUTIONS WITHIN POPULATIONS OF DIFFERENT SAMPLE SIZE BY DIFFERENT SETS OF REGRESSION SLOPES OF FREQUENCY OF F-RATIOS EXCEEDING $_{.95}F_{k-1,\ N-k-1}$

k = 2
Regression Coefficients

		.5,.5	.4,.6	.3,.7	.2,.8	.1,.9
	20	93	98	98	73	44
Sample	30	82	106	90	75	58
Sizes	40	88	89	102	73	60
(equal n)	100	96	88	91	79	57
	200	105	86	91	87	52

Chi-square = 14.49 df = 16 p > .5

TABLE III--Continued

k = 3
Regression Coefficients

		.5,.5,.5	.4,.5,.6	.3,.5,.7	.2,.5,.8	.1,.5,.9
	20	102	89	72	77	69
Sample	30	93	104	101	81	89
Sizes	40	106	84	80	80	72
(equal n)	100	94	97	78	91	66
	200	89	89	103	85	57

Chi-square = 18.55 df = 16 p > .25

k = 5
Regression Coefficients

		.5,.5,.5, .5,.5	.4,.4,.5, .6,.6	.3,.4,.5, .6,.7	.2,.4,.5, .6,.8	.1,.3,.5, .7,.9
	20	111	100	102	91	89
Sample	30	83	98	89	88	79
Sizes	40	106	91	87	96	78
(equal n)	100	109	100	81	75	85
	200	87	91	92	87	72

Chi-square = 10.47 df = 16 p > .8

exceeding $_{.95}F_{k-1,\ N-k-1}$ from different sample sizes across different regression coefficients. The null hypothesis that the frequencies are distributed the same across sets of regression slopes for all sample sizes was retained in each case, k = 2, 3, and 5. These results indicate that the frequency of F-ratios exceeding $_{.95}F_{k-1,\ N-k-1}$ follows the same pattern for each sample size.

Finally, the relationship between size of k, or the number of treatment groups, and the frequency of actual significance levels differing significantly from nominal levels was considered. Chi-square tests of independence between sample size and levels of significance were performed for both equal sample sizes and unequal sample sizes. Table IV, exhibits the contingency tables for these tests. The first contingency

TABLE IV

CHI-SQUARE TESTS FOR THE INDEPENDENCE OF ACTUAL LEVELS OF SIGNIFICANCE DIFFERING SIGNIFICANTLY FROM NOMINAL SIGNIFICANCE LEVELS AND SAMPLE SIZES

		Levels of Significance			
		.200	.100	.050	.010
	20	8	6	5	2
Sample	30	6	7	4	1
Sizes	40	6	5	6	1
(equal n)	100	8	3	5	2
k = 2, 3, 5	200	9	7	3	1
Chi-square = 5.37		df = 12		p > .9	
	20,30	7	5	4	1
Sample	20,40	8	9	9	5
Sizes	30,40	3	2	1	1
(unequal n)					
Chi-square = 2.44		df = 6		p > .8	

table employed only equal n experiments while the second used experiments with unequal sample sizes for combinations of 20 and 30, 20 and 40, and 30 and 40. In each case the chi-square statistic led to the hypothesis of independence being retained. Thus, there seems to be no relationship between size of sample and the frequency of actual significant levels differing from nominal significance levels significantly at different nominal levels of significance.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Under certain experimental conditions, the effect of violation of the assumption of homogeneity of regression slopes upon the F-test of the one-way fixed-effects analysis of covariance model was investigated. A computer simulation was employed to control certain parameters and to vary the regression slopes systematically within an experiment. Both equal sample sizes and unequal sample sizes and varying numbers of treatment groups were used.

In each of the simulations an empirical distribution of F-ratios was compared with the theoretical distribution. This was done by a comparison of actual significance levels with the corresponding nominal significance levels of .20, .10, .05, and .01. Graphical comparisons of the two distributions were also made in each simulation.

In each simulation the null condition was assumed, combined with a particular set of experimental conditions.

The following conclusions appear to be appropriate:

1. For equal sample sizes within an experiment, the one-way fixed-effects analysis of covariance model is robust to violation of the homogeneous regression slopes assumption under

all but the most heterogeneous slopes. In agreement with Peckham's study (1, p. 52) with smaller samples, as the degree of heterogeneity of the slopes increases the test becomes more conservative with respect to making a Type I error. Consequently, if in an experiment with equal sample sizes a researcher rejects the null hypothesis at the α level under conditions of heterogeneous regression slopes, there is less than an α probability of rejecting a true null hypothesis.

2. With unequal sample sizes within an experiment no generalization can be made. The direction of bias in such experiments with heterogeneous regression coefficients cannot be predicted and consequently the effect on the Type I error cannot be determined.

Each of these conclusions seems to extend across all sample sizes and numbers of treatment groups considered.

Further investigation should be made of the relationship between sample size and regression slope in an attempt to derive some general statement about the interaction of the two when the regression slopes are unequal.

Additional studies should be done on the effect of violation of the homogeneous regression slopes assumption in a covariance model with more than one covariable or with multi-factor experiments. Other studies involving the effects of violation of additional assumptions upon the analysis of covariance model are suggested by Peckham (1, pp. 61-62).

CHAPTER BIBLIOGRAPHY

1. Peckham, Percy D., "An Investigation of the Effects of Non-homogeneity of Regression Slopes Upon the F-test of Analysis of Covariance," unpublished doctoral dissertation, School of Education, University of Colorado, Boulder, Colorado, 1968.

APPENDIX A

SIMULATION CONDITIONS, PARAMETERS, AND CONSTANTS

TABLE V

SIMULATIONS BY NUMBER WITH THEIR CONDITIONS

$E(Y._j) = \bar{X}._j = 0 \quad \sigma_y^2 = MS_x = 1$

Simulation Number	Number of Groups, k	Sample Sizes n_j	Regression Coefficients β_j
1	2	20,20	.5,.5
2	2	20,20	.4,.6
3	2	20,20	.3,.7
4	2	20,20	.2,.8
5	2	20,20	.1,.9
6	2	20,30	.5,.5
7	2	20,30	.4,.6
8	2	20,30	.3,.7
9	2	20,30	.2,.8
10	2	20,30	.1,.9
11	2	20,40	.5,.5
12	2	20,40	.4,.6
13	2	20,40	.3,.7
14	2	20,40	.2,.8
15	2	20,40	.1,.9
16	2	30,40	.5,.5
17	2	30,40	.4,.6
18	2	30,40	.3,.7
19	2	30,40	.2,.8
20	2	30,40	.1,.9
21	2	30,30	.5,.5
22	2	30,30	.4,.6
23	2	30,30	.3,.7
24	2	30,30	.2,.8
25	2	30,30	.1,.9
26	2	40,40	.5,.5
27	2	40,40	.4,.6
28	2	40,40	.3,.7
29	2	40,40	.2,.8
30	2	40,40	.1,.9

TABLE V--Continued

Simulation Number	Number of Groups, k	Sample Sizes n_j	Regression Coefficients β_j
31	3	20,20,20	.5,.5,.5
32	3	20,20,20	.4,.5,.6
33	3	20,20,20	.3,.5,.7
34	3	20,20,20	.2,.5,.8
35	3	20,20,20	.1,.5,.9
36	3	20,20,30	.5,.5,.5
37	3	20,20,30	.4,.5,.6
38	3	20,20,30	.3,.5,.7
39	3	20,20,30	.2,.5,.8
40	3	20,20,30	.1,.5,.9
41	3	20,20,40	.5,.5,.5
42	3	20,20,40	.4,.5,.6
43	3	20,20,40	.3,.5,.7
44	3	20,20,40	.2,.5,.8
45	3	20,20,40	.1,.5,.9
46	3	20,30,30	.5,.5,.5
47	3	20,30,30	.4,.5,.6
48	3	20,30,30	.3,.5,.7
49	3	20,30,30	.2,.5,.8
50	3	20,30,30	.1,.5,.9
51	3	30,40,40	.5,.5,.5
52	3	30,40,40	.4,.5,.6
53	3	30,40,40	.3,.5,.7
54	3	30,40,40	.2,.5,.8
55	3	30,40,40	.1,.5,.9
56	3	30,30,40	.5,.5,.5
57	3	30,30,40	.4,.5,.6
58	3	30,30,40	.3,.5,.7
59	3	30,30,40	.2,.5,.8
60	3	30,30,40	.1,.5,.9
61	3	30,30,30	.5,.5,.5
62	3	30,30,30	.4,.5,.6
63	3	30,30,30	.3,.5,.7
64	3	30,30,30	.2,.5,.8
65	3	30,30,30	.1,.5,.9

TABLE V--Continued

Simulation Number	Number of Groups, k	Sample Sizes n_j	Regression Coefficients β_j
66	3	40,40,40	.5,.5,.5
67	3	40,40,40	.4,.5,.6
68	3	40,40,40	.3,.5,.7
69	3	40,40,40	.2,.5,.8
70	3	40,40,40	.1,.5,.9
71	3	20,40,40	.5,.5,.5
72	3	20,40,40	.4,.5,.6
73	3	20,40,40	.3,.5,.7
74	3	20,40,40	.2,.5,.8
75	3	20,40,40	.1,.5,.9
76	3	20,30,40	.5,.5,.5
77	3	20,30,40	.4,.5,.6
78	3	20,30,40	.3,.5,.7
79	3	20,30,40	.2,.5,.8
80	3	20,30,40	.1,.5,.9
81	5	20,20,20,20,20	.5,.5,.5,.5,.5
82	5	20,20,20,20,20	.4,.4,.5,.6,.6
83	5	20,20,20,20,20	.3,.4,.5,.6,.7
84	5	20,20,20,20,20	.2,.4,.5,.6,.8
85	5	20,20,20,20,20	.1,.3,.5,.7,.9
86	5	20,20,20,30,30	.5,.5,.5,.5,.5
87	5	20,20,20,30,30	.4,.4,.5,.6,.6
88	5	20,20,20,30,30	.3,.4,.5,.6,.7
89	5	20,20,20,30,30	.2,.4,.5,.6,.8
90	5	20,20,20,30,30	.1,.3,.5,.7,.9
91	5	20,20,20,30,40	.5,.5,.5,.5,.5
92	5	20,20,20,30,40	.4,.4,.5,.6,.6
93	5	20,20,20,30,40	.3,.4,.5,.6,.7
94	5	20,20,20,30,40	.2,.4,.5,.6,.8
95	5	20,20,20,30,40	.1,.3,.5,.7,.9
96	5	20,20,30,30,30	.5,.5,.5,.5,.5
97	5	20,20,30,30,30	.4,.4,.5,.6,.6
98	5	20,20,30,30,30	.3,.4,.5,.6,17
99	5	20,20,30,30,30	.2,.4,.5,.6,.8
100	5	20,20,30,30,30	.1,.3,.5,.7,.9

TABLE V--Continued

Simulation Number	Number of Groups, k	Sample Sizes n_j	Regression Coefficients β_j
101	5	20,20,30,40,40	.5,.5,.5,.5,.5
102	5	20,20,30,40,40	.4,.4,.5,.6,.6
103	5	20,20,30,40,40	.3,.4,.5,.6,.7
104	5	20,20,30,40,40	.2,.4,.5,.6,.8
105	5	20,20,30,40,40	.1,.3,.5,.7,.9
106	5	20,20,40,40,40	.5,.5,.5,.5,.5
107	5	20,20,40,40,40	.4,.4,.5,.6,.6
108	5	20,20,40,40,40	.3,.4,.5,.6,.7
109	5	20,20,40,40,40	.2,.4,.5,.6,.8
110	5	20,20,40,40,40	.1,.3,.5,.7,.9
111	5	20,30,30,30,40	.5,.5,.5,.5,.5
112	5	20,30,30,30,40	.4,.4,.5,.6,.6
113	5	20,30,30,30,40	.3,.4,.5,.6,.7
114	5	20,30,30,30,40	.2,.4,.5,.6,.8
115	5	20,30,30,30,40	.1,.3,.5,.7,.9
116	5	20,30,40,40,40	.5,.5,.5,.5,.5
117	5	20,30,40,40,40	.4,.4,.5,.6,.6
118	5	20,30,40,40,40	.3,.4,.5,.6,.7
119	5	20,30,40,40,40	.2,.4,.5,.6,.8
120	5	20,30,40,40,40	.1,.3,.5,.7,.9
121	5	30,30,30,30,30	.5,.5,.5,.5,.5
122	5	30,30,30,30,30	.4,.4,.5,.6,.6
123	5	30,30,30,30,30	.3,.4,.5,.6,.7
124	5	30,30,30,30,30	.2,.4,.5,.6,.8
125	5	30,30,30,30,30	.1,.3,.5,.7,.9
126	5	40,40,40,40,40	.5,.5,.5,.5,.5
127	5	40,40,40,40,40	.4,.4,.5,.6,.6
128	5	40,40,40,40,40	.3,.4,.5,.6,.7
129	5	40,40,40,40,40	.2,.4,.5,.6,.8
130	5	40,40,40,40,40	.1,.3,.5,.7,.9
131	5	30,30,40,40,40	.5,.5,.5,.5,.5
132	5	30,30,40,40,40	.4,.4,.5,.6,.6
133	5	30,30,40,40,40	.3,.4,.5,.6,.7
134	5	30,30,40,40,40	.2,.4,.5,.6,.8
135	5	30,30,40,40,40	.1,.3,.5,.7,.9

TABLE V--Continued

Simulation Number	Number of Groups, k	Sample Sizes n_j	Regression Coefficients β_j
136	2	100,100	.5,.5
137	2	100,100	.4,.6
138	2	100,100	.3,.7
139	2	100,100	.2,.8
140	2	100,100	.1,.9
141	2	200,200	.5,.5
142	2	200,200	.4,.6
143	2	200,200	.3,.7
144	2	200,200	.2,.8
145	2	200,200	.1,.9
146	3	100,100,100	.5,.5,.5
147	3	100,100,100	.4,.5,.6
148	3	100,100,100	.3,.5,.7
149	3	100,100,100	.2,.5,.8
150	3	100,100,100	.1,.5,.9
151	3	200,200,200	.5,.5,.5
152	3	200,200,200	.4,.5,.6
153	3	200,200,200	.3,.5,.7
154	3	200,200,200	.2,.5,.8
155	3	200,200,200	.1,.5,.9
156	5	100,100,100,100,100	.5,.5,.5,.5,.5
157	5	100,100,100,100,100	.4,.4,.5,.6,.6
158	5	100,100,100,100,100	.3,.4,.5,.6,.7
159	5	100,100,100,100,100	.2,.4,.5,.6,.8
160	5	100,100,100,100,100	.1,.3,.5,.7,.9
161	5	200,200,200,200,200	.5,.5,.5,.5,.5
162	5	200,200,200,200,200	.4,.4,.5,.6,.6
163	5	200,200,200,200,200	.3,.4,.5,.6,.7
164	5	200,200,200,200,200	.2,.4,.5,.6,.8
165	5	200,200,200,200,200	.1,.3,.5,.7,.9

Choice of Concomitant Variable Values

The values of X, the concomitant variable (covariate), were fixed for a given sample size n. This necessitated the selection of five sets of X values of sizes corresponding to the five sample sizes used in the study. Each set was used each time its corresponding sample size was specified for a given simulation. The sets were generated by the computer using the subroutine RANDU with the same trigonometric transformations explained in Chapter III, page twenty-seven. Parameters specified for the generation were that the resulting samples of random numbers should be from a normal distribution with zero mean and variance equal to one.

Several sets of each size were generated initially. The set having the best combined standard deviation and probability of being from a normal distribution was selected from the several for each sample size. The probability of being from a normal distribution was obtained using a Kilmogorov-Smirnov test for goodness-of-fit. The five sets were then each transformed linearly in such a manner so as to give means exactly zero. Table VI gives the five sets used along with their statistics. It can be noted that each standard deviation is greater than or equal to .99 and each set has a probability greater than .95 of being from a normal distribution. The samples can be seen to be generally symmetric also. Since these sets remained fixed throughout each replication within a simulation it was felt that these approximations to the parameters were satisfactory.

TABLE VI

VALUES OF THE CONCOMITANT VARIABLE WITH CORRESPONDING STATISTICS

Sample Size	Mean $\bar{X}_{.j}$	Standard Deviation	D*	P**	Values of X			
20	0.00	.9921	.1047	.9807	-1.936468	-1.360229	-0.997819	-0.955770
					-0.603340	-0.589310	-0.513810	-0.488230
					-0.389120	-0.110570	0.023260	0.069390
					0.246650	0.658670	0.721180	0.737940
					0.957410	1.034949	1.509829	1.993779
30	0.00	.9930	.0904	.9669	-1.864868	-1.658560	-1.496460	-1.310019
					-1.113238	-1.009660	-0.987410	-0.674220
					-0.542530	-0.540590	-0.346710	-0.275520
					-0.200090	0.043880	0.143620	0.162260
					0.189950	0.196650	0.259040	0.311590
					0.462430	0.705820	0.730360	0.888810
					0.934520	0.952360	1.127850	1.328038
					1.377819	2.204988	. . . . .	. . . . .
40	0.00	.98775	.0794	.9626	-2.073619	-2.005699	-1.672009	-1.537829
					-1.346588	-1.127579	-0.950099	-0.790490
					-0.763850	-0.630970	-0.576510	-0.539890
					-0.507380	-0.413210	-0.305450	-0.216470
					-0.180860	-0.126480	-0.094260	0.034670
					0.050240	0.152190	0.166780	0.194010
					0.244760	0.438940	0.473300	0.482510
					0.489960	0.640390	0.774910	1.055420
					1.104970	1.112559	1.136020	1.299719
					1.305389	1.454259	1.512889	1.735259

TABLE VI--Continued

Sample Size	Mean $\bar{X}_{.j}$	Standard Deviation	D*	P**	Values of X			
100	0.00	.9923	.0519	.9502	-2.312960	-2.054778	-1.990520	-1.952028
					-1.928699	-1.525040	-1.451859	-1.402068
					-1.394489	-1.317359	-1.267988	-1.222149
					-1.123420	-1.055169	-0.920770	-0.899760
					-0.883270	-0.857980	-0.853160	-0.812870
					-0.775860	-0.774370	-0.762070	-0.756700
					-0.660960	-0.652100	-0.649080	-0.602030
					-0.599500	-0.590060	-0.541330	-0.502800
					-0.473910	-0.473170	-0.468800	-0.461820
					-0.446350	-0.433610	-0.412060	-0.366780
					-0.351280	-0.335730	-0.295660	-0.261420
					-0.249800	-0.229770	-0.206860	-0.119780
					-0.027590	-0.009140	-0.004790	0.049880
					0.054270	0.124390	0.142630	0.148670
					0.158100	0.231300	0.280380	0.307290
					0.320580	0.346490	0.368050	0.435120
					0.442090	0.447510	0.475090	0.509320
					0.518260	0.532890	0.554800	0.564470
					0.591360	0.591510	0.606050	0.646770
					0.794710	0.812380	0.864680	0.897070
					0.908609	0.933770	0.992920	0.995000
					1.037709	1.087919	1.088619	1.146779
					1.315729	1.319599	1.378189	1.503529
					1.564699	1.593670	1.664469	1.681549
					1.683749	1.835499	1.864899	2.307949
200	0.00	.9979	.0360	.9580	-2.530599	-2.486719	-2.432540	-2.228579
					-2.213428	-2.141149	-2.119959	-1.759329
					-1.714478	-1.630969	-1.540419	-1.528529
					-1.459719	-1.452339	-1.341799	-1.321029

TABLE VI--Continued

Sample Size	Mean $\overline{X}_{.j}$	Standard Deviation	D*	P**	Values of X			
					-1.301969	-1.290519	-1.243129	-1.237569
					-1.236139	-1.232149	-1.195379	-1.192209
					-1.113919	-1.076500	-1.018120	-0.992610
					-0.957670	-0.934380	-0.933200	-0.933050
					-0.901990	-0.861580	-0.854170	-0.853330
					-0.832250	-0.826910	-0.793720	-0.776330
					-0.767710	-0.736610	-0.715280	-0.694070
					-0.681910	-0.665640	-0.636680	-0.627680
					-0.627650	-0.618770	-0.615460	-0.605460
					-0.599070	-0.598730	-0.595430	-0.592400
					-0.564470	-0.563660	-0.556360	-0.551230
					-0.532000	-0.501980	-0.489110	-0.484550
					-0.480250	-0.475500	-0.448700	-0.446210
					-0.431100	-0.427050	-0.405250	-0.404860
					-0.403860	-0.373450	-0.361510	-0.361470
					-0.352690	-0.338100	-0.330130	-0.323290
					-0.302460	-0.275120	-0.261390	-0.261250
					-0.259290	-0.254700	-0.226650	-0.209580
					-0.209170	-0.207420	-0.204410	-0.179360
					-0.176800	-0.154100	-0.130970	-0.118380
					-0.112020	-0.107160	-0.093580	-0.069820
					-0.068440	-0.057700	-0.023940	-0.000760
					0.010800	0.023610	0.026360	0.036460
					0.043740	0.048660	0.076490	0.097290
					0.104580	0.128650	0.133660	0.150090
					0.159710	0.174730	0.176970	0.180240
					0.181870	0.216840	0.219990	0.233350
					0.238140	0.242660	0.274570	0.290990
					0.301450	0.350700	0.355270	0.378880
					0.386450	0.390900	0.422960	0.430820

TABLE VI--Continued

Sample Size	Mean $\overline{X}_{.j}$	Standard Deviation	D*	P**	Values of X			
					0.434100	0.474240	0.481240	0.488750
					0.523020	0.527420	0.531440	0.538570
					0.574870	0.690480	0.603790	0.623400
					0.697700	0.709870	0.741170	0.780860
					0.799690	0.801630	0.804200	0.828080
					0.886010	0.897500	0.900360	0.916070
					0.917630	0.959580	0.964550	0.981200
					0.982999	0.986249	1.017570	1.052658
					1.094039	1.096009	1.100120	1.158799
					1.188989	1.211008	1.230559	1.234349
					1.260759	1.270020	1.305389	1.332479
					1.341389	1.352888	1.382289	1.417878
					1.453749	1.467649	1.492379	1.539169
					1.579589	1.605479	1.609319	1.616260
					1.616379	1.732259	1.743149	1.801238
					2.165050	2.185849	2.320588	3.030139

*Kolmogorov-Smirnov D statistic.

**Probability that sample is from a normal distribution.

APPENDIX B

SUMMARY DATA FROM SIMULATIONS

The simulation constants, parameters, and statistics of the 183 simulations are summarized in Table VII. Table VIII presents a comparison of significance levels among simulations with like coefficients.

Included in Table VII are the constants set, namely the number of treatment groups, k, and sample sizes, n_j. The parameters set for each simulation were the regression slopes, β_j. Statistics obtained for each simulation were regression slopes, B_j, means of the dependent variable, M_j, and standard deviations of the dependent variable, SD_j. Also presented in Table VII are the appropriate F values with their corresponding nominal significance levels, α, and actual significance levels, α'.

Values of the actual significance levels which are significantly different from the nominal significance levels are marked with one or two asterisks. One asterisk indicates that the actual significance level is significantly smaller than the nominal level. An actual significance level which is significantly larger than the nominal level is denoted by two asterisks.

Table VIII presents data for comparison of the actual significance levels with the corresponding nominal significance levels for simulations with like coefficients.

TABLE VII

SUMMARY DATA FROM SIMULATIONS WITH REGRESSION COEFFICIENTS, MEANS AND STANDARD DEVIATIONS OF DEPENDENT VARIABLE, F VALUES, NOMINAL AND ACTUAL SIGNIFICANCE LEVELS

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
1	2	20	.5	.4996	-.0040	.9834	1.703	.200	.2040
		20	.5	.4975	-.0037	.9860	2.846	.100	.1045
		..	..	. . .	. . .	. . .	4.106	.050	.0465
		..	..	. . .	. . .	. . .	7.372	.010	.0120
2	2	20	.4	.3935	.0012	.9907	1.703	.200	.1870
		20	.6	.5978	-.0041	.9832	2.846	.100	.0990
		..	..	. . .	. . .	. . .	4.106	.050	.0490
		..	..	. . .	. . .	. . .	7.372	.010	.0085
3	2	20	.3	.3003	-.0102	.9885	1.703	.200	.1895
		20	.7	.6976	-.0022	.9891	2.846	.100	.0895
		..	..	. . .	. . .	. . .	4.106	.050	.0490
		..	..	. . .	. . .	. . .	7.372	.010	.0105
4	2	20	.2	.2006	.0089	.9861	1.703	.200	.1620*
		20	.8	.8015	-.0010	.9885	2.846	.100	.0735*
		..	..	. . .	. . .	. . .	4.106	.050	.0365*
		..	..	. . .	. . .	. . .	7.372	.010	.0050*
5	2	20	.1	.0972	.0045	.9840	1.703	.200	.1495*
		20	.9	.9022	.0015	.9939	2.846	.100	.0595*
		..	..	. . .	. . .	. . .	4.106	.050	.0220*
		..	..	. . .	. . .	. . .	7.372	.010	.0040*
6	2	20	.5	.4905	.0006	.9860	1.690	.200	.1915
		30	.5	.5030	-.0032	.9948	2.815	.100	.0920
		..	..	. . .	. . .	. . .	4.047	.050	.0460
		..	..	. . .	. . .	. . .	7.207	.010	.0085
7	2	20	.4	.3970	.0058	.9851	1.690	.200	.2160
		30	.6	.6002	.0041	.9902	2.815	.100	.1065
		..	..	. . .	. . .	. . .	4.047	.050	.0565
		..	..	. . .	. . .	. . .	7.207	.010	.0115

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
8	2	20	.3	.2986	.0108	.9897	1.690	.200	.2255**
		30	.7	.6993	-.0031	.9908	2.815	.100	.1195**
		..	..	. . .	. . .	. . .	4.047	.050	.0595
		..	..	. . .	. . .	. . .	7.207	.010	.0140
9	2	20	.2	.1986	.0019	.9819	1.690	.200	.2210**
		30	.8	.7981	.0036	.9920	2.815	.100	.1085
		..	..	. . .	. . .	. . .	4.047	.050	.0560
		..	..	. . .	. . .	. . .	7.207	.010	.0085
10	2	20	.1	.1078	.0062	.9812	1.690	.200	.2240**
		30	.9	.9010	-.0016	.9894	2.815	.100	.1190**
		..	..	. . .	. . .	. . .	4.047	.050	.0630**
		..	..	. . .	. . .	. . .	7.207	.010	.0110
11	2	20	.5	.4955	-.0041	.9804	1.681	.200	.1990
		40	.5	.4900	.0028	.9898	2.796	.100	.1000
		..	..	. . .	. . .	. . .	4.010	.050	.0525
		..	..	. . .	. . .	. . .	7.102	.010	.0075
12	2	20	.4	.3934	-.0022	.9920	1.681	.200	.2140
		40	.6	.5983	-.0049	.9950	2.796	.100	.1050
		..	..	. . .	. . .	. . .	4.010	.050	.0480
		..	..	. . .	. . .	. . .	7.102	.010	.0110
13	2	20	.3	.2914	.0057	.9847	1.681	.200	.2465**
		40	.7	.6983	.0015	.9875	2.796	.100	.1270**
		..	..	. . .	. . .	. . .	4.010	.050	.0735**
		..	..	. . .	. . .	. . .	7.102	.010	.0235**
14	2	20	.2	.1891	.0006	.9892	1.681	.200	.2500**
		40	.8	.7982	.0019	.9889	2.796	.100	.1350**
		..	..	. . .	. . .	. . .	4.010	.050	.0785**
		..	..	. . .	. . .	. . .	7.102	.010	.0160
15	2	20	.1	.1050	.0004	.9890	1.681	.200	.2345**
		40	.9	.8992	.0022	.9876	2.796	.100	.1295**
		..	..	. . .	. . .	. . .	4.010	.050	.0720**
		..	..	. . .	. . .	. . .	7.102	.010	.0200**
16	2	30	.5	.4925	-.0026	.9911	1.675	.200	.2025
		40	.5	.4944	-.0025	.9873	2.782	.100	.0965
		..	..	. . .	. . .	. . .	3.984	.050	.0445
		..	..	. . .	. . .	. . .	7.029	.010	.0150

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
17	2	30	.4	.3988	-.0005	.9944	1.675	.200	.1955
		40	.6	.5934	-.0016	.9921	2.782	.100	.0940
		. .	. .	. . .	. . .	. . .	3.984	.050	.0445
		. .	. .	. . .	. . .	. . .	7.029	.010	.0090
18	2	30	.3	.2948	-.0047	.9868	1.675	.200	.1895
		40	.7	.6977	-.0062	.9916	2.782	.100	.1005
		. .	. .	. . .	. . .	. . .	3.984	.050	.0495
		. .	. .	. . .	. . .	. . .	7.029	.010	.0115
19	2	30	.2	.1938	-.0065	.9967	1.675	.200	.1870
		40	.8	.7986	-.0004	.9868	2.782	.100	.0980
		. .	. .	. . .	. . .	. . .	3.984	.050	.0485
		. .	. .	. . .	. . .	. . .	7.029	.010	.0100
20	2	30	.1	.0993	.0016	.9901	1.675	.200	.1880
		40	.9	.8995	-.0016	.9900	2.782	.100	.0920
		. .	. .	. . .	. . .	. . .	3.984	.050	.0420
		. .	. .	. . .	. . .	. . .	7.029	.010	.0070
21	2	30	.5	.4949	-.0017	.9907	1.681	.200	.2020
		30	.5	.4953	.0074	.9899	2.796	.100	.0940
		. .	. .	. . .	. . .	. . .	4.010	.050	.0410
		. .	. .	. . .	. . .	. . .	7.102	.010	.0115
22	2	30	.4	.3940	.0102	.9918	1.681	.200	.2000
		30	.6	.6024	-.0009	.9908	2.796	.100	.1040
		. .	. .	. . .	. . .	. . .	4.010	.050	.0530
		. .	. .	. . .	. . .	. . .	7.102	.010	.0085
23	2	30	.3	.2956	-.0003	.9938	1.681	.200	.2050
		30	.7	.6988	-.0051	.9926	2.796	.100	.0990
		. .	. .	. . .	. . .	. . .	4.010	.050	.0450
		. .	. .	. . .	. . .	. . .	7.102	.010	.0065
24	2	30	.2	.1893	-.0032	.9942	1.681	.200	.1855
		30	.8	.7992	.0044	.9887	2.796	.100	.0850*
		. .	. .	. . .	. . .	. . .	4.010	.050	.0375*
		. .	. .	. . .	. . .	. . .	7.102	.010	.0080
25	2	30	.1	.1017	.0045	.9939	1.681	.200	.1510*
		30	.9	.9007	-.0005	.9920	2.796	.100	.0665*
		. .	. .	. . .	. . .	. . .	4.010	.050	.0290*
		. .	. .	. . .	. . .	. . .	7.102	.010	.0040*

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
26	2	40	.5	.4944	.0012	.9934	1.671	.200	.2070
		40	.5	.4949	-.0003	.9900	2.772	.100	.0910
		..	..	. . .	. . .	. . .	3.965	.050	.0440
		..	..	. . .	. . .	. . .	6.976	.010	.0115
27	2	40	.4	.3936	.0023	.9923	1.671	.200	.2075
		40	.6	.5973	-.0010	.9867	2.772	.100	.1055
		..	..	. . .	. . .	. . .	3.965	.050	.0445
		..	..	. . .	. . .	. . .	6.976	.010	.0100
28	2	40	.3	.2955	.0004	.9932	1.671	.200	.1910
		40	.7	.6936	.0010	.9899	2.772	.100	.0990
		..	..	. . .	. . .	. . .	3.965	.050	.0510
		..	..	. . .	. . .	. . .	6.976	.010	.0070
29	2	40	.2	.2035	.0028	.9939	1.671	.200	.1755*
		40	.8	.7993	.0001	.9907	2.772	.100	.0785*
		..	..	. . .	. . .	. . .	3.965	.050	.0365*
		..	..	. . .	. . .	. . .	6.976	.010	.0060
30	2	40	.1	.0919	.0074	.9887	1.671	.200	.1600*
		40	.9	.8998	.0005	.9893	2.772	.100	.0705*
		..	..	. . .	. . .	. . .	3.965	.050	.0300*
		..	..	. . .	. . .	. . .	6.976	.010	.0040*
31	3	20	.5	.5042	-.0025	.9871	1.657	.200	.2000
		20	.5	.4973	.0026	.9883	2.400	.100	.0990
		20	.5	.4952	-.0022	.9837	3.162	.050	.0510
		..	..	. . .	. . .	. . .	5.006	.010	.0105
32	3	20	.4	.3924	-.0034	.9836	1.657	.200	.1925
		20	.5	.4910	.0063	.9827	2.400	.100	.0920
		20	.6	.5881	-.0010	.9891	3.162	.050	.0445
		..	..	. . .	. . .	. . .	5.006	.010	.0095
33	3	20	.3	.2863	.0097	.9794	1.657	.200	.1600*
		20	.5	.4924	-.0014	.9905	2.400	.100	.0755*
		20	.7	.6994	-.0024	.9837	3.162	.050	.0360*
		..	..	. . .	. . .	. . .	5.006	.010	.0100
34	3	20	.2	.1934	-.0041	.9797	1.657	.200	.1670*
		20	.5	.5000	-.0006	.9881	2.400	.100	.0845*
		20	.8	.8009	-.0039	.9876	3.162	.050	.0385*
		..	..	. . .	. . .	. . .	5.006	.010	.0080

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
35	3	20	.1	.0909	.0046	.9871	1.657	.200	.1475*
		20	.5	.4892	-.0016	.9845	2.400	.100	.0720*
		20	.9	.9010	-.0012	.9861	3.162	.050	.0345*
		..	..	. . .	. . .	. . .	5.006	.010	.0075
36	3	20	.5	.4992	.0029	.9869	1.649	.200	.1980
		20	.5	.4985	.0058	.9843	2.385	.100	.1010
		30	.5	.4967	.0016	.9903	3.136	.050	.0555
		..	..	. . .	. . .	. . .	4.942	.010	.0155
37	3	20	.4	.3895	-.0034	.9834	1.649	.200	.2250**
		20	.5	.4940	.0027	.9870	2.385	.100	.1175**
		30	.6	.5968	.0003	.9879	3.136	.050	.0575
		..	..	. . .	. . .	. . .	4.942	.010	.0110
38	3	20	.3	.2927	-.0006	.9864	1.649	.200	.1940
		20	.5	.4926	.0011	.9869	2.385	.100	.0990
		30	.7	.6952	.0057	.9866	3.136	.050	.0490
		..	..	. . .	. . .	. . .	4.942	.010	.0085
39	3	20	.2	.1877	-.0029	.9832	1.649	.200	.2210**
		20	.5	.4953	.0053	.9919	2.385	.100	.1150**
		30	.8	.8002	-.0024	.9864	3.136	.050	.0635**
		..	..	. . .	. . .	. . .	4.942	.010	.0145
40	3	20	.1	.0987	.0011	.9879	1.649	.200	.2020
		20	.5	.4961	.0001	.9839	2.385	.100	.1060
		30	.9	.9000	-.0001	.9910	3.136	.050	.0575
		..	..	. . .	. . .	. . .	4.942	.010	.0110
41	3	20	.5	.4922	-.0038	.9839	1.644	.200	.1960
		20	.5	.4934	-.0053	.9851	2.374	.100	.1010
		40	.5	.4926	-.0030	.9898	3.117	.050	.0510
		..	..	. . .	. . .	. . .	4.896	.010	.0150
42	3	20	.4	.4058	.0050	.9883	1.644	.200	.2195**
		20	.5	.5018	.0006	.9912	2.374	.100	.1165**
		40	.6	.5962	-.0052	.9946	3.117	.050	.0600
		..	..	. . .	. . .	. . .	4.896	.010	.0140
43	3	20	.3	.2912	-.0027	.9910	1.644	.200	.2385**
		20	.5	.4906	.0014	.9817	2.374	.100	.1190**
		40	.7	.6975	-.0008	.9889	3.117	.050	.0635**
		..	..	. . .	. . .	. . .	4.896	.010	.0130

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
44	3	20	.2	.2025	.0014	.9929	1.644	.200	.2270**
		20	.5	.4940	-.0061	.9861	2.374	.100	.1150**
		40	.8	.7976	.0006	.9889	3.117	.050	.0635**
		. .	. .	. . .	. . .	. . .	4.896	.010	.0165**
45	3	20	.1	.0951	-.0013	.9904	1.644	.200	.2125
		20	.5	.4962	.0004	.9795	2.374	.100	.1160**
		40	.9	.9000	.0014	.9885	3.117	.050	.0660**
		. .	. .	. . .	. . .	. . .	4.896	.010	.0185**
46	3	20	.5	.4989	-.0026	.9944	1.644	.200	.1910
		30	.5	.4993	-.0038	.9950	2.374	.100	.0940
		30	.5	.5035	.0043	.9957	3.117	.050	.0450
		. .	. .	. . .	. . .	. . .	4.896	.010	.0085
47	3	20	.4	.3954	-.0003	.9930	1.644	.200	.2070
		30	.5	.4953	-.0025	.9950	2.374	.100	.1050
		30	.6	.5934	.0005	.9922	3.117	.050	.0555
		. .	. .	. . .	. . .	. . .	4.896	.010	.0145
48	3	20	.3	.3004	.0063	.9871	1.644	.200	.2070
		30	.5	.4988	.0027	.9878	2.374	.100	.1055
		30	.7	.6969	.0006	.9921	3.117	.050	.0525
		. .	. .	. . .	. . .	. . .	4.896	.010	.0110
49	3	20	.2	.1954	-.0074	.9864	1.644	.200	.1805*
		30	.5	.5007	.0047	.9910	2.374	.100	.0895
		30	.8	.8010	.0074	.9877	3.117	.050	.0495
		. .	. .	. . .	. . .	. . .	4.896	.010	.0090
50	3	20	.1	.0976	.0073	.9803	1.644	.200	.1920
		30	.5	.4940	-.0018	.9930	2.374	.100	.1035
		30	.9	.9005	-.0022	.9898	3.117	.050	.0475
		. .	. .	. . .	. . .	. . .	4.896	.010	.0150
51	3	30	.5	.4965	.0010	.9894	1.634	.200	.1985
		40	.5	.5006	-.0004	.9927	2.353	.100	.1015
		40	.5	.4983	.0023	.9951	3.082	.050	.0470
		. .	. .	. . .	. . .	. . .	4.811	.010	.0115
52	3	30	.4	.3907	-.0002	.9891	1.634	.200	.1965
		40	.5	.4945	.0030	.9913	2.353	.100	.0965
		40	.6	.5969	-.0010	.9929	3.082	.050	.0520
		. .	. .	. . .	. . .	. . .	4.811	.010	.0145

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
53	3	30	.3	.3004	.0026	.9936	1.634	.200	.2005
		40	.5	.4977	.0058	.9879	2.353	.100	.1020
		40	.7	.6956	.0007	.9891	3.082	.050	.0495
		..	..	. . .	. . .	. . .	4.811	.010	.0075
54	3	30	.2	.2020	.0021	.9928	1.634	.200	.1965
		40	.5	.4946	.0008	.9926	2.353	.100	.0950
		40	.8	.7980	-.0013	.9869	3.082	.050	.0455
		..	..	. . .	. . .	. . .	4.811	.010	.0090
55	3	30	.1	.1016	-.0034	.9887	1.634	.200	.1735*
		40	.5	.4922	-.0038	.9930	2.353	.100	.0880
		40	.9	.8999	.0009	.9896	3.082	.050	.0415
		..	..	. . .	. . .	. . .	4.811	.010	.0040*
56	3	30	.5	.4963	-.0037	.9942	1.637	.200	.2055
		30	.5	.4978	-.0004	.9888	2.359	.100	.1030
		40	.5	.4932	-.0044	.9956	3.091	.050	.0480
		..	..	. . .	. . .	. . .	4.833	.010	.0080
57	3	30	.4	.4011	.0000	.9964	1.637	.200	.1955
		30	.5	.5022	-.0028	.9936	2.359	.100	.0975
		40	.6	.5978	-.0058	.9915	3.091	.050	.0550
		..	..	. . .	. . .	. . .	4.833	.010	.0125
58	3	30	.3	.2951	.0023	.9896	1.637	.200	.2080
		30	.5	.4996	-.0012	.9896	2.359	.100	.1075
		40	.7	.6973	.0000	.9880	3.091	.050	.0530
		..	..	. . .	. . .	. . .	4.833	.010	.0110
59	3	30	.2	.1965	-.0108	.9921	1.637	.200	.1965
		30	.5	.4939	.0062	.9856	2.359	.100	.1080
		40	.8	.7987	.0036	.9888	3.091	.050	.0560
		..	..	. . .	. . .	. . .	4.833	.010	.0085
60	3	30	.1	.0942	.0025	.9902	1.637	.200	.1960
		30	.5	.4986	-.0002	.9897	2.359	.100	.0930
		40	.9	.9000	.0018	.9917	3.091	.050	.0445
		..	..	. . .	. . .	. . .	4.833	.010	.0075
61	3	30	.5	.4992	.0035	.9909	1.640	.200	.2040
		30	.5	.4937	-.0066	.9913	2.365	.100	.1005
		30	.5	.4950	-.0061	.9900	3.103	.050	.0465
		..	..	. . .	. . .	. . .	4.861	.010	.0115

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
62	3	30	.4	.3861	-.0042	.9891	1.640	.200	.2110
		30	.5	.4990	-.0010	.9943	2.365	.100	.1060
		30	.6	.5991	.0005	.9900	3.103	.050	.0520
		..	..	. . .	. . .	. . .	4.861	.010	.0090
63	3	30	.3	.2935	-.0034	.9865	1.640	.200	.2040
		30	.5	.4972	-.0003	.9938	2.365	.100	.1080
		30	.7	.7005	-.0004	.9890	3.103	.050	.0505
		..	..	. . .	. . .	. . .	4.861	.010	.0065
64	3	30	.2	.2036	.0002	.9939	1.640	.200	.1705*
		30	.5	.4917	-.0048	.9906	2.365	.100	.0865*
		30	.8	.8008	-.0000	.9886	3.103	.050	.0405*
		..	..	. . .	. . .	. . .	4.861	.010	.0085
65	3	30	.1	.1004	-.0030	.9921	1.640	.200	.1590*
		30	.5	.4933	.0026	.9899	2.365	.100	.0805*
		30	.9	.9019	.0028	.9917	3.103	.050	.0445
		..	..	. . .	. . .	. . .	4.861	.010	.0105
66	3	40	.5	.4984	-.0025	.9891	1.632	.200	.2020
		40	.5	.4947	.0015	.9872	2.349	.100	.1035
		40	.5	.4944	.0033	.9933	3.074	.050	.0530
		..	..	. . .	. . .	. . .	4.793	.010	.0120
67	3	40	.4	.3941	.0041	.9945	1.632	.200	.1890
		40	.5	.4971	-.0008	.9897	2.349	.100	.0890
		40	.6	.5966	.0034	.9878	3.074	.050	.0420
		..	..	. . .	. . .	. . .	4.793	.010	.0065
68	3	40	.3	.2899	-.0036	.9911	1.632	.200	.1895
		40	.5	.4950	-.0003	.9913	2.349	.100	.0885
		40	.7	.7002	.0021	.9909	3.074	.050	.0400*
		..	..	. . .	. . .	. . .	4.793	.010	.0115
69	3	40	.2	.1936	.0038	.9939	1.632	.200	.1625*
		40	.5	.4936	.0000	.9930	2.349	.100	.0830*
		40	.8	.7971	-.0004	.9877	3.074	.050	.0400*
		..	..	. . .	. . .	. . .	4.793	.010	.0085
70	3	40	.1	.0981	-.0003	.9886	1.632	.200	.1665*
		40	.5	.4935	.0065	.9941	2.349	.100	.0775*
		40	.9	.8993	-.0006	.9896	3.074	.050	.0360*
		..	..	. . .	. . .	. . .	4.793	.010	.0080

TABLE VII--<u>Continued</u>

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
71	3	20	.5	.4956	-.0041	.9887	1.637	.200	.2215**
		40	.5	.4944	-.0002	.9902	2.359	.100	.1080
		40	.5	.4905	-.0013	.9837	3.091	.050	.0490
		..	..	. . .	. . .	. . .	4.833	.010	.0095
72	3	20	.4	.3984	.0007	.9889	1.637	.200	.2020
		40	.5	.4975	.0020	.9896	2.359	.100	.0975
		40	.6	.5942	-.0024	.9852	3.091	.050	.0455
		..	..	. . .	. . .	. . .	4.833	.010	.0070
73	3	20	.3	.2972	-.0051	.9802	1.637	.200	.2120
		40	.5	.4938	.0046	.9938	2.359	.100	.1135
		40	.7	.6967	-.0023	.9913	3.091	.050	.0620**
		..	..	. . .	. . .	. . .	4.833	.010	.0125
74	3	20	.2	.1943	-.0030	.9851	1.637	.200	.2055
		40	.5	.4941	-.0023	.9911	2.359	.100	.1120
		40	.8	.7988	.0045	.9876	3.091	.050	.0605
		..	..	. . .	. . .	. . .	4.833	.010	.0150
75	3	20	.1	.0925	.0031	.9849	1.637	.200	.1990
		40	.5	.4047	-.0005	.9920	2.359	.100	.1100
		40	.9	.8988	.0031	.9885	3.091	.050	.0570
		..	..	. . .	. . .	. . .	4.833	.010	.0160
76	3	20	.5	.4972	.0078	.9820	1.640	.200	.1930
		30	.5	.4964	.0051	.9889	2.365	.100	.0885
		40	.5	.4919	-.0003	.9918	3.103	.050	.0455
		..	..	. . .	. . .	. . .	4.861	.010	.0085
77	3	20	.4	.3979	.0002	.9838	1.640	.200	.2070
		30	.5	.4958	-.0045	.9916	2.365	.100	.1050
		40	.6	.5951	.0007	.9891	3.103	.050	.0540
		..	..	. . .	. . .	. . .	4.861	.010	.0140
78	3	20	.3	.2978	-.0015	.9815	1.640	.200	.2235**
		30	.5	.4969	.0014	.9914	2.365	.100	.1145**
		40	.7	.6954	.0015	.9897	3.103	.050	.0700**
		..	..	. . .	. . .	. . .	4.861	.010	.0165**
79	3	20	.2	.1920	-.0012	.9868	1.640	.200	.2210**
		30	.5	.4883	.0002	.9851	2.365	.100	.1210**
		40	.8	.7999	-.0045	.9898	3.103	.050	.0595
		..	..	. . .	. . .	. . .	4.861	.010	.0155

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
80	3	20	.1	.0919	-.0046	.9878	1.640	.200	.2210**
		30	.5	.4981	-.0027	.9920	2.365	.100	.1215**
		40	.9	.8998	.0006	.9866	3.103	.050	.0590
		..	..	. . .	. . .	. . .	4.861	.010	.0150
81	5	20	.5	.4956	.0009	.9887	1.529	.200	.2025
		20	.5	.5000	.0032	.9933	2.006	.100	.1010
		20	.5	.4882	-.0022	.9878	2.469	.050	.0555
		20	.5	.4952	-.0036	.9823	3.526	.010	.0090
		20	.5	.4929	.0012	.9876	. . .	. . .	. . .
82	5	20	.4	.3848	-.0020	.9836	1.529	.200	.1795*
		20	.4	.3971	-.0051	.9847	2.006	.100	.0950
		20	.5	.4960	.0033	.9896	2.469	.050	.0500
		20	.6	.5941	-.0012	.9804	3.526	.010	.0115
		20	.6	.5972	-.0021	.9854	. . .	. . .	. . .
83	5	20	.3	.2872	-.0037	.9843	1.529	.200	.1925
		20	.4	.3987	.0022	.9872	2.006	.100	.0975
		20	.5	.4891	-.0007	.9863	2.469	.050	.0510
		20	.6	.5948	-.0037	.9811	3.526	.010	.0105
		20	.7	.6984	-.0001	.9907	. . .	. . .	. . .
84	5	20	.2	.1874	.0016	.9864	1.529	.200	.1730*
		20	.4	.3958	.0029	.9879	2.006	.100	.0965
		20	.5	.4956	-.0015	.9852	2.469	.050	.0455
		20	.6	.6027	.0026	.9855	3.526	.010	.0085
		20	.8	.8033	-.0016	.9880	. . .	. . .	. . .
85	5	20	.1	.0900	.0005	.9864	1.529	.200	.1685*
		20	.3	.3042	.0054	.9917	2.006	.100	.0815*
		20	.5	.4984	.0031	.9889	2.469	.050	.0445
		20	.7	.6999	-.0005	.9877	3.526	.010	.0100
		20	.9	.9006	.0023	.9853	. . .	. . .	. . .
86	5	20	.5	.4972	.0015	.9876	1.524	.200	.2070
		20	.5	.4980	-.0064	.9812	1.995	.100	.1065
		20	.5	.4927	.0104	.9898	2.451	.050	.0560
		30	.5	.5005	.0020	.9891	3.488	.010	.0120
		30	.5	.4976	-.0025	.9913	. . .	. . .	. . .

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
87	5	20	.4	.3924	-.0011	.9830	1.524	.200	.2165
		20	.4	.3989	-.0016	.9892	1.995	.100	.1210**
		20	.5	.4961	-.0001	.9897	2.451	.050	.0635**
		30	.6	.5912	-.0030	.9843	3.488	.010	.0165**
		30	.6	.5990	.0031	.9901	. . .	. . .	. . .
88	5	20	.3	.2867	.0015	.9885	1.524	.200	.2200**
		20	.4	.3914	-.0021	.9830	1.995	.100	.1105
		20	.5	.4899	-.0047	.9897	2.451	.050	.0575
		30	.6	.5972	-.0017	.9925	3.488	.010	.0115
		30	.7	.6962	-.0001	.9872	. . .	. . .	. . .
89	5	20	.2	.1945	-.0064	.9869	1.524	.200	.1980
		20	.4	.3948	-.0015	.9840	1.995	.100	.1125
		20	.5	.4997	.0012	.9871	2.451	.050	.0620**
		30	.6	.6004	.0009	.9916	3.488	.010	.0125
		30	.8	.8023	-.0005	.9907	. . .	. . .	. . .
90	5	20	.1	.1020	.0000	.9842	1.524	.200	.1885
		20	.3	.2972	.0006	.9886	1.995	.100	.0985
		20	.5	.4975	.0017	.9861	2.451	.050	.0505
		30	.7	.7014	.0028	.9900	3.488	.010	.0110
		30	.9	.9009	-.0026	.9912	. . .	. . .	. . .
91	5	20	.5	.5026	-.0044	.9913	1.521	.200	.1965
		20	.5	.4976	-.0045	.9828	1.991	.100	.1060
		20	.5	.4948	-.0038	.9837	2.445	.050	.0565
		30	.5	.4960	-.0007	.9925	3.475	.010	.0070
		40	.5	.4963	.0009	.9887	. . .	. . .	. . .
92	5	20	.4	.3885	.0096	.9800	1.521	.200	.2230**
		20	.4	.3923	-.0011	.9766	1.991	.100	.1175**
		20	.5	.4948	.0004	.9871	2.445	.050	.0590
		30	.6	.5973	-.0045	.9903	3.475	.010	.0145
		40	.6	.6005	.0009	.9912	. . .	. . .	. . .
93	5	20	.3	.2940	-.0026	.9927	1.521	.200	.2230**
		20	.4	.3958	.0007	.9861	1.991	.100	.1145**
		20	.5	.4995	.0083	.9832	2.445	.050	.0615**
		30	.6	.5929	.0043	.9898	3.475	.010	.0110
		40	.7	.6941	-.0032	.9863	. . .	. . .	. . .

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
94	5	20	.2	.1910	-.0003	.9810	1.521	.200	.2225**
		20	.4	.3968	-.0023	.9895	1.991	.100	.1210**
		20	.5	.4985	-.0016	.9838	2.445	.050	.0605
		30	.6	.5947	-.0011	.9918	3.475	.010	.0135
		40	.8	.7974	-.0027	.9905	. . .	. . .	. . .
95	5	20	.1	.1022	-.0029	.9897	1.521	.200	.2145
		20	.3	.2948	-.0062	.9889	1.991	.100	.1105
		20	.5	.4908	.0026	.9802	2.445	.050	.0620**
		30	.7	.7007	-.0011	.9902	3.475	.010	.0140
		40	.9	.8993	.0007	.9888	. . .	. . .	. . .
96	5	20	.5	.4942	-.0020	.9900	1.521	.200	.1975
		20	.5	.4940	-.0028	.9865	1.991	.100	.0960
		30	.5	.4965	-.0013	.9910	2.445	.050	.0460
		30	.5	.4943	-.0046	.9953	3.475	.010	.0115
		30	.5	.4901	.0040	.9836	. . .	. . .	. . .
97	5	20	.4	.3915	-.0027	.9844	1.521	.200	.2015
		20	.4	.3917	-.0064	.9887	1.991	.100	.0965
		30	.5	.5015	-.0008	.9912	2.445	.050	.0470
		30	.6	.5934	.0003	.9878	3.475	.010	.0130
		30	.6	.5979	-.0028	.9919	. . .	. . .	. . .
98	5	20	.3	.3013	-.0089	.9865	1.521	.200	.2100
		20	.4	.3918	-.0008	.9926	1.991	.100	.1135
		30	.5	.4976	-.0003	.9907	2.445	.050	.0610
		30	.6	.5960	.0017	.9925	3.475	.010	.0130
		30	.7	.6966	.0000	.9908	. . .	. . .	. . .
99	5	20	.2	.2033	-.0034	.9855	1.521	.200	.1910
		20	.4	.4004	.0033	.9878	1.991	.100	.0955
		30	.5	.5025	-.0016	.9934	2.445	.050	.0440
		30	.6	.5954	.0016	.9911	3.475	.010	.0115
		30	.8	.8038	-.0034	.9906	. . .	. . .	. . .
100	5	20	.1	.0984	-.0004	.9917	1.521	.200	.1945
		20	.3	.2977	.0038	.9904	1.991	.100	.1075
		30	.5	.4982	.0019	.9931	2.445	.050	.0570
		30	.7	.6968	.0018	.9930	3.475	.010	.0130
		30	.9	.9001	-.0013	.9894	. . .	. . .	. . .

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
101	5	20	.5	.4979	-.0024	.9923	1.518	.200	.2135
		20	.5	.5011	-.0013	.9821	1.984	.100	.1060
		30	.5	.4953	-.0004	.9882	2.435	.050	.0540
		40	.5	.4952	.0055	.9928	3.453	.010	.0065
		40	.5	.4935	.0012	.9880	. . .	. . .	. . .
102	5	20	.4	.3874	-.0022	.9821	1.518	.200	.2185**
		20	.4	.3999	-.0033	.9811	1.984	.100	.1135
		30	.5	.4998	-.0067	.9926	2.435	.050	.0615**
		40	.6	.5956	-.0002	.9923	3.453	.010	.0135
		40	.6	.5947	-.0013	.9864	. . .	. . .	. . .
103	5	20	.3	.2980	.0034	.9904	1.518	.200	.2305**
		20	.4	.3973	-.0014	.9896	1.984	.100	.1260**
		30	.5	.4952	-.0025	.9880	2.435	.050	.0615**
		40	.6	.5971	.0074	.9904	3.453	.010	.0145
		40	.7	.6964	-.0003	.9916	. . .	. . .	. . .
104	5	20	.2	.1915	.0009	.9898	1.518	.200	.2285**
		20	.4	.3982	.0013	.9875	1.984	.100	.1215**
		30	.5	.4953	-.0006	.9905	2.435	.050	.0695**
		40	.6	.5945	-.0008	.9942	3.453	.010	.0150
		40	.8	.7972	-.0022	.9915	. . .	. . .	. . .
105	5	20	.1	.0920	-.0064	.9840	1.518	.200	.2165
		20	.3	.3004	.0098	.9878	1.984	.100	.1175**
		30	.5	.4990	-.0035	.9954	2.435	.050	.0685**
		40	.7	.6980	-.0034	.9874	3.453	.010	.0185**
		40	.9	.8994	.0014	.9894	. . .	. . .	. . .
106	5	20	.5	.4911	.0068	.9835	1.517	.200	.1960
		20	.5	.4949	-.0027	.9871	1.982	.100	.0980
		40	.5	.4971	.0008	.9907	2.430	.050	.0530
		40	.5	.4920	-.0013	.9934	3.444	.010	.0075
		40	.5	.4932	-.0055	.9882	. . .	. . .	. . .
107	5	20	.4	.3973	.0039	.9867	1.517	.200	.2130
		20	.4	.3971	.0052	.9835	1.982	.100	.1160**
		40	.5	.4900	-.0040	.9927	2.430	.050	.0595
		40	.6	.5949	.0022	.9912	3.444	.010	.0130
		40	.6	.5975	-.0008	.9932	. . .	. . .	. . .

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
108	5	20	.3	.2984	.0023	.9833	1.517	.200	.2045
		20	.4	.3959	-.0049	.9782	1.982	.100	.1090
		40	.5	.4932	.0043	.9891	2.430	.050	.0630**
		40	.6	.5961	.0036	.9885	3.444	.010	.0110
		40	.7	.6960	.0020	. . .	. . .	. . .	. . .
109	5	20	.2	.2021	.0013	.9934	1.517	.200	.2335**
		20	.4	.3945	-.0009	.9803	1.982	.100	.1240**
		40	.5	.4914	.0033	.9851	2.430	.050	.0645**
		40	.6	.5955	.0030	.9895	3.444	.010	.0135
		40	.8	.7981	-.0036	.9881	. . .	. . .	. . .
110	5	20	.1	.0982	.0058	.9855	1.517	.200	.2090
		20	.3	.2908	.0066	.9839	1.982	.100	.1095
		40	.5	.4896	.0026	.9847	2.430	.050	.0605
		40	.7	.6975	.0035	.9902	3.444	.010	.0215**
		40	.9	.8995	-.0025	.9880	. . .	. . .	. . .
111	5	20	.5	.4937	.0001	.9812	1.518	.200	.1800*
		30	.5	.4927	.0012	.9960	1.984	.100	.0940
		30	.5	.4944	-.0008	.9843	2.435	.050	.0510
		30	.5	.4996	-.0017	.9889	3.453	.010	.0095
		40	.5	.4926	.0033	.9878	. . .	. . .	. . .
112	5	20	.4	.3943	.0043	.9853	1.518	.200	.2240**
		30	.4	.3973	-.0009	.9891	1.984	.100	.1130
		30	.5	.4972	-.0012	.9873	2.435	.050	.0565
		30	.6	.5971	-.0033	.9858	3.453	.010	.0130
		40	.6	.5984	-.0029	. . .	. . .	. . .	. . .
113	5	20	.3	.2910	.0018	.9867	1.518	.200	.2150
		30	.4	.3945	-.0006	.9874	1.984	.100	.1075
		30	.5	.4989	-.0003	.9907	2.435	.050	.0535
		30	.6	.5990	-.0006	.9902	3.453	.010	.0095
		40	.7	.6969	-.0013	.9895	. . .	. . .	. . .
114	5	20	.2	.1951	.0027	.9867	1.518	.200	.2095
		30	.4	.3976	-.0001	.9873	1.984	.100	.1030
		30	.5	.4925	-.0030	.9817	2.435	.050	.0530
		30	.6	.5985	.0019	.9891	3.453	.010	.0135
		40	.8	.7963	-.0017	.9881	. . .	. . .	. . .

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal a	Actual a'
115	5	20	.1	.0998	.0013	.9897	1.518	.200	.1995
		30	.3	.2974	.0012	.9898	1.984	.100	.1115
		30	.5	.4944	-.0025	.9911	2.435	.050	.0615**
		30	.7	.6993	.0019	.9909	3.453	.010	.0120
		40	.9	.8991	.0012	.9882	. . .	. . .	. . .
116	5	20	.5	.4915	-.0001	.9853	1.516	.200	.2060
		30	.5	.4945	-.0049	.9929	1.980	.100	.1035
		40	.5	.4931	-.0027	.9927	2.427	.050	.0485
		40	.5	.4962	-.0021	.9921	3.436	.010	.0075
		40	.5	.4922	.0009	.9898	. . .	. . .	. . .
117	5	20	.4	.3945	-.0030	.9866	1.516	.200	.1910
		30	.4	.3832	-.0044	.9877	1.980	.100	.0970
		40	.5	.4984	-.0017	.9913	2.427	.050	.0455
		40	.6	.5915	.0055	.9881	3.436	.010	.0080
		40	.6	.5951	-.0028	.9870	. . .	. . .	. . .
118	5	20	.3	.2858	.0047	.9834	1.516	.200	.2220**
		30	.4	.4027	-.0000	.9970	1.980	.100	.1070
		40	.5	.4933	.0008	.9911	2.427	.050	.0590
		40	.6	.5973	.0008	.9884	3.436	.010	.0110
		40	.7	.6988	-.0053	.9877	. . .	. . .	. . .
119	5	20	.2	.1965	.0025	.9818	1.516	.200	.2060
		30	.4	.3953	.0085	.9944	1.980	.100	.1085
		40	.5	.4954	-.0023	.9918	2.427	.050	.0595
		40	.6	.5963	.0035	.9886	3.436	.010	.0145
		40	.8	.7967	-.0010	.9878	. . .	. . .	. . .
120	5	20	.1	.0964	.0028	.9849	1.516	.200	.2080
		30	.3	.3011	-.0039	.9939	1.980	.100	.1120
		40	.5	.4987	-.0048	.9946	2.427	.050	.0525
		40	.7	.6981	-.0035	.9908	3.436	.010	.0130
		40	.9	.8991	.0018	.9869	. . .	. . .	. . .
121	5	30	.5	.4949	.0057	.9899	1.518	.200	.1935
		30	.5	.4953	-.0007	.9960	1.984	.100	.0850*
		30	.5	.4953	-.0059	.9868	2.435	.050	.0415
		30	.5	.4910	.0008	.9910	3.453	.010	.0115
		30	.5	.5025	.0004	.9944	. . .	. . .	. . .

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
122	5	30	.4	.3952	-.0055	.9854	1.518	.200	.1980
		30	.4	.3982	-.0033	.9887	1.984	.100	.0960
		30	.5	.4964	-.0039	.9906	2.435	.050	.0490
		30	.6	.5957	.0002	.9867	3.453	.010	.0160
		30	.6	.5989	.0001	.9921	. . .	. . .	. . .
123	5	30	.3	.2981	-.0034	.9887	1.518	.200	.1815*
		30	.4	.3962	-.0041	.9860	1.984	.100	.0935
		30	.5	.4939	-.0028	.9903	2.435	.050	.0445
		30	.6	.5984	-.0020	.9895	3.453	.010	.0065
		30	.7	.7005	.0014	.9917	. . .	. . .	. . .
124	5	30	.2	.1981	-.0029	.9885	1.518	.200	.1780*
		30	.4	.3995	-.0037	.9919	1.984	.100	.0855*
		30	.5	.4964	.0035	.9870	2.435	.050	.0440
		30	.6	.5999	-.0023	.9901	3.453	.010	.0095
		30	.8	.7992	.0031	.9883	. . .	. . .	. . .
125	5	30	.1	.0993	-.0008	.9893	1.518	.200	.1595*
		30	.3	.2949	-.0008	.9878	1.984	.100	.0755*
		30	.5	.4998	-.0028	.9928	2.435	.050	.0395*
		30	.7	.7007	-.0006	.9871	3.453	.010	.0090
		30	.9	.9004	.0014	.9934	. . .	. . .	. . .
126	5	40	.5	.4943	.0052	.9890	1.513	.200	.2005
		40	.5	.5024	-.0038	.9938	1.974	.100	.1070
		40	.5	.4945	.0015	.9936	2.418	.050	.0530
		40	.5	.4956	.0014	.9924	3.418	.010	.0140
		40	.5	.4952	-.0019	.9877	. . .	. . .	. . .
127	5	40	.4	.3959	-.0074	.9945	1.513	.200	.1950
		40	.4	.3929	-.0025	.9925	1.974	.100	.0980
		40	.5	.4977	.0023	.9904	2.418	.050	.0455
		40	.6	.5950	-.0043	.9905	3.418	.010	.0085
		40	.6	.5930	-.0014	.9888	. . .	. . .	. . .
128	5	40	.3	.2966	.0014	.9899	1.513	.200	.1885
		40	.4	.3950	.0089	.9900	1.974	.100	.0985
		40	.5	.4942	-.0023	.9907	2.418	.050	.0435
		40	.6	.5924	.0021	.9888	3.418	.010	.0100
		40	.7	.6971	-.0024	.9875	. . .	. . .	. . .

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
129	5	40	.2	.1939	.0006	.9900	1.513	.200	.1745*
		40	.4	.3954	.0030	.9986	1.974	.100	.0945
		40	.5	.4970	-.0036	.9964	2.418	.050	.0480
		40	.6	.5974	-.0041	.9915	3.418	.010	.0095
		40	.8	.7967	.0032	.9906	. . .	. . .	. . .
130	5	40	.1	.0909	-.0045	.9907	1.513	.200	.1615*
		40	.3	.2917	-.0036	.9970	1.974	.100	.0780*
		40	.5	.4904	-.0011	.9901	2.418	.050	.0390*
		40	.7	.6986	-.0023	.9901	3.418	.010	.0070
		40	.9	.8987	-.0008	.9883	. . .	. . .	. . .
131	5	30	.5	.4976	.0017	.9902	1.514	.200	.1820*
		30	.5	.4959	.0048	.9921	1.978	.100	.0855*
		40	.5	.4908	-.0037	.9874	2.424	.050	.0425
		40	.5	.4892	.0062	.9888	3.428	.010	.0080
		40	.5	.4923	.0012	.9890	. . .	. . .	. . .
132	5	30	.4	.3938	.0025	.9884	1.514	.200	.1950
		30	.4	.3894	-.0036	.9892	1.978	.100	.0995
		40	.5	.4933	.0022	.9923	2.424	.050	.0515
		40	.6	.5964	.0018	.9845	3.428	.010	.0100
		40	.6	.5993	-.0014	.9925	. . .	. . .	. . .
133	5	30	.3	.2958	-.0002	.9874	1.514	.200	.1945
		30	.4	.3994	.0021	.9926	1.978	.100	.0935
		40	.5	.4920	-.0002	.9919	2.424	.050	.0425
		40	.6	.5937	.0031	.9879	3.428	.010	.0095
		40	.7	.6979	.0014	.9896	. . .	. . .	. . .
134	5	30	.2	.1967	.0001	.9927	1.514	.200	.1955
		30	.4	.3917	.0044	.9886	1.978	.100	.0970
		40	.5	.4962	-.0033	.9887	2.424	.050	.0490
		40	.6	.5959	-.0005	.9904	3.428	.010	.0110
		40	.8	.7993	.0025	.9902	. . .	. . .	. . .
135	5	30	.1	.1014	.0013	.9876	1.514	.200	.1685*
		30	.3	.2964	.0079	.9886	1.978	.100	.0830*
		40	.5	.4986	-.0046	.9927	2.424	.050	.0375*
		40	.7	.6982	-.0023	.9910	3.428	.010	.0065
		40	.9	.8985	-.0016	.9866	. . .	. . .	. . .

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
136	2	100	.5	.4967	-.0046	.9972	1.653	.200	.2000
		100	.5	.4941	.0001	.9982	2.731	.100	.1045
		...	..	. . .	. . .	. . .	3.889	.050	.0480
		...	..	. . .	. . .	. . .	6.765	.010	.0095
137	2	100	.4	.3975	-.0024	.9970	1.653	.200	.1945
		100	.6	.5960	-.0034	.9942	2.731	.100	.0920
		...	..	. . .	. . .	. . .	3.889	.050	.0440
		...	..	. . .	. . .	. . .	6.765	.010	.0105
138	2	100	.3	.3008	.0001	.9992	1.653	.200	.2025
		100	.7	.6974	.0031	.9926	2.731	.100	.0945
		...	..	. . .	. . .	. . .	3.889	.050	.0455
		...	..	. . .	. . .	. . .	6.765	.010	.0095
139	2	100	.2	.1992	-.0008	.9955	1.653	.200	.1780*
		100	.8	.7986	.0008	.9928	2.731	.100	.0885
		...	..	. . .	. . .	. . .	3.889	.050	.0395*
		...	..	. . .	. . .	. . .	6.765	.010	.0075
140	2	100	.1	.0966	.0035	.9998	1.653	.200	.1545*
		100	.9	.8997	.0011	.9935	2.731	.100	.0660*
		...	..	. . .	. . .	. . .	3.889	.050	.0285*
		...	..	. . .	. . .	. . .	6.765	.010	.0025*
141	2	200	.5	.4998	.0018	.9981	1.653	.200	.2135
		200	.5	.4981	.0008	.9971	2.726	.100	.1100
		...	..	. . .	. . .	. . .	3.876	.050	.0525
		...	..	. . .	. . .	. . .	6.730	.010	.0060
142	2	200	.4	.3983	-.0005	.9980	1.653	.200	.1865
		200	.6	.5977	.0007	.9980	2.726	.100	.0965
		...	..	. . .	. . .	. . .	3.876	.050	.0430
		...	..	. . .	. . .	. . .	6.730	.010	.0090
143	2	200	.3	.2994	.0021	.9990	1.653	.200	.1945
		200	.7	.6994	.0002	.9994	2.726	.100	.0975
		...	..	. . .	. . .	. . .	3.876	.050	.0455
		...	..	. . .	. . .	. . .	6.730	.010	.0075
144	2	200	.2	.1980	-.0008	.9986	1.653	.200	.1725*
		200	.8	.8005	-.0011	.9977	2.726	.100	.0880
		...	..	. . .	. . .	. . .	3.876	.050	.0435
		...	..	. . .	. . .	. . .	6.730	.010	.0080

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
145	2	200	.1	.0997	-.0011	.9993	1.653	.200	.1390*
		200	.9	.8997	.0005	.9977	2.726	.100	.0585*
		...	..	...	...	...	3.876	.050	.0260*
		...	..	...	...	...	6.730	.010	.0040*
146	3	100	.5	.4972	-.0000	.9943	1.622	.200	.2000
		100	.5	.4960	.0022	.9979	2.325	.100	.0975
		100	.5	.4992	.0010	.9962	3.036	.050	.0470
		...	..	...	...	...	4.700	.010	.0100
147	3	100	.4	.3975	-.0001	.9969	1.622	.200	.1885
		100	.5	.4968	.0027	.9961	2.325	.100	.0995
		100	.6	.5986	-.0017	.9939	3.036	.050	.0485
		...	..	...	...	...	4.700	.010	.0095
148	3	100	.3	.2990	-.0004	.9986	1.622	.200	.1800*
		100	.5	.4974	.0011	.9948	2.325	.100	.0880
		100	.7	.6974	.0009	.9963	3.036	.050	.0390*
		...	..	...	...	...	4.700	.010	.0045*
149	3	100	.2	.1947	.0010	.9942	1.622	.200	.1820*
		100	.5	.4979	.0002	.9975	2.325	.100	.0855
		100	.7	.7999	.0004	.9945	3.036	.050	.0455
		...	..	...	...	...	4.700	.010	.0105
150	3	100	.1	.0950	-.0023	.9977	1.622	.200	.1415*
		100	.5	.4977	-.0016	.9933	2.325	.100	.0750*
		100	.9	.9003	.0017	.9926	3.036	.050	.0330*
		...	..	...	...	...	4.700	.010	.0080
151	3	200	.5	.4979	-.0009	.9973	1.622	.200	.2085
		200	.5	.4998	-.0011	.9975	2.314	.100	.0965
		200	.5	.4989	.0014	.9986	3.020	.050	.0445
		...	..	...	...	...	4.661	.010	.0110
152	3	200	.4	.4003	.0003	.9982	1.622	.200	.1805*
		200	.5	.4995	.0010	.9976	2.314	.100	.0860*
		200	.6	.5979	-.0002	.9987	3.020	.050	.0445
		...	..	...	...	...	4.661	.010	.0110
153	3	200	.3	.2965	.0020	.9977	1.622	.200	.1820*
		200	.5	.5014	.0020	.9986	2.314	.100	.0945
		200	.7	.6990	.0008	.9970	3.020	.050	.0515
		...	..	...	...	...	4.661	.010	.0085

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
154	3	200	.2	.2014	-.0013	.9986	1.622	.200	.1780*
		200	.5	.4970	.0011	.9971	2.314	.100	.0830*
		200	.8	.7990	-.0011	.9966	3.020	.050	.0425
		...	..	. . .	. . .	. . .	4.661	.010	.0130
155	3	200	.1	.1002	.0017	1.0005	1.622	.200	.1530*
		200	.5	.4985	-.0006	1.0000	2.314	.100	.0670*
		200	.9	.8997	-.0004	.9974	3.020	.050	.0285*
		...	..	. . .	. . .	. . .	4.661	.010	.0095
156	5	100	.5	.4992	-.0015	.9968	1.512	.200	.2090
		100	.5	.4976	-.0003	.9962	1.961	.100	.1140
		100	.5	.4967	-.0024	.9944	2.399	.050	.0545
		100	.5	.4983	.0029	.9943	3.379	.010	.0120
		100	.5	.4977	.0004	.9949	. . .	. . .	. . .
157	5	100	.4	.3971	-.0009	.9936	1.512	.200	.1905
		100	.4	.3953	-.0013	.9967	1.961	.100	.0985
		100	.5	.4954	-.0020	.9940	2.399	.050	.0500
		100	.6	.5958	.0030	.9945	3.379	.010	.0125
		100	.6	.5974	-.0017	.9950	. . .	. . .	. . .
158	5	100	.3	.2997	-.0016	.9962	1.512	.200	.1805*
		100	.4	.3984	.0016	.9961	1.961	.100	.0915
		100	.5	.4944	-.0035	.9927	2.399	.050	.0405
		100	.6	.5971	.0005	.9994	3.379	.010	.0095
		100	.7	.6966	.0019	.9925	. . .	. . .	. . .
159	5	100	.2	.1973	-.0013	.9979	1.512	.200	.1660*
		100	.4	.3977	-.0011	.9984	1.961	.100	.0805*
		100	.5	.4991	.0006	1.0004	2.399	.050	.0375*
		100	.6	.5965	.0020	.9942	3.379	.010	.0065
		100	.8	.7985	.0002	.9944	. . .	. . .	. . .
160	5	100	.1	.1007	.0011	.9973	1.512	.200	.1615*
		100	.3	.2968	-.0018	.9974	1.961	.100	.0870
		100	.5	.4957	-.0013	.9963	2.399	.050	.0425
		100	.7	.6966	-.0005	.9955	3.379	.010	.0095
		100	.9	.8993	.0005	.9929	. . .	. . .	. . .
161	5	200	.5	.4999	.0004	.9884	1.500	.200	.1990
		200	.5	.4985	-.0016	.9979	1.940	.100	.1020
		200	.5	.4984	.0002	.9987	2.370	.050	.0435
		200	.5	.5006	-.0006	.9970	3.320	.010	.0075
		200	.5	.4985	-.0009	.9993	. . .	. . .	. . .

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
162	5	200	.4	.3975	.0011	.9978	1.500	.200	.2030
		200	.4	.3969	-.0007	.9956	1.940	.100	.0990
		200	.5	.4995	.0012	.9975	2.370	.050	.0455
		200	.6	.5993	.0010	.9984	3.320	.010	.0085
		200	.6	. . .	.0005	.9984	. . .	. . .	. . .
163	5	200	.3	.2983	.0030	.9954	1.500	.200	.1755*
		200	.4	.3985	.0002	.9971	1.940	.100	.0835*
		200	.5	.4998	.0001	.9995	2.370	.050	.0460
		200	.6	.5988	-.0014	.9998	3.320	.010	.0090
		200	.7	.6998	.0010	.9999	. . .	. . .	. . .
164	5	200	.2	.2022	.0006	.9978	1.500	.200	.1595*
		200	.4	.3984	.0002	.9977	1.940	.100	.0810*
		200	.5	.5022	-.0015	.9995	2.370	.050	.0435
		200	.6	.5991	.0011	.9984	3.320	.010	.0100
		200	.8	.8000	.0006	.9989	. . .	. . .	. . .
165	5	200	.1	.1007	-.0016	.9998	1.500	.200	.1640*
		200	.3	.3002	.0017	.9994	1.940	.100	.0805*
		200	.5	.4997	.0007	.9986	2.370	.050	.0360*
		200	.7	.6993	.0018	.9956	3.320	.010	.0065
		200	.9	.9001	.0005	.9988	. . .	. . .	. . .
166	2	30	.5	.4925	-.0026	.9911	1.675	.200	.2025
		40	.5	.4944	-.0025	.9873	2.782	.100	.0965
		..	..	. . .	. . .	. . .	3.984	.050	.0445
		..	..	. . .	. . .	. . .	7.029	.010	.0105
167	2	30	.6	.5989	-.0004	.9939	1.675	.200	.1810*
		40	.4	.3928	-.0019	.9948	2.782	.100	.0805*
		..	..	. . .	. . .	. . .	3.984	.050	.0325*
		..	..	. . .	. . .	. . .	7.029	.010	.0075
168	2	30	.7	.6993	-.0035	.9868	1.675	.200	.1570*
		40	.3	.2982	-.0082	.9946	2.782	.100	.0735*
		..	..	. . .	. . .	. . .	3.984	.050	.0335*
		..	..	. . .	. . .	. . .	7.029	.010	.0035*
169	2	30	.8	.7977	-.0040	.9915	1.675	.200	.1500*
		40	.2	.1958	-.0006	.9905	2.782	.100	.0625*
		..	..	. . .	. . .	. . .	3.984	.050	.0300*
		..	..	. . .	. . .	. . .	7.029	.010	.0040*

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
170	2	30	.9	.9012	.0007	.9916	1.675	.200	.1235*
		40	.1	.1029	-.0037	.9955	2.782	.100	.0450*
		..	..	. . .	. . .	. . .	3.984	.050	.0190*
		..	..	. . .	. . .	. . .	7.029	.010	.0015*
171	2	20	.5	.4955	-.0041	.9804	1.681	.200	.1990
		40	.5	.4900	.0028	.9898	2.796	.100	.1000
		..	..	. . .	. . .	. . .	4.010	.050	.0525
		..	..	. . .	. . .	. . .	7.102	.010	.0075
172	2	20	.6	.5947	-.0020	.9904	1.681	.200	.1790*
		40	.4	.3995	-.0056	.9965	2.796	.100	.0750*
		..	..	. . .	. . .	. . .	4.010	.050	.0305*
		..	..	. . .	. . .	. . .	7.102	.010	.0075
173	2	20	.7	.6984	.0043	.9839	1.681	.200	.1670*
		40	.3	.2962	.0021	.9895	2.796	.100	.0710*
		..	..	. . .	. . .	. . .	4.010	.050	.0350*
		..	..	. . .	. . .	. . .	7.102	.010	.0075
174	2	20	.8	.7984	.0003	.9852	1.681	.200	.1190*
		40	.2	.1967	.0032	.9938	2.796	.100	.0475*
		..	..	. . .	. . .	. . .	4.010	.050	.0160*
		..	..	. . .	. . .	. . .	7.102	.010	.0005*
175	2	20	.9	.9025	.0002	.9923	1.681	.200	.0705*
		40	.1	.0975	.0050	.9941	2.796	.100	.0255*
		..	..	. . .	. . .	. . .	4.010	.050	.0075*
		..	..	. . .	. . .	. . .	7.102	.010	.0000*
176	3	20	.5	.4922	-.0038	.9839	1.644	.200	.1960
		20	.5	.4934	-.0053	.9851	2.374	.100	.1010
		40	.5	.4926	-.0030	.9898	3.117	.050	.0510
		..	..	. . .	. . .	. . .	4.896	.010	.0105
177	3	20	.6	.6045	.0044	.9900	1.644	.200	.1880
		20	.5	.5018	.0006	.9912	2.374	.100	.0930
		40	.4	.3969	-.0059	.9967	3.117	.050	.0425
		..	..	. . .	. . .	. . .	4.896	.010	.0075
178	3	20	.7	.6962	-.0020	.9878	1.644	.200	.1710*
		20	.5	.4906	.0014	.9817	2.374	.100	.0750*
		40	.3	.2971	-.0011	.9914	3.117	.050	.0385
		..	..	. . .	. . .	. . .	4.896	.010	.0080

TABLE VII--Continued

Simulation Number	k	n_j	β_j	B_j	M_j	SD_j	F	Nominal α	Actual α'
179	3	20	.8	.8012	.0009	.9925	1.644	.200	.1275*
		20	.5	.4940	-.0061	.9861	2.374	.100	.0600*
		40	.2	.1960	.0010	.9947	3.117	.050	.0220*
		..	..	. . .	. . .	. . .	4.896	.010	.0050*
180	3	20	.9	.9009	-.0006	.9883	1.644	.200	.1145*
		20	.5	.4962	.0004	.9795	2.374	.100	.0455*
		40	.1	.1008	.0031	.9920	3.117	.050	.0200*
		..	..	. . .	. . .	. . .	4.896	.010	.0030*
181	3	20	-.4	-.3894	-.0027	.9790	1.657	.200	.1635*
		20	0	.0028	.0029	.9885	2.400	.100	.0755*
		20	.4	.3942	-.0023	.9839	3.162	.050	.0305*
		..	..	. . .	. . .	. . .	5.006	.010	.0035*
182	3	20	.1	.0909	.0046	.9871	1.657	.200	.1420*
		20	.1	.0899	-.0019	.9891	2.400	.100	.0630*
		20	.9	.9010	-.0012	.9861	3.162	.050	.0345*
		..	..	. . .	. . .	. . .	5.006	.010	.0070
183	3	20	.1	.0958	-.0095	.9797	1.657	.200	.1720*
		20	.2	.1926	.0110	.9860	2.400	.100	.0820*
		20	.7	.6991	-.0007	.9873	3.162	.050	.0420
		..	..	. . .	. . .	. . .	5.006	.010	.0100

*Actual significance level is significantly smaller than the nominal significance level.

**Actual significance level is significantly larger than the nominal significance level.

TABLE VIII

COMPARISON OF ACTUAL SIGNIFICANCE LEVEL IN SIMULATIONS WITH NOMINAL SIGNIFICANCE LEVEL FOR SIMULATIONS WITH LIKE REGRESSION COEFFICIENTS

Simulation Number	Sample Sizes	Regression Coefficients	Actual Significance Levels Corresponding to Nominal Significance Levels of .200	.100	.050	.010
1	20,20	.5,.5	.2040	.1045	.0465	.0120
6	20,30		.1915	.0920	.0460	.0085
11	20,40		.1990	.1000	.0525	.0075
16	30,40		.2025	.0965	.0445	.0150
21	30,30		.2020	.0940	.0410	.0115
26	40,40		.2070	.0910	.0440	.0115
136	100,100		.2000	.1045	.0480	.0095
141	200,200		.2135	.1100	.0525	.0060
2	20,20	.4,.6	.1870	.0990	.0490	.0090
7	20,30		.2160	.1065	.0565	.0115
12	20,40		.2140	.1050	.0480	.0110
17	30,40		.1995	.0940	.0445	.0090
22	30,30		.2000	.1040	.0530	.0085
27	40,40		.2075	.1055	.0445	.0100
137	100,100		.1945	.0920	.0440	.0105
142	200,200		.1865	.0965	.0430	.0090
3	20,20	.3,.7	.1895	.0895	.0490	.0105
8	20,30		.2255	.1195	.0595	.0140
13	20,40		.2465	.1270	.0735	.0235
18	30,40		.1895	.1005	.0495	.0115
23	30,40		.2050	.0990	.0450	.0065
28	40,40		.1910	.0990	.0510	.0070
138	100,100		.2025	.0945	.0455	.0095
143	200,200		.1945	.0975	.0455	.0075

TABLE VIII--Continued

Simulation Number	Sample Sizes	Regression Coefficients	Actual Significance Levels Corresponding to Nominal Significance Levels of .200	.100	.050	.010
4	20,20	.2,.8	.1620	.0735	.0365	.0050
9	20,30		.2210	.1085	.0560	.0085
14	20,40		.2500	.1350	.0785	.0160
19	30,40		.1870	.0980	.0485	.0100
24	30,30		.1855	.0850	.0375	.0080
29	40,40		.1755	.0785	.0365	.0060
139	100,100		.1780	.0885	.0395	.0075
144	200,200		.1725	.0880	.0435	.0080
5	20,20	.1,.9	.1495	.0595	.0220	.0040
10	20,30		.2240	.1190	.0630	.0110
15	20,40		.2345	.1295	.0720	.0200
20	30,40		.1880	.0920	.0420	.0070
25	30,30		.1510	.0665	.0290	.0040
30	40,40		.1600	.0705	.0300	.0040
140	100,100		.1545	.0660	.0285	.0025
145	200,200		.1390	.0585	.0260	.0040
31	20,20,20	.5,.5,.5	.2000	.0990	.0510	.0105
36	20,20,30		.1980	.1010	.0555	.0155
41	20,20,40		.1960	.1010	.0510	.0150
46	20,30,30		.1910	.0940	.0450	.0085
51	30,40,40		.1985	.1015	.0470	.0115
56	30,30,40		.2055	.1030	.0480	.0080
61	30,30,30		.2040	.1005	.0465	.0115
66	40,40,40		.2020	.1035	.0530	.0120
71	20,40,40		.2215	.1080	.0490	.0095
76	20,30,40		.1930	.0885	.0455	.0085
146	100,100,100		.2000	.0975	.0470	.0100
151	200,200,200		.2085	.0965	.0445	.0110
32	20,20,20	.4,.5,.6	.1925	.0920	.0445	.0095
37	20,20,30		.2250	.1175	.0575	.0110
42	20,20,40		.2195	.1165	.0600	.0140
47	20,30,30		.2070	.1050	.0555	.0145

TABLE VIII--Continued

Simulation Number	Sample Sizes	Regression Coefficients	Actual Significance Levels Corresponding to Nominal Significance Levels of .200	.100	.050	.010
52	30,40,40		.1965	.0965	.0520	.0145
57	30,30,40		.1955	.0975	.0550	.0125
62	30,30,30		.2110	.1060	.0520	.0090
67	40,40,40		.1890	.0890	.0420	.0065
72	20,40,40		.2020	.0975	.0455	.0070
77	20,30,40		.2070	.1050	.0540	.0140
147	100,100,100		.1885	.0995	.0485	.0095
152	200,200,200		.1805	.0860	.0445	.0110
33	20,20,20	.3,.5,.7	.1600	.0755	.0360	.0100
38	20,20,30		.1940	.0990	.0490	.0085
43	20,20,40		.2385	.1190	.0635	.0130
48	20,30,30		.2070	.1055	.0525	.0110
53	30,40,40		.2005	.1020	.0495	.0075
58	30,30,40		.2080	.1075	.0530	.0110
63	30,30,30		.2040	.1080	.0505	.0065
68	40,40,40		.1895	.0885	.0400	.0115
73	20,40,40		.2120	.1135	.0620	.0125
78	20,30,40		.2235	.1145	.0700	.0165
148	100,100,100		.1800	.0880	.0390	.0045
153	200,200,200		.1820	.0945	.0515	.0085
34	20,20,20	.2,.5,.8	.1670	.0845	.0385	.0080
39	20,20,30		.2210	.1150	.0635	.0145
44	20,20,40		.2270	.1150	.0635	.0165
49	20,30,30		.1805	.0895	.0495	.0090
54	30,40,40		.1965	.0950	.0455	.0090
59	30,30,40		.1965	.1080	.0560	.0085
64	30,30,30		.1705	.0865	.0405	.0085
69	40,40,40		.1625	.0830	.0400	.0085
74	20,40,40		.2055	.1120	.0605	.0150
79	20,30,40		.2210	.1210	.0595	.0155
149	100,100,100		.1820	.0855	.0455	.0105
154	200,200,200		.1780	.0830	.0425	.0130

TABLE VIII--Continued

Simulation Number	Sample Sizes	Regression Coefficients	Actual Significance Levels Corresponding to Nominal Significance Levels of .200	.100	.050	.010
35	20,20,20	.1,.5,.9	.1475	.0720	.0345	.0075
40	20,20,30		.2020	.1060	.0575	.0110
45	20,20,40		.2125	.1160	.0660	.0185
50	20,30,30		.1920	.1035	.0475	.0150
55	30,40,40		.1735	.0880	.0415	.0040
60	30,30,40		.1960	.0930	.0445	.0075
65	30,30,30		.1590	.0805	.0445	.0105
70	40,40,40		.1665	.0775	.0360	.0080
75	20,40,40		.1990	.1100	.0570	.0160
80	20,30,40		.2210	.1215	.0590	.0150
150	100,100,100		.1415	.0750	.0330	.0080
155	200,200,200		.1530	.0670	.0285	.0095
81	20,20,20,20,20	.5,.5,.5,.5, .5	.2025	.1010	.0555	.0090
86	20,20,20,30,30		.2070	.1065	.0560	.0120
91	20,20,20,30,40		.1965	.1060	.0565	.0070
96	20,20,30,30,30		.1975	.0960	.0460	.0115
101	20,20,30,40,40		.2135	.1060	.0540	.0065
106	20,20,40,40,40		.1960	.0980	.0530	.0075
111	20,30,30,30,40		.1800	.0940	.0510	.0095
116	20,30,40,40,40		.2060	.1035	.0485	.0075
121	30,30,30,30,30		.1935	.0850	.0415	.0115
126	40,40,40,40,40		.2005	.1070	.0530	.0140
131	30,30,40,40,40		.1820	.0855	.0425	.0080
156	100,100,100,100,100		.2090	.1140	.0545	.0120
161	200,200,200,200,200		.1990	.1020	.0435	.0075
82	20,20,20,20,20	.4,.4,.5,.6, .6	.1795	.0950	.0500	.0115
87	20,20,20,30,30		.2165	.1210	.0635	.0165
92	20,20,20,30,40		.2230	.1175	.0590	.0145
97	20,20,30,30,30		.2015	.0965	.0470	.0130
102	20,20,30,40,40		.2185	.1135	.0615	.0135
107	20,20,40,40,40		.2130	.1160	.0595	.0130
112	20,30,30,30,40		.2240	.1130	.0565	.0130

TABLE VIII--Continued

Simulation Number	Sample Sizes	Regression Coefficients	Actual Significance Levels Corresponding to Nominal Significance Levels of .200	.100	.050	.010
117	20,30,40,40,40		.1910	.0970	.0455	.0080
122	30,30,30,30,30		.1980	.0960	.0490	.0160
127	40,40,40,40,40		.1950	.0980	.0455	.0085
132	30,30,40,40,40		.1950	.0995	.0515	.0100
157	100,100,100,100,100		.1905	.0985	.0500	.0125
162	200,200,200,200,200		.2030	.0990	.0455	.0085
83	20,20,20,20,20	.3,.4,.5,.6,.7	.1925	.0975	.0510	.0105
88	20,20,20,30,30		.2200	.1105	.0575	.0115
93	20,20,20,30,40		.2230	.1145	.0615	.0110
98	20,20,30,30,30		.2100	.1135	.0610	.0130
103	20,20,30,40,40		.2305	.1260	.0615	.0145
108	20,20,40,40,40		.2045	.1090	.0630	.0110
113	20,30,30,30,40		.2150	.1075	.0535	.0095
118	20,30,40,40,40		.2220	.1070	.0590	.0110
123	30,30,30,30,30		.1815	.0935	.0445	.0065
128	40,40,40,40,40		.1885	.0985	.0435	.0100
133	30,30,40,40,40		.1945	.0935	.0425	.0095
138	100,100,100,100,100		.1805	.0915	.0405	.0095
163	200,200,200,200,200		.1755	.0835	.0460	.0090
84	20,20,20,20,20	.2,.4,.5,.6,.8	.1730	.0965	.0455	.0085
89	20,20,20,30,30		.1980	.1125	.0620	.0125
94	20,20,20,30,40		.2225	.1210	.0605	.0135
99	20,20,30,30,30		.1910	.0955	.0440	.0115
104	20,20,30,40,40		.2285	.1215	.0695	.0150
109	20,20,40,40,40		.2335	.1240	.0645	.0135
114	20,30,30,30,40		.2095	.1030	.0530	.0135
119	20,30,40,40,40		.2060	.1085	.0595	.0145
124	30,30,30,30,30		.1780	.0855	.0440	.0095
129	40,40,40,40,40		.1745	.0945	.0480	.0095
134	30,30,40,40,40		.1955	.0970	.0490	.0110
159	100,100,100,100,100		.1660	.0805	.0375	.0065
164	200,200,200,200,200		.1595	.0810	.0435	.0100

TABLE VIII--Continued

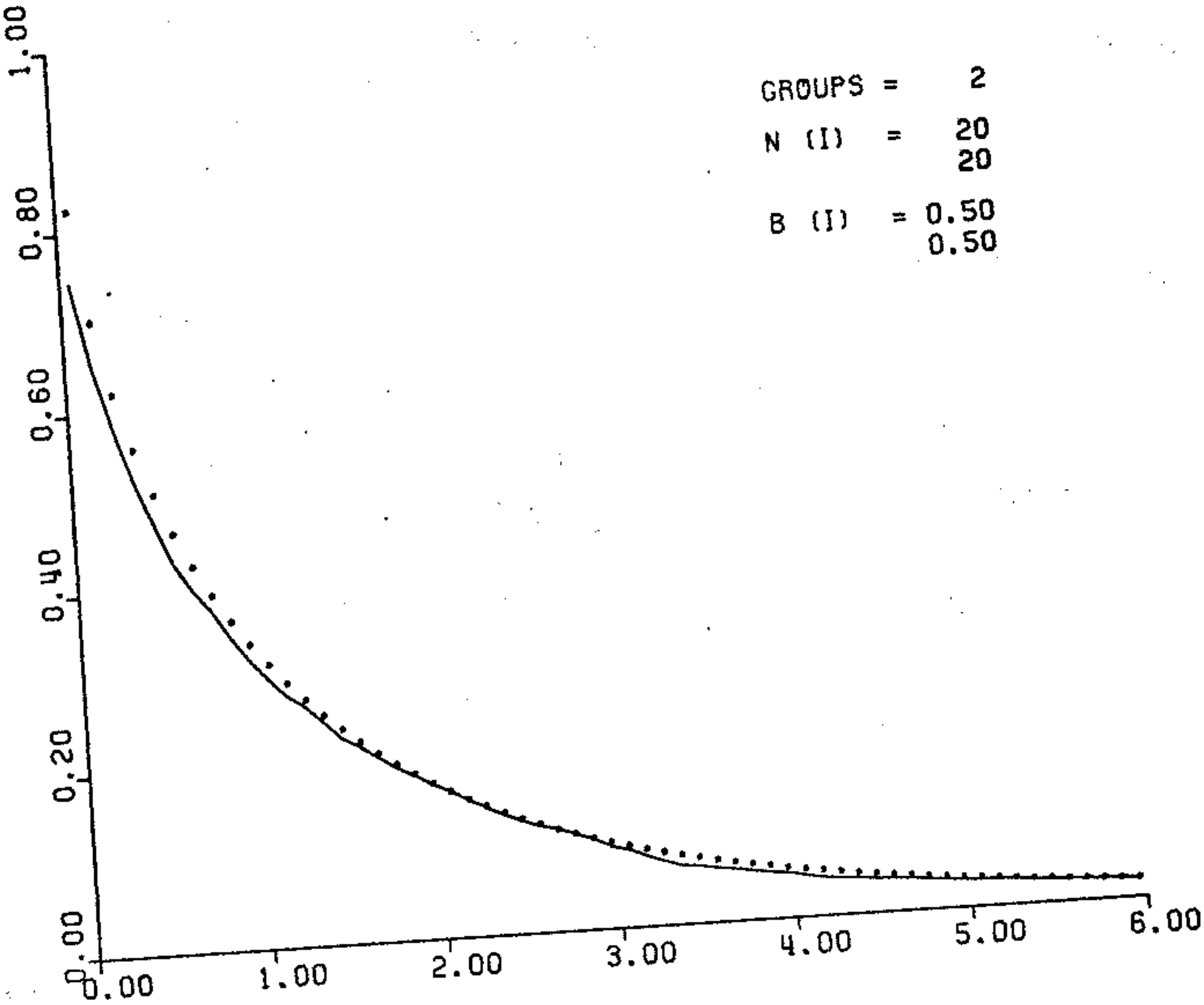
Simulation Number	Sample Sizes	Regression Coefficients	Actual Significance Levels Corresponding to Nominal Significance Levels of .200	.100	.050	.010
85	20,20,20,20,20	.1,.3,.5,.7, .9	.1685	.0815	.0445	.0100
90	20,20,20,30,30		.1885	.0985	.0505	.0110
95	20,20,20,30,40		.2145	.1105	.0620	.0140
100	20,20,30,30,30		.1945	.1075	.0570	.0130
105	20,20,30,40,40		.2165	.1175	.0685	.0185
110	20,20,40,40,40		.2090	.1095	.0605	.0215
115	20,30,30,30,40		.1995	.1115	.0615	.0120
120	20,30,40,40,40		.2080	.1120	.0525	.0130
125	30,30,30,30,30		.1595	.0755	.0395	.0090
130	40,40,40,40,40		.1615	.0780	.0390	.0070
135	30,30,40,40,40		.1685	.0830	.0375	.0065
160	100,100,100,100,100		.1615	.0870	.0425	.0095
165	200,200,200,200,200		.1595	.0755	.0395	.0090

APPENDIX C

COMPARISONS OF THE THEORETICAL AND EMPIRICAL DISTRIBUTIONS

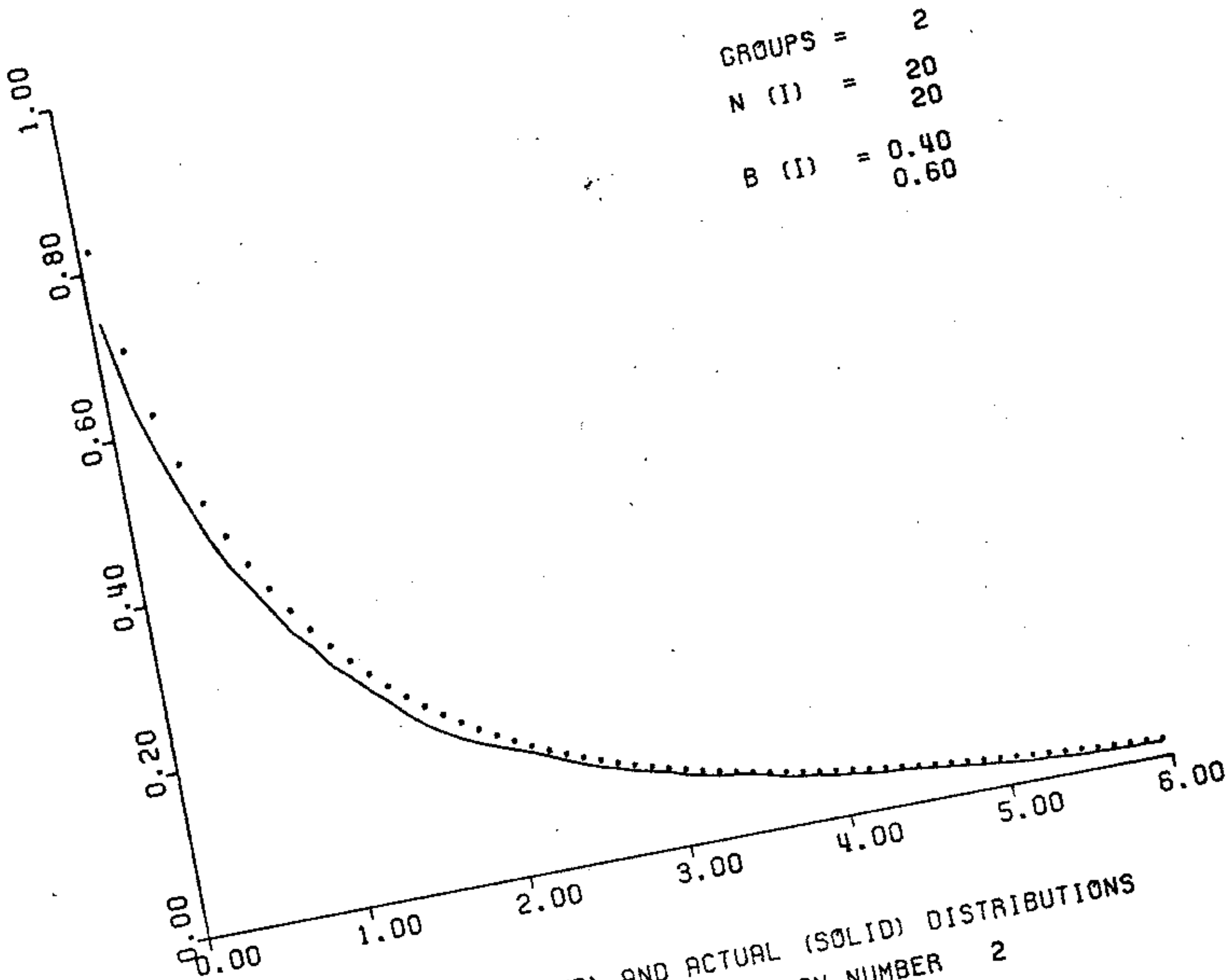
Appendix C contains the computer plotted graph for each simulation in this study. Each graph shows the relationship of the empirically derived distribution to the theoretical one. The theoretical distribution is a probability function showing the probability of obtaining an F value greater than or equal to a given F value on the horizontal axis. The vertical axis indicates the probability. The empirical or actual distribution superimposed upon the theoretical shows the proportion of F values obtained in the simulation which are greater than are equal to a value of F on the horizontal axis. These two distributions provide a graphical comparison of actual and nominal significance levels.

