A DETERMINATION OF THE BOTHE DEPRESSION FACTOR
FOR DISCS IN WATER

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CHAPTER I

INTRODUCTION

The purpose of this work is to determine experimentally the depression of the neutron density by a detecting foil. The depression factor is known as the "self-shading" of the foil.

The use of metal foils, such as indium, as neutron detectors depends upon the absorption of the neutrons by the detector. When a neutron is absorbed by the detector, the neutron density is lowered in that region. Since the absorption of neutrons by the detector depends on the neutron density, the absorption is not as large as it should be to indicate the true neutron density.

Obviously, the magnitude of this effect must be known if measurements involving absorbing detectors are to be accurate and dependable. The extent of the depression of the neutron density in the region of the detector is a function of the medium and the detector. The larger the detector, the greater is the absorption by the detector and the greater is the depression of the neutron density. The medium affects the depression by its ability to scatter neutrons.

Bothe[1] derived an expression from which the value of the depression factor may be found. The general expression is:

$$f = \frac{1}{1 + \frac{\alpha}{2}\left(\frac{R}{\lambda s} \quad \frac{3L}{2R + 3L} \quad - 1\right)} \qquad \text{for } R \gg \lambda s$$

and

$$f = \frac{1}{1 + 0.23 \ \frac{R}{\lambda s}} \qquad \text{for } R \ll \lambda s,$$

where f is the depression or self-shading factor, _ is the probability of absorption of a neutron in one cressing of the detector, R is the radius of the detector, s is the scattering mean free path of the medium, and L is the diffusion length of the medium.

Bothe considered spherical shell detectors for purposes of developing the formula; while disc shaped detectors are used almost exclusively in practice. To correct for this Bothe assumed that a disc would have the same effect as a sphere of radius two-thirds that of the disc.

Tittle[2] has found that it is more appropriate to use the transport mean free path λ_{Tr} than the scattering mean free

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[1] W. Bothe, "Zur Methodik der Neutronensonden," *Zeits. f. Physik*, CXX (1943), 437.

[2] C. W. Tittle, unpublished research, North Texas State College.

path λ_s. The transport mean free path has a value[3] of 0.433 ± 0.01 cm. in water and the scattering mean free path has a value[4] of 0.3 cm. in water. This difference is large enough to change the value of f significantly.

The question arises as to when to use the formula for $R \gg \lambda_{tr}$ and when to use the formula for $R \ll \lambda_{tr}$. It is desirable to know what restrictions must be placed on the relation of R to λ_{tr}. Experimental data would be useful to determine what the relation of R to λ_{tr} must be for the formulas to be valid.

In water, which has a transport mean free path of 0.433 ± 0.01 cm., it is difficult to know whether or not the formula for $R \gg \lambda_{tr}$ applies to foils of radius 1.0 cm.

Bothe did some experiments to verify his expressions for f. His method was based on choosing a medium which had a large mean free path so that the factor f would approach unity and the error of neglecting it would be vanishingly small. For detectors with a radius of 1.4 cm. or less, aluminum, for which the mean free path is about 10 cm., is good. By comparing the detection of neutrons with Dy_2O_3 detectors of radii 0.7 cm. and 1.4 cm. in aluminum with the detection in paraffin, Bothe found that his formulas were good. The detectors were made in three thicknesses to provide data on the dependence of f upon foil thickness as well as the dependence of f upon radii of the detector.

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[3] Ibid.

[4] Bothe, op. cit., p. 1.

Unfortunately, Bothe did not take data over a very wide range. Consequently, the formulas are not verified in some regions that are of interest.

Dacey, Paine, and Goodman[5] did an experiment using another method for determining f. Their measurements were made in a water medium only. They were able to determine f by first, activating a single foil and counting its activation; second, placing a similar foil on each side of the original foil and repeating the measurements, counting only the activation of the original foil; and third, placing two similar foils on each side of the original foil and repeating the measurements, again counting only the activation of the original foil.

It is clear that the neutron flux through the original foil is lowered by the presence of other foils. From this reduction of neutron flux, it is possible to find the value of f.

The theoretical formula by Bothe and the two experiments by Bothe and Dacey, Paine, and Goodman are the extent of the major determinations of depression factors. Since Bothe's experiments were performed in paraffin, it is difficult to apply his data to measurements made in water. This difficulty is

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[5] John E. Dacey, R. W. Paine, and C. Goodman, M.I.T. Lab. for Nuc. Sci. and Eng. Tech. Report No. 23, Oct. 20, 1949, p. 46.

due to the fact that transport mean free path for paraffin is not definite but varies from sample to sample because of variations in the chemical composition of the samples.

CHAPTER II

EXPERIMENTAL ARRANGEMENT AND PROCEDURE

The determination of the depression factor <u>f</u> was based upon a comparison of foil activations obtained in the medium under study with activations obtained under similar experimental conditions in a medium in which the factor <u>f</u> is only slightly different from unity. Since water is used extensively as a slowing-down material in neutron studies, the measurements to be described were carried out with water as the medium of interest. A large tank of water (2 ft. x 2 ft. x 4 ft.) was used as the medium.

There are few materials that have a transport mean free path of sufficient size to reduce the depression factor to unity. Aluminum is one such material, but was not available in sufficient quantity to make its use feasible. The next best substitute which was available was white sand (SiO_2), which has a transport mean free path of about 6.5 cm.[1]

For the activations in sand, the sand and the foils were inclosed in an 8-inch box constructed of thin sheet iron and water-proofed. The box in turn was submerged in the water tank for activation of the foils. The details of the construction are shown in Figure 1.

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[1] H. R. Kroeger, "Thermal Neutron Cross Sections," <u>Nucleonics</u>, Oct, 1949.

The distribution of thermal neutrons in water is a very critical function of the distance from the source. For this reason, some means of supporting the detecting foils in a definite position that could be reproduced with precision was necessary. The supporting device should have slowing-down properties similar to chose of the medium in order for the experiment to give a correct value for f. Hence, sheet aluminum was used for supporting the foils in the sand (Figure 5). After calculations were made on the properties of several plastics, Lucite, which has the chemical formula[2] $CH_2:C(CH_3)COOCH_3$, was found to have approximately the same transport mean free path as water. Details of the construction of the Lucite support used for the measurements in water are shown in Figure 6.

The detectors were made of a special grade of purified indium. The specifications of the foils are given in Table 1. The supply of indium was a little less than 100 mg/cm^2 thick. In order to have thinner foils, a set of rollers was made to roll the foil to the desired thickness. Thin foils were combined to obtain thicker ones. Since the accuracy of the experiment depended on the accuracy to which the foil area was known, a set of foil cutters was made to stamp out foils of precise radii. The foil thickness was determined by weighing on an analytical balance.

The neutrons were supplied by a 50 mg. radium-beryllium

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[2] Handbook of Physics and Chemistry, Chemical Rubber Publishing Company, Thirtieth Edition, 1279-1289.

TABLE 1

FOIL SPECIFICATIONS

Foil	Diameter (cm)	δ (mg/cm^2)	$\mu\delta$
E + F	2.0	190.47	0.1700
D	2.0	98.92	0.0876
K	2.0	45.19	0.0403
L	2.0	23.17	0.0207
A + B	2.5	192.82	0.1720
Z	2.5	98.16	0.0876
6	2.5	53.57	0.0477
8	2.5	29.91	0.0267
G + I	3.0	193.83	0.1730
H	3.0	98.42	0.0888
O	3.0	43.17	0.0385
P	3.0	26.81	0.0303

source inclosed in a monel metal cylinder. The source was held by an aluminum tube which was reamed so that the source position was well defined.