Data printed above each graph indicate the conditions simulated. The number of treatment groups is given along with each group sample size, N(I), and regression coefficient, B(I), paired in order of listing.



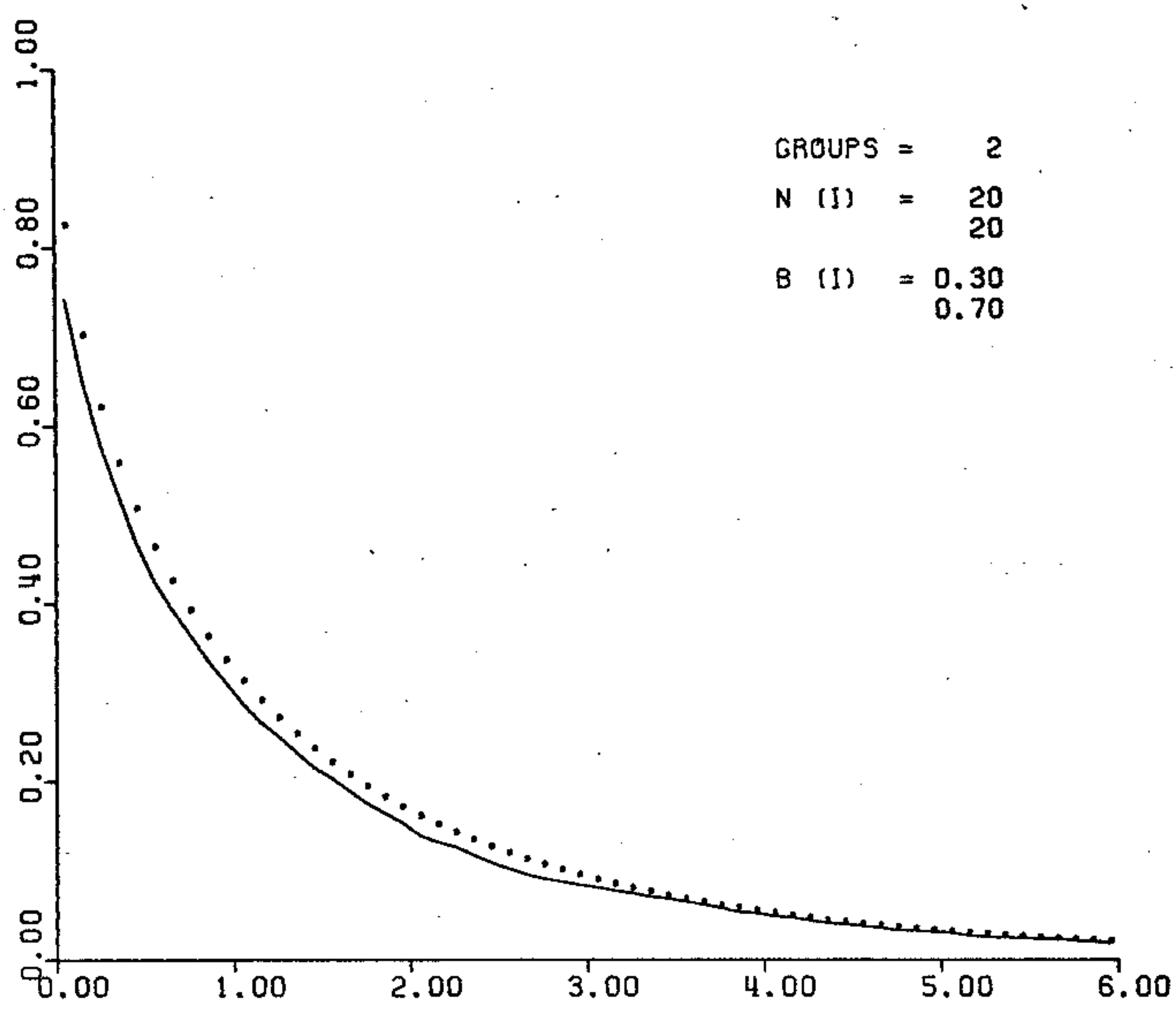
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 1 : SIMULATION NUMBER 1



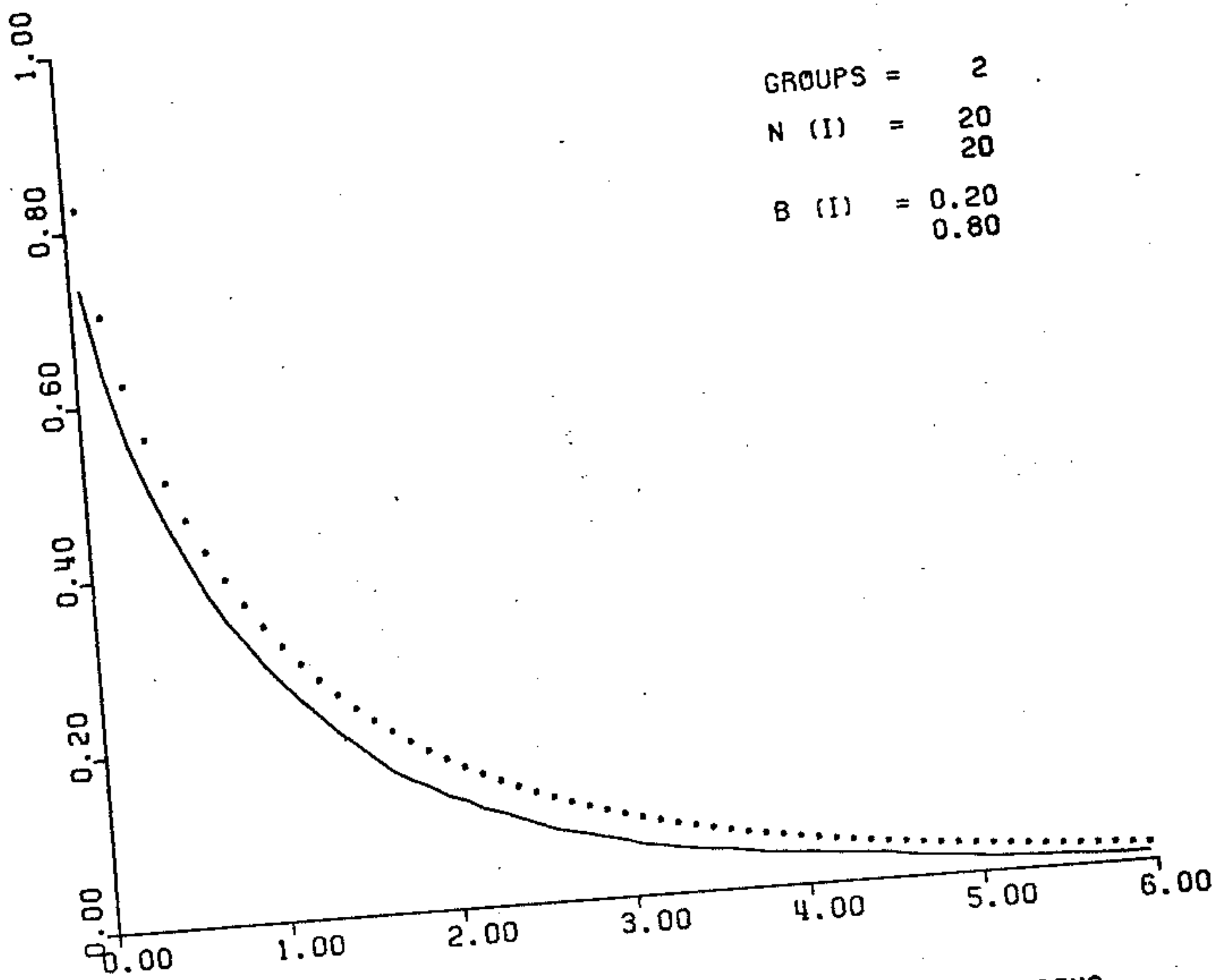
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 2 : SIMULATION NUMBER 2

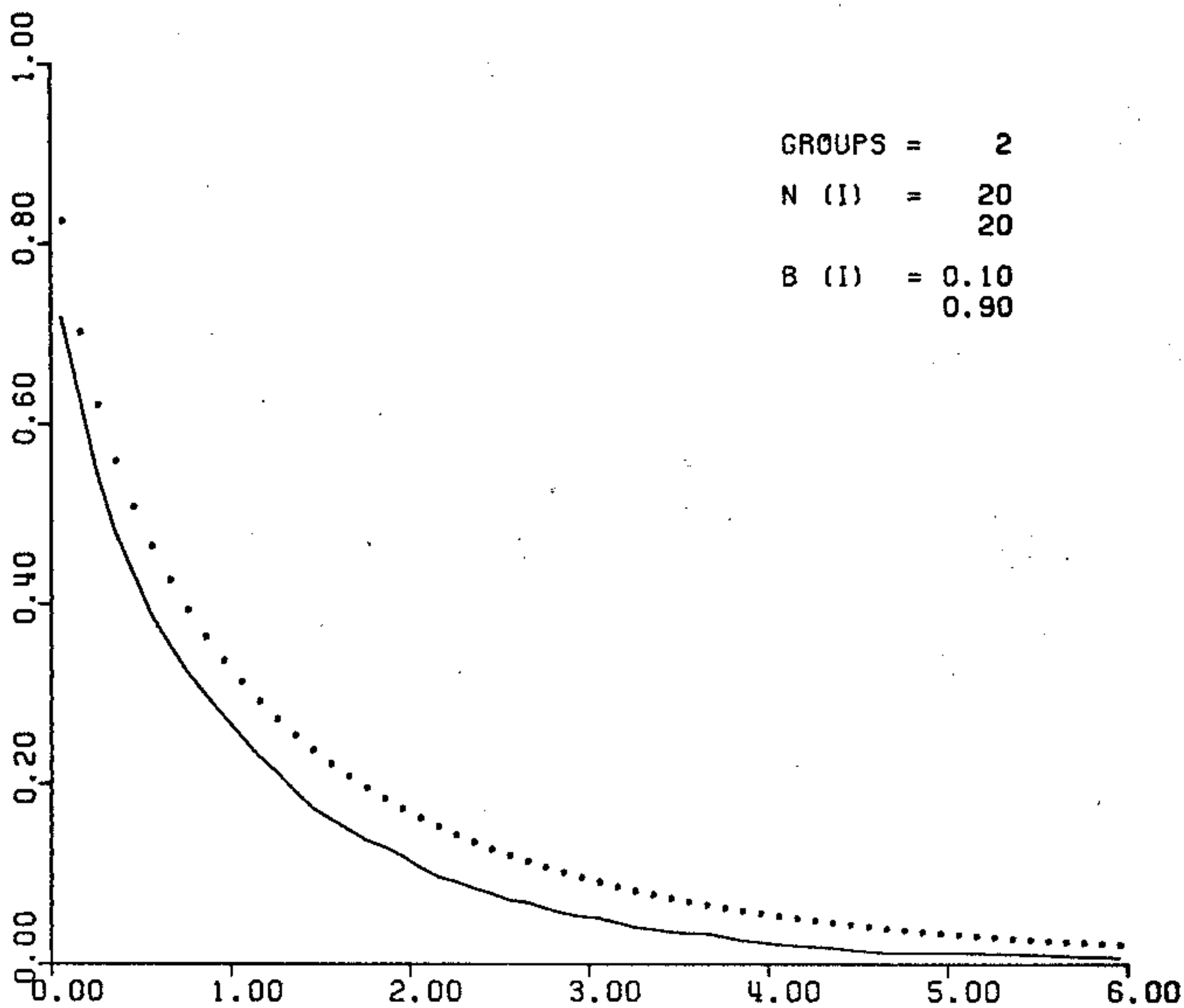


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

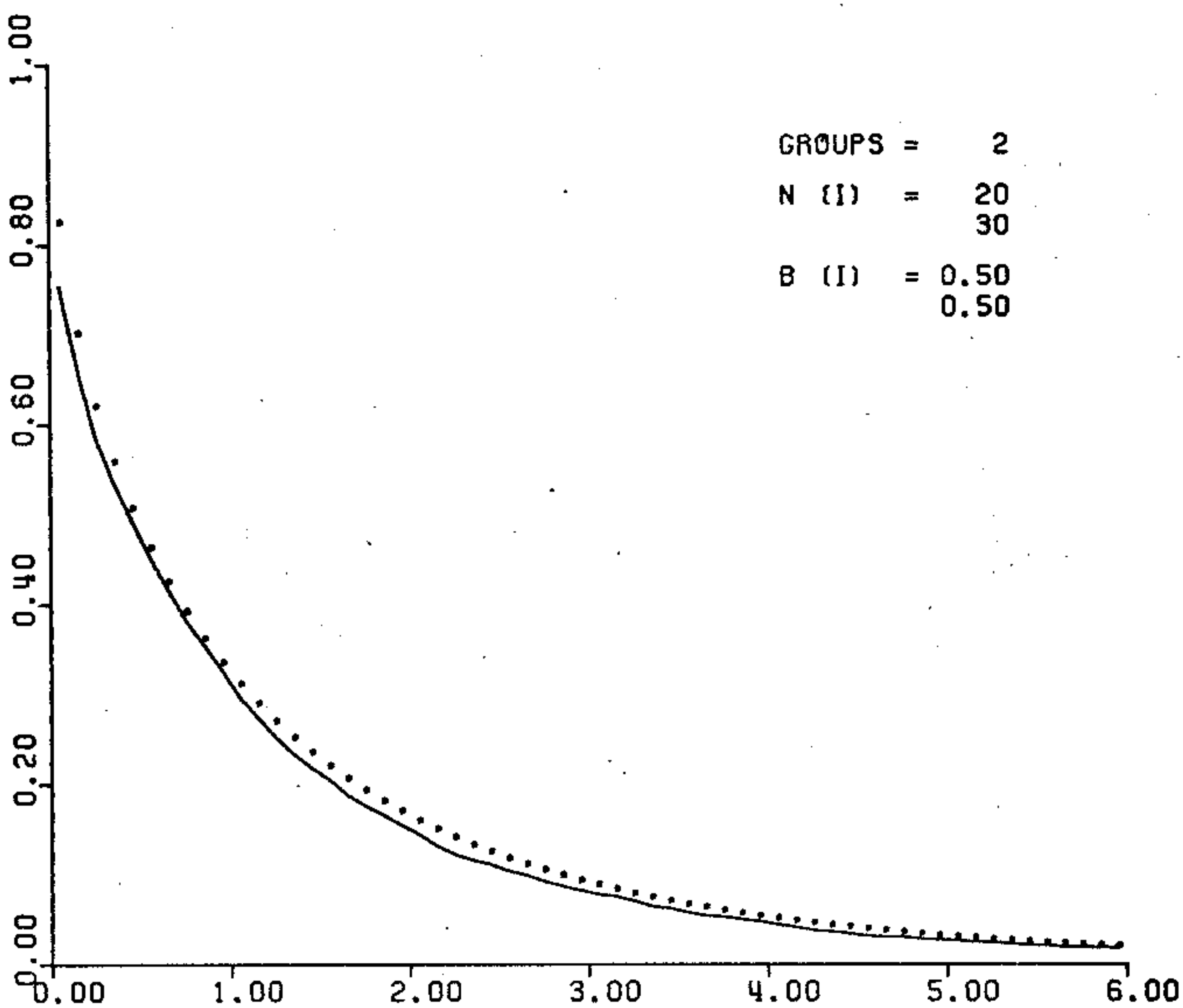
FIGURE 3 : SIMULATION NUMBER 3



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 4 : SIMULATION NUMBER 4

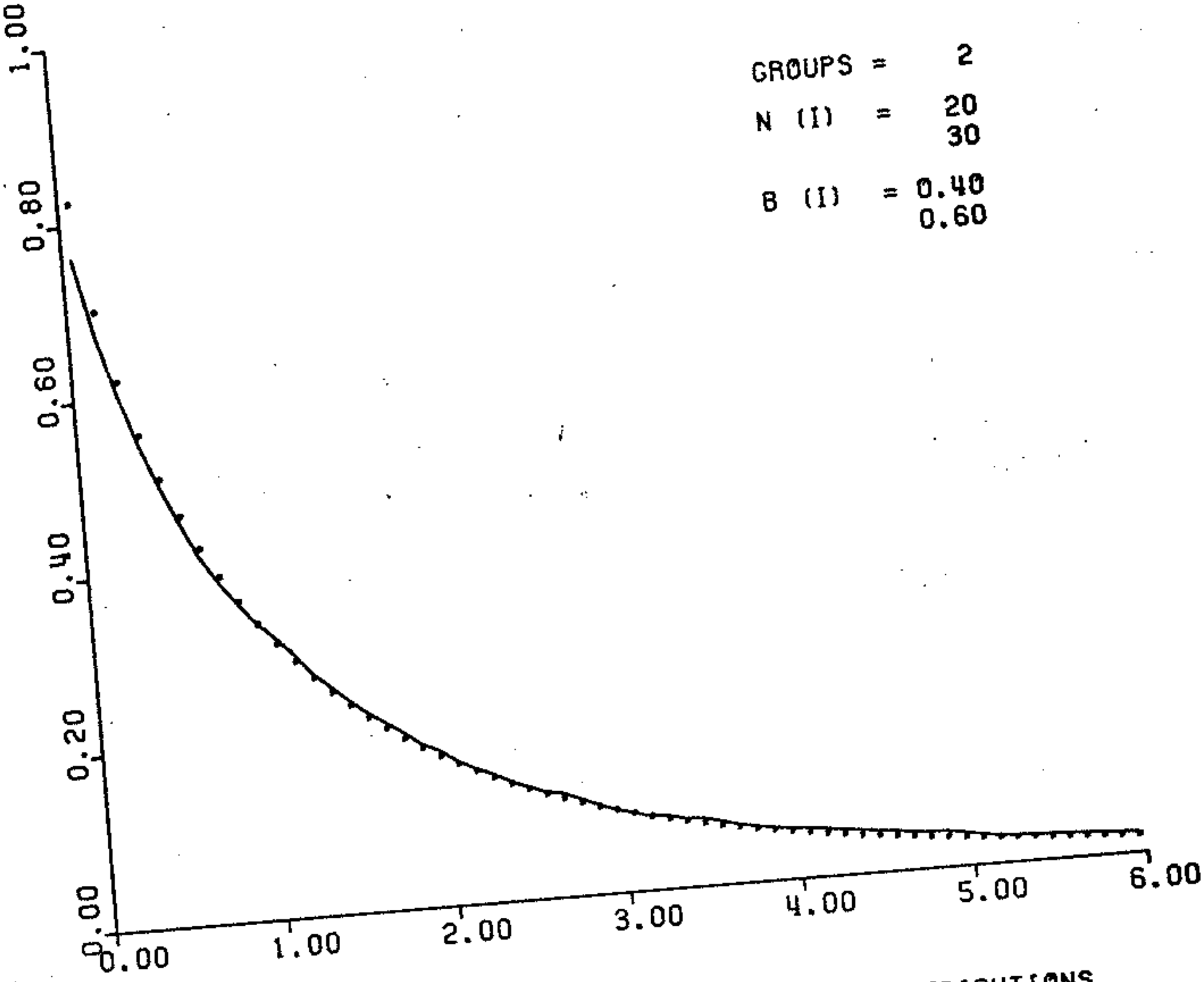


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 5 : SIMULATION NUMBER 5



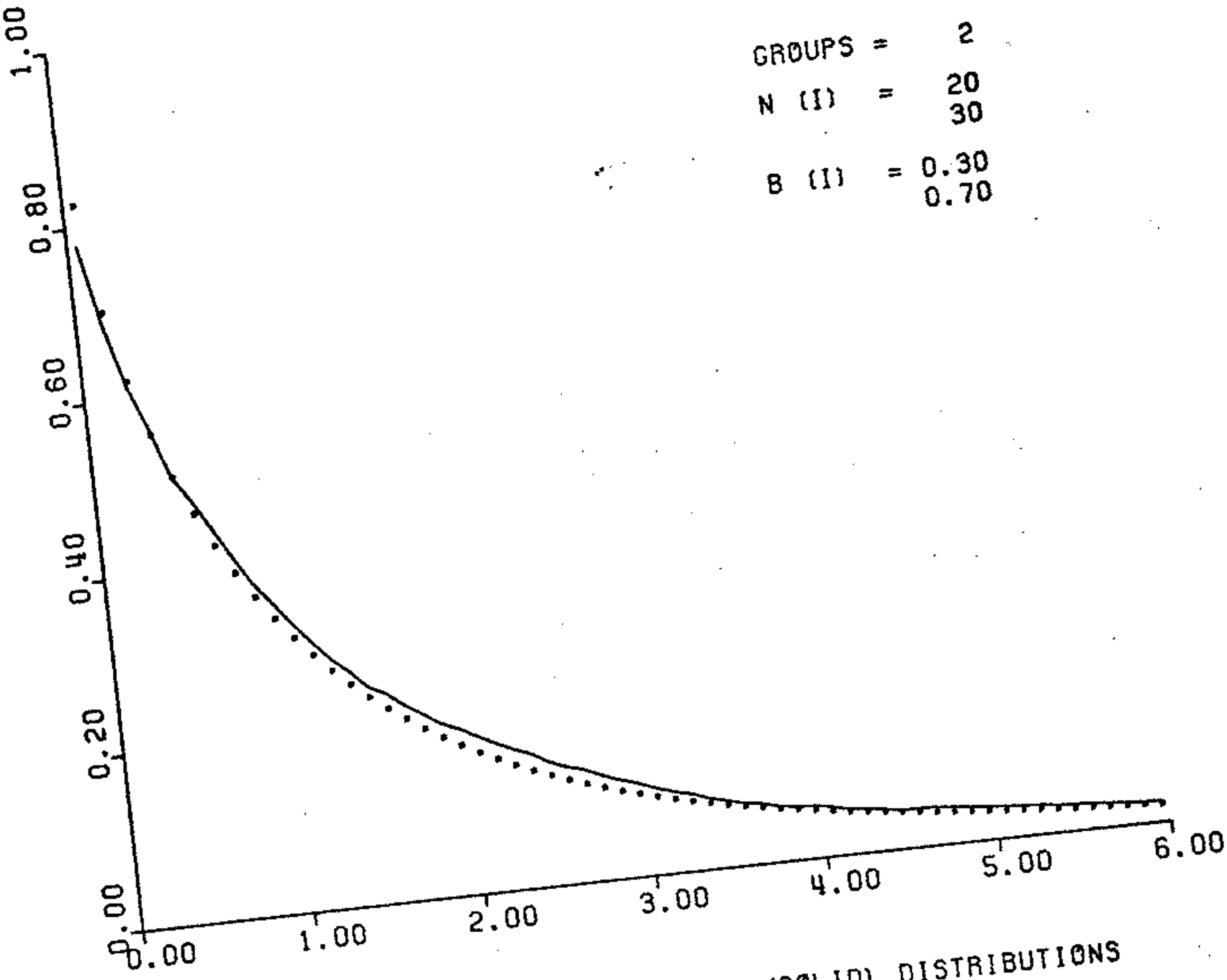
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 6 : SIMULATION NUMBER 6

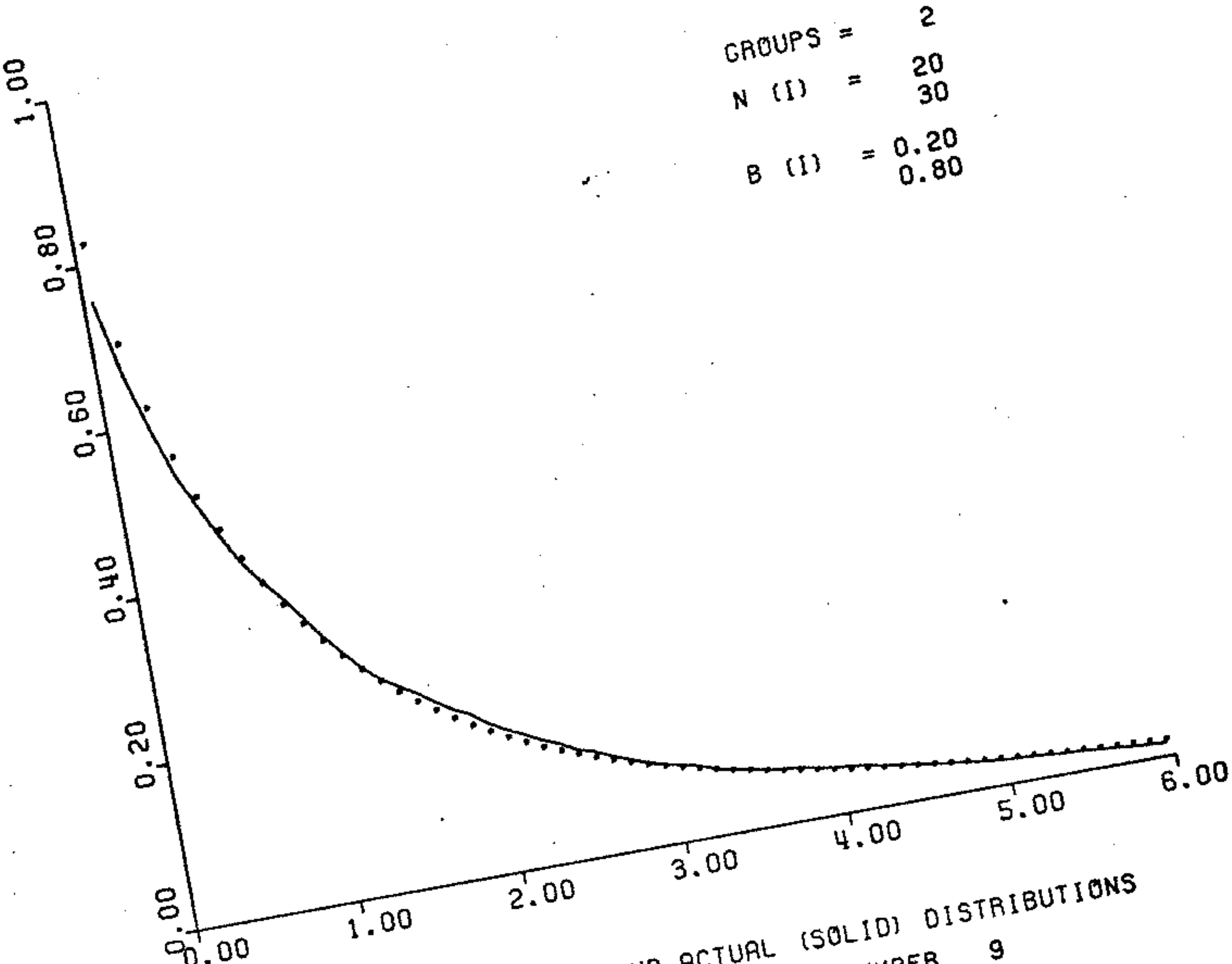


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 7 : SIMULATION NUMBER 7

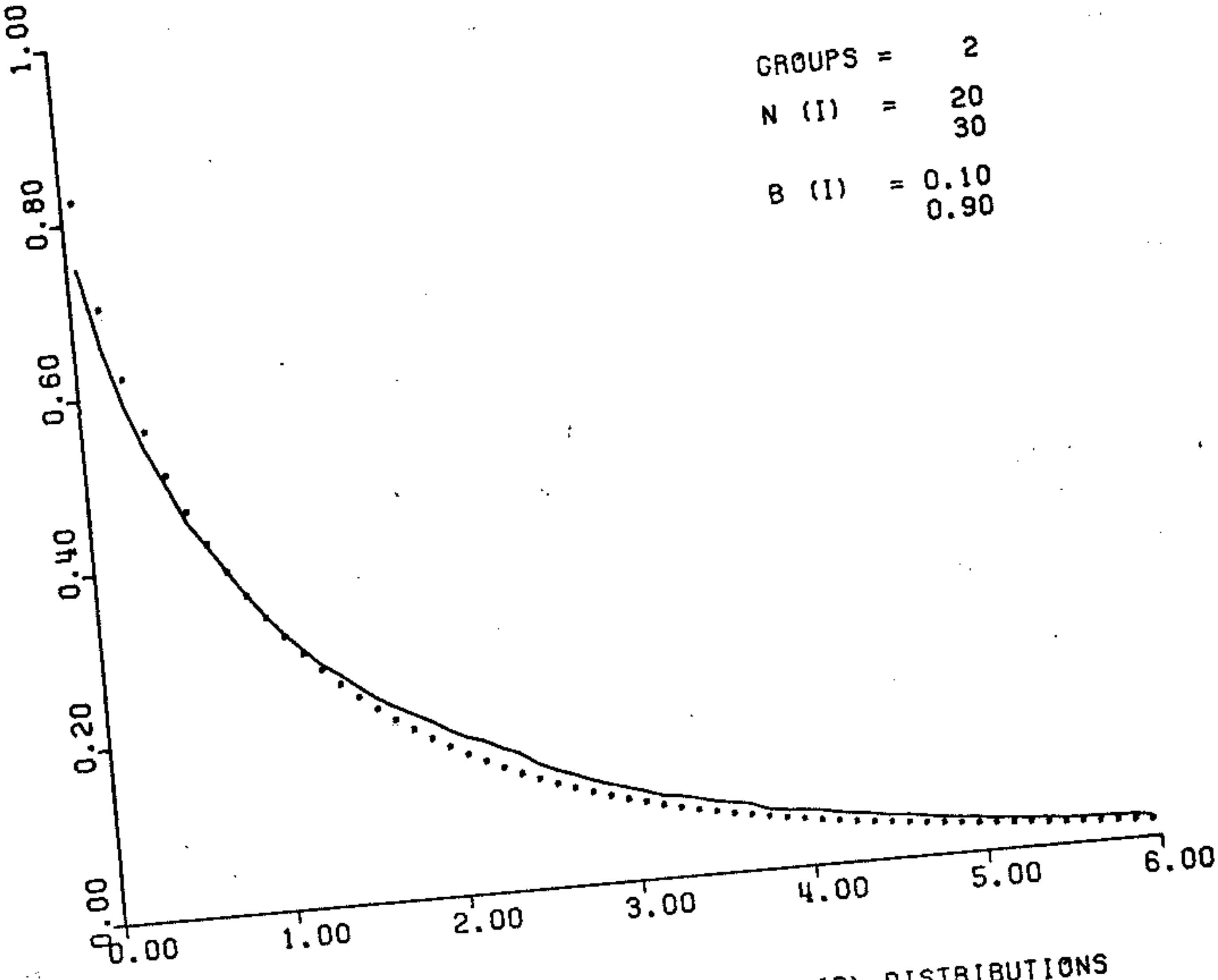


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 8 : SIMULATION NUMBER 8

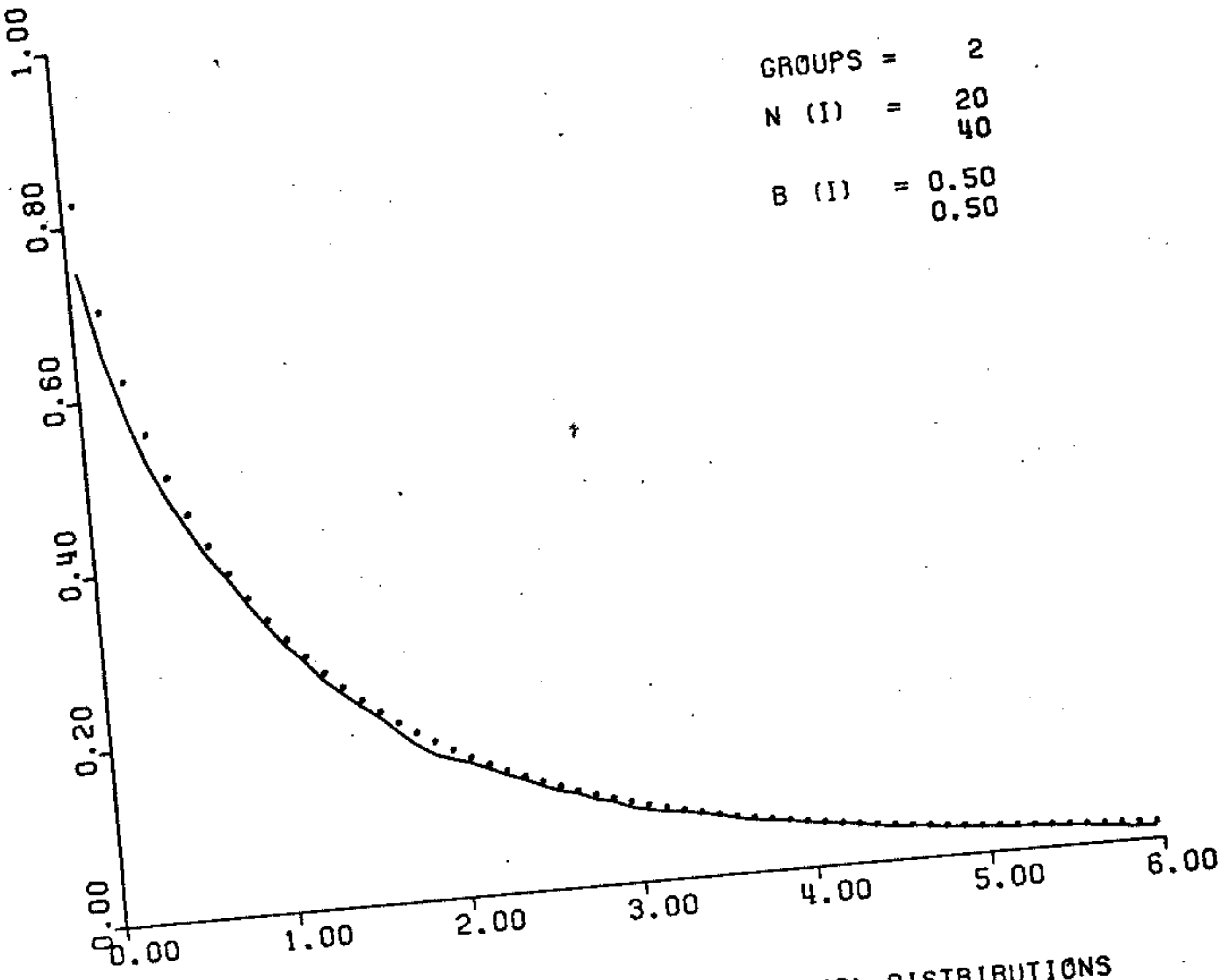


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 9 : SIMULATION NUMBER 9

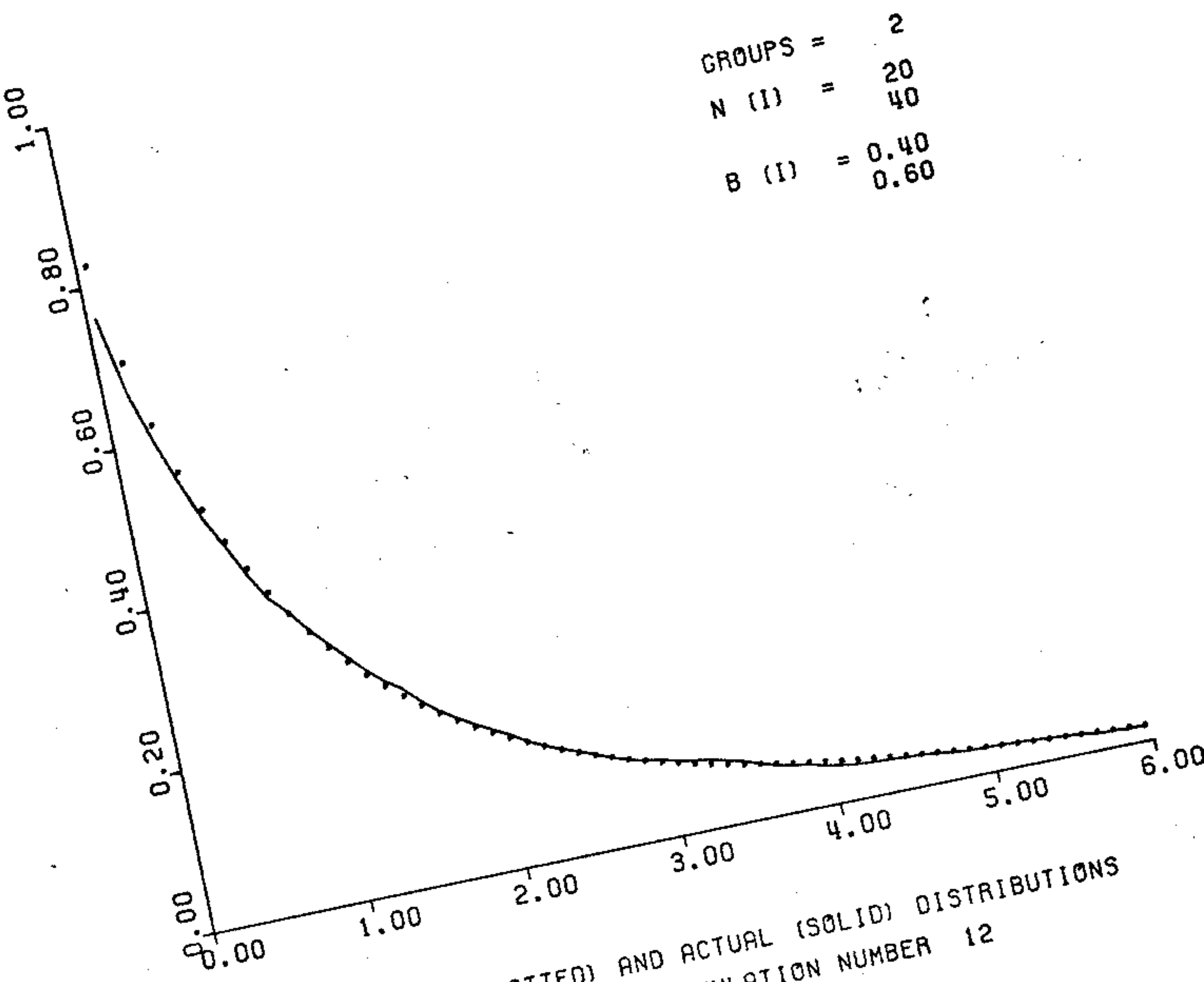


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 10 : SIMULATION NUMBER 10

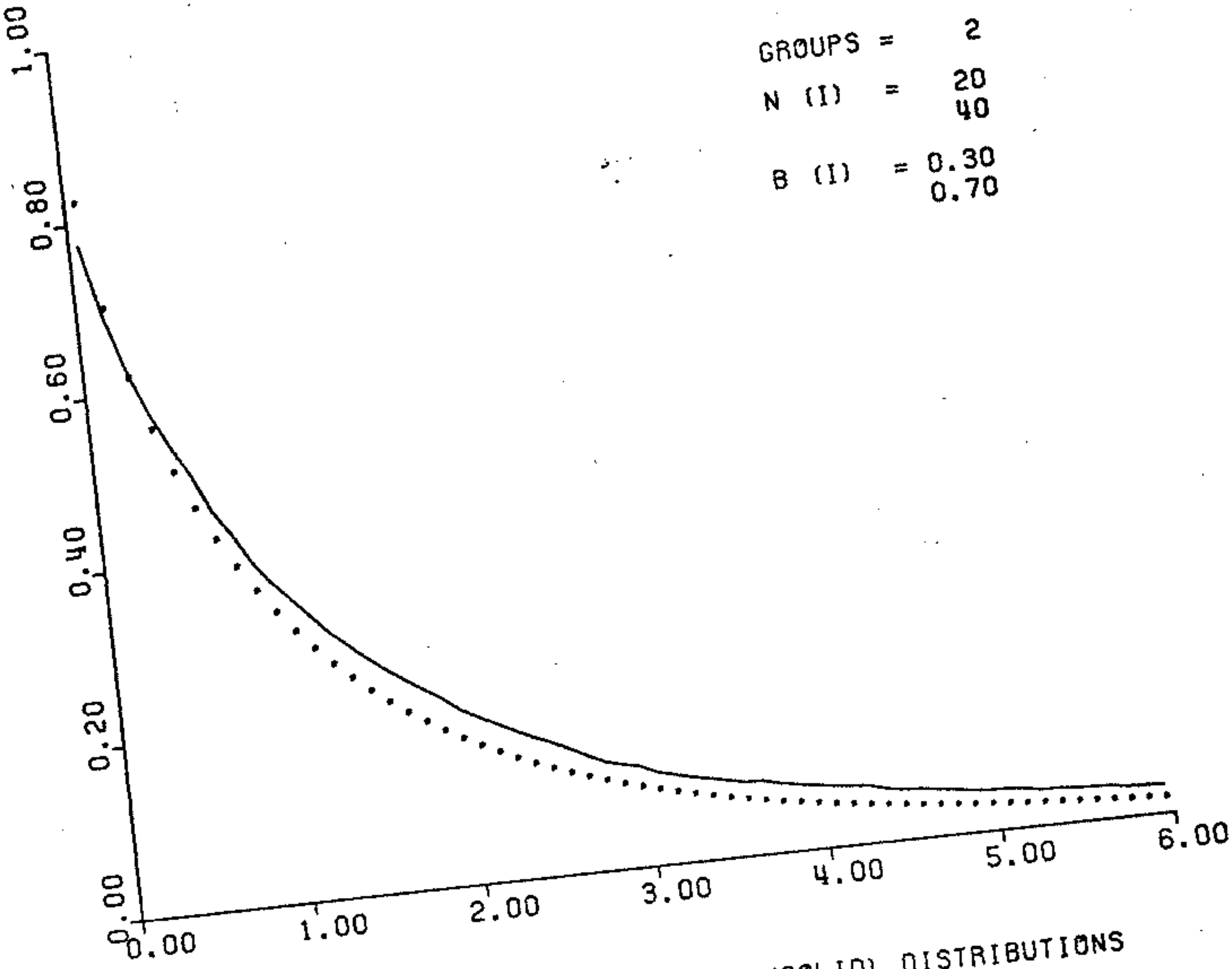


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 11 : SIMULATION NUMBER 11

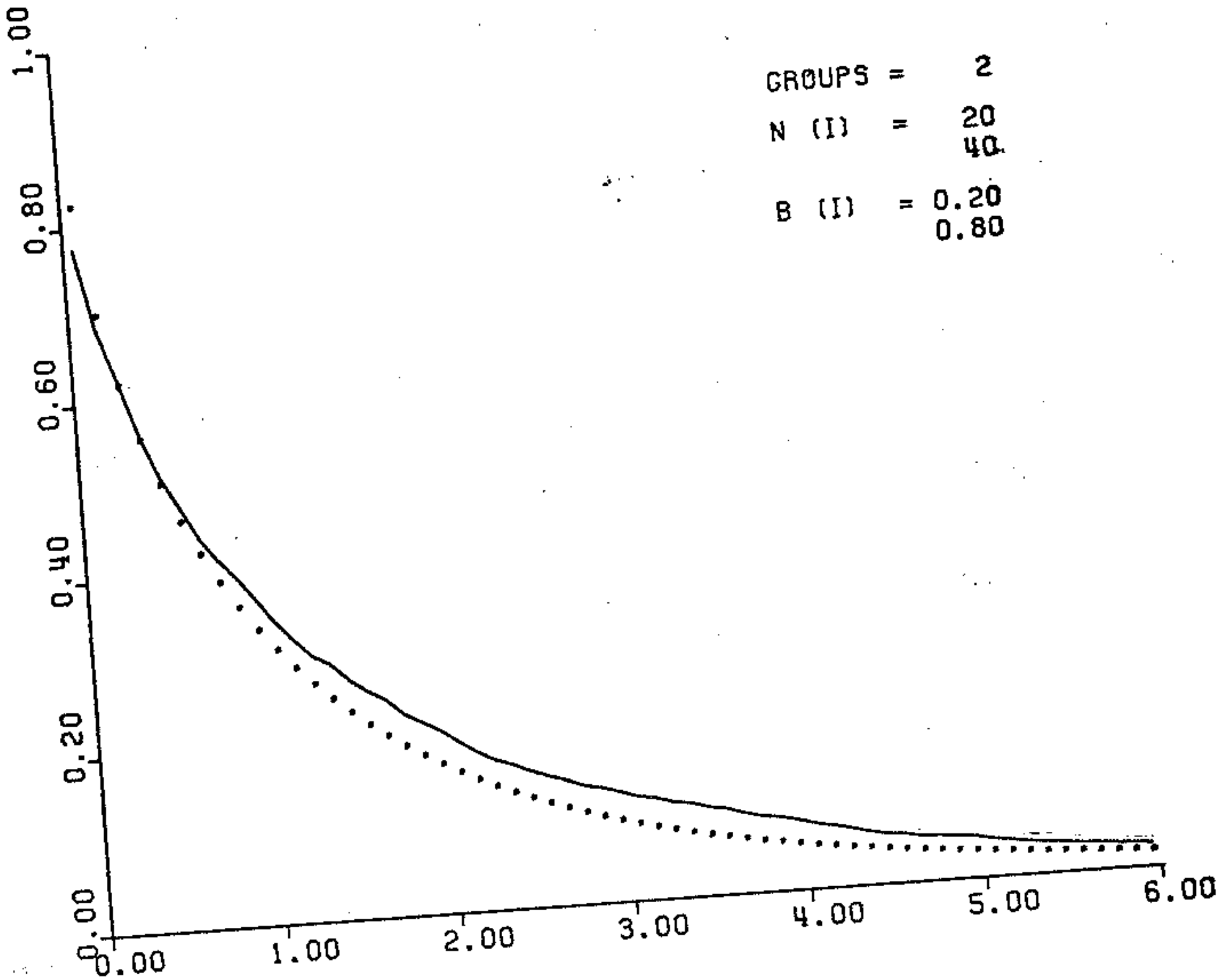


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 12 : SIMULATION NUMBER 12

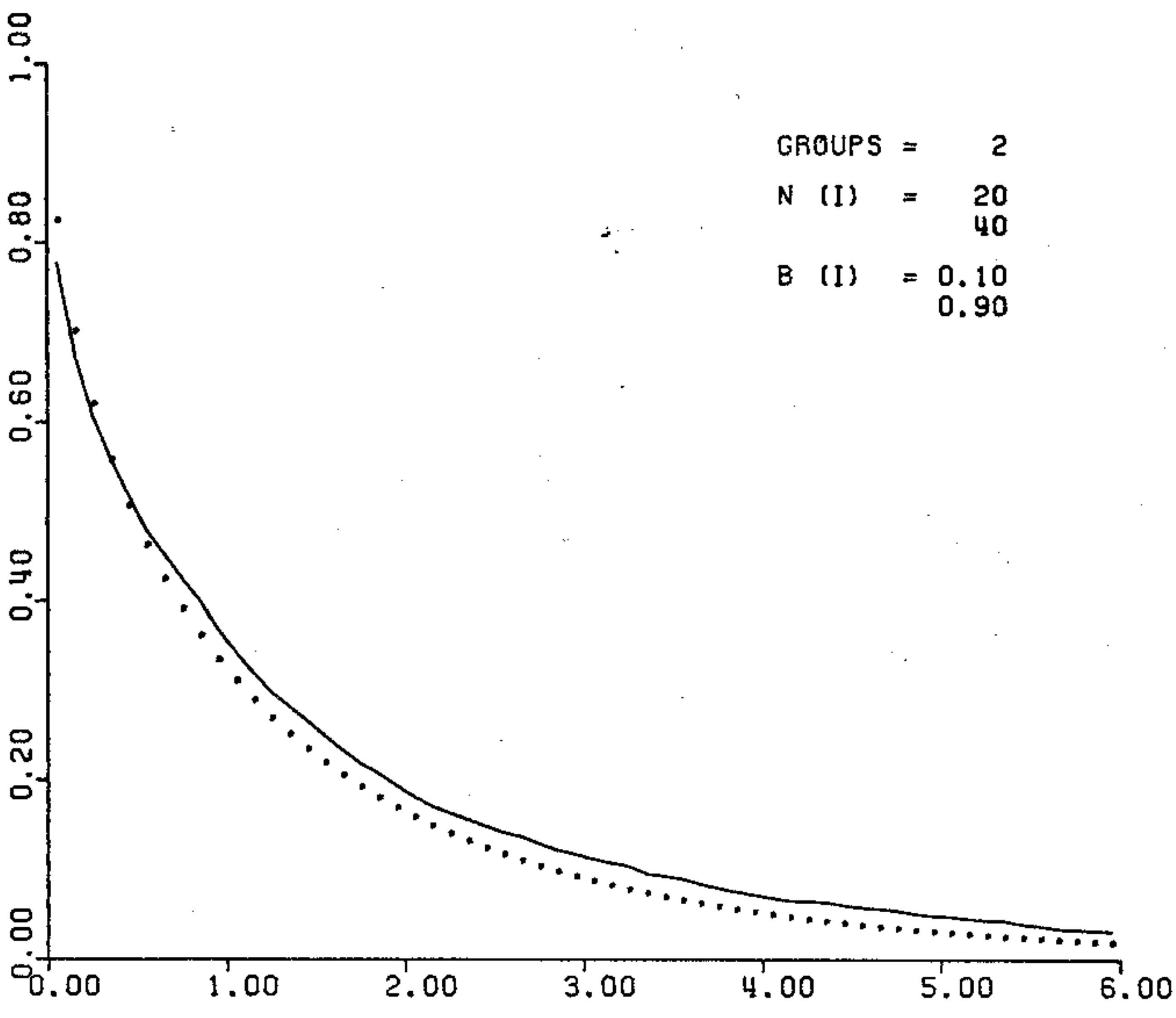


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 13 : SIMULATION NUMBER 13

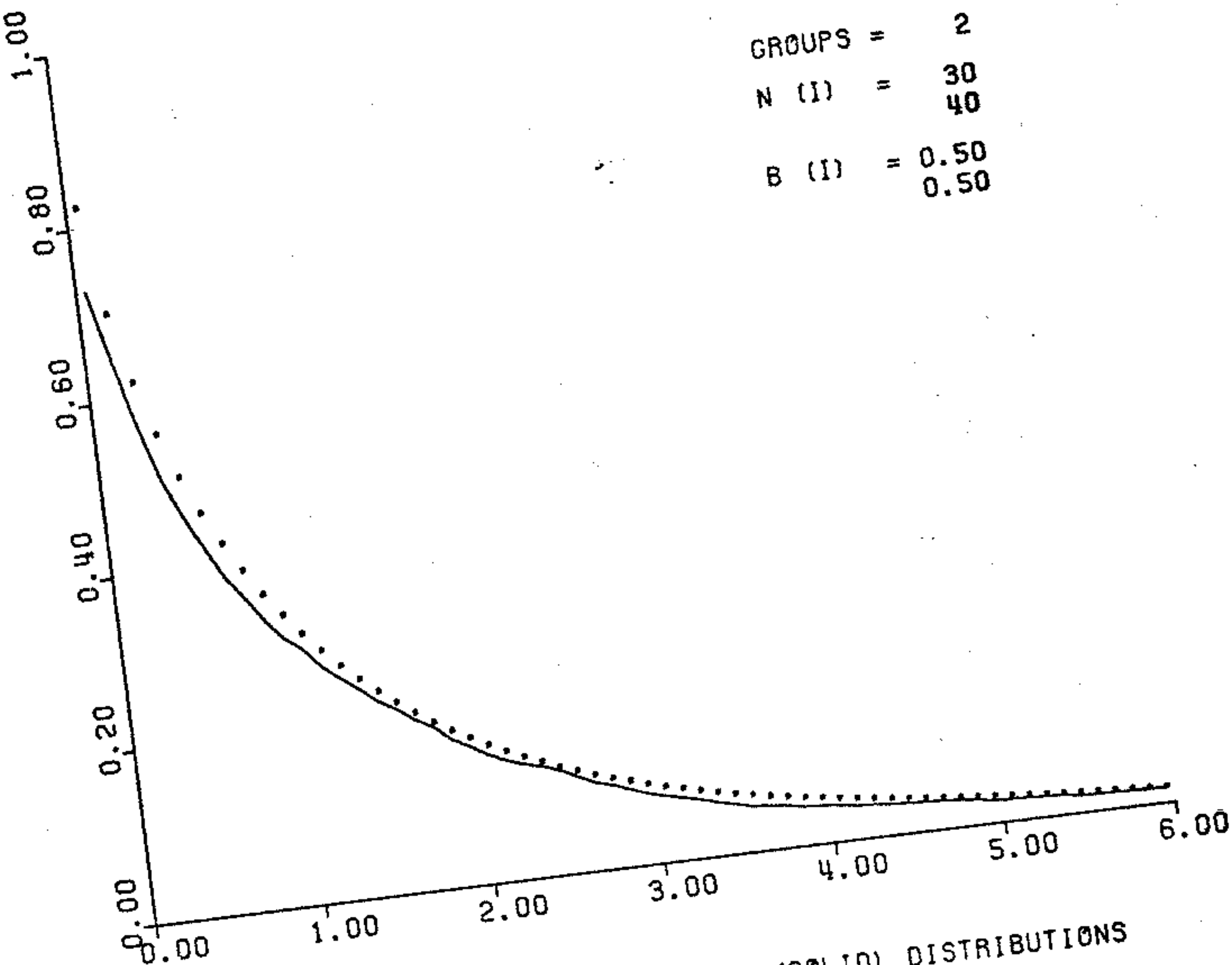


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 14 : SIMULATION NUMBER 14



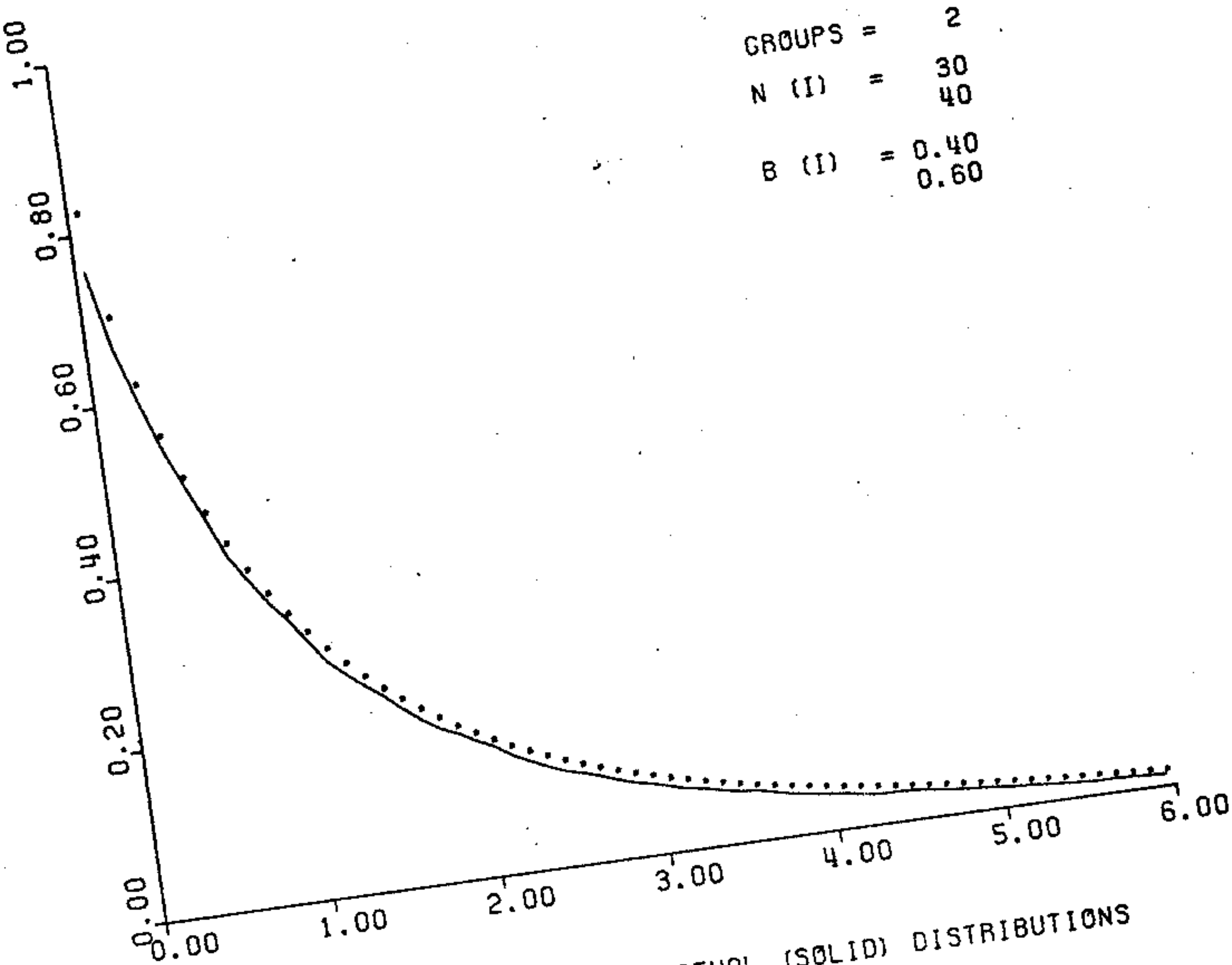
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 15 : SIMULATION NUMBER 15



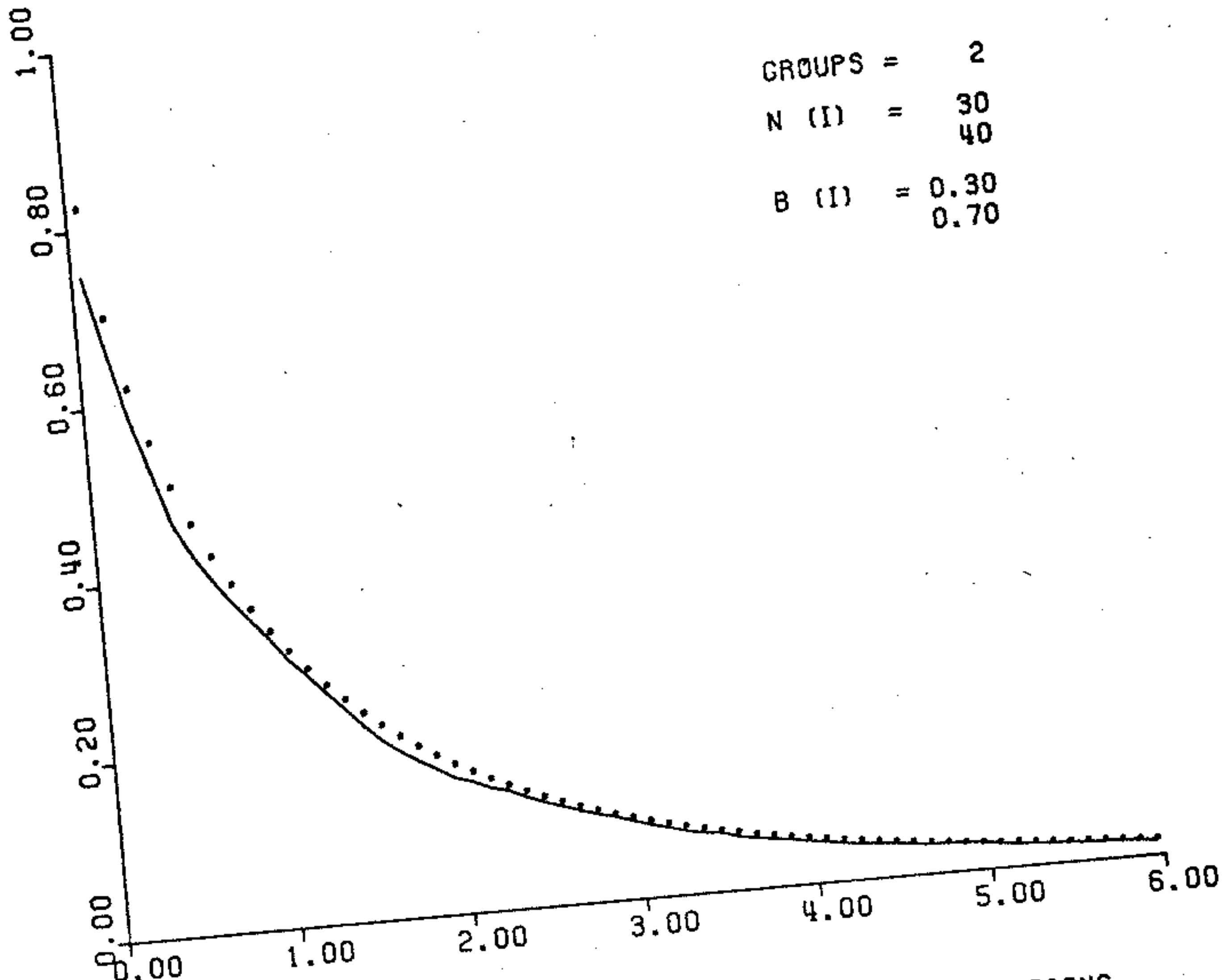
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 16 : SIMULATION NUMBER 16



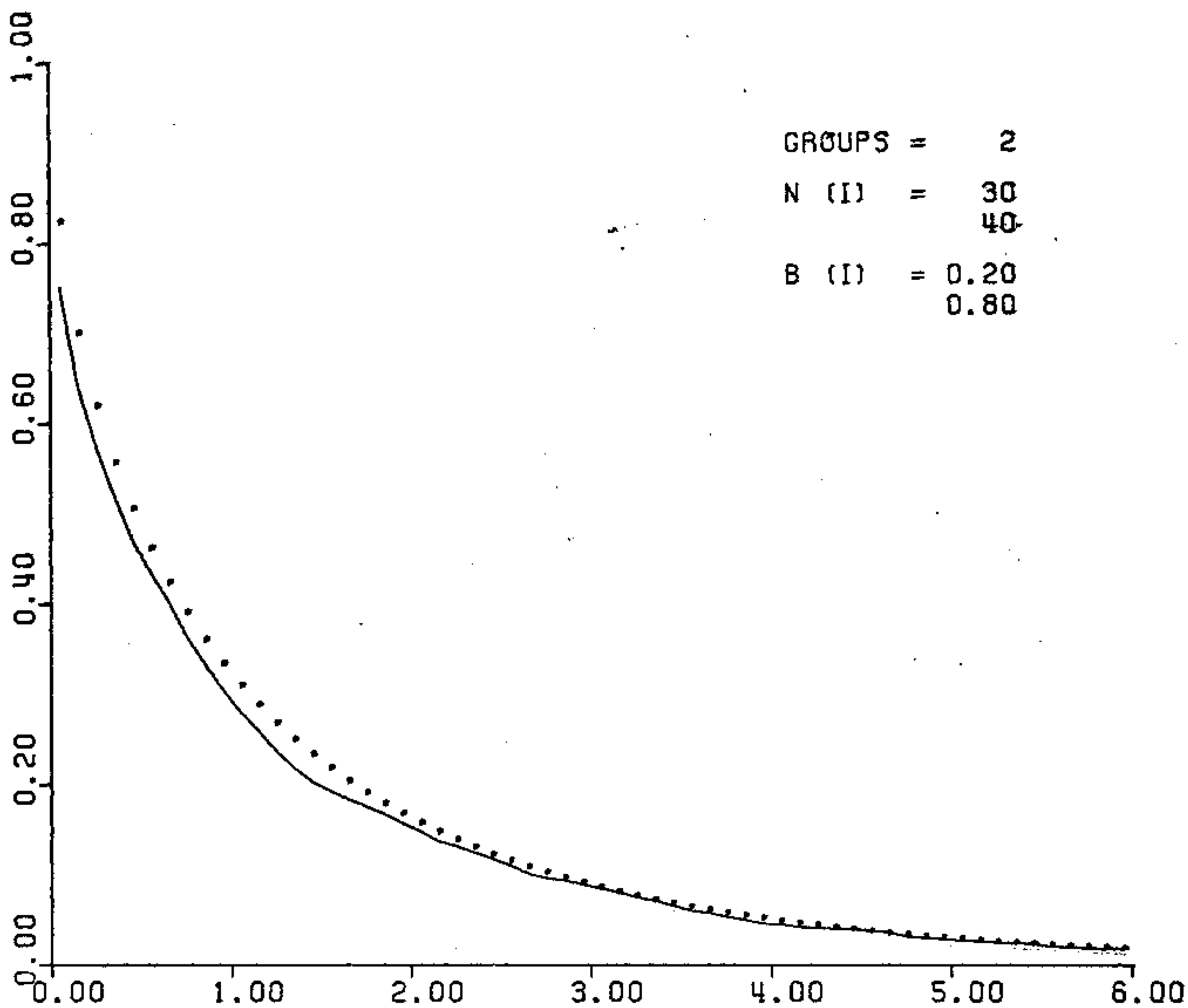
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 17 : SIMULATION NUMBER 17



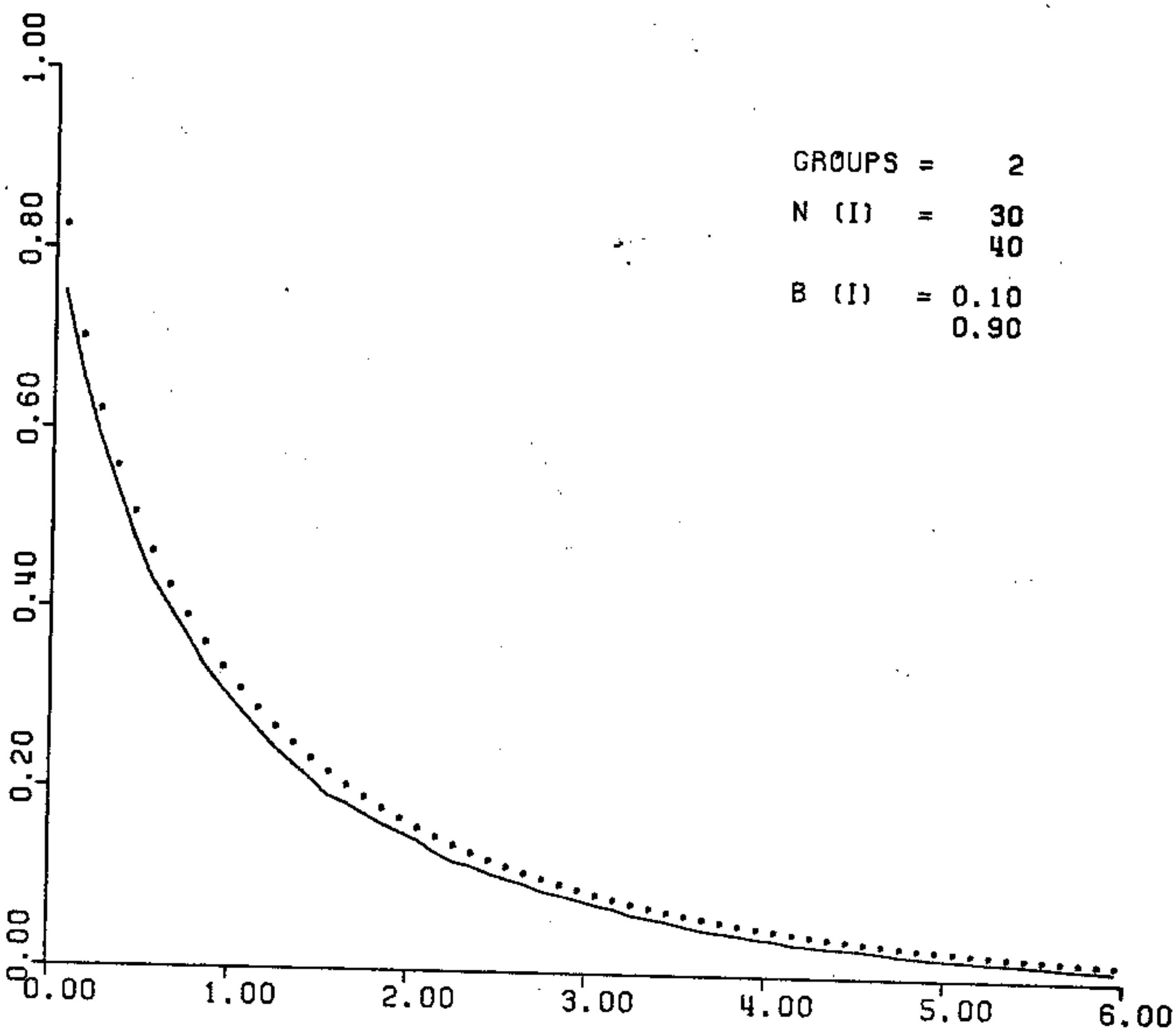
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 18 : SIMULATION NUMBER 18

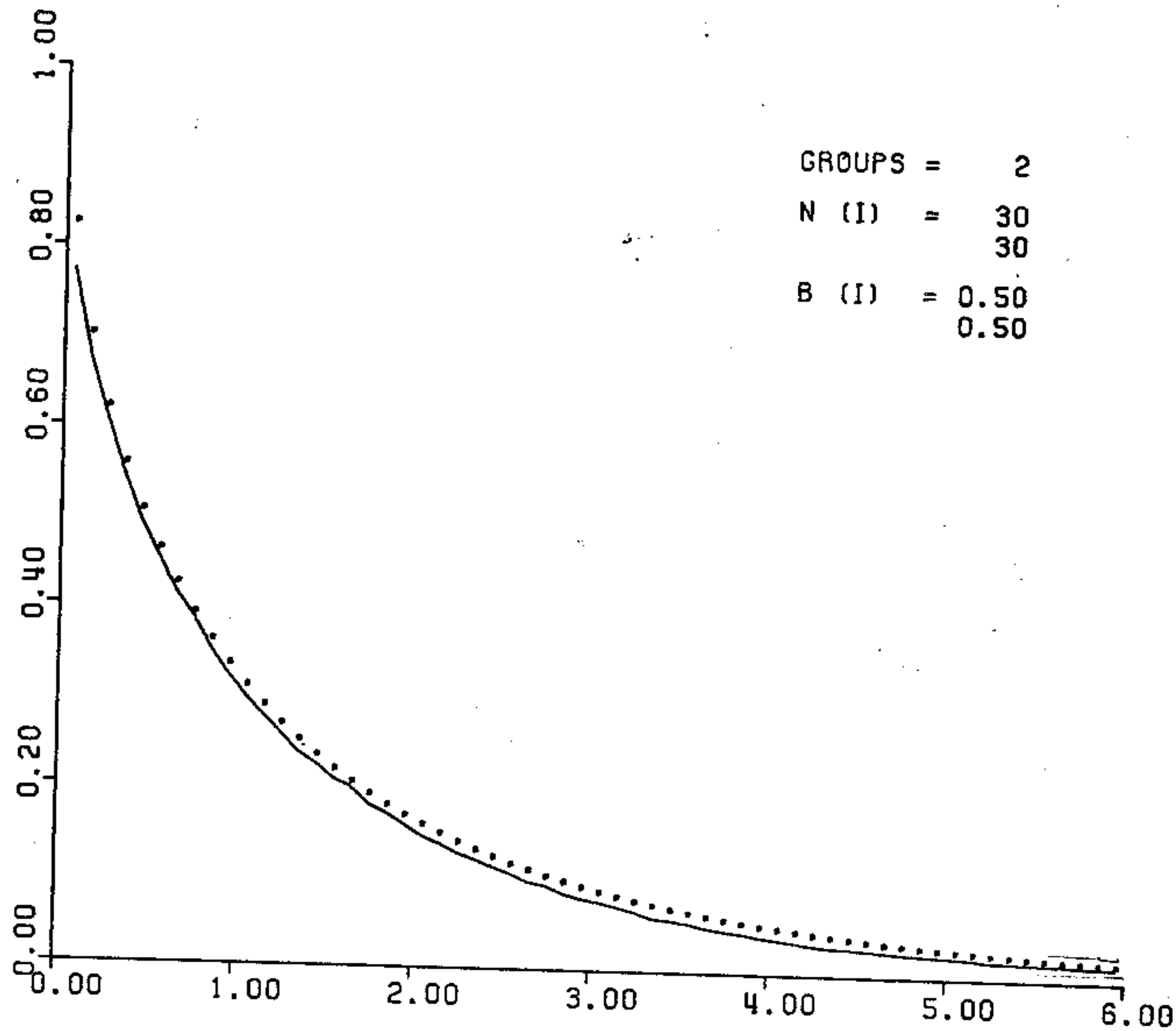


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

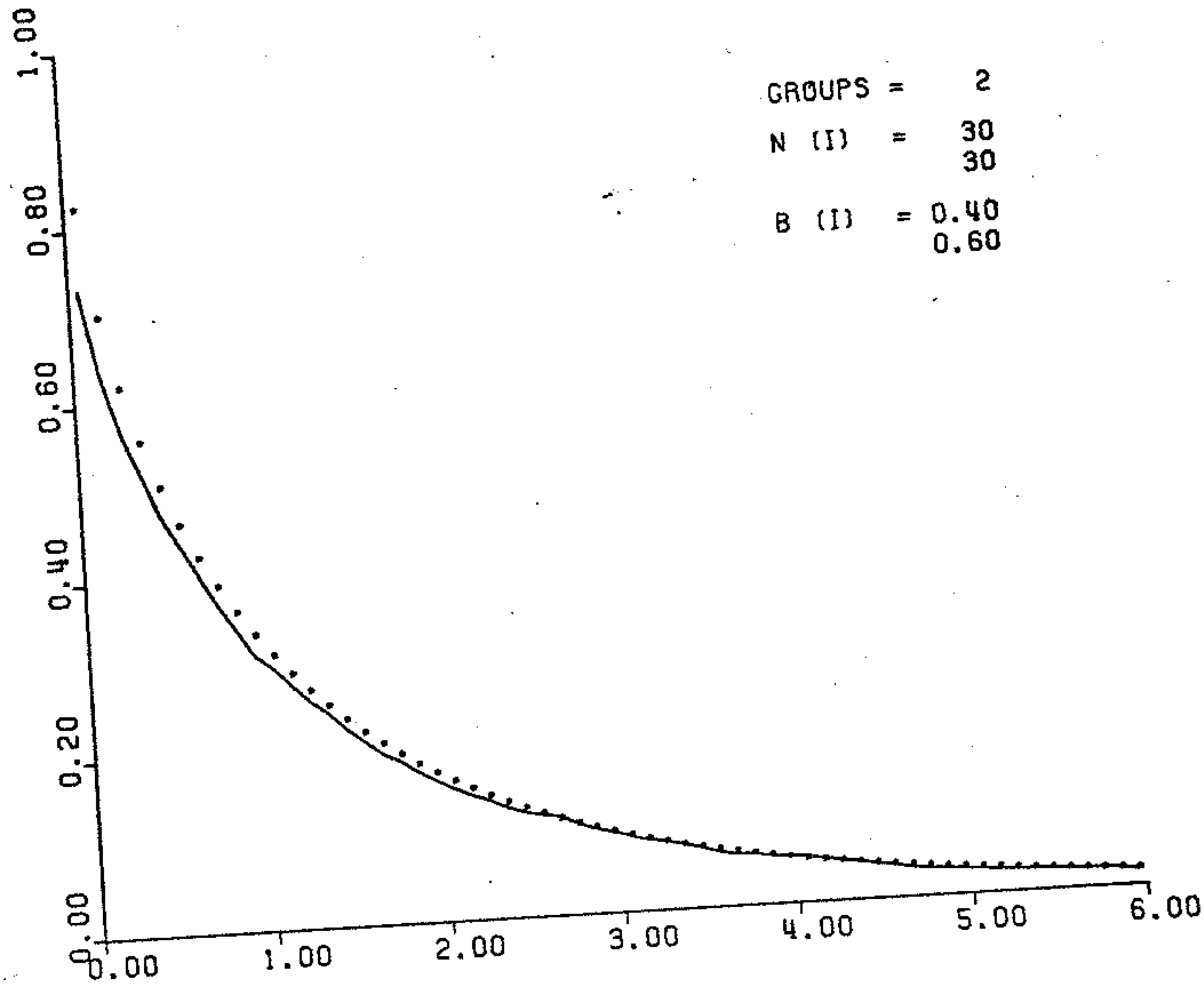
FIGURE 19 : SIMULATION NUMBER 19



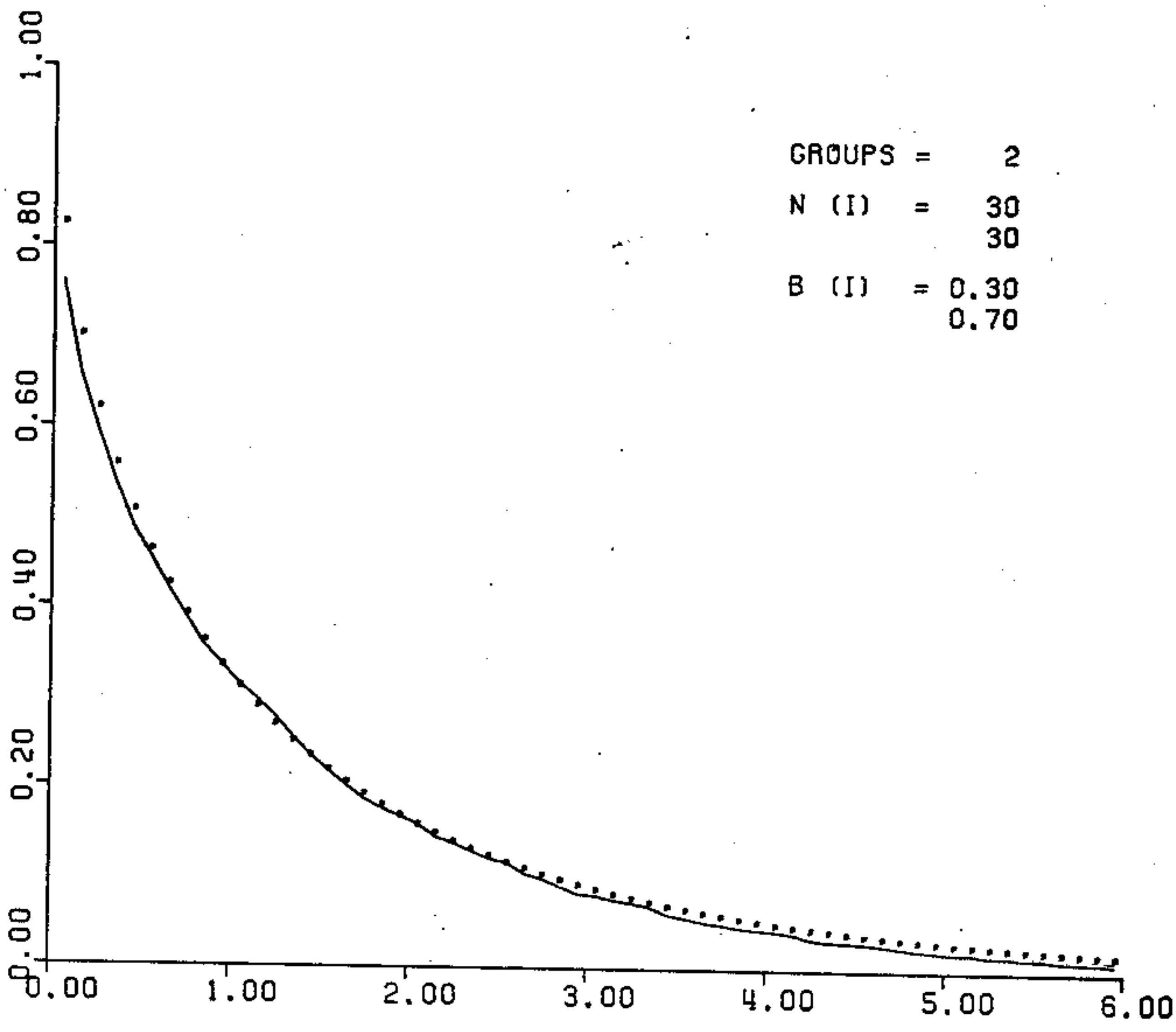
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 20 : SIMULATION NUMBER 20



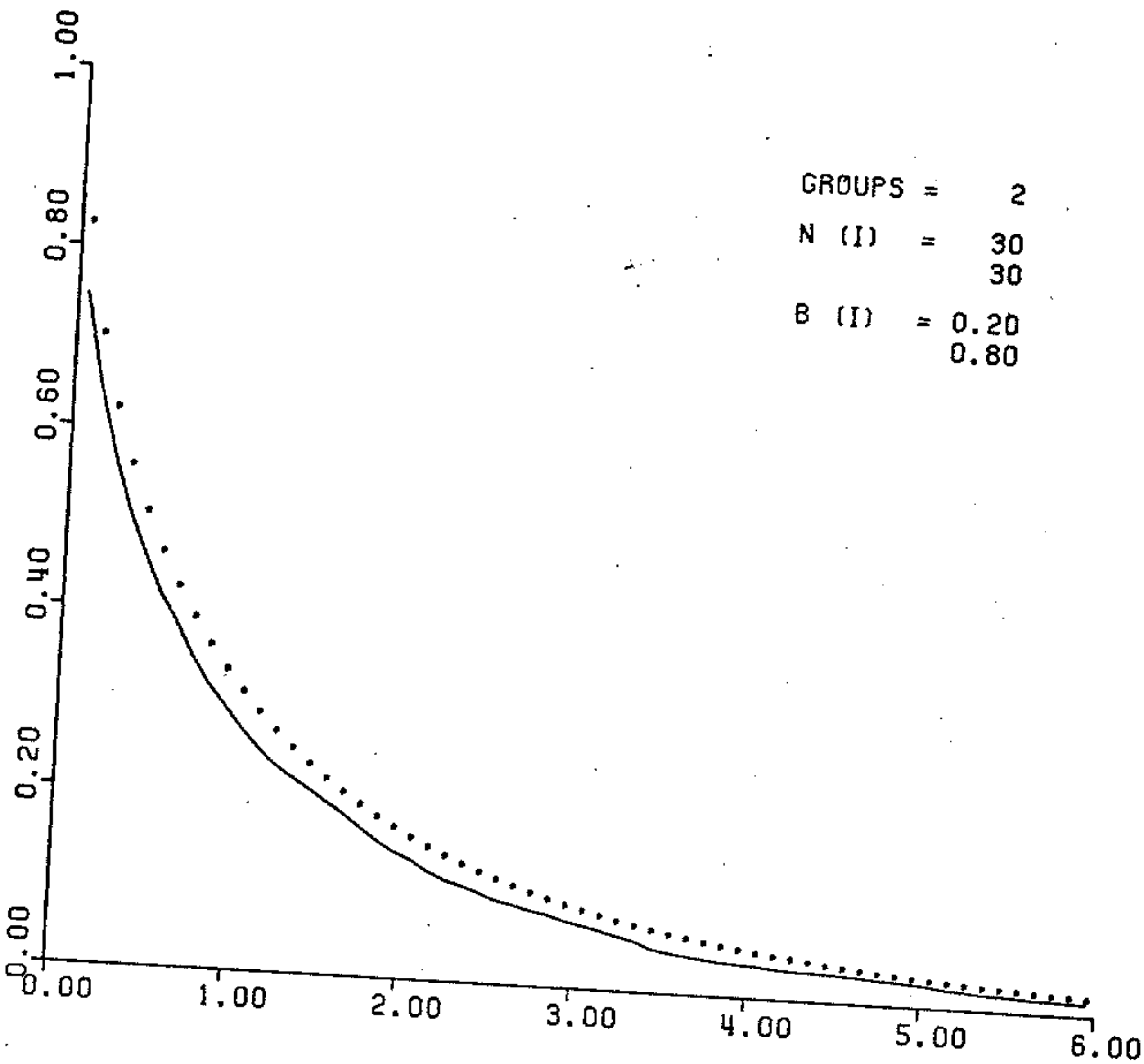
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 21 : SIMULATION NUMBER 21



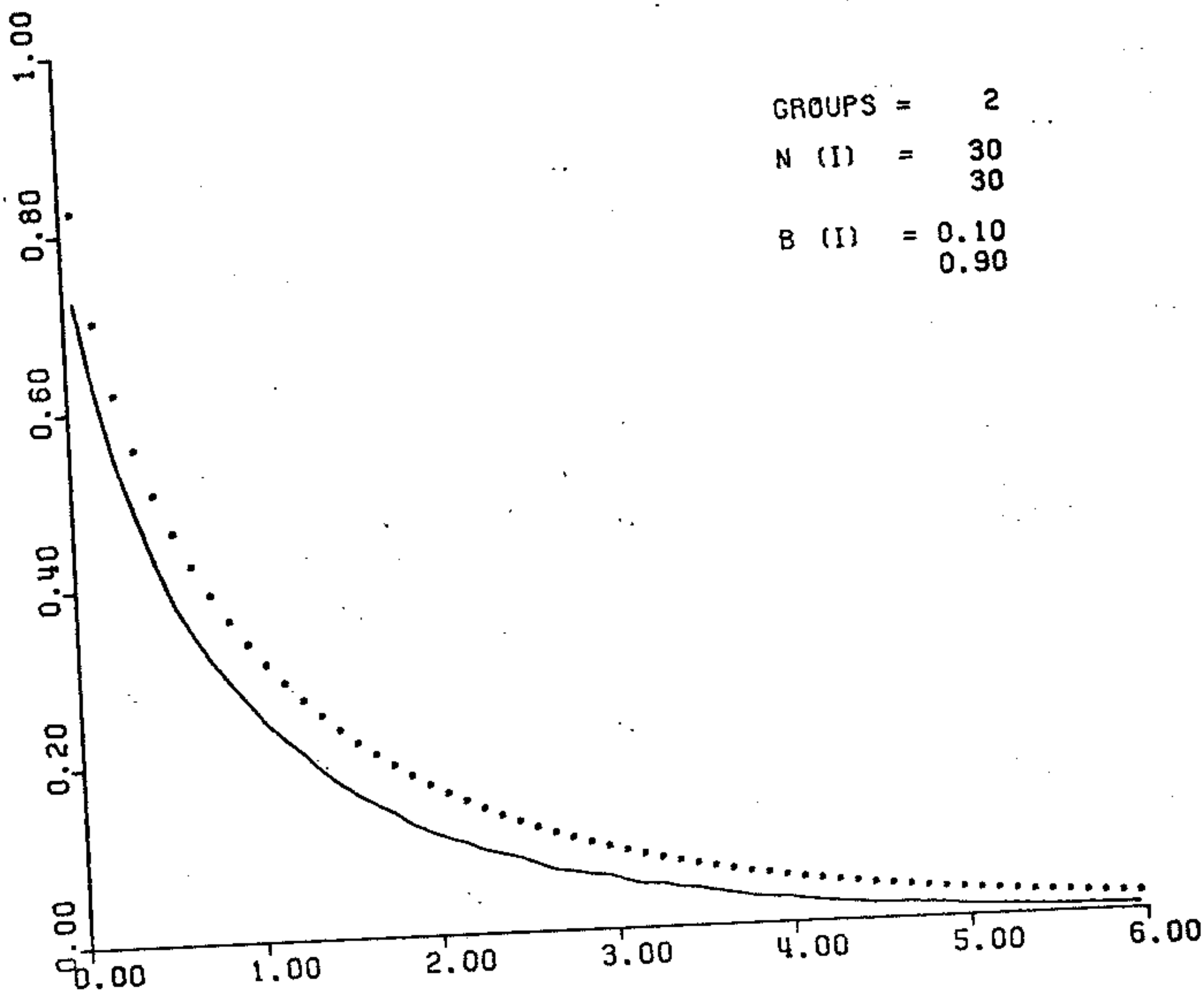
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 22 : SIMULATION NUMBER 22



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 23 : SIMULATION NUMBER 23

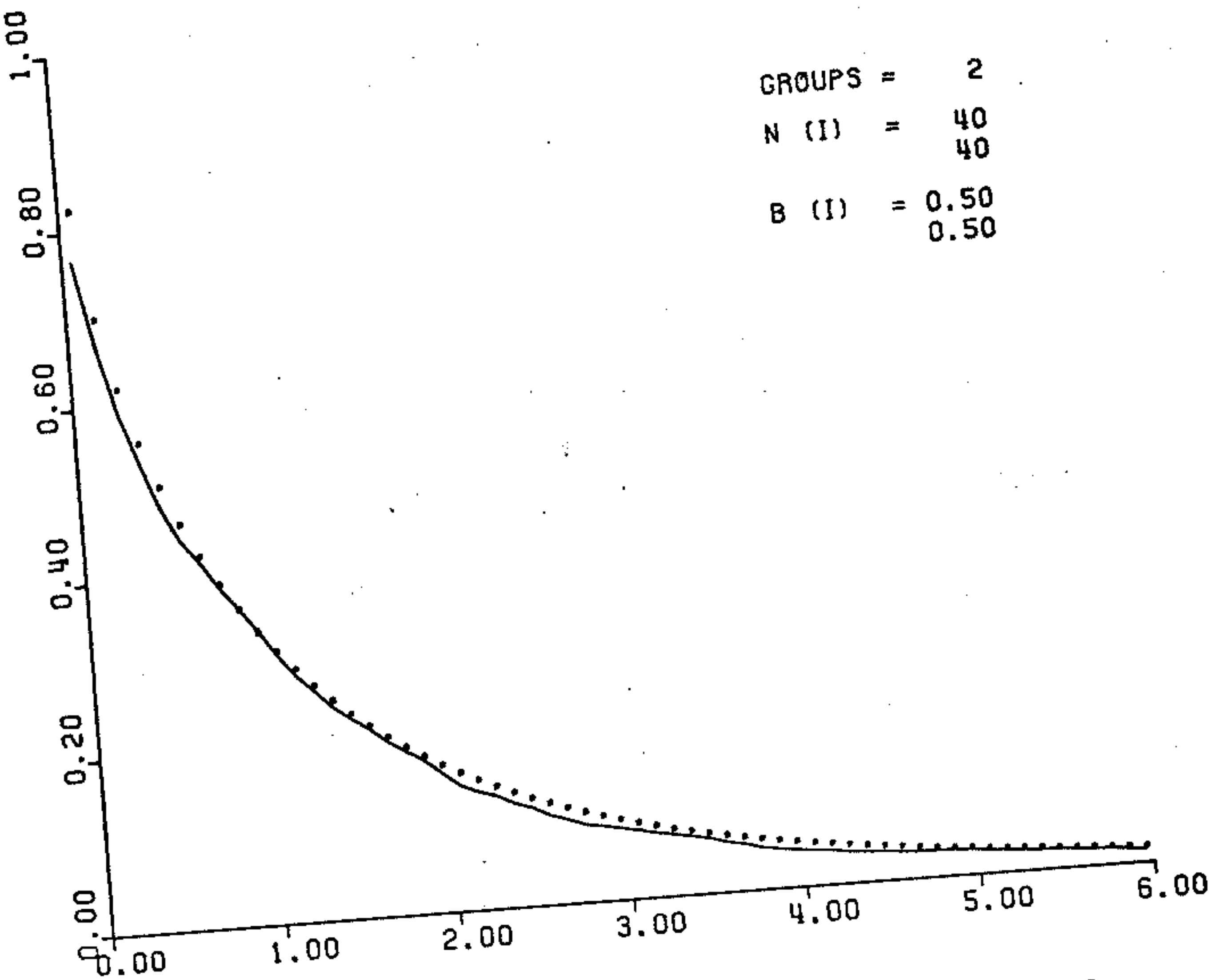


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 24 : SIMULATION NUMBER 24



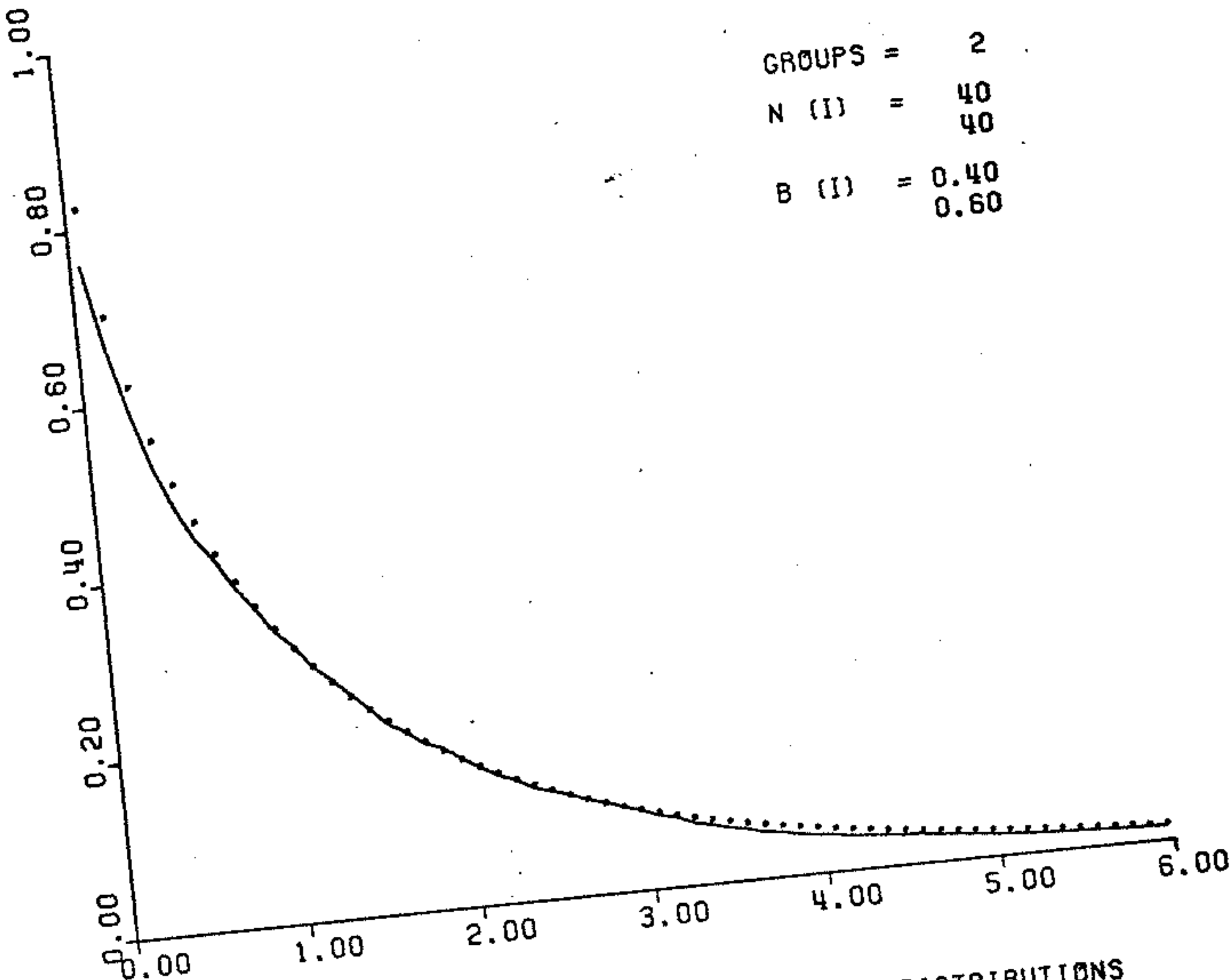
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 25 : SIMULATION NUMBER 25



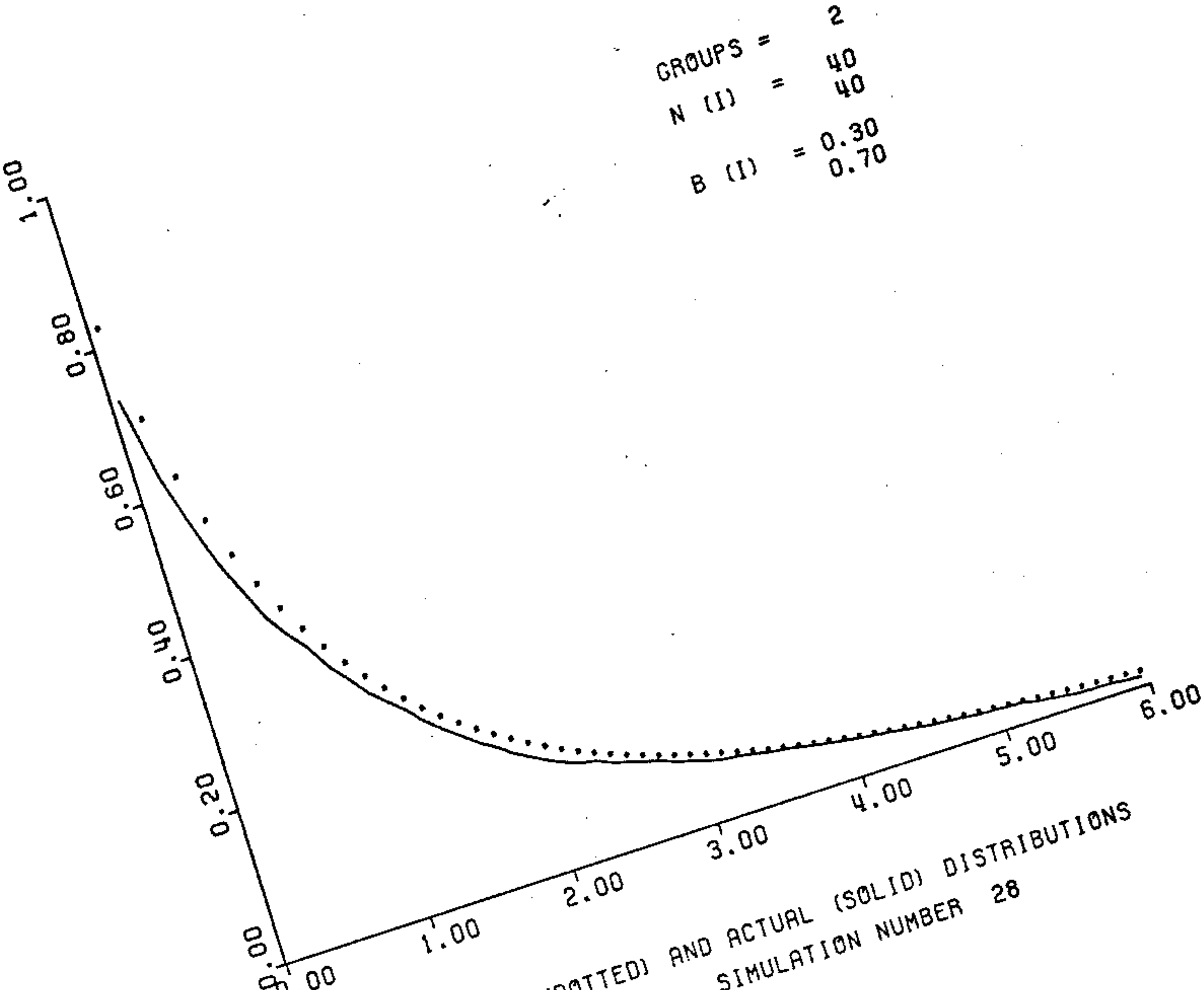
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 26 : SIMULATION NUMBER 26

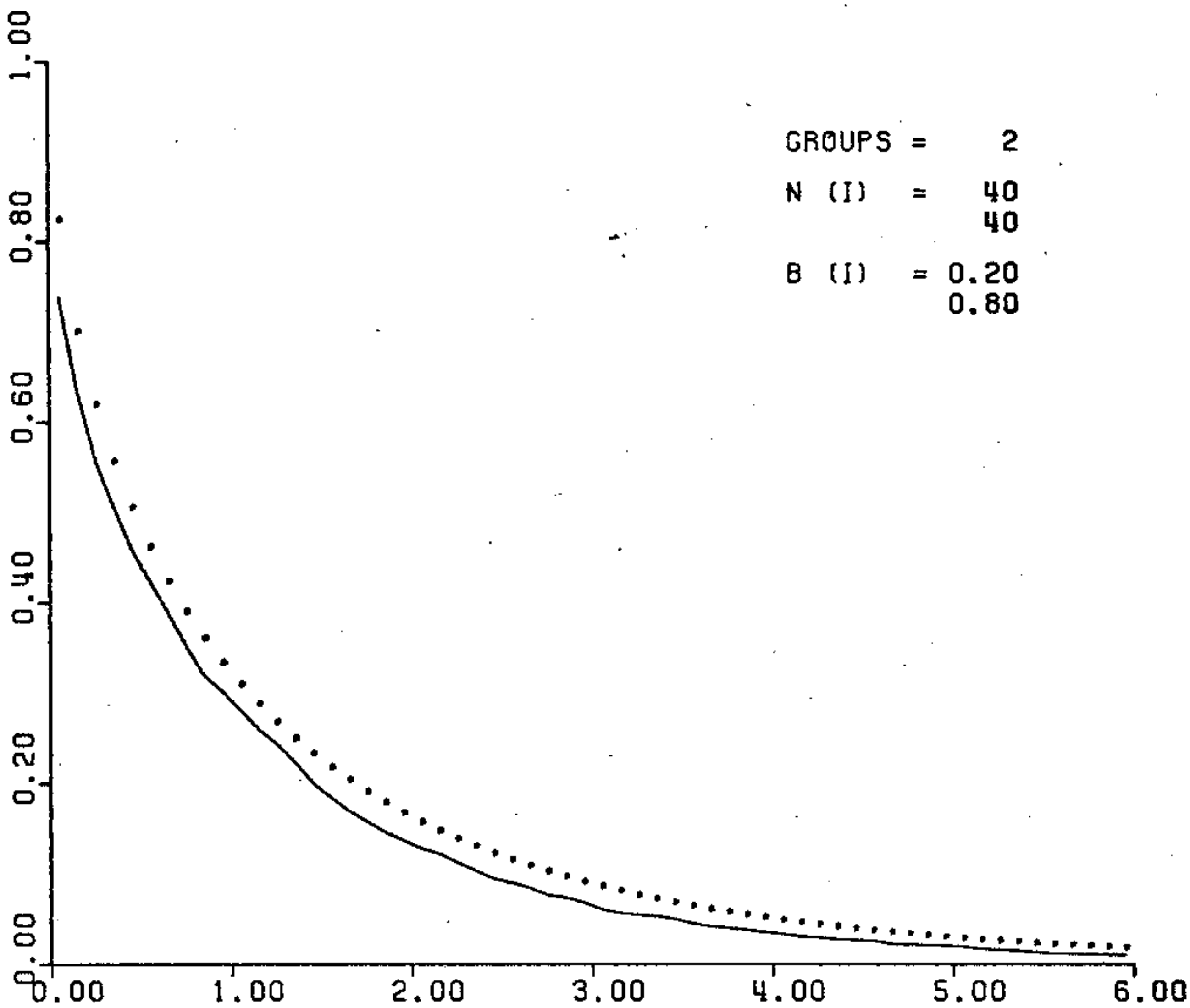


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 27 : SIMULATION NUMBER 27

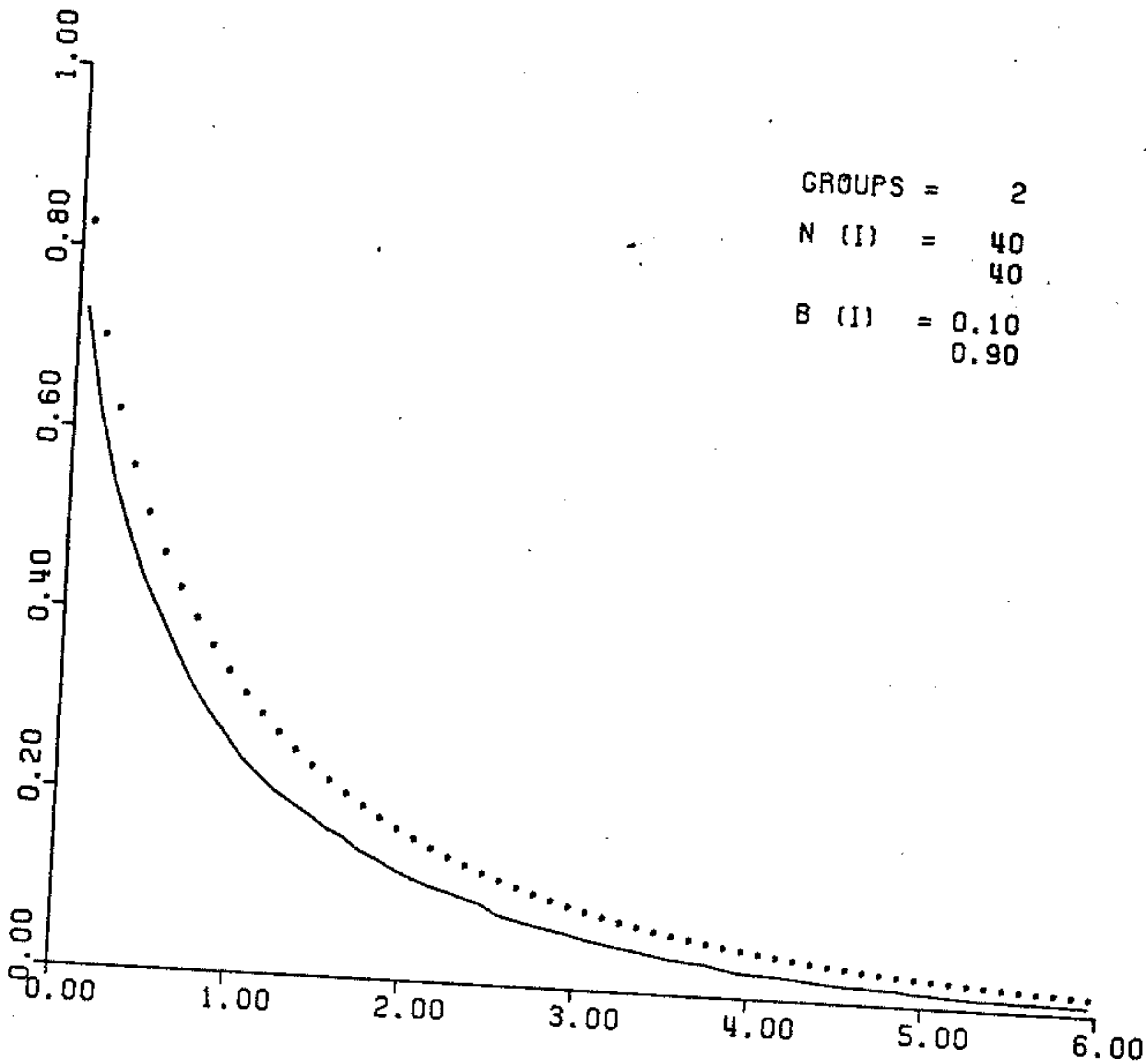


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 28 : SIMULATION NUMBER 28

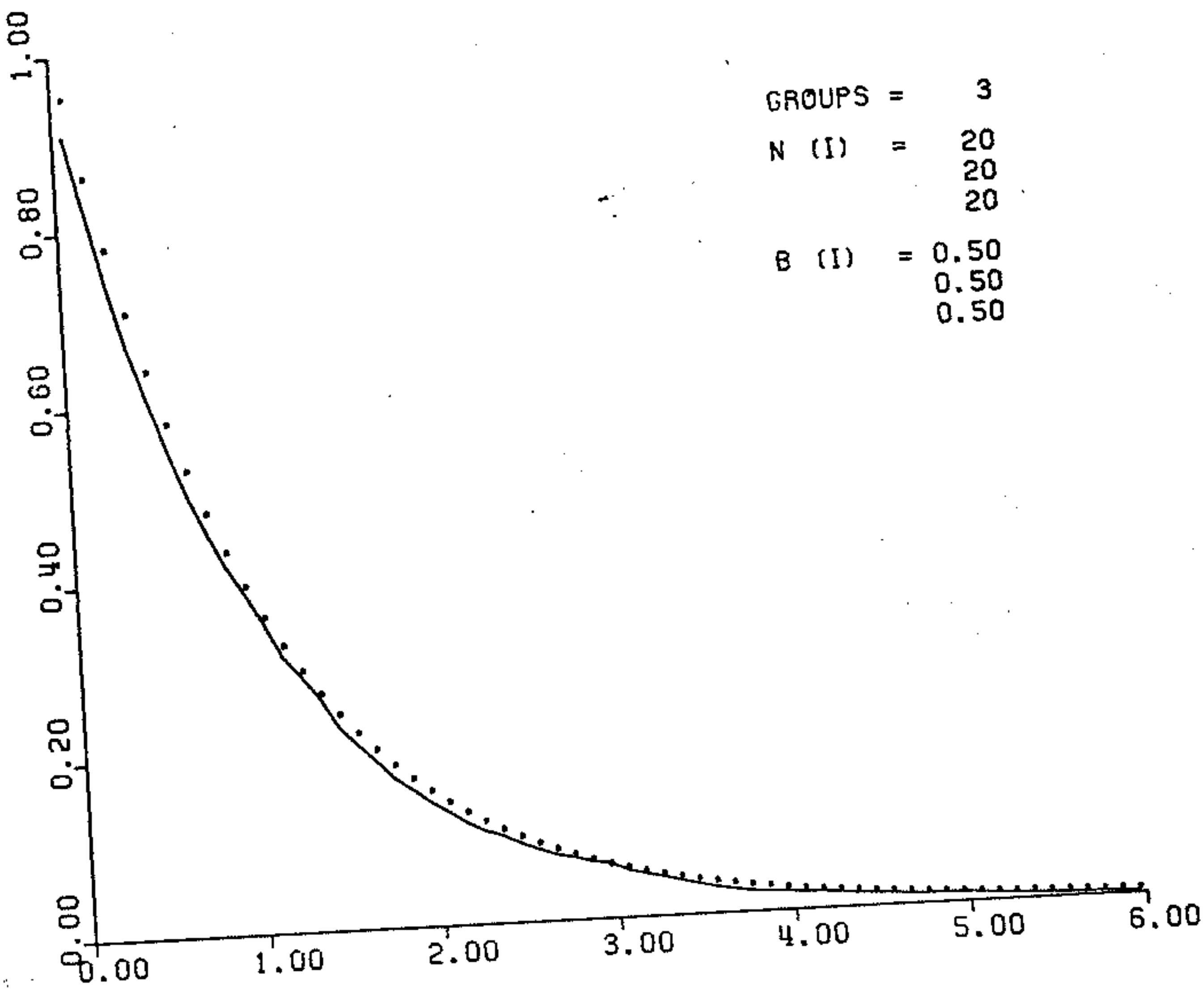


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 29 : SIMULATION NUMBER 29

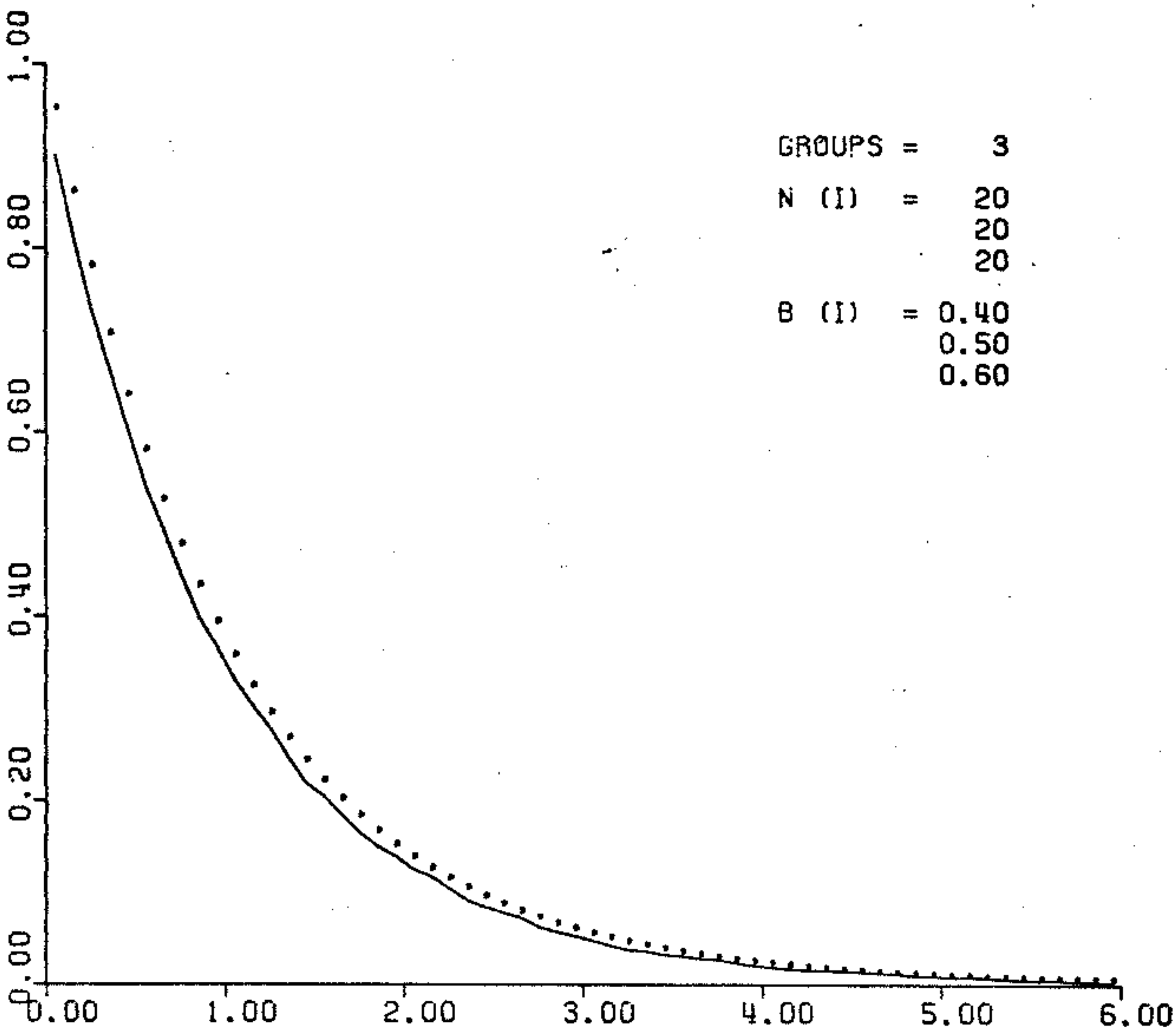


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 30 : SIMULATION NUMBER 30



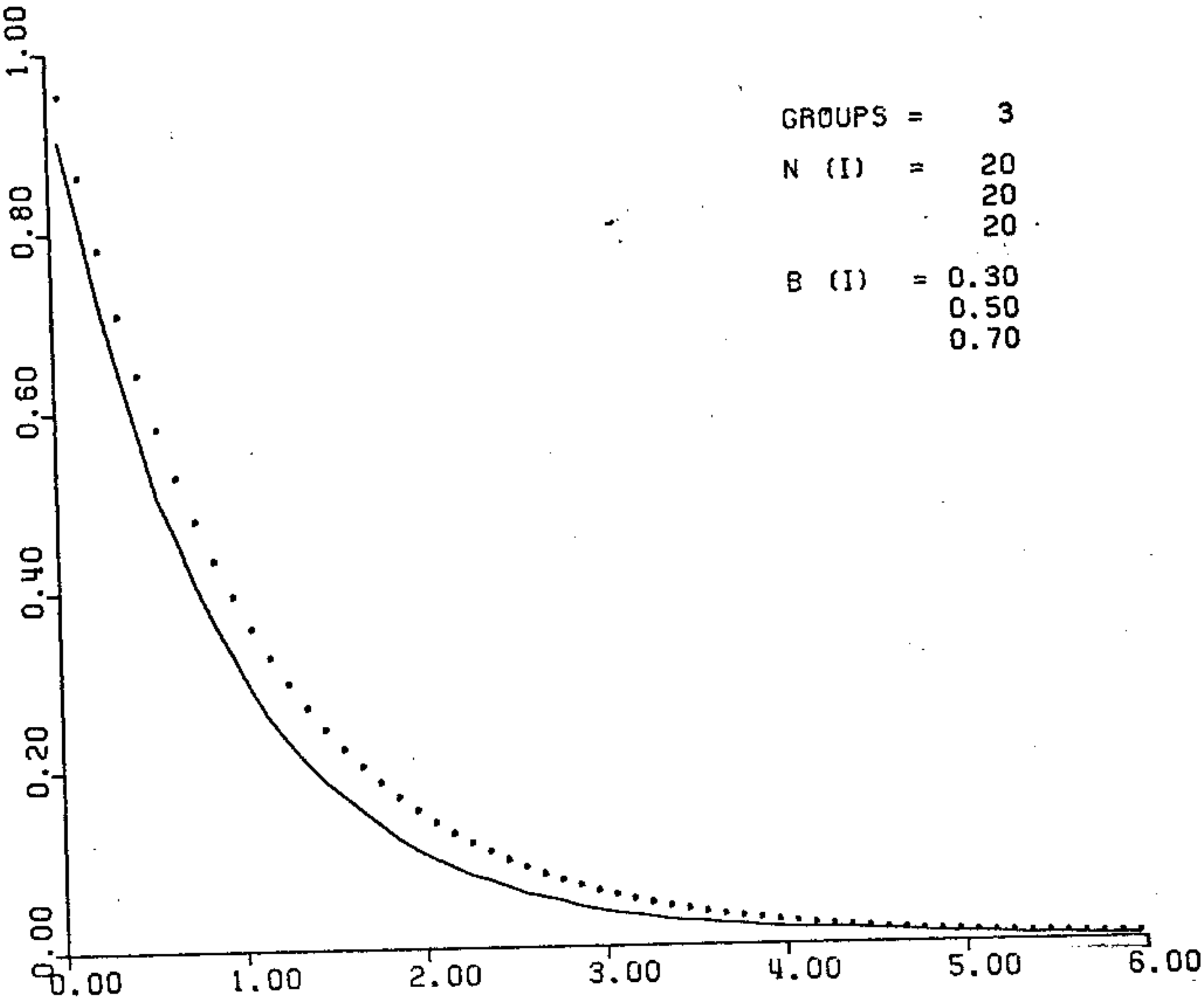
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 31 : SIMULATION NUMBER 31



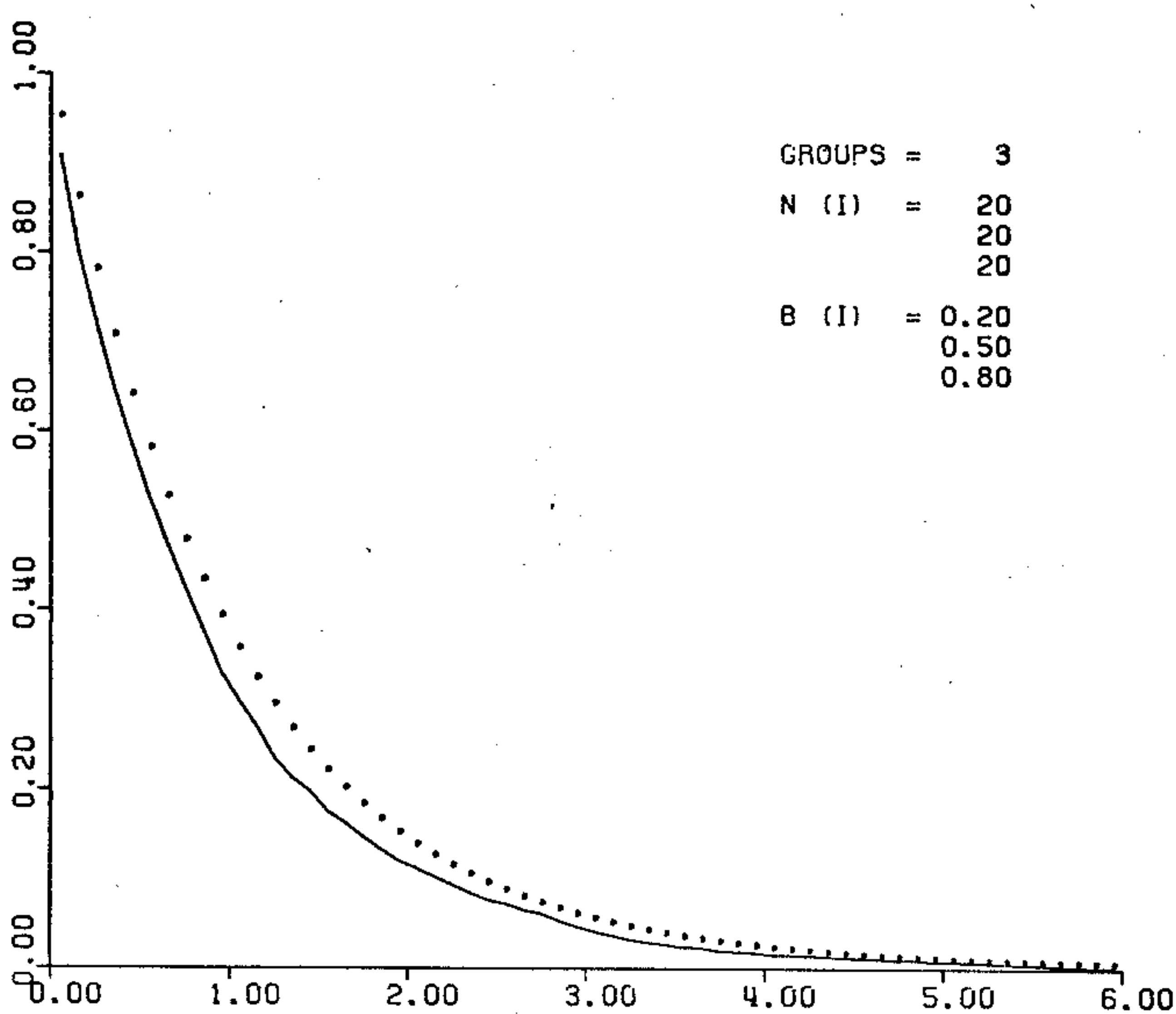
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 32 : SIMULATION NUMBER 32

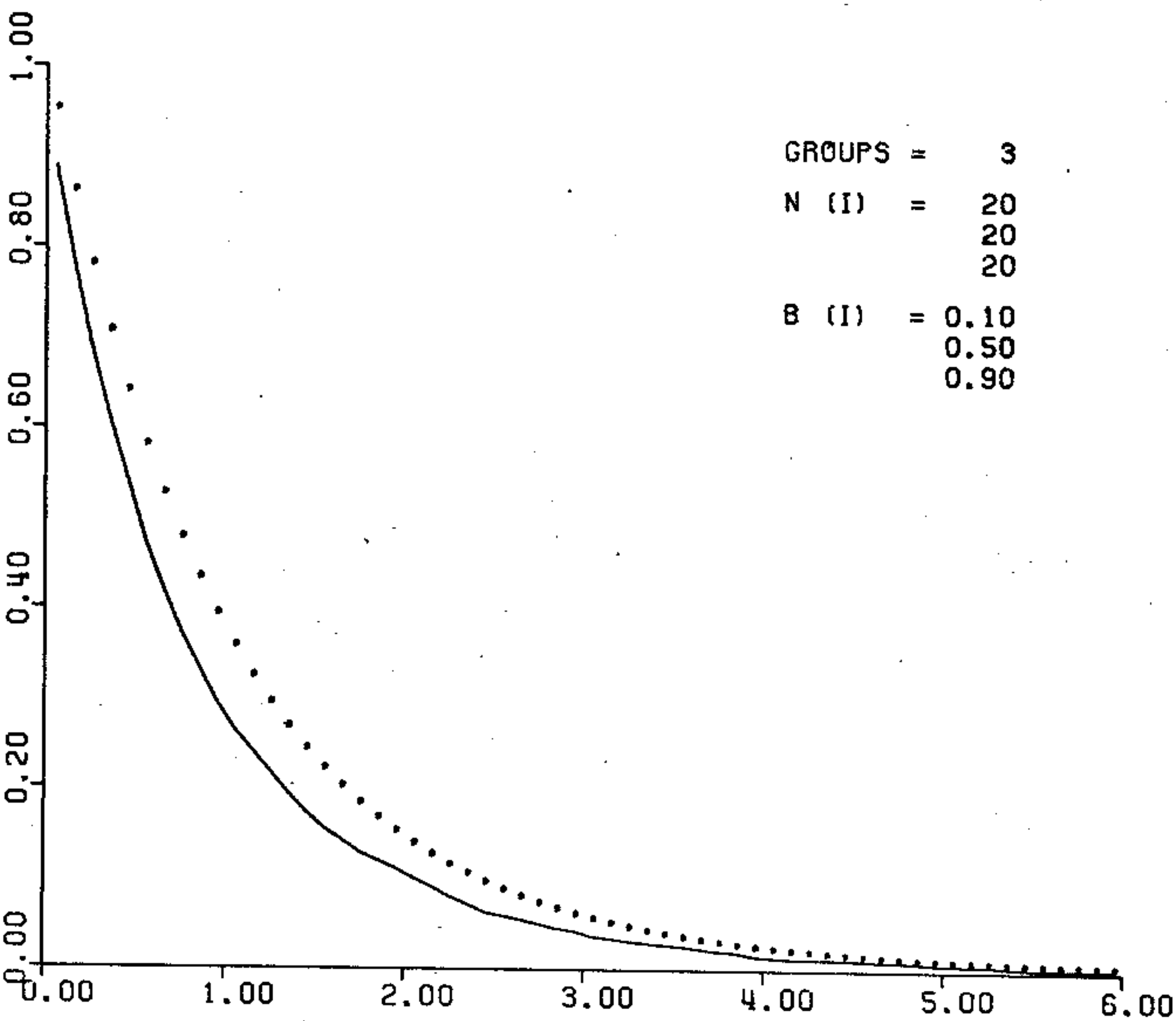


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 33 : SIMULATION NUMBER 33

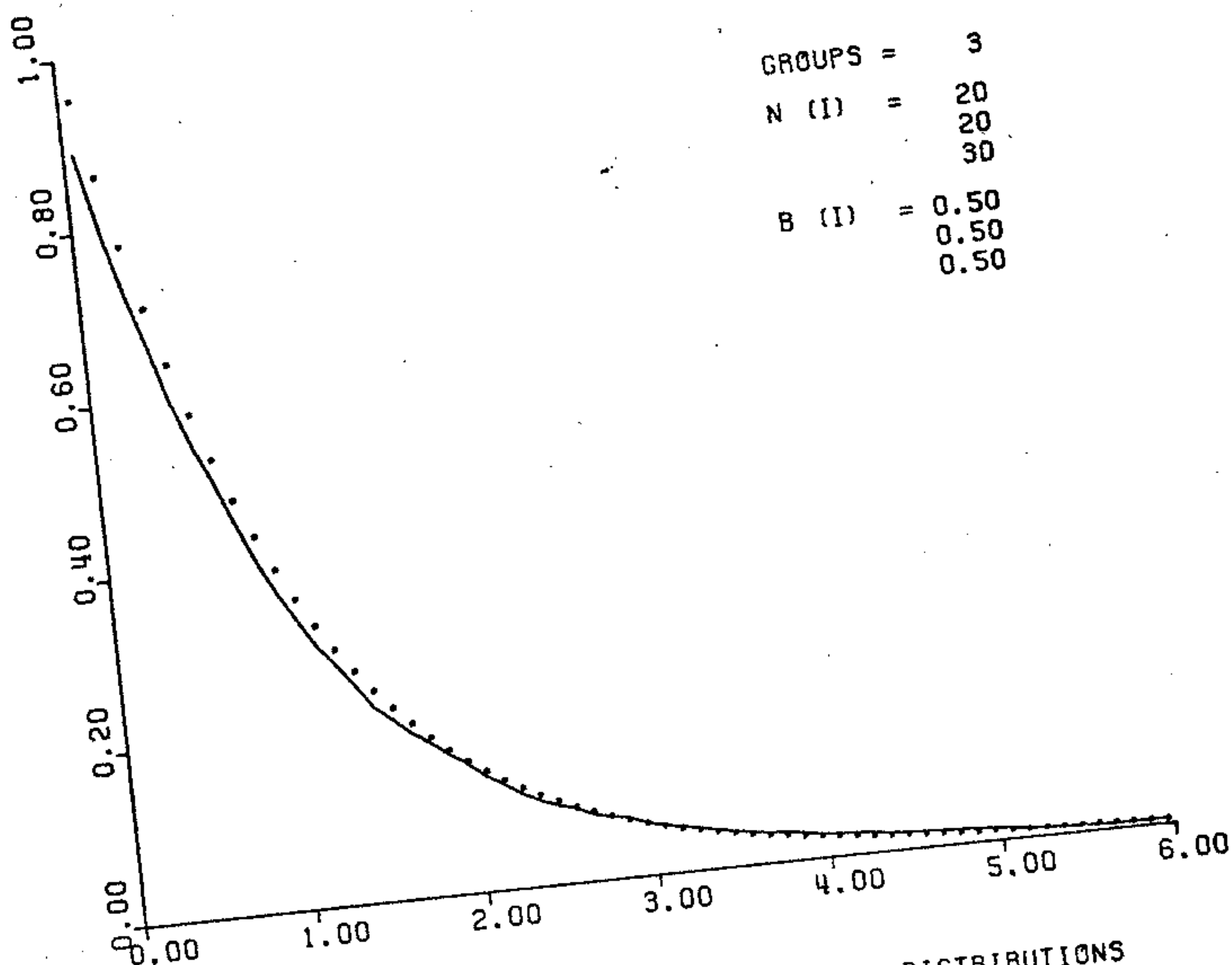


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 34 : SIMULATION NUMBER 34

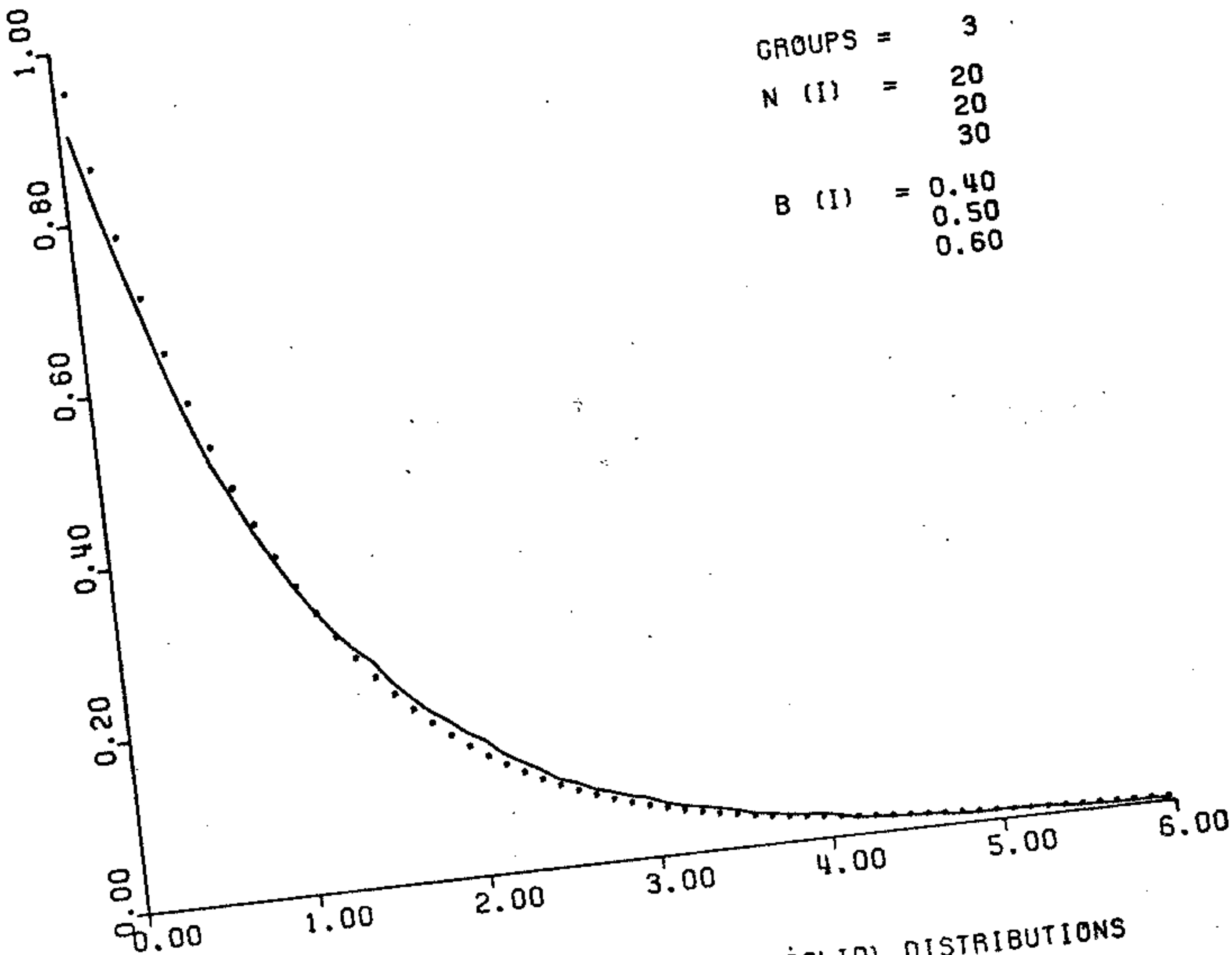


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 35 : SIMULATION NUMBER 35

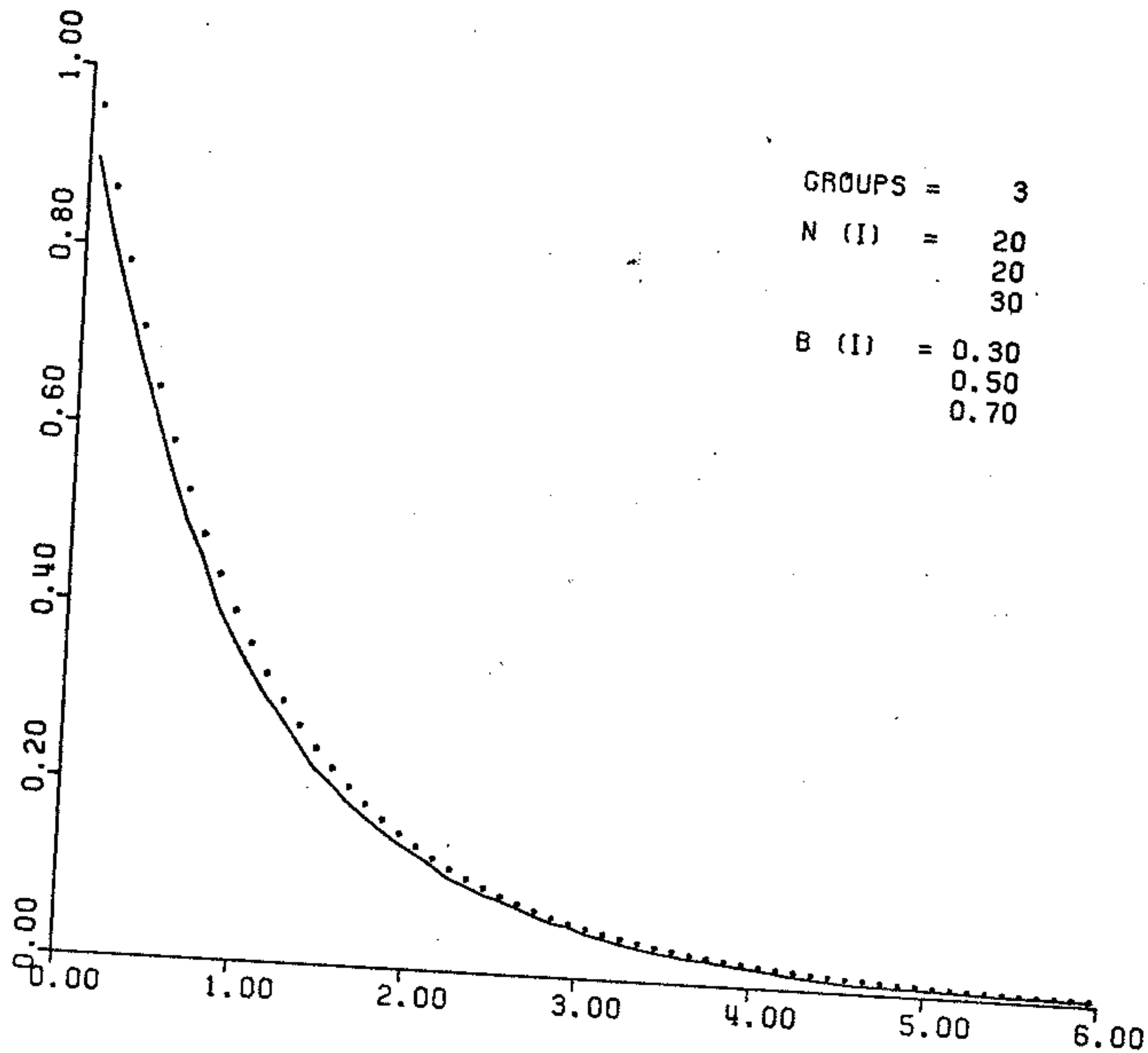


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 36 : SIMULATION NUMBER 36

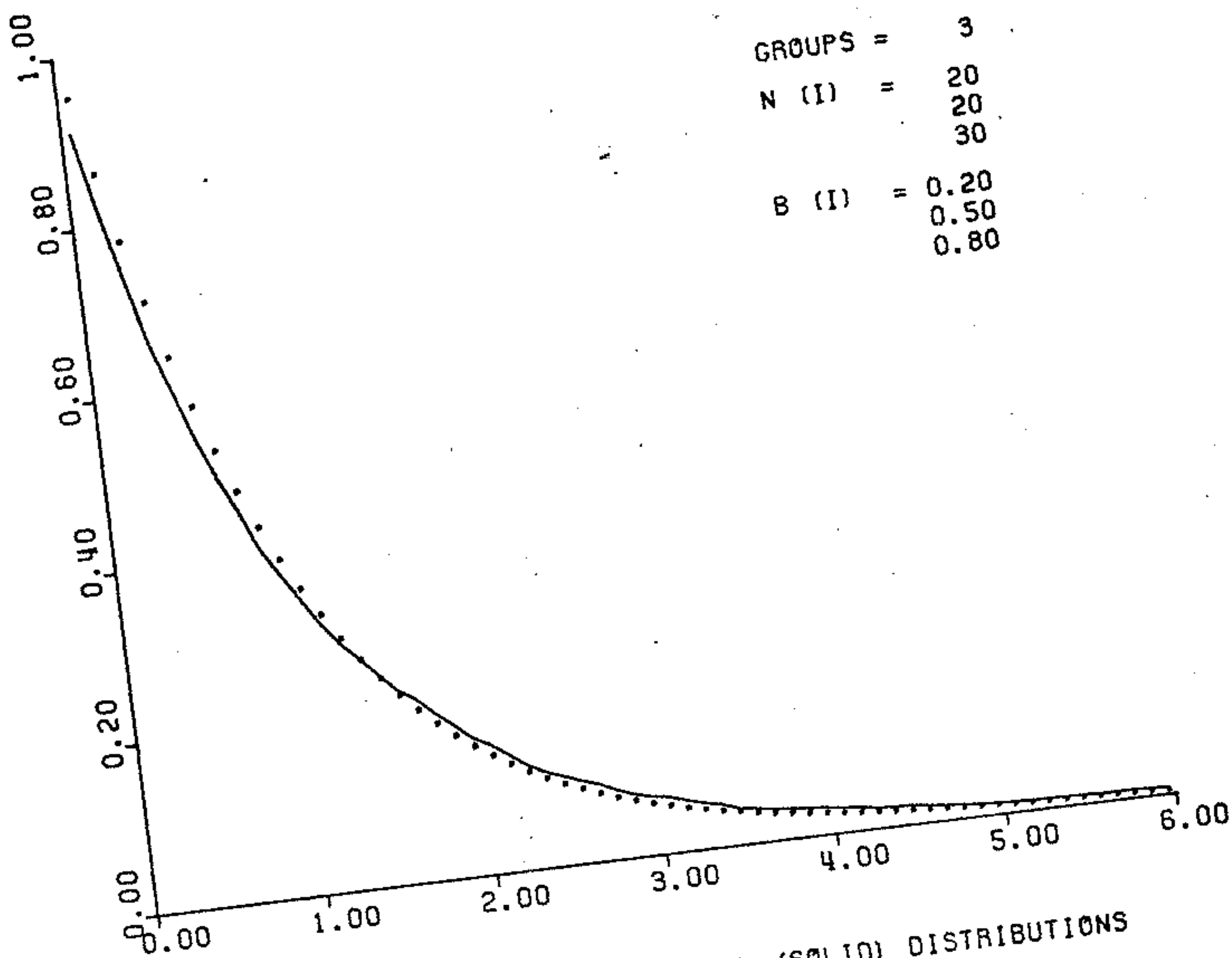


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 37 : SIMULATION NUMBER 37

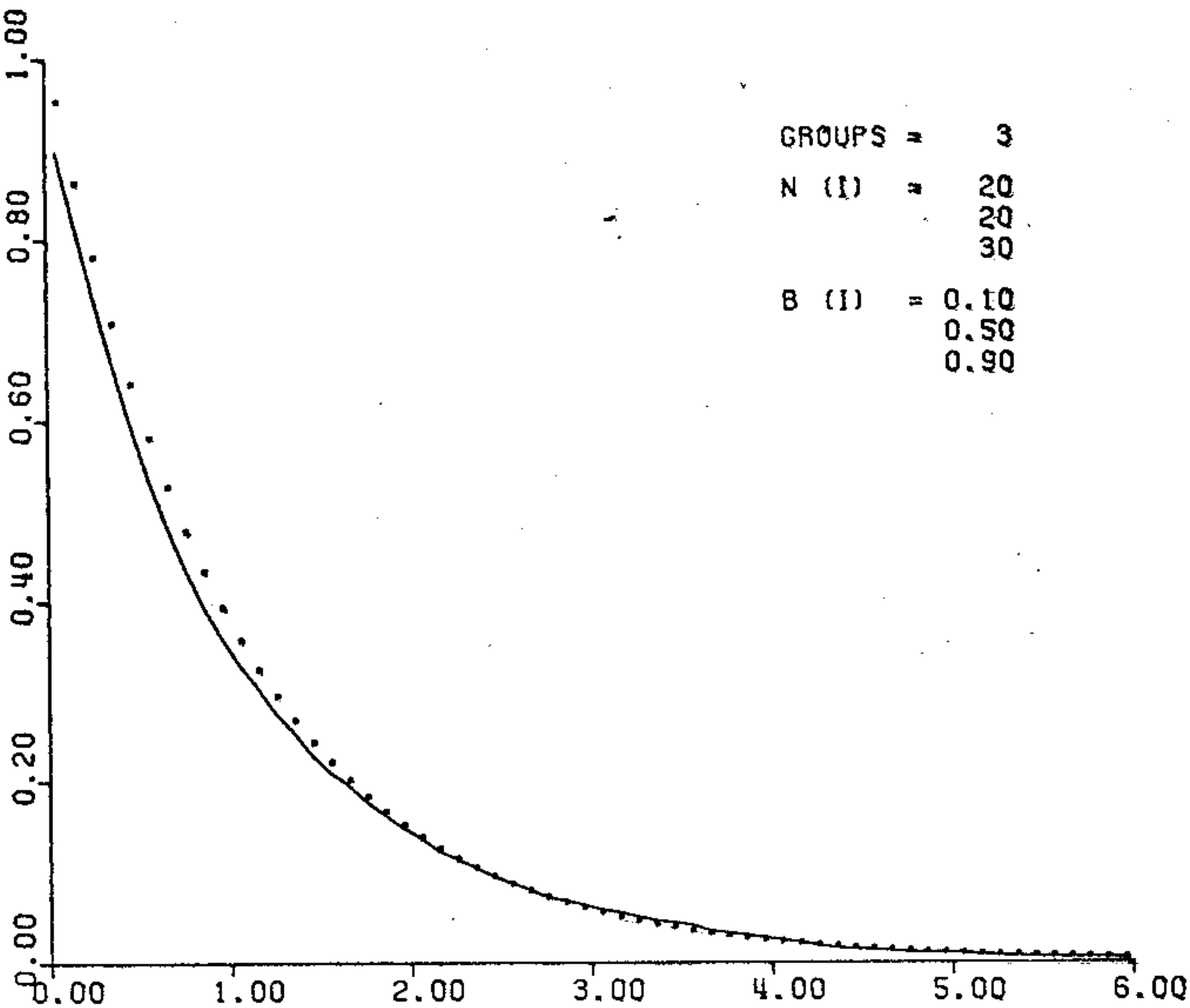


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 38 : SIMULATION NUMBER 38



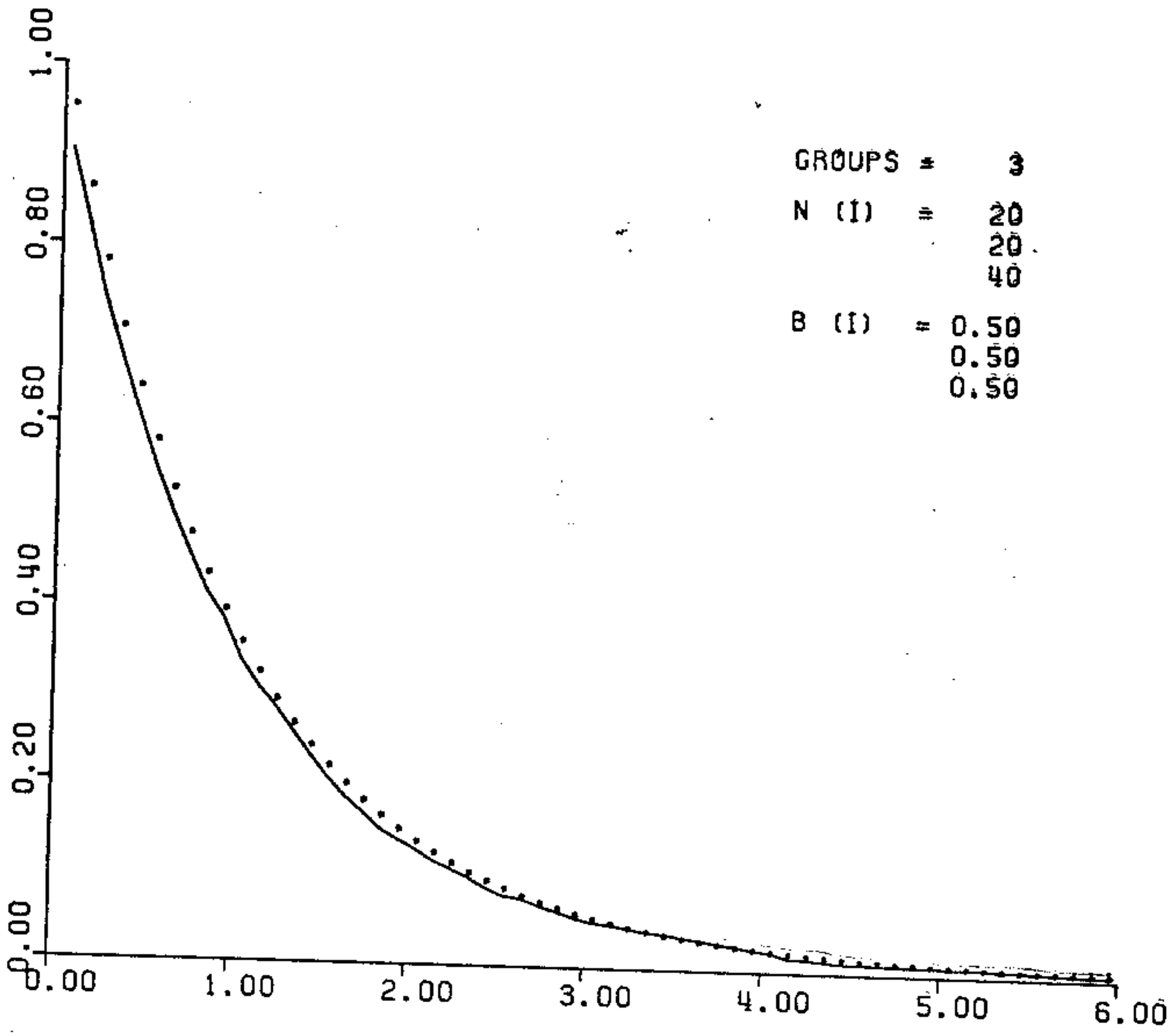
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 39 : SIMULATION NUMBER 39

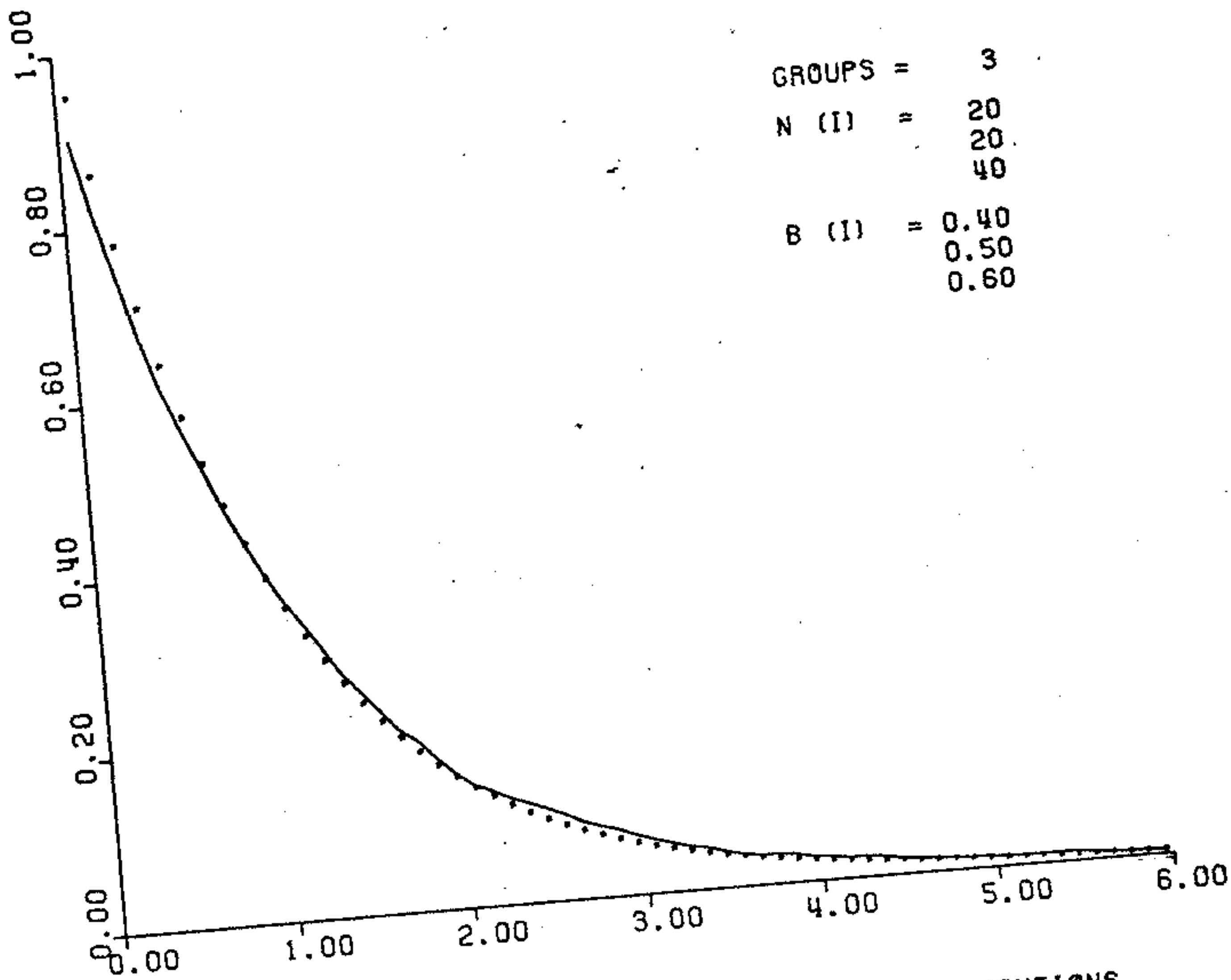


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 40 : SIMULATION NUMBER 40

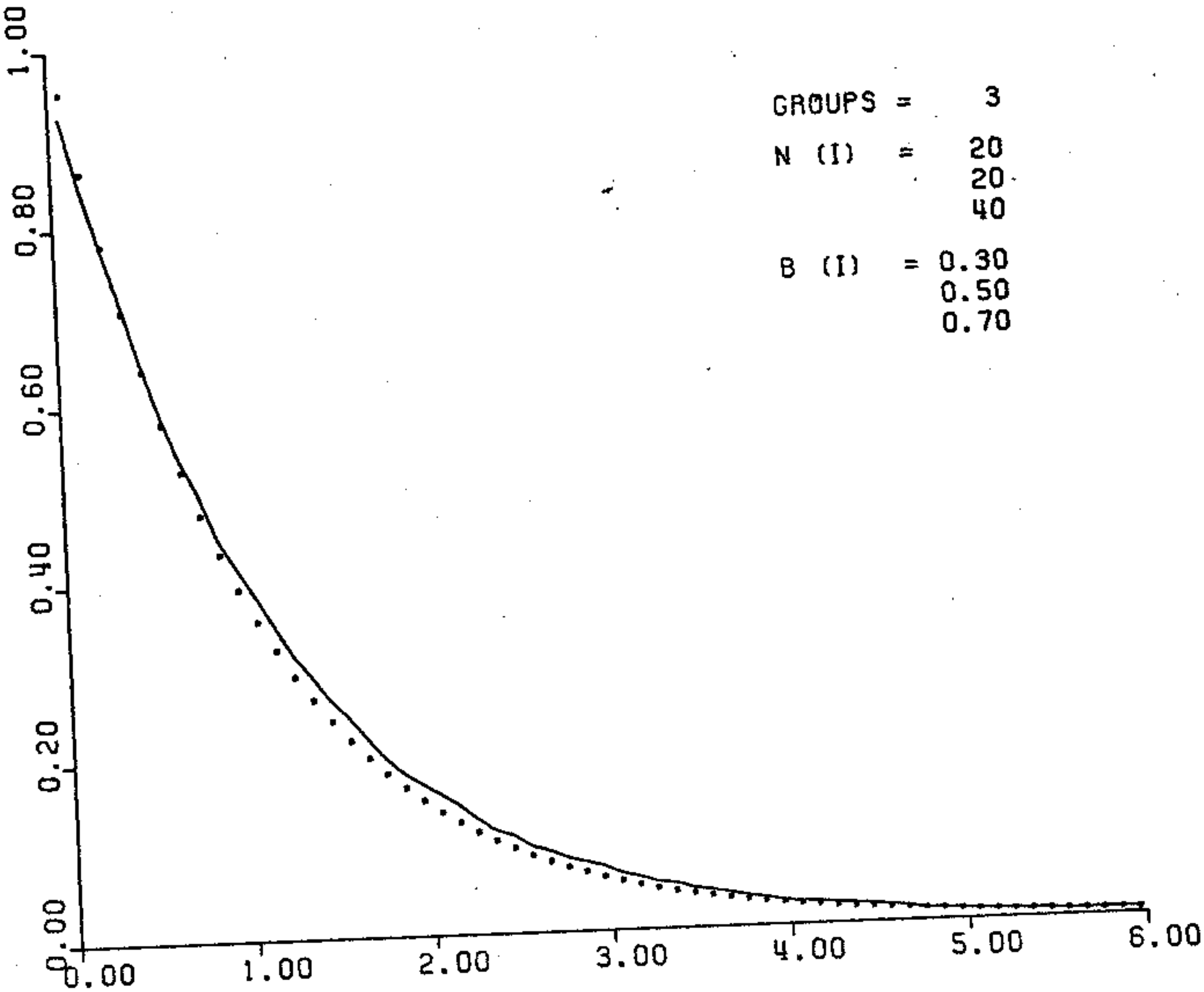


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 41 : SIMULATION NUMBER 41



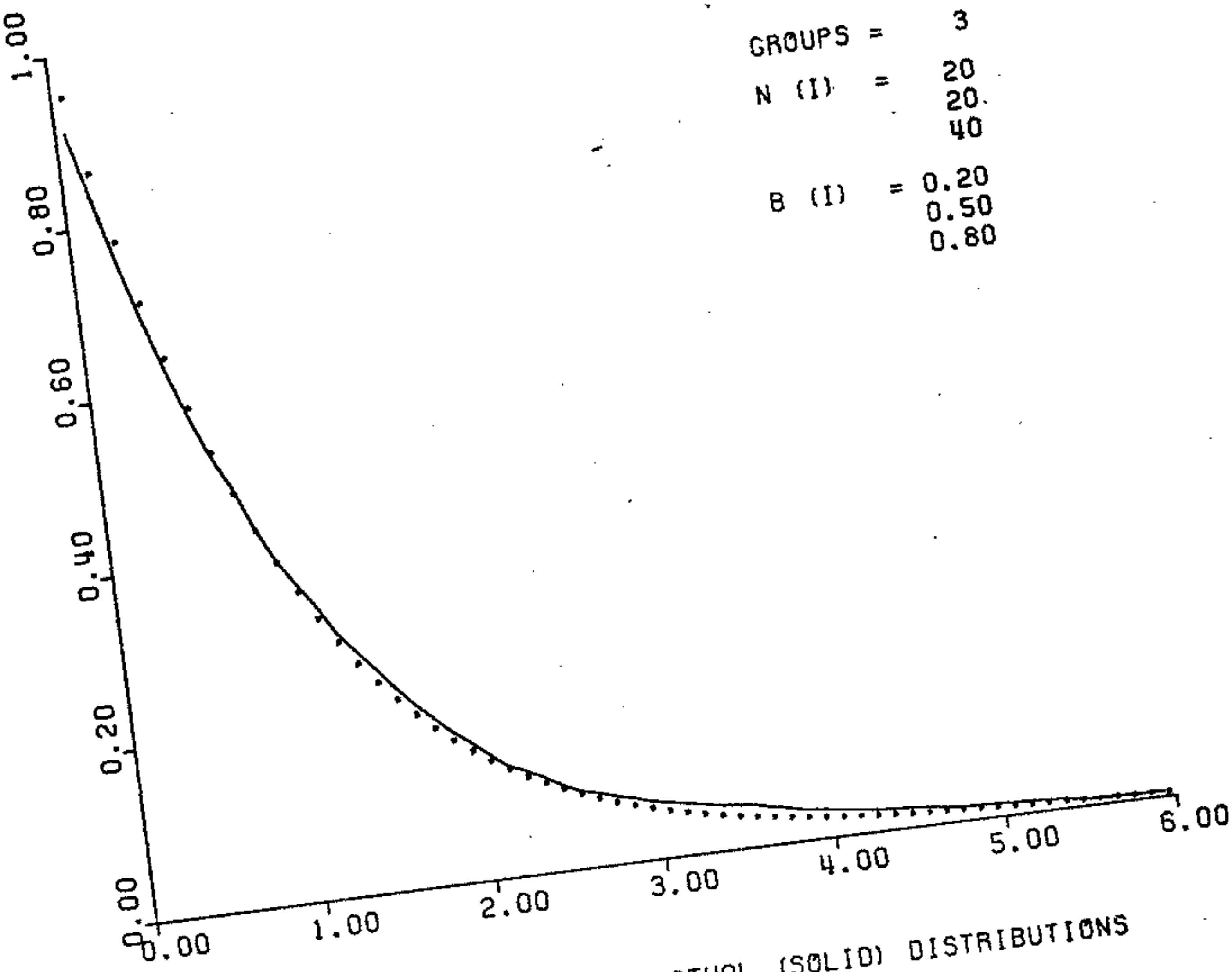
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 42 : SIMULATION NUMBER 42

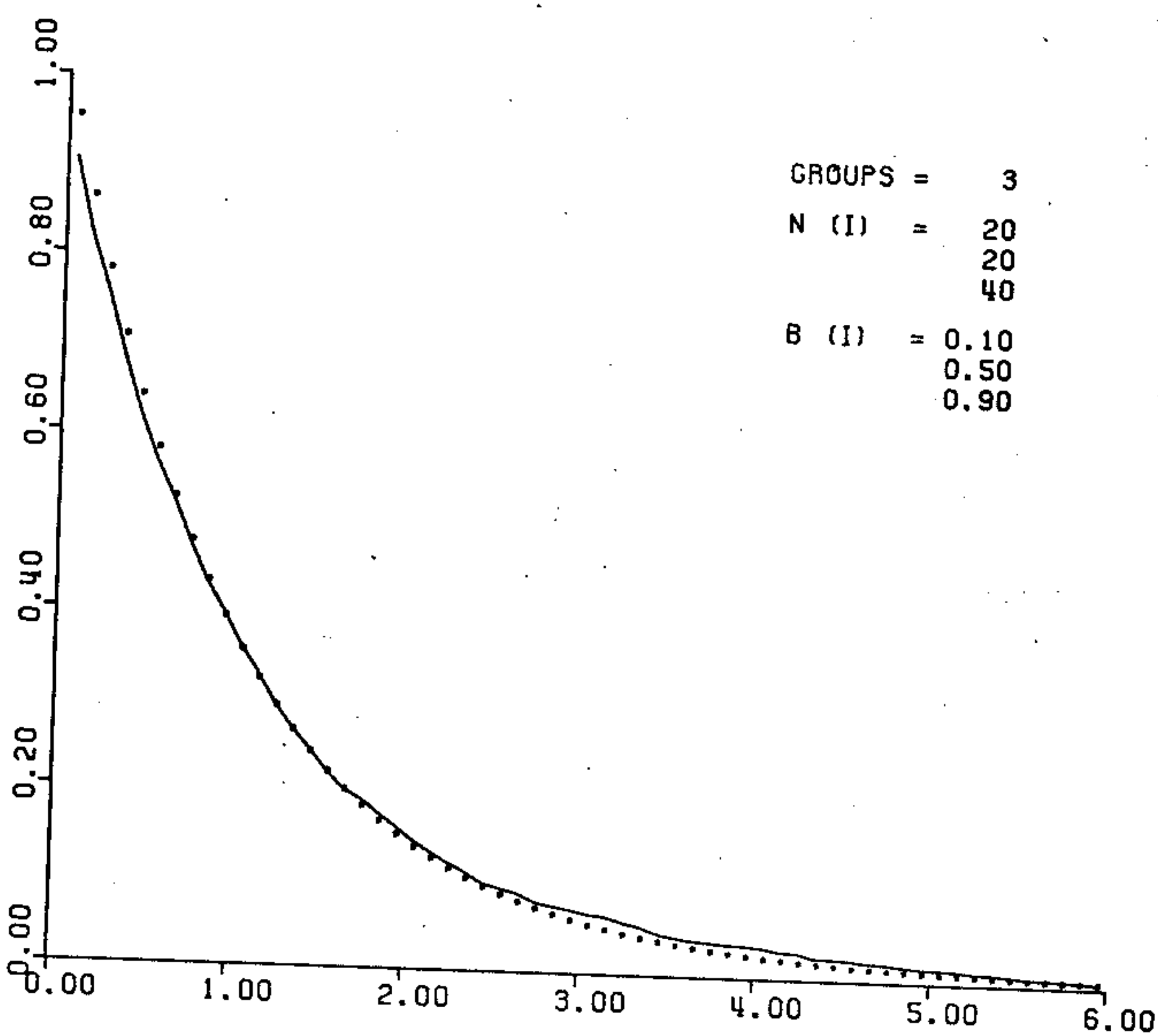


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

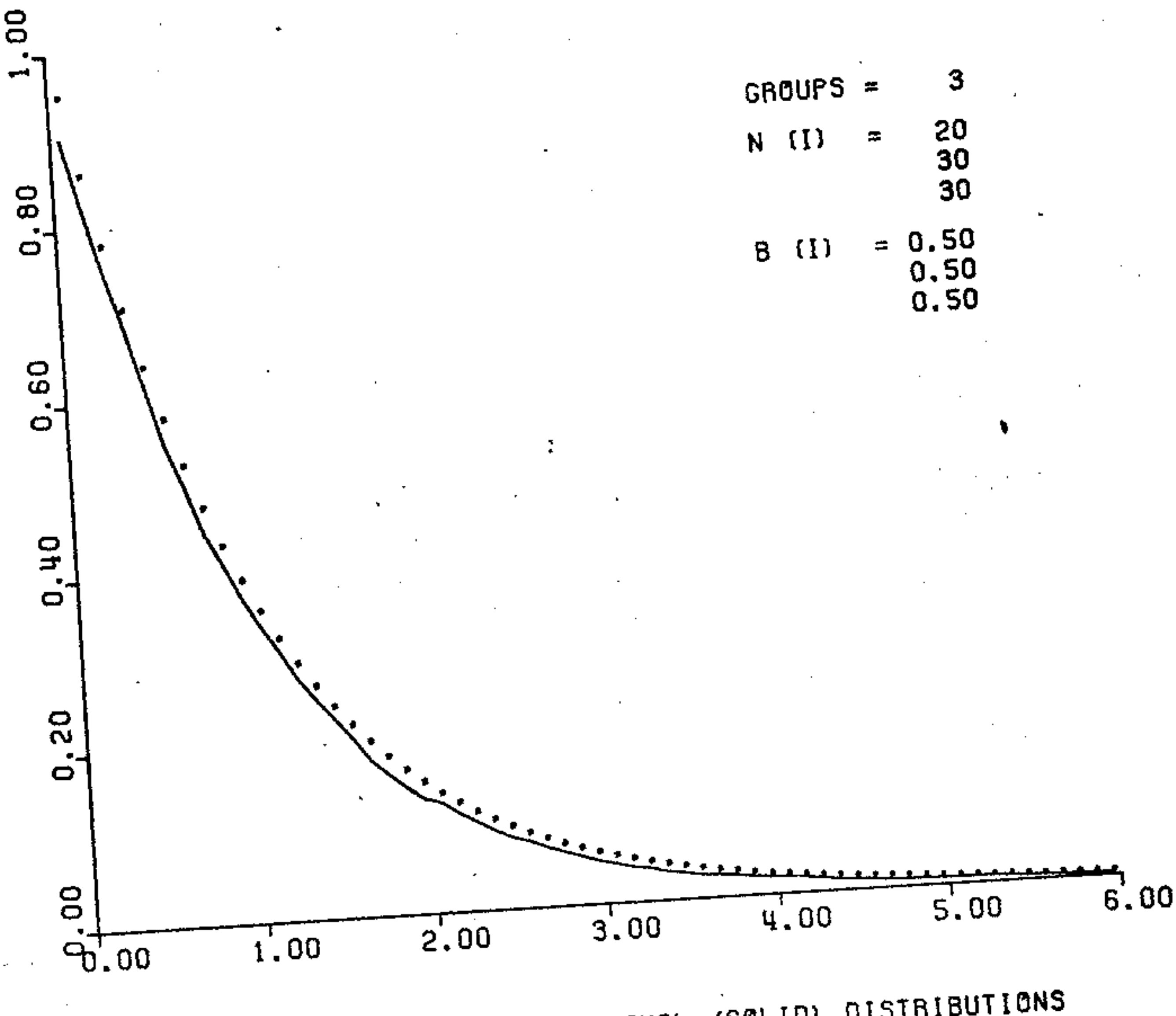
FIGURE 43 : SIMULATION NUMBER 43



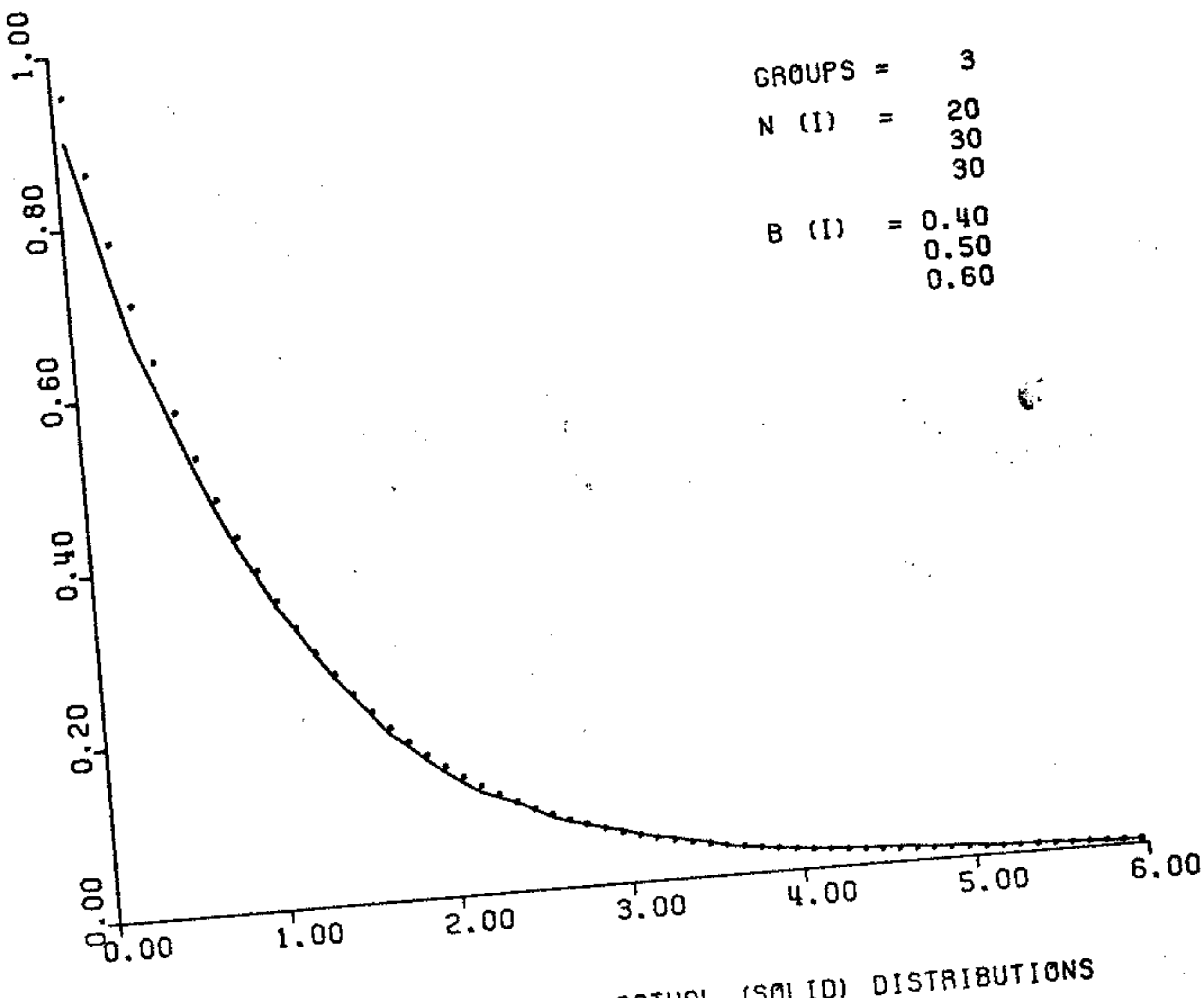
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 44 : SIMULATION NUMBER 44



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 45 : SIMULATION NUMBER 45

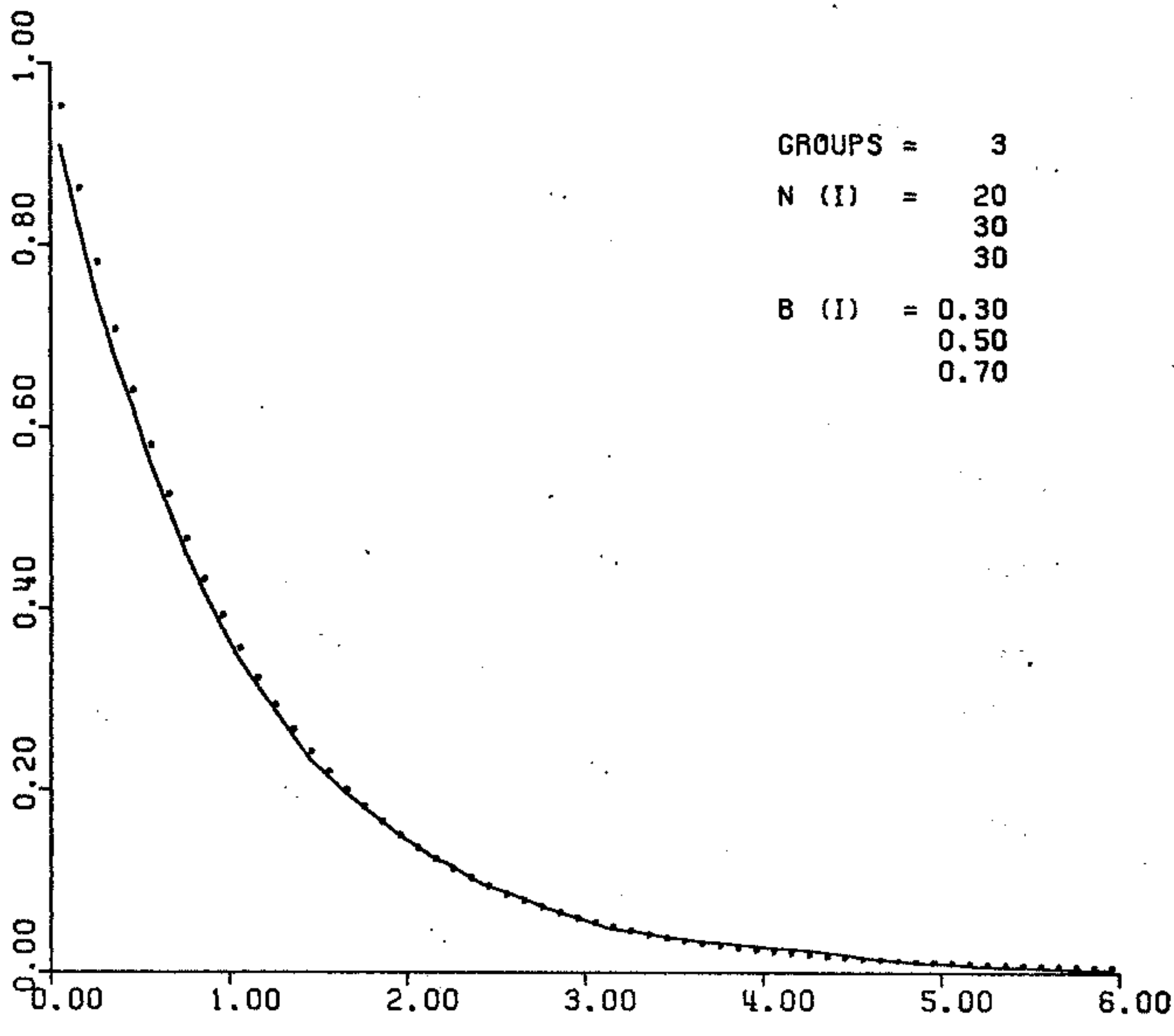


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 46 : SIMULATION NUMBER 46

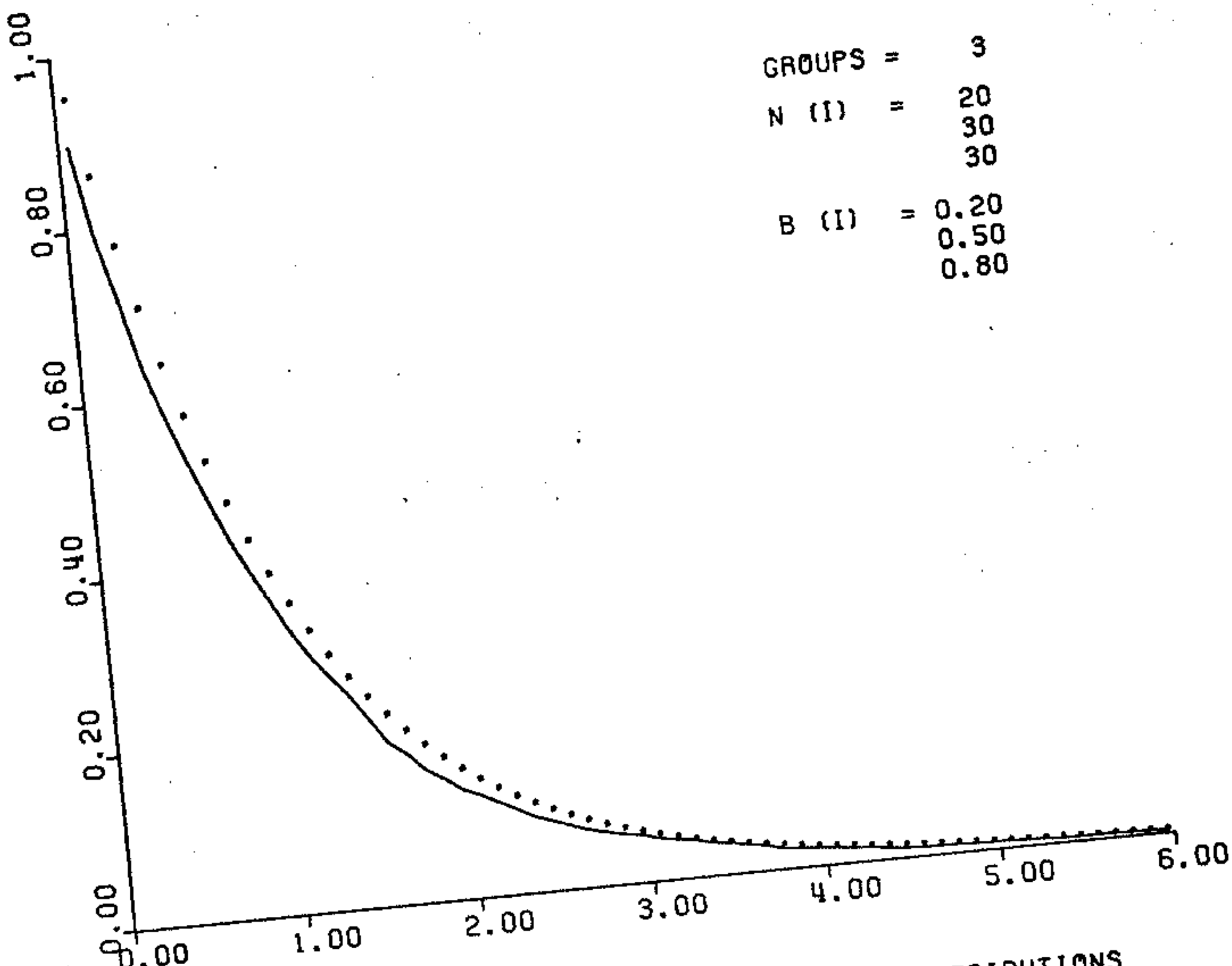


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

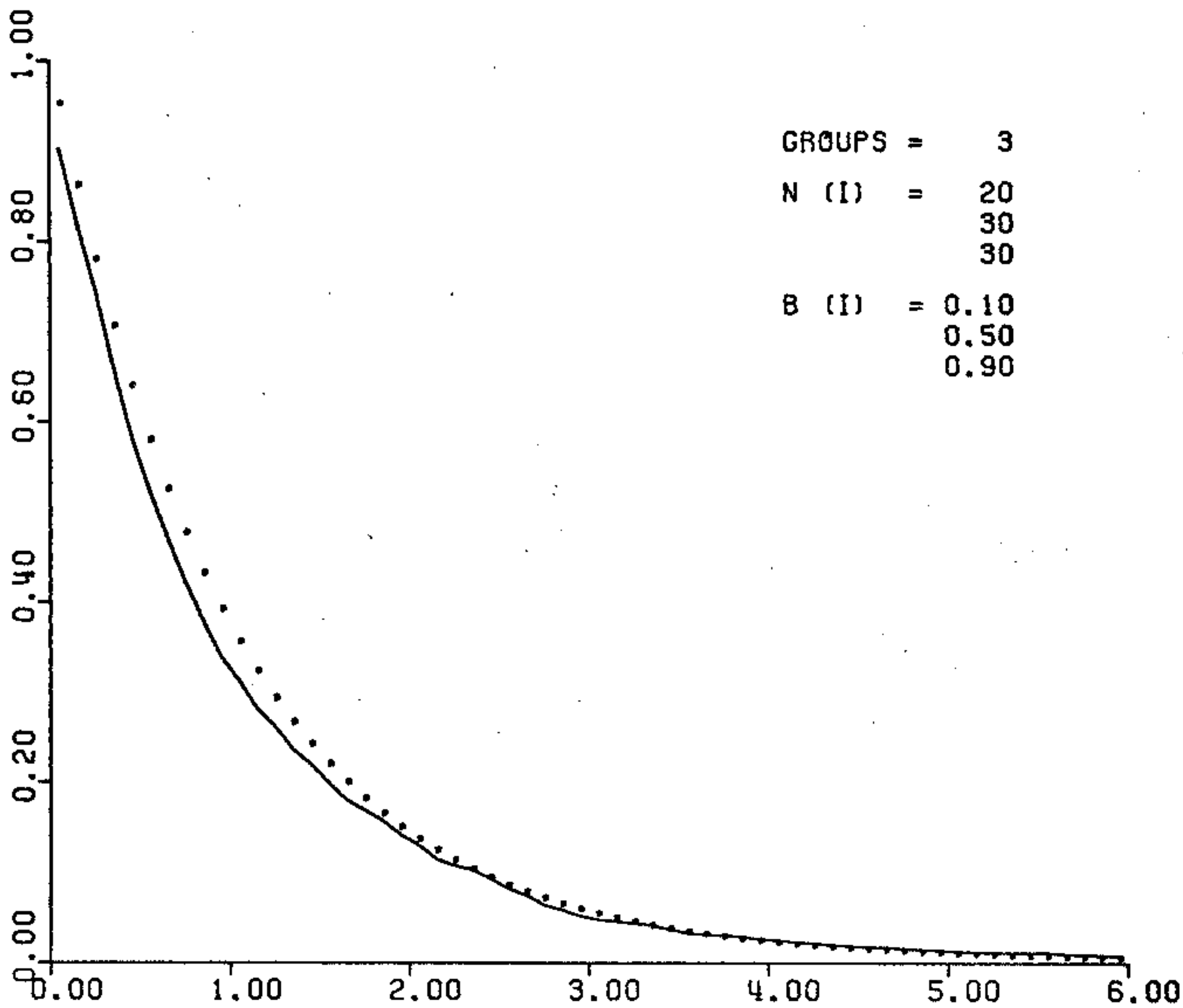
FIGURE 47 : SIMULATION NUMBER 47



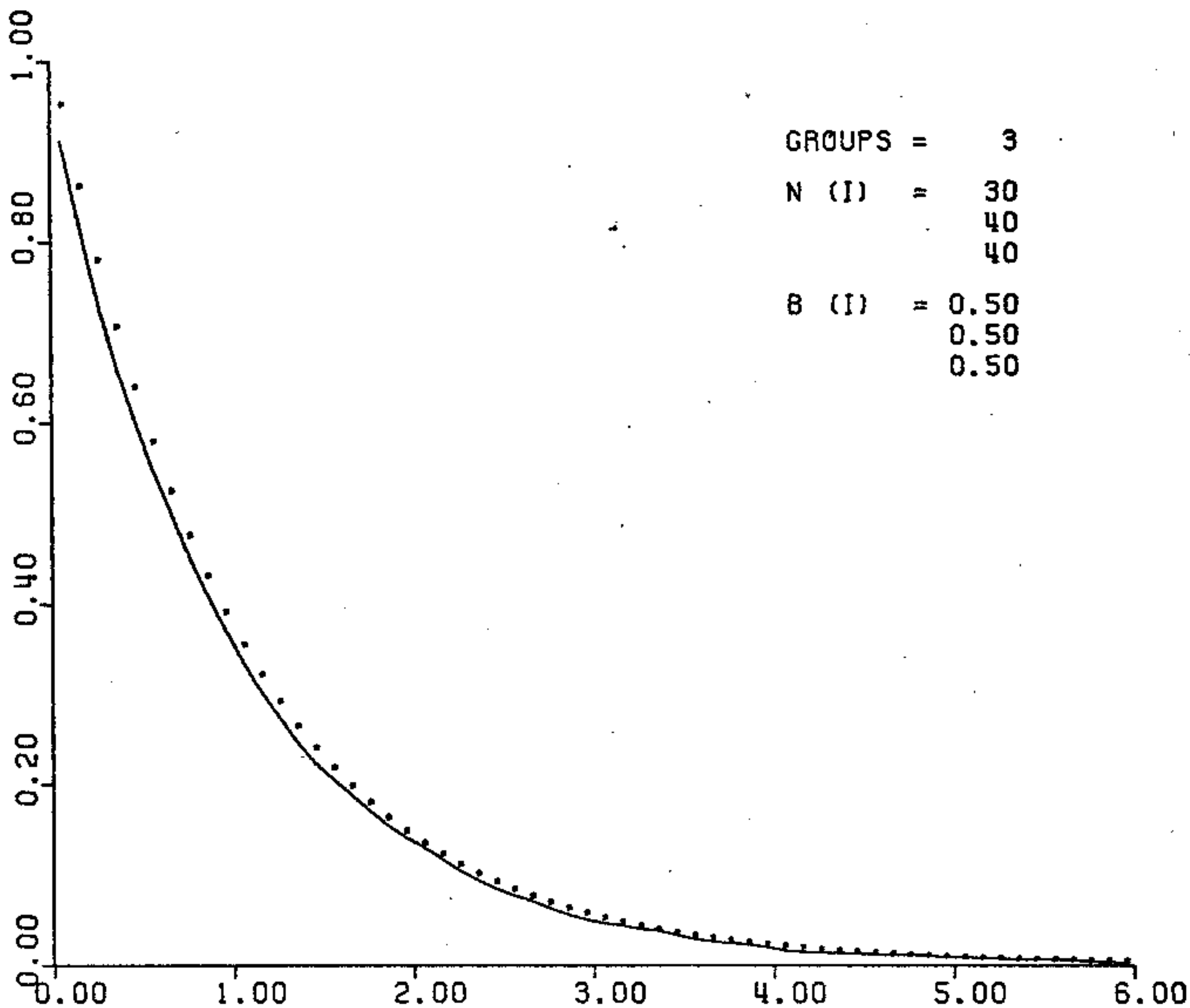
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 48 : SIMULATION NUMBER 48



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 49 : SIMULATION NUMBER 49

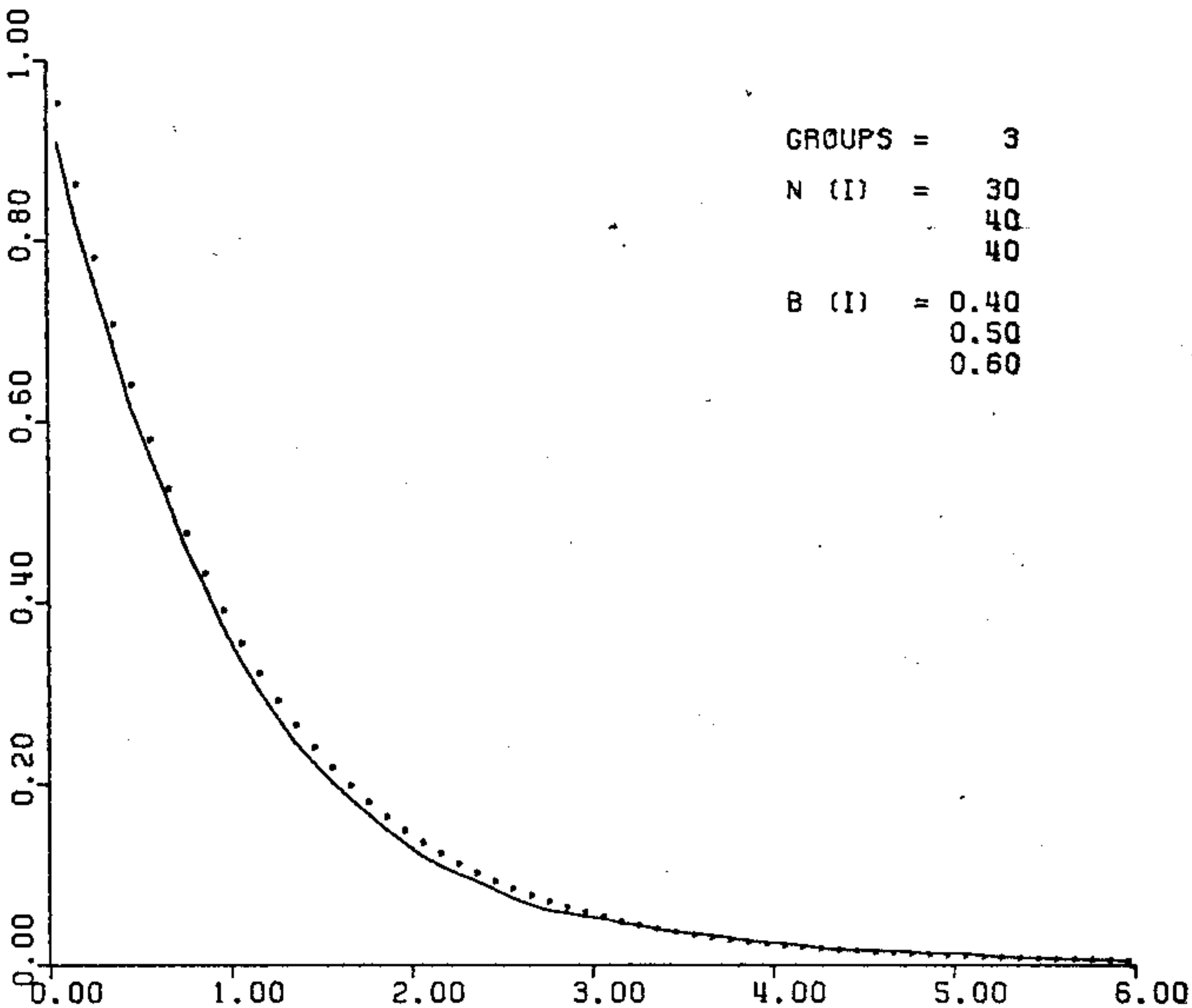


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 50 : SIMULATION NUMBER 50



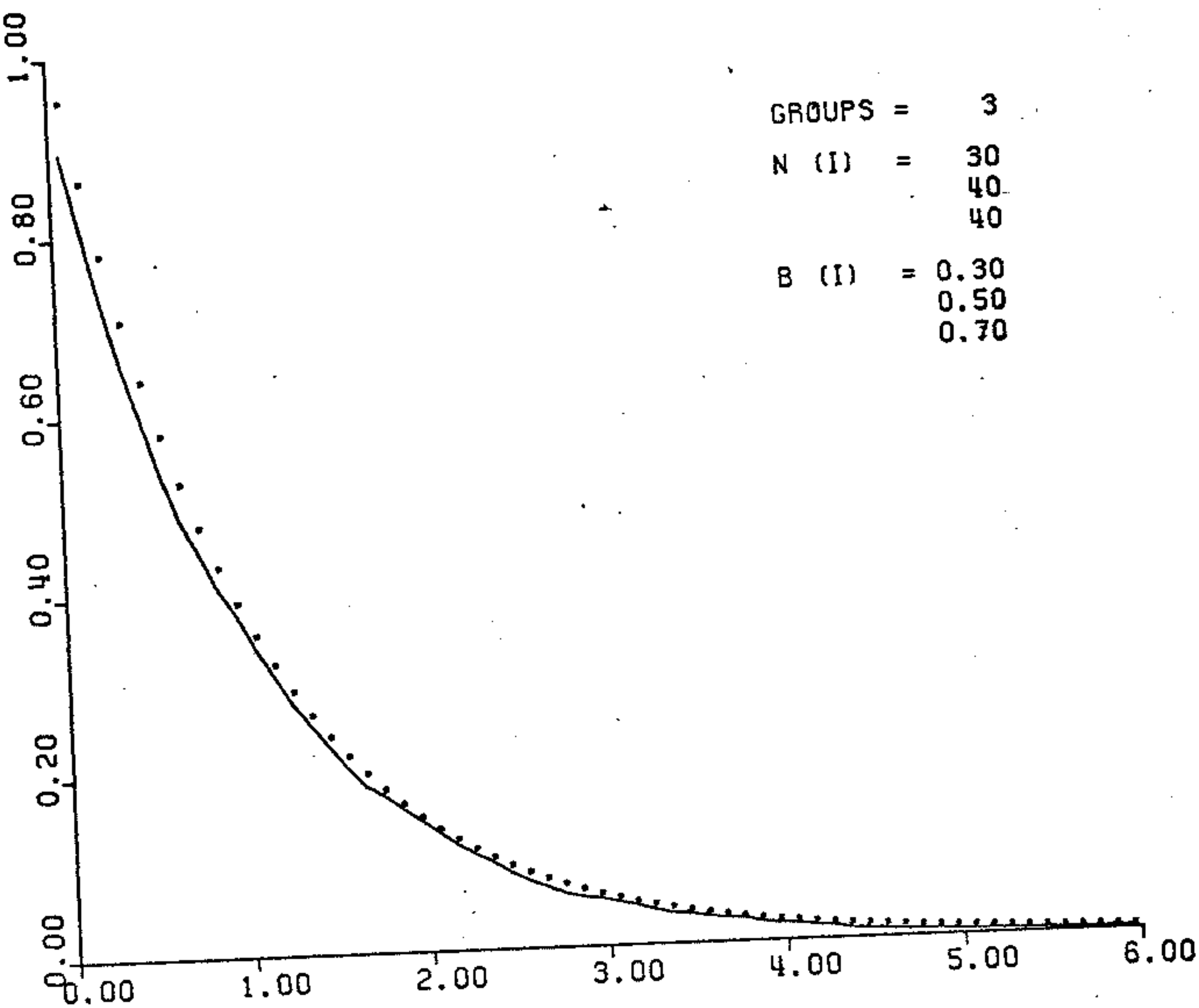
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 51 : SIMULATION NUMBER 51



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 52 : SIMULATION NUMBER 52



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

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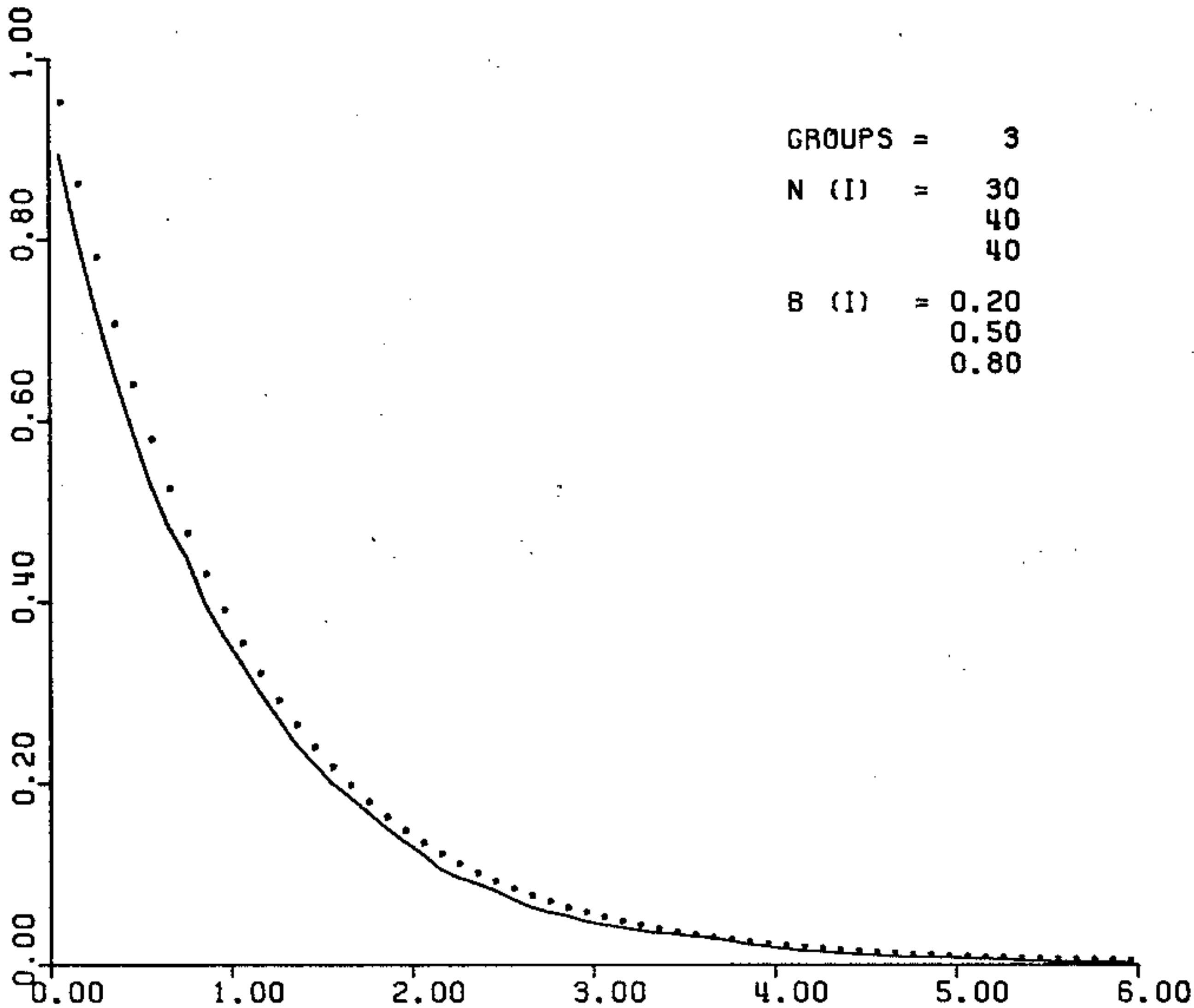

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

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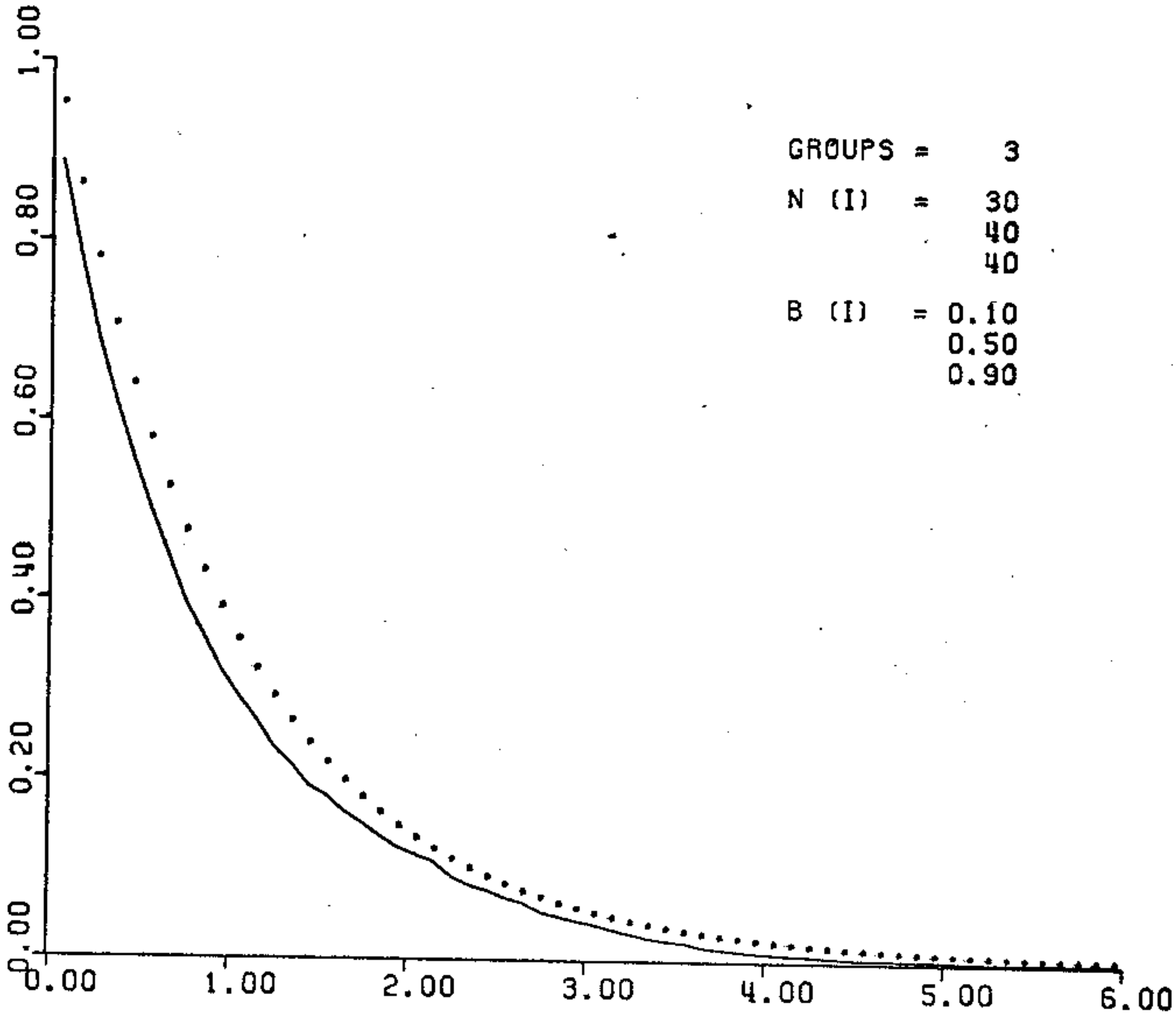