The Ra-Be source emits both groups of fast and slow neutrons. The initially fast neutrons will be slowed down by water and will eventually reach thermal energies. In the neighborhood of the foil, then, there will be neutrons of every energy from the highest emitted by the source down to and including those with thermal energies. An indium detector will respond primarily to neutrons having the indium resonance energy (1.44 electron volts) and to thermal neutrons. In this case we were interested in the density of thermal neutrons in the vicinity of the foil. In order to realize this, two sets of measurements were carried out. One set of exposures was made with bare indium foils as detectors (to give thermal plus resonance activation) and a second was made with the foil holder surrounded by a cadmium shield. Since 1 mm. of cadmium absorbs practically all thermal neutrons that pass through it, the foil activation in the second case corresponded to resonance activation alone. By subtraction we were able to obtain the activation due to the thermal neutrons. The cadmium shield used around the supports is shown in Figure 4. The absorption of the neutrons results in the production of In^{116}, which has two radioactive isotopes, one with a half-life of 13 seconds; the other, 54 minutes. The 13-second activity was eliminated by waiting at least 4 minutes before counting the foils.

The activation of the foils was measured by counting the beta particles emitted with a Tracerlab TGC-2 Geiger-Muller tube together with the accompanying Tracerlab "64 Scaler." The tube had a 1.94 mg/cm^2 mica window. A UX$_2$ standard source was used to check the sensitivity of the counting arrangement and to determine the drift from one time to another.

It was necessary that the backscattering be constant for a given thickness. To insure this, foil adptors were made for the counter set-up. The adaptors were identical with each other except for the radii of the recess that held the foils in position (Figure 3).

An exposure and counting schedule was adopted that would allow the Ra-Be neutron source to be kept in the water during counting periods. This was done so that the background counting rate would be as nearly constant as possible.

The activation of the foils was not the same on both sides. The average activation of the two sides is the activation that is proportional to _, the probability of absorption of a neutron on one crossing of the foil. A counting rate proportional to the average activation was taken in each case by turning the foil over several times during the course of counting. The schedule was to count one side of the foil for a period of five minutes, wait one minute during which time the foil was turned over, count five minutes, wait one minute during which time the foil was turned back over, etc. This procedure was followed until sufficient counts were made to keep the value of the statistical probable error 0.5 per cent. This was not quite true for the small thin foils because the counting rate was too low.

The foil adaptors were made so that they would just slide into the Tracerlab unsheilded holder. This arrangement permitted the foil to be in close proximity to the Geiger tube, which gave higher counting rates than could have been attained otherwise.