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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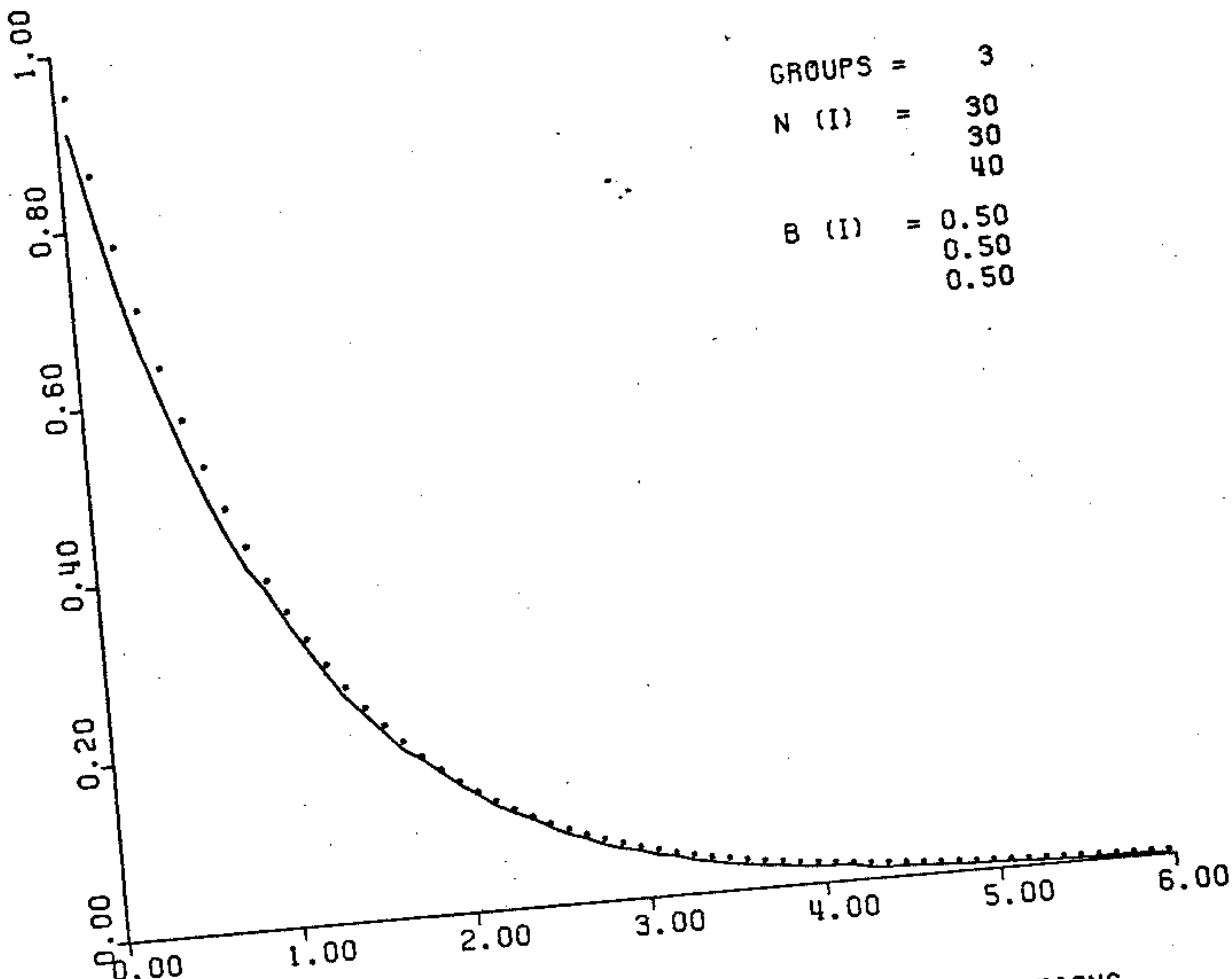

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

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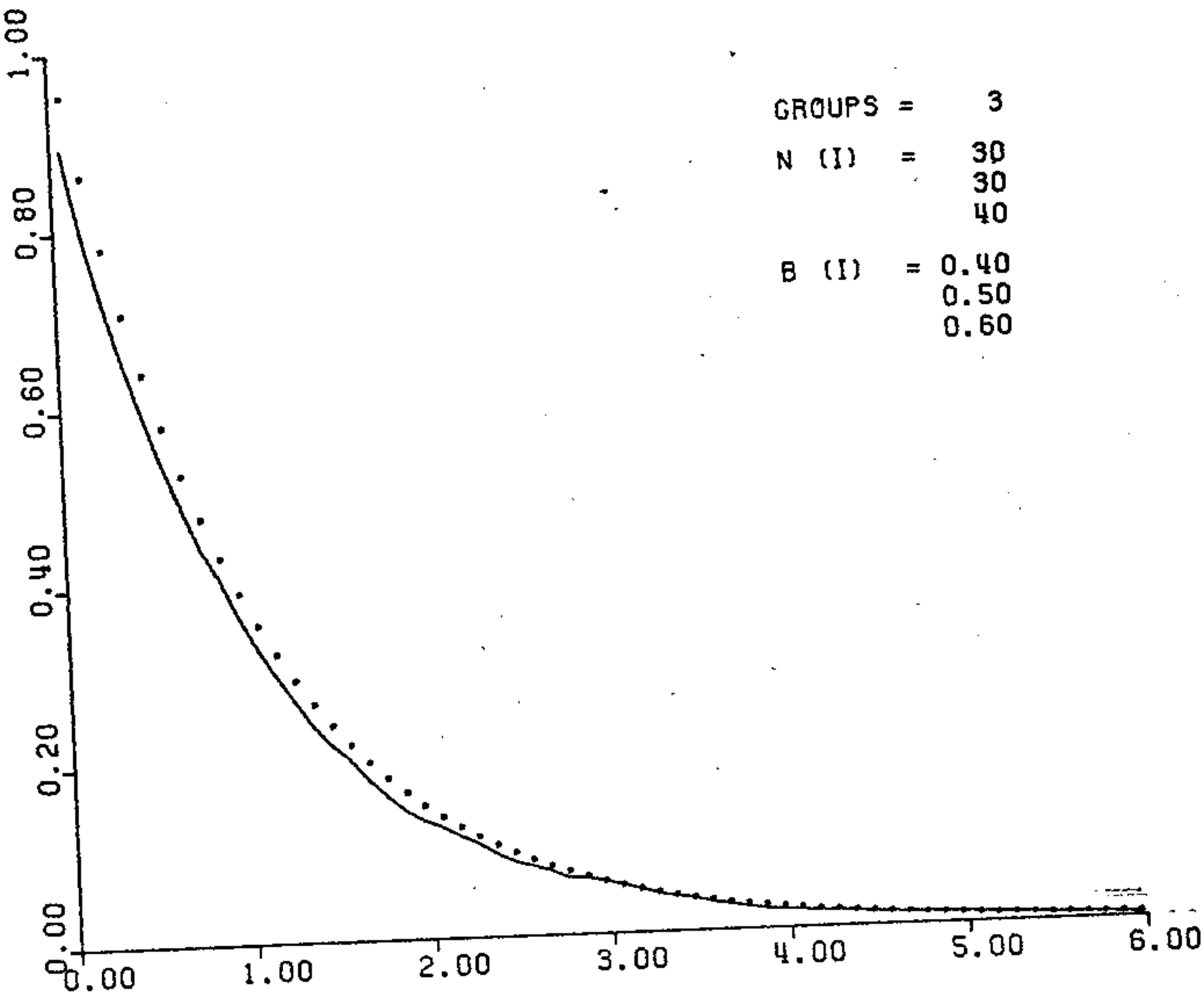
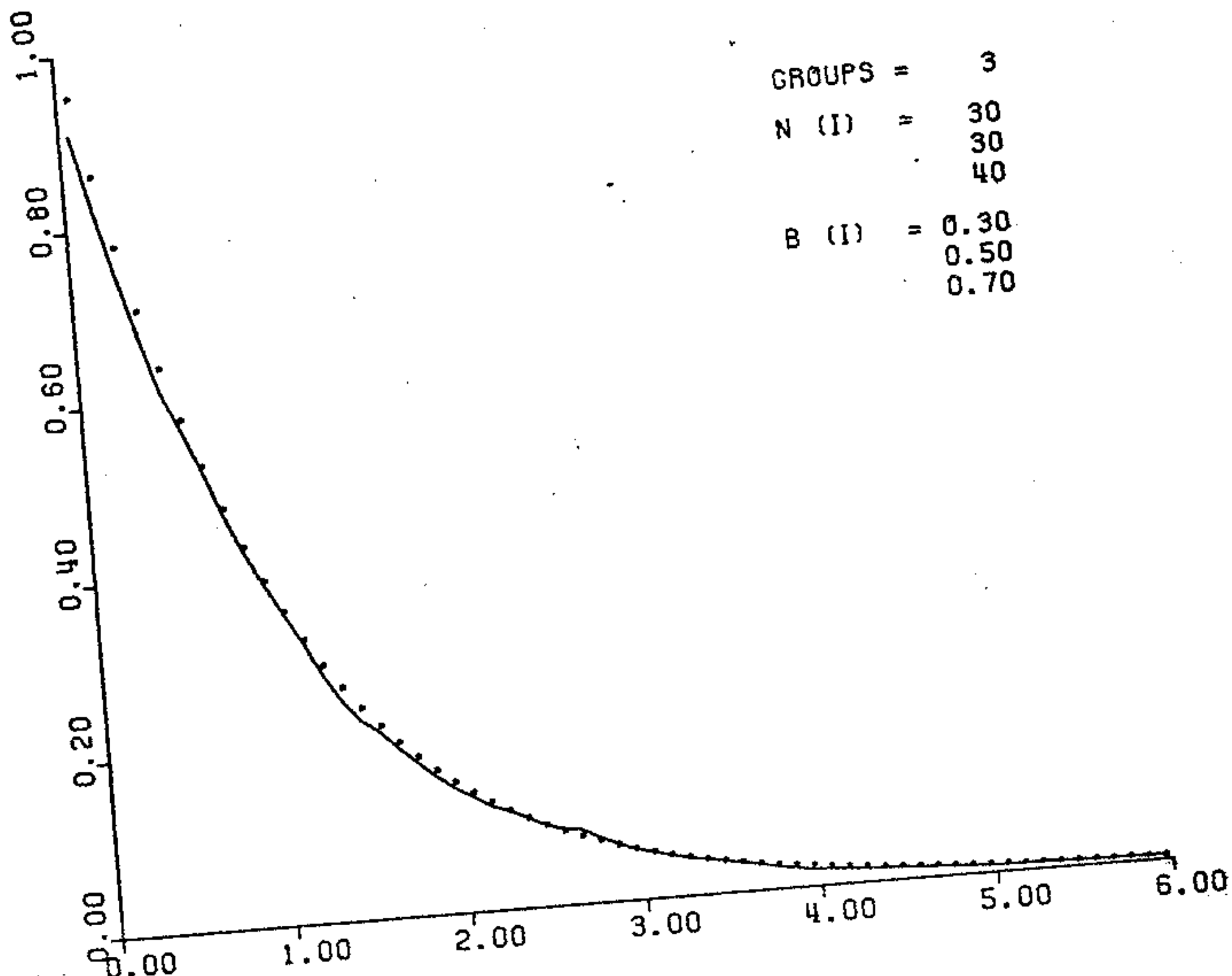
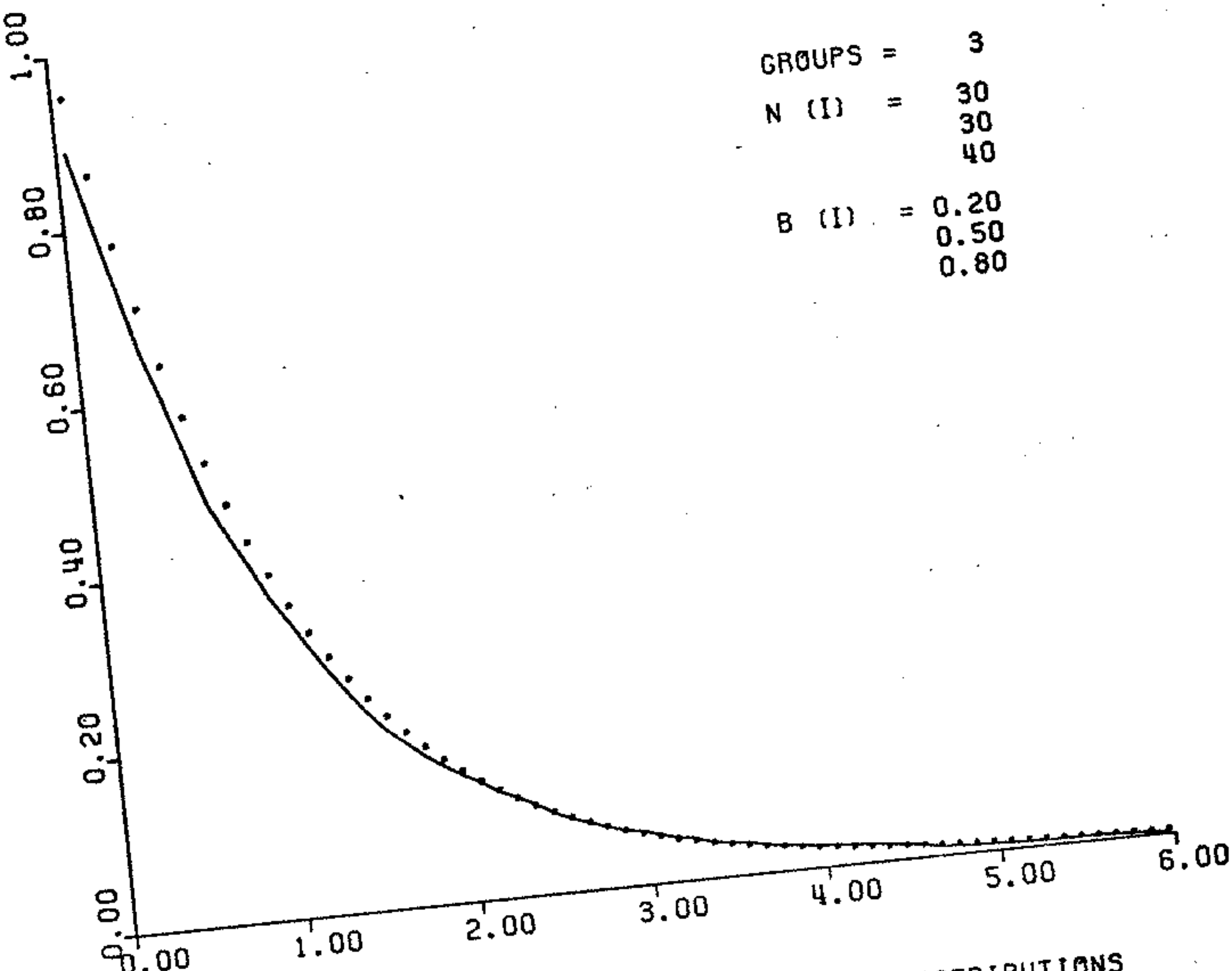


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 57 : SIMULATION NUMBER 57



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 58 : SIMULATION NUMBER 58



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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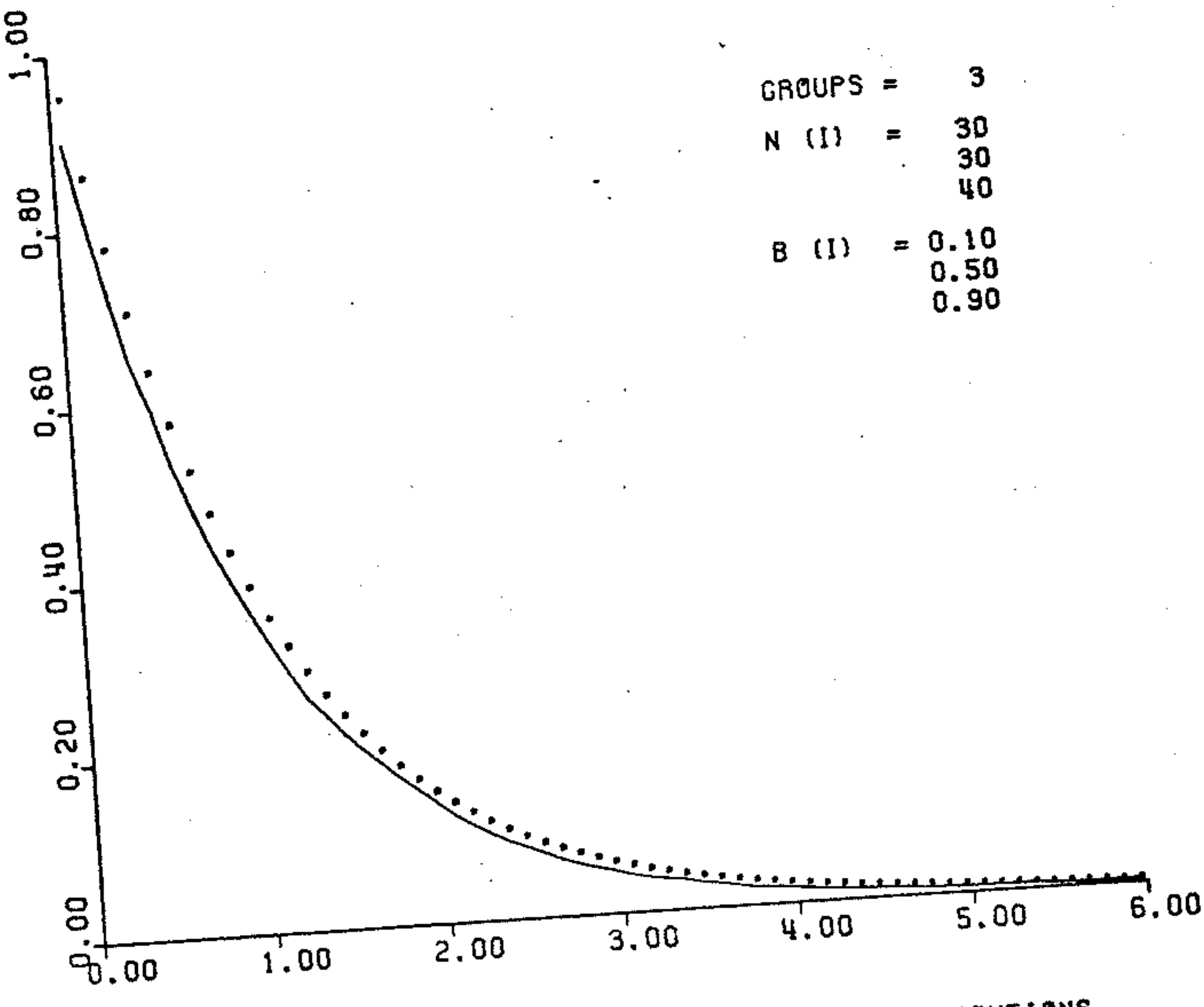

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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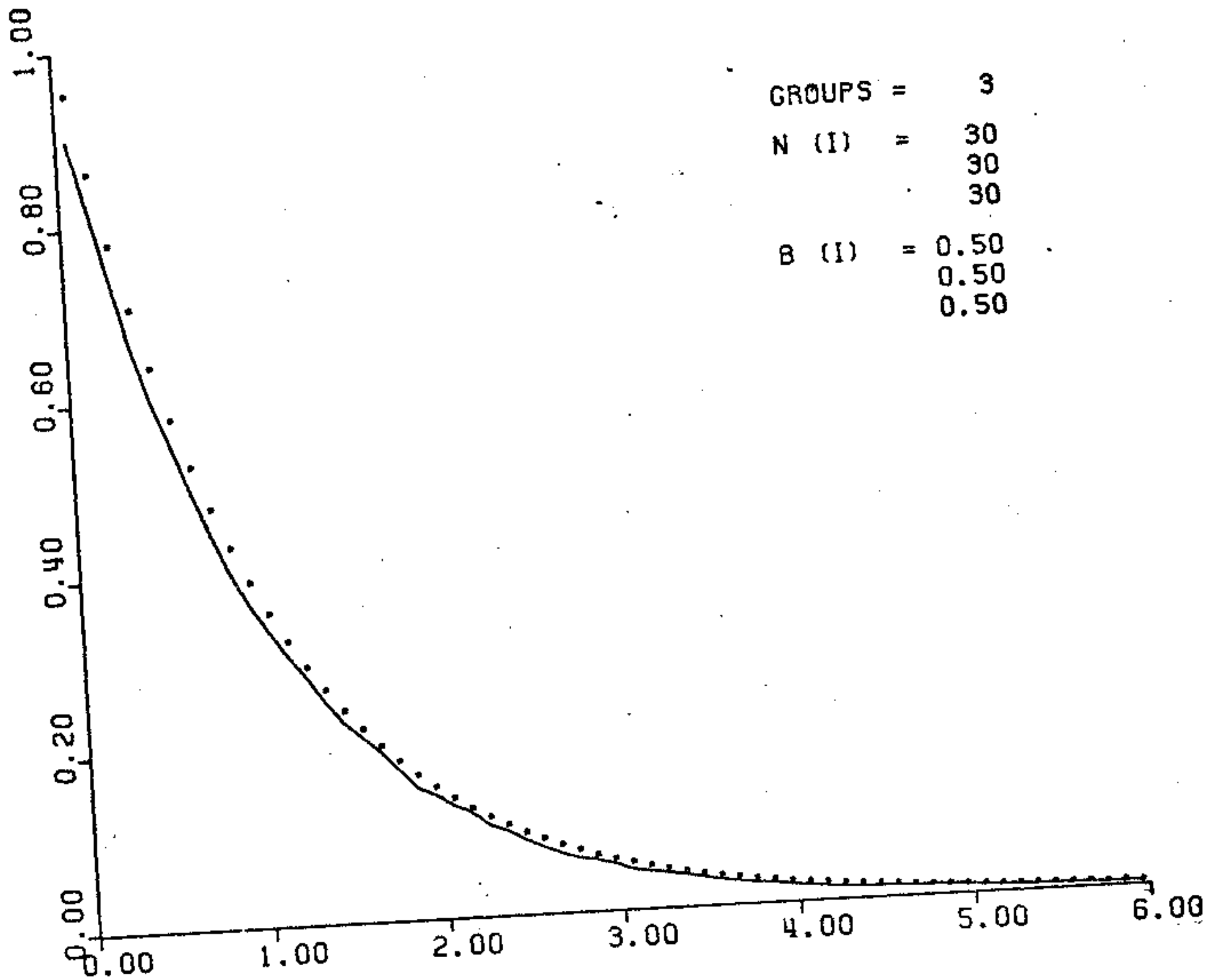
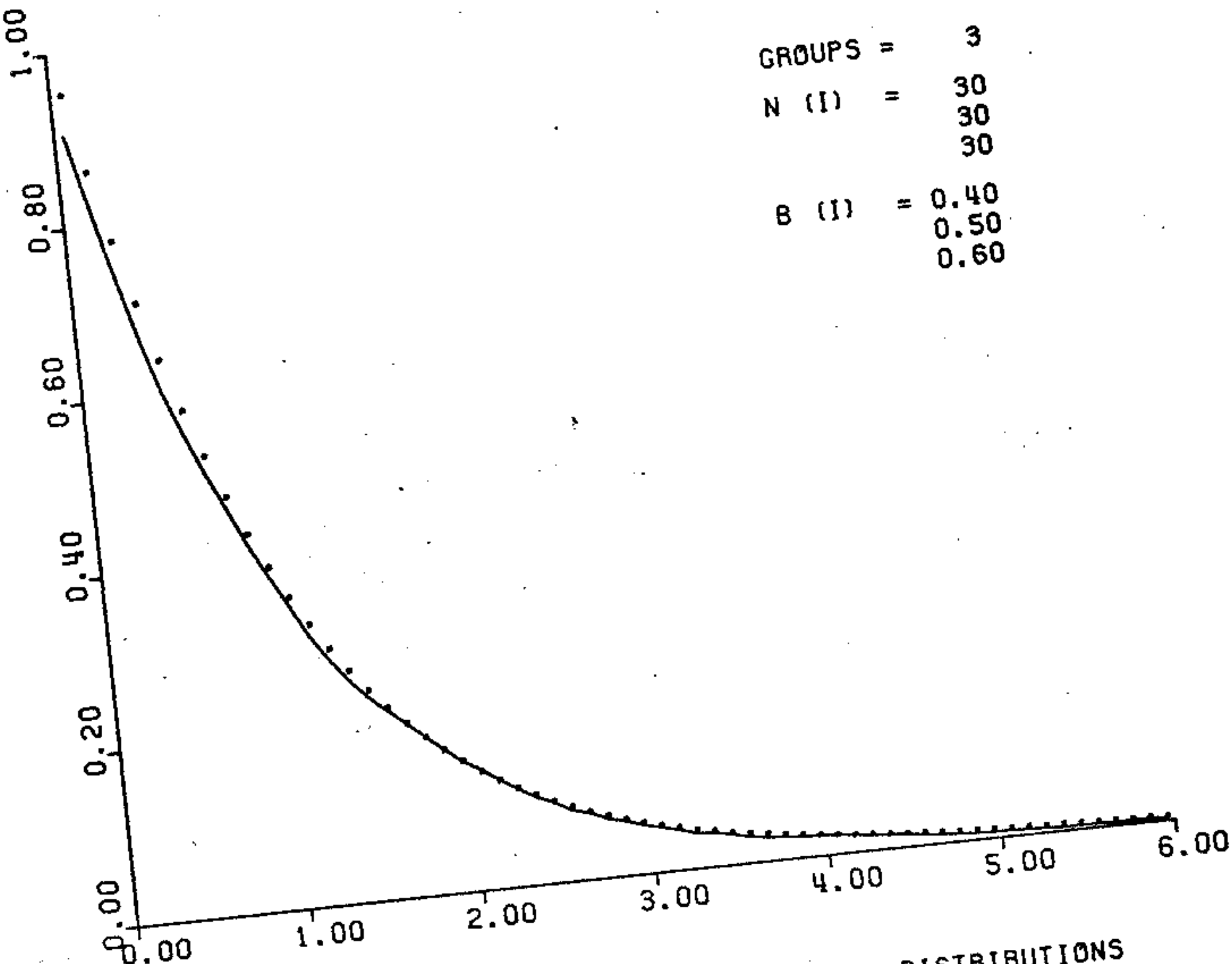


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 61 : SIMULATION NUMBER 61



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

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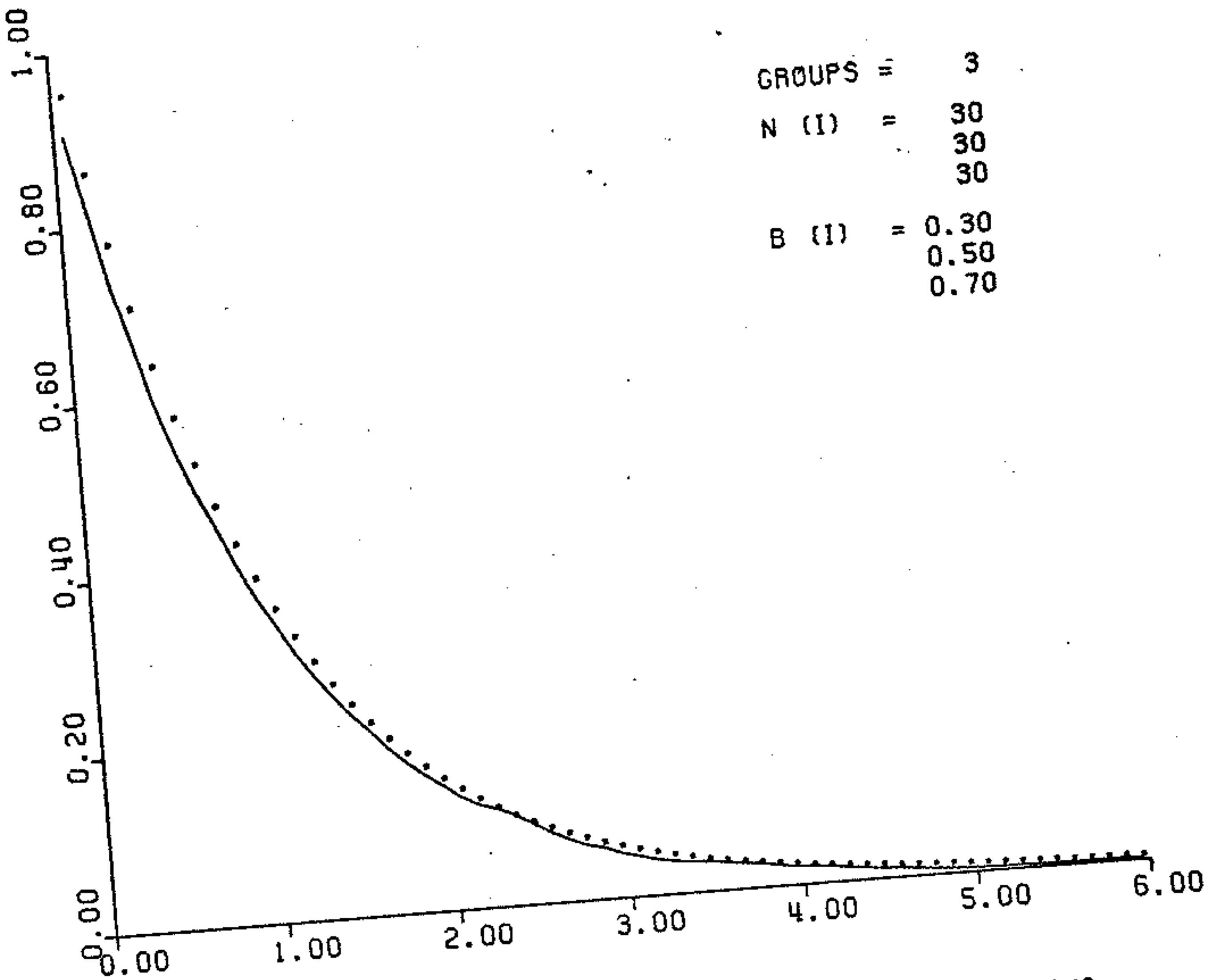

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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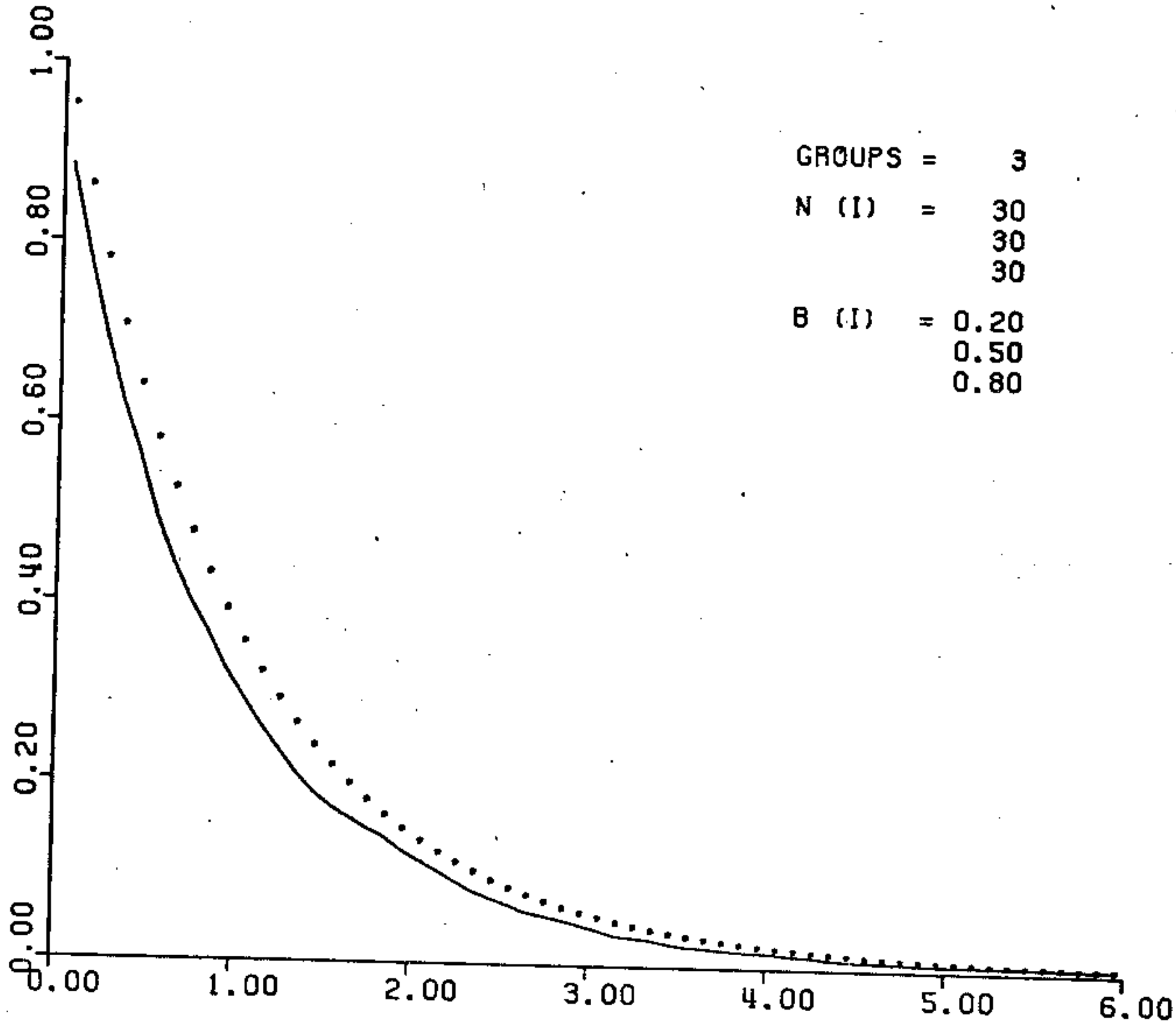
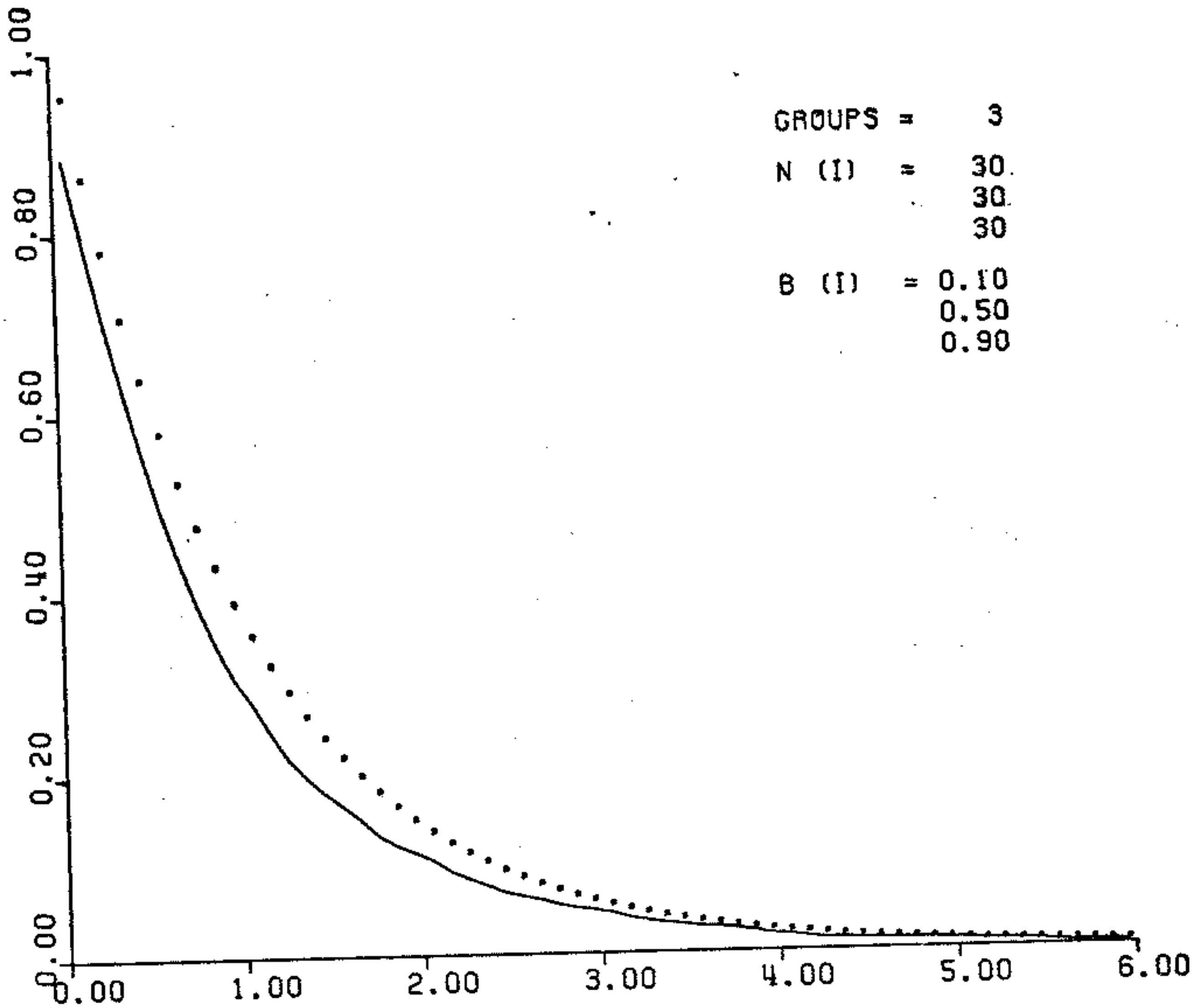
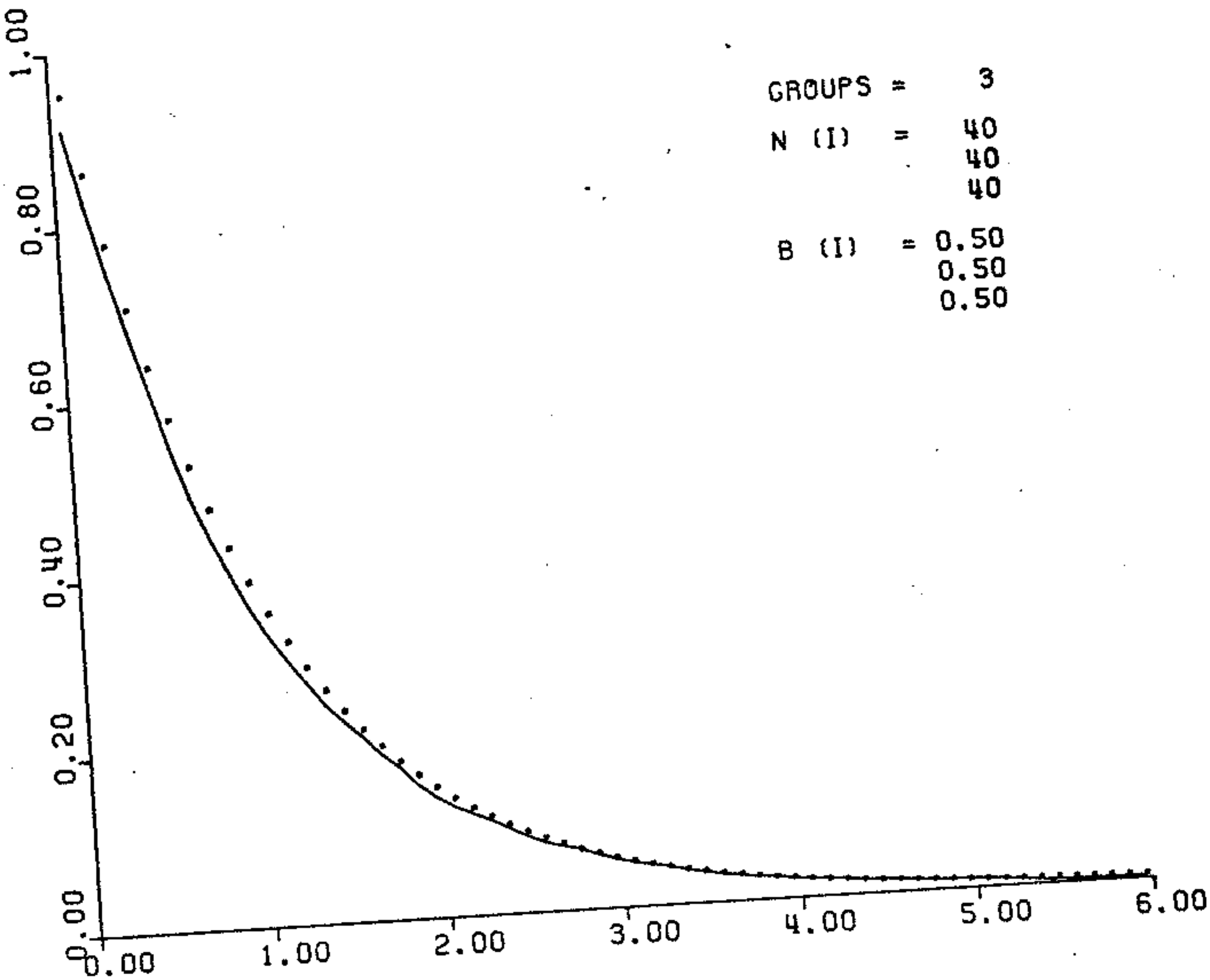


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 64 : SIMULATION NUMBER 64



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 65 : SIMULATION NUMBER 65



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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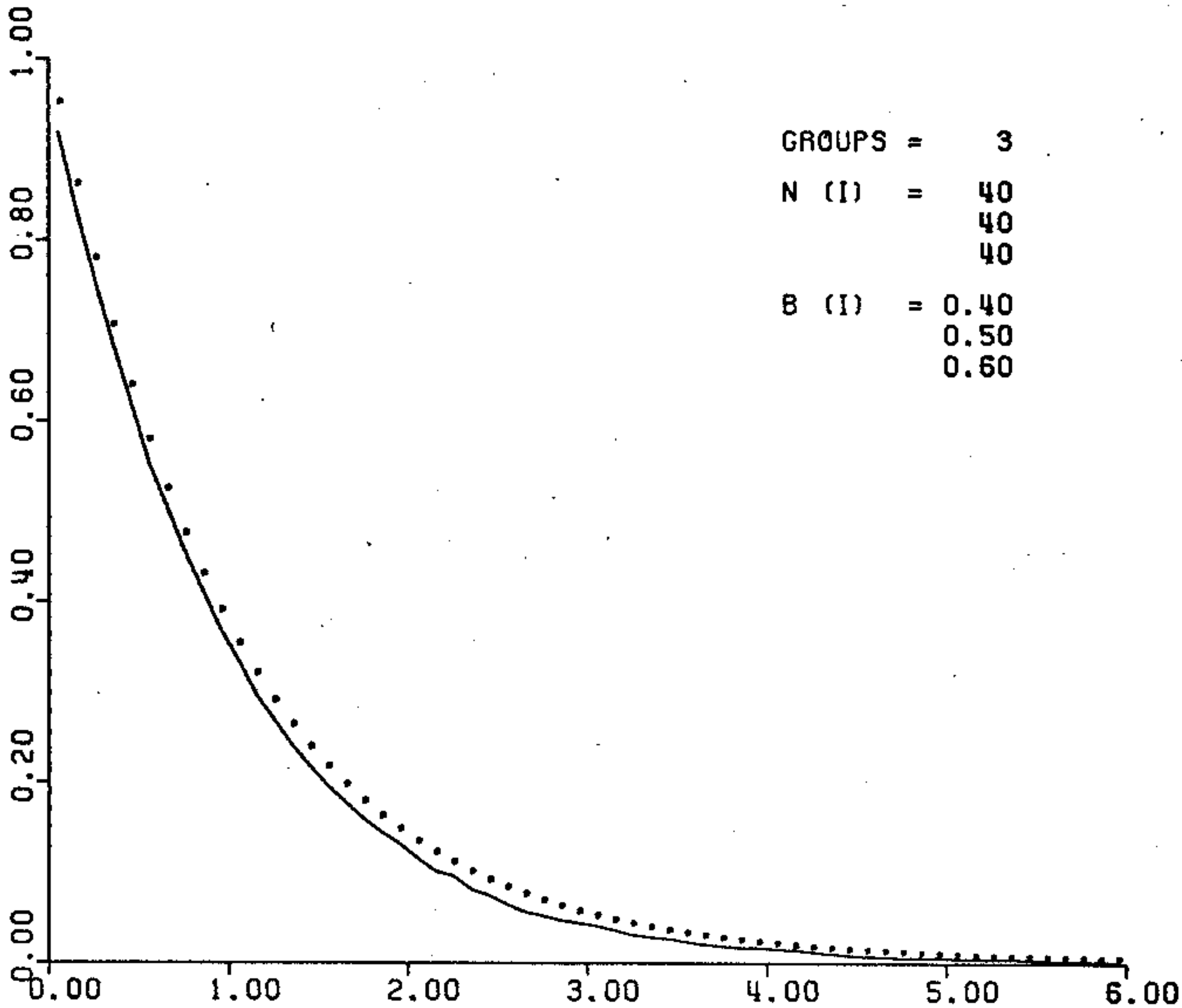

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

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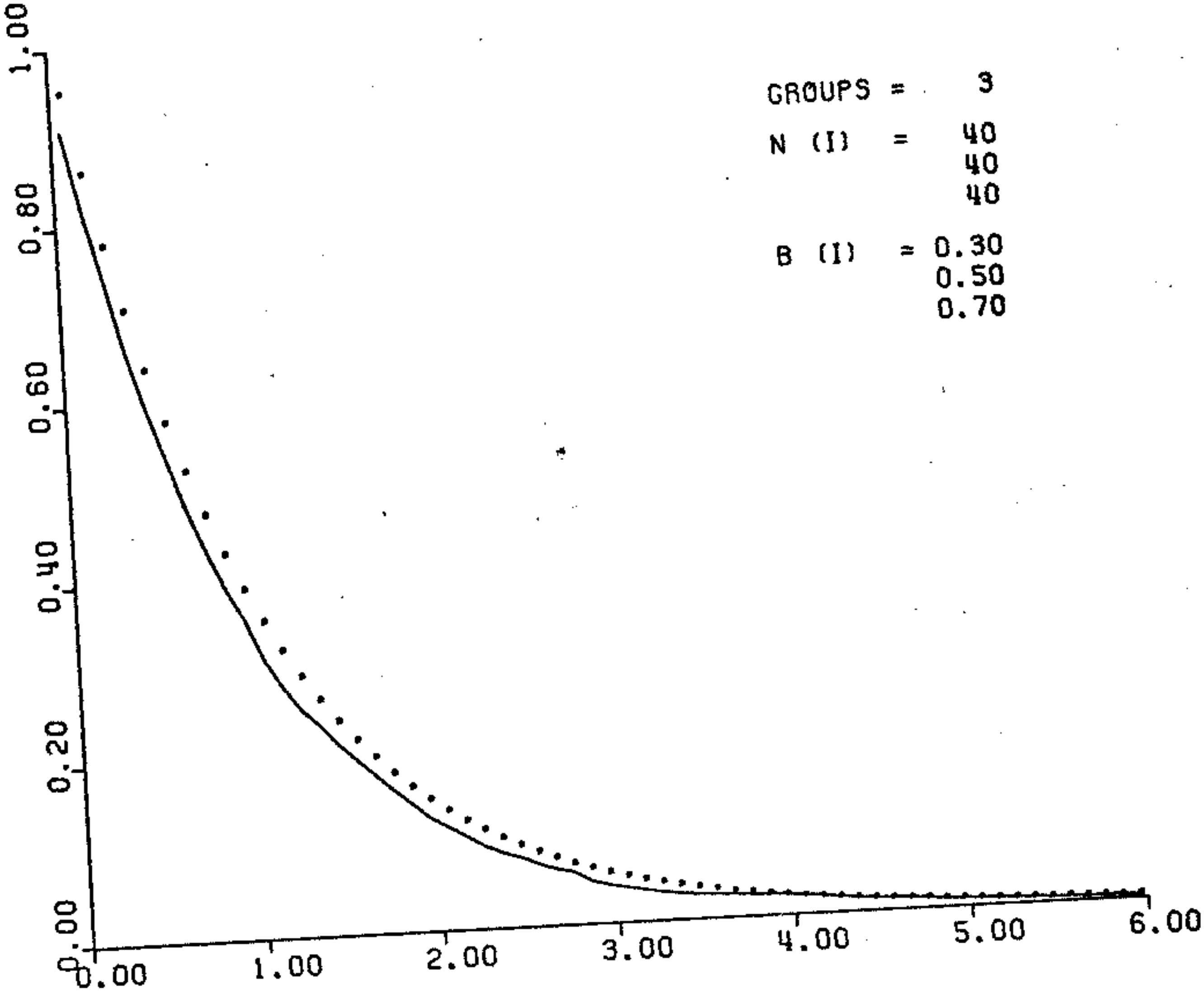
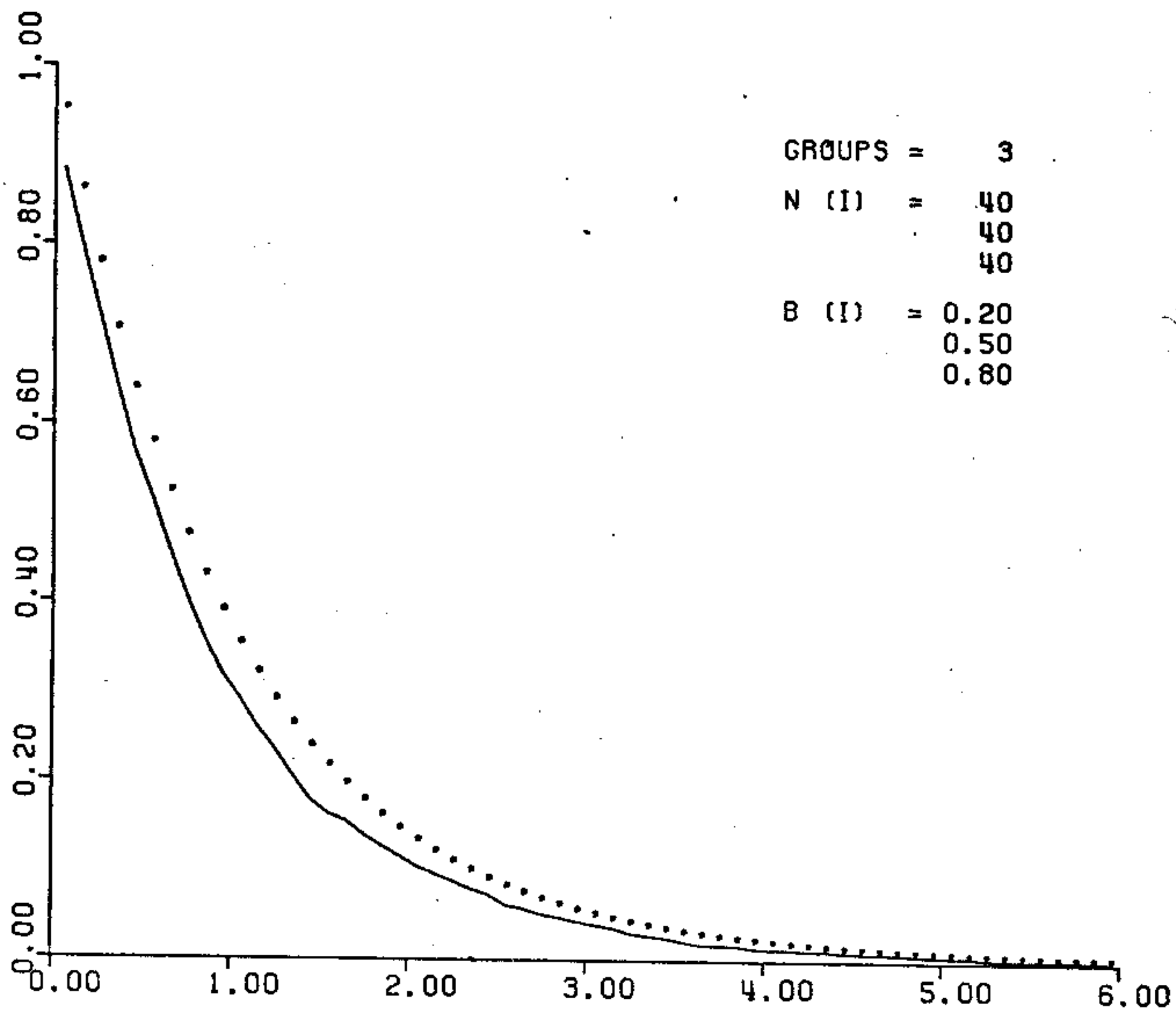


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 68 : SIMULATION NUMBER 68



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

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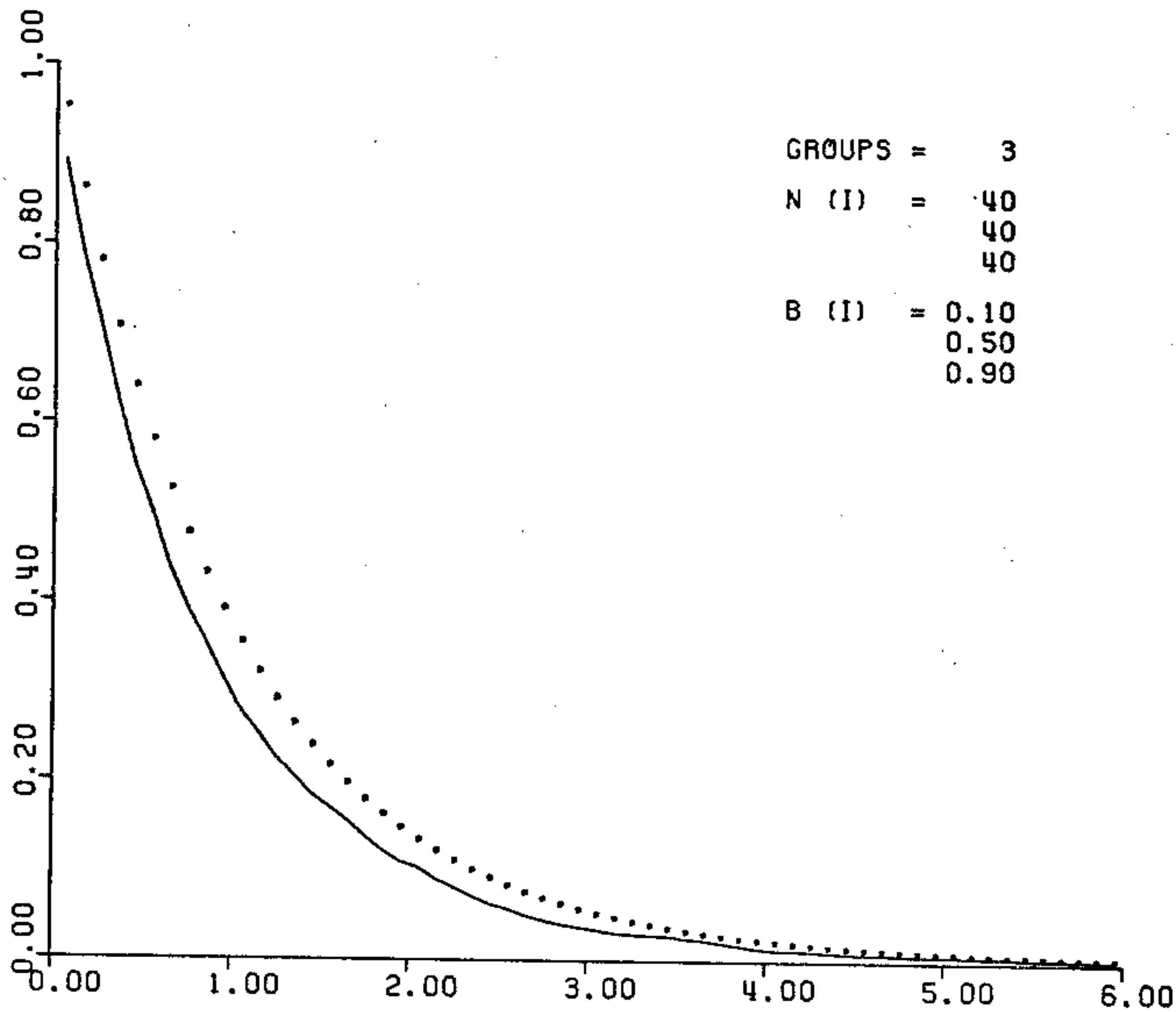
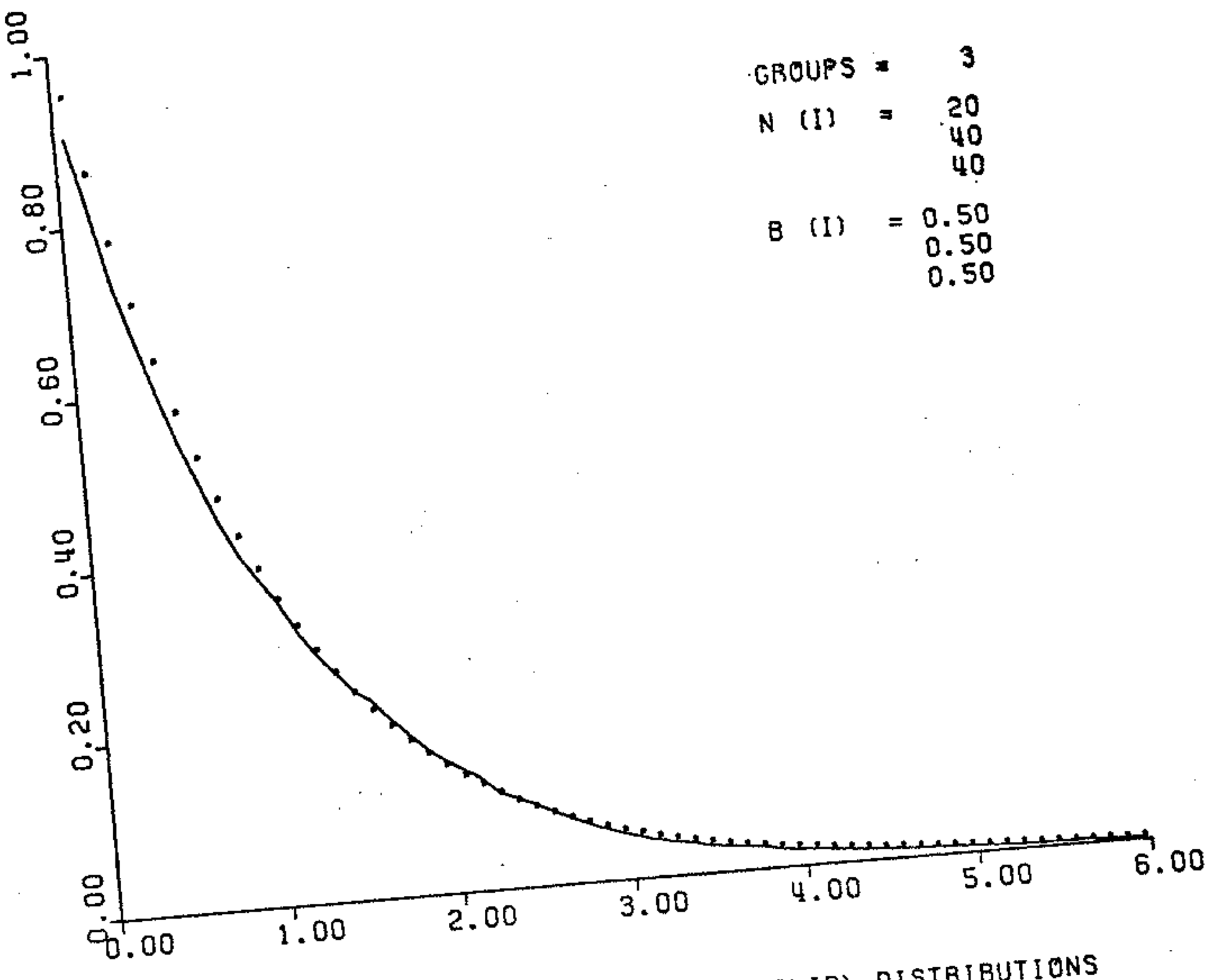
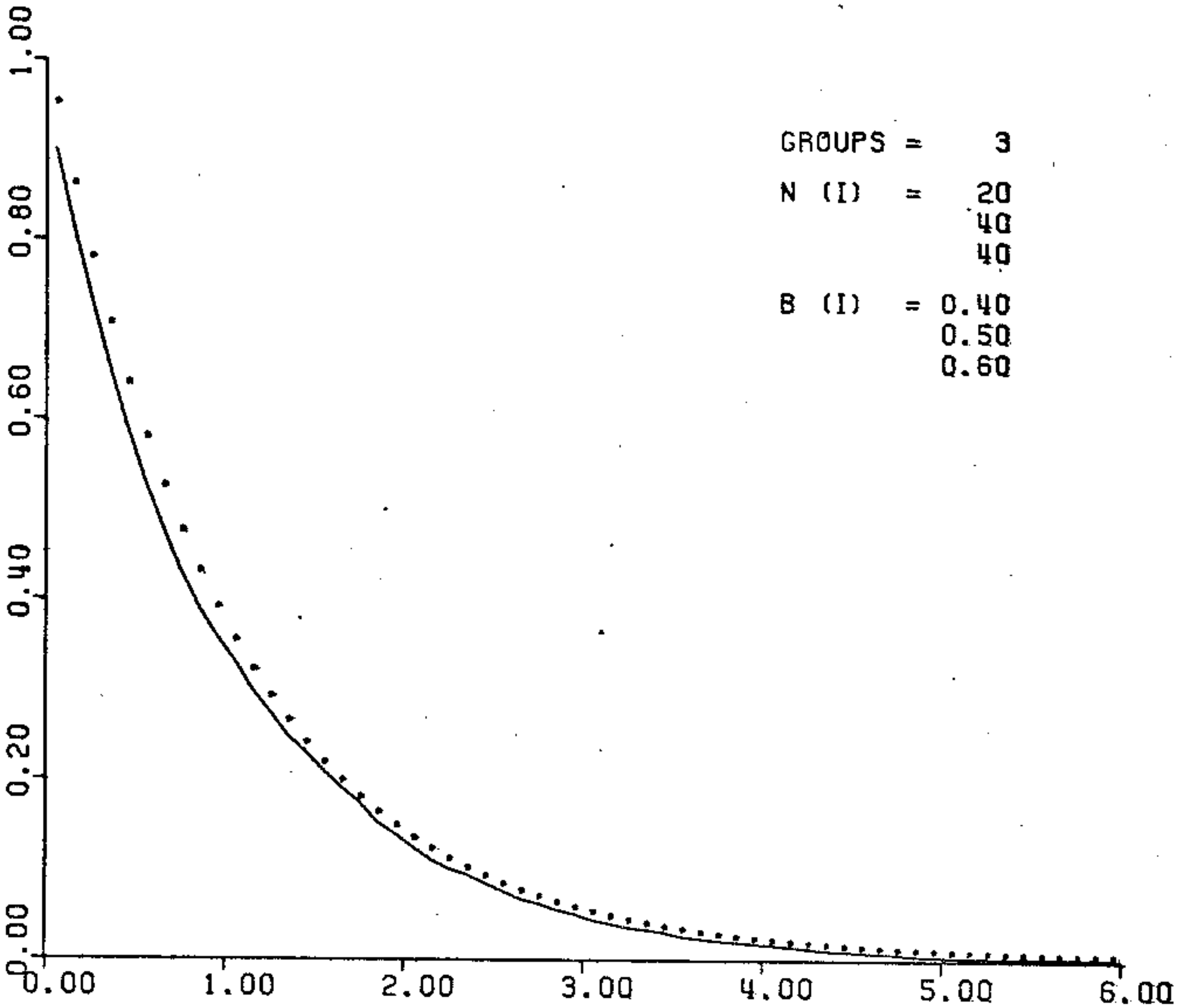


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 70 : SIMULATION NUMBER 70



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 71 : SIMULATION NUMBER 71



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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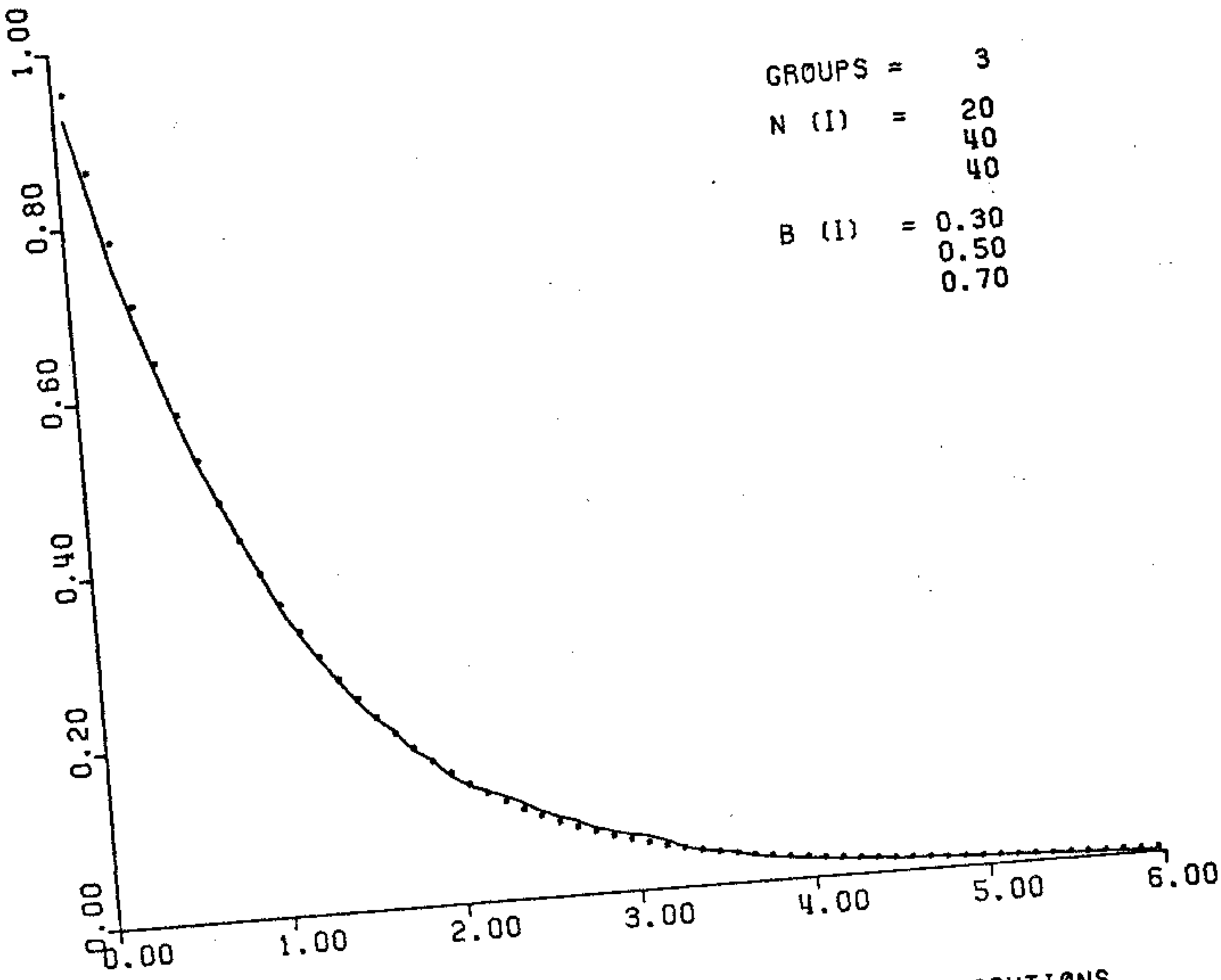

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

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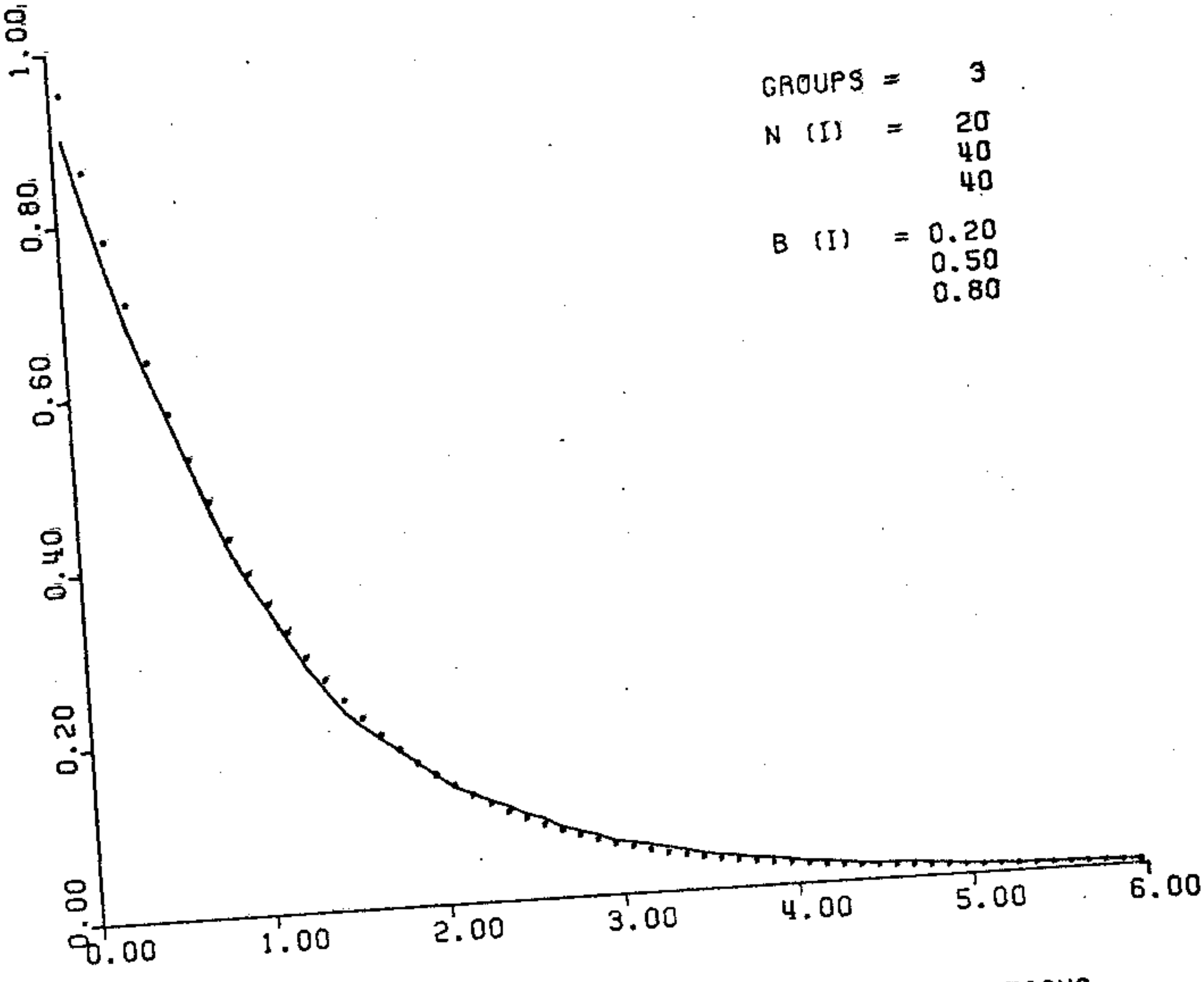
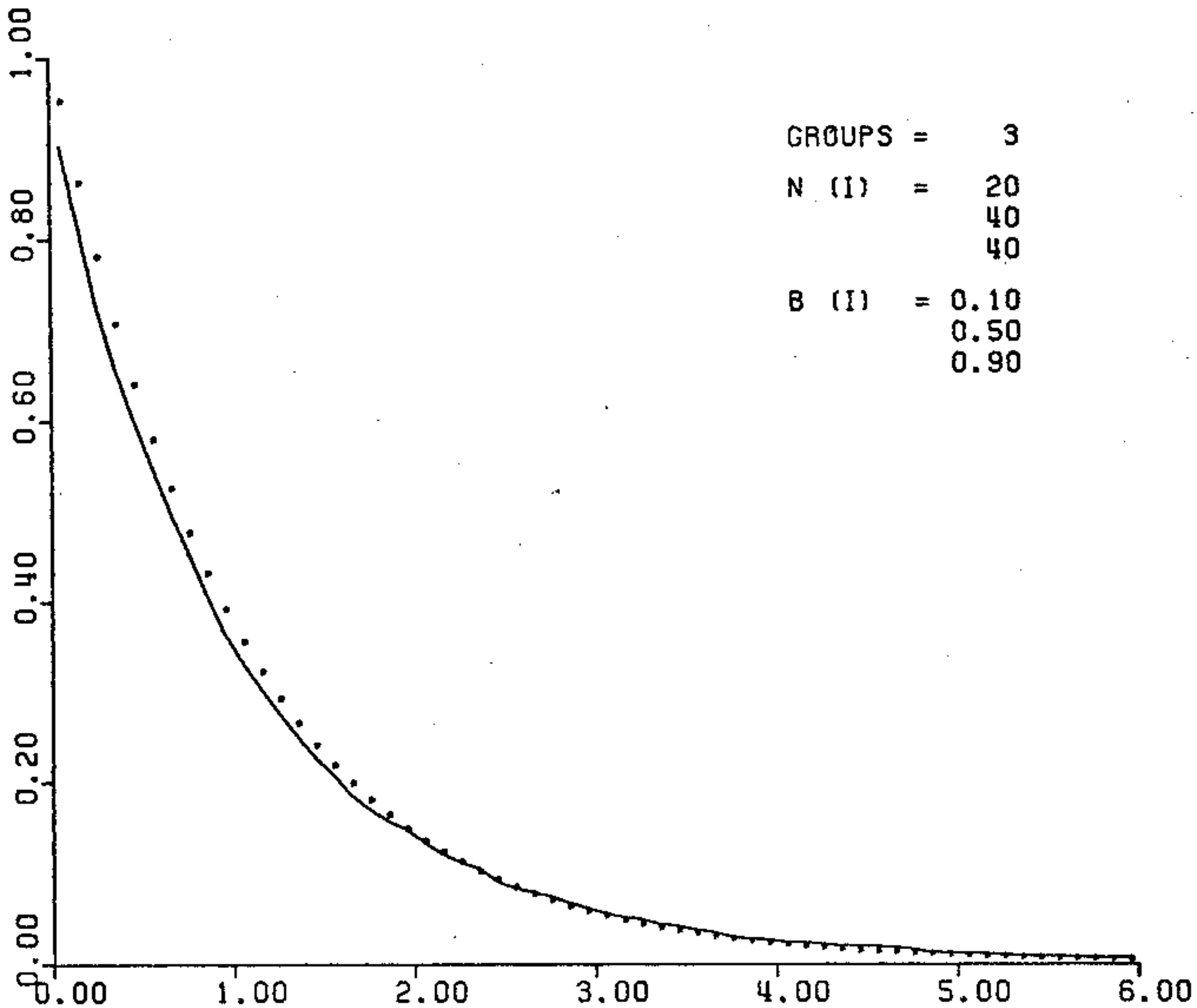


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 74 : SIMULATION NUMBER 74



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

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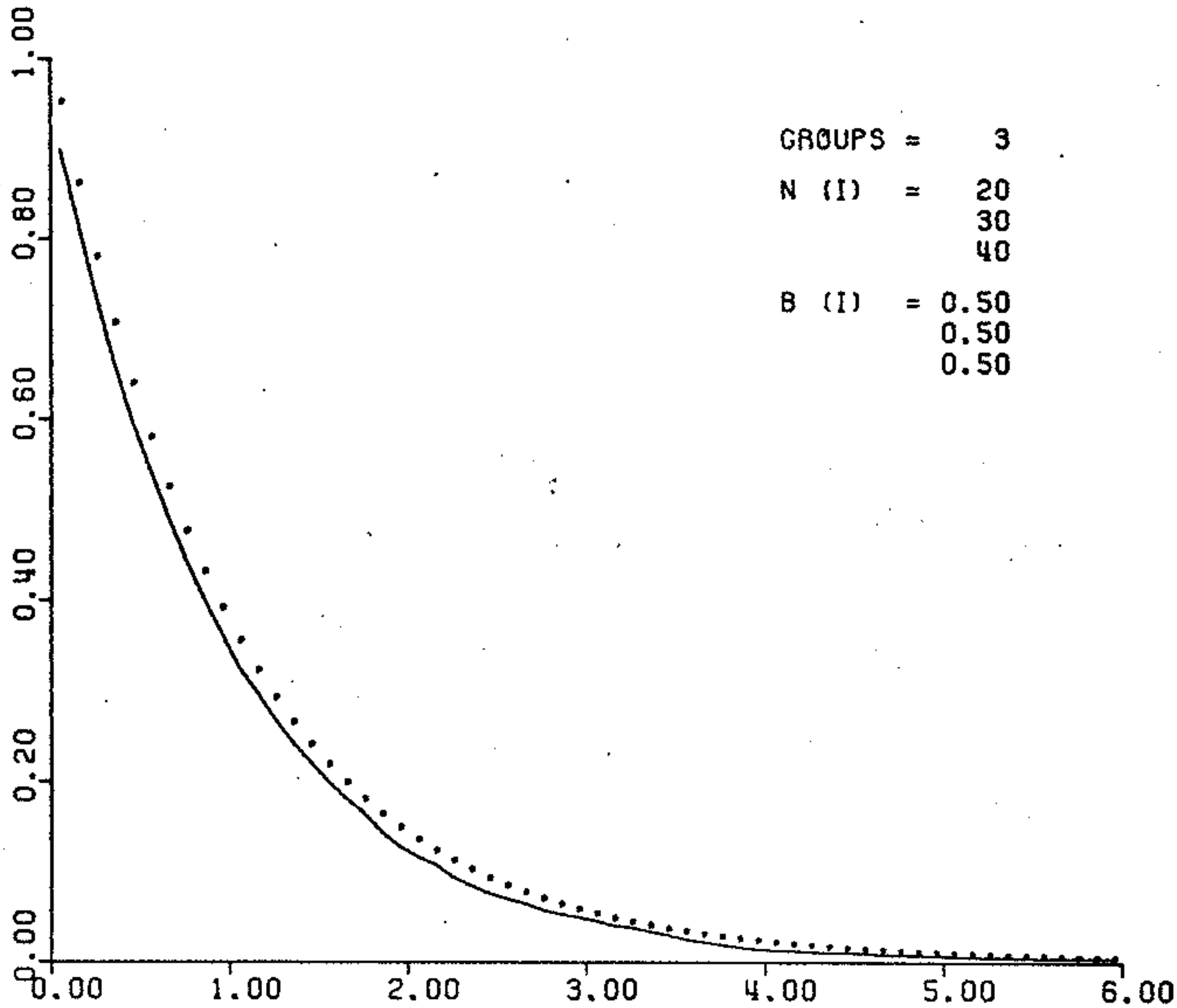

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

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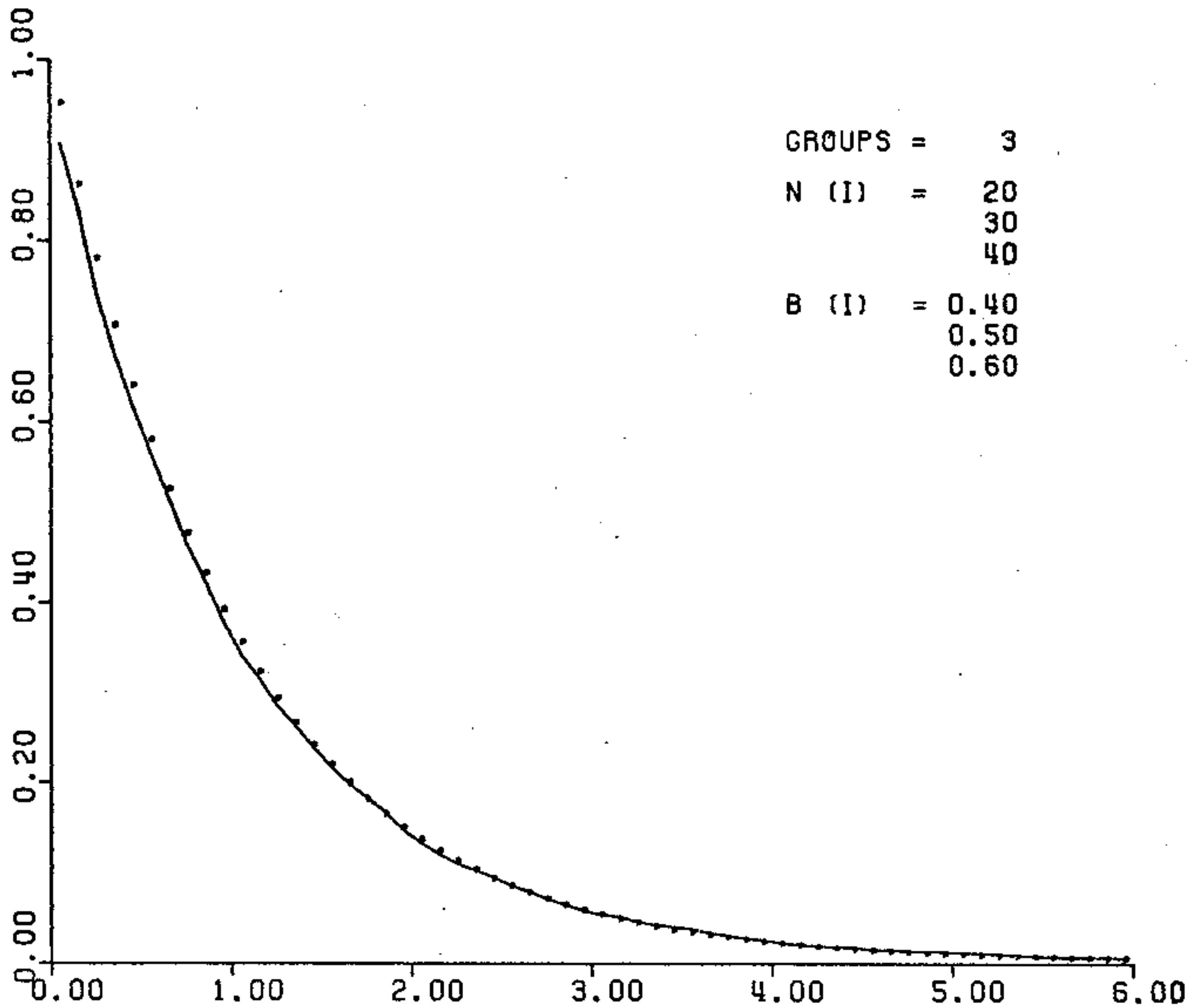
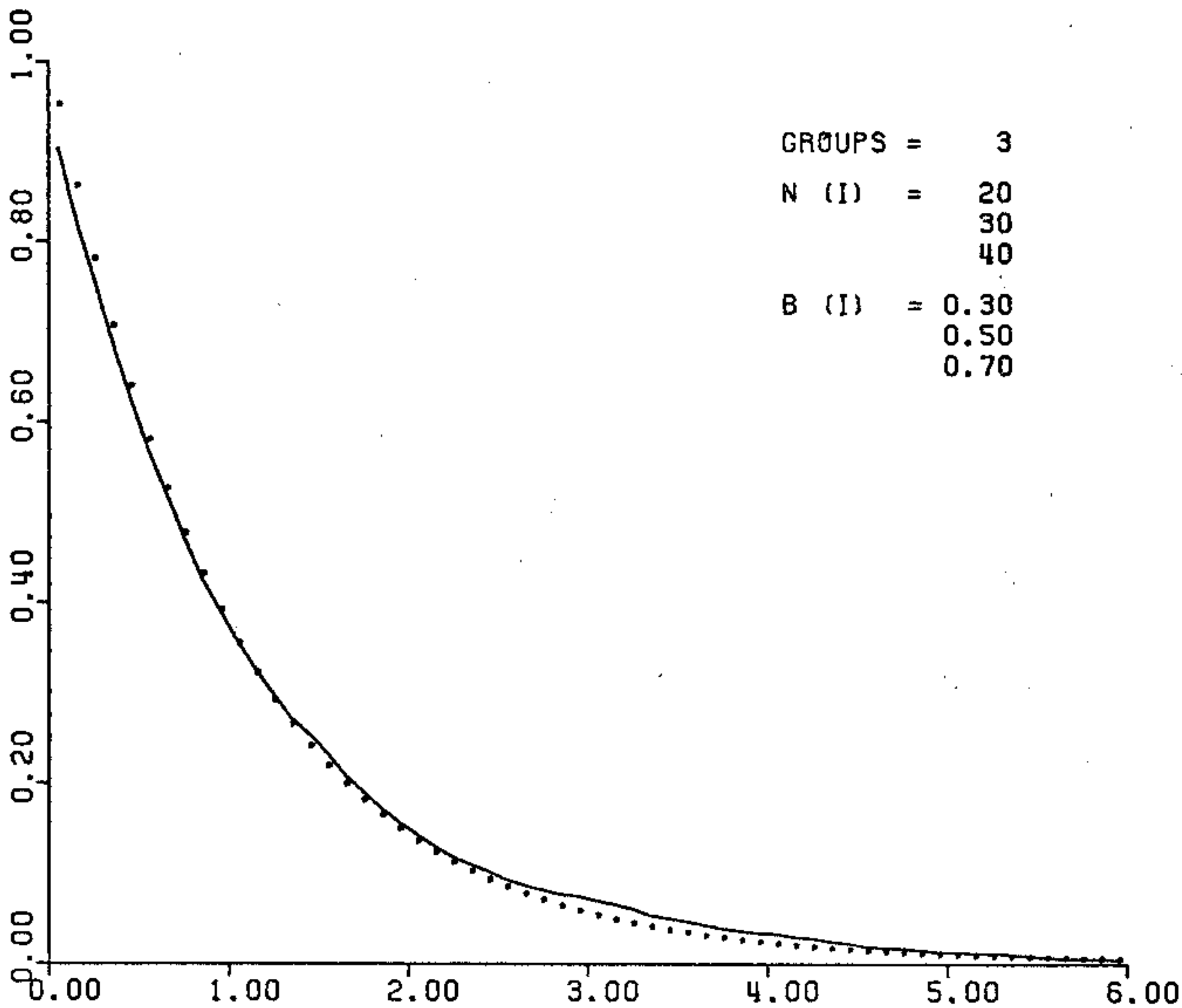


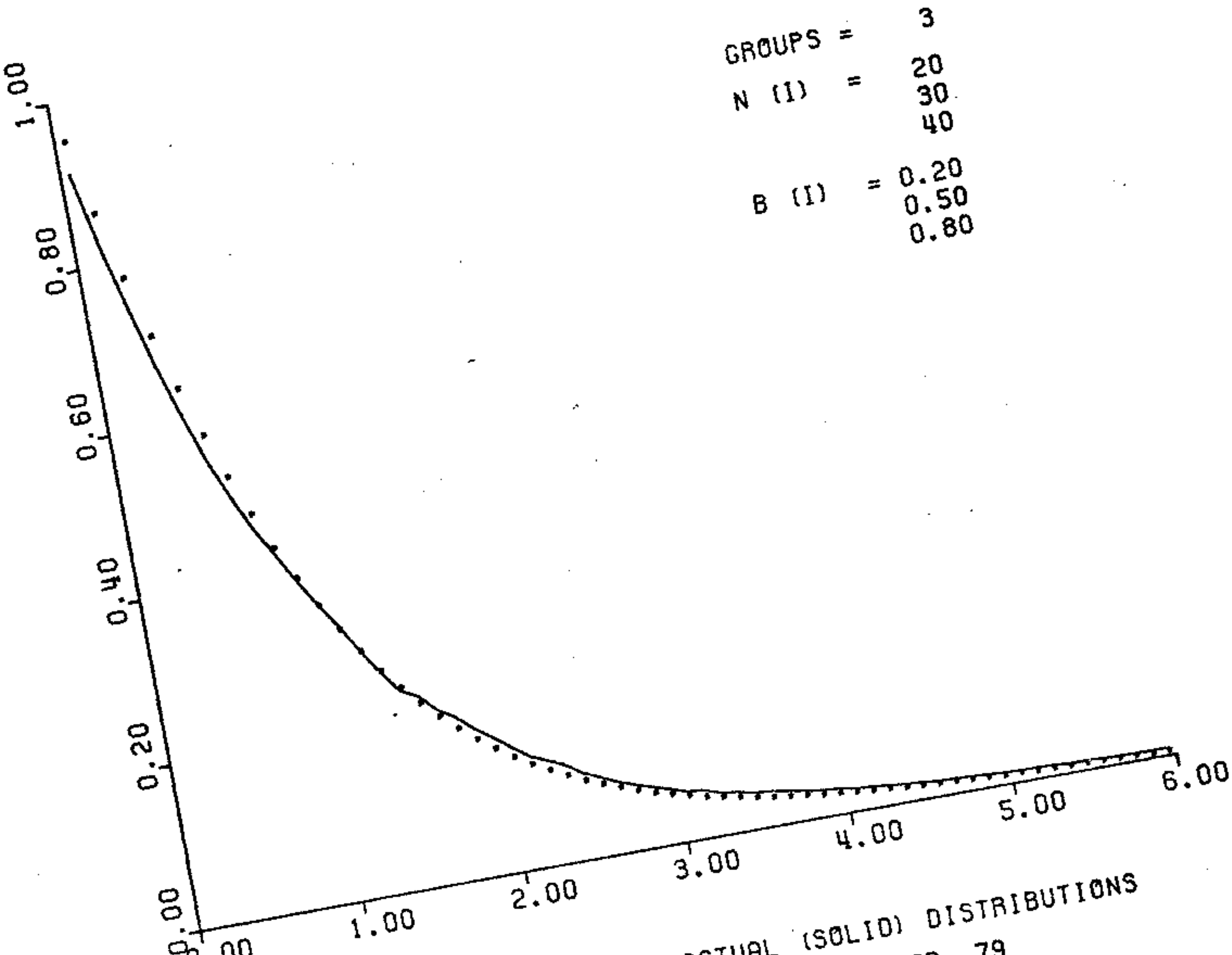
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 77 : SIMULATION NUMBER 77



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 78 : SIMULATION NUMBER 78



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

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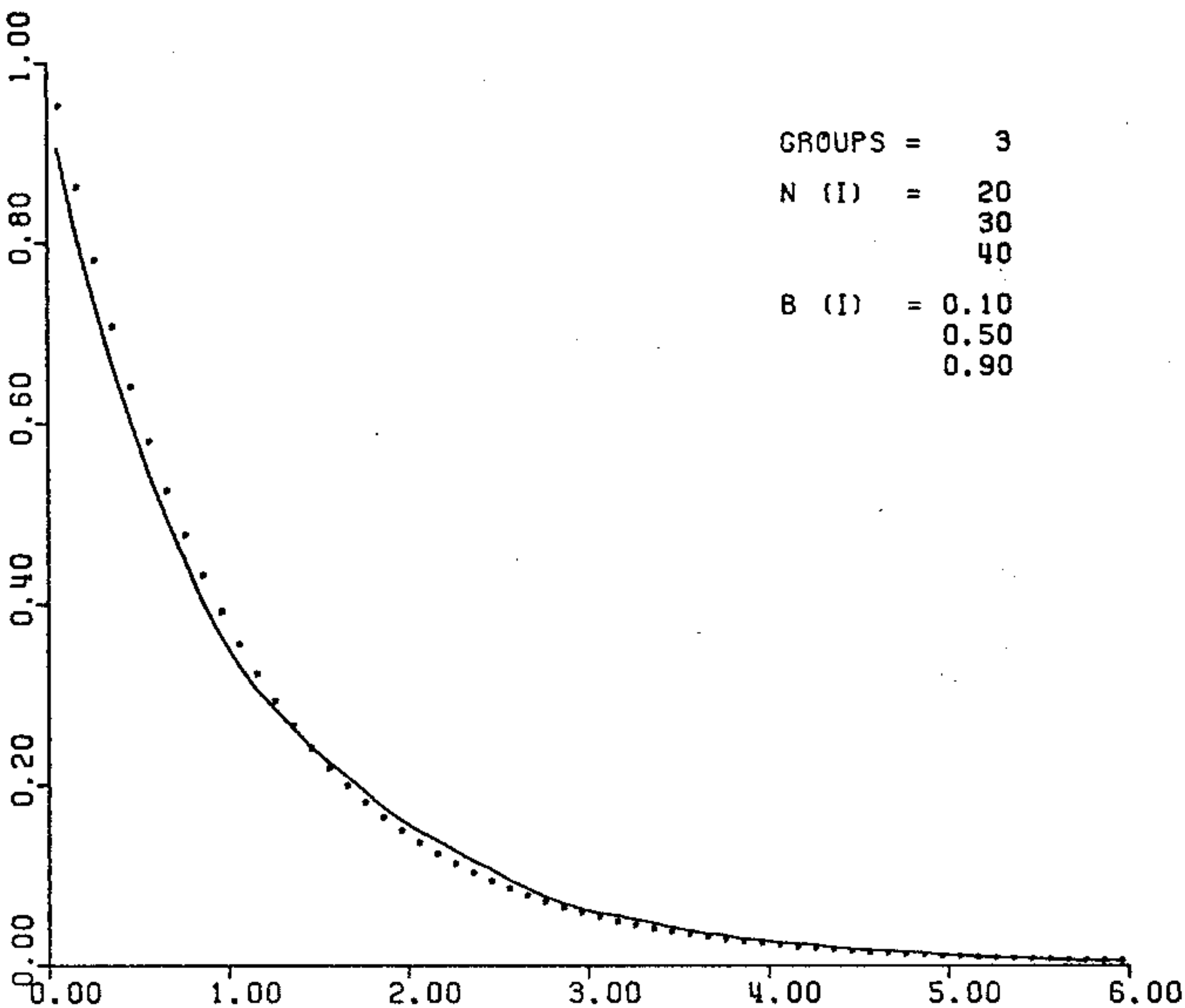

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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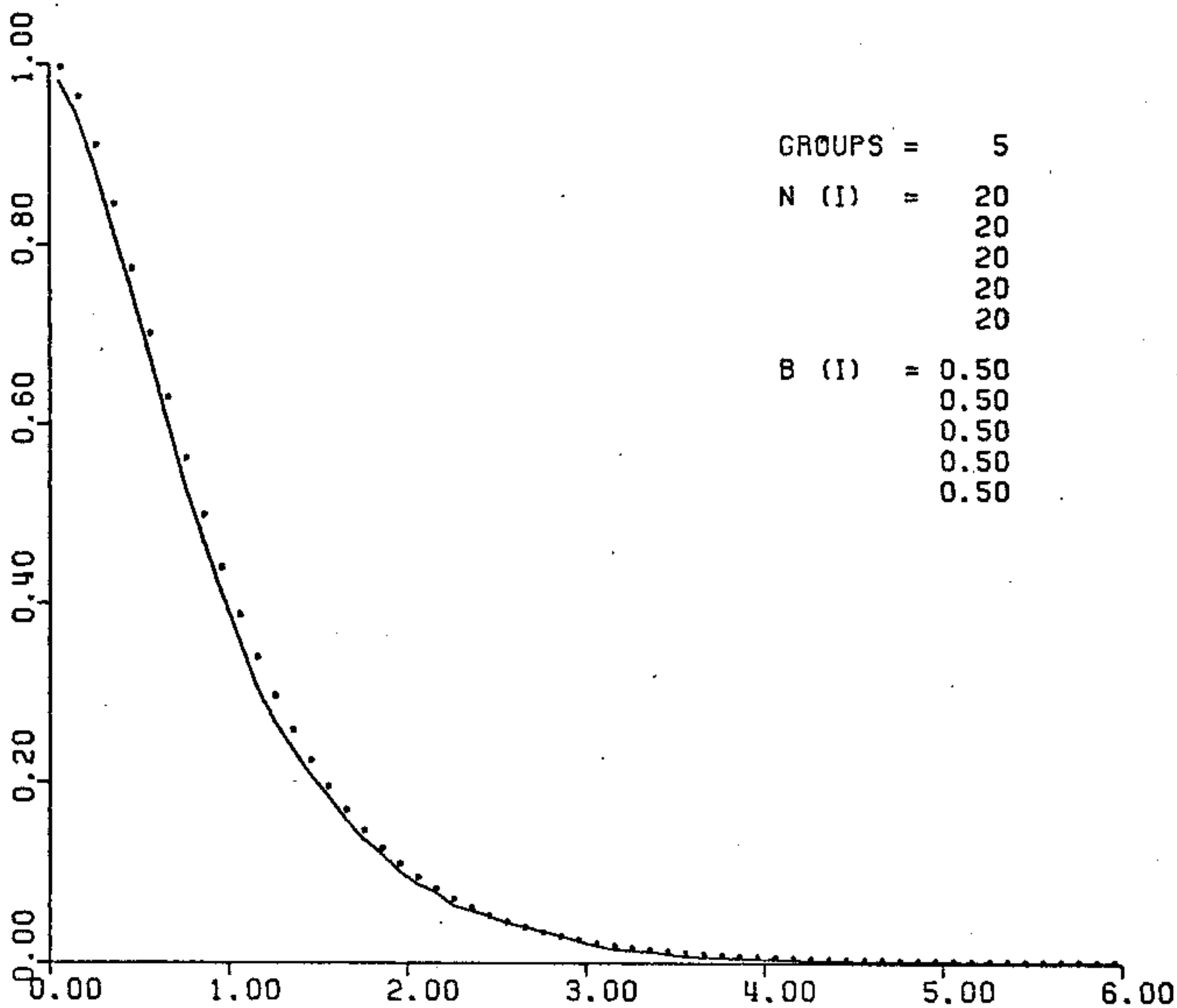

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

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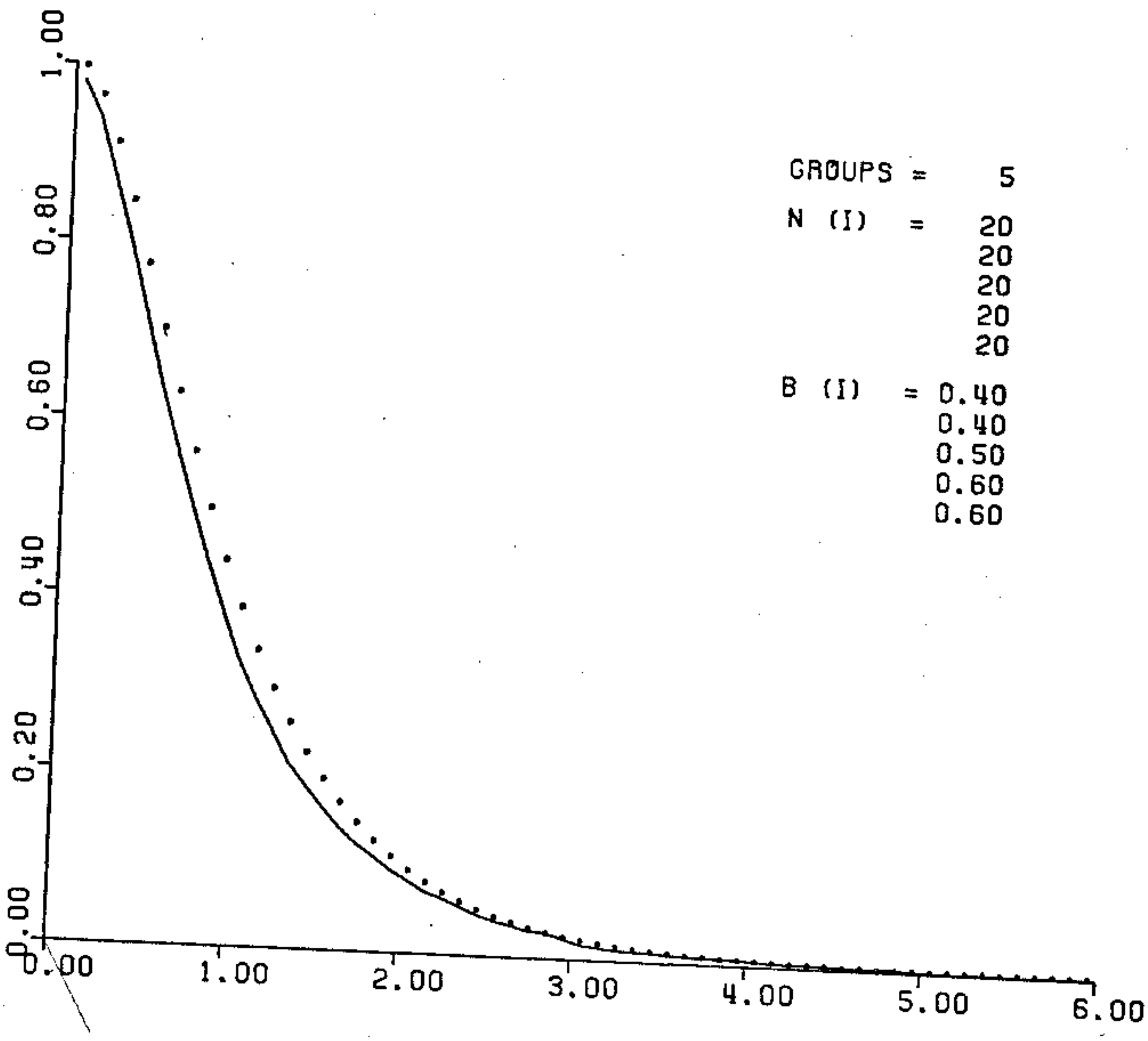