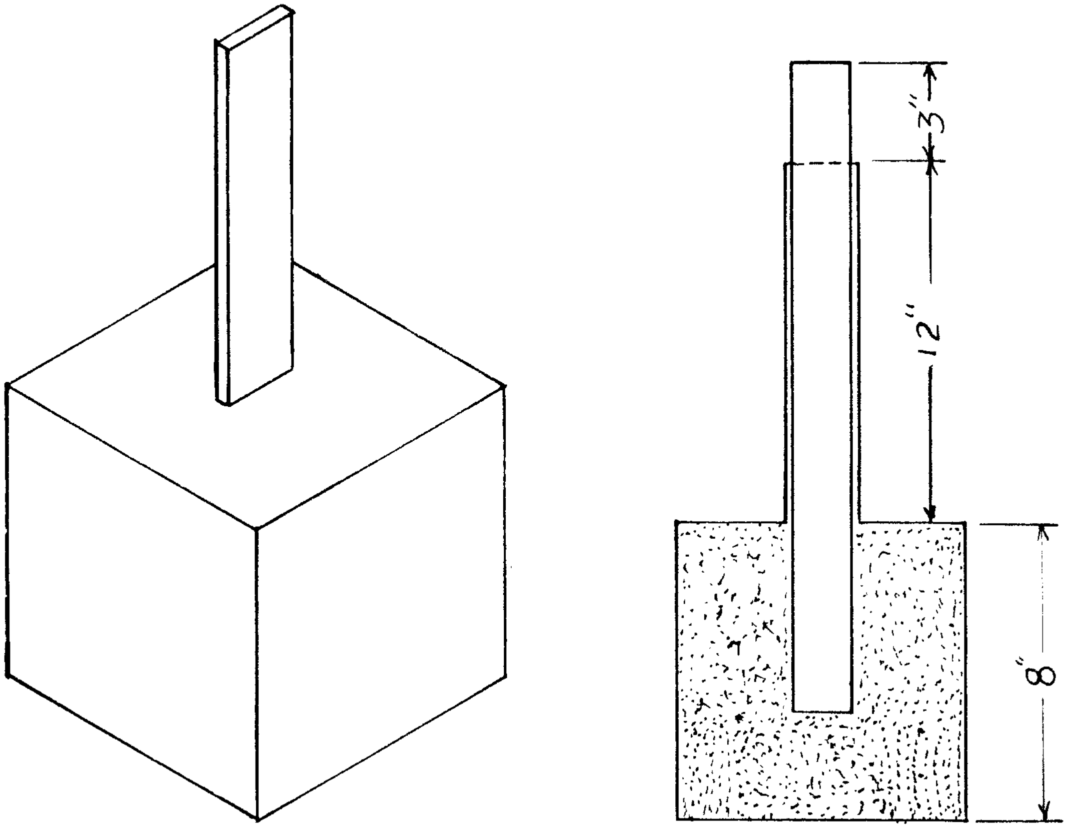

Fig. 1. Sand Box

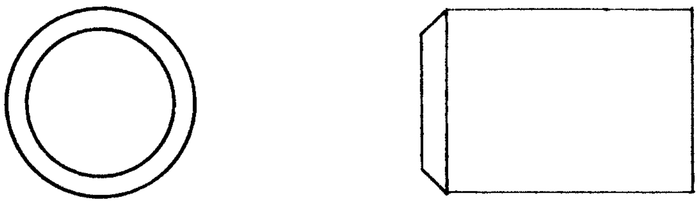

Fig. 2. Foil Cutter

Fig. 3. Foil adaptor

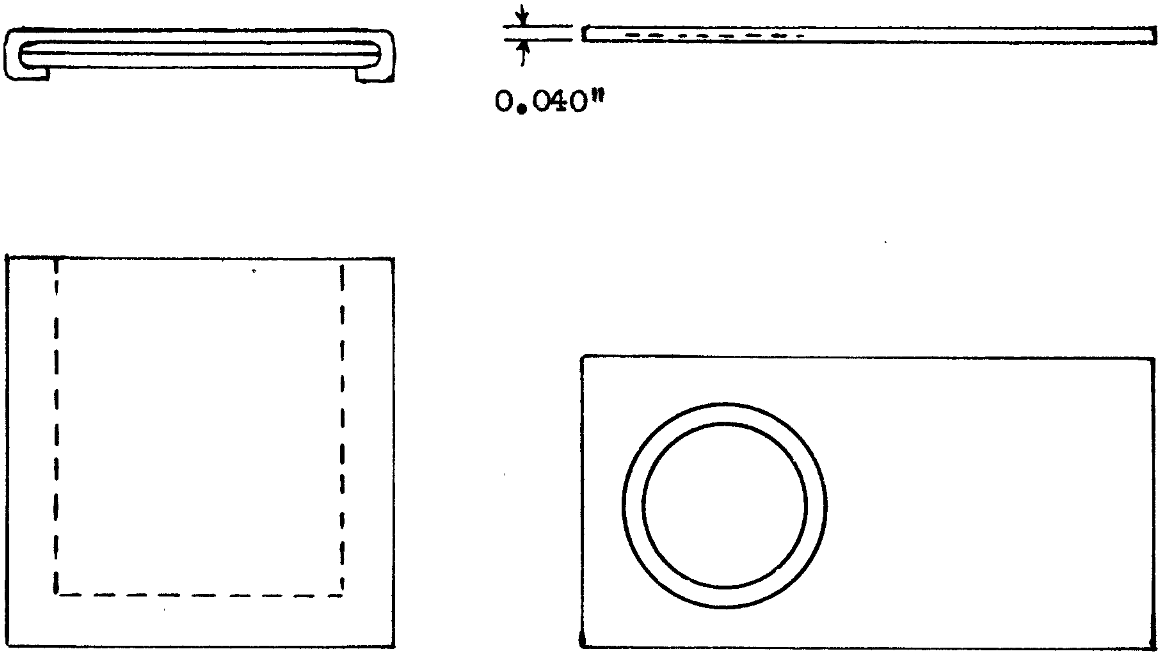

Fig. 4. Cadmium shield

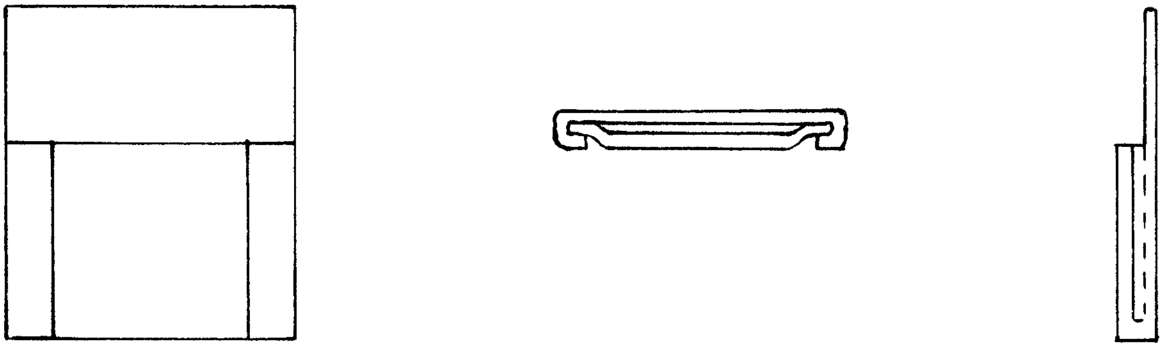

Fig. 5. Aluminum foil holder

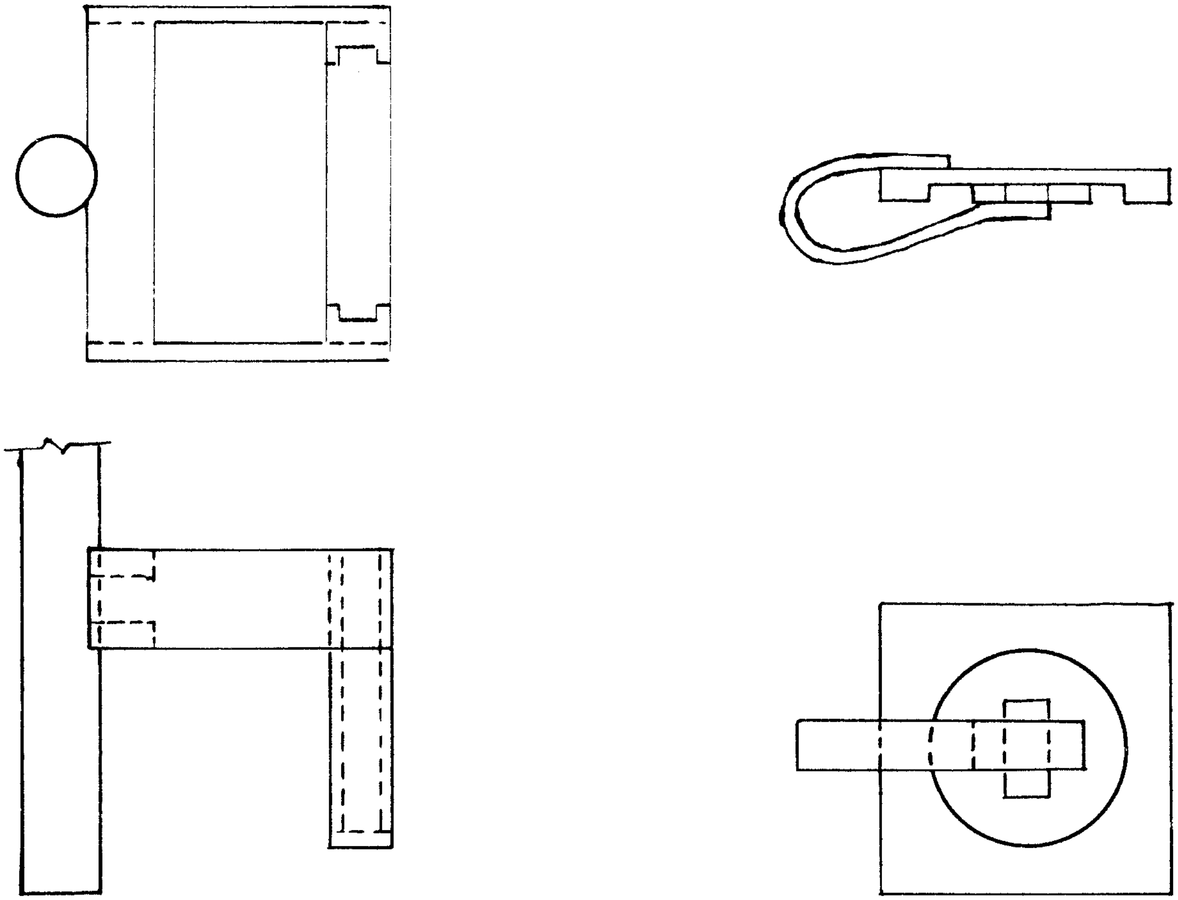

Fig. 6. Lucite foil holder

CHAPTER III

THEORY AND DATA

The data taken according to Chapter II had to be reduced to a form that would allow comparison of one measurement with another.

Comparison of data must be made through initial saturation counting rates. The initial saturation counting rate, C_o sat., can be calculated from raw data.

The initial counting rate C_o is given by

$$C_o = \frac{\text{Total Counts}}{\tau\left(e^{-\frac{t_1}{\tau}} - e^{-\frac{t_2}{\tau}}\right)}$$

and the initial saturation counting rate is

$$C_o \text{ sat} = \frac{C_o}{\left(1 - e^{-\frac{t}{\tau}}\right)}$$