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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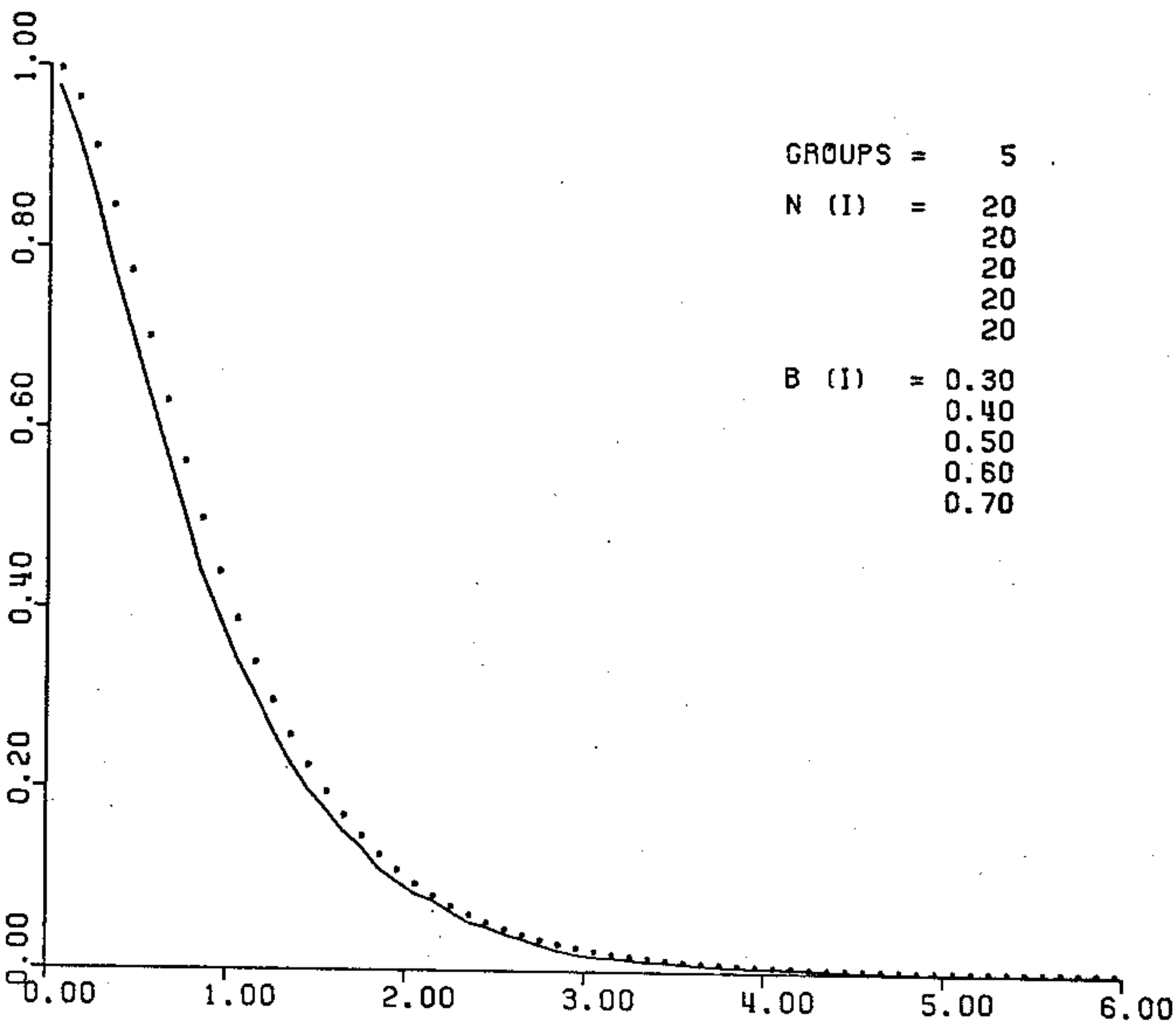
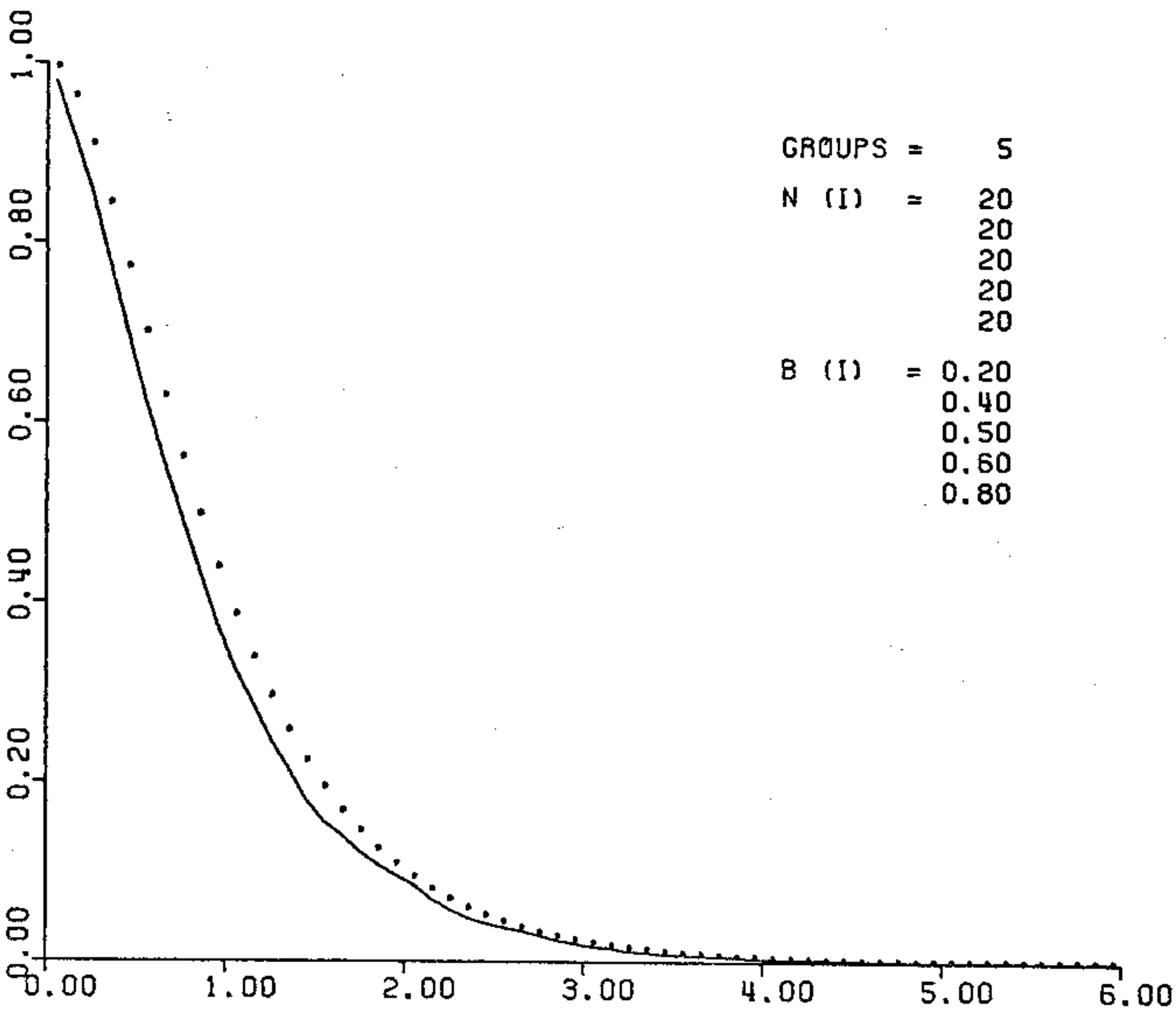
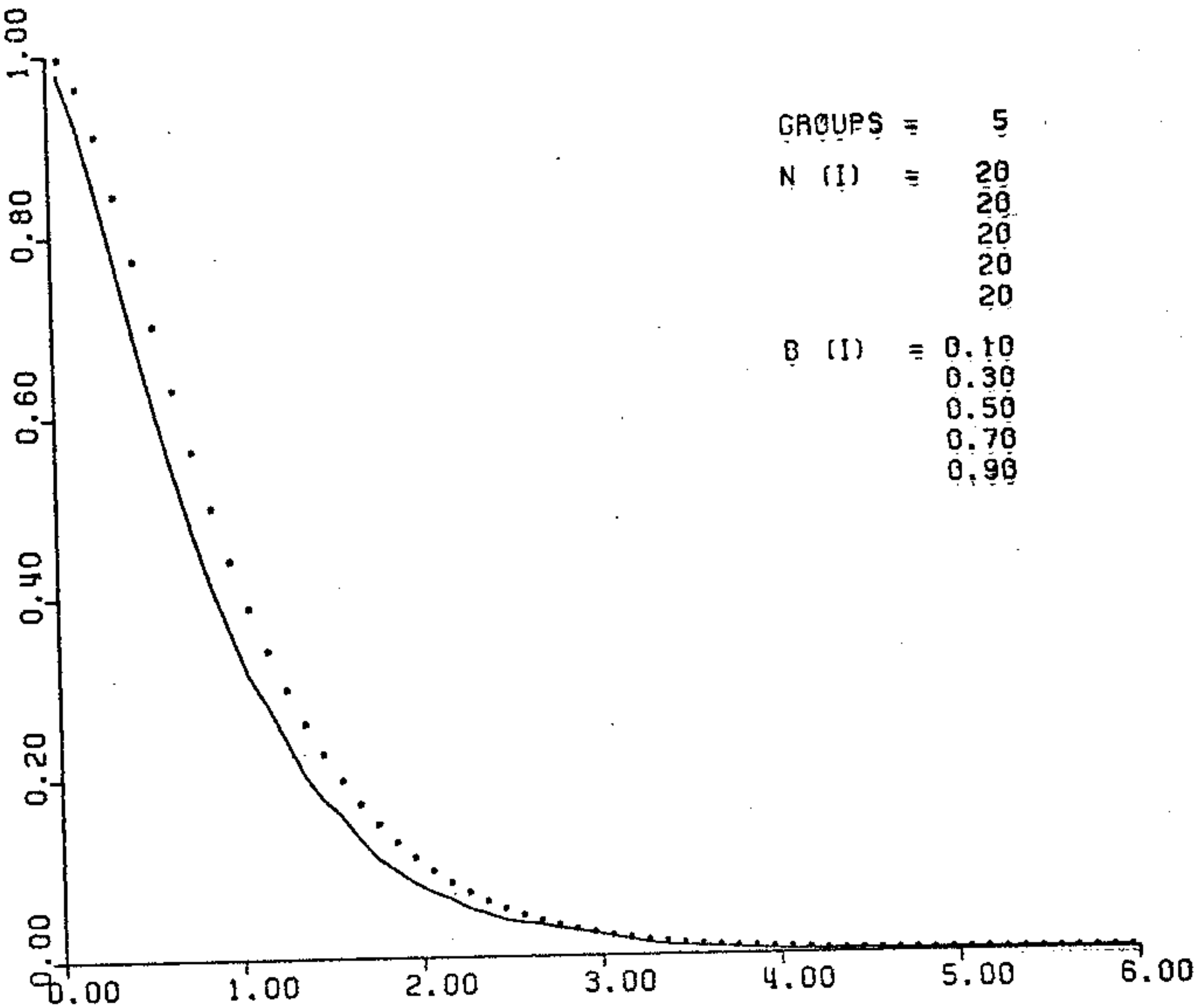


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 83 : SIMULATION NUMBER 83



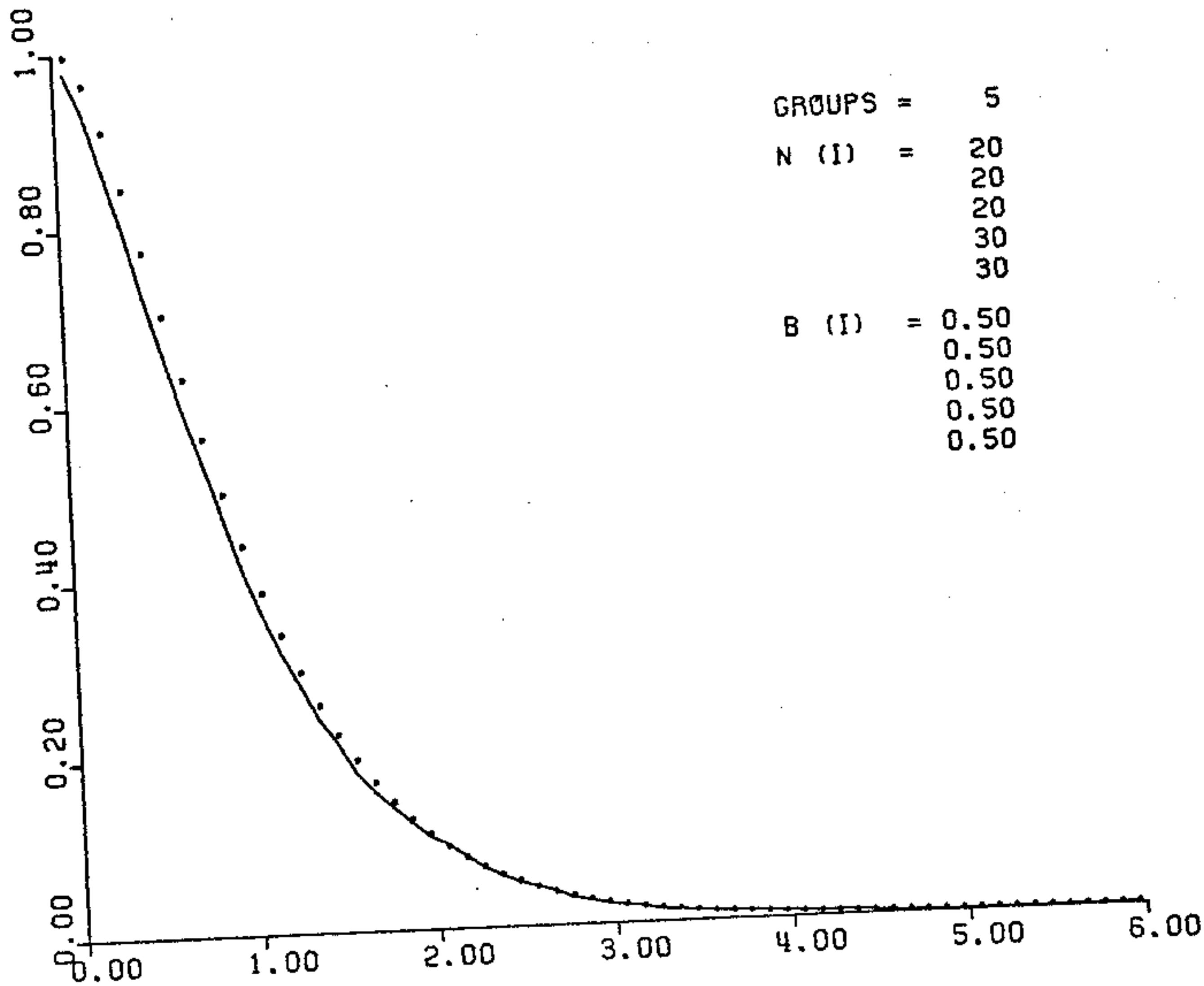
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 84 : SIMULATION NUMBER 84



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 85 : SIMULATION NUMBER 85



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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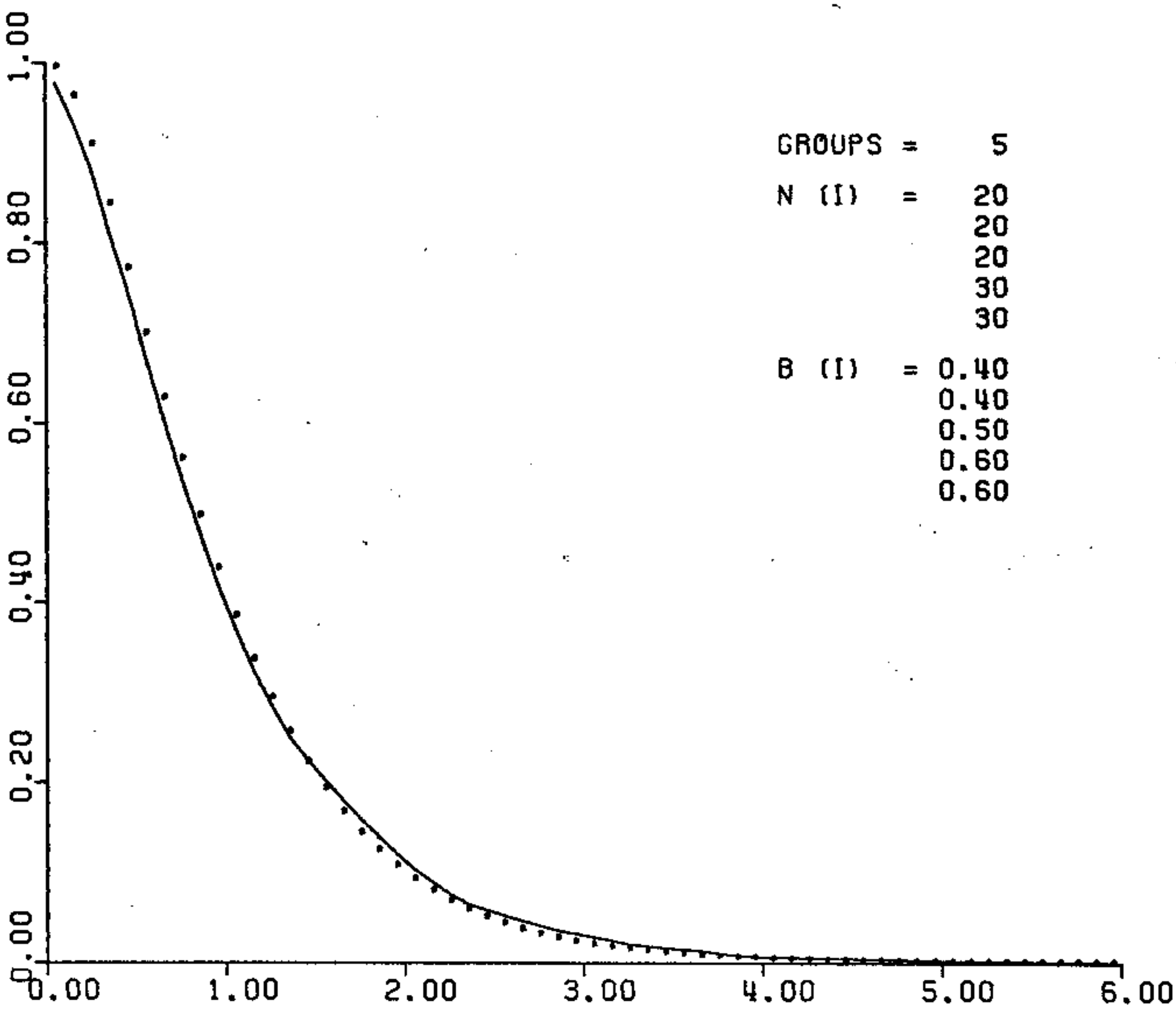

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

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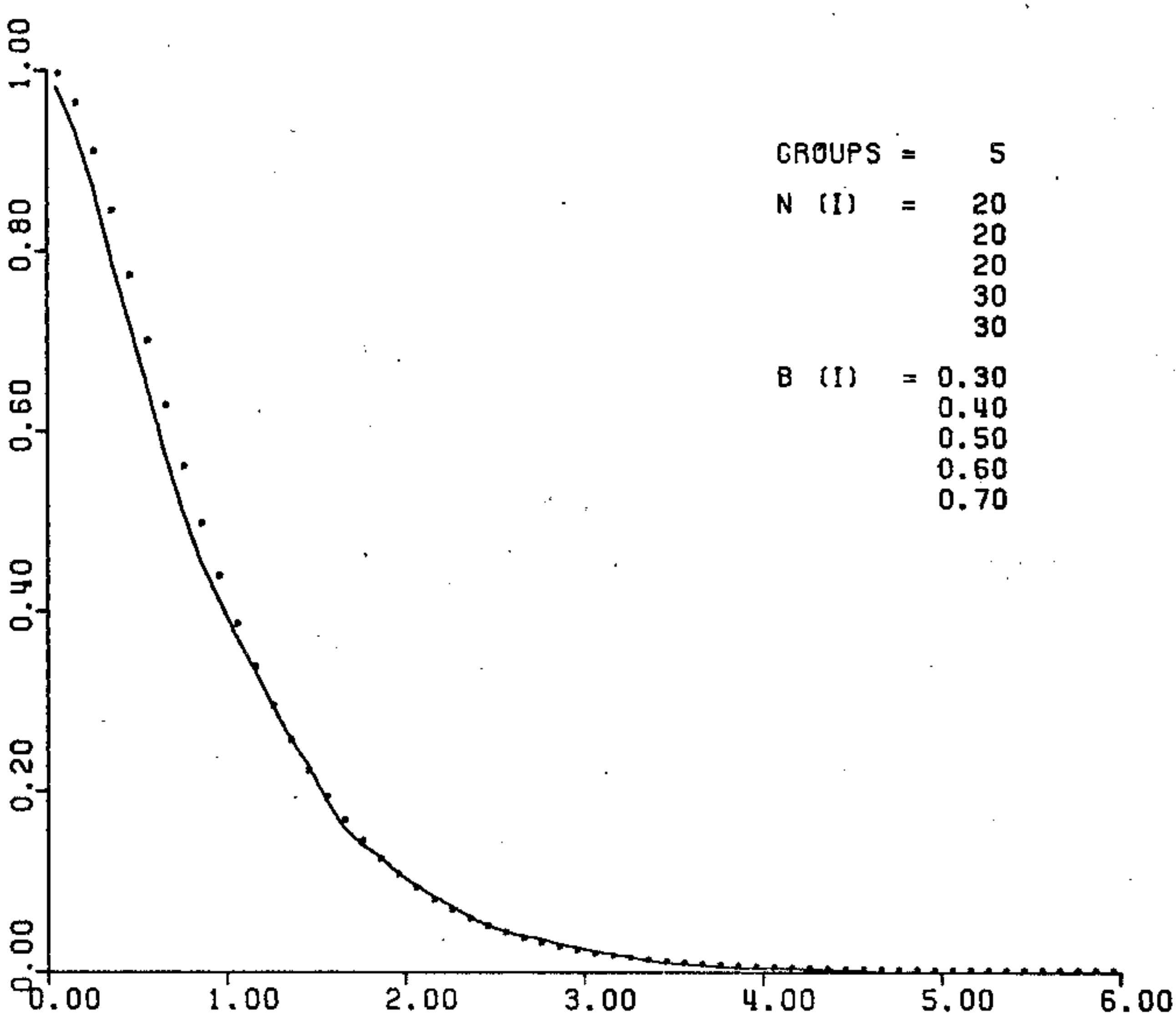

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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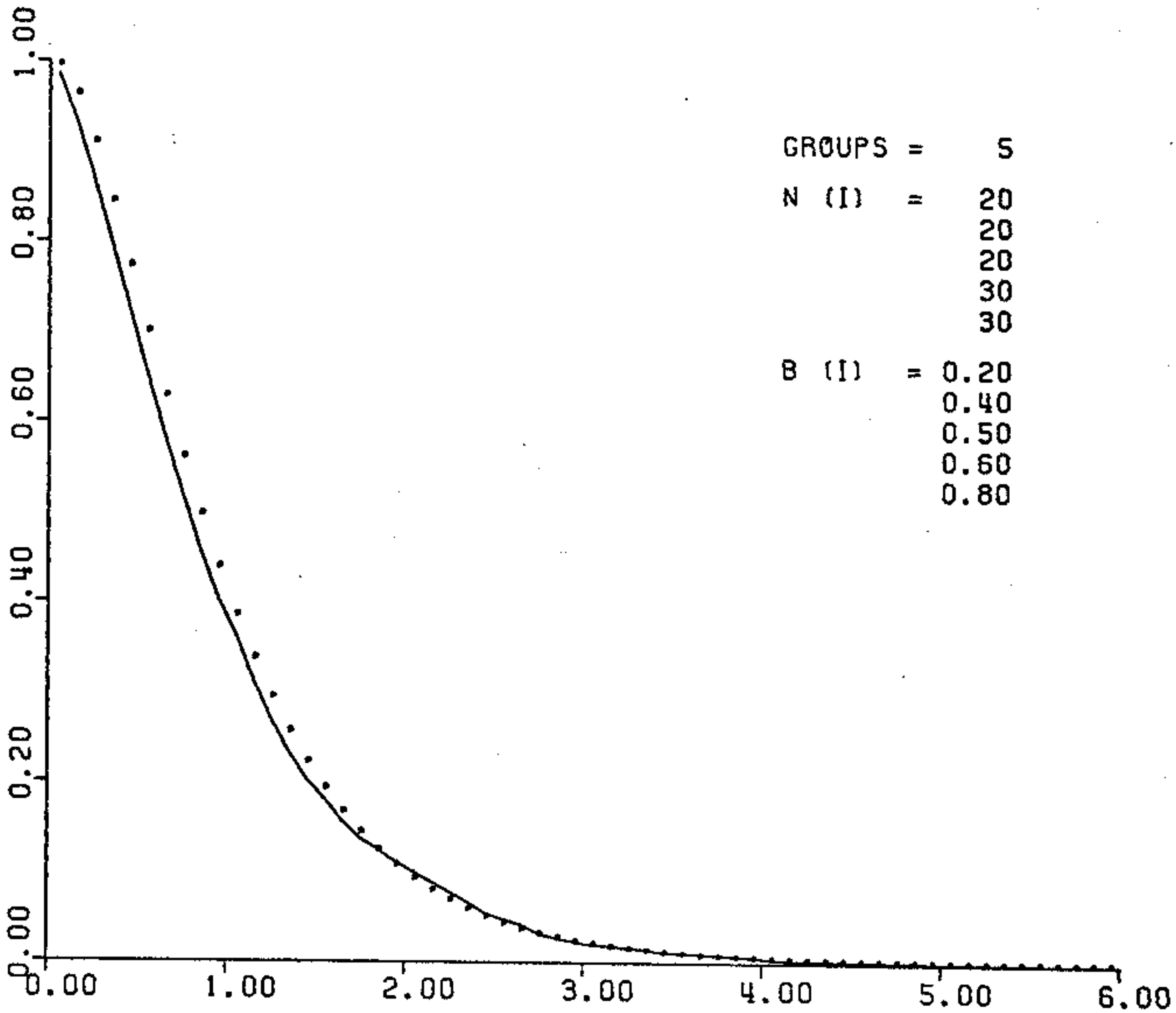

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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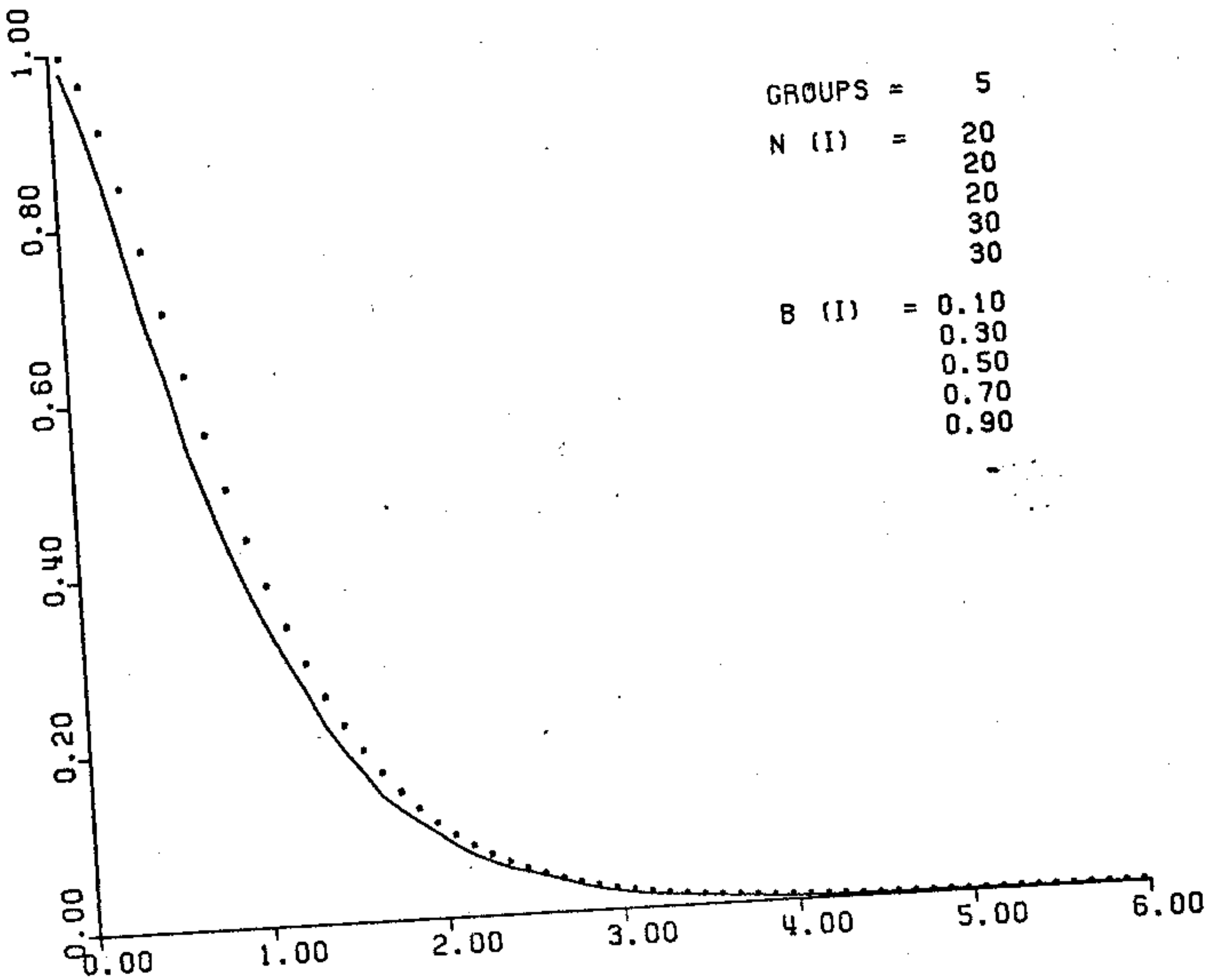

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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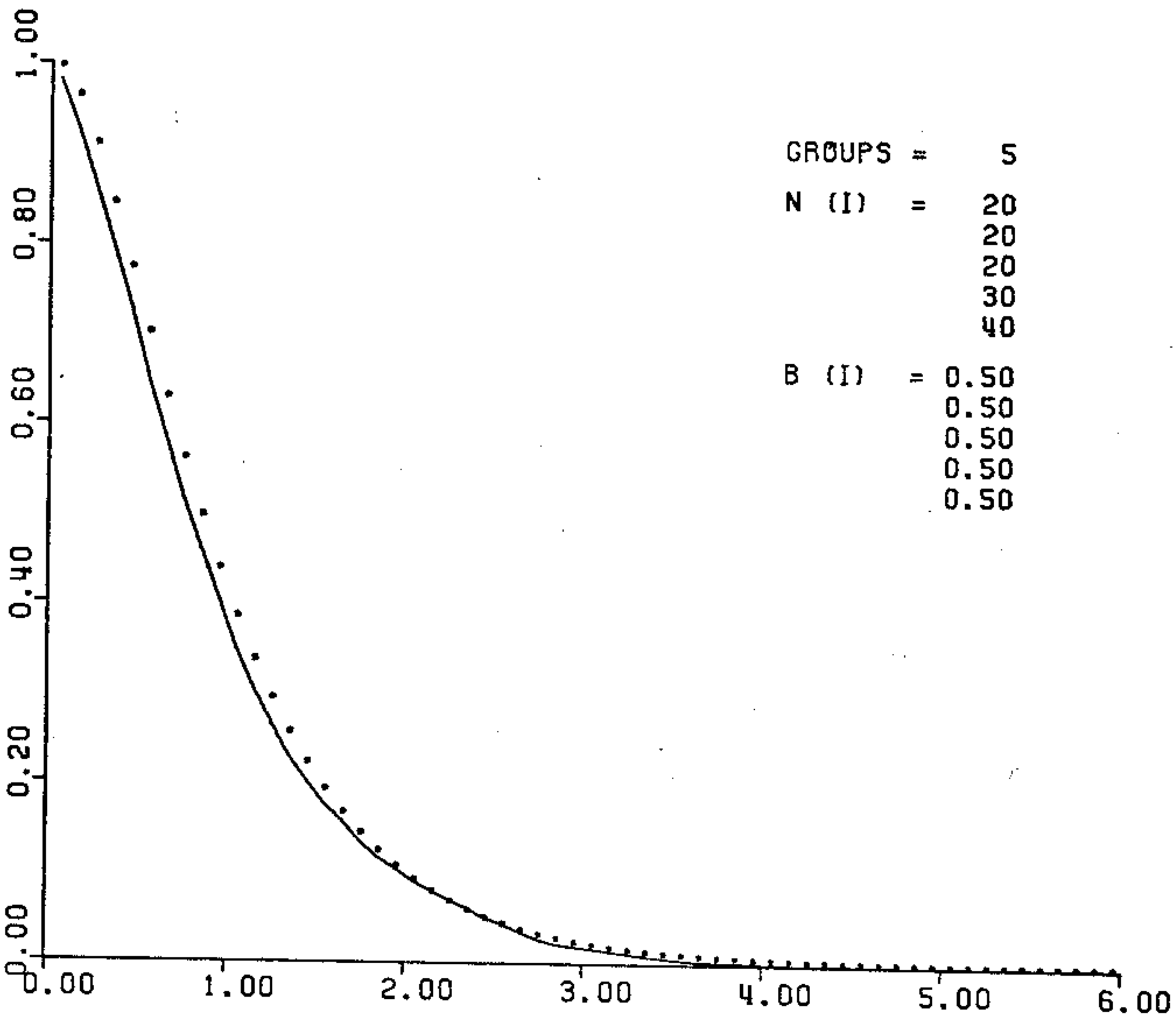

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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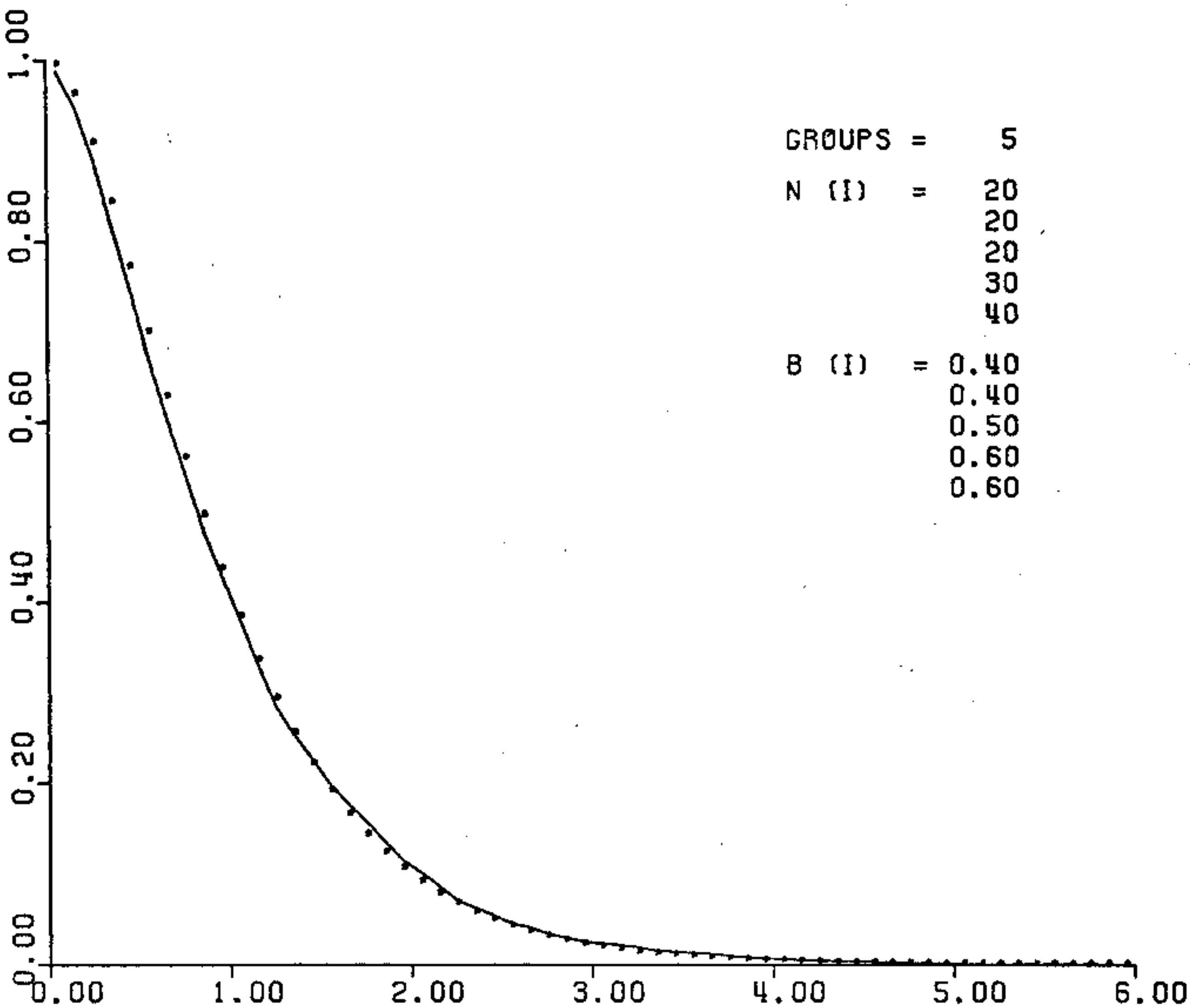
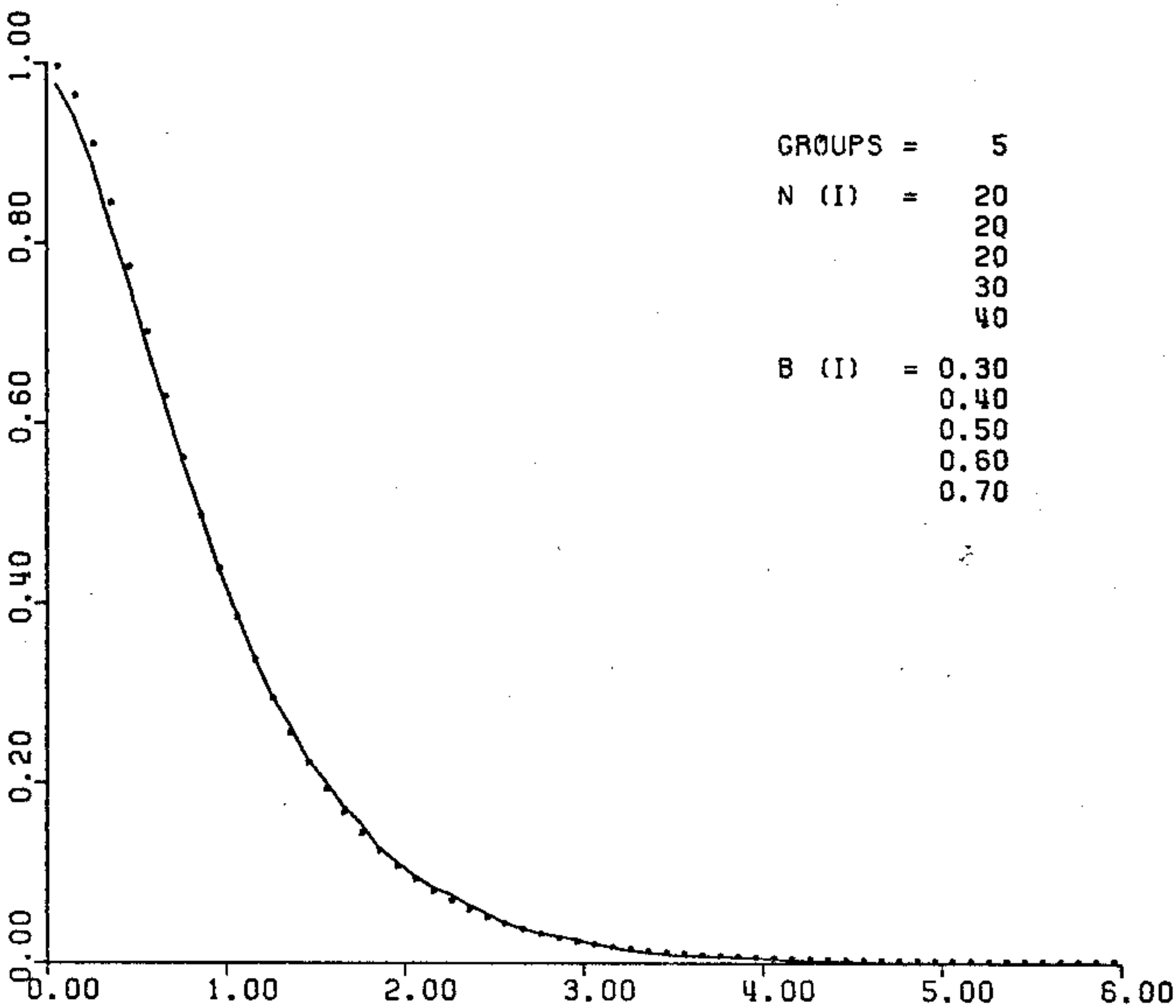


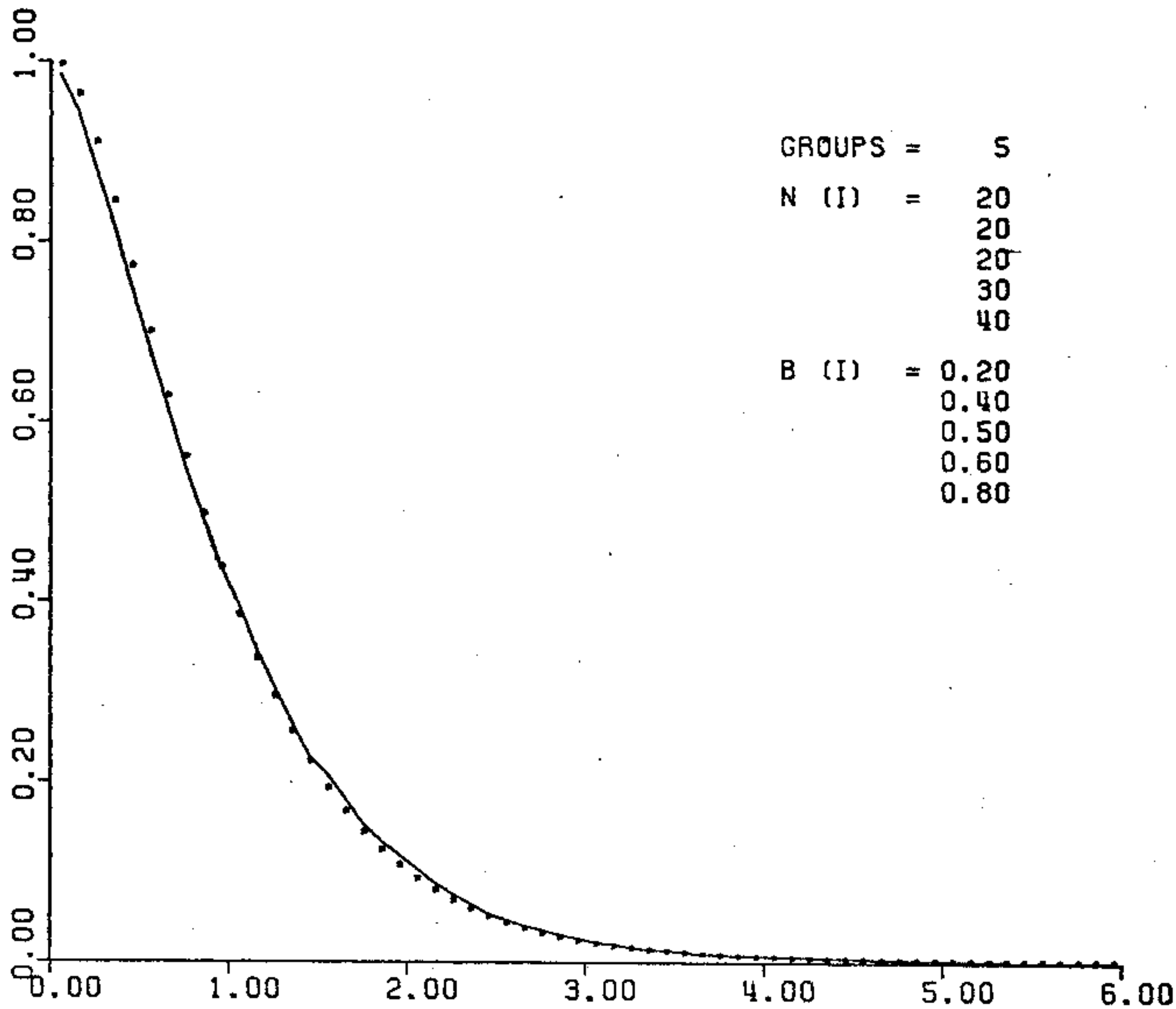
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 92 : SIMULATION NUMBER 92



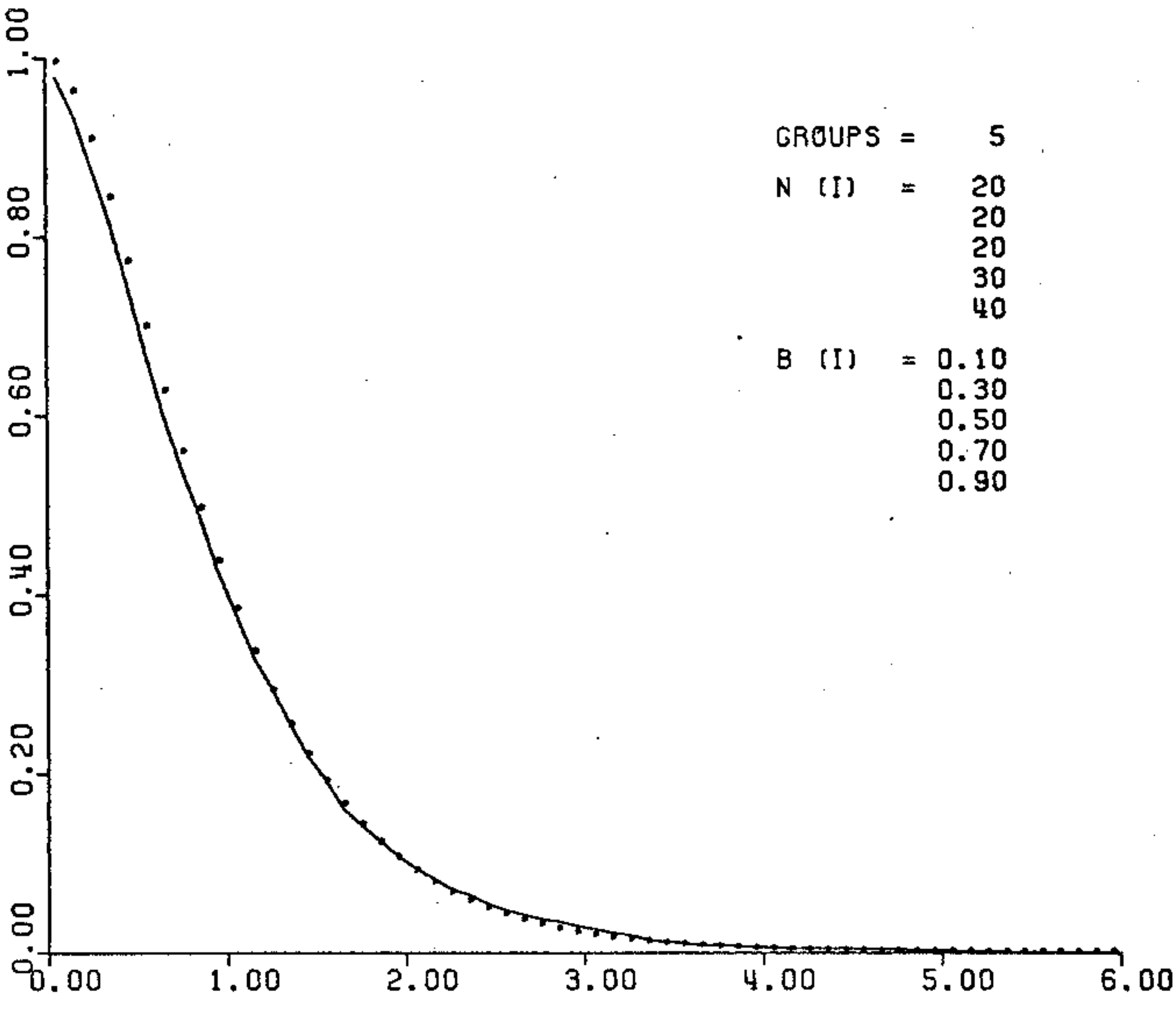
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 93 : SIMULATION NUMBER 93



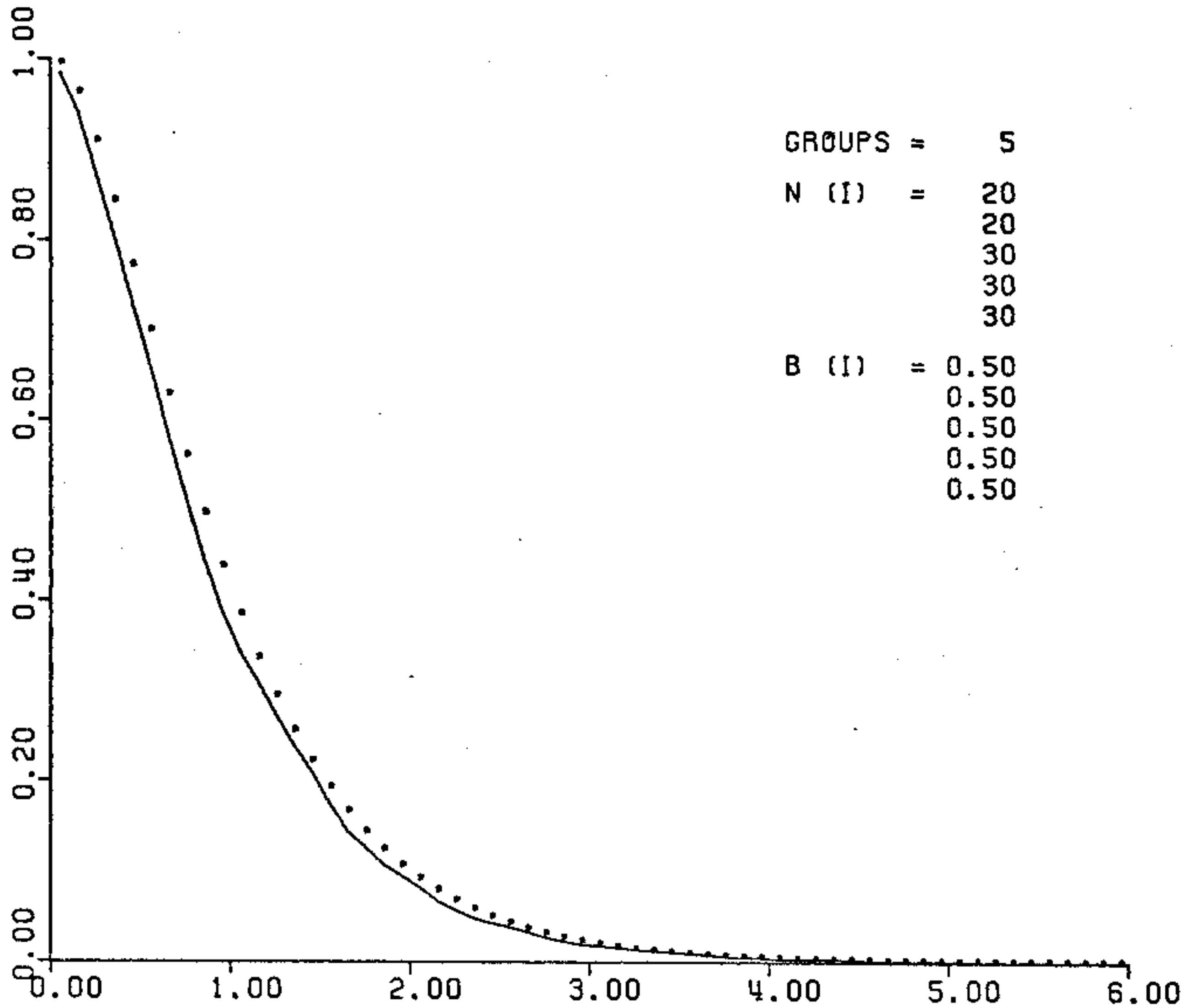
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 94 : SIMULATION NUMBER 94



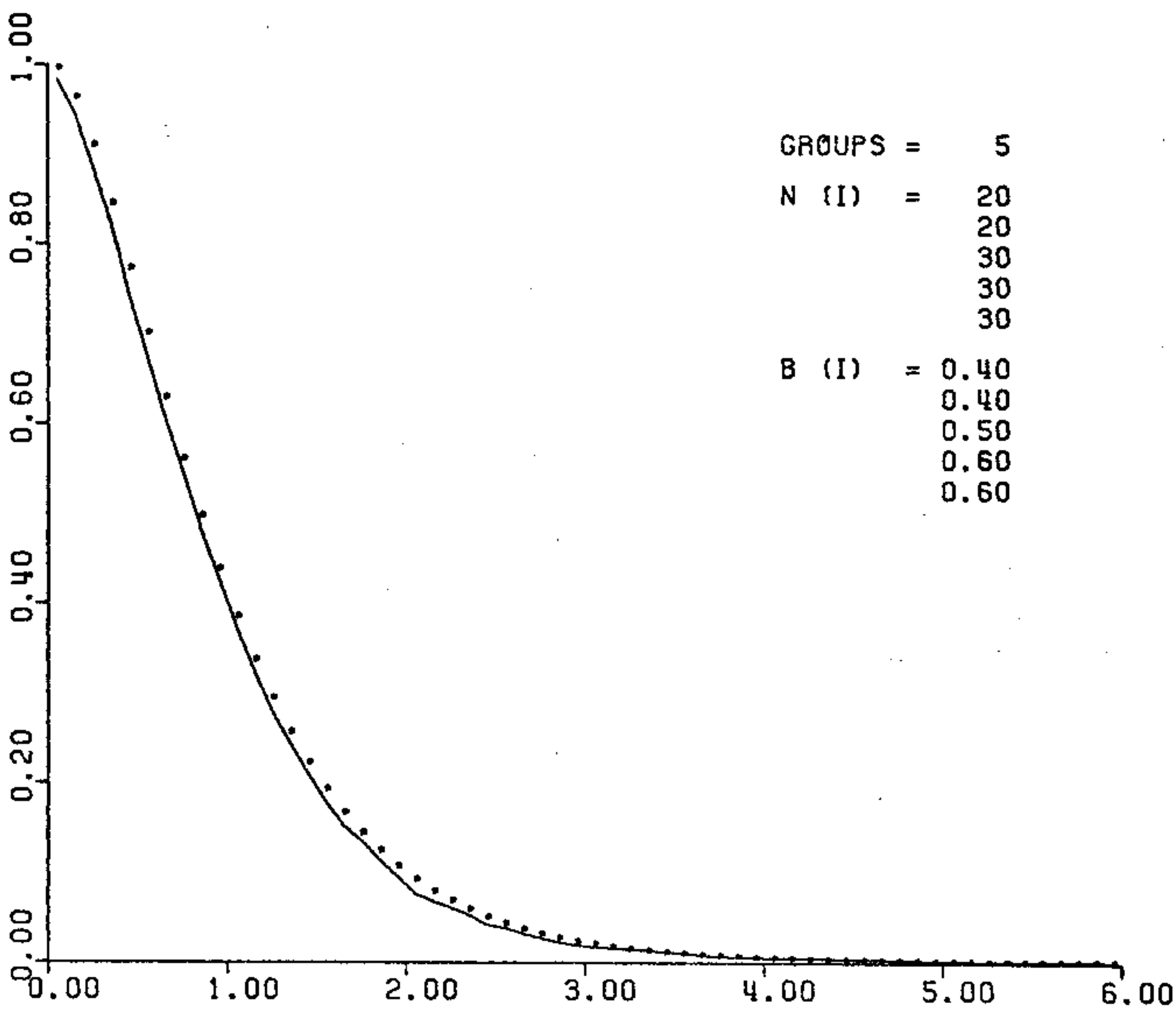
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 95 : SIMULATION NUMBER 95

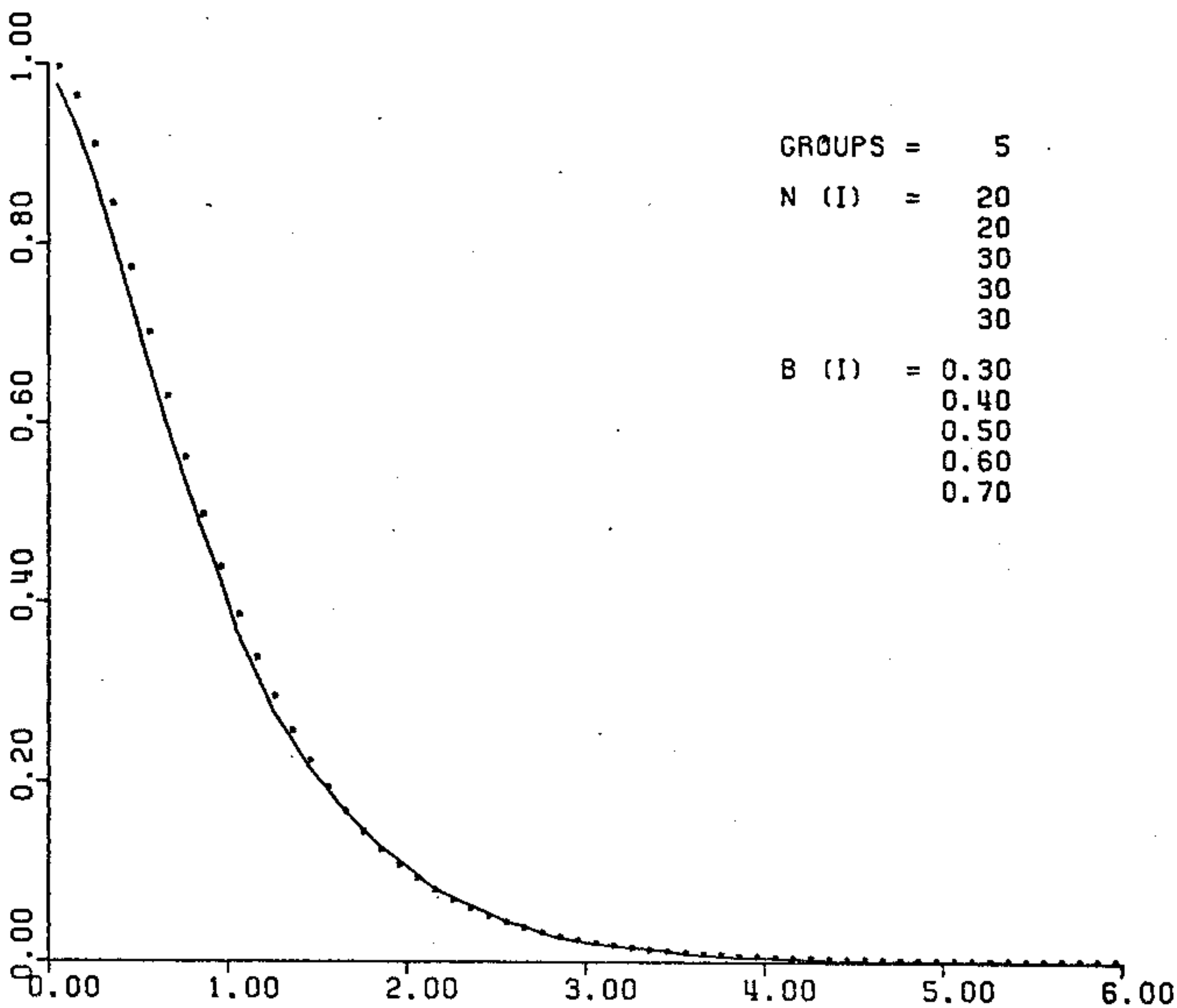


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

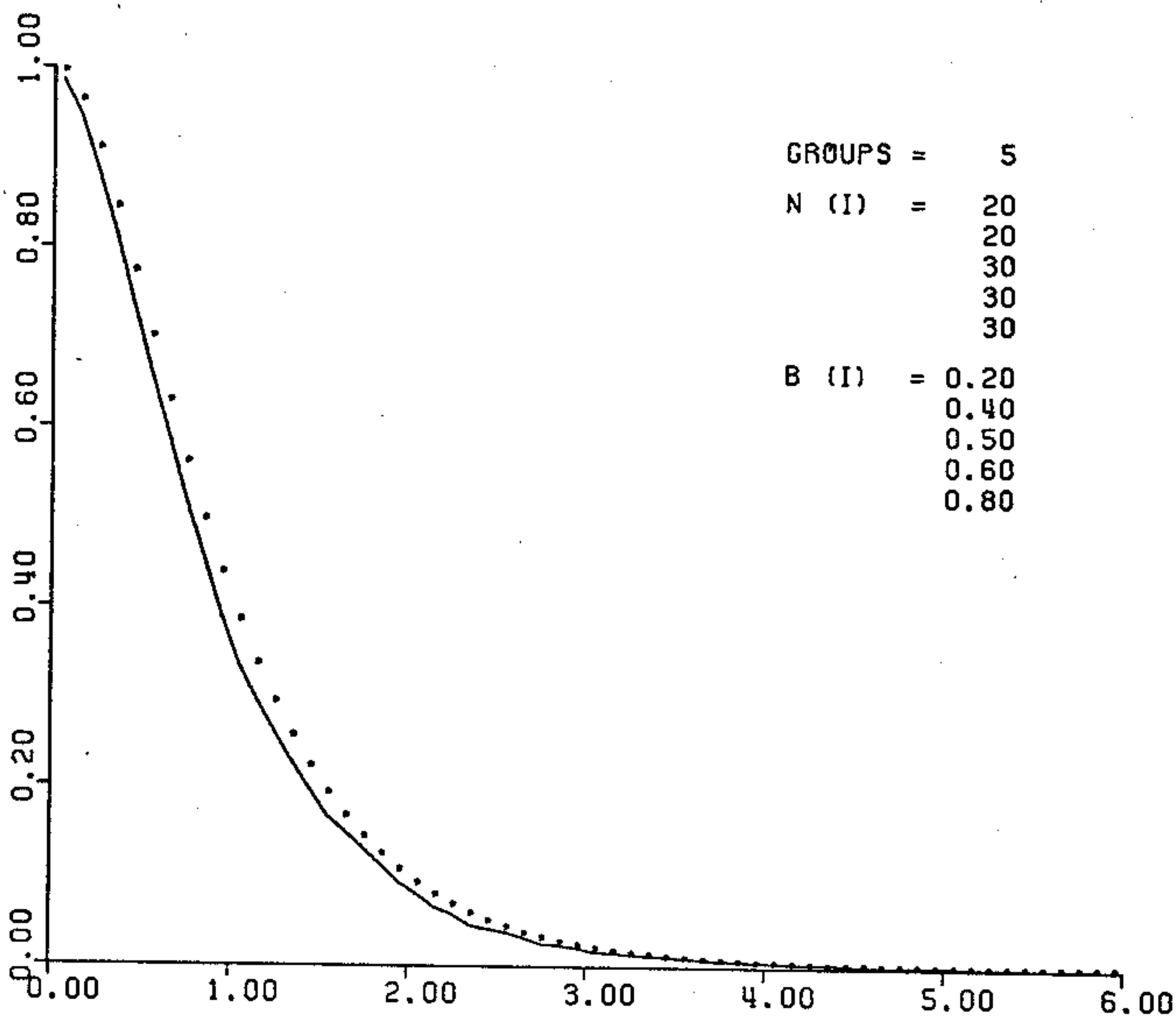
FIGURE 96 : SIMULATION NUMBER 96



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 97 : SIMULATION NUMBER 97

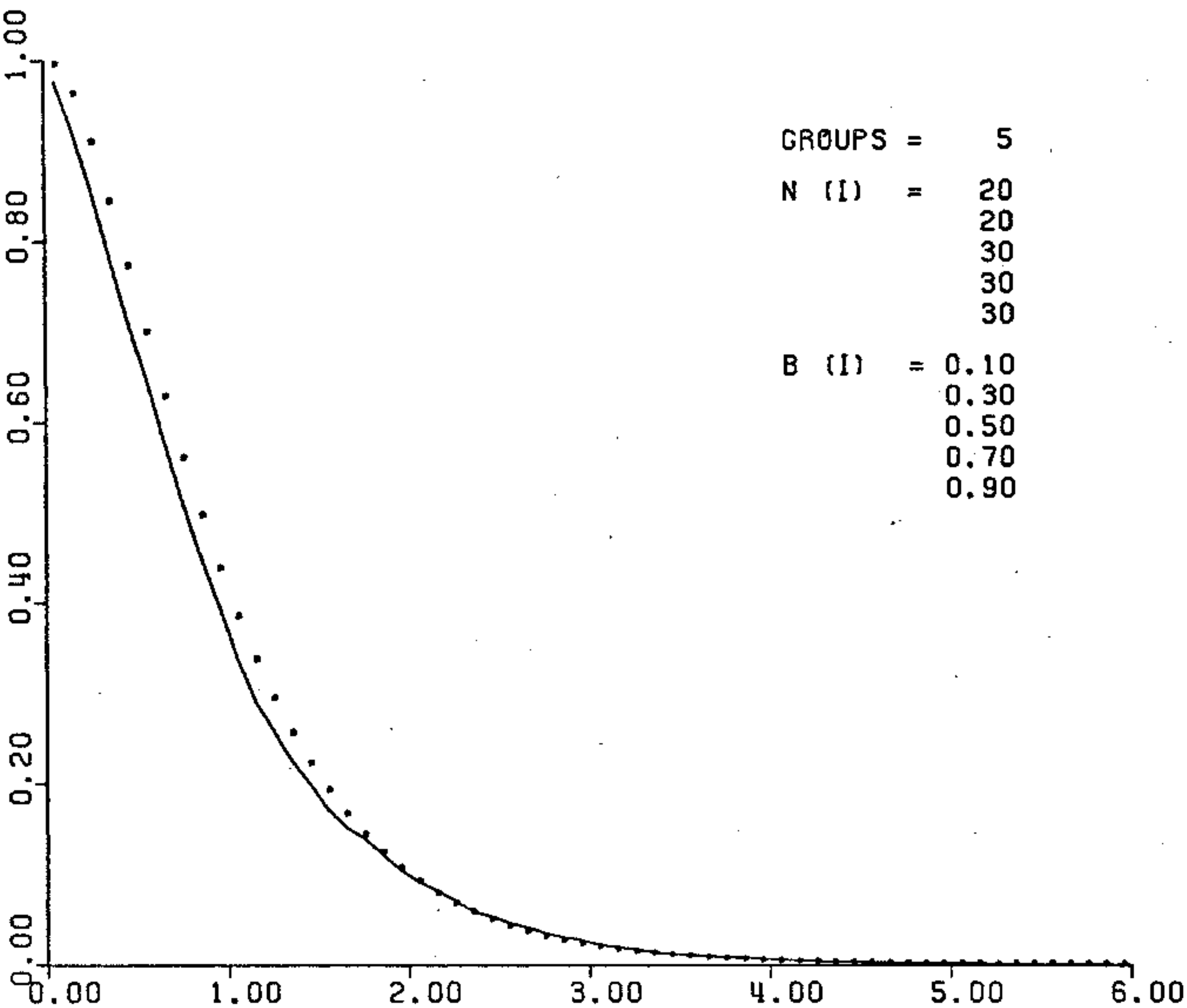


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 98 : SIMULATION NUMBER 98

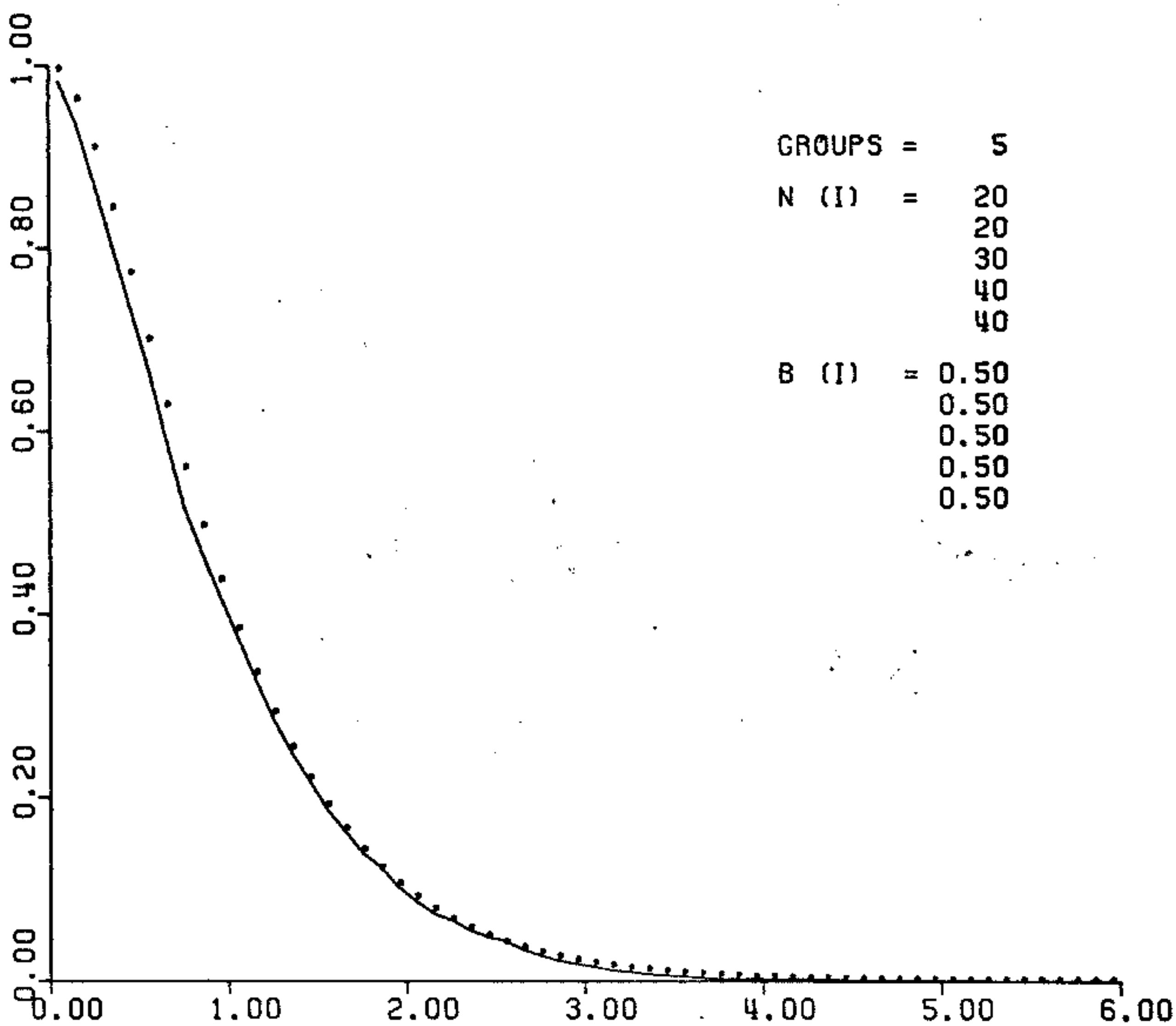


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 99 : SIMULATION NUMBER 99

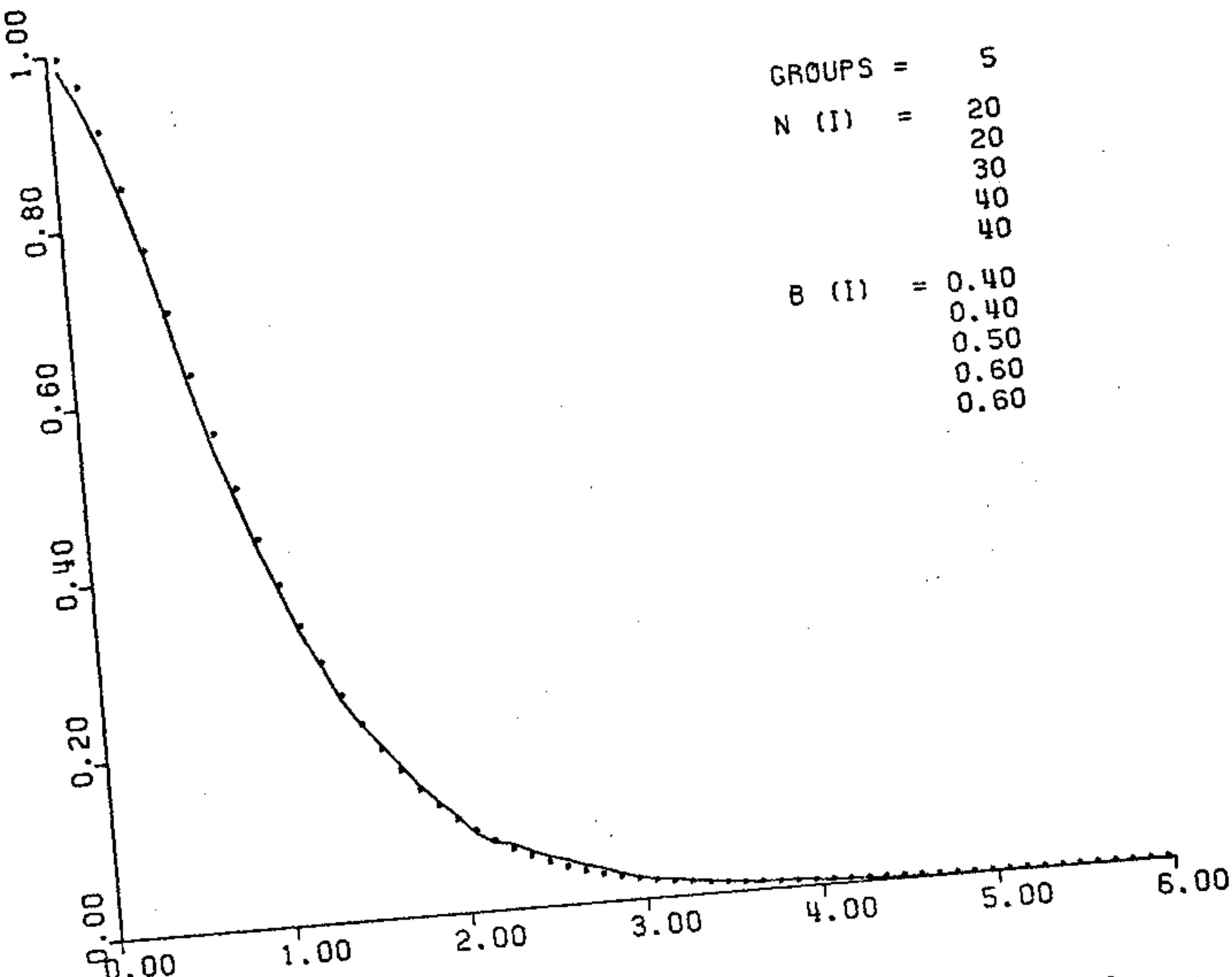


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 100 : SIMULATION NUMBER 100

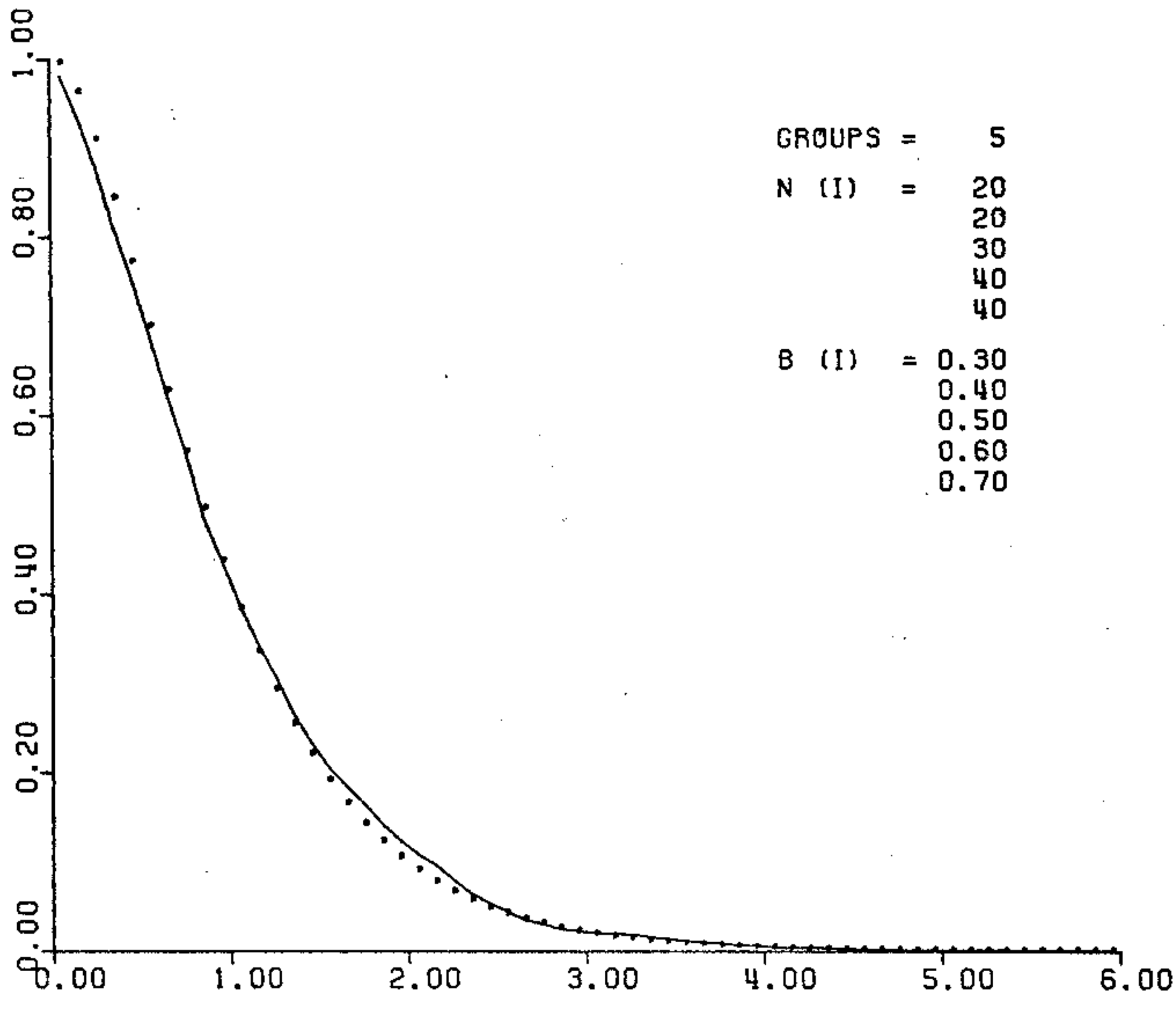


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 101 : SIMULATION NUMBER 101

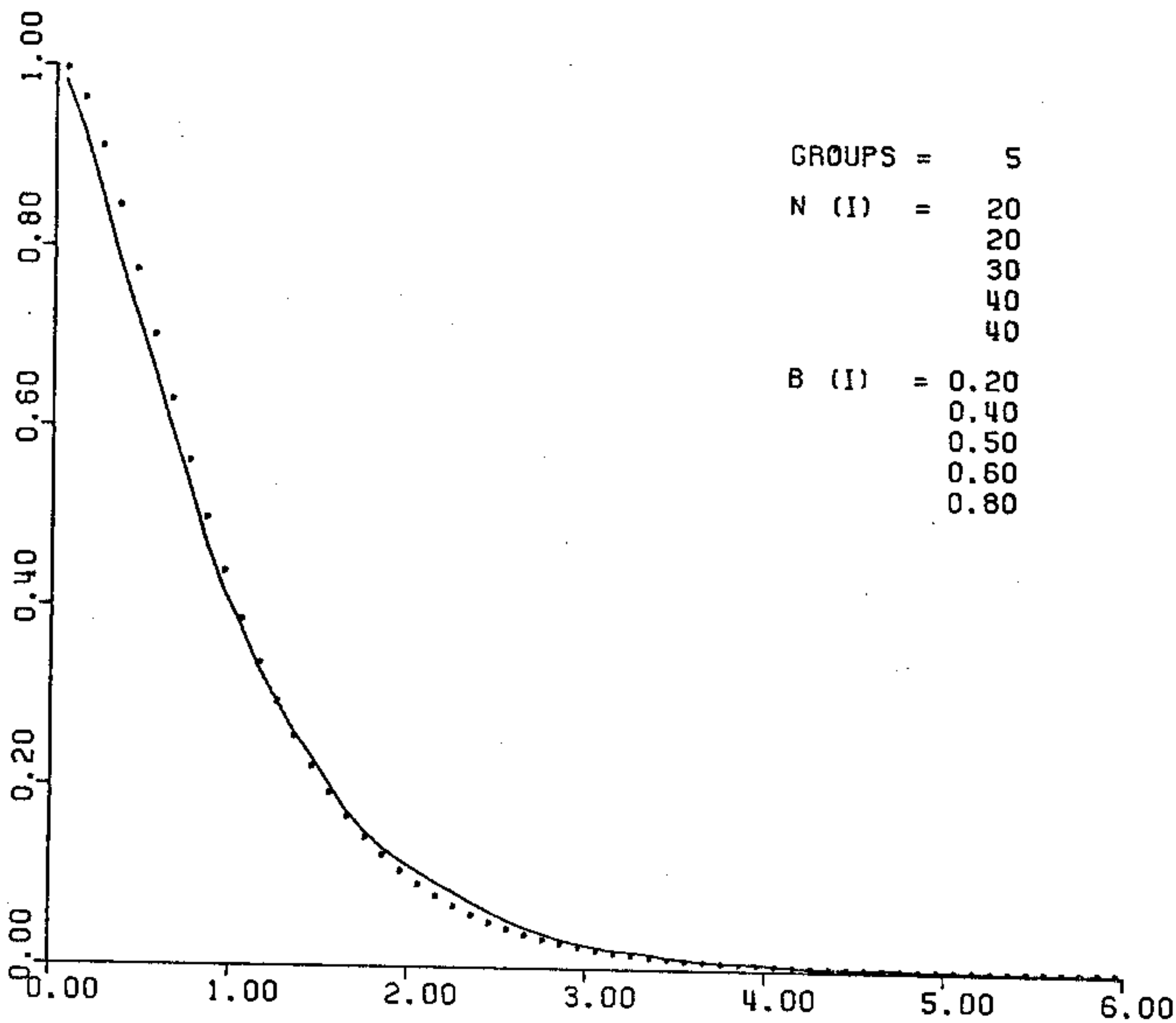


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 102 : SIMULATION NUMBER 102



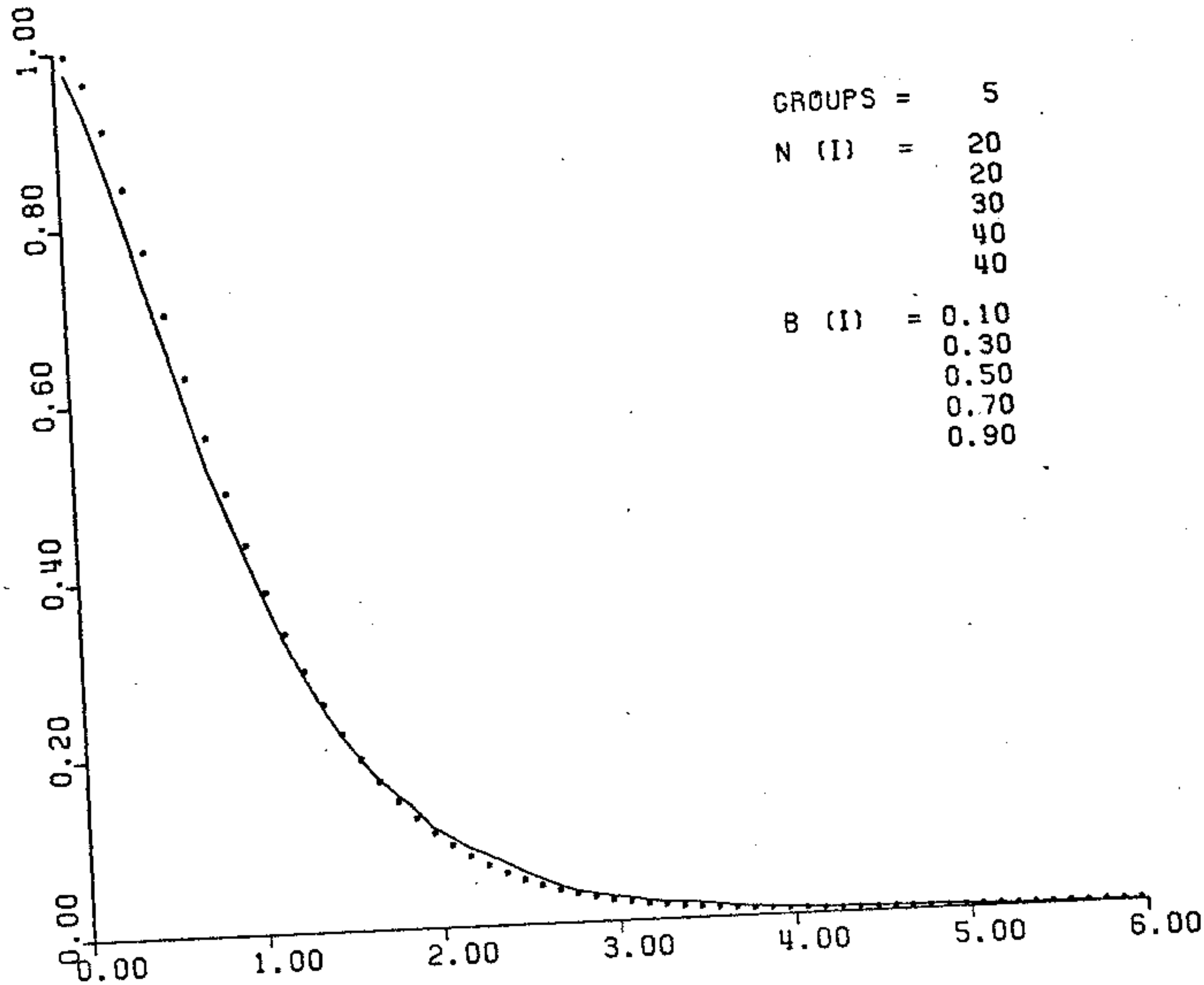
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 103 : SIMULATION NUMBER 103



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 104 : SIMULATION NUMBER 104



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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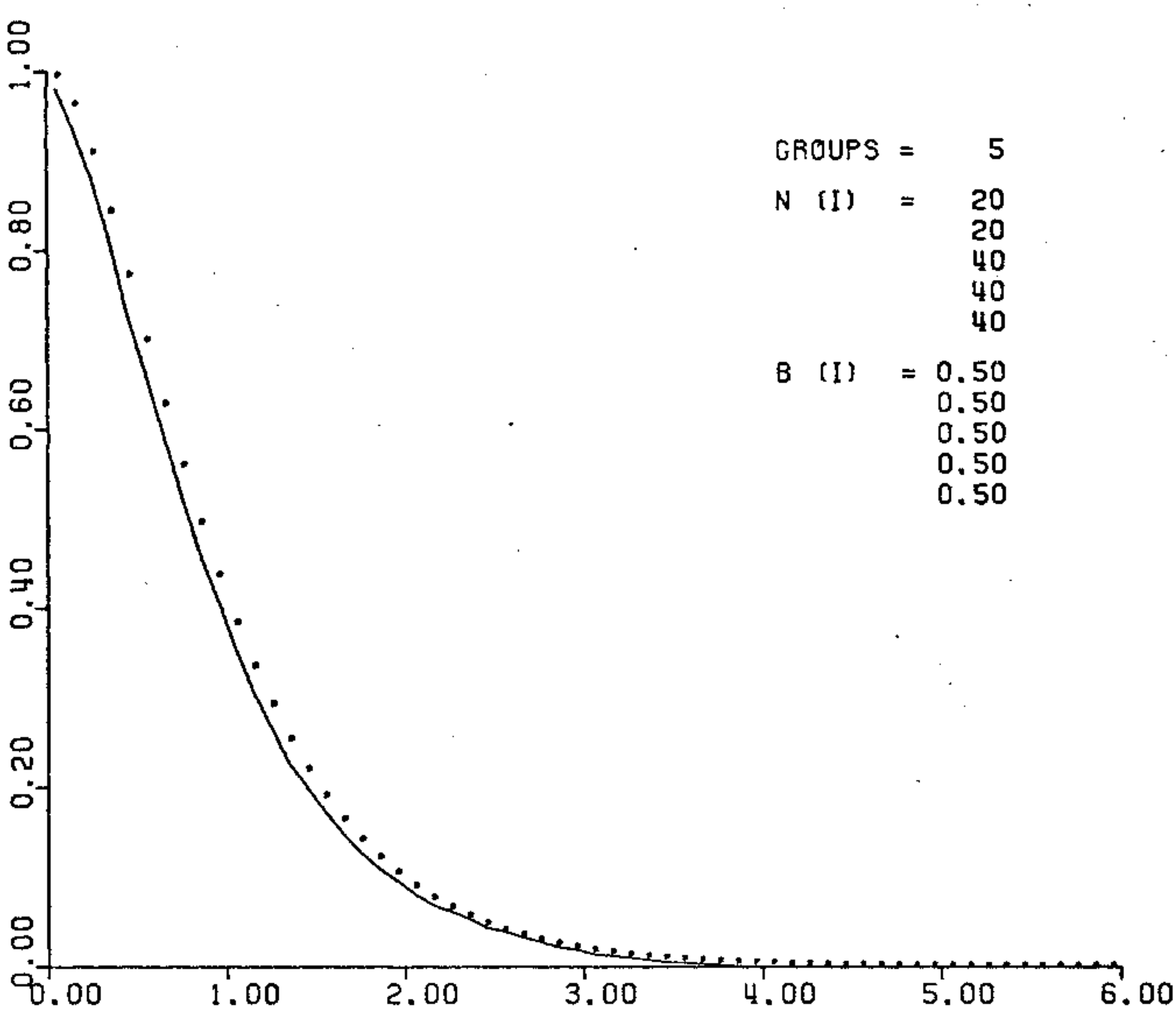

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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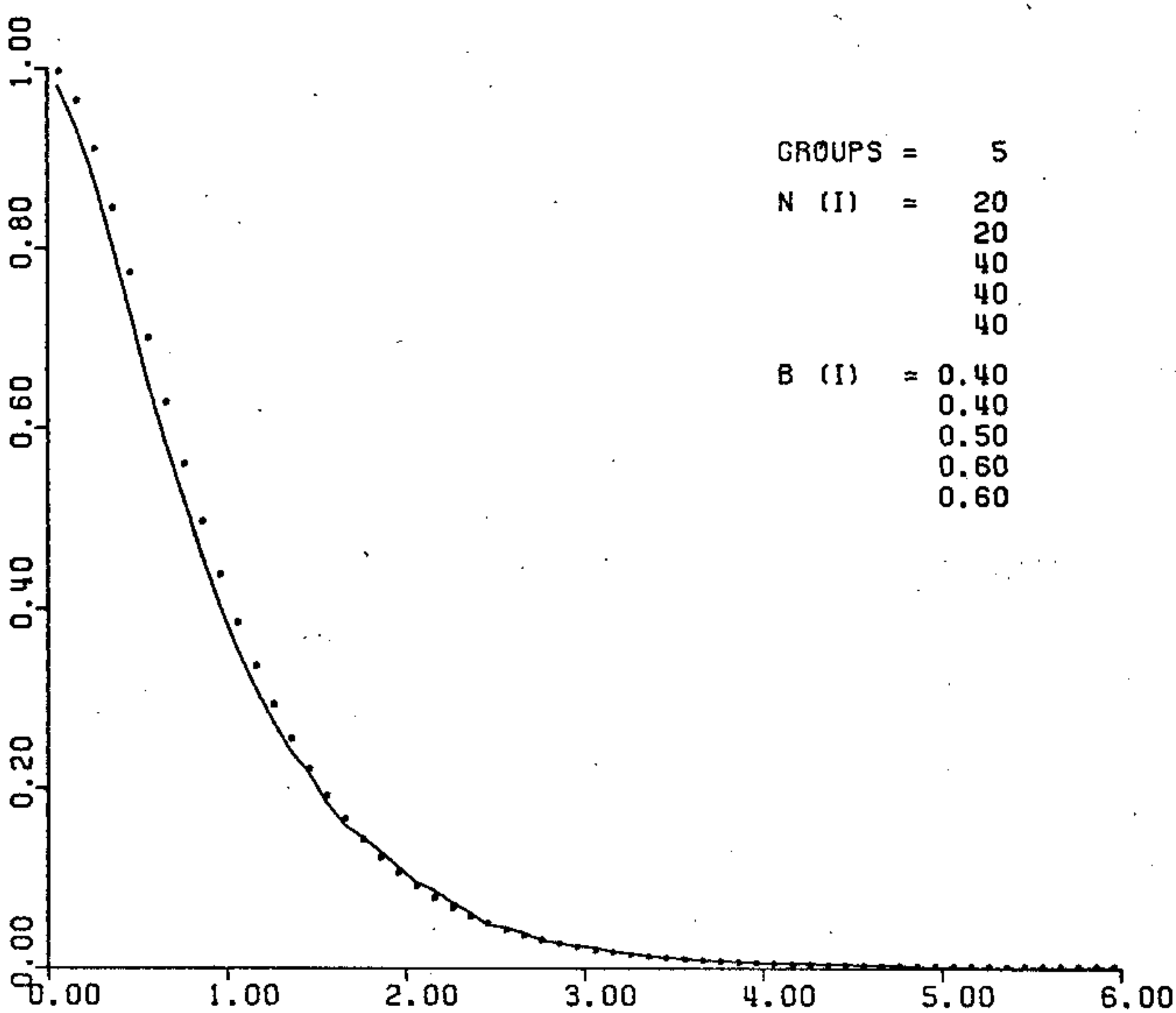
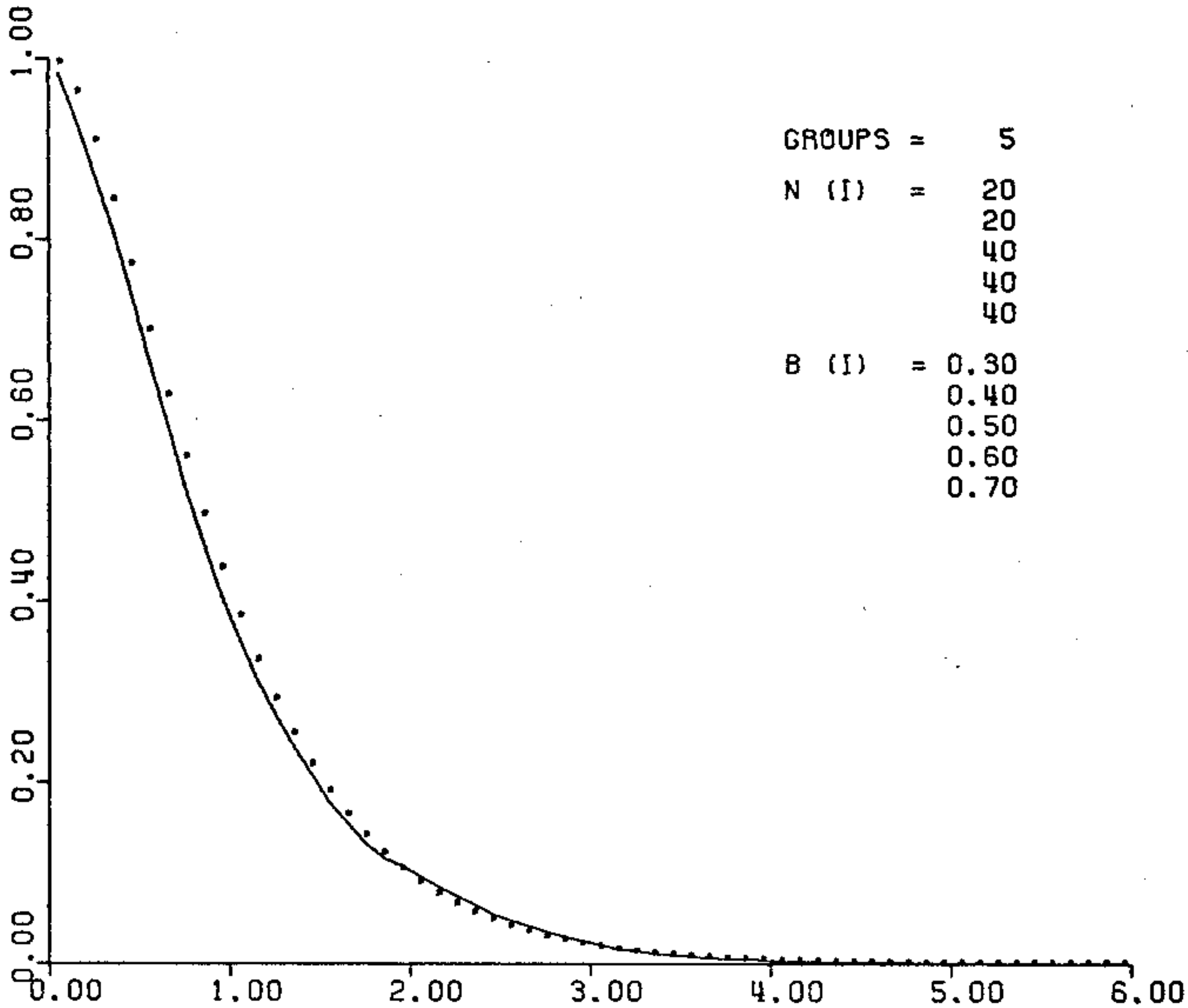


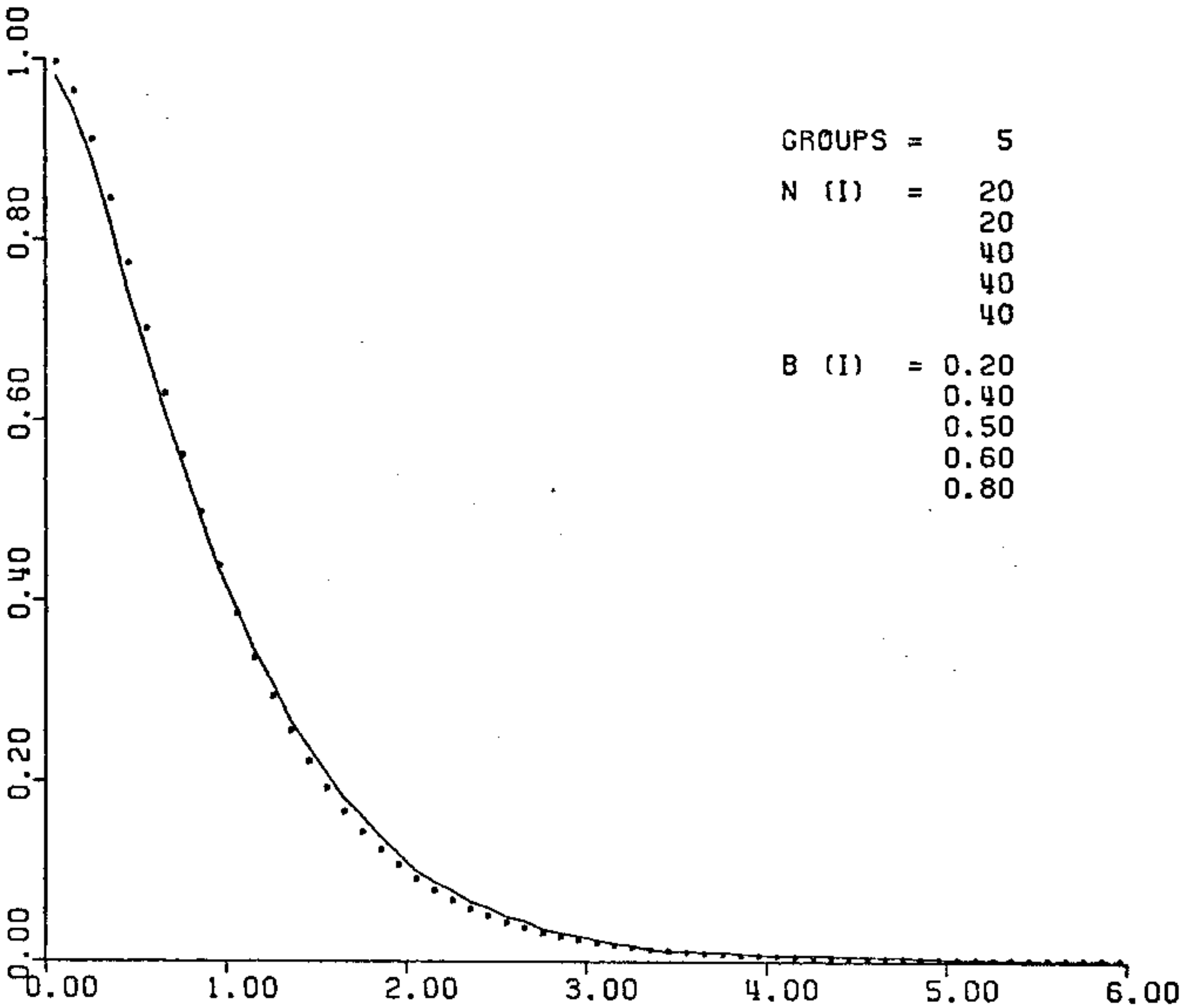
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 107 : SIMULATION NUMBER 107



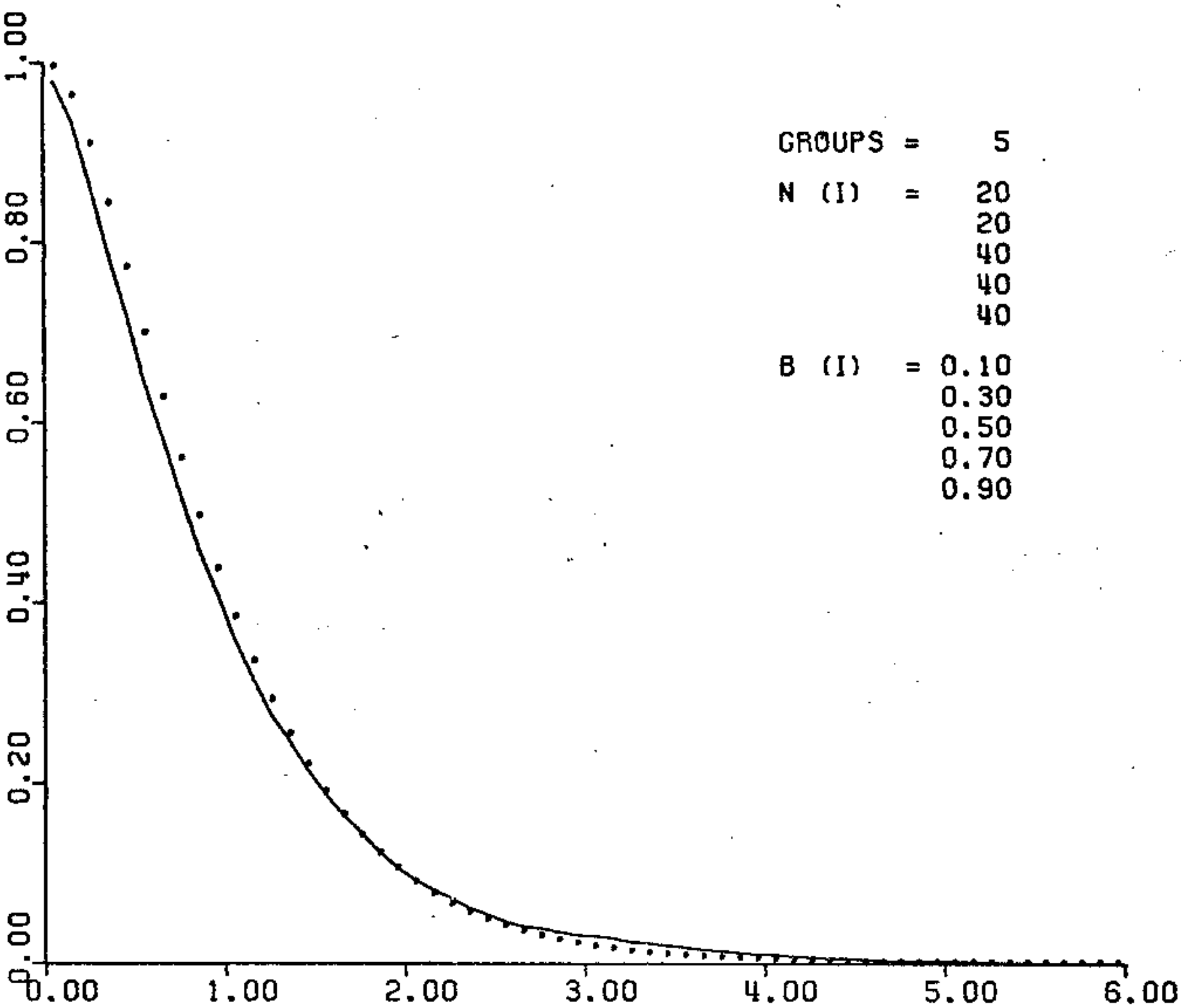
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 108 : SIMULATION NUMBER 108



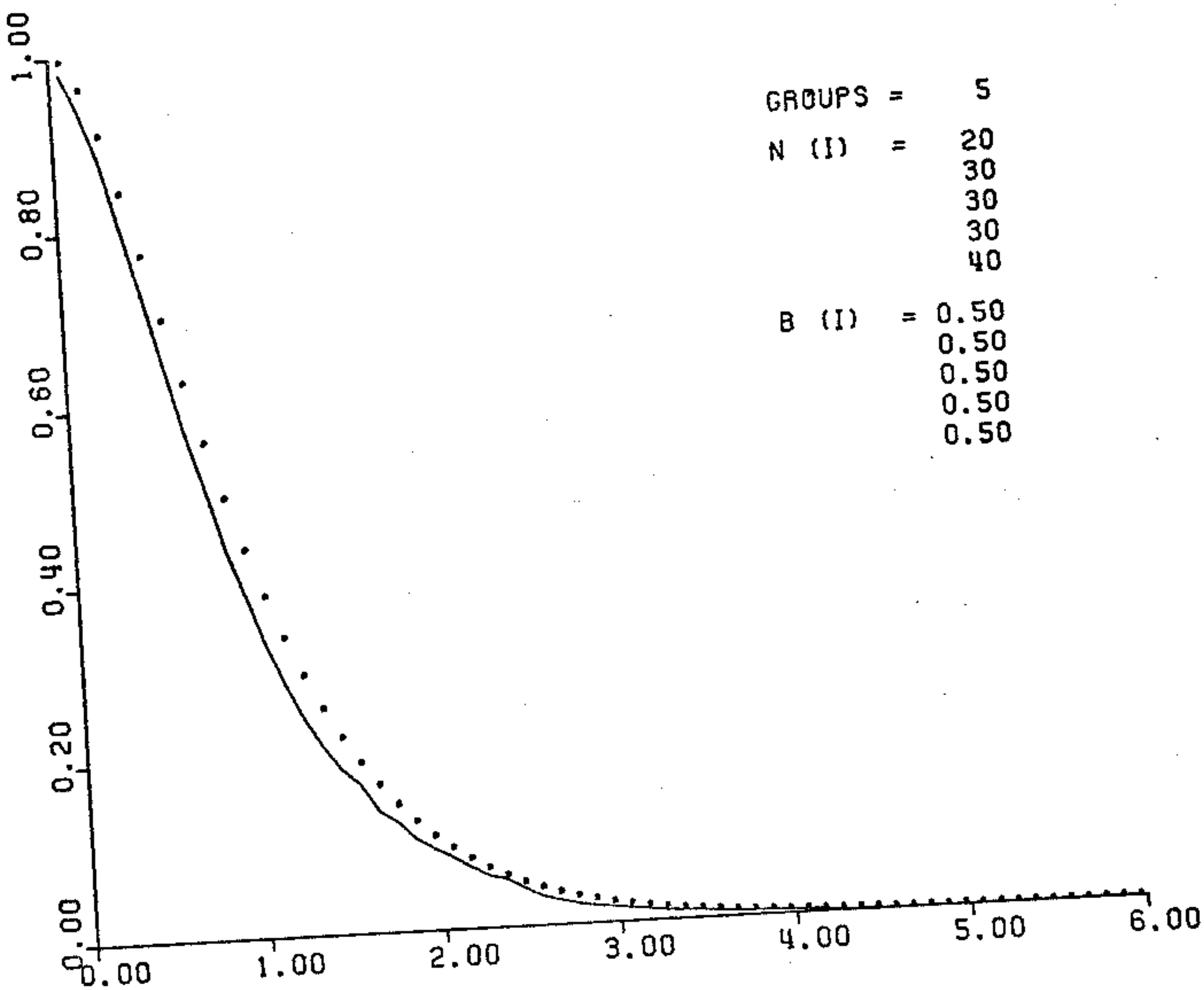
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 109 : SIMULATION NUMBER 109

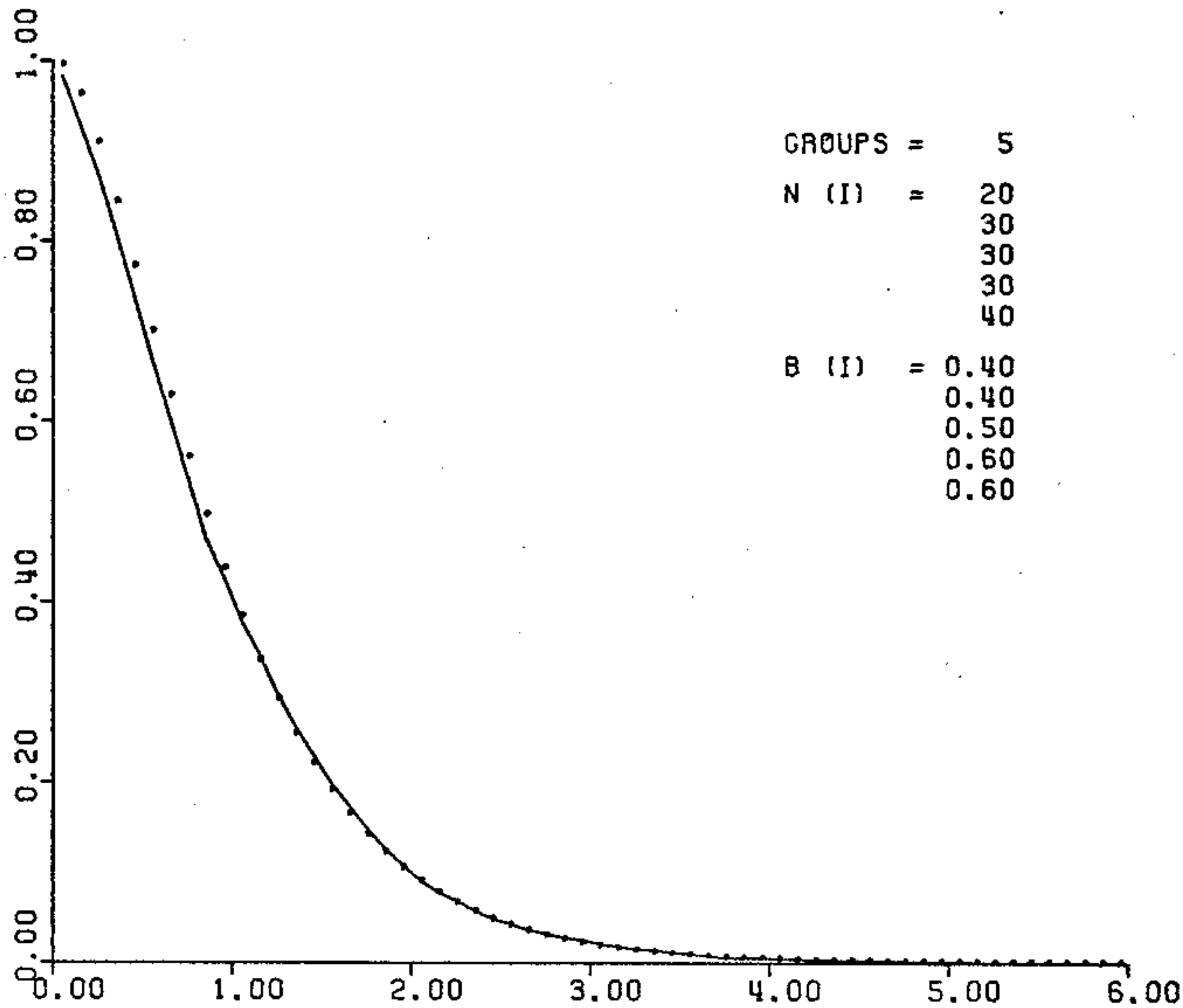


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 110 : SIMULATION NUMBER 110

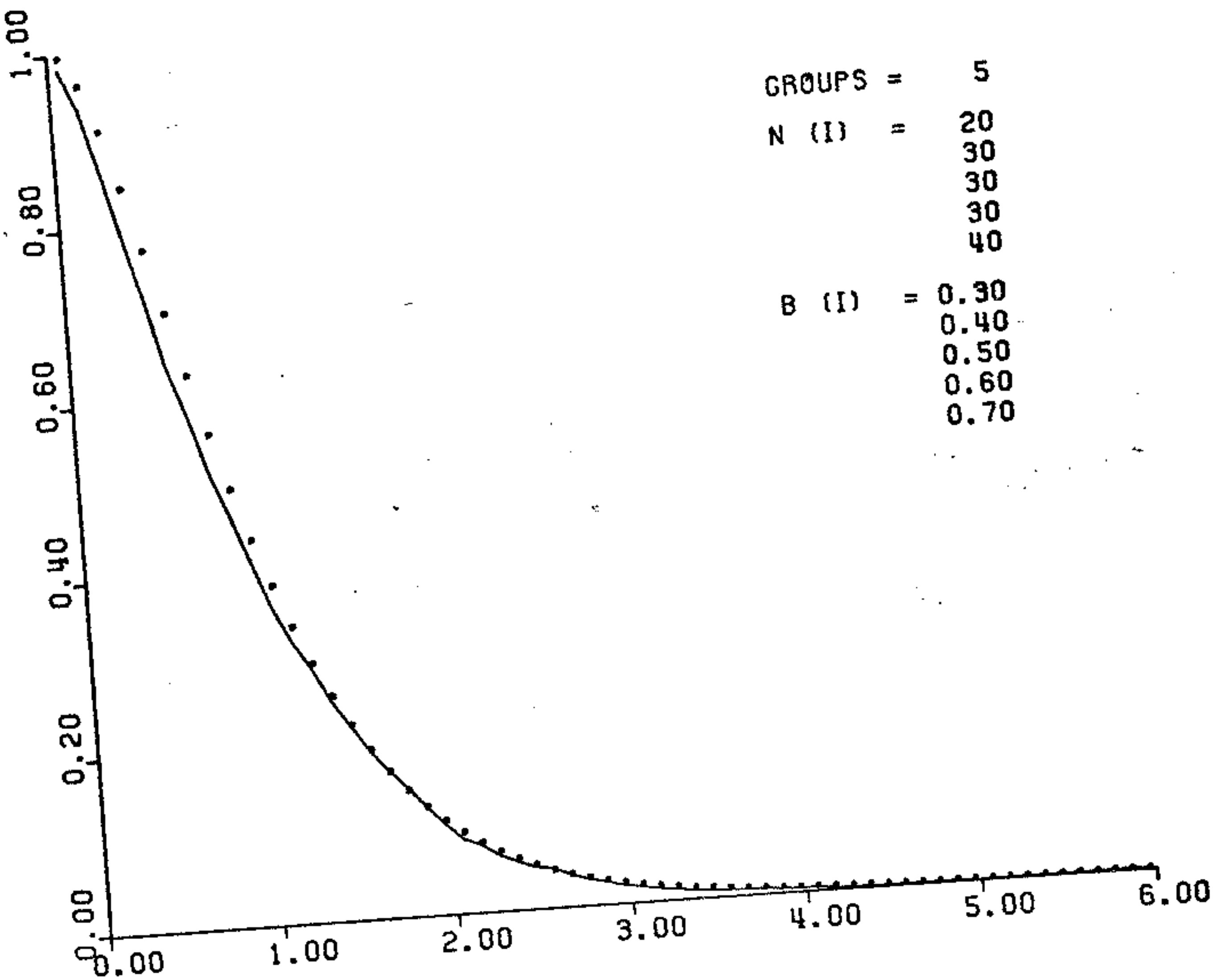


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 111 : SIMULATION NUMBER 111



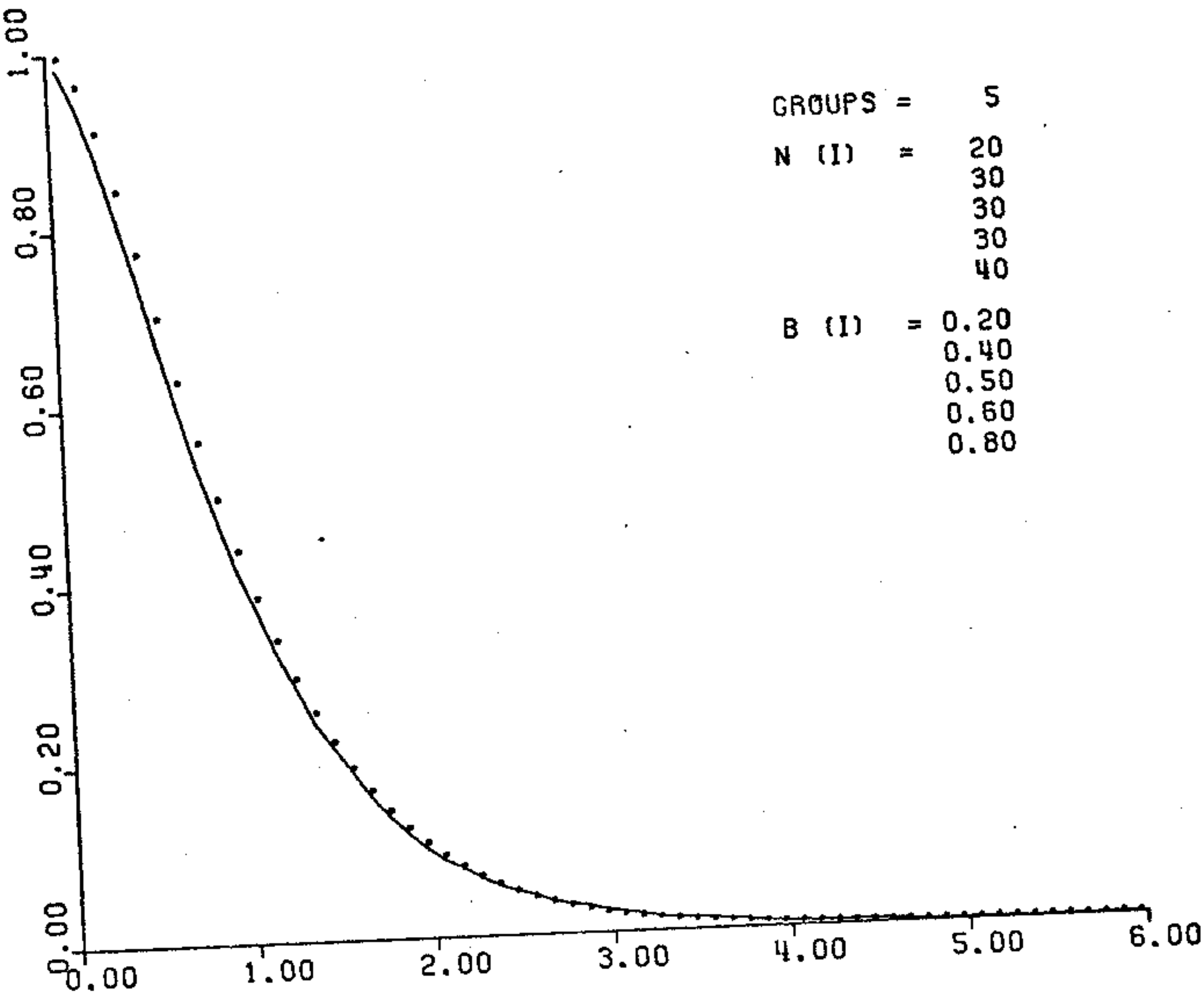
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 112 : SIMULATION NUMBER 112



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 113 : SIMULATION NUMBER 113



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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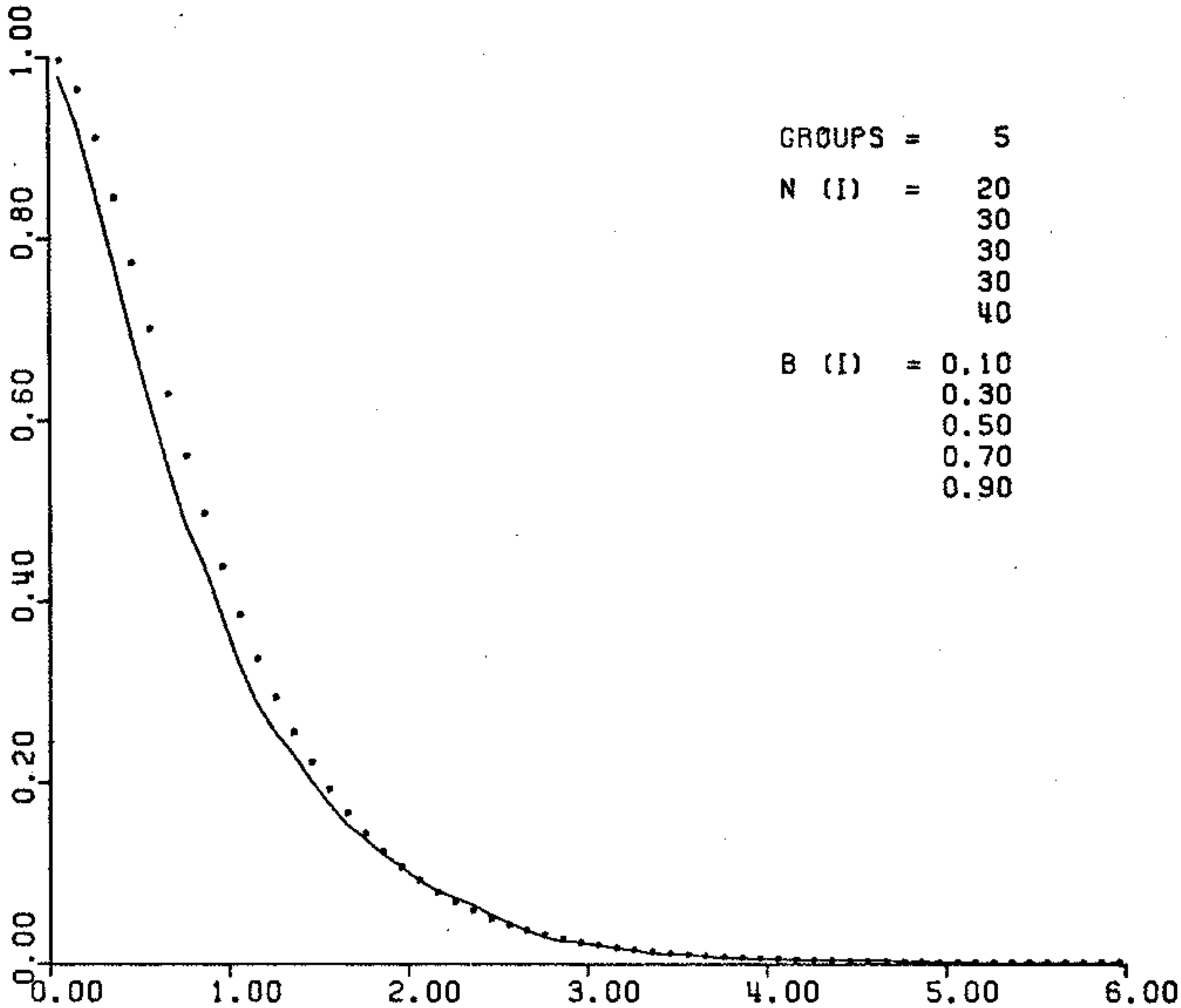

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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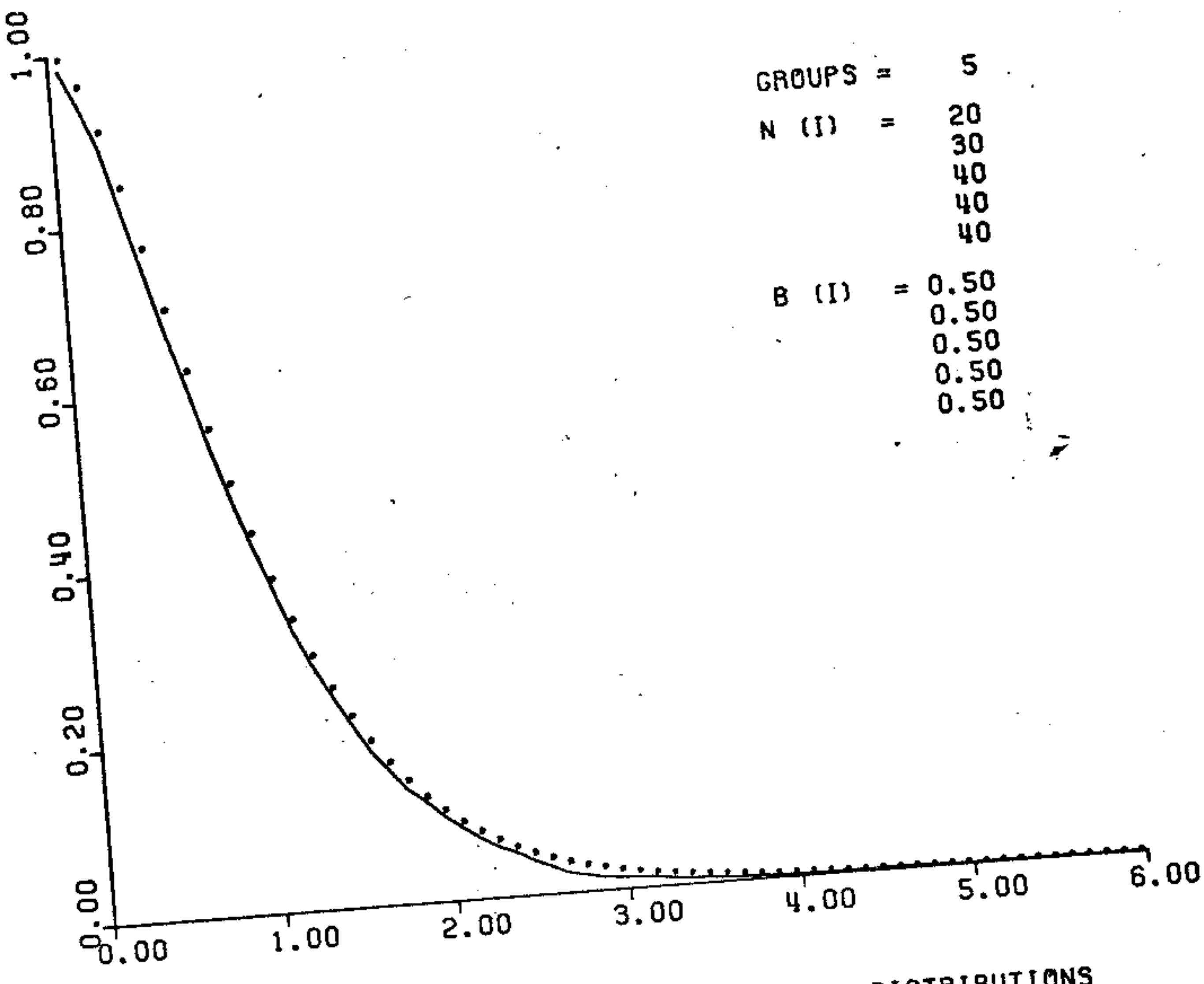

THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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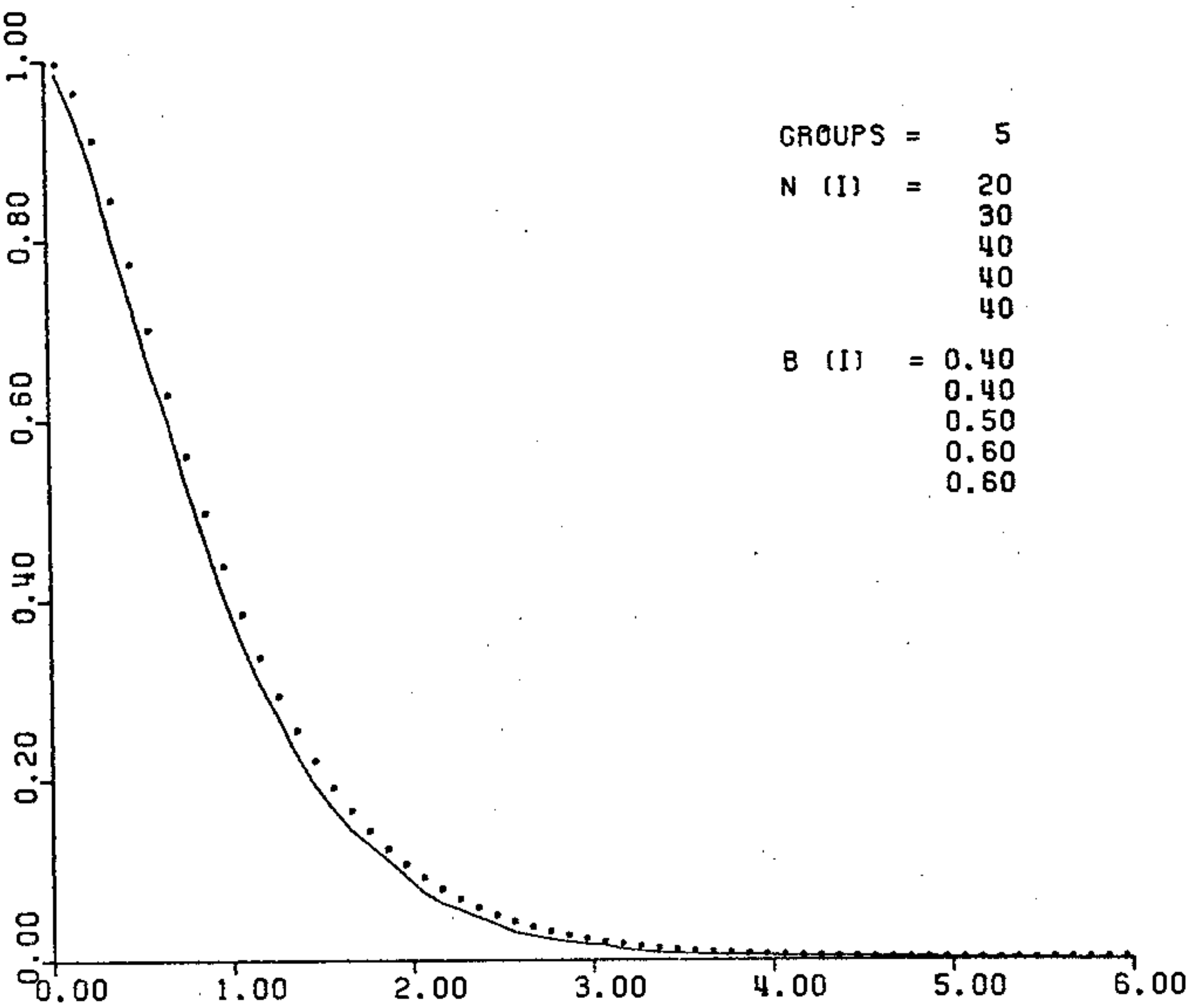
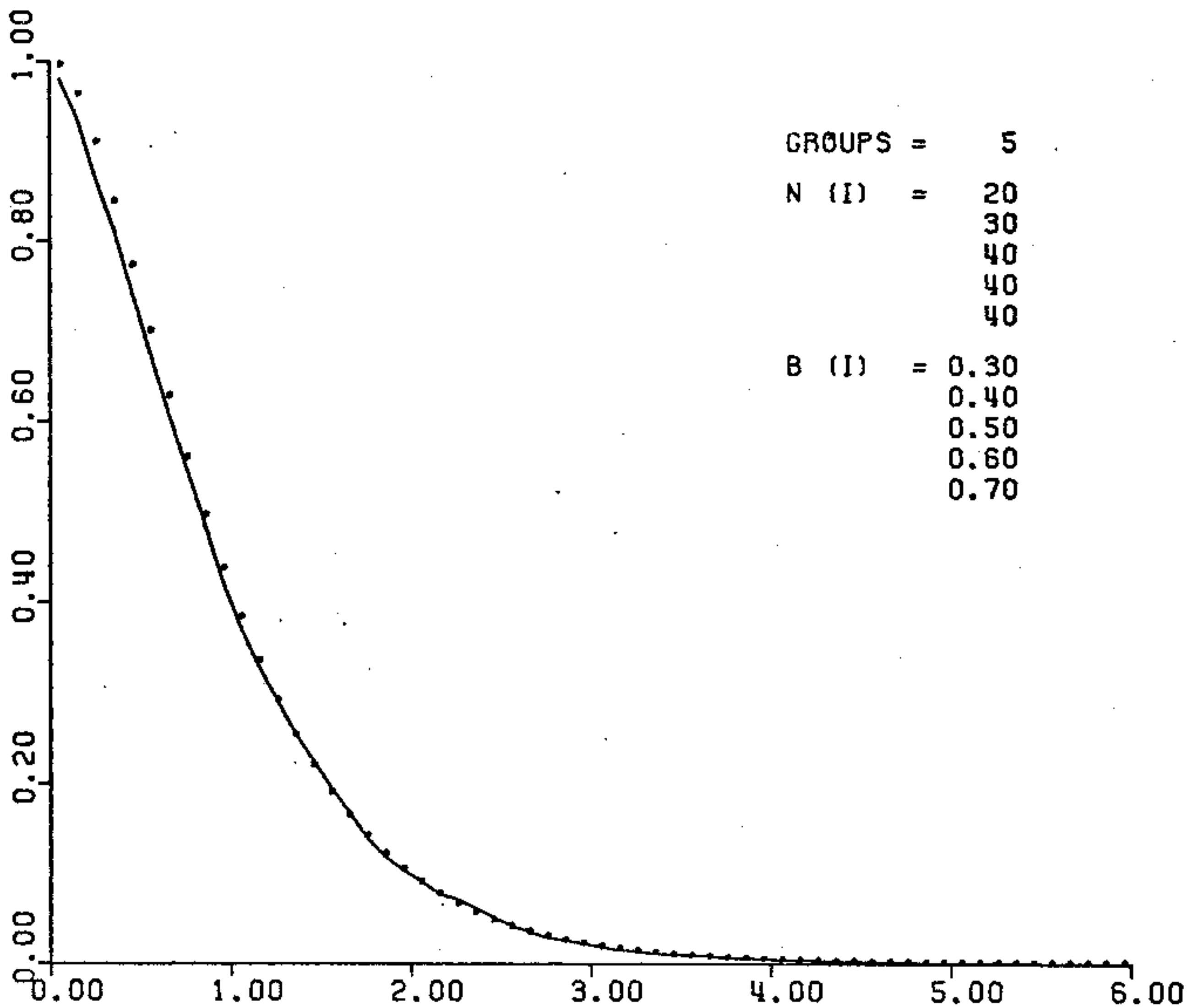


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 117 : SIMULATION NUMBER 117



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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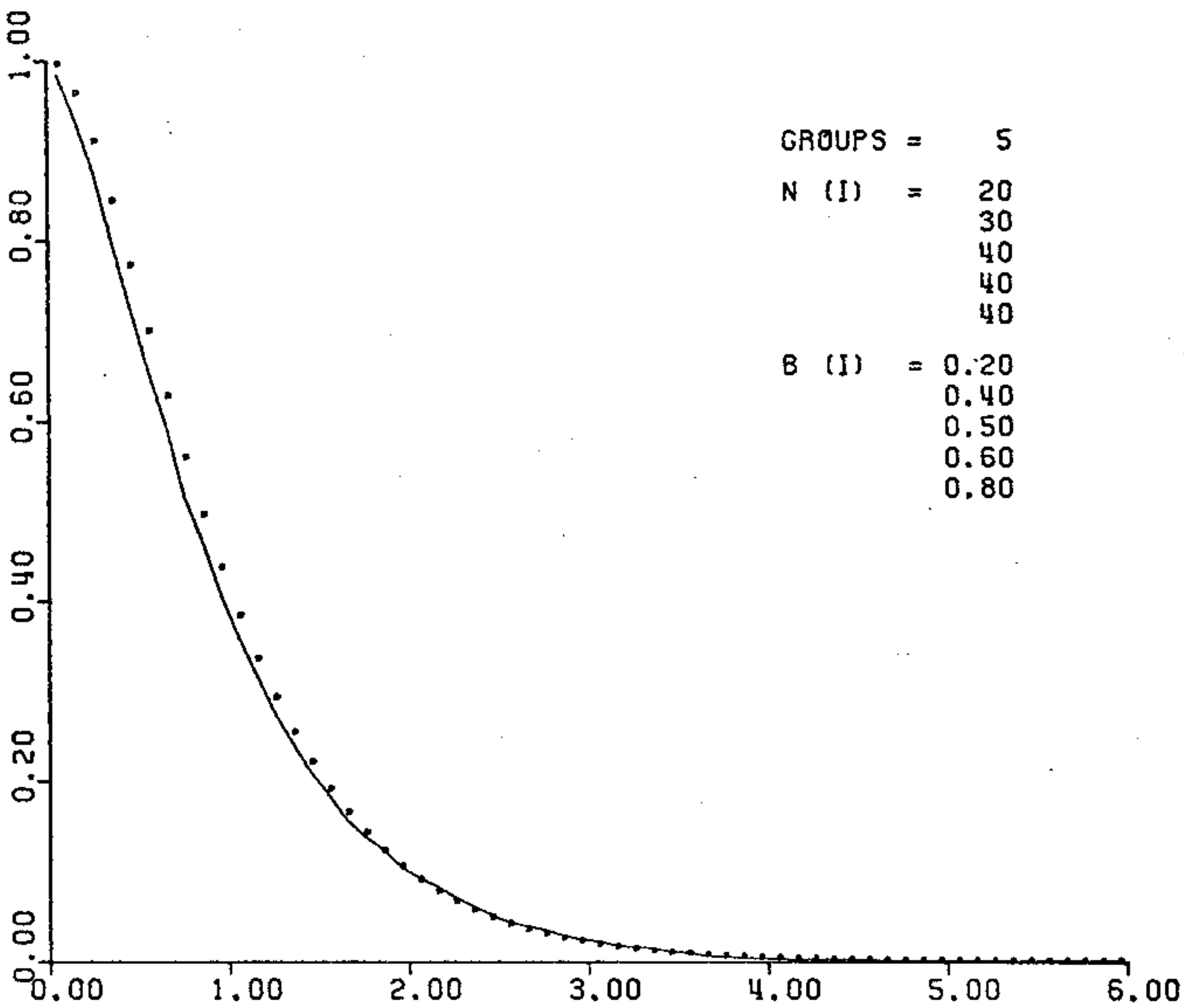
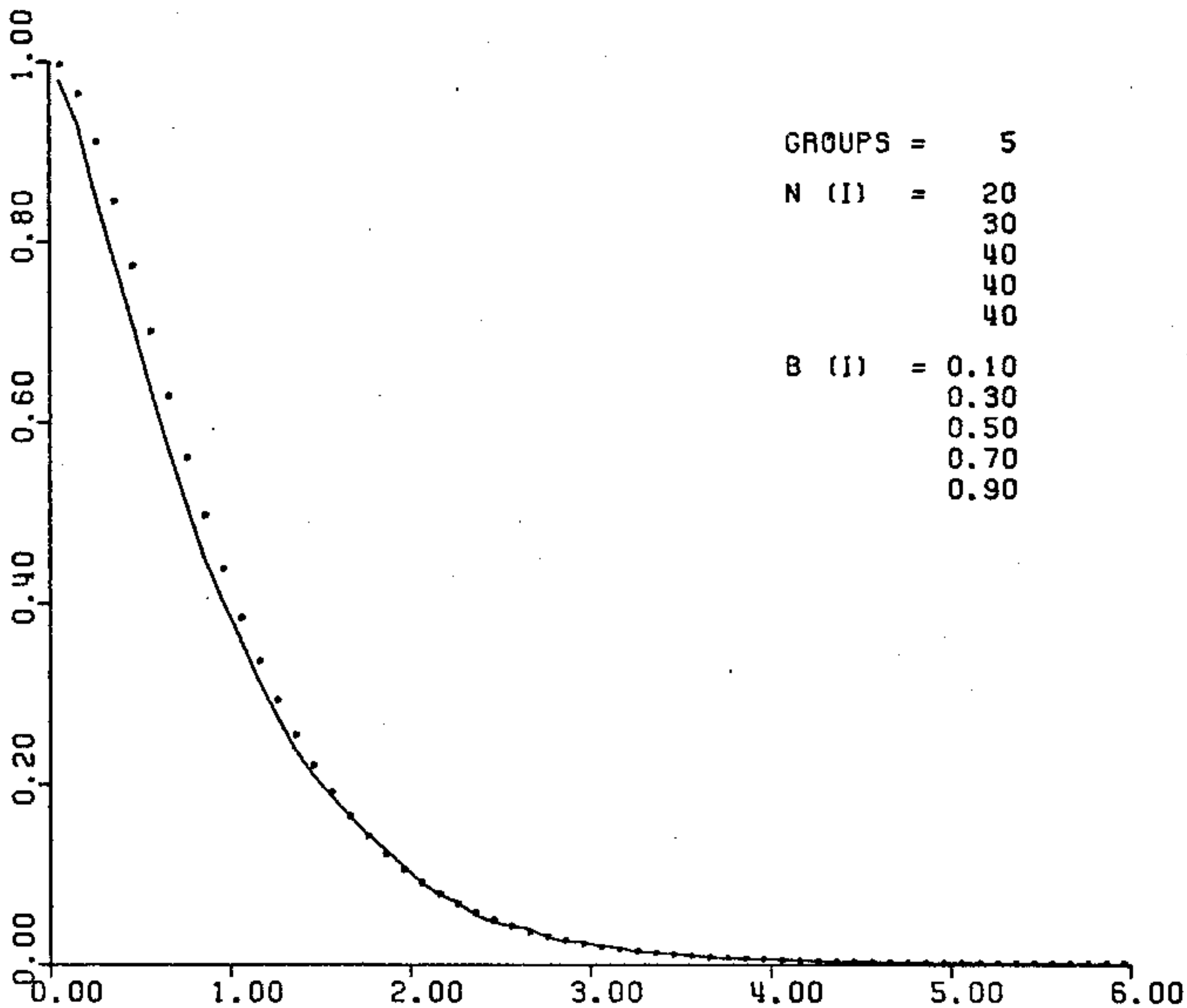
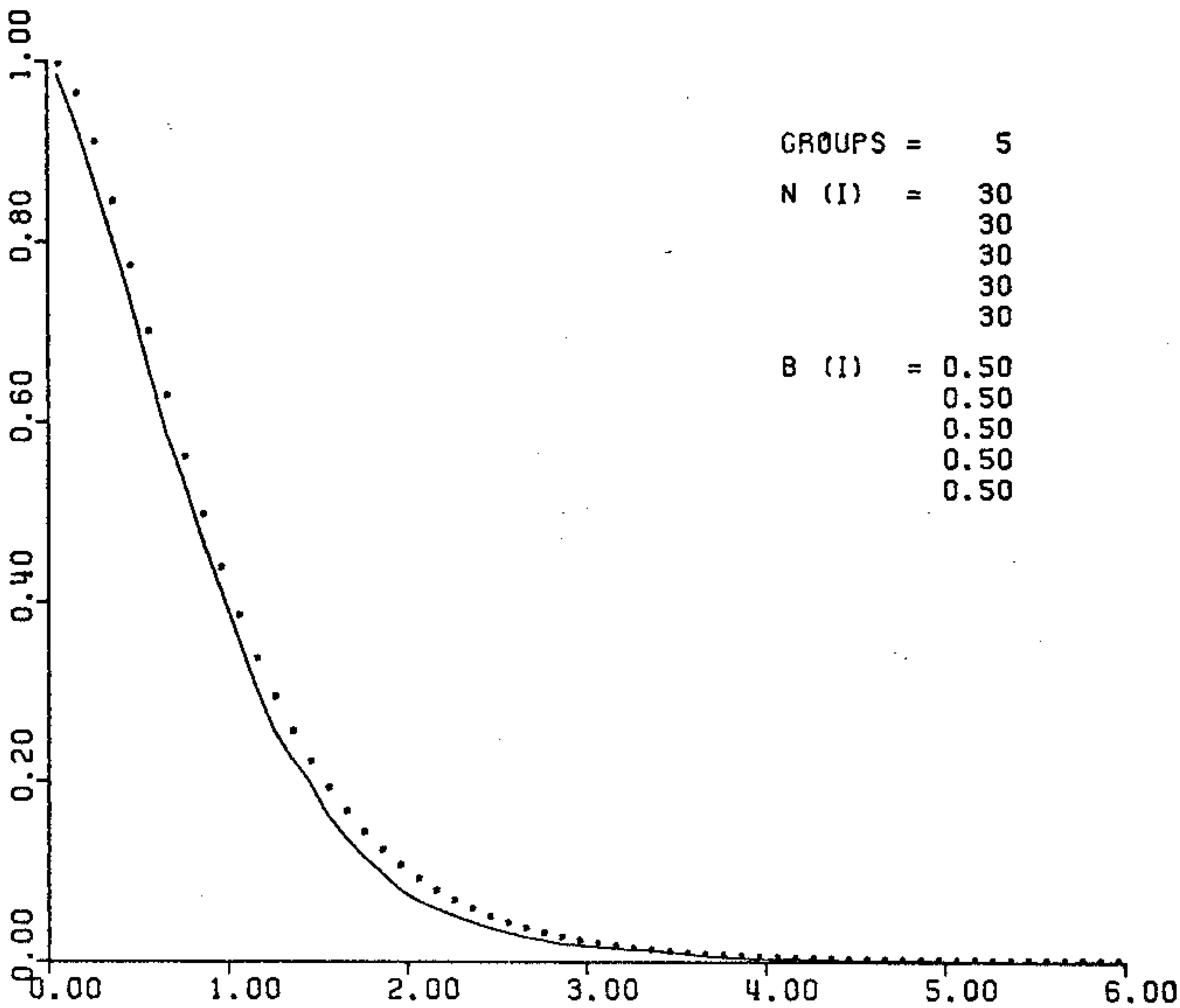


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

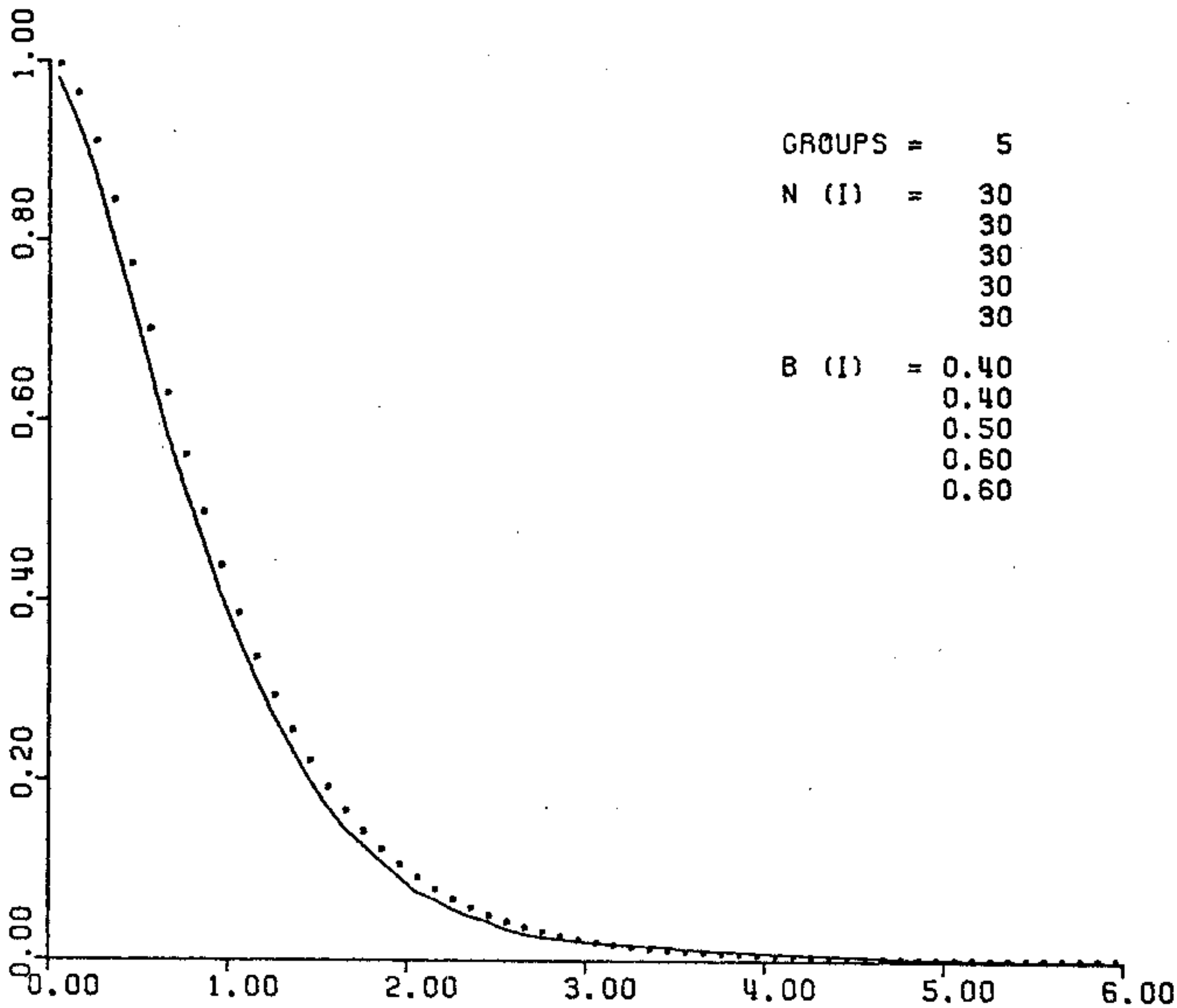
FIGURE 119 : SIMULATION NUMBER 119



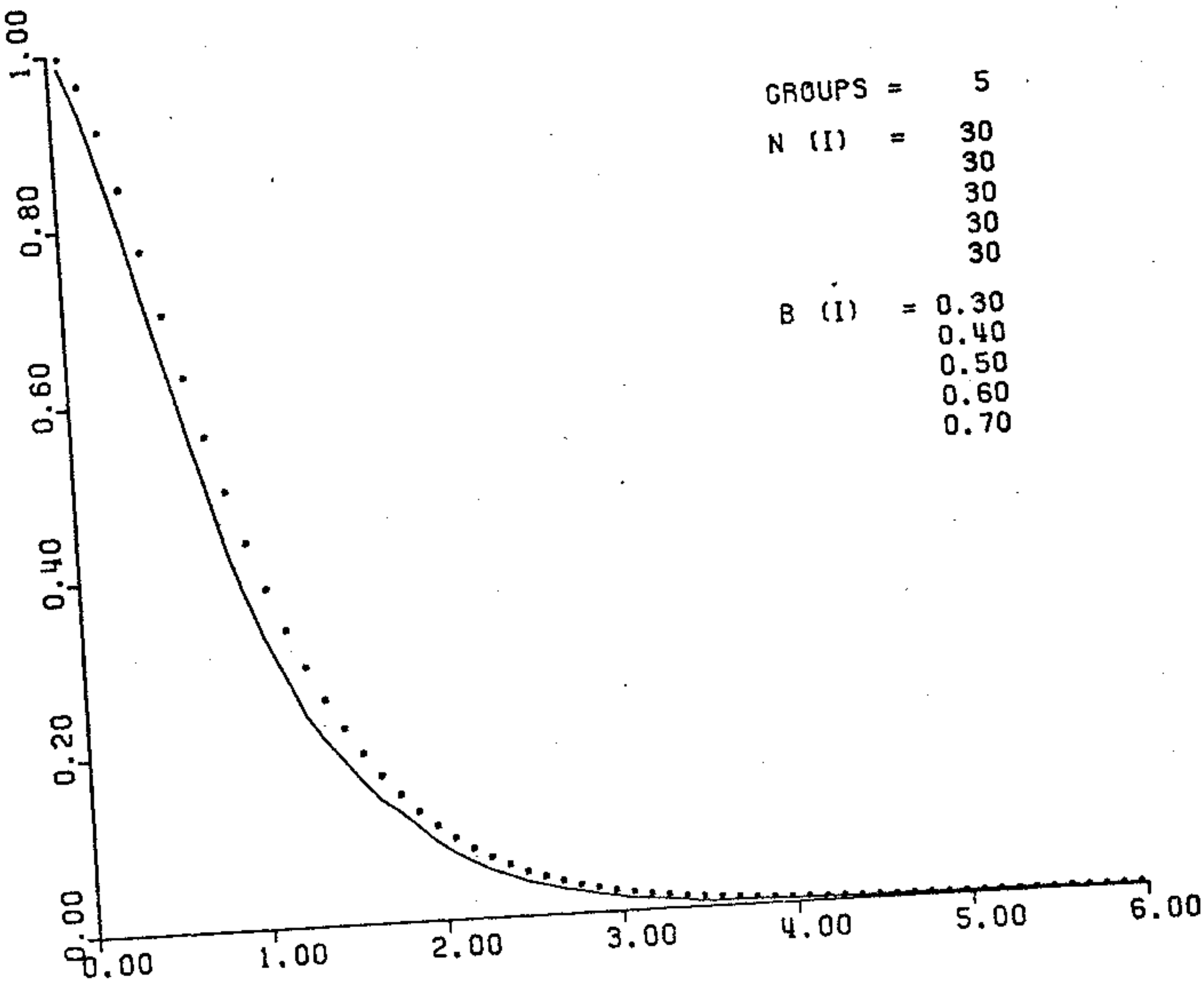
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 120 : SIMULATION NUMBER 120



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 121 : SIMULATION NUMBER 121

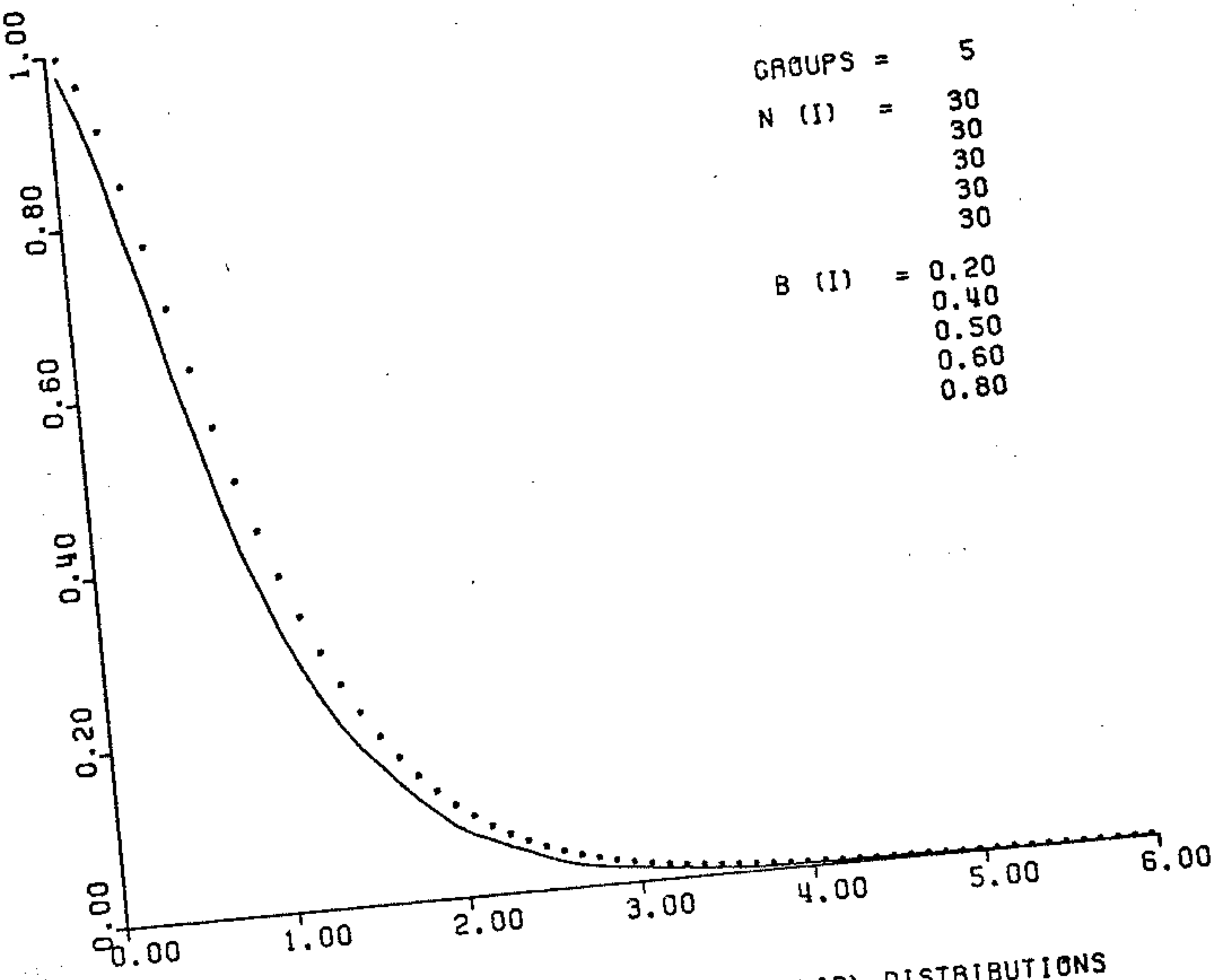


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 122 : SIMULATION NUMBER 122

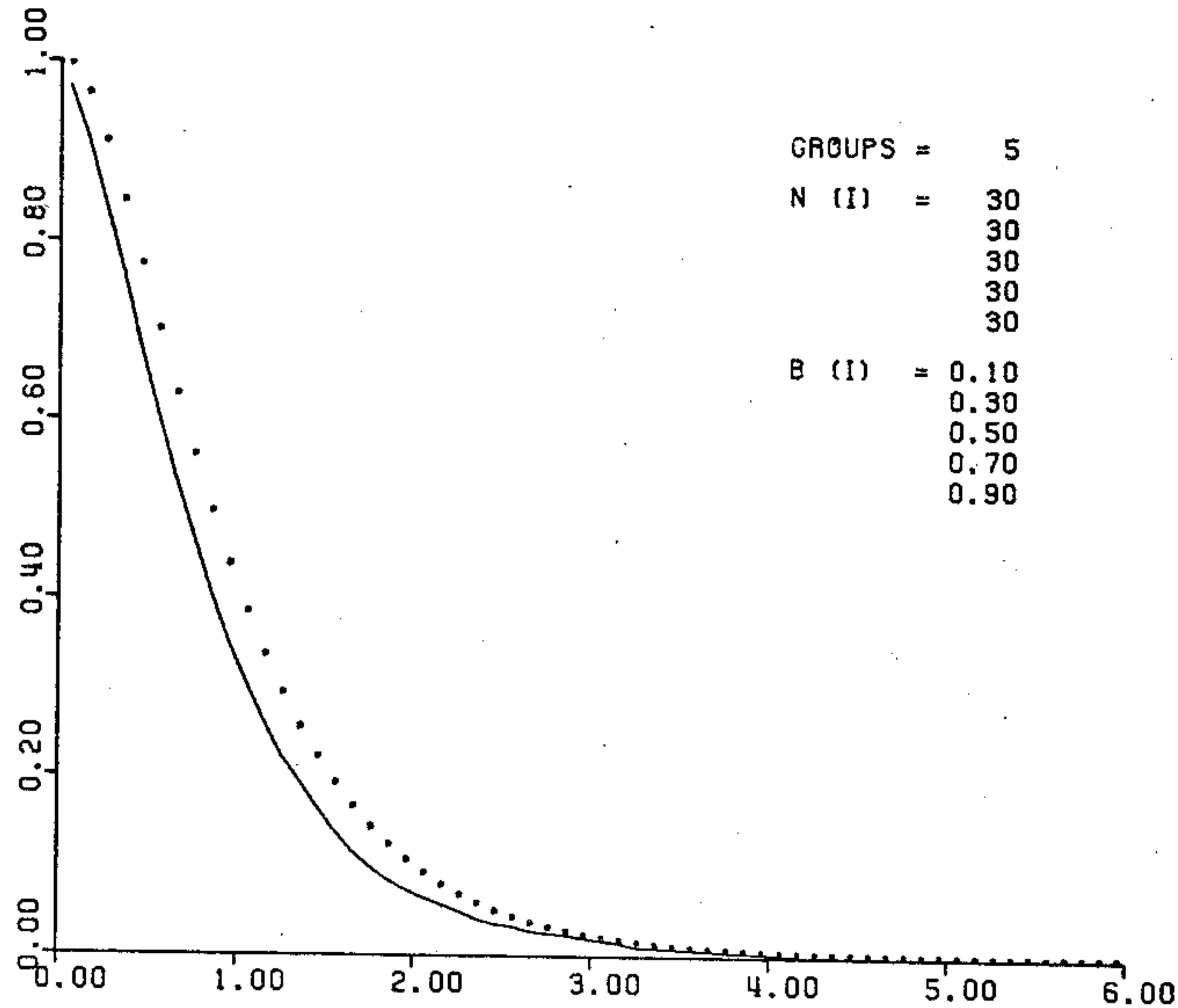


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 123 : SIMULATION NUMBER 123



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 124 : SIMULATION NUMBER 124



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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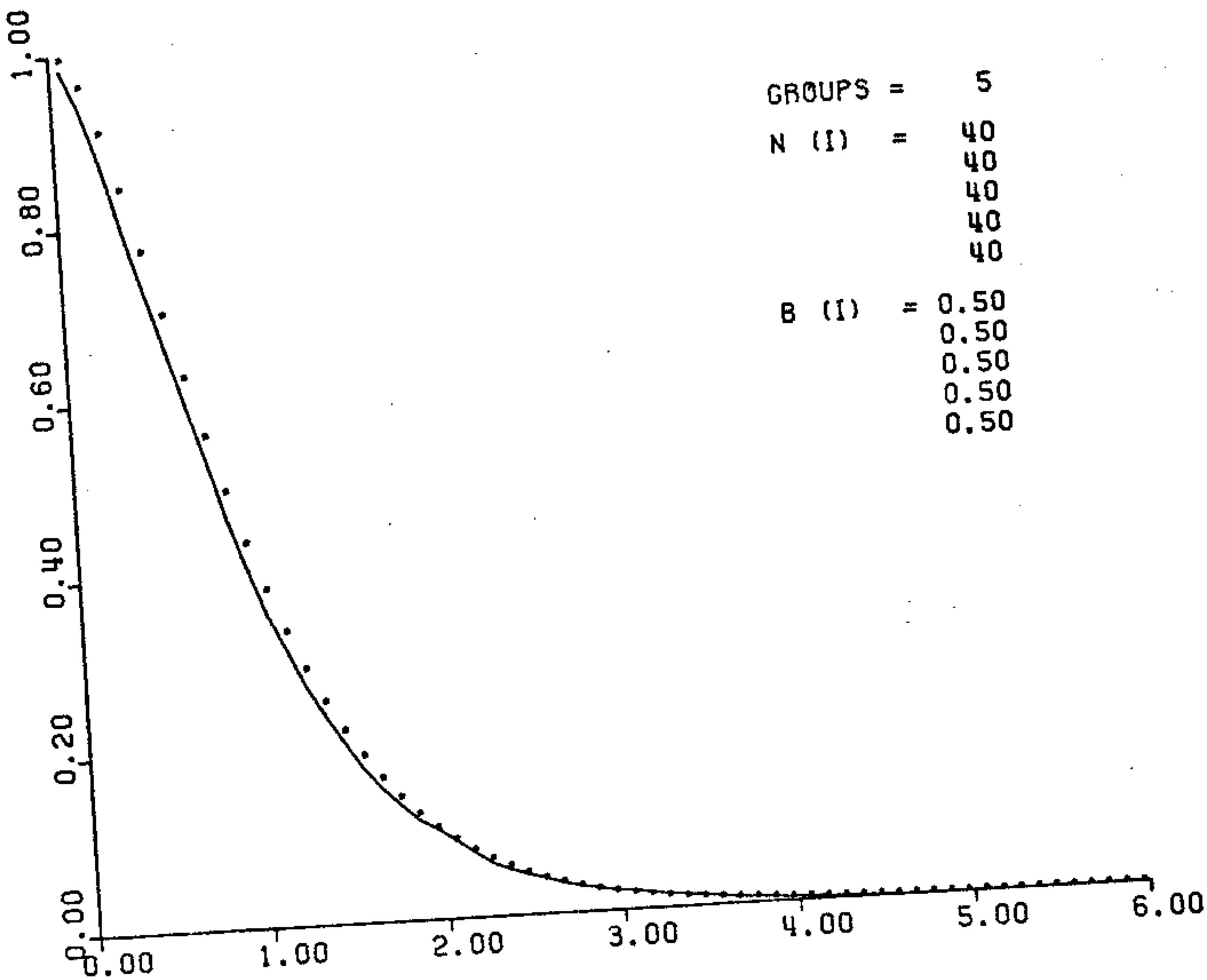
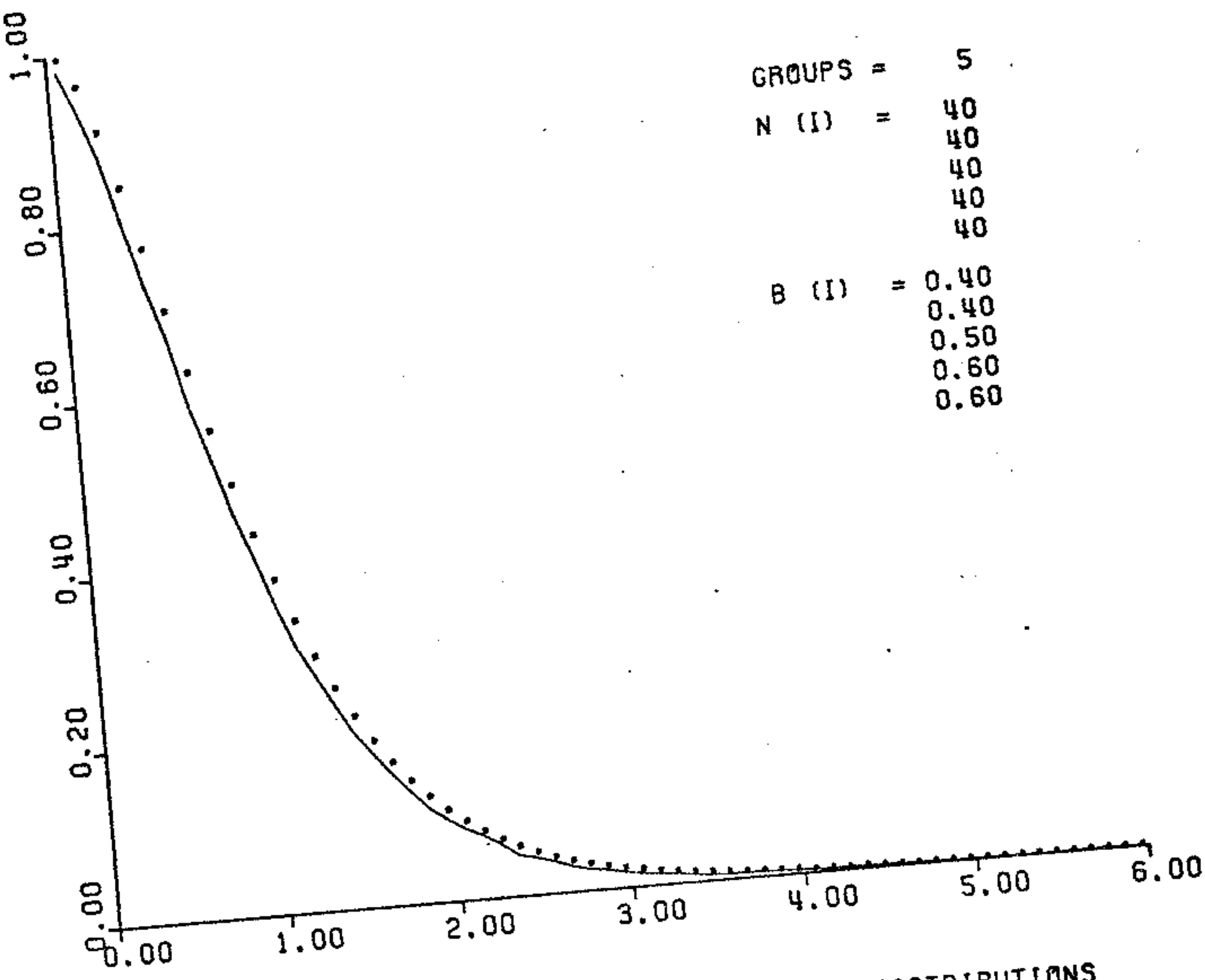
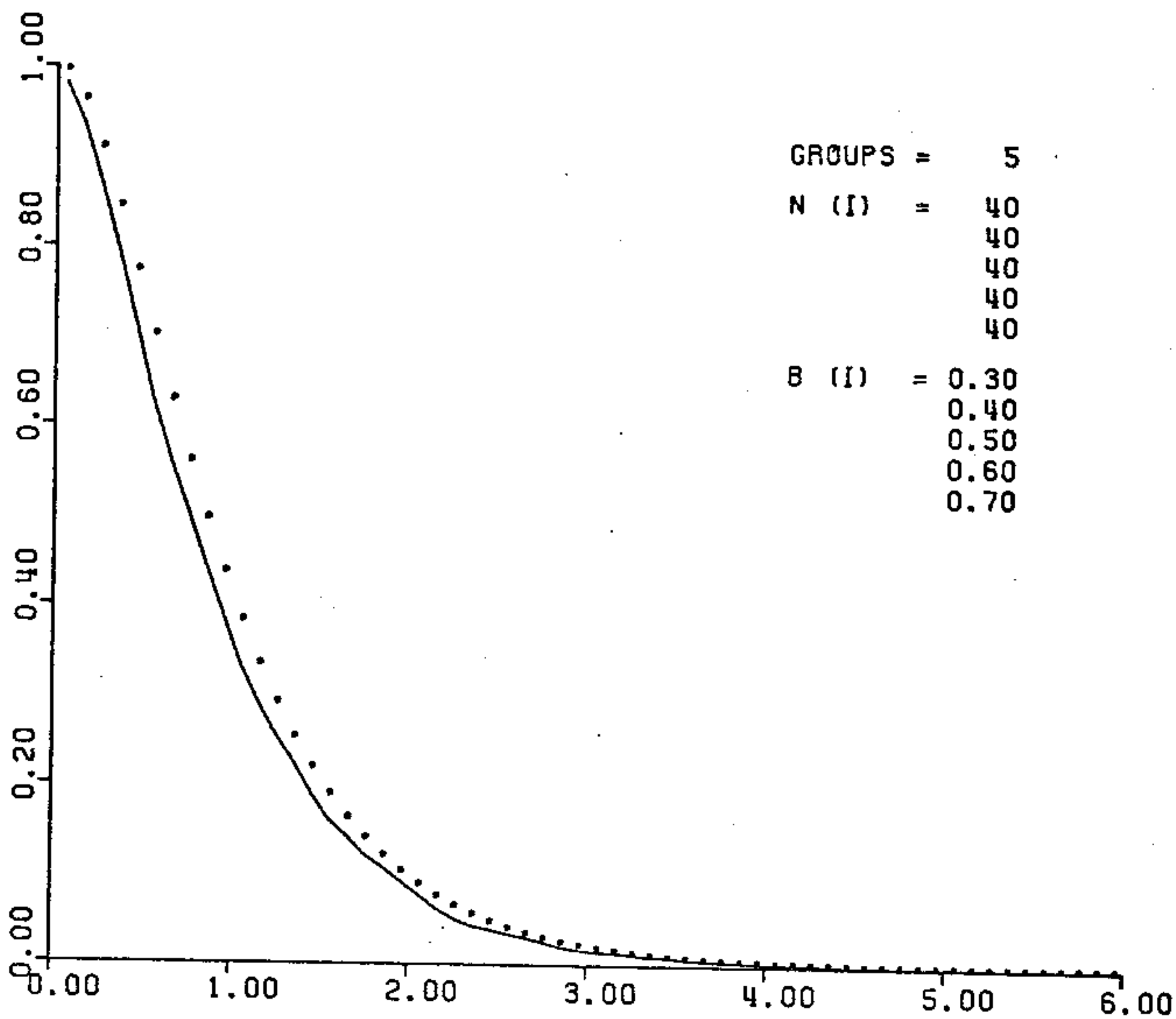


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 126 : SIMULATION NUMBER 126

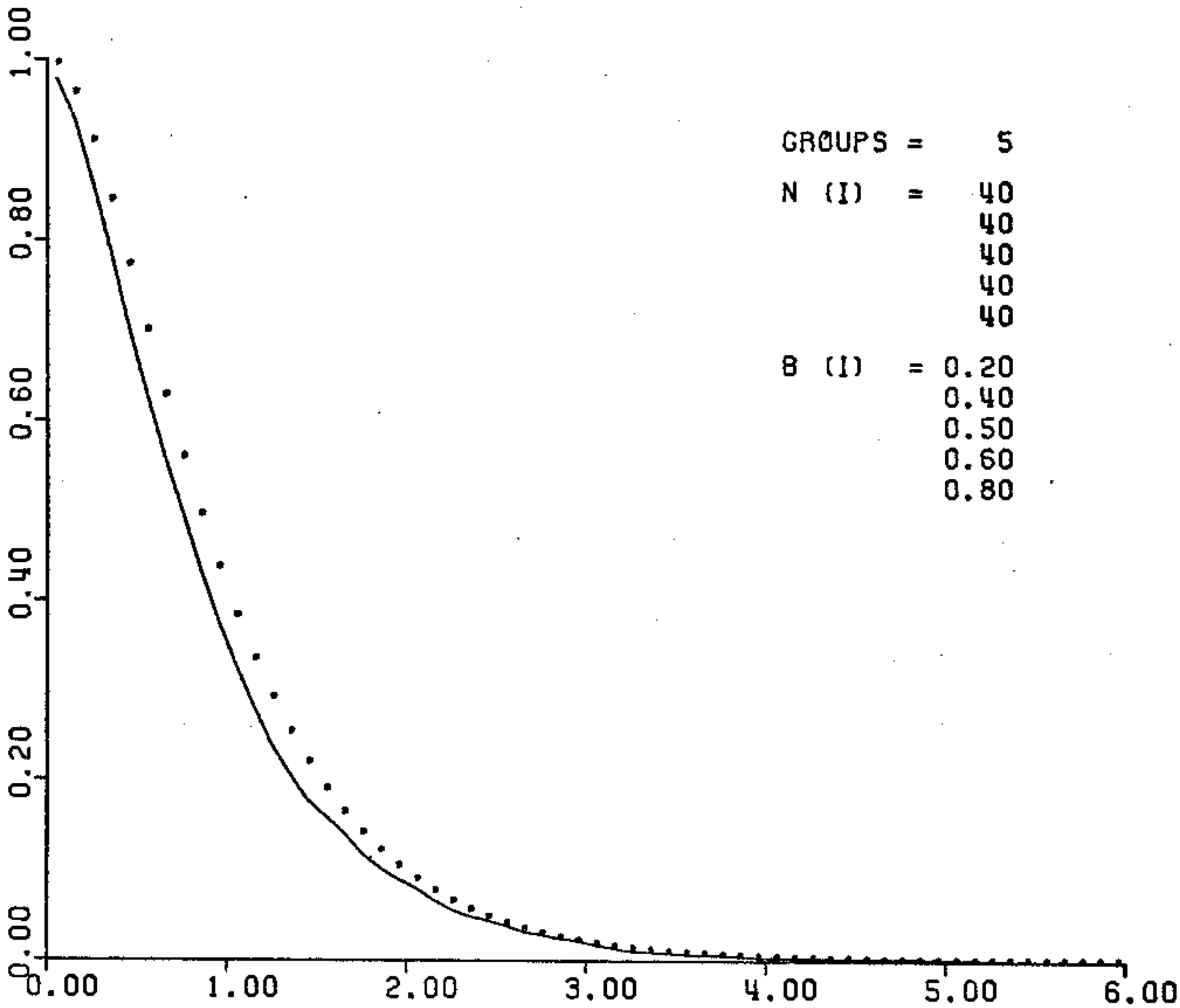


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

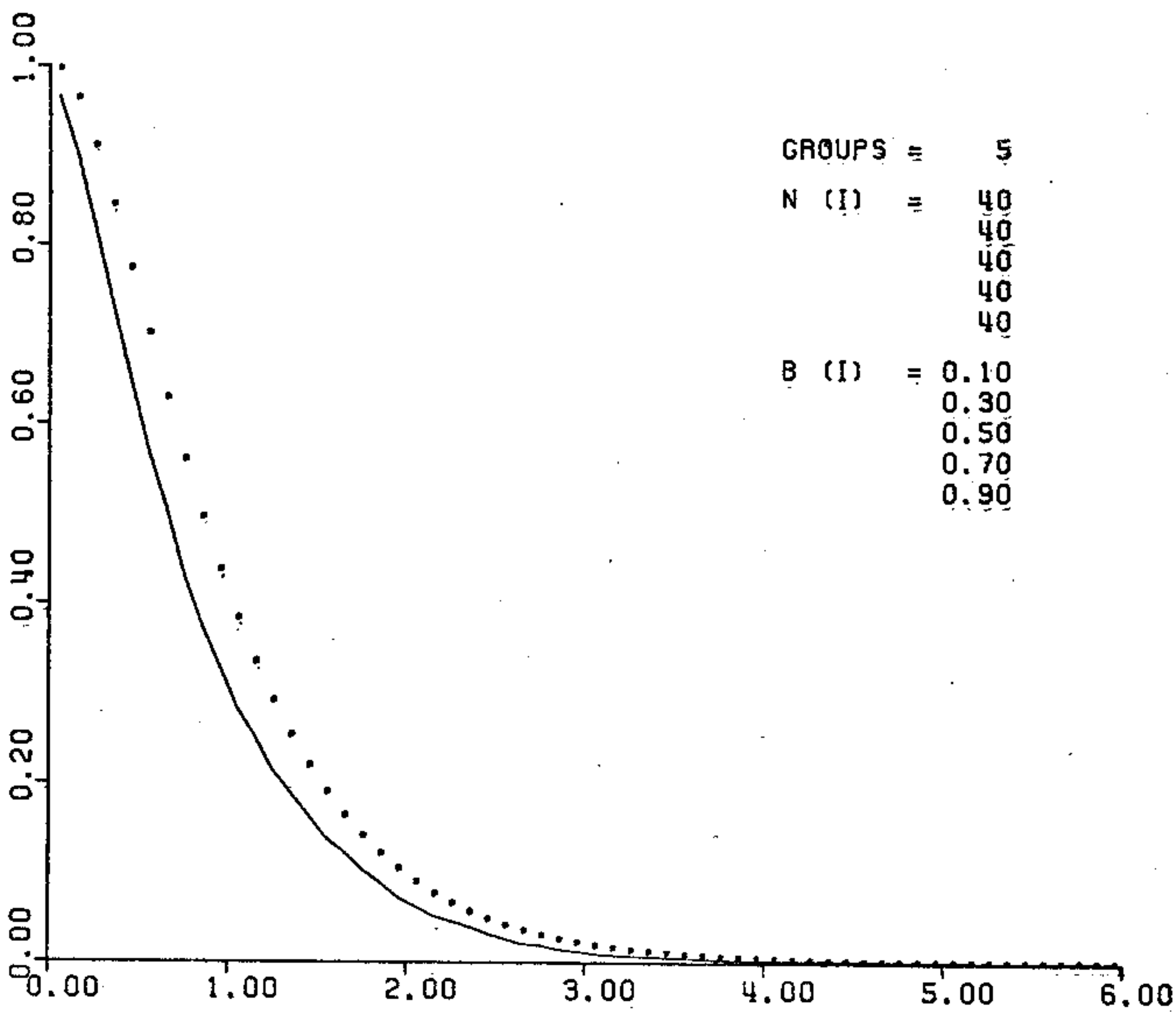
FIGURE 127 : SIMULATION NUMBER 127



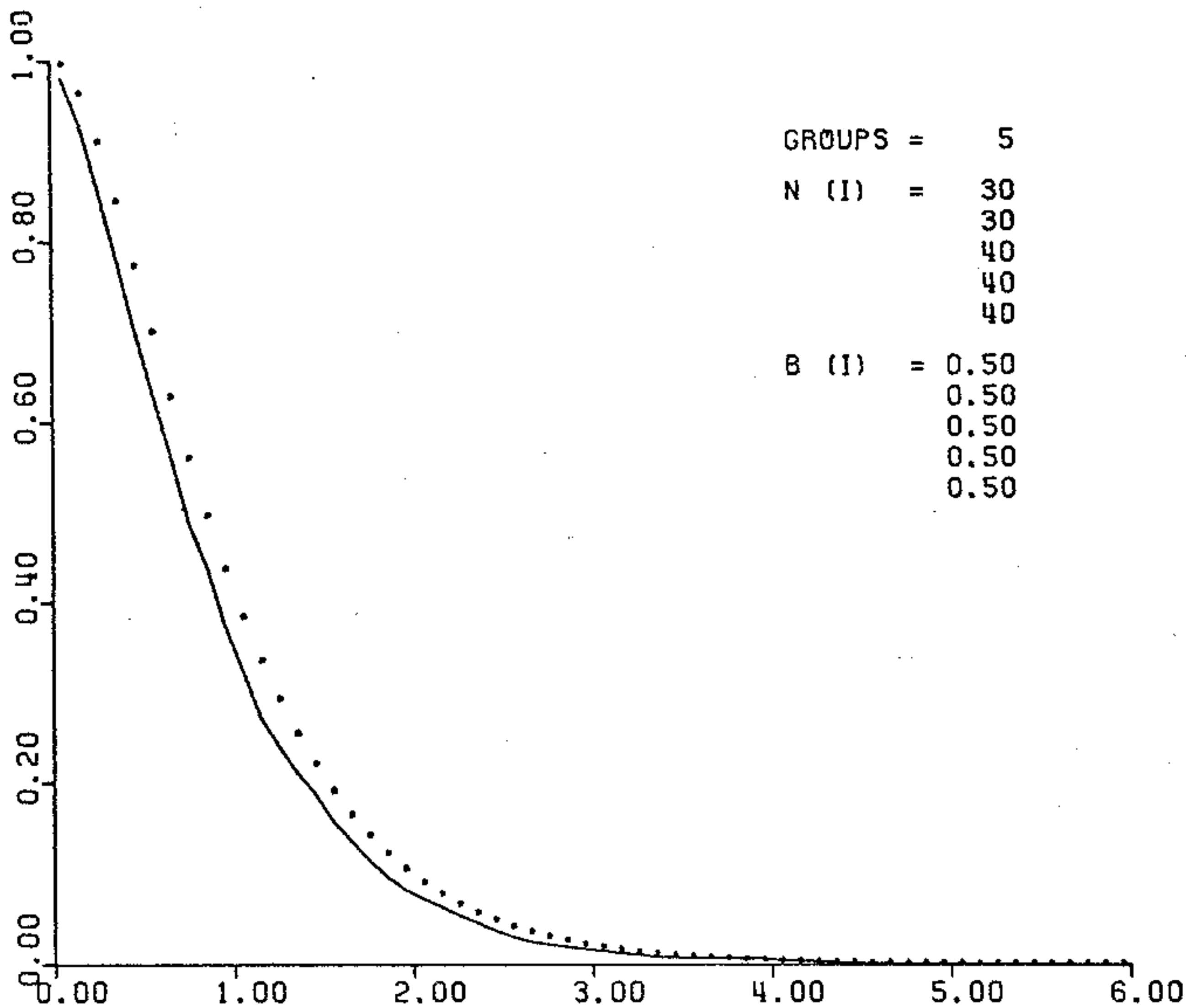
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 128 : SIMULATION NUMBER 128



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 129 : SIMULATION NUMBER 129

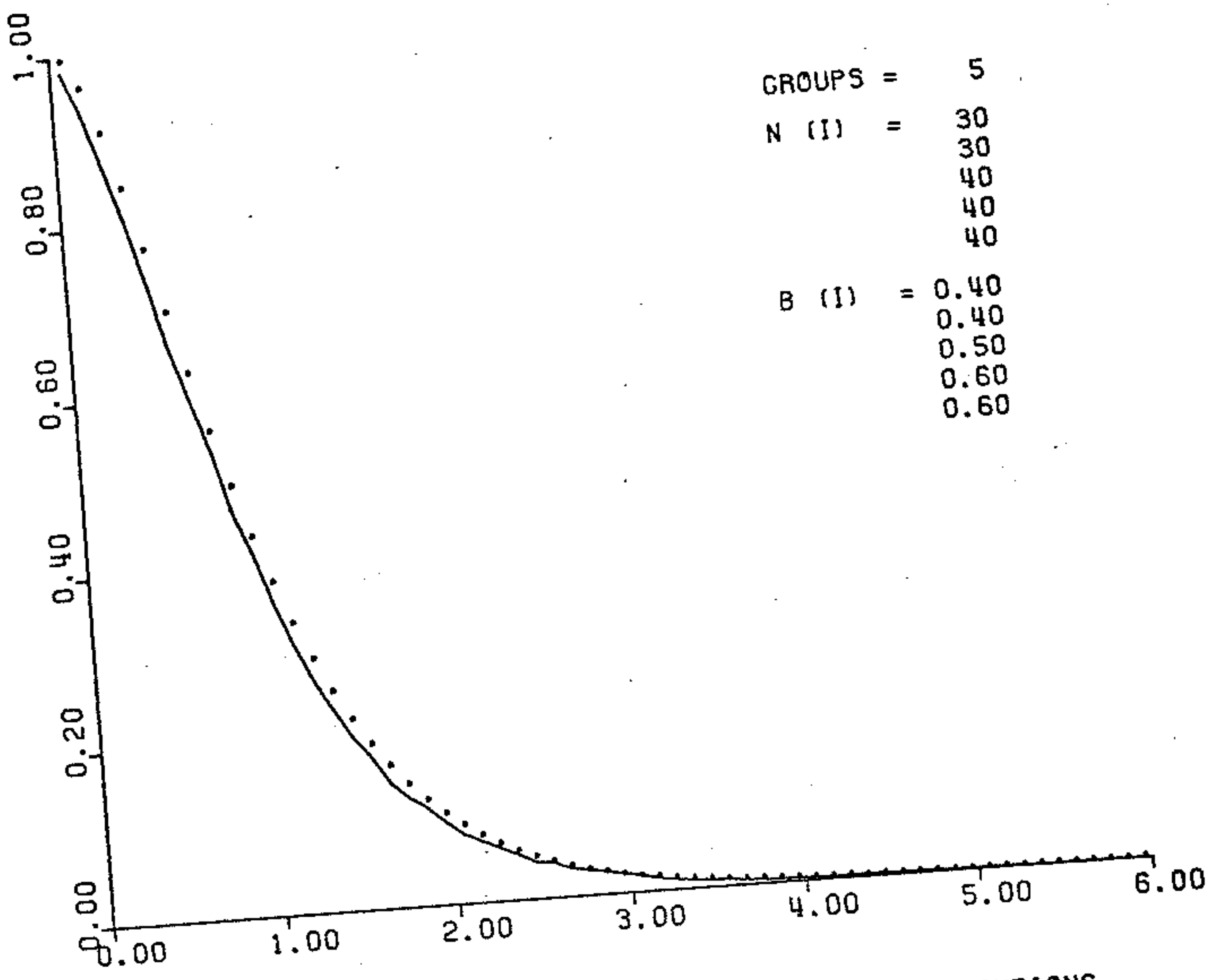


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 130 : SIMULATION NUMBER 130



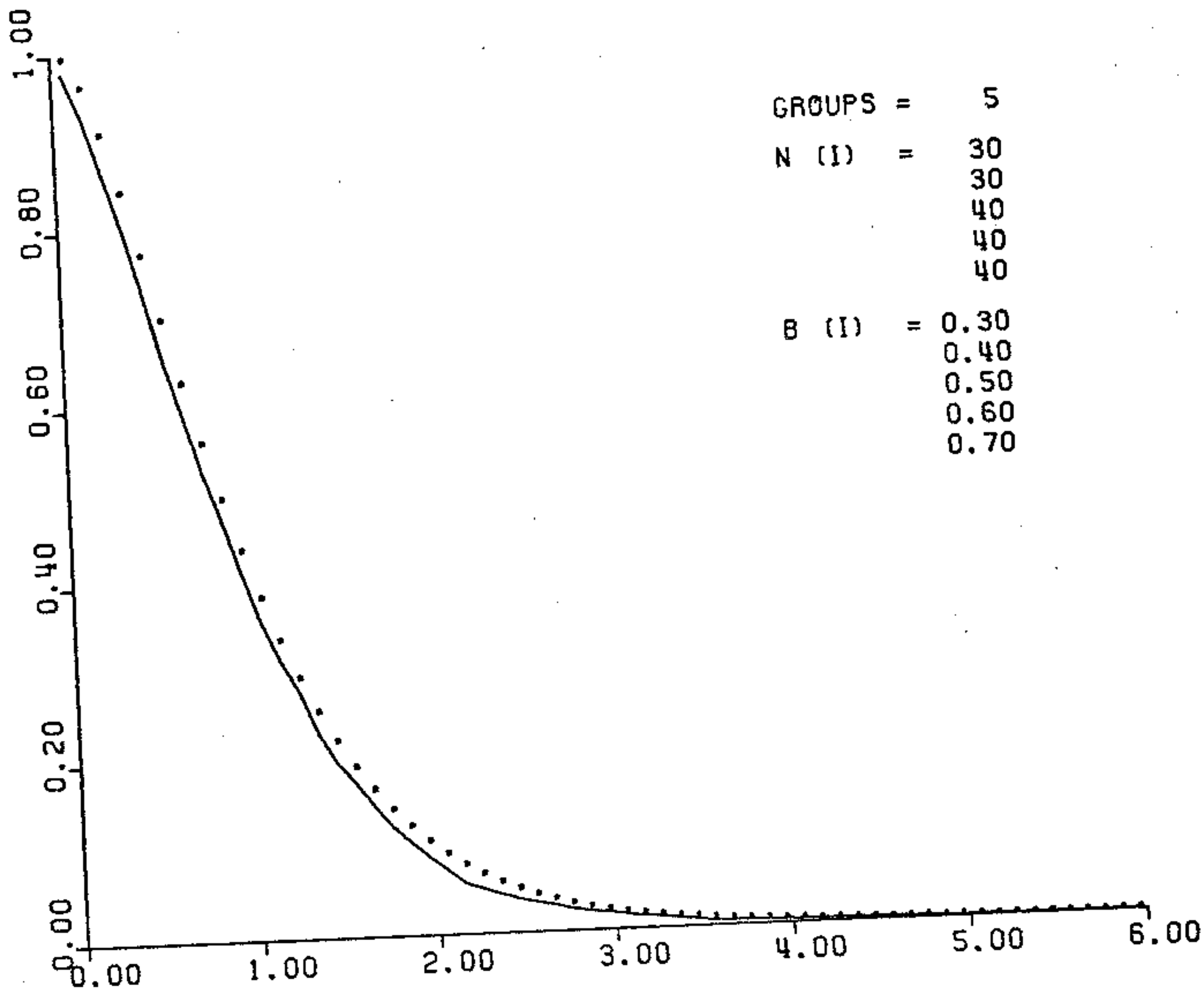
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 131 : SIMULATION NUMBER 131

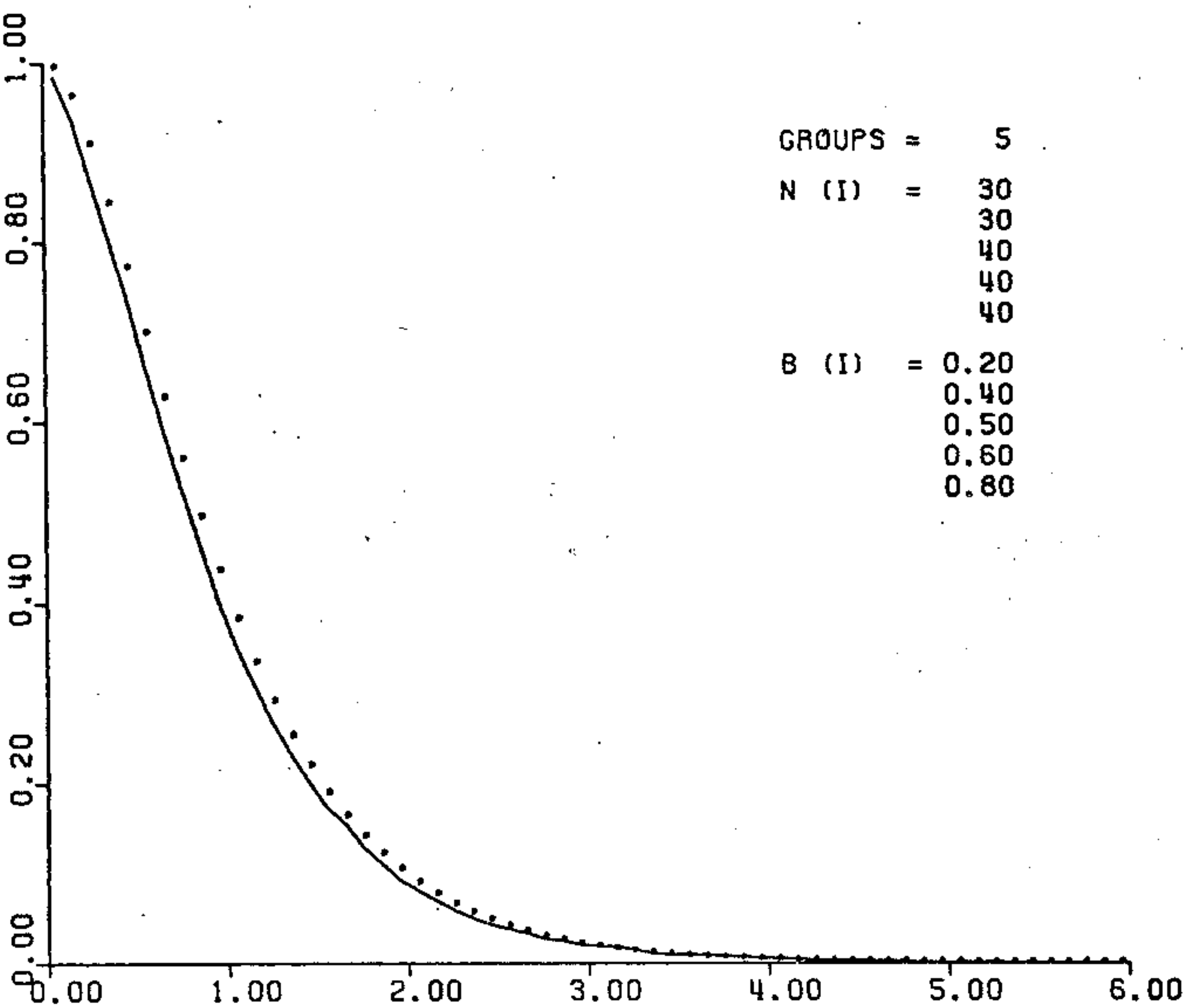


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 132 : SIMULATION NUMBER 132



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 133 : SIMULATION NUMBER 133