or

$$C_o \text{ sat} = \frac{\text{Total Counts}}{\tau\left(1 - e^{-\frac{t_2}{\tau}}\right)\left(e^{-\frac{t_1}{\tau}} - e^{-\frac{t_2}{\tau}}\right)} ,$$

where is the mean life of the artificially activated In^{115}, t_1 is the time elapsed from the end of the activation to the beginning of the count, e is the base of the natural logarithms, and t_2 is the time elapsed from the end of the activation to the end of the count.

All the data taken was corrected to C_o sat. One other correction was made before the thermal plus resonance data was compared with the resonance data. The cadmium sheild in which the resonance data was taken had cadmium walls 0.040 inch thick. Cadmium has small cross-section for indium resonance neutrons but it captures enough of the resonance neutron that a correction must be made. The value of the correction for this effect was taken from Figure 7.

C_o sat for thermal neutrons was obtained by subtracting the corrected C_o sat for thermal plus resonance neutrons. The resulting data is compiled in Table 2.

The data in Table 2 had to be plotted before it could be interpreted. Figure 8 is the plot of the data in Table 2. The data could not be plotted until it was normalized. The neutron flux did not have the same value for all activations, the efficiency of the counter was different for different radii of foils, and the efficiency of the counter was different for different thicknesses of foils. All these things had to be corrected for so that the counting rates could be compared directly. The process of correcting the data in this respect is called normalization. If CR is the counting rate; E(r), the radius dependent efficiency of the counter; E(t), the thickness dependent efficienty; F_s, the neutron flux