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

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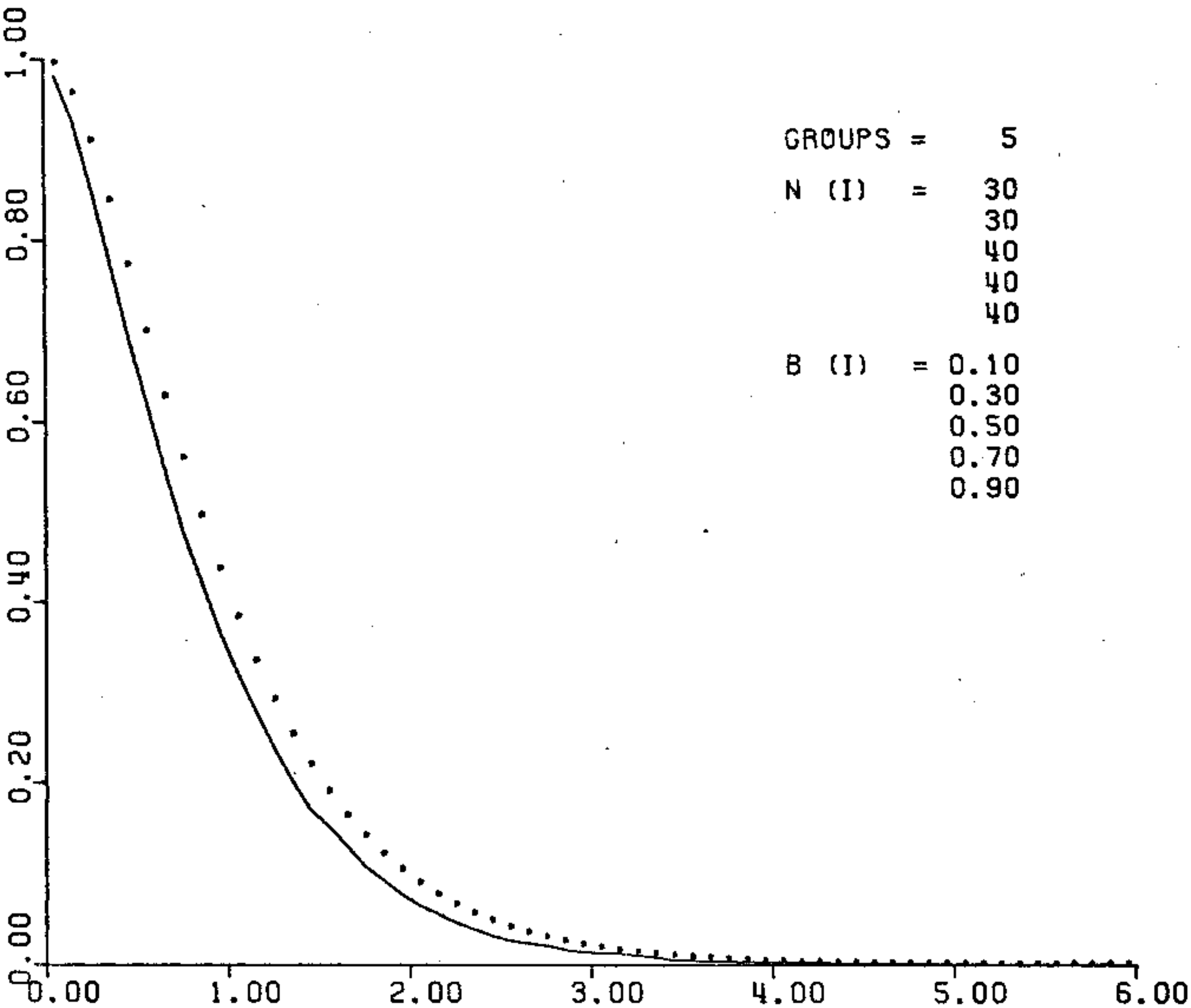
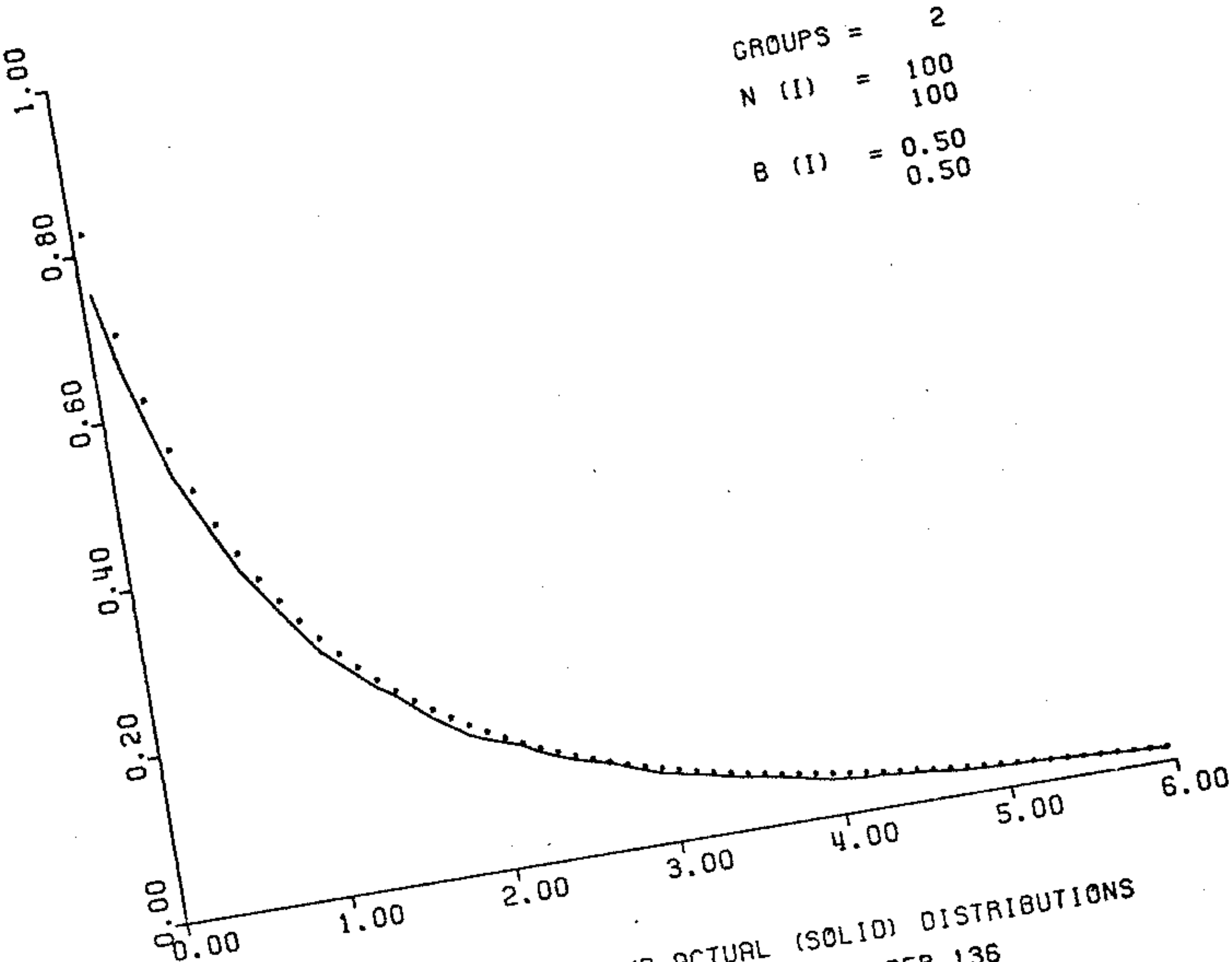


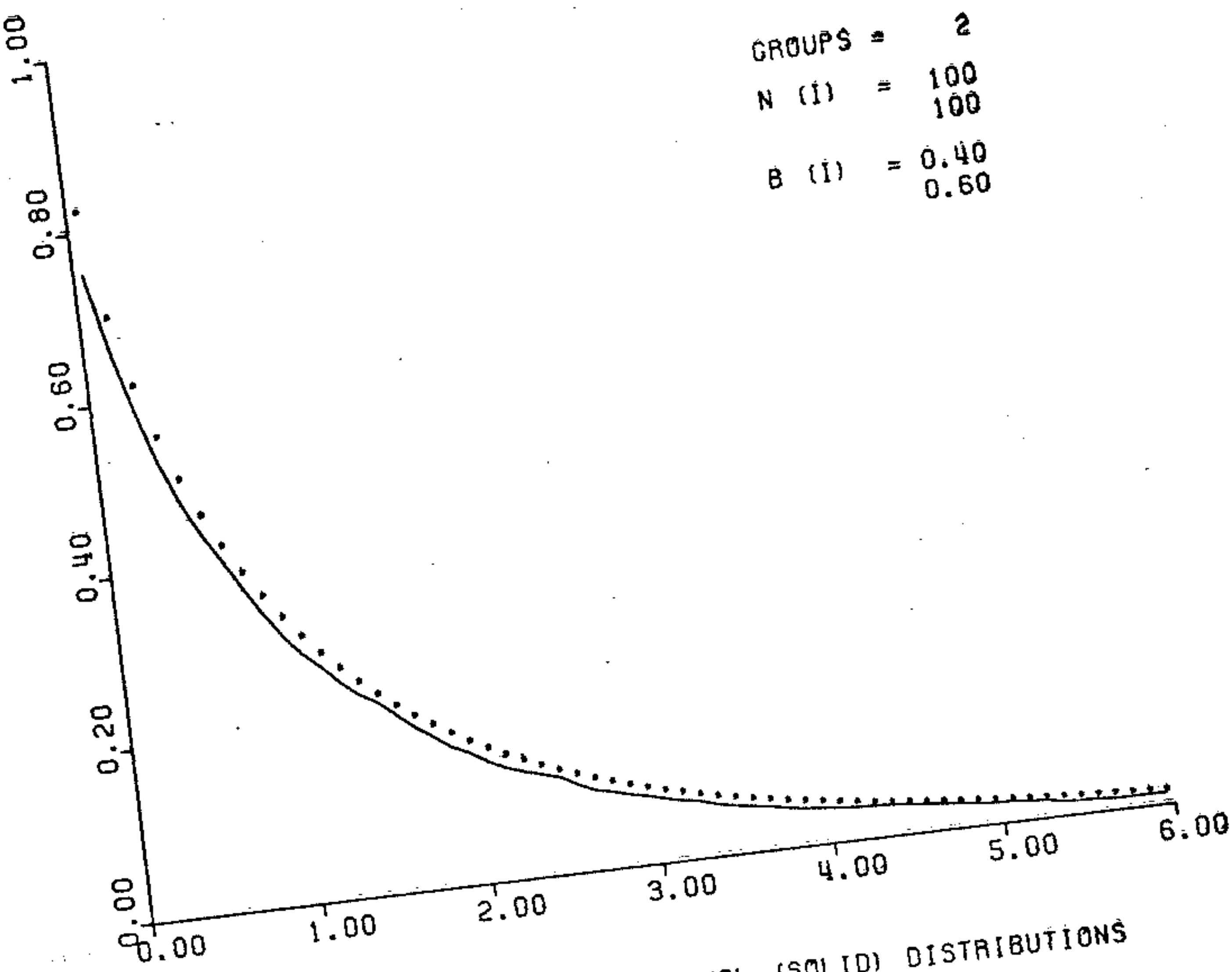
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 135 : SIMULATION NUMBER 135

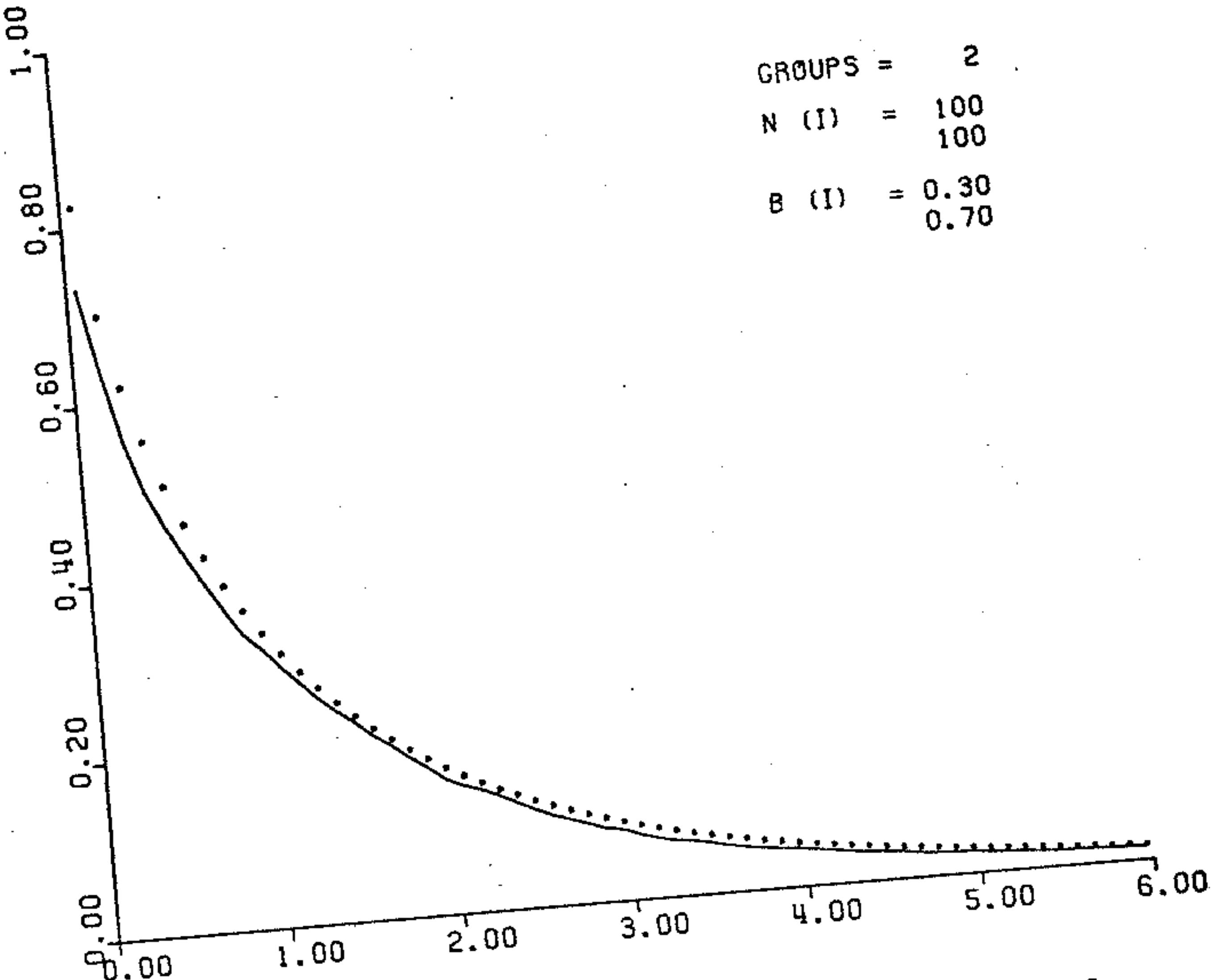


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 136 : SIMULATION NUMBER 136

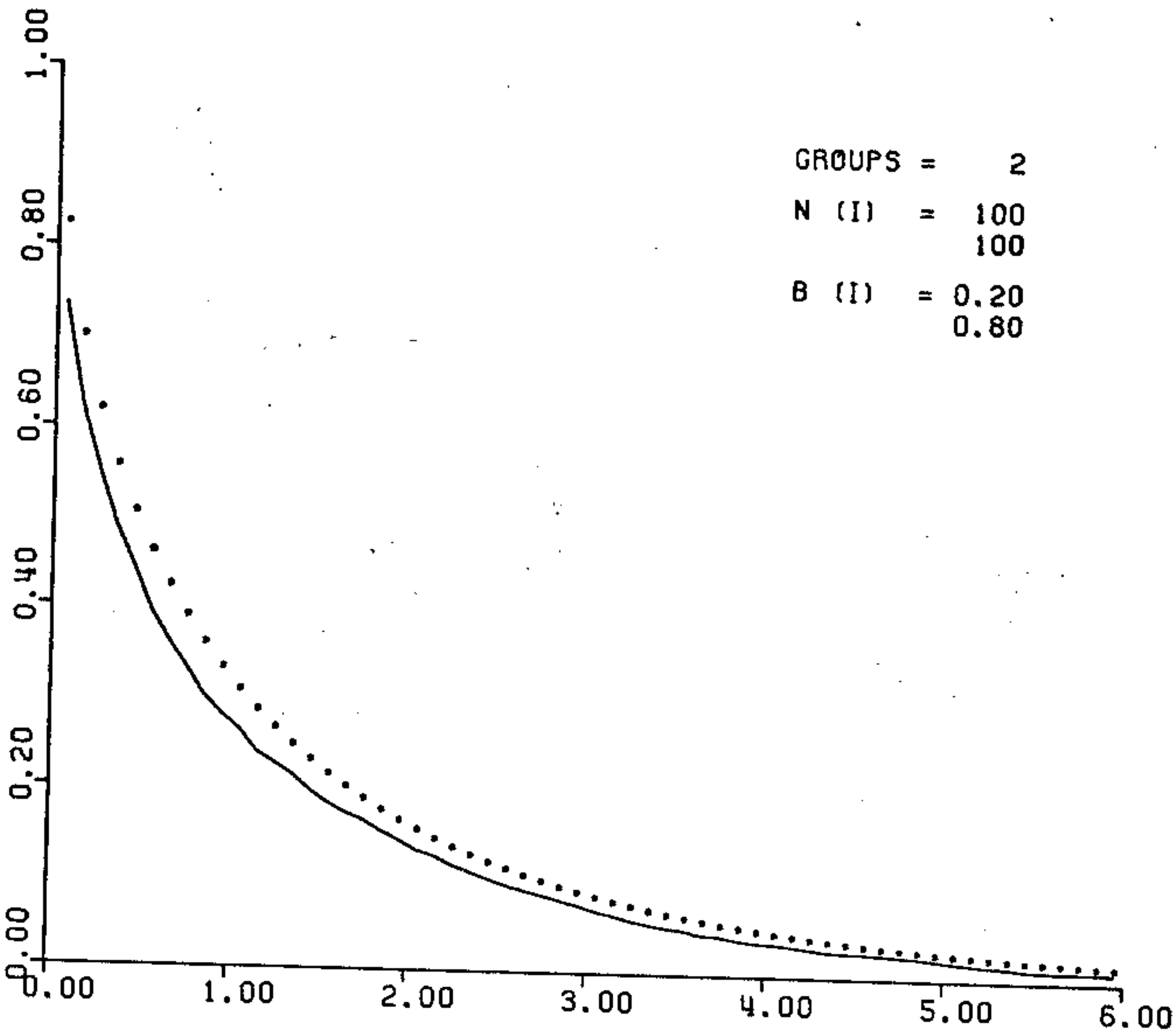


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 137 : SIMULATION NUMBER 137

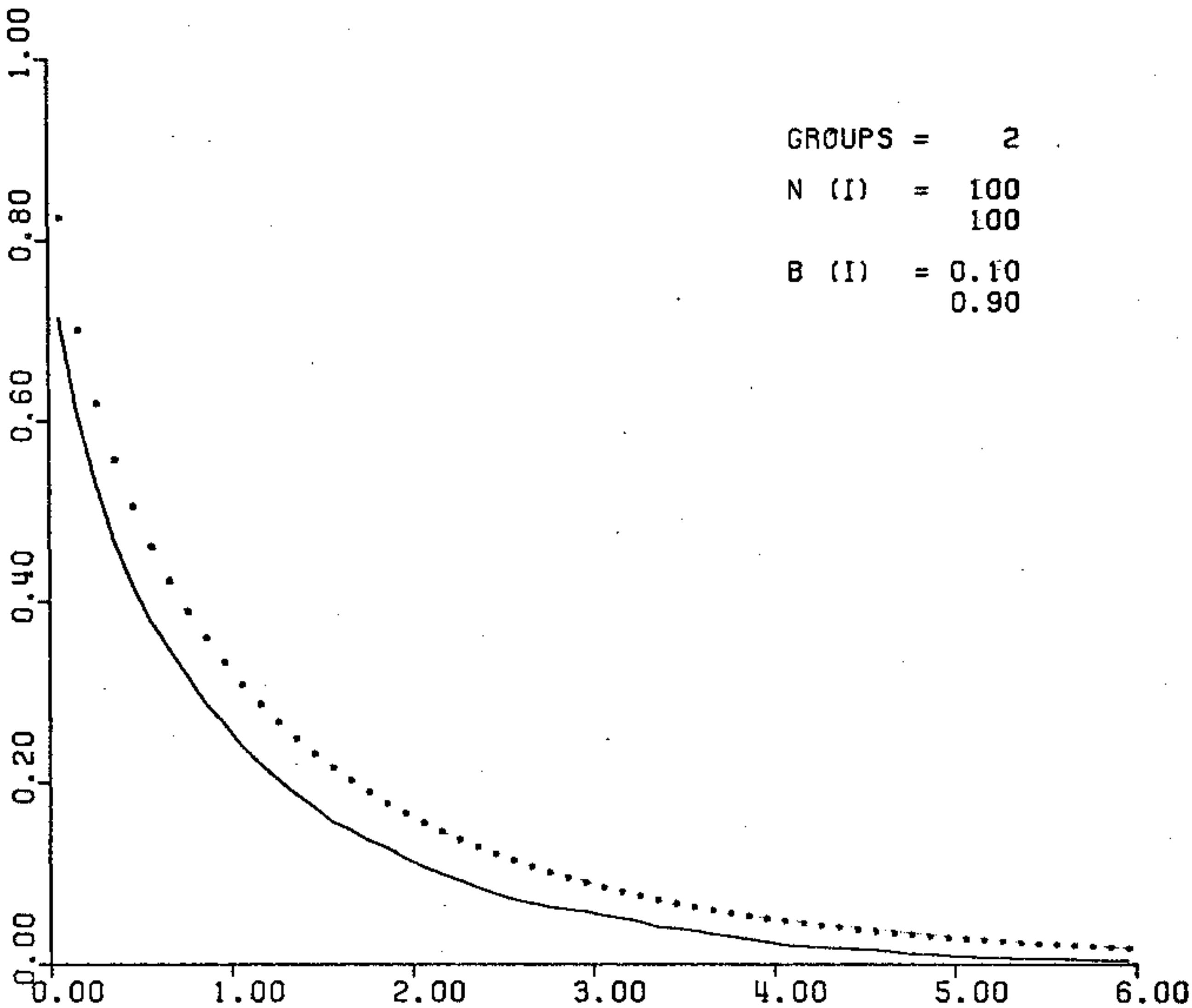


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

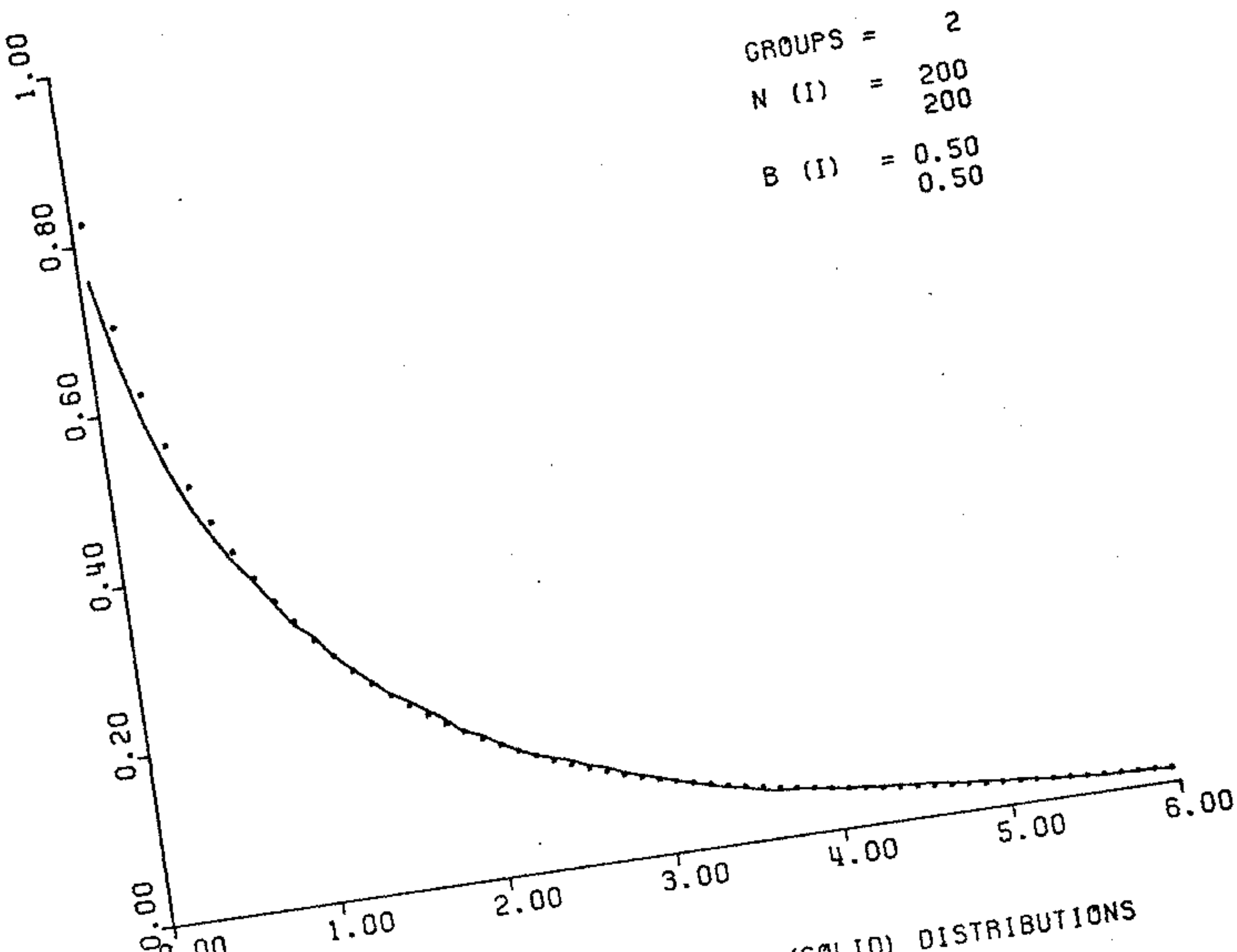
FIGURE 138 : SIMULATION NUMBER 138



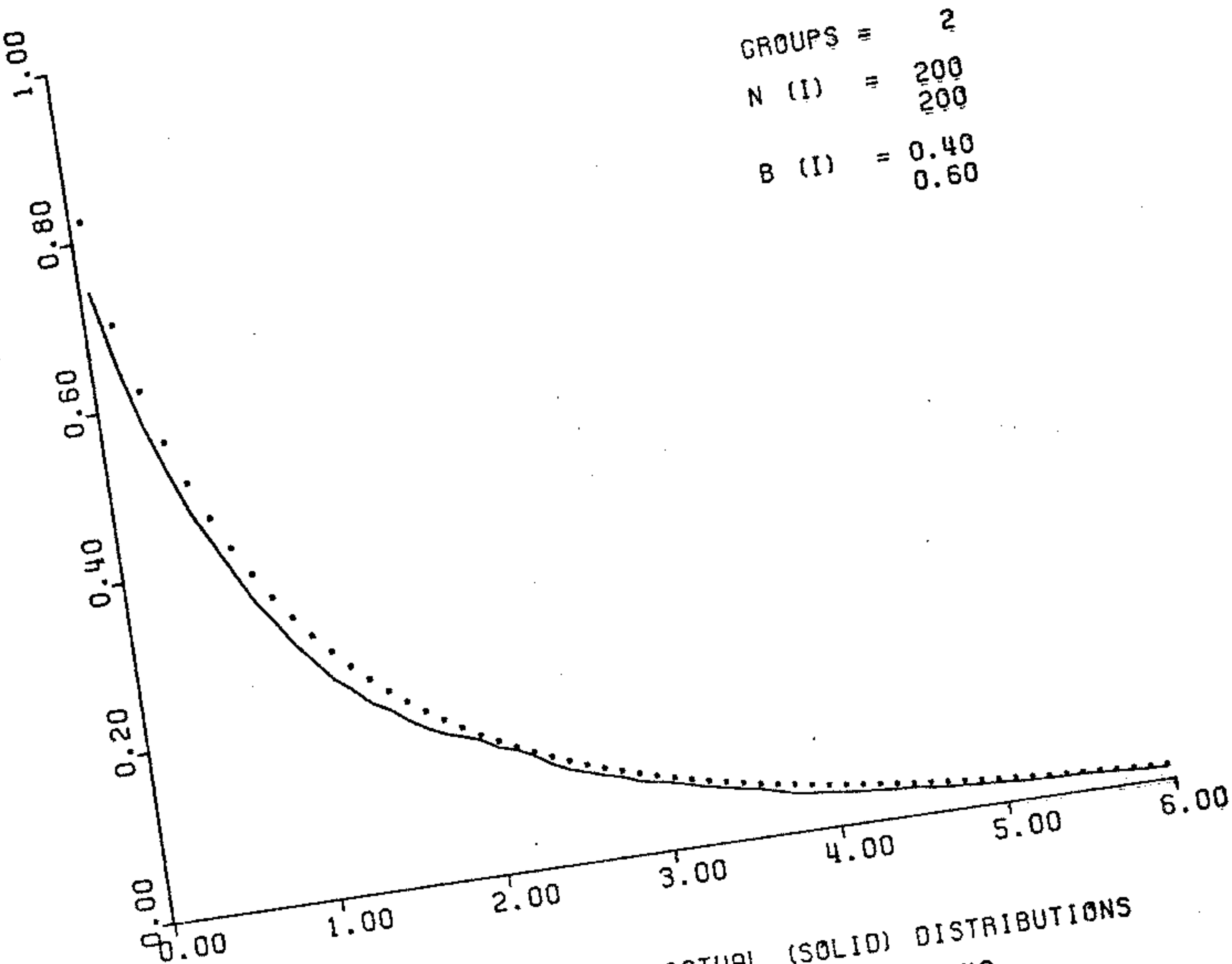
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 139 : SIMULATION NUMBER 139



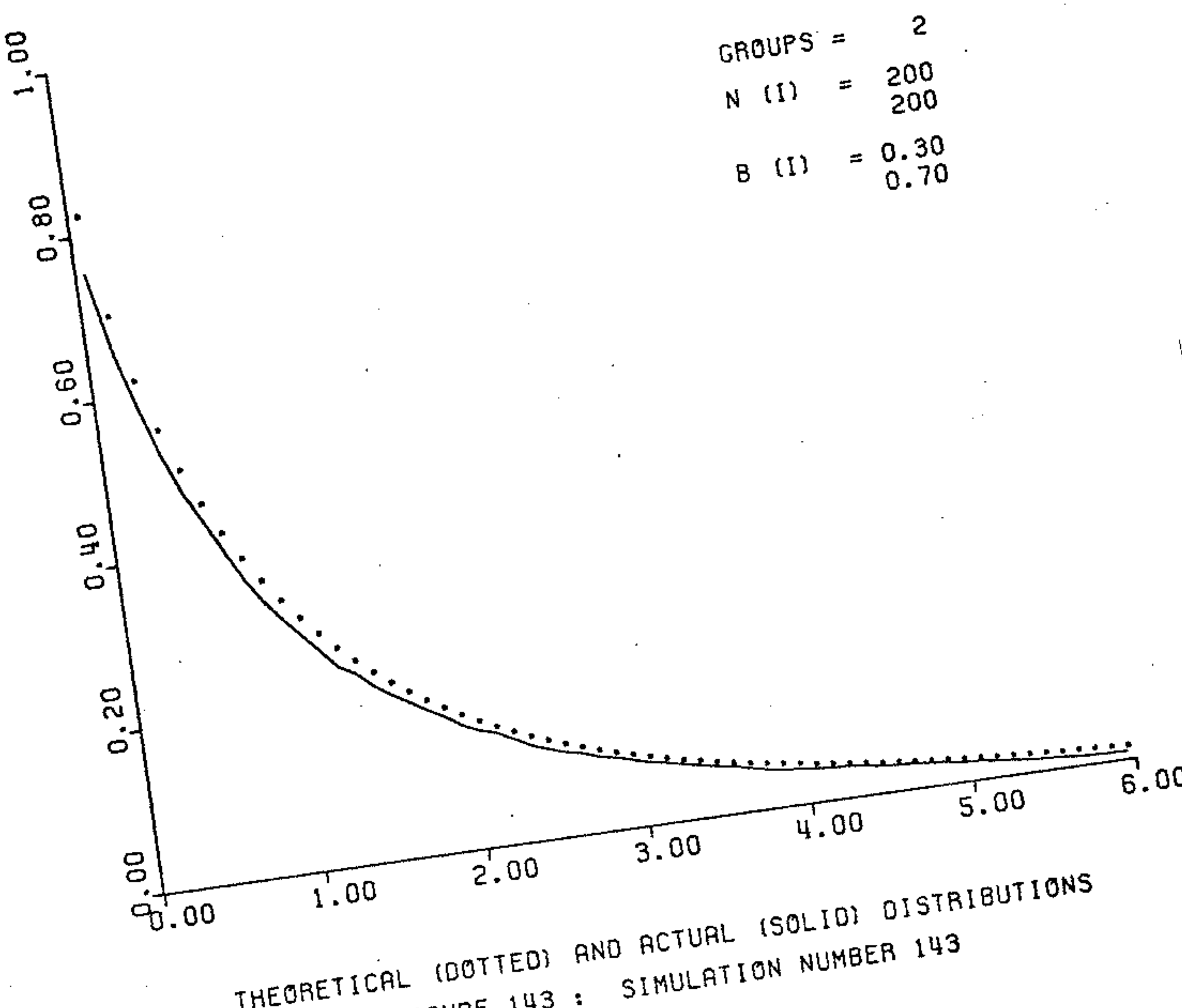
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 140 : SIMULATION NUMBER 140



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 141 : SIMULATION NUMBER 141



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 142 : SIMULATION NUMBER 142



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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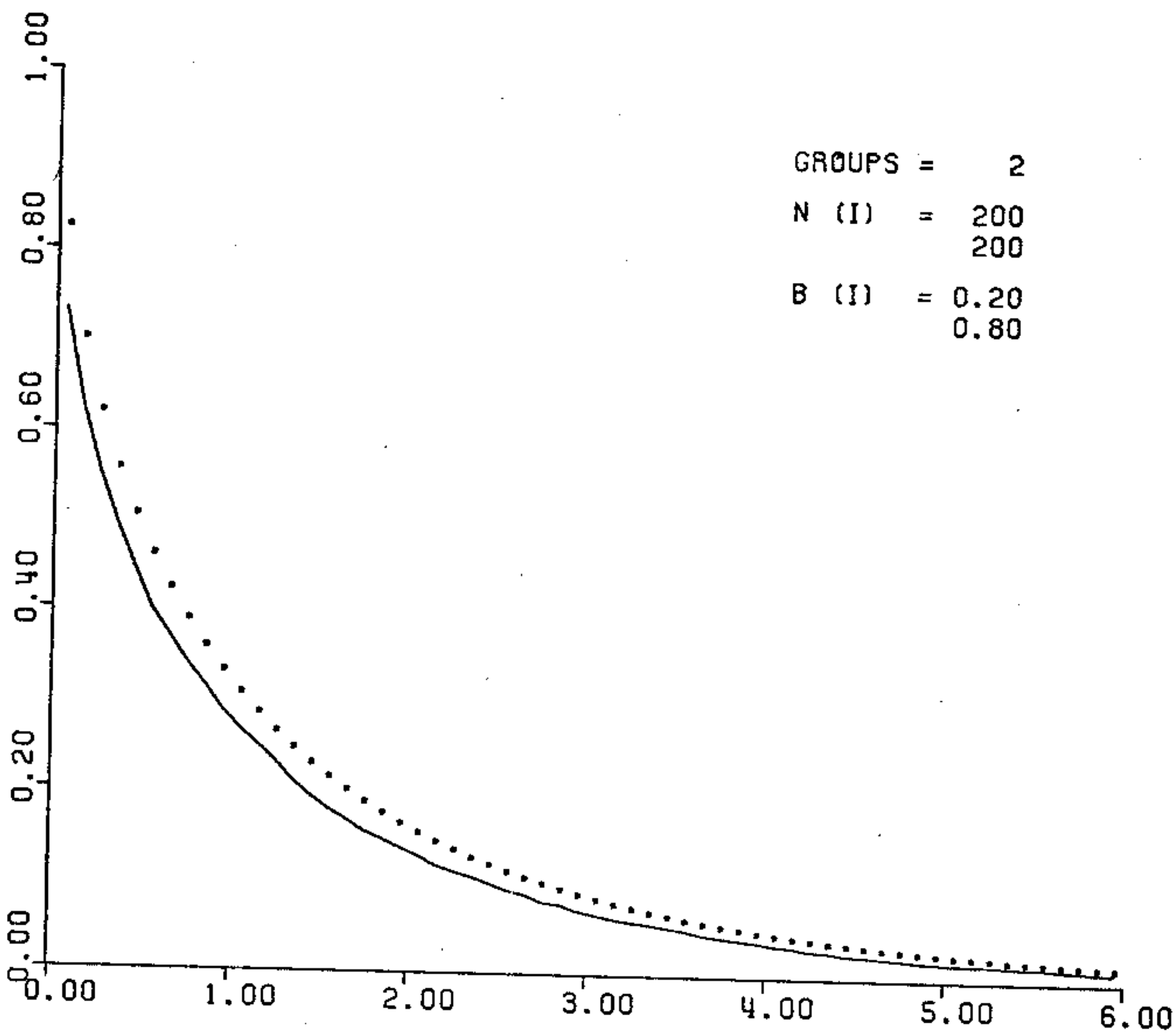
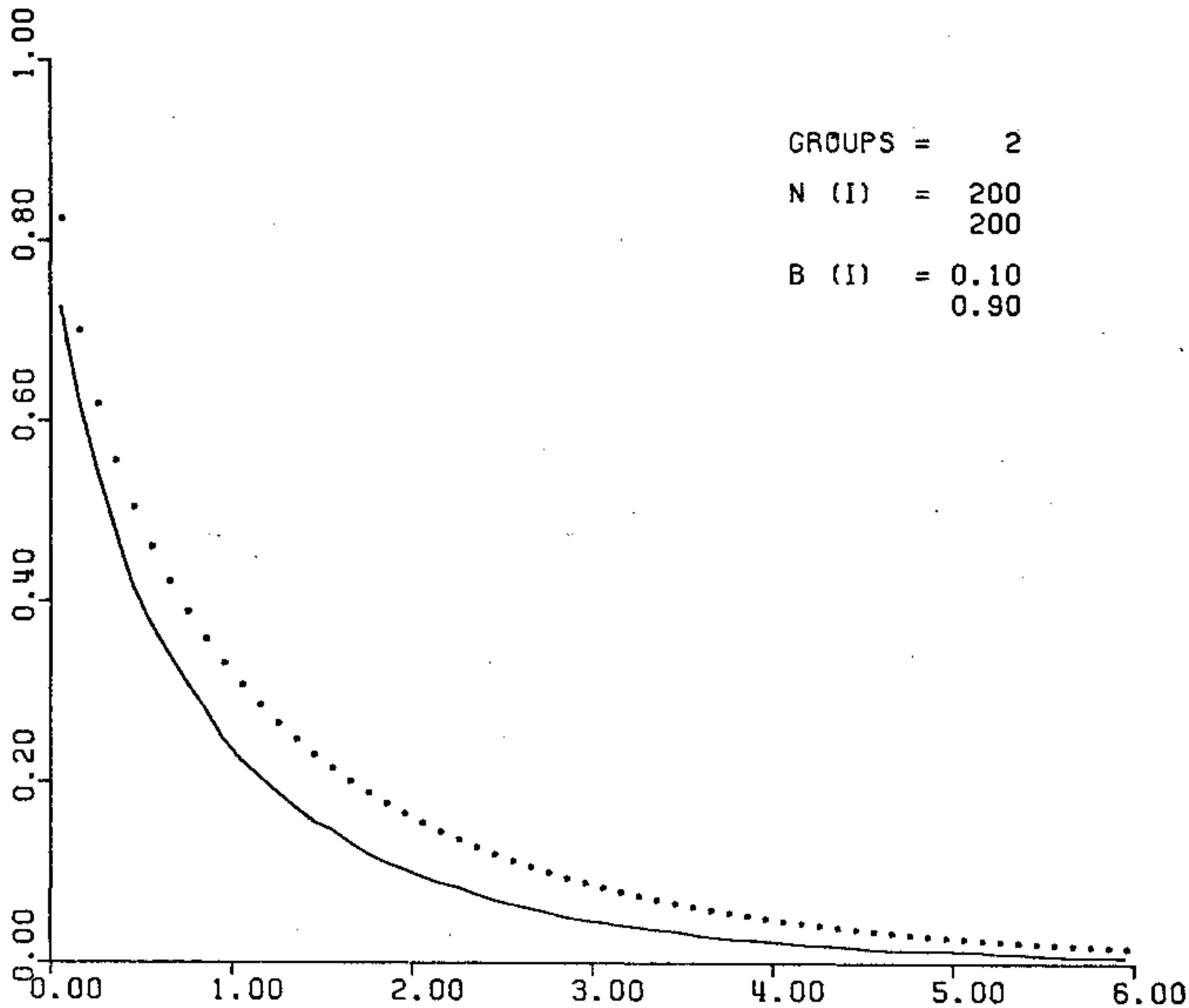
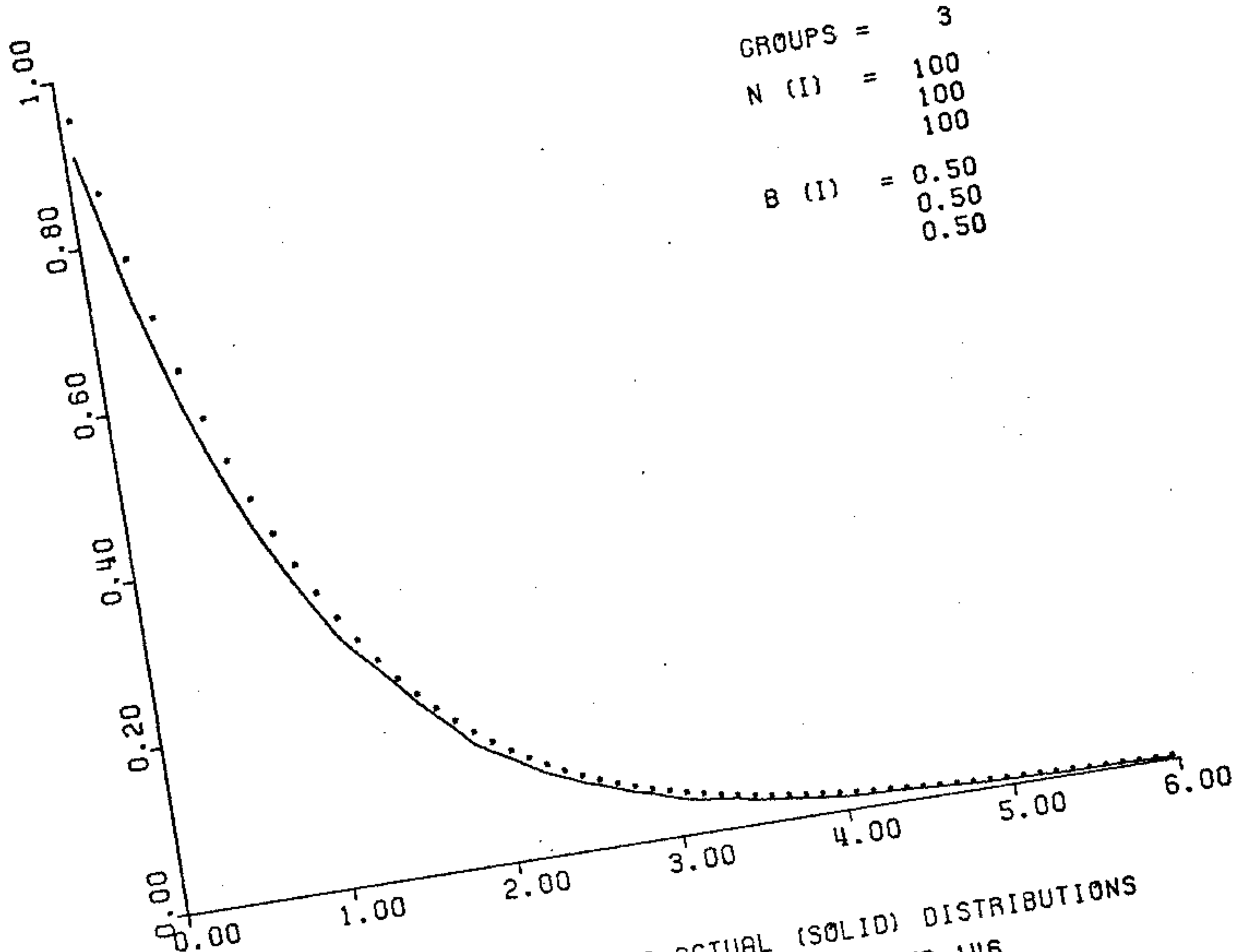


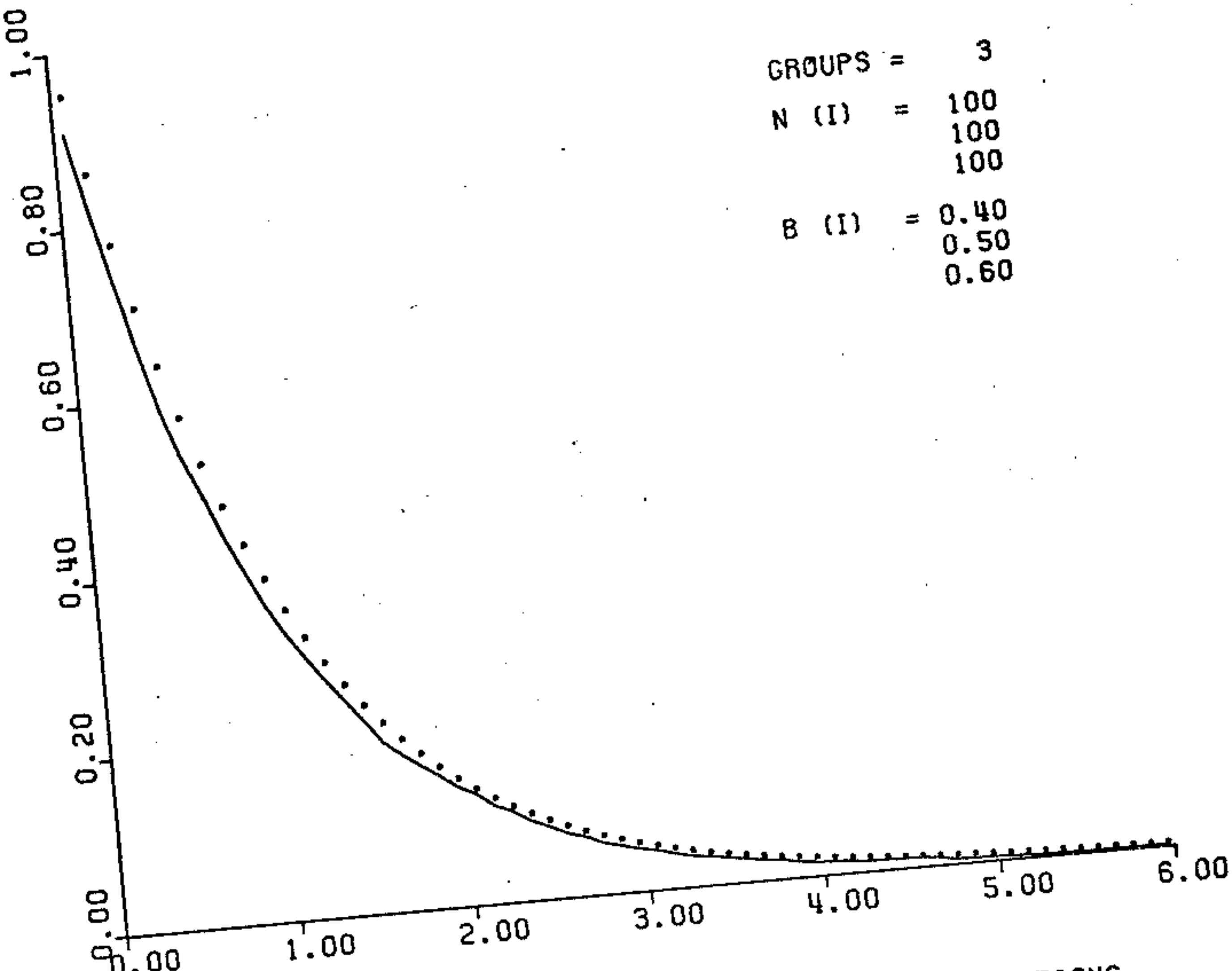
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS .
FIGURE 144 : SIMULATION NUMBER 144



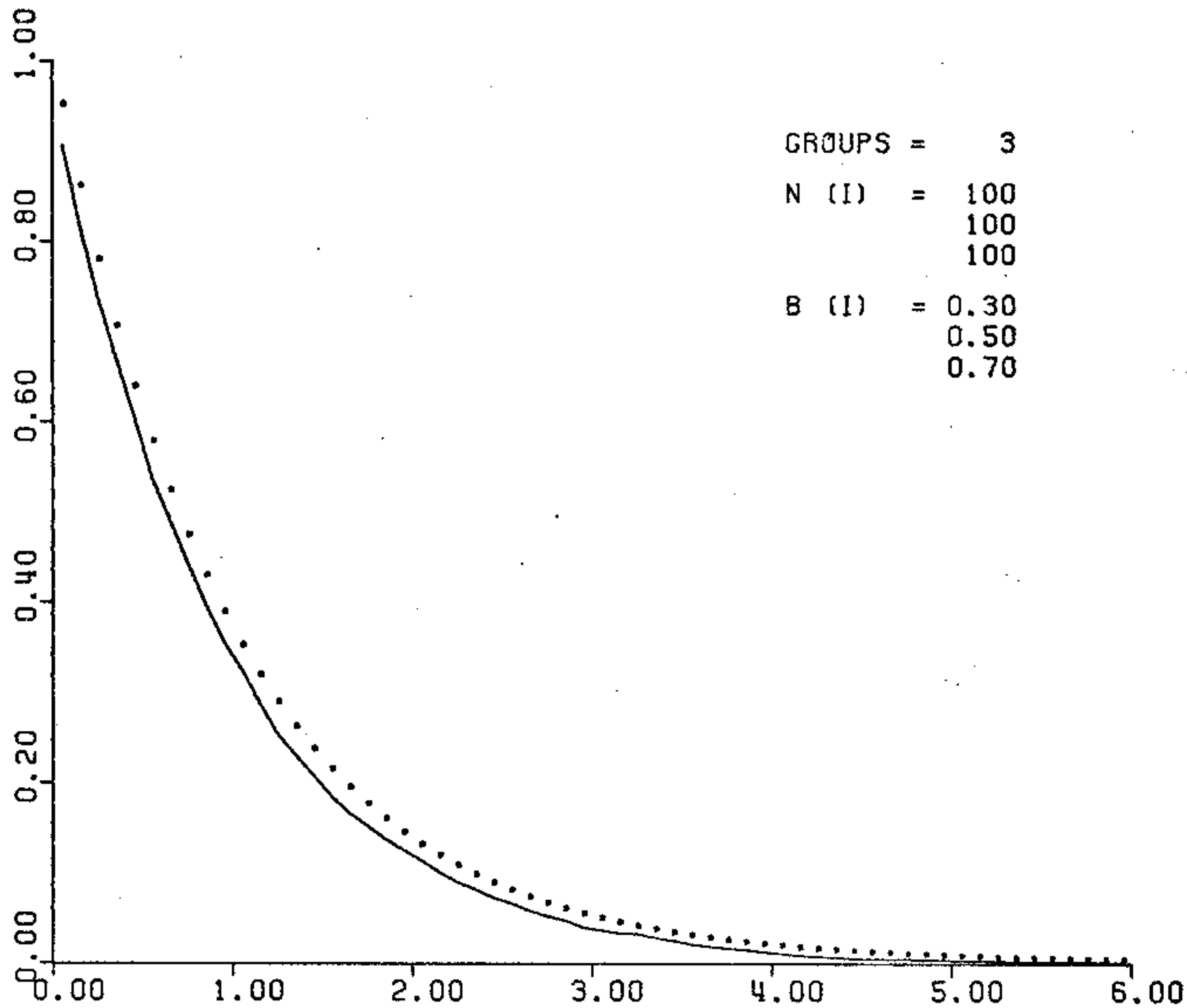
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 145 : SIMULATION NUMBER 145



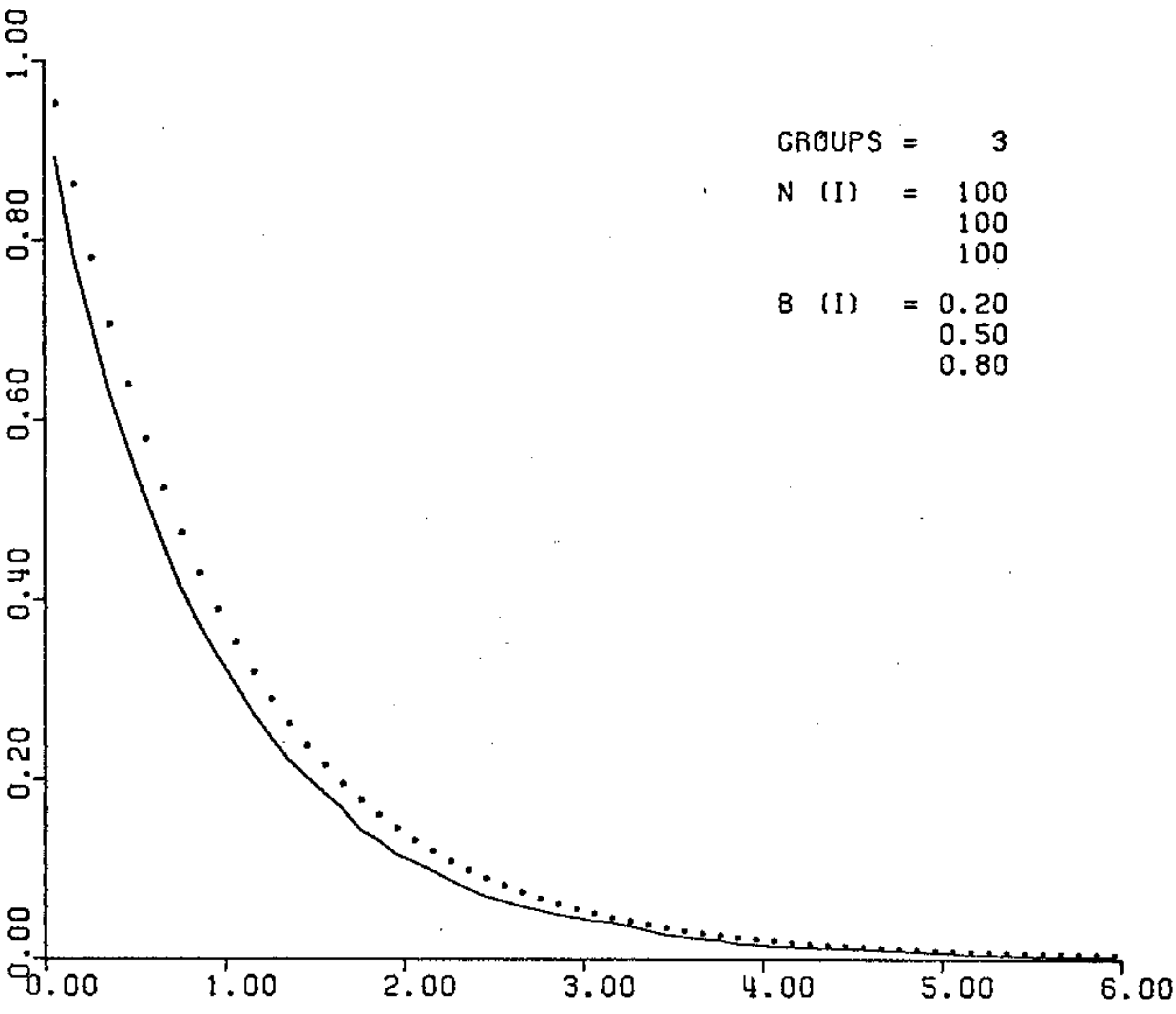
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 146 : SIMULATION NUMBER 146



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 147 : SIMULATION NUMBER 147

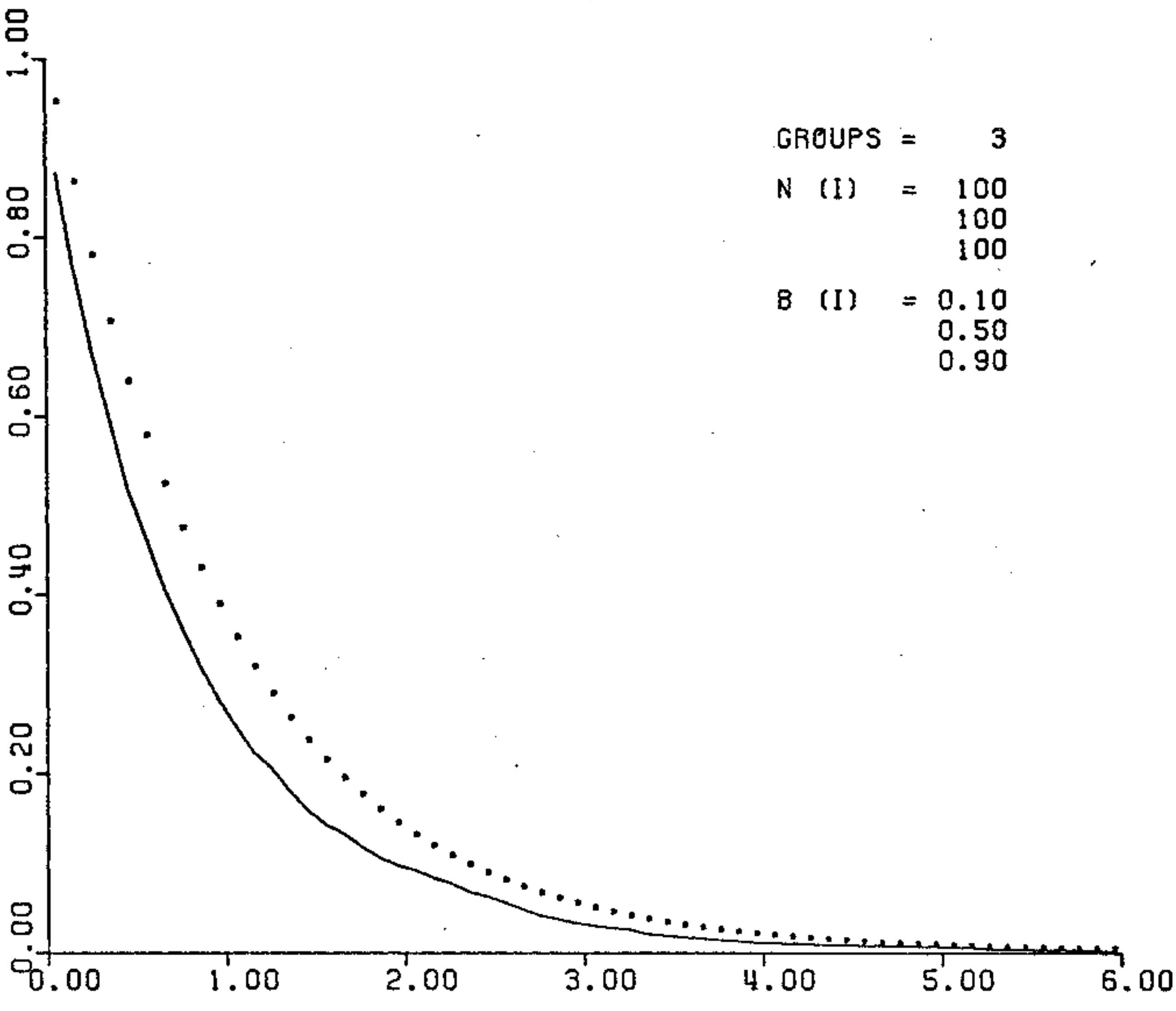


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 148 : SIMULATION NUMBER 148

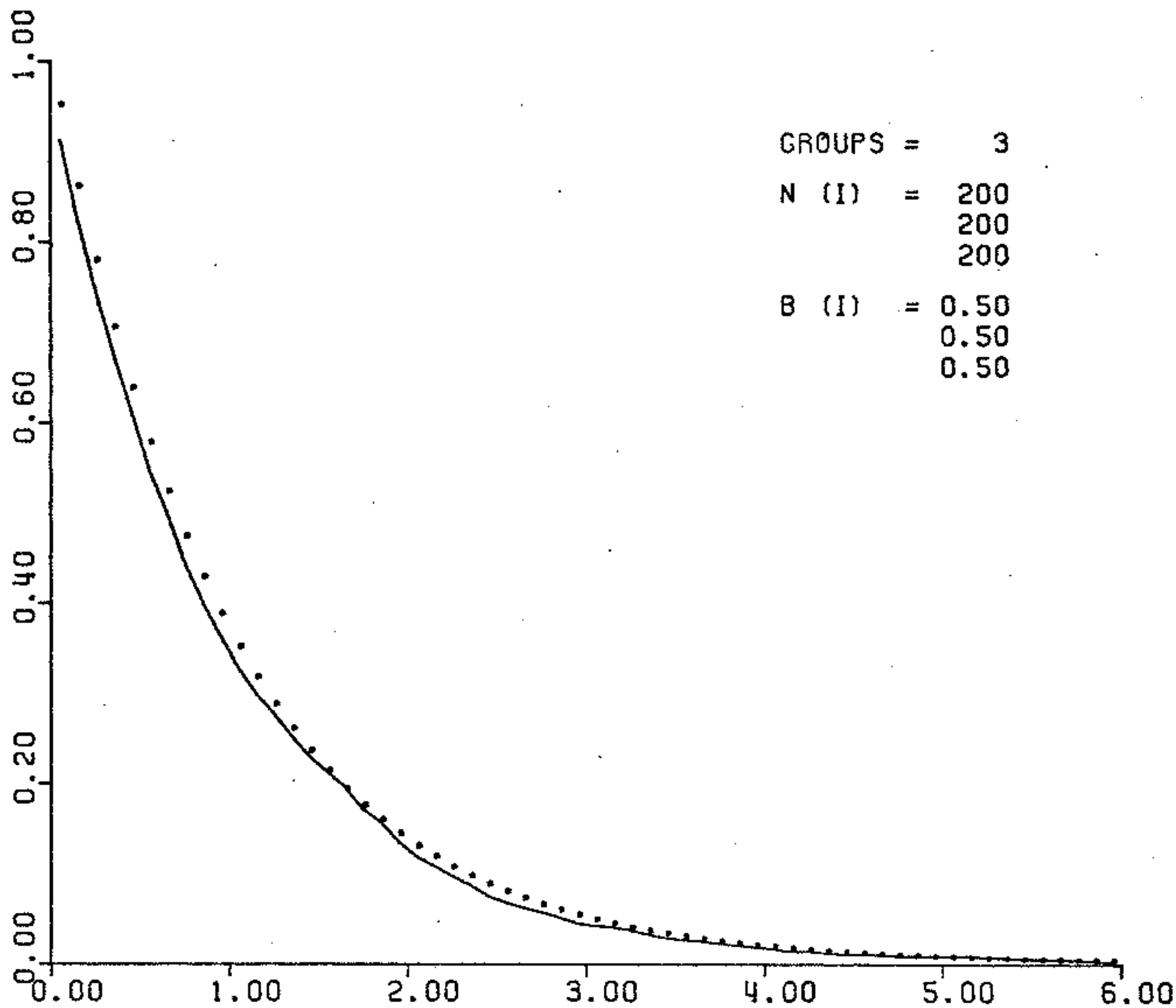


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 149 : SIMULATION NUMBER 149

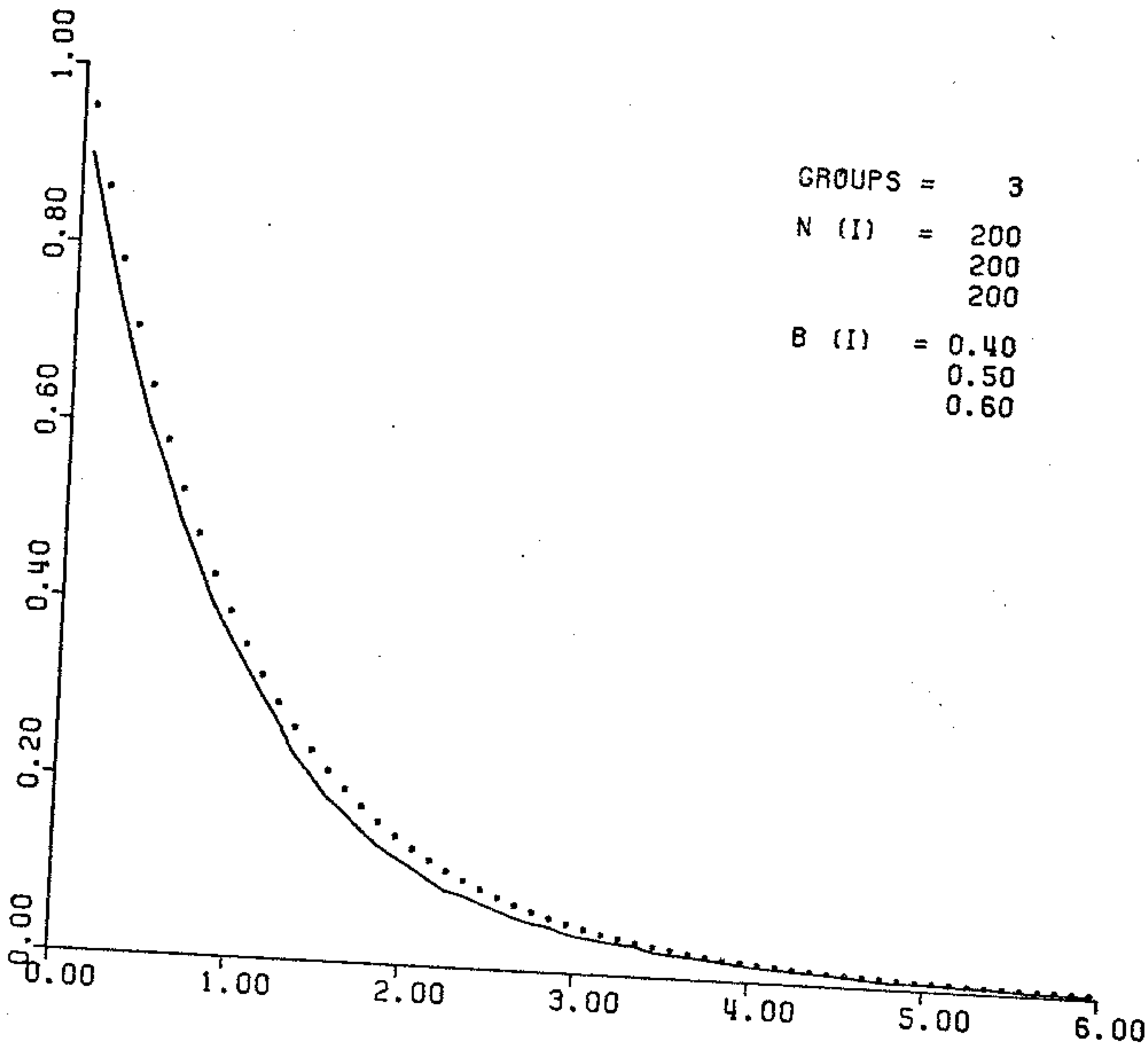


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 150 : SIMULATION NUMBER 150

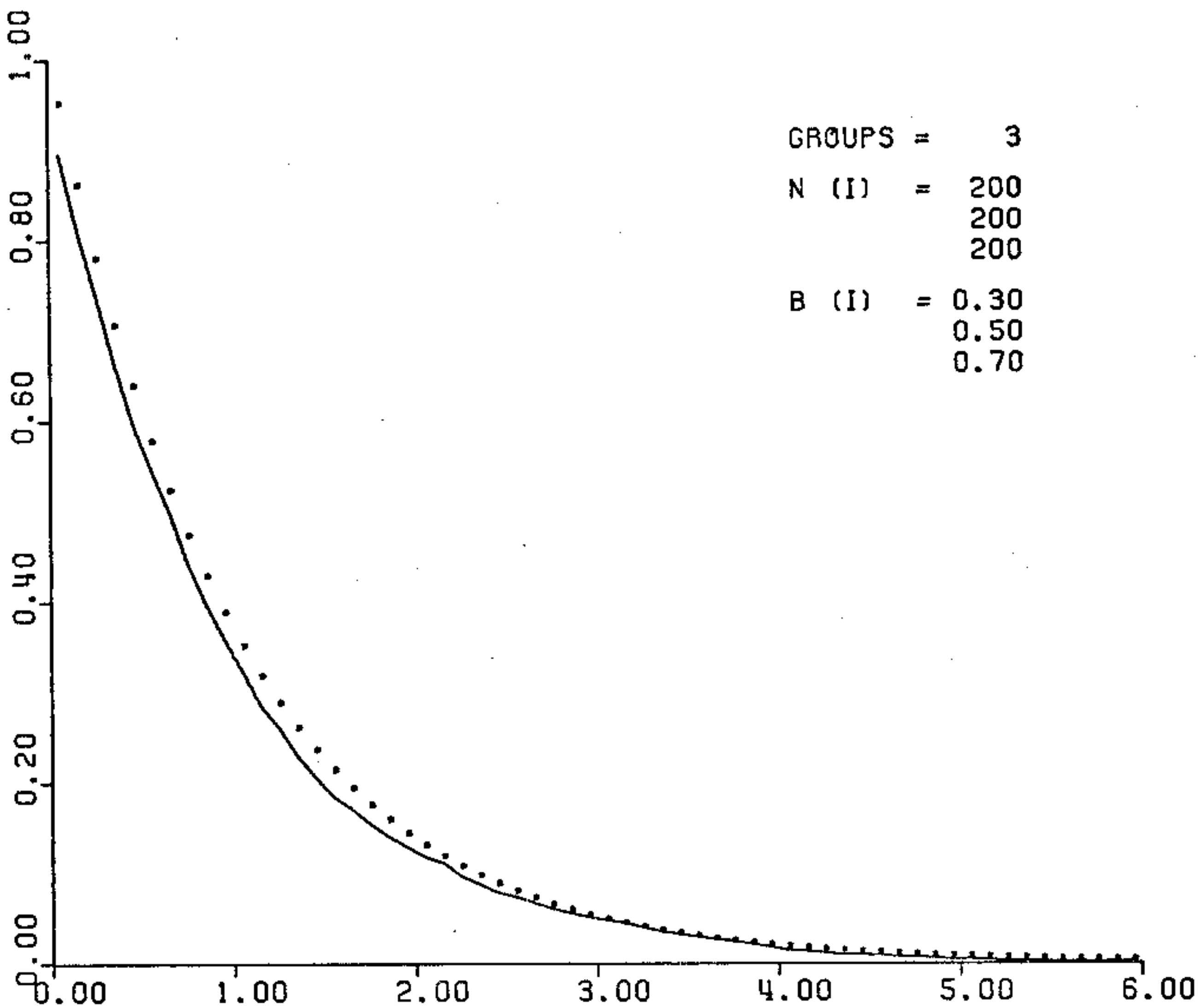


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 151 : SIMULATION NUMBER 151

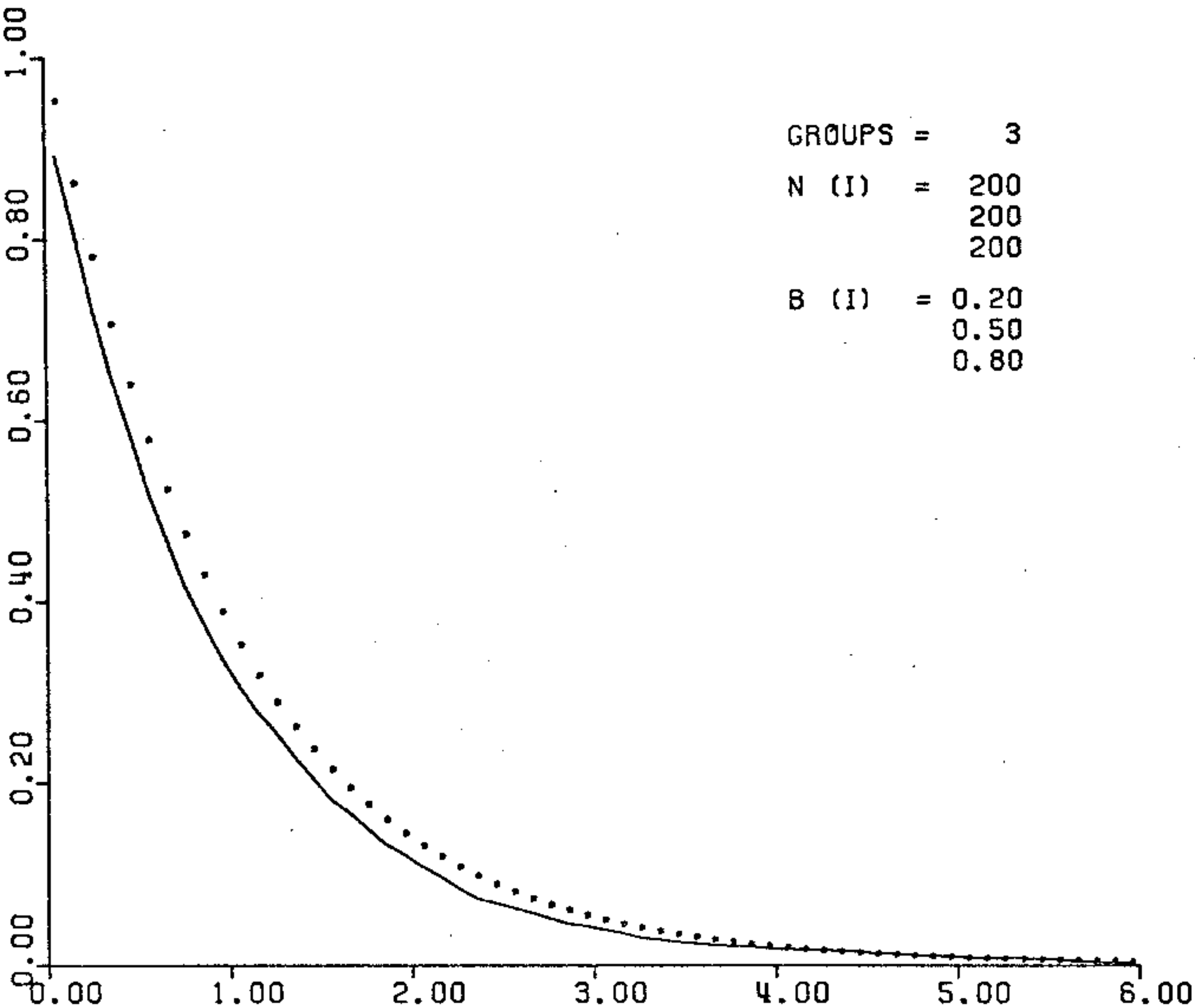


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 152 : SIMULATION NUMBER 152



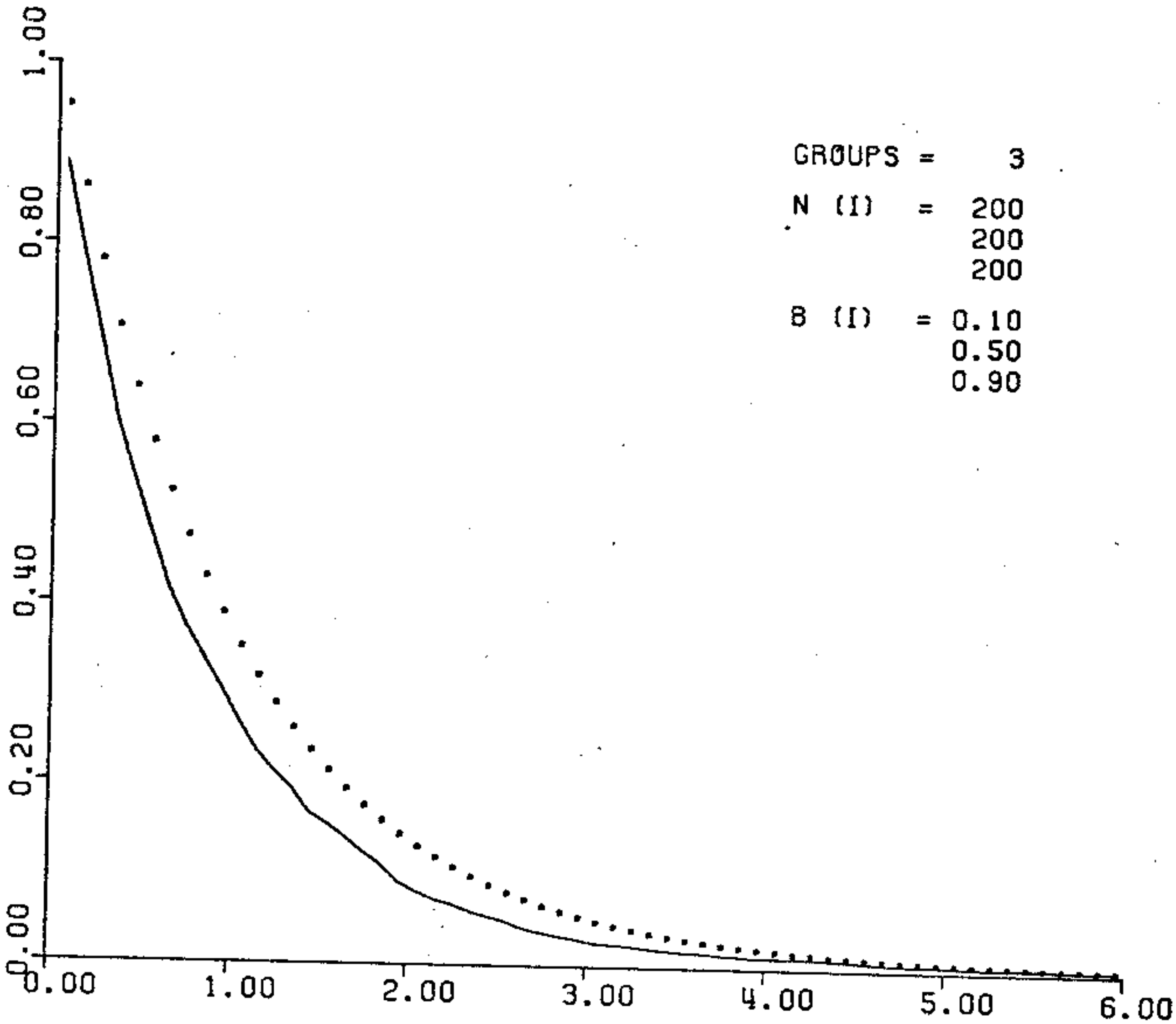
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 153 : SIMULATION NUMBER 153

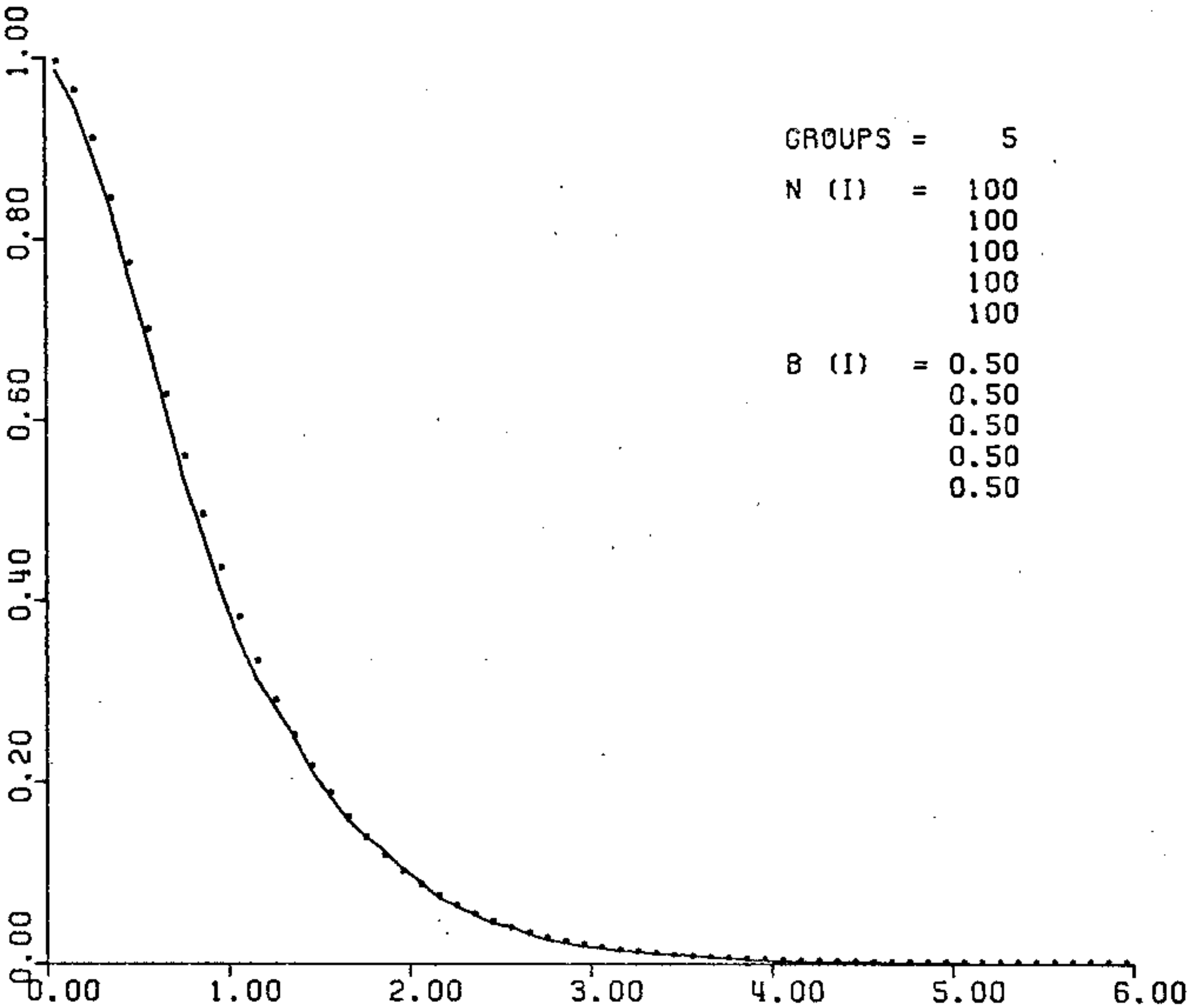


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 154 : SIMULATION NUMBER 154

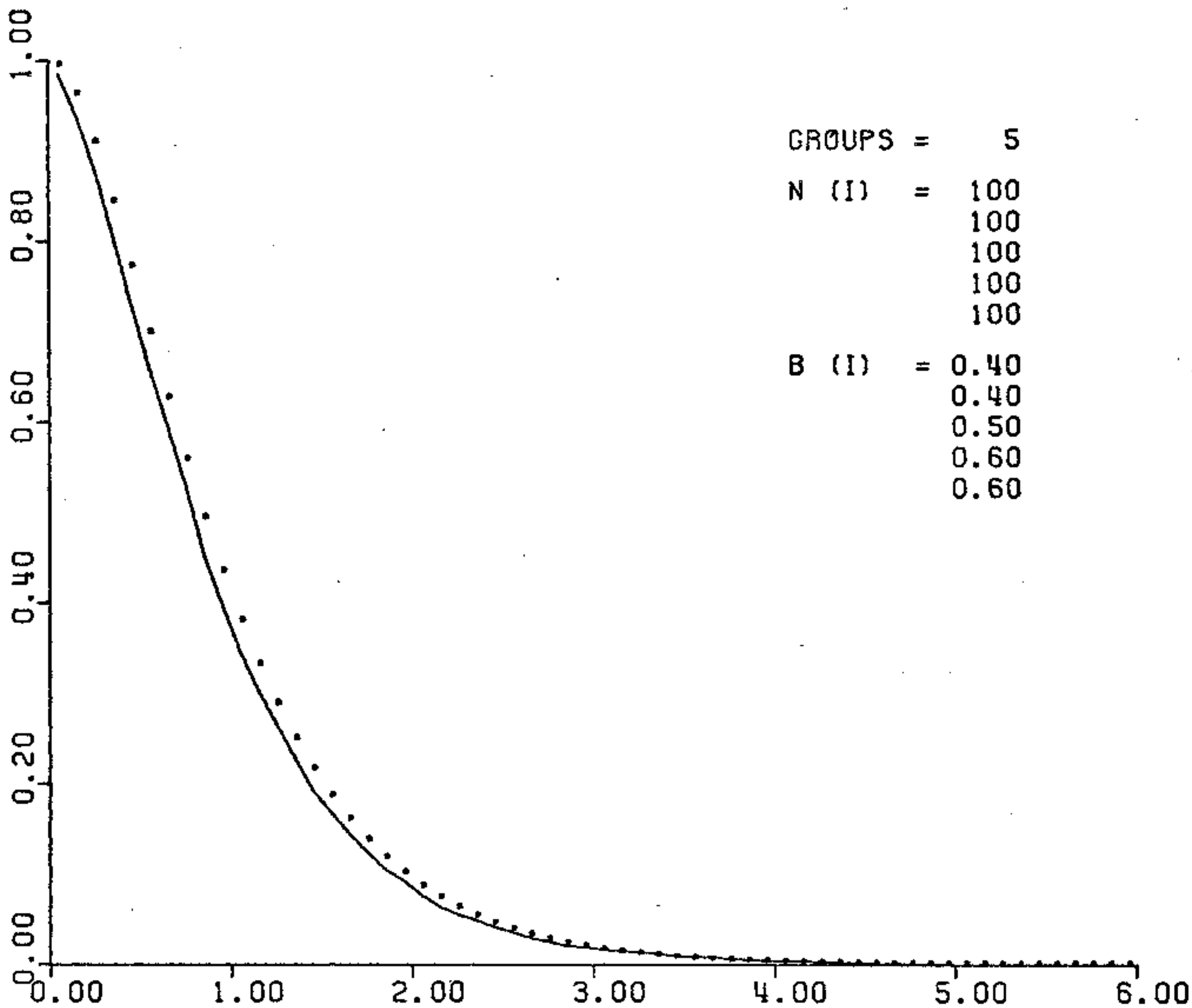


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 155 : SIMULATION NUMBER 155

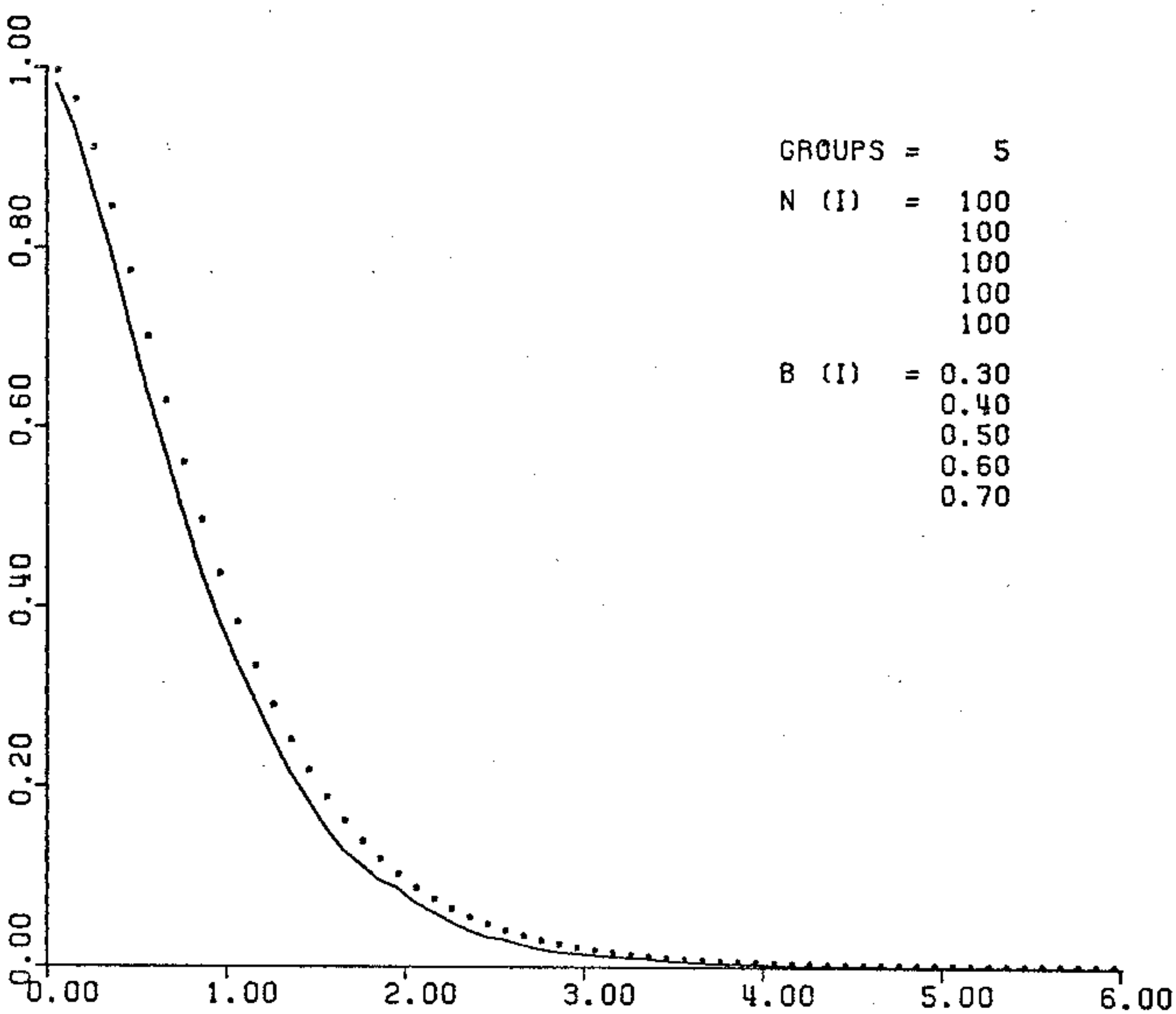


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 156 : SIMULATION NUMBER 156

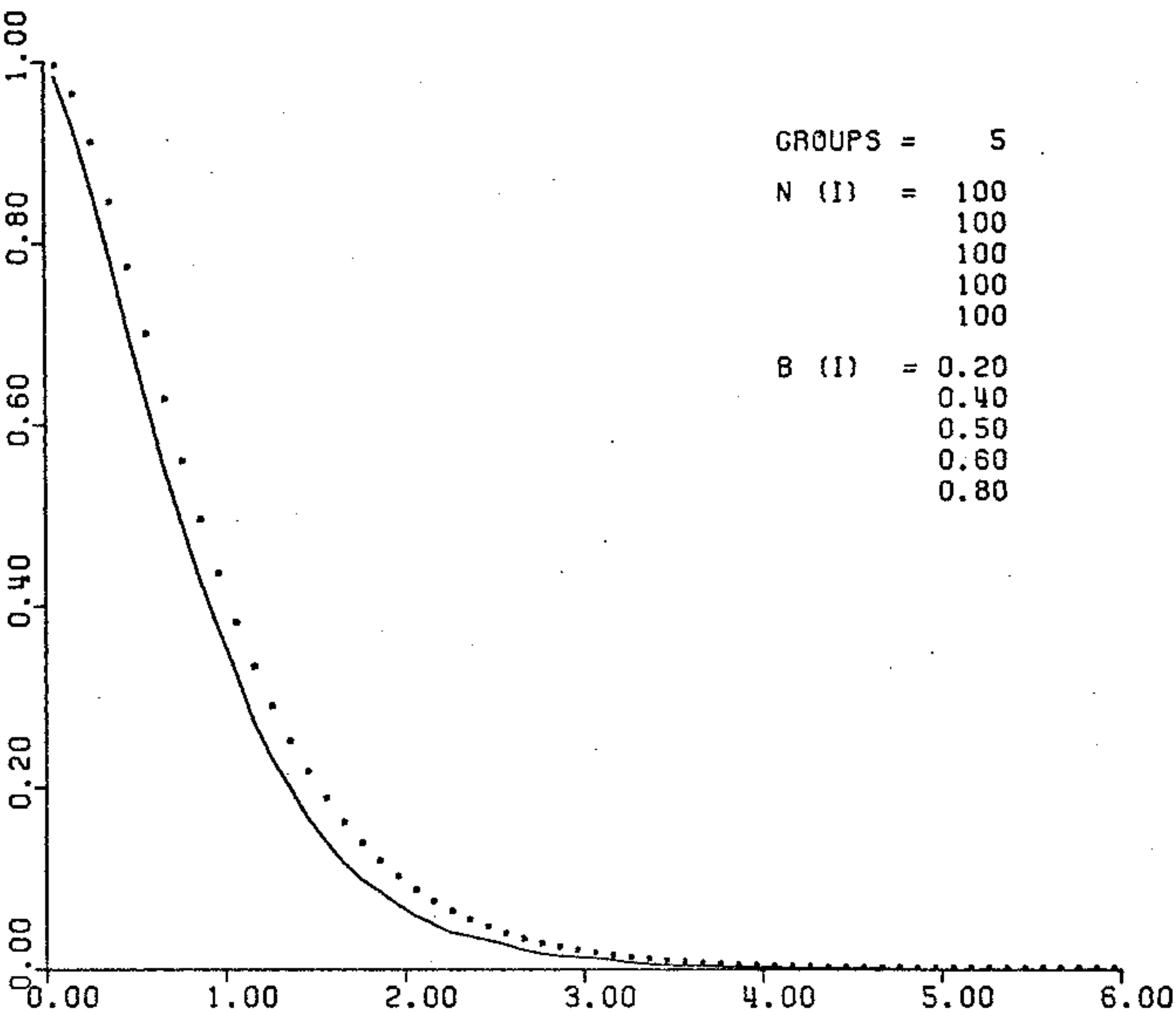


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 157 : SIMULATION NUMBER 157

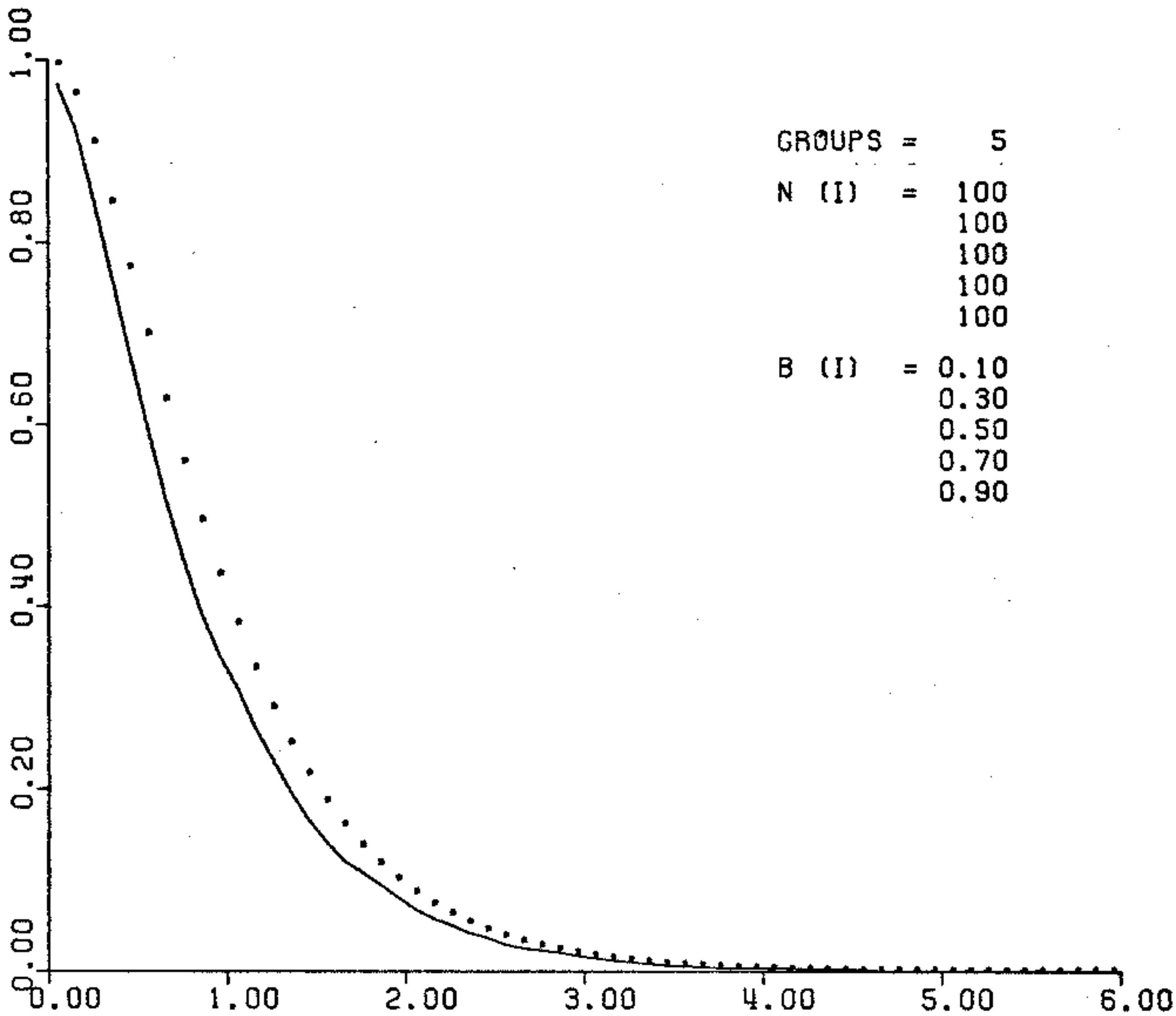


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 158 : SIMULATION NUMBER 158

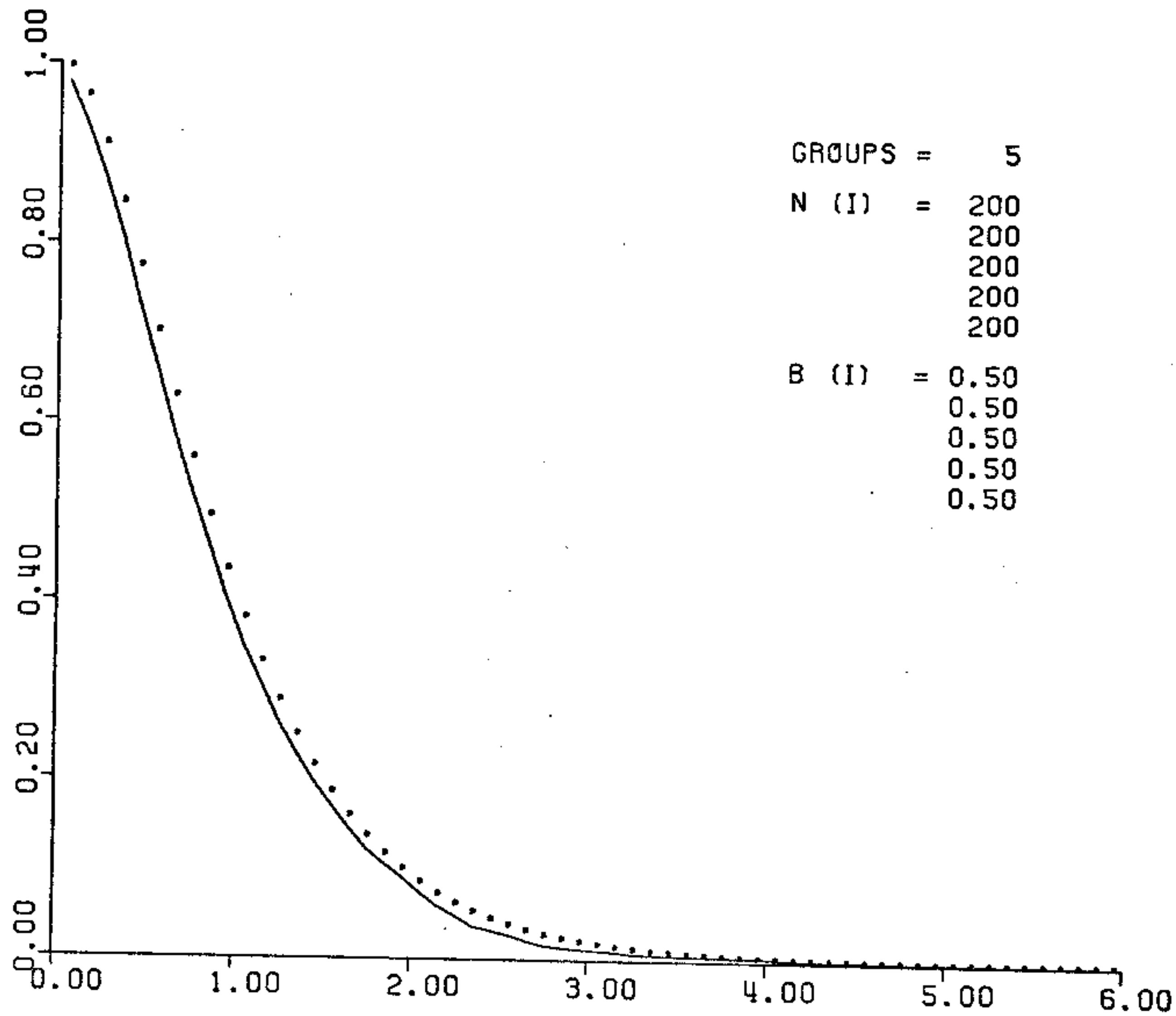


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 159 : SIMULATION NUMBER 159

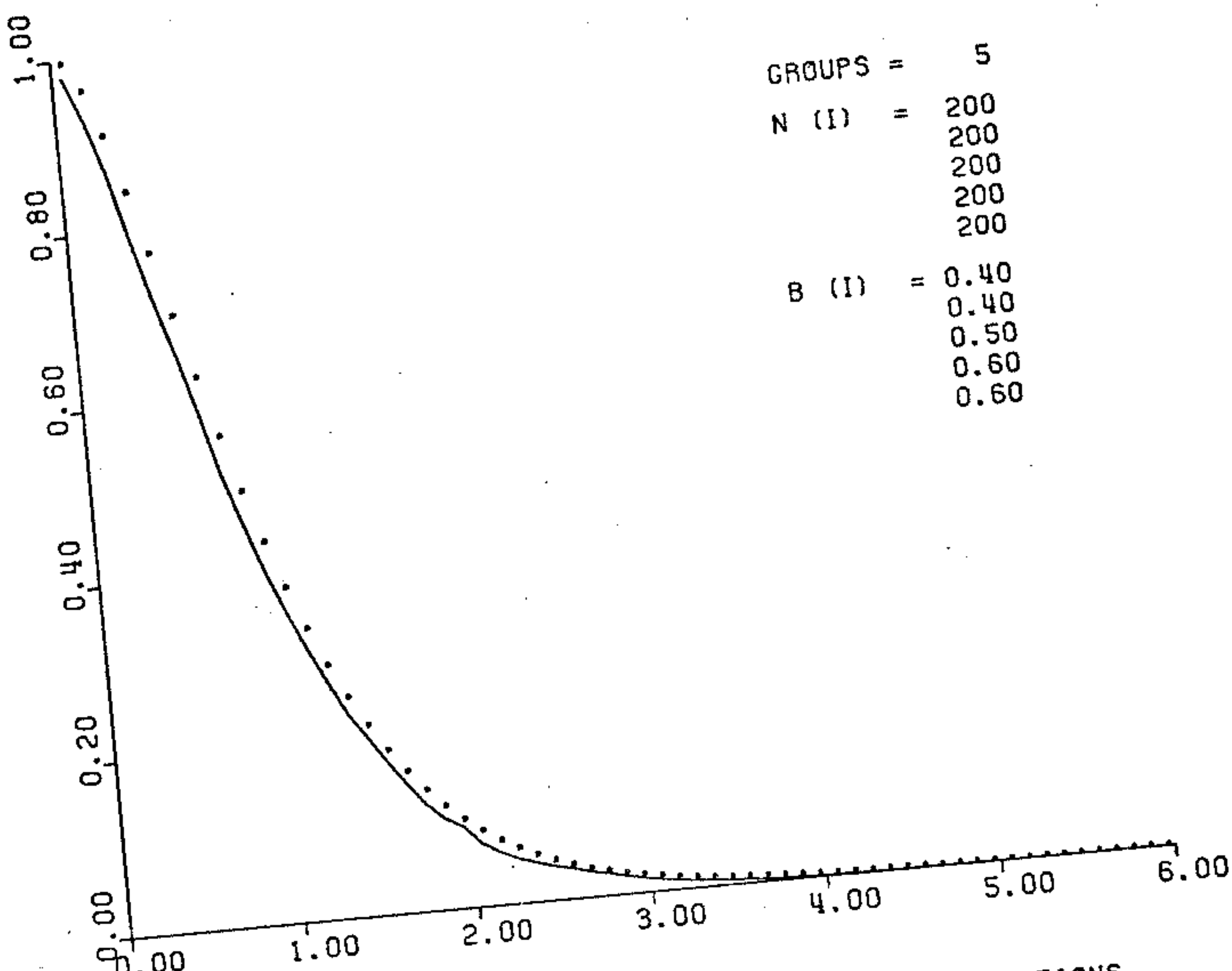


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 160 : SIMULATION NUMBER 160