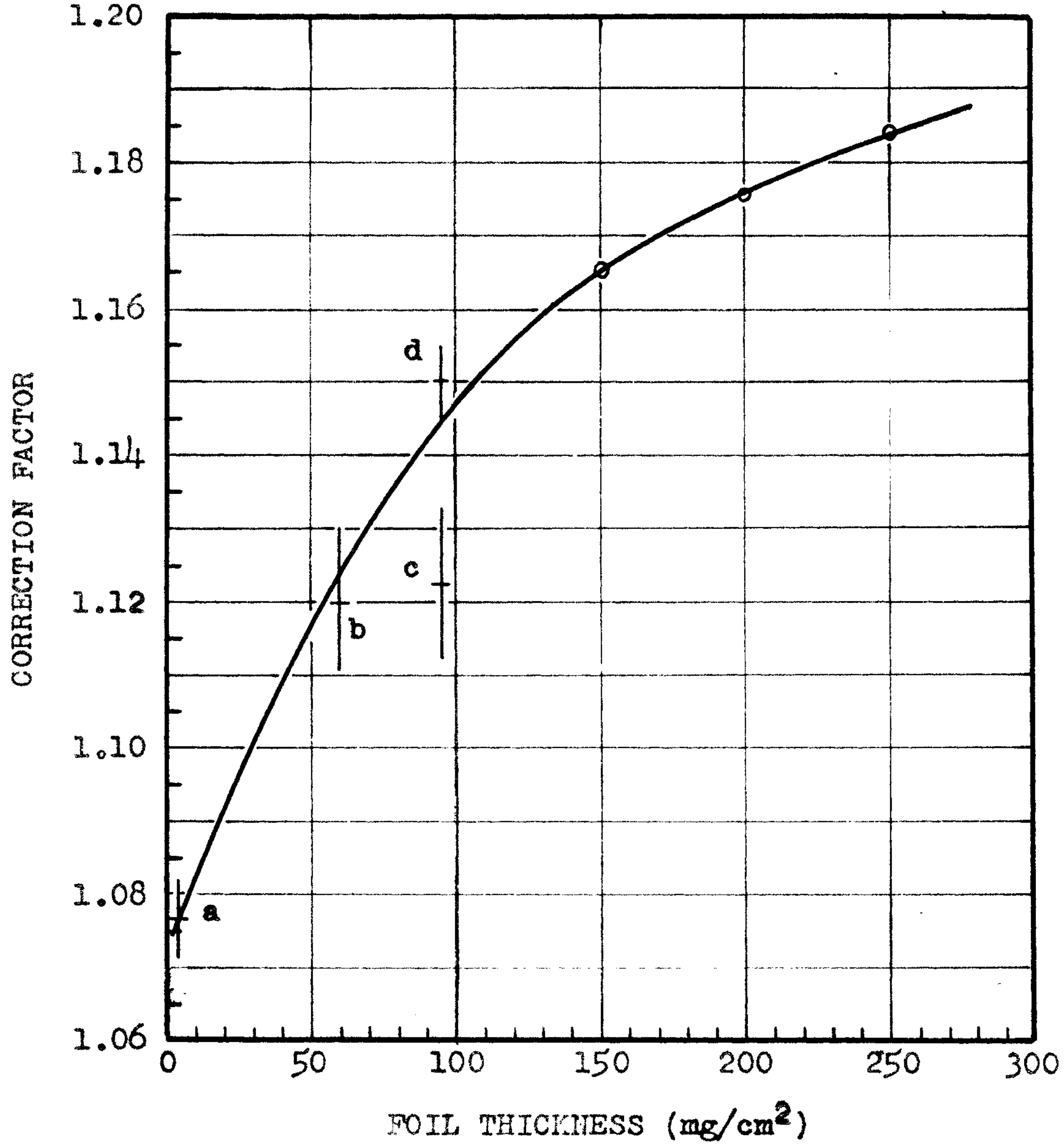

Fig. 7--Correction factor for absorption of indium resonance neutrons in 0.04 inch cadmium.

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a. Data for this point obtained from: R. L. Walker, MDDC-414, 43.

b. Data for this point obtained from: J. E. Dacey, R. W. Paine, and C. Goodman, MIT Lab. Nuc. Sci. Eng. Tech. Report No. 23, 46 (Oct., 1949).

c. Data for this point obtained from: J. H. Rush, Phys. Rev., 73, 271 (1948).

d. Data for this point obtained from: J. Kunstadter, Phys. Rev., 78, 484 (1950).