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 161 : SIMULATION NUMBER 161



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
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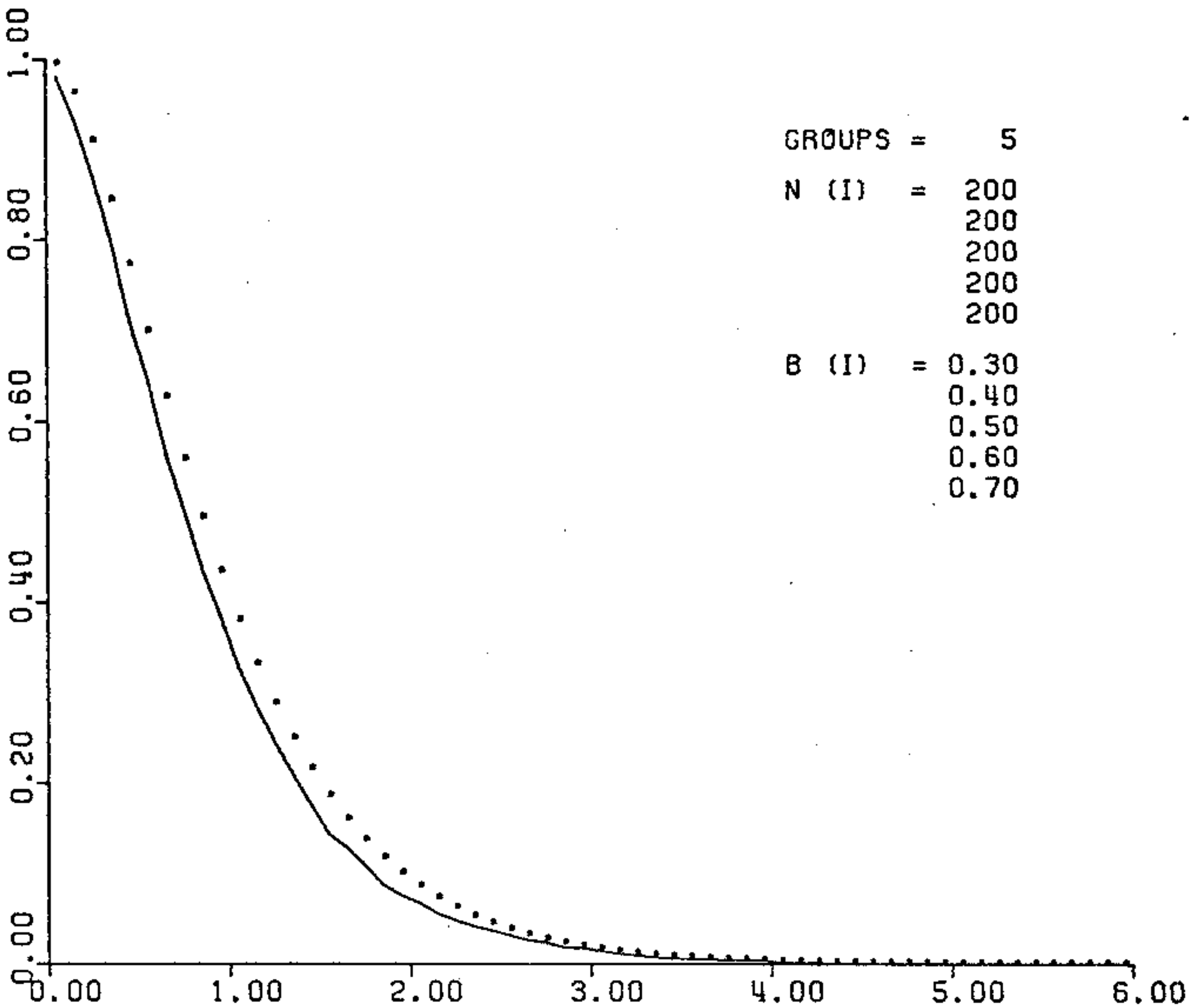
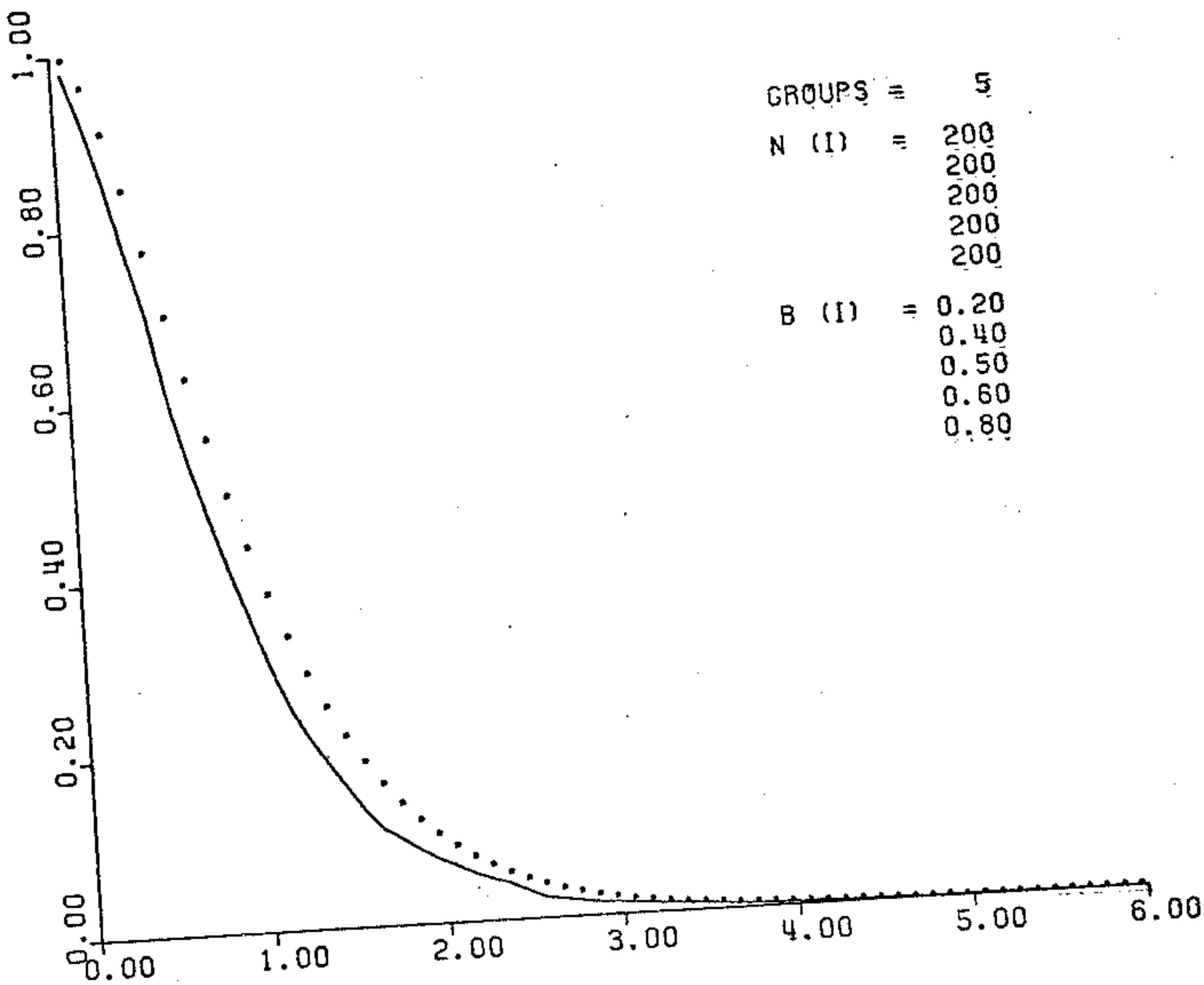


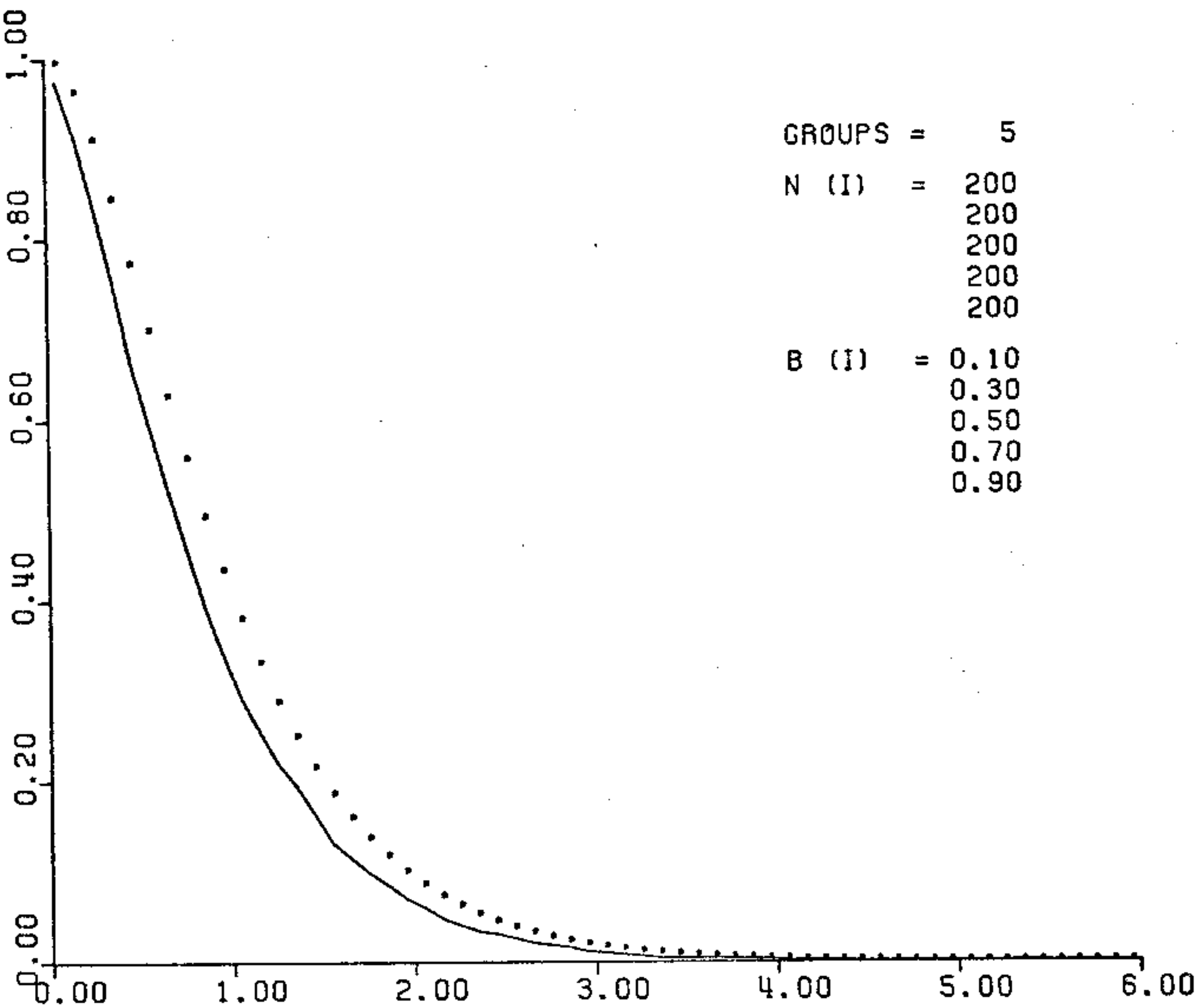
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 163 : SIMULATION NUMBER 163



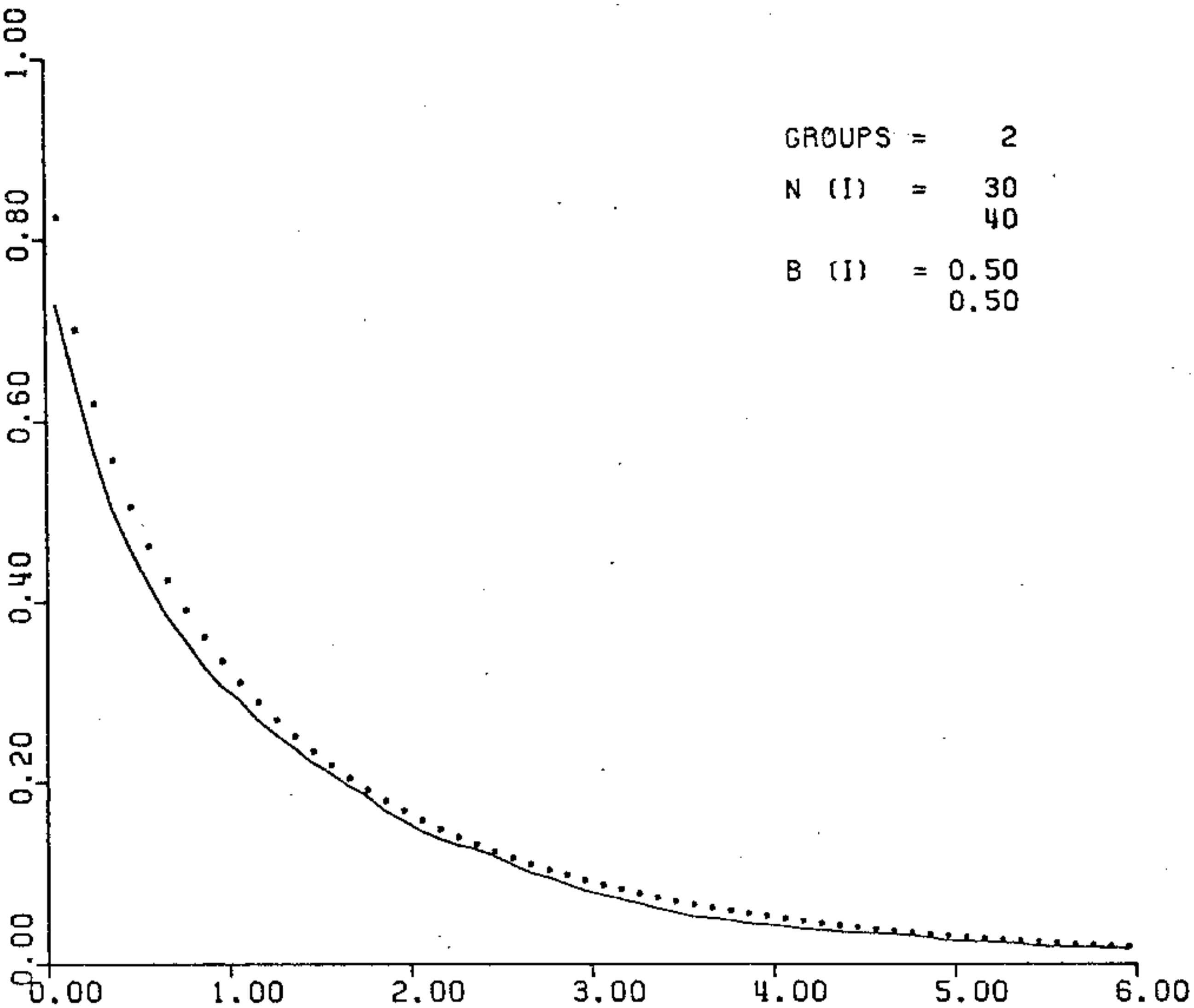
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 164 : SIMULATION NUMBER 164



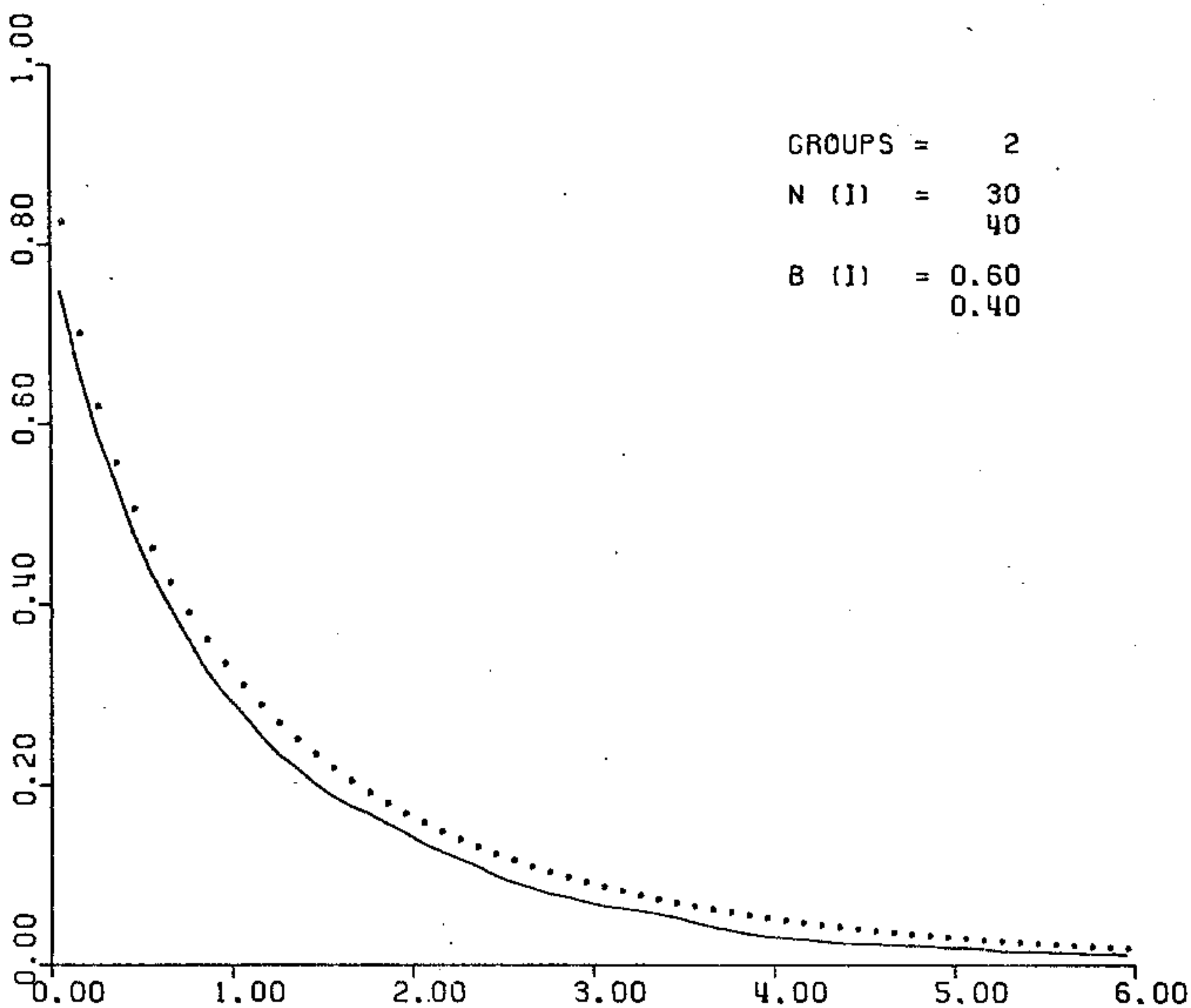
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 165 : SIMULATION NUMBER 165



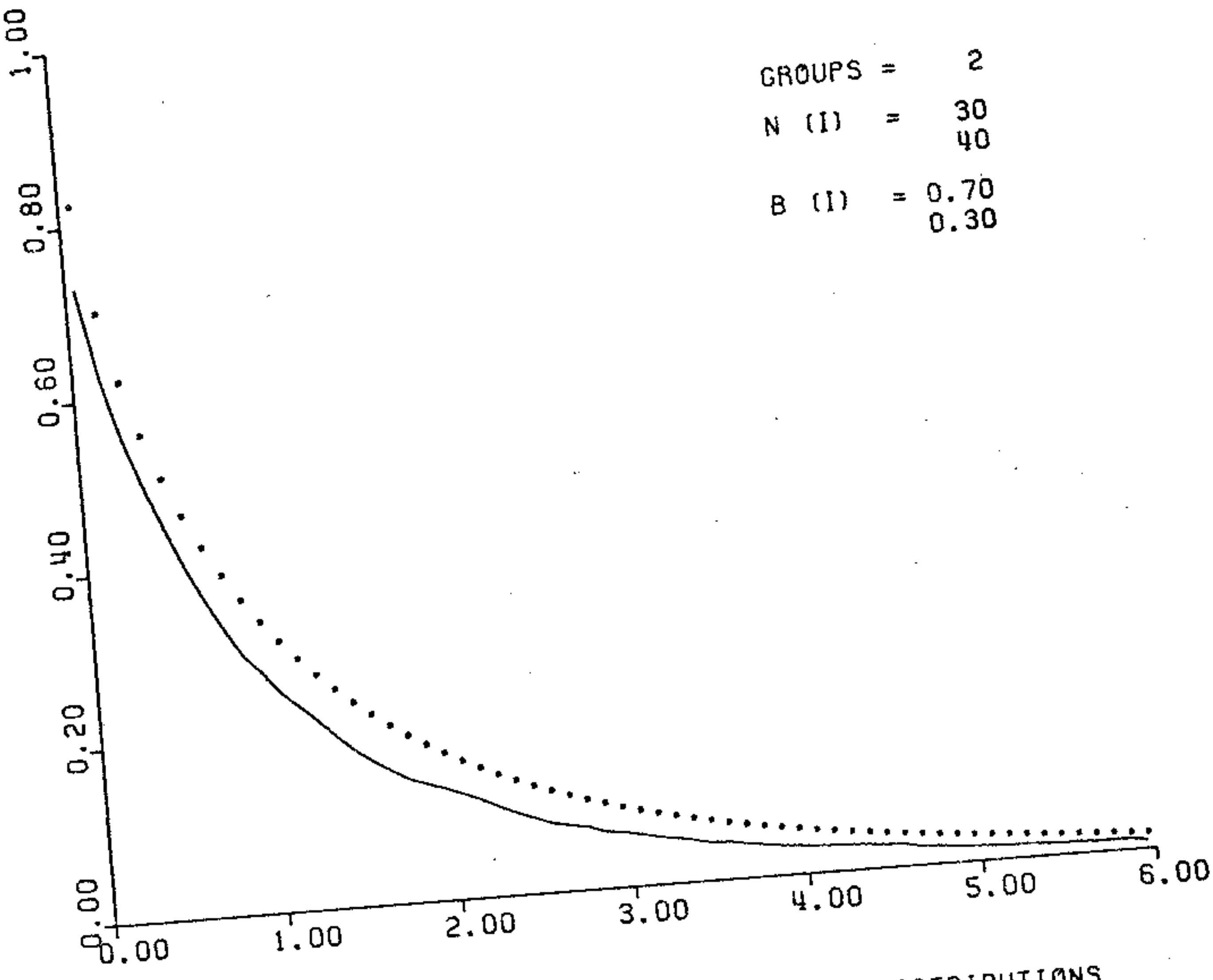
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 166 : SIMULATION NUMBER 166

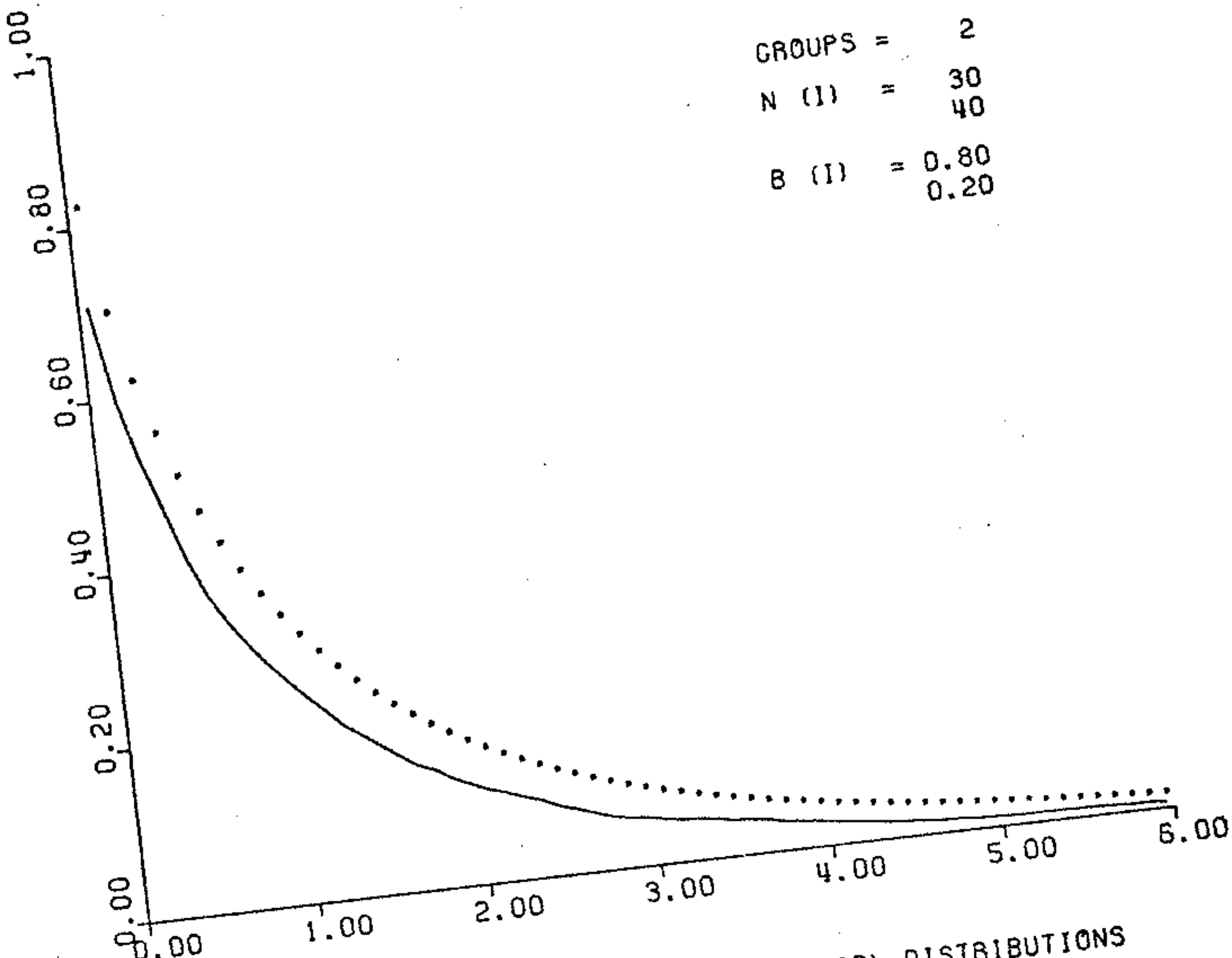


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

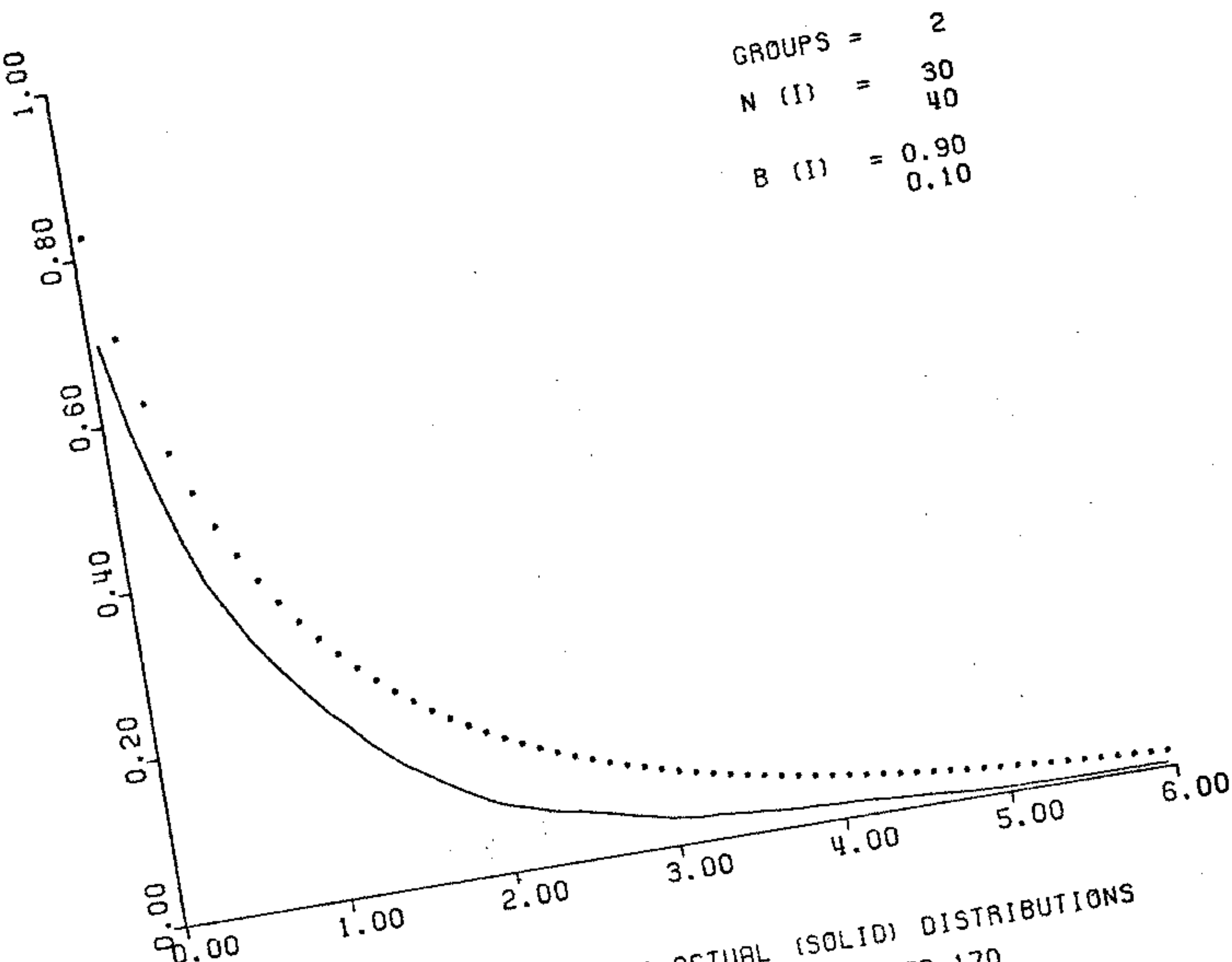
FIGURE 167 : SIMULATION NUMBER 167



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 168 : SIMULATION NUMBER 168

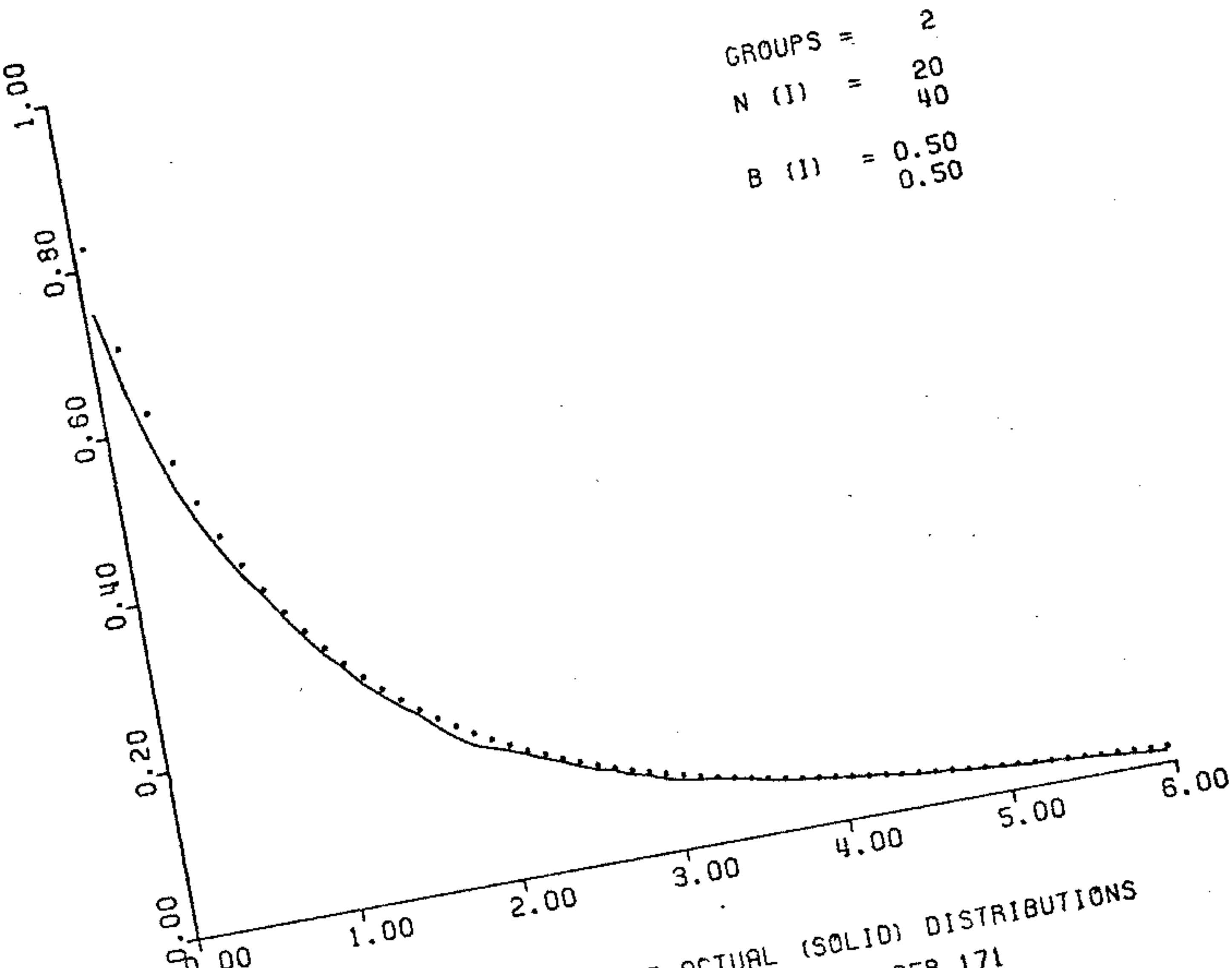


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 169 : SIMULATION NUMBER 169

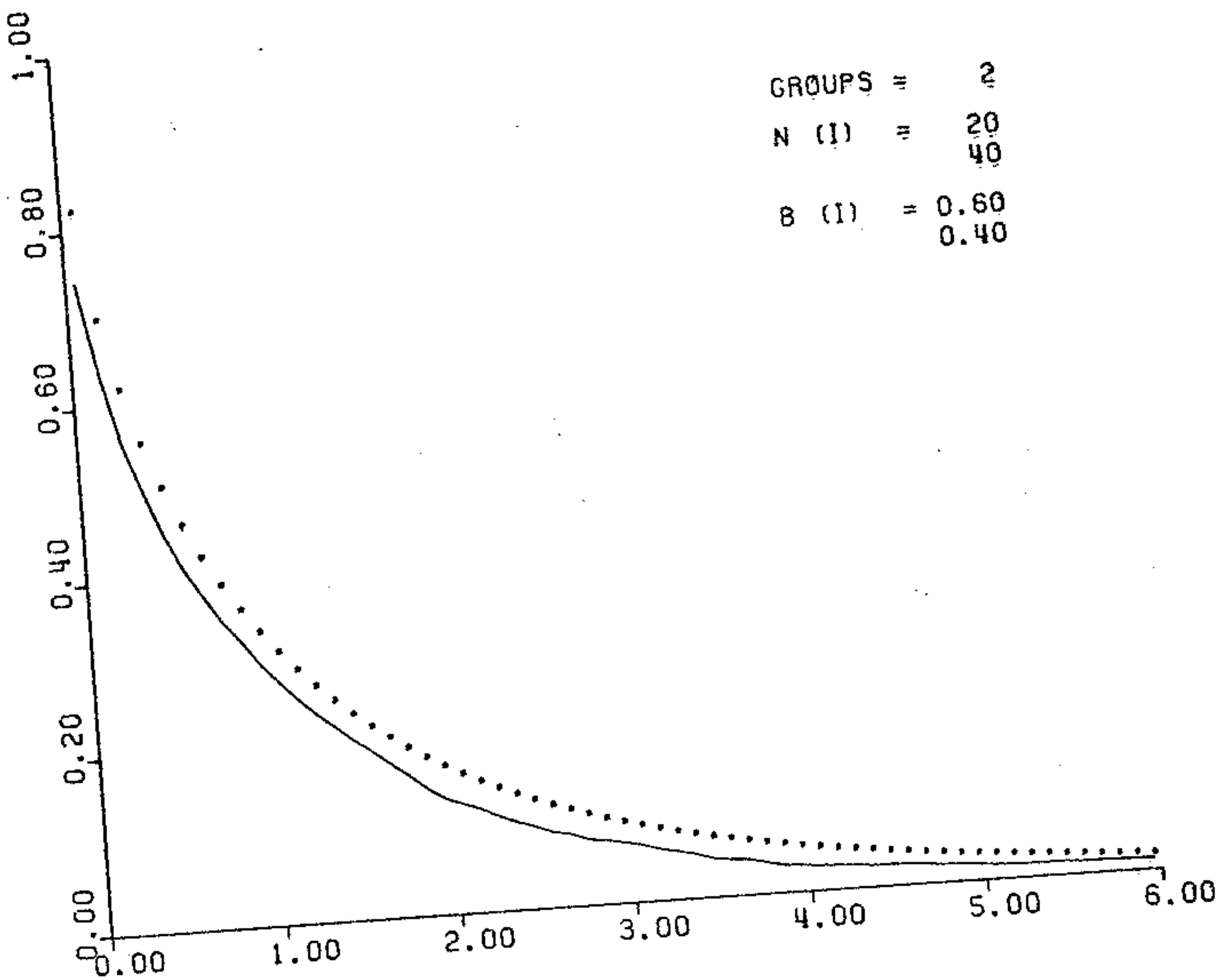


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

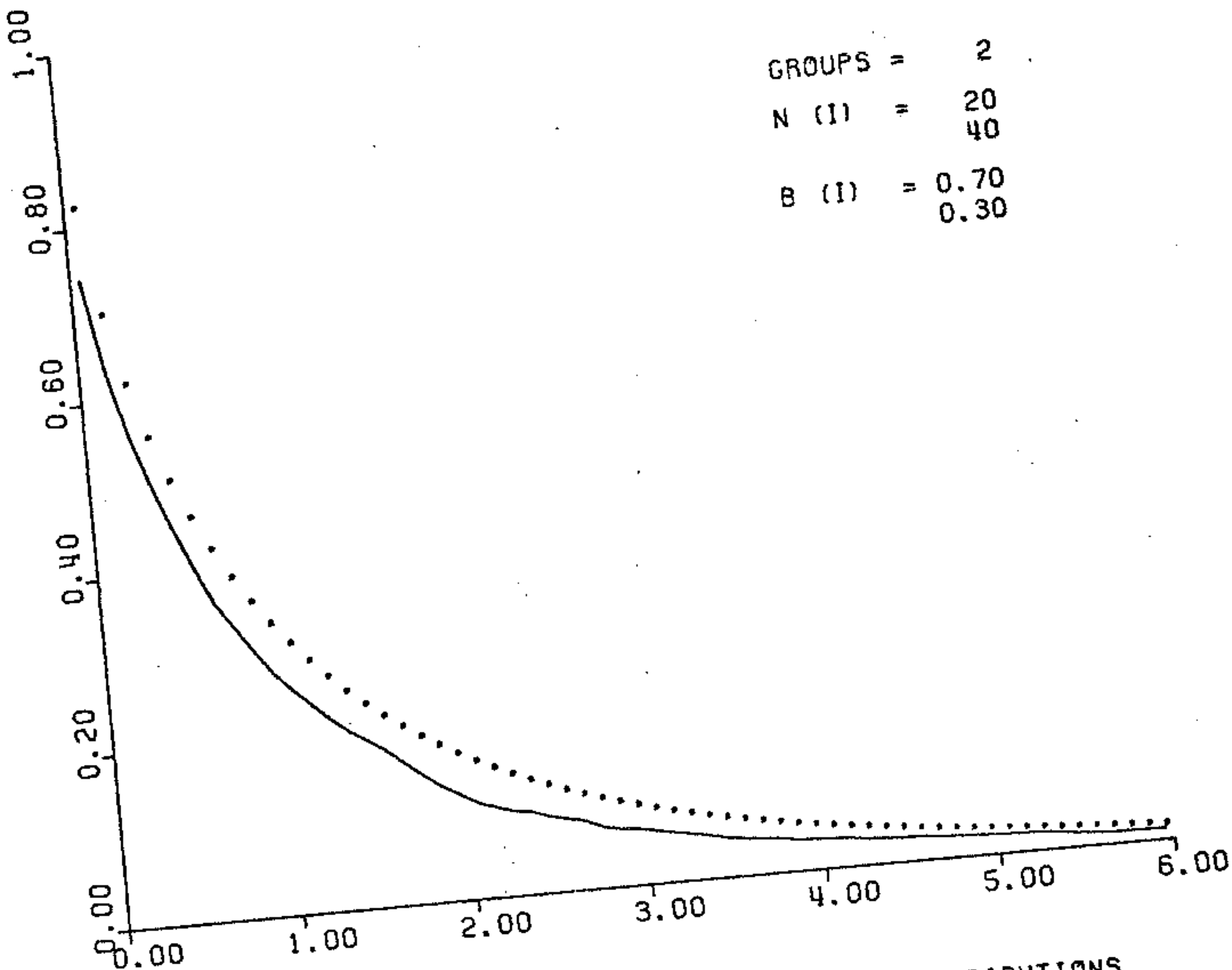
FIGURE 170 : SIMULATION NUMBER 170



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 171 : SIMULATION NUMBER 171

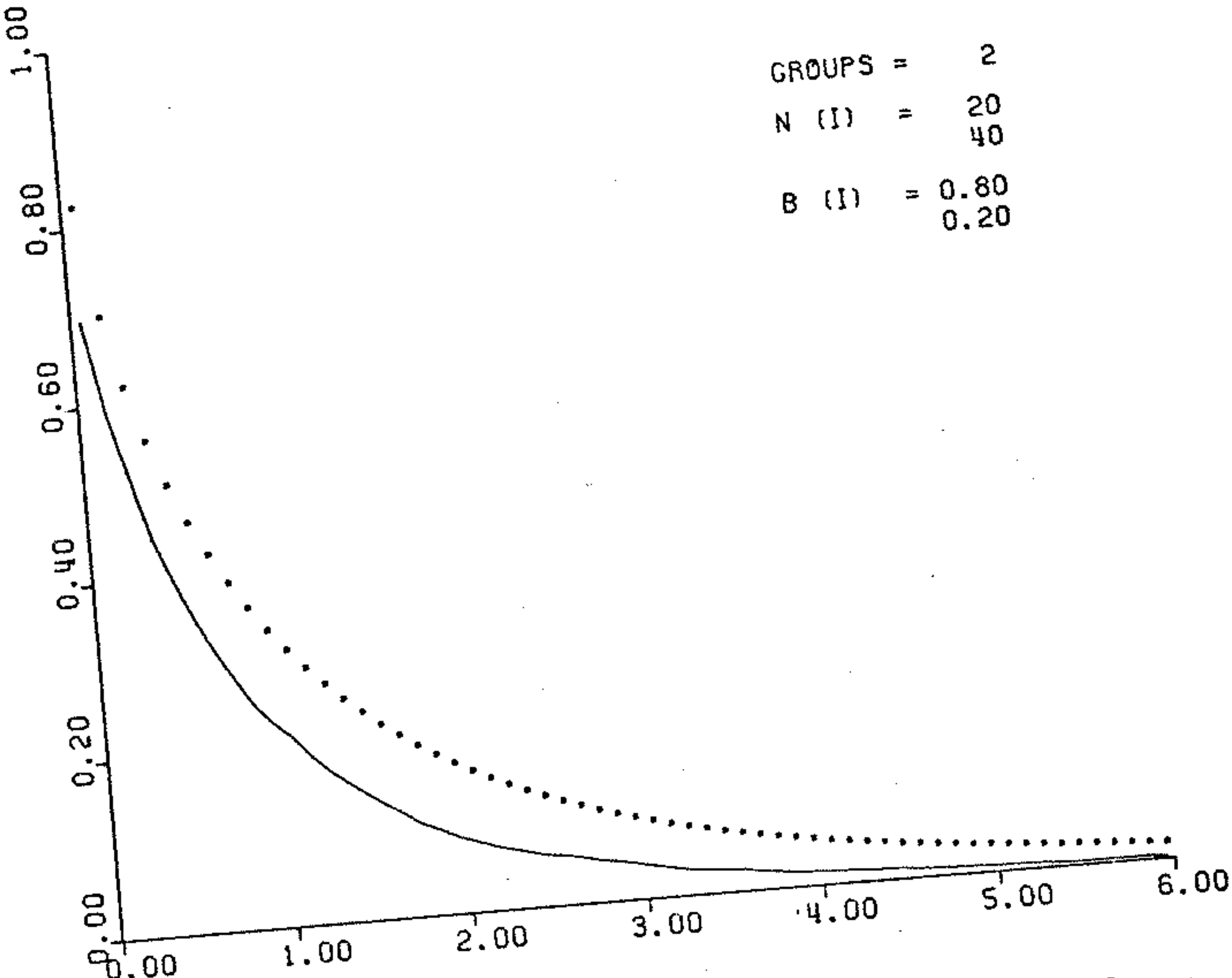


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 172 : SIMULATION NUMBER 172



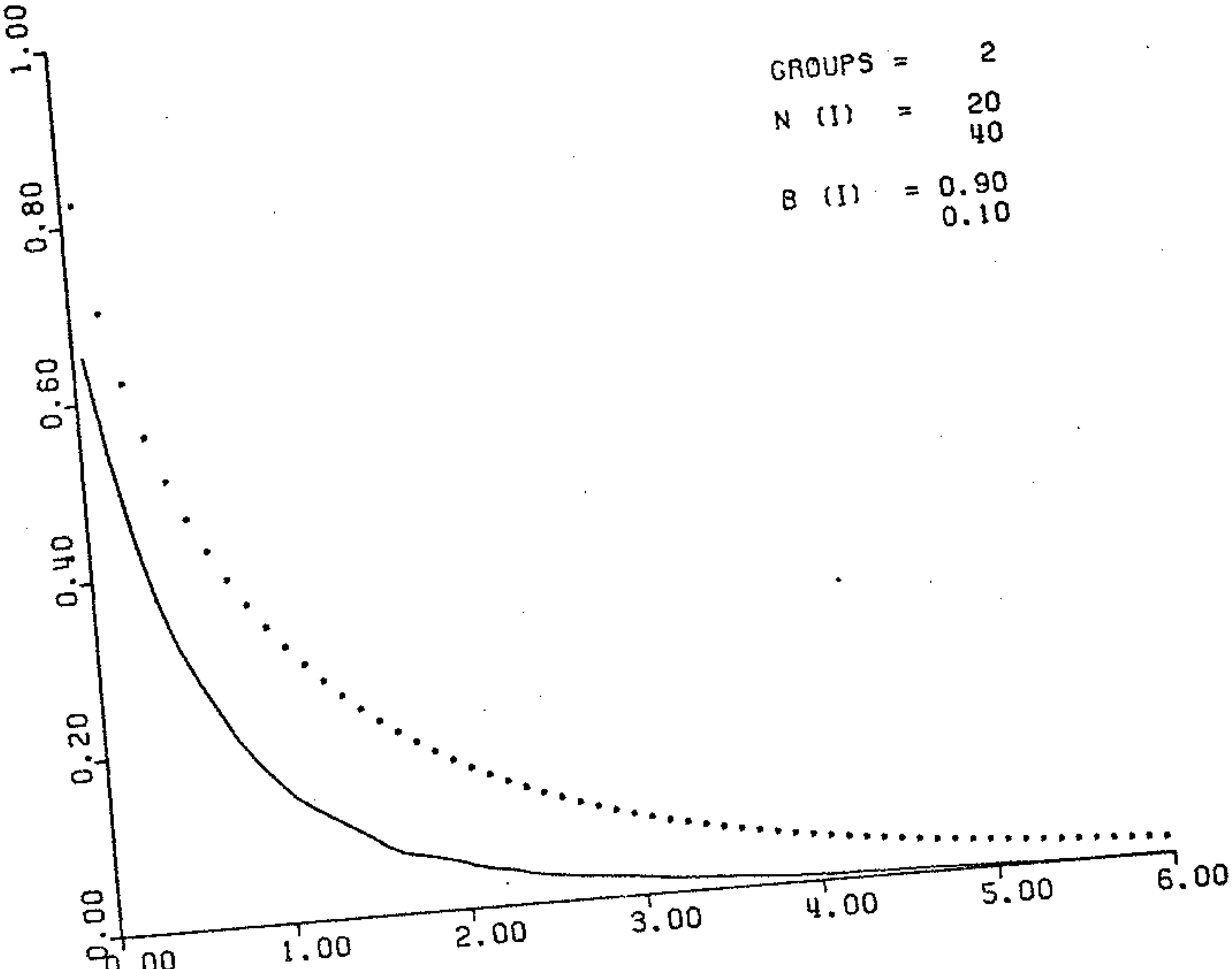
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 173 : SIMULATION NUMBER 173



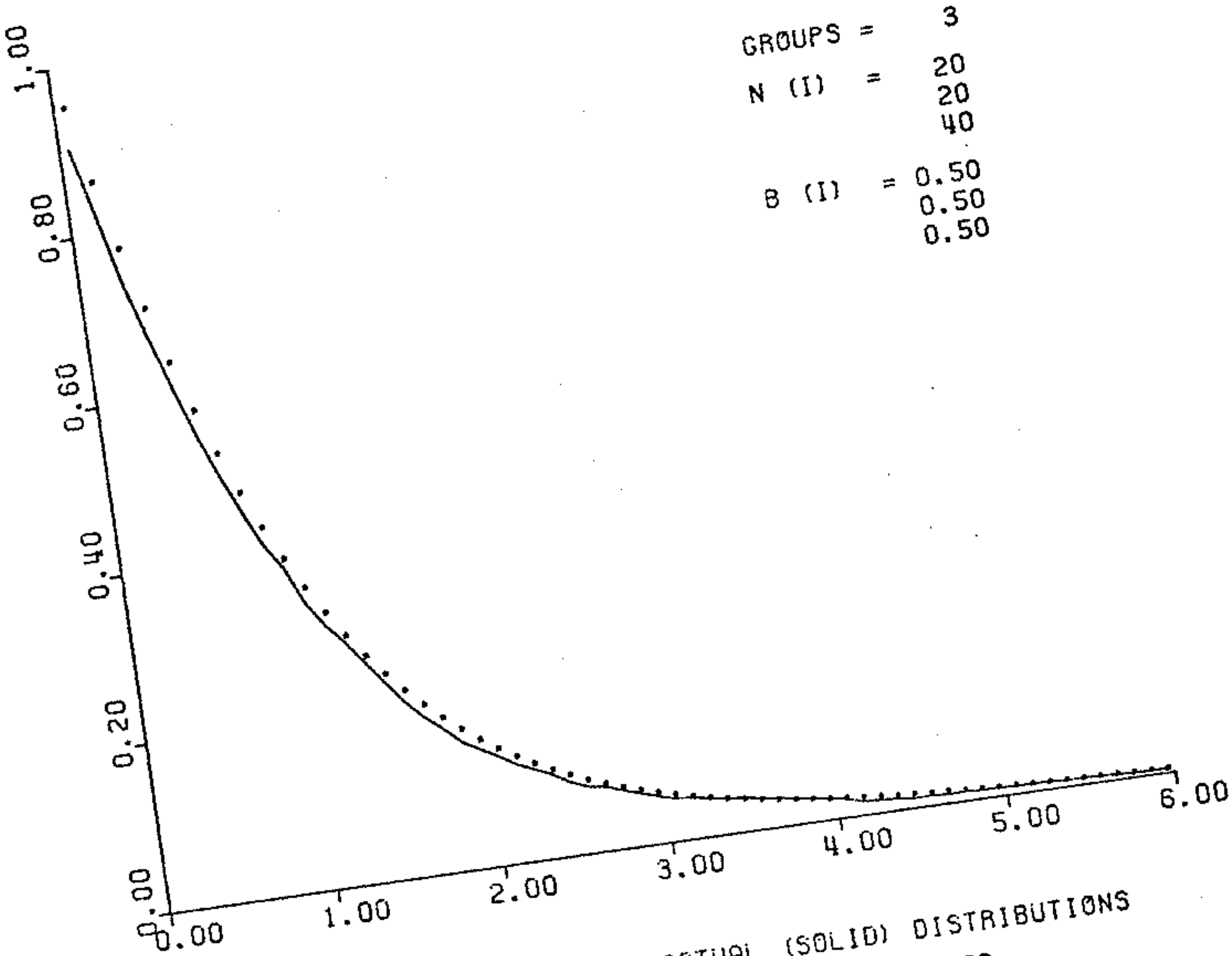
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 174 : SIMULATION NUMBER 174

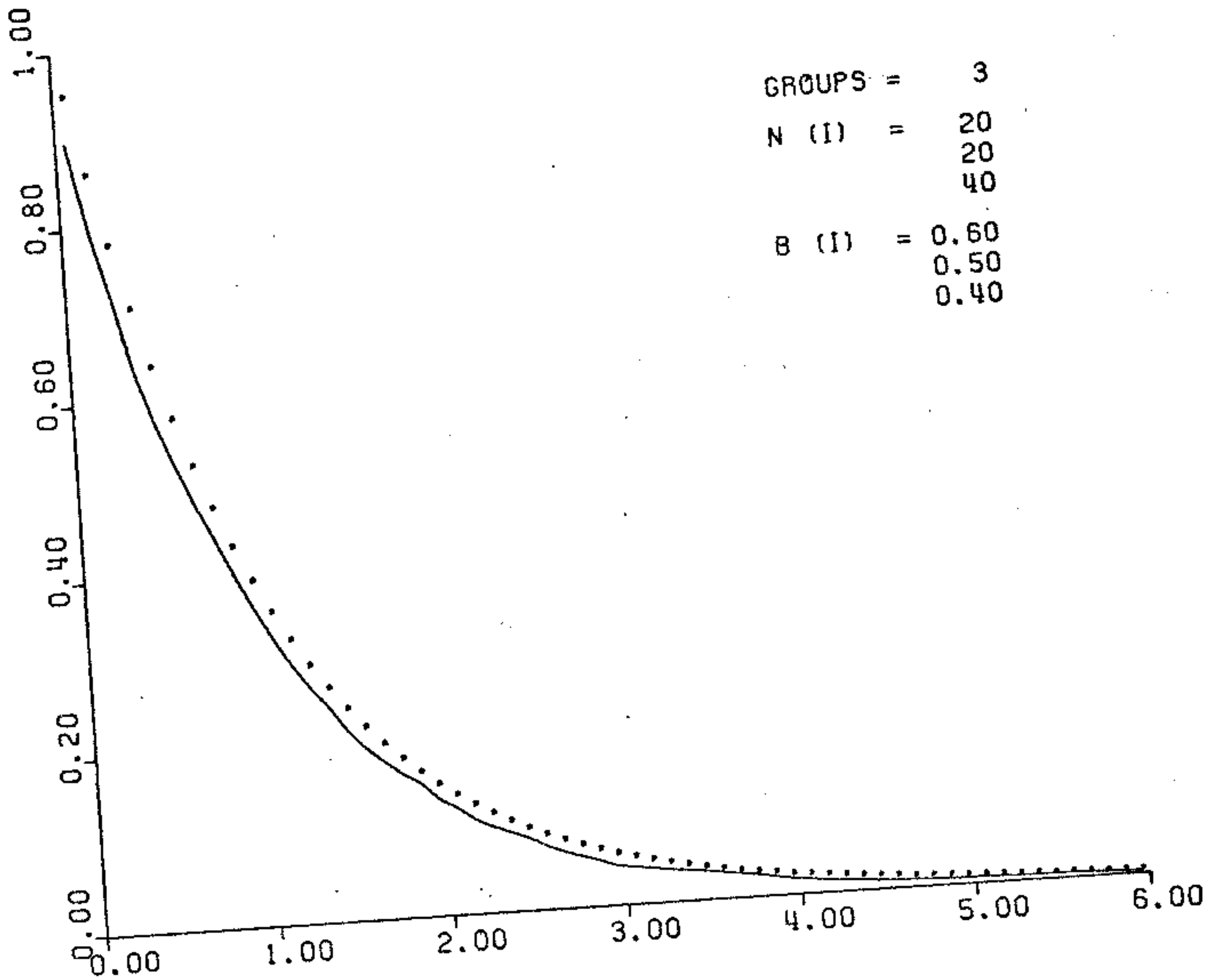


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

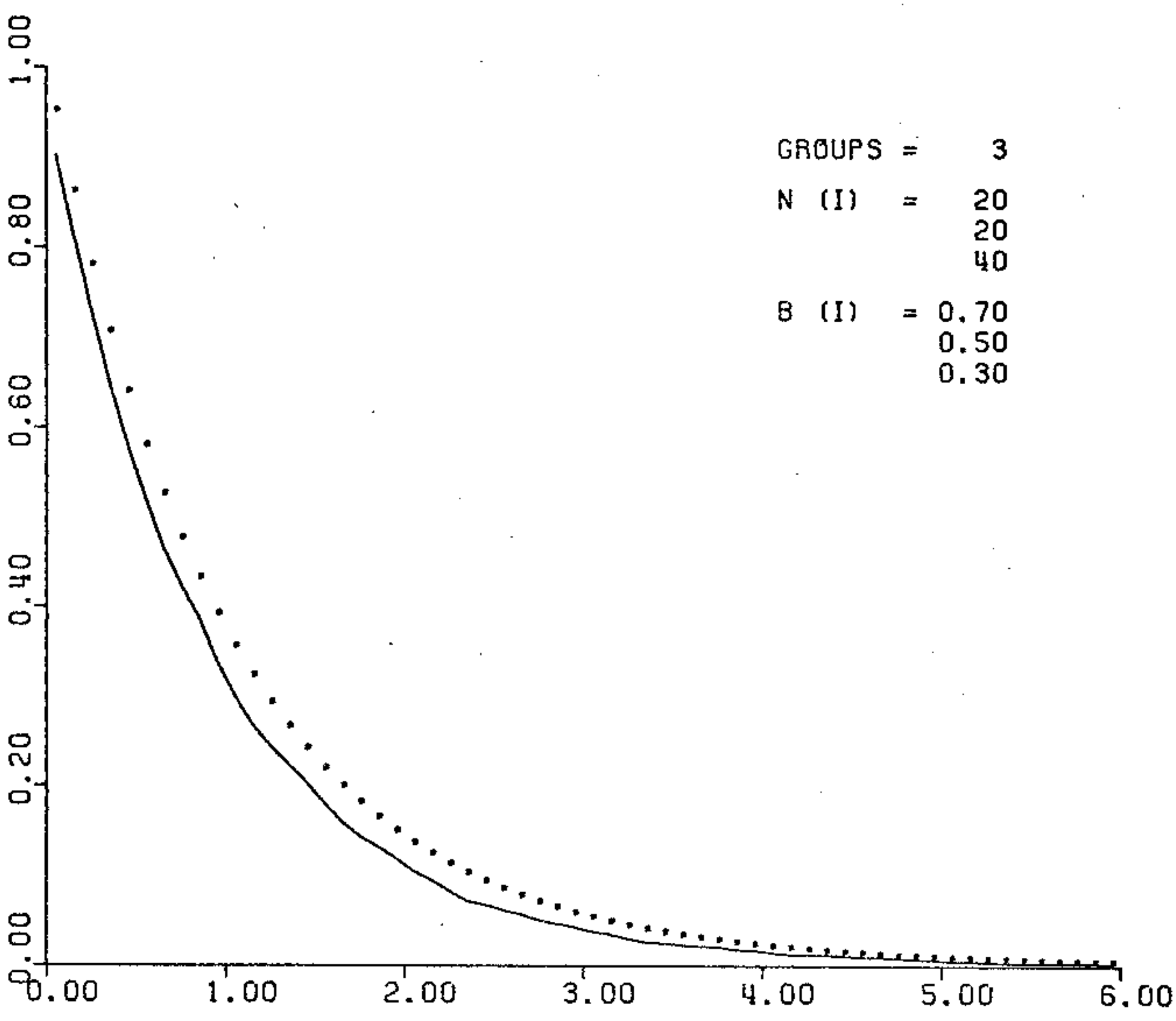
FIGURE 175 : SIMULATION NUMBER 175



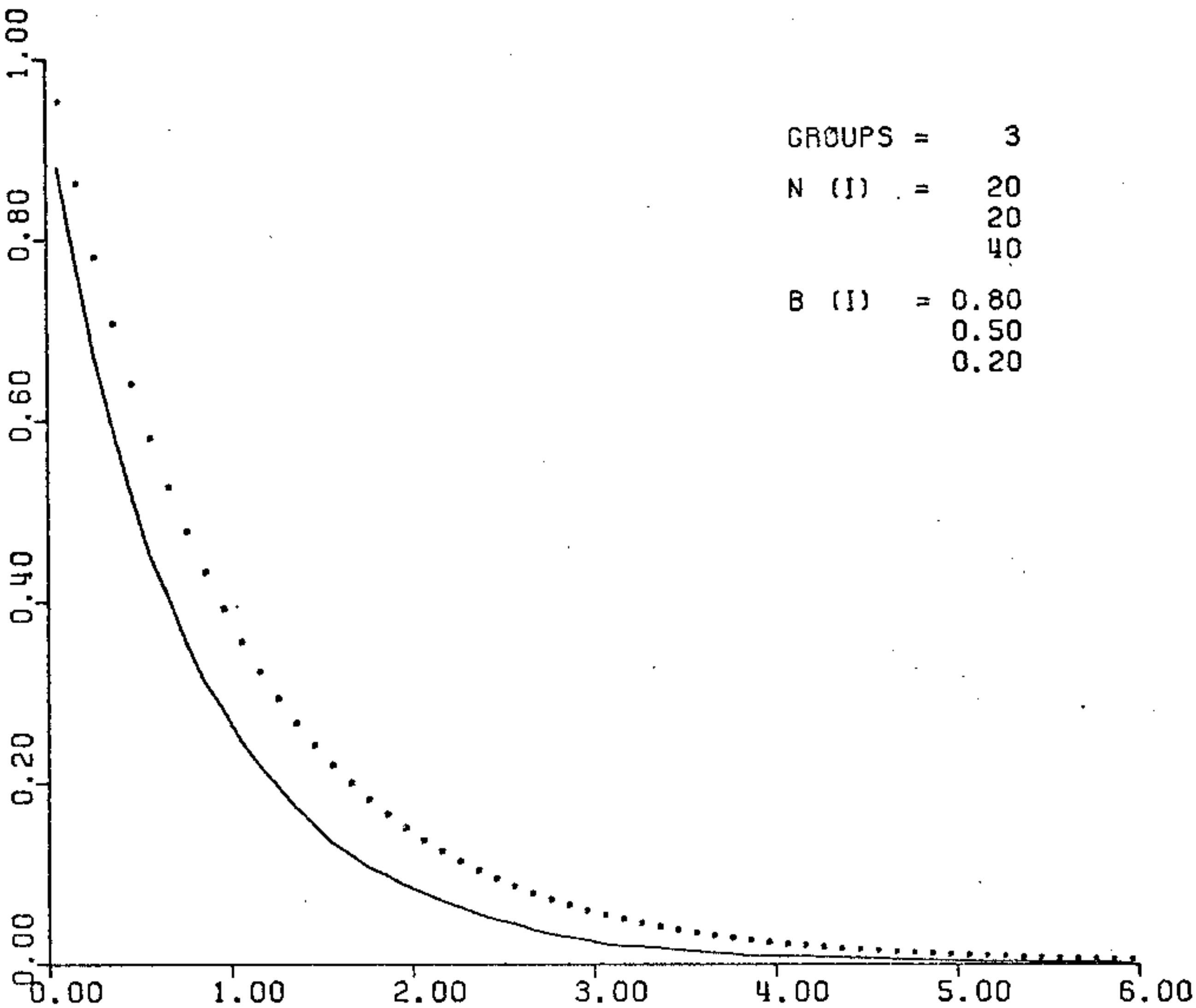
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 176 : SIMULATION NUMBER 176



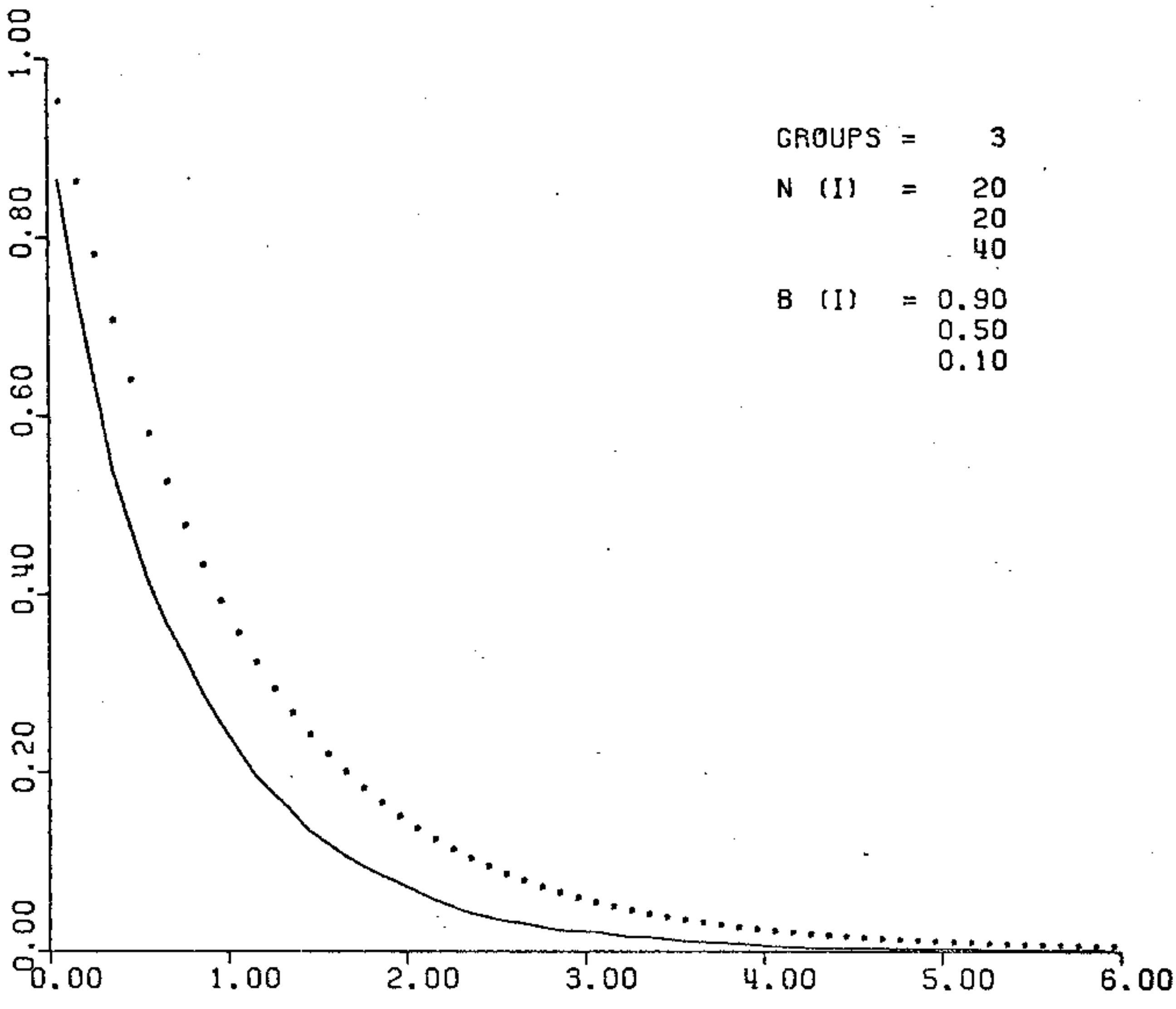
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 177 : SIMULATION NUMBER 177



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 178 : SIMULATION NUMBER 178

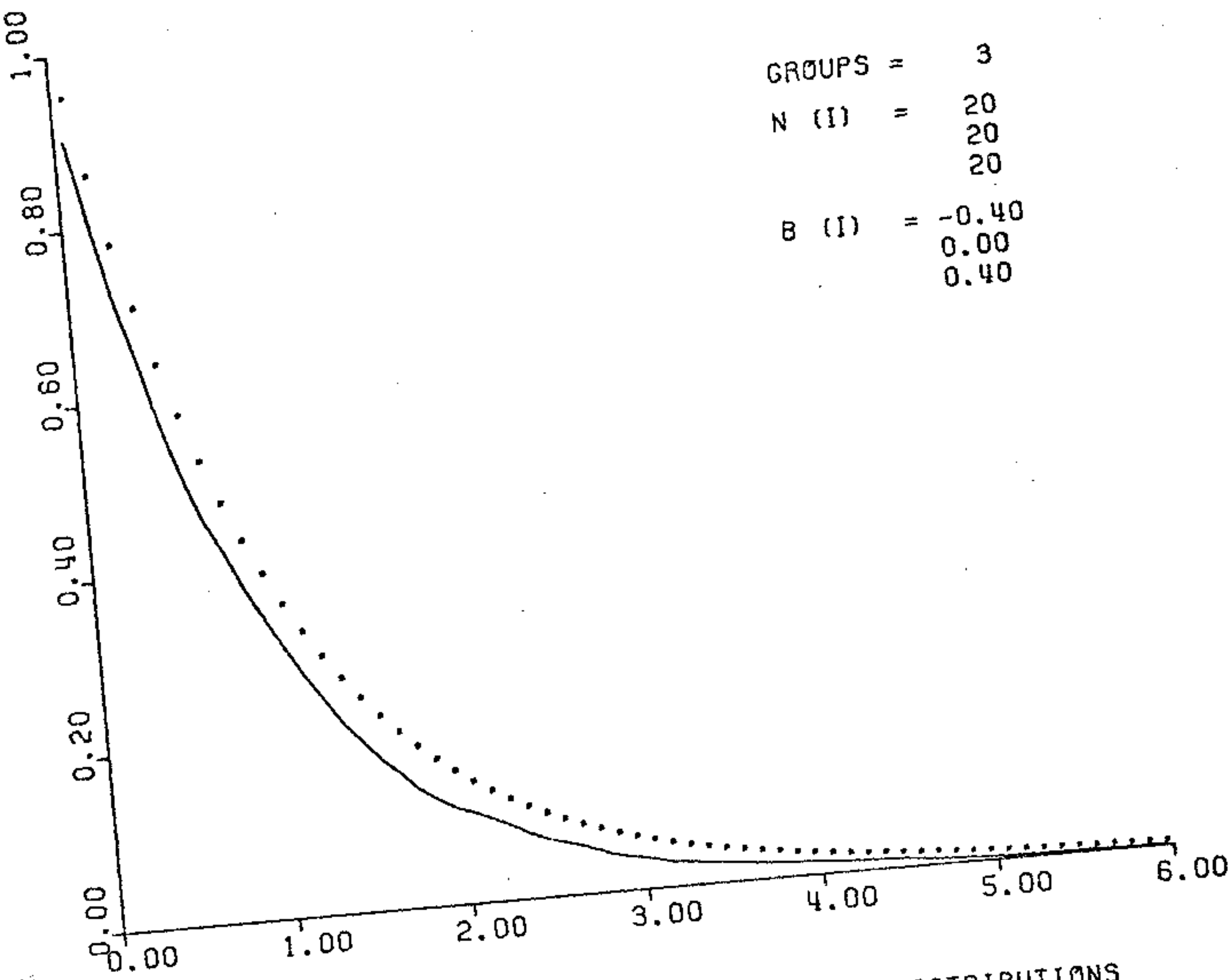


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 179 : SIMULATION NUMBER 179



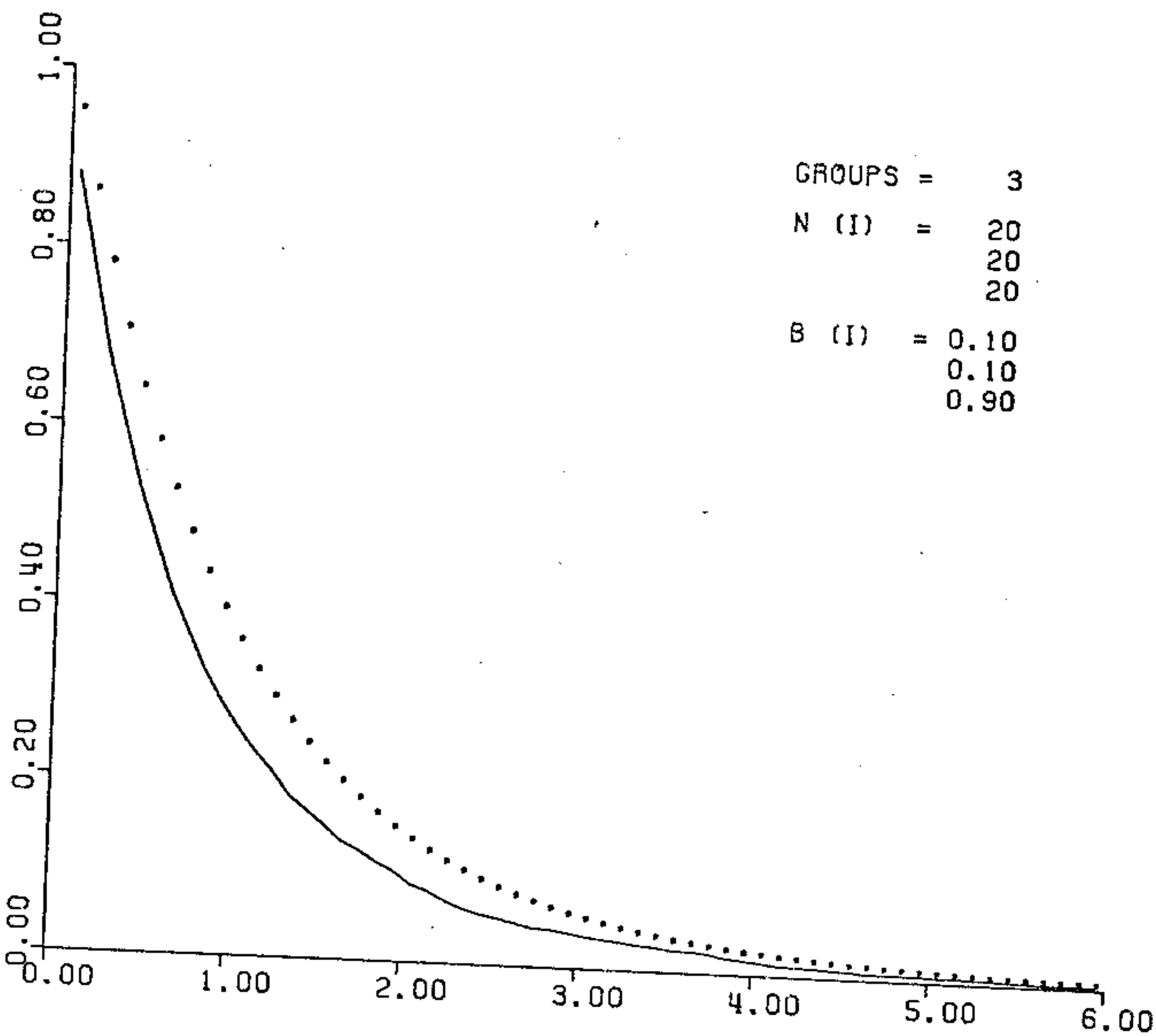
THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 180 : SIMULATION NUMBER 180

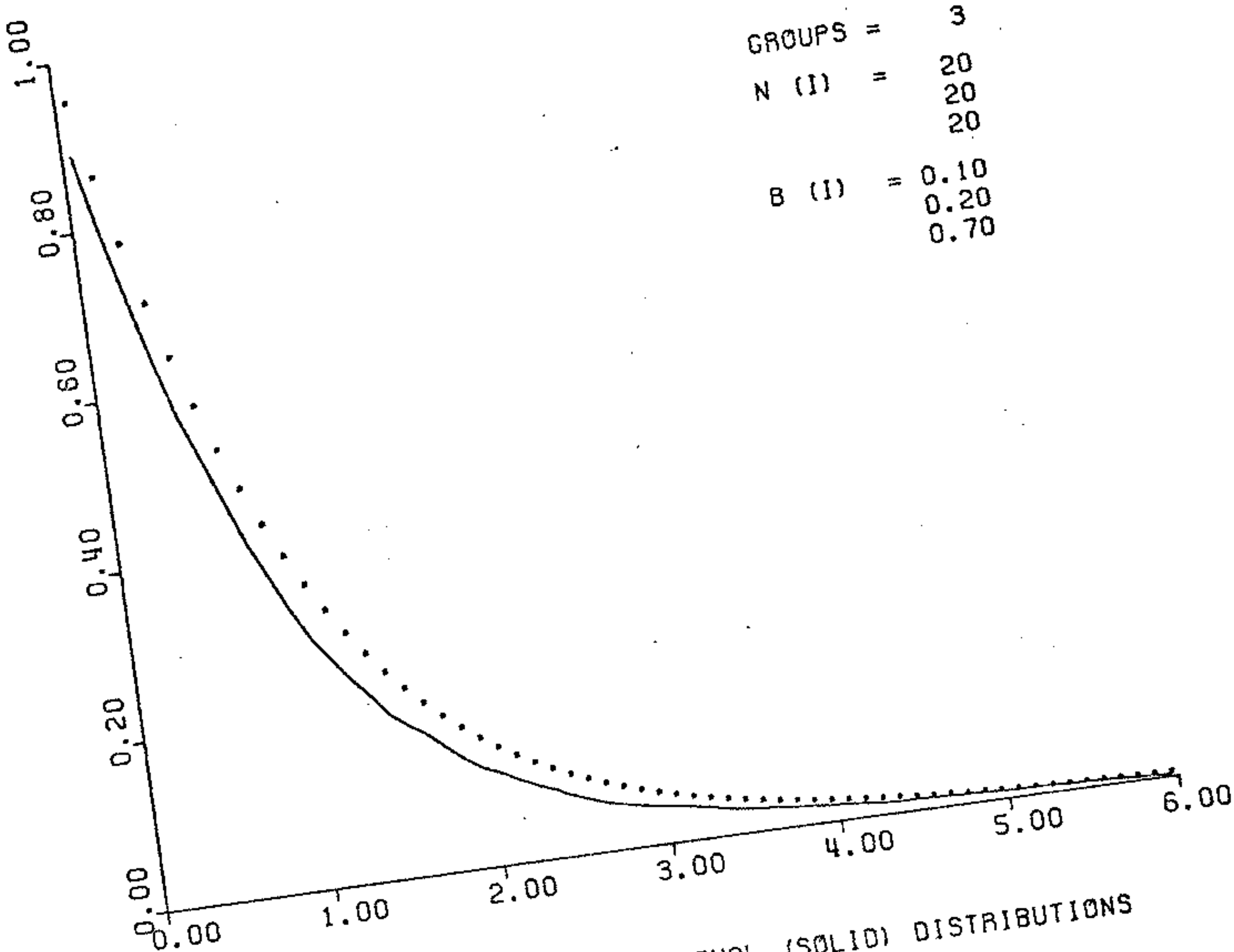


THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS

FIGURE 181 : SIMULATION NUMBER 181



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 182 : SIMULATION NUMBER 182



THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS
FIGURE 183 : SIMULATION NUMBER 183

APPENDIX D

TESTS AND APPLICATIONS OF THE RANDOM NUMBER GENERATORS

Random number generators tested were IBM subroutines GAUSS and RANDU. The purpose of GAUSS is to compute a normally distributed random number sequence with a given mean and standard deviation. Subroutine RANDU is required by GAUSS. The GAUSS subroutine uses twelve uniform random numbers generated by RANDU to compute a normal random number by the Central Limit Theorem. The formula employed is:

$$(1) \qquad Y = \frac{\sum_{i=1}^{K} (X_i - \frac{K}{2})}{\sqrt{K/12}}$$

where X_i is a uniformly distributed random number, $0 < X_i < 1$. K is the number of values X_i to be used. Y approaches a true normal distribution assymptotically as K approaches infinity. For this subroutine K was chosen as 12. Equation (1) reduces to

$$(2) \qquad Y = \sum_{i=1}^{12} (X_i - 6.0).$$

The resulting normal random number obtained by equation (2) was then adjusted to match the given mean and standard deviation using the formula:

$$Y' = Y \cdot S + AM$$

where Y' is the required normally distributed random number, S is the required standard deviation, and AM is the required mean.

The tests of the random number sequence generated by GAUSS were the frequency test and the runs above and below the means test. The frequency test outlined in Naylor was used to check the normal distribution of the sequence. Naylor also explains the use of the runs above and below the means test as a test for the oscillatory nature of sequences. It was assumed that satisfactory results on these two tests on GAUSS implied that the RANDU subroutine was functioning properly since it was employed by GAUSS.

Frequency Test

One-hundred sets of 1000 random numbers each with normal distribution, zero mean, and standard deviation of one were generated using GAUSS. Each set of 1000 numbers was placed in equi-probable intervals where the expected number of random numbers in each subinterval was 100. The subintervals were determined using the unit normal distribution with mean of zero. The actual frequencies were calculated for each set of 1000 numbers and chi-square computed using the formula

$$\chi_1^2 = \frac{\sum_{j=1}^{10} (f_j - 100)^2}{100},$$

where f_j is the observed frequency in subinterval j. If the entire sequence of 100,000 random numbers is composed of "truly" random observations on a variable normally distributed with a mean of zero and standard deviation of one, then the 100 χ_1^2's would have approximately a chi-square distribution with 9 degrees of freedom. The goodness-of-fit of the empirical distribution of the 100 χ_1^2's to the theoretical chi-square distribution was tested by placing each χ_1^2 into one of ten equi-probable intervals and computing the statistic

$$\chi_F^2 = \frac{\sum_{j=1}^{10} (F_j - 10)^2}{10}$$

where F_j is the observed frequency of χ_1^2's in subinterval j. The results of this test are given in Table IX.

TABLE IX

CHI-SQUARE TEST FOR THE NORMAL DISTRIBUTION OF A POPULATION BASED 100,000 PSEUDORANDOM NUMBERS

Equi-probable Interval	Frequency
0.00 - 4.17	6
4.18 - 5.38	12
5.39 - 6.39	7
6.40 - 7.36	8
7.37 - 8.34	6
8.35 - 9.41	11
9.42 - 10.66	16
10.67 - 12.24	8
12.25 - 14.68	15
14.69 - 00.00	11

$\chi^2 = 11.6$ df = 9 p = .25

The χ_F^2 statistic would also be distributed as chi-square with nine degrees of freedom. A value of 11.6 was obtained for χ_F^2 with the critical value for rejection of the hypothesis that the statistic χ_F^2 is distributed as chi-square is 16.919 at the .05 level. Thus, no significant difference was found between the observed and expected frequency distributions. It was therefore concluded that the 100,000 pseudorandom numbers could be from the unit normal distribution with mean of zero, and the frequency test of GAUSS produced satisfactory results.

Runs Test

One-hundred samples (sequences) of 500 random numbers were generated, using the GAUSS subroutine, with a unit normal distribution and mean of zero.

A run is defined as a subsequence of numbers in which each term is either above the mean or each term is below the mean. Since for the purposes of this test the mean was zero, a run is a subsequence of consecutive positive terms or consecutive negative terms. The number of terms in a run is called the length of the run.

For each of the 100 sequences the number of runs of lengths 1, 2, 3, . . . 7 and of length greater than or equal to 8 were counted. The expected frequency of runs of each of these lengths was calculated by the expression for expected frequency of runs of length k, $(N - k + 3)2^{-k-1}$, where N is the number of terms of the sequence.

A chi-square goodness-of-fit statistic was computed on each sequence comparing the observed frequency of runs of each length to the expected frequency. The 100 values of chi-square thus obtained were themselves tested by a chi-square goodness-of-fit test using the ten equi-probable intervals with expected frequencies of ten listed in Table X. The χ^2 of 2.2 with nine degrees of freedom was not significant. Therefore, the test indicated no significant difference between the observed

TABLE X

CHI-SQUARE GOODNESS-OF-FIT TEST FOR RUNS ABOVE AND BELOW THE MEAN BASED ON 50,000 PSEUDORANDOM NUMBERS

Equi-probable Interval	Frequency
0.00 - 4.17	13
4.18 - 5.38	9
5.39 - 6.39	11
6.40 - 7.36	10
7.37 - 8.34	9
8.35 - 9.41	9
9.42 - 10.66	9
10.67 - 12.24	8
12.25 - 14.68	10
14.69 - 00.00	12

$\chi^2 = 2.2$ df = 9 p = .98 $R_{.05}$: $\chi^2 \geq 16.919$

frequency distribution and expected frequency distribution. The results of the test for runs above and below the mean, consequently, were satisfactory indicating a random oscillatory nature of the sequence of pseudorandom numbers generated by GAUSS.

Lagged Product Test

A final test on the random number generator RANDU was the lagged product test, a measure of the independence of the pseudorandom numbers. The subroutine RANDU was used to generate 500 sets of 100 random numbers, uniformly distributed with zero mean and standard deviation of one. For each set, or sequence, of 100 random numbers the lagged product statistic C_k, was computed for k=1, 2, . . . 10, where

$$C_k = \frac{\sum_{i=1}^{100-k} r_i r_{i+k}}{100-k}, \quad k > 0.$$

The factor r_i represents the ith term of the sequence. According to Naylor, if there is no correlation between r_i and r_{i+k}, the values of C_k will be approximately normally distributed with mean of 0.25 and standard deviation equal to $\sqrt{13N - 19k} / 12(100 - k)$. Each value of C_k was converted to a standardized unit normal value, G_k, with mean of zero by the formula:

$$G_k = \frac{C_k - .25}{\sqrt{13N - 19K} / 12\ (100 - k)}, \quad k > 0.$$

Hence, if the G_k's for a particular value of k are normally distributed with zero mean and standard deviation equal to one, then there is no correlation between r_i and r_{i+k}. For each k, the 500 values of G_k were checked for unit normal distribution with zero mean using a Kolmogorov-Smirnov test of goodness-of-fit. The results of these tests are given in Table XI. The test of lag nine indicated that the goodness-of-fit hypothesis should be rejected at the .05 level. Generally, however, the subroutine RANDU performance with respect to independence of pseudorandom numbers was acceptable.

TABLE XI

KOLMOGOROV-SMIRNOV GOODNESS-OF-FIT TEST
FOR THE LAGGED PRODUCT STATISTIC
TO THE NORMAL DISTRIBUTION

Lag (k)	D	Probability**
1	0.033156	0.6417
2	0.045417	0.2537
3	0.034660	0.5853
4	0.032473	0.6674
5	0.036577	0.5153
6	0.033777	0.6183
7	0.049077	0.1798
8	0.053820	0.1104
9	0.066681*	0.0234
10	0.046190	0.2364

*Significant at .05 level. $R_{.05}$: $D \geq .060821$.

**Approximate due to limitations of Kolmogorov-Smirnov computer program.

APPENDIX E

THE COMPUTER PROGRAM

PROGRAM 10*

GENERATE CRITERION DATA & PERFORM ANALYSIS OF COVARIANCE.

```
IX   = SEED FOR RANDOM NUMBER GENERATOR.
NGPS = NUMBER OF GROUPS.
N    = VECTOR CONTAINING NUMBER OF OBSERVATIONS IN EACH GROUP.
BT   = VECTOR OF BETA WEIGHTS FOR EACH GROUP.
XX   = ARRAY OF COVARIATE DATA.
F    = F-VALUE FROM THE ANALYSIS.
XMS  = VECTOR OF COMPUTED MEANS FOR EACH GROUP.
STDS = VECTOR OF COMPUTED STANDARD DEVIATIONS FOR EACH GROUP.
BS   = VECTOR OF COMPUTED BETA WEIGHTS FOR EACH GROUP.
XYZ  = TRAILER VALUE.

      IMPLICIT REAL*8 (A-H,O-Z)
      REAL*4 XX,Z,R1,R2,A,B,XMS,STDS,BS,XYZ,F
      DIMENSION SUMW(5,2),SUMX2(5,2),XMS(5),STDS(5),BS(5),XN(5),
     *XX(5,200),BT(5),N(5)
      REWIND 30
      XYZ=999.
    1 READ (5,10,END=999) IX,NGPS,N
   10 FORMAT(I9,I2,5I5)
      READ (5,20) BT
   20 FORMAT(5F5.0)
      DO 55 I=1,NGPS
      M=N(I)
   55 READ (5,30)(XX(I,J),J=1,M)
   30 FORMAT(10F8.6)
      WRITE (6,50) NGPS,(N(I),I=1,NGPS)
   50 FORMAT(I2,' GROUPS',5X,'N''S =',5I5)
      WRITE (6,60) (BT(J),J=1,NGPS)
   60 FORMAT(' BETAS = ',5F8.3)
      WRITE (6,70) IX
   70 FORMAT(' IX = ',I9////)
      DO 8 II=1,2000
      DO 11 I=1,NGPS
      SUMW(I,1)=0.D0
      SUMW(I,2)=0.D0
      SUMX2(I,1)=0.D0
      SUMX2(I,2)=0.D0
   11 XN(I)=0.D0
      TOT=0.D0
      SUMX=0.D0
      SUMY=0.D0
      C11=0.D0
      C12=0.D0
      C22=0.D0
      DO 3 I=1,NGPS
      BB=BT(I)
      VAR=DSQRT(1.D00-BB*BB)
      SUMXY=0.D0
      M=N(I)
      DO 333 J=1,M
      IF(MOD(J,2) .EQ. 0) GO TO 203
      IX=IX*65539
      IF(IX .LT. 0) IX=IX+2147483647+1
      R1=FLOAT(IX)*.4656613E-9
      IX=IX*65539
```

```
      IF(IX .LT. 0) IX=IX+2147483647+1
      R2=FLOAT(IX)*.4656613E-9
      A=SQRT(-2.*ALOG(R1))
      B=6.283185*R2
      Z=A*SIN(B)
      GO TO 204
  203 Z=A*COS(B)
  204 X=XX(I,J)
      Y=BB*X+Z*VAR
      SUMW(I,1)=SUMW(I,1)+Y
      SUMW(I,2) =SUMW(I,2)+X
      XY=X*Y
      X=X*X
      Y=Y*Y
      SUMX2(I,1)=SUMX2(I,1)+Y
      SUMX2(I,2)=SUMX2(I,2)+X
      SUMXY=SUMXY+XY
      C11=C11+X
      C22=C22+Y
      C12=C12+XY
      XN(I)=XN(I)+1.D0
  333 CONTINUE
      BS(I)=(SUMXY-SUMW(I,1)*SUMW(I,2)/XN(I))/(SUMX2(I,2)-SUMW(I,2)**2
     */XN(I))
    3 CONTINUE
      DO 5 I=1,NGPS
      SUMY=SUMY+SUMW(I,1)
      SUMX=SUMX+SUMW(I,2)
    5 TOT=TOT+XN(I)
      D11=C11-SUMX*SUMX/TOT
      D12=C12-SUMX*SUMY/TOT
      D22=C22-SUMY*SUMY/TOT
      DO 112 I=1,NGPS
      ZZ=XN(I)
      FACT=ZZ/(ZZ-1.D0)
      X=SUMW(I,2)/ZZ
      Y=SUMW(I,1)/ZZ
      STDX=DSQRT((SUMX2(I,2)/ZZ-X*X)*FACT)
      STDY=DSQRT((SUMX2(I,1)/ZZ-Y*Y)*FACT)
      XMS(I)=Y
      BS(I)=BS(I)*STDX/STDY
      STDS(I)=STDY
      C11=C11-X*X*ZZ
      C12=C12-X*Y*ZZ
  112 C22=C22-Y*Y*ZZ
      XNGPS=NGPS
      ANS1=D22-D12*D12/D11
      ANS2=C22-C12*C12/C11
      F=((ANS1-ANS2)/(XNGPS-1.D0))/(ANS2/(TOT-XNGPS-1.D0))
      WRITE (30,80) F,(XMS(I),STDS(I),BS(I),I=1,NGPS)
   80 FORMAT(16A4)
    8 CONTINUE
      WRITE(30) XYZ,(XYZ,XYZ,XYZ,I=1,NGPS)
      GO TO 1
  999 STOP 0000
      END
```

```
PROGRAM TO:

   COMPUTE & PLOT (ON CALCOMP PLOTTER) DISTRIBUTIONS.

ISIM  = SIMULATION NUMBER.
NG    = NUMBER OF GROUPS.
XN    = VECTOR OF NUMBER OF OBSERVATIONS IN EACH GROUP.
B     = VECTOR OF BETA WEIGHTS FOR EACH GROUP.
F     = F-VALUE FROM EACH ANALYSIS.
DUMMY = VALUE FOR READING TRAILER RECORD.
PRBF  = FUNCTION FOR COMPUTING PROBABILITY OF AN F >= X.
LINE1 = SUBROUTINE FOR PLOTTING POINTS.

OTHER SUBROUTINES ARE FROM THE CALCOMP SOFTWARE PACKAGE.

   DIMENSION X(62),PA(62),PT(62),XN(5),B(5),F(2000),LBLE(14),
  * IBUF(2000)
   DATA X/.05,.15,.25,.35,.45,.55,.65,.75,.85,.95,1.05,1.15,1.25,
  * 1.35,1.45,1.55,1.65,1.75,1.85,1.95,2.05,2.15,2.25,2.35,2.45,2.55,
  * 2.65,2.75,2.85,2.95,3.05,3.15,3.25,3.35,3.45,3.55,3.65,3.75,3.85,
  * 3.95,4.05,4.15,4.25,4.35,4.45,4.55,4.65,4.75,4.85,4.95,5.05,
  * 5.15,5.25,5.35,5.45,5.55,5.65,5.75,5.85,5.95/,
  * LBLE/'THEORETICAL (DOTTED) AND ACTUAL (SOLID) DISTRIBUTIONS   '/
   CALL PLOTS(IBUF,2000,6)
   XL=6.
   YL=5.
   SP=.10
   XS=XL-13.*SP-.6
   N=2000
   XX=2000.
   II=0
   REWIND 30
 1 READ (5,10,END=999) ISIM,NG,XN
10 FORMAT(2I5,5F5.0)
   II=II+1
   READ (5,20) B
20 FORMAT(5F5.0)
   READ (30,30) F
30 FORMAT(A4)
   READ (30,30) DUMMY
   DO 2 I=1,60
 2 PA(I)=0.
   XNN=0.
   DO 3 I=1,NG
 3 XNN=XNN+XN(I)
   ID1=NG-1
   ID2=XNN-ID1
   XNG=NG
   DO 5 I=1,N
   J=IFIX(10.*F(I))+1
   IF(J .GT. 60) GO TO 5
   PA(J)=PA(J)+1.
 5 CONTINUE
   SUM=0.
   DO 6 I=1,60
   WRITE (6,60) I,PA(I)
60 FORMAT(I3,F10.0)
   SUM=SUM+PA(I)
```

```
      PT(I)=MDFRI(ID.,ID2,DBLE(A(I)))
      WRITE (6,70) PA(I),PT(I)
   70 FORMAT('+',15X,2F15.4)
    6 CONTINUE
      M=N-1
      DO 51 I=1,M
      LO=I+1
      DO 51 J=LO,N
      IF(F(I) .LE. F(J)) GO TO 51
      TEMP=F(I)
      F(I)=F(J)
      F(J)=TEMP
   51 CONTINUE
      WRITE (6,80)
   80 FORMAT('1')
      WRITE (6,50) (I,F(I),I=1,N)
   50 FORMAT(10(I5,F8.4))
      X(62)=6./XL
      X(61)=0.
      PA(61)=0.
      PA(62)= 1./YL
      PT(62)=PA(62)
      LL=MOD(II,2)
      IF(LL .EQ. 1) CALL PLOT(0., -30., -3)
      CALL PLOT(-1.625, 3.+(1-LL)*7., -3)
      CALL RECT(0., 0., 11., 8.5, 0., 3)
      CALL PLOT(1.625, 3.5, -3)
      CALL AXIS(0., 0., ' ', -1, XL, 0., 0., X(62))
      CALL AXIS(0., 0., ' ', 1, YL, 90., 0., PA(62))
      CALL SYMBOL (.35, -.75, SP, LBLE, 0., 53)
      XP=(XL-34.*SP)/2.
      CALL SYMBOL(XP, -1., SP, 'FIGURE ', 0., 7)
      NB=0
      XSIM=ISIM
      IF(ISIM .LT. 100) NB=1
      IF(ISIM .LT. 10) NB=2
      IF(NB .EQ. 0) GO TO 11
      DO 12 I=1,NB
   12 CALL SYMBOL(999., -1., SP, ' ', 0., 1)
   11 CALL NUMBER(999., -1., SP, XSIM, 0., -1)
      CALL SYMBOL(999., -1., SP, ' :  SIMULATION NUMBER ', 0., 22)
      IF(NB .EQ. 0) GO TO 14
      DO 13 I=1,NB
   13 CALL SYMBOL(999., -1., SP, ' ', 0., 1)
   14 CALL NUMBER(999., -1., SP, XSIM, 0., -1)
      YP=YL-.5
      CALL SYMBOL(XS, YP, SP, 'GROUPS =    ', 0., 12)
      CALL NUMBER(999., YP, SP, XNG, 0., -1)
      YP=YP-.28
      CALL SYMBOL(XS, YP, SP, 'N (I)  =', 0., 8)
      DO 7 I=1,NG
      XP=XL-3.*SP-.6
      IF(XN(I) .GE. 100.) GO TO 9
      XP=XP+SP
    9 CALL NUMBER(XP, YP, SP, XN(I), 0., -1)
    7 YP=YP-SP-.07
      YP=YP+SP-.21
      CALL SYMBOL(XS, YP, SP, 'B (I)  =', 0., 8)
      XP=XL-3.*SP-.7
```

```
      CALL NUMBER(XP, YP, SP, B(I), 0., 2)
    8 YP=YP-SP-.07
      CALL LINE1(X,PT,62)
      CALL LINE(X, PA, 60, 1, 0, 0)
      IF(LL .EQ. 0) CALL PLOT(XL+5., 0., -3)
      GO TO 1
  999 CALL PLOT(XL+5., -30., -3)
      CALL PLOT(0., 0., 999)
      STOP
      END
      SUBROUTINE LINE1(X,Y,N)
      DIMENSION X(1),Y(1)
      XDEL=X(N)
      YDEL=Y(N)
      M=N-2
      DO 5 I=1,M
      XX=X(I)/XDEL
      YY=Y(I)/YDEL
      CALL PLOT(XX, YY, 3)
    5 CALL SYMBOL(XX, YY, .03, 14, 0., -1)
      RETURN
      END
```

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