TABLE 2

INITIAL SATURATION COUNTING RATE CORRECTED FOR RESONANCE ACTIVATION AND BACKGROUND

Foil	C_o sat (in sand)	C_o sat (in water)
G + I	1736	6494
H	1647	6962
O	1224	5952
P	968	4746
A + B	1385	5339
Z	1316	5800
6	1097	5080
8	756	3832
E + F	909	3757
D	837	4227
K	660	3304
L	441	2294

in the sand box; F_w, the neutron flux in the water; α, average probability of absorption of the neutron on one crossing of the foil, and subscript positions on CR and f represent radius, foil thickness, and medium, respectively, the normalization factor for the sand medium can be obtained by the following equations:

$$CR_{11s} = E(r_1)\, E(t_1)\, F_s \alpha(t_1) \,,$$

$$CR_{21s} = E(r_2)\, E(t_1)\, F_s \alpha(t_1)$$

and

$$\frac{CR_{11s}}{CR_{21s}} = \frac{E(r_1)}{E(r_2)} \,. \qquad (1)$$

Similarly,

$$\frac{CR_{11s}}{CR_{31s}} = \frac{E(r_1)}{E(r_3)} \,.$$

Equation (1) shows that for a given foil thickness the counting rate is a function of $E(r)$ only. Then a normalization factor $E(r_1)/E(r_2)$ may be applied at a constanf foil thickness. The argument may be extended to other thicknesses by noting that the same $E(r)$ applies at any constant thickness since the ratio of the radii of the foils is constant. The data taken in sand was normalized by the above formulas. The resulting curves show good agreement (Figure 8).

For a water and sand medium

$$CR_{11w} = \frac{1}{f_{11}} E(r_1)\, E(t_1)\, F_w \alpha(t_1)$$

$$CR_{11s} = E(r_1)\, E(t_1)\, F_s \alpha(t_1)$$

and

$$CR_{11w} = CR_{11s} \frac{F_w}{F_s} f_{11}. \qquad (2)$$

Inspection of equation (2) shows that the counting rate in water is proportional to the counting rate in sand, since F_w, F_s, and f_{11} are constants. If the counting rate in water is multiplied by the ratio of the neutron flux in water to the neutron flux in sand, the measurements in sand and the measurements in water will be normalized at $\underline{f} = 1$. Since data for a point at which $F = 1$ could not be taken, the measurements made in water could not be normalized by a direct mathematical method.

If the ratio of the neutron fluxes in the two media had been known from some other experiment, the problem would have been solved since any detecting foil would depress the neutron flux in water near the foil.

The measurements taken in water were normalized by carefully choosing a value for the ratio of the neutron flux. After the counting rates were plotted against foil thickness, the value of the ratio of the neutron fluxes in the two media was chosen so that the curves drawn through points of constant radii converged smoothly to zero counting rate at zero foil thickness.

The normalization constants used to normalize the data are as follows:

For measurements taken in sand medium:

$\frac{1647}{837}$ x counting rate (2 cm. foils)

$\frac{1647}{1316}$ x counting rate (2.5 cm. foils)

$\frac{1647}{1647}$ x counting rate (3.0 cm. foils)

For measurements taken in water medium:

$\frac{1647}{837}$ x counting rate (2 cm. foils)

$\frac{1647}{1316}$ x counting rate (2.5 cm. foils)

$\frac{1647}{1647}$ x counting rate (3.0 cm. foils)

From equation (2) the depression factor f_{11} is the ratio of CR_{11w} to CR_{11s} after CR_{11w} and CR_{11s} have been normalized. The depression factors (f experimental) found in Table 3 were obtained by this method.

The values of f given by the theoretical formula listed in Chapter I for which $R \gg \lambda_{Tr}$ may be compared with the experimental f. To calculate f it is necessary to calculate α. Bothe's[1] expression for α is

$$\alpha = 1-(1-E^{-\mu\delta})e^{-\mu\delta} + \mu^2\delta^2 \,Ei(-\mu\delta),$$

$$\mu = N\sigma_t ,$$

where δ is the foil thickness, N is Avagadro's number, σ_t is the In^{115} cross-section for thermal neutrons, and $Ei(-\mu\delta)$ is the exponential integral function of $-\mu\delta$. The value of

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[1] Bothe, op. cit., p. 1.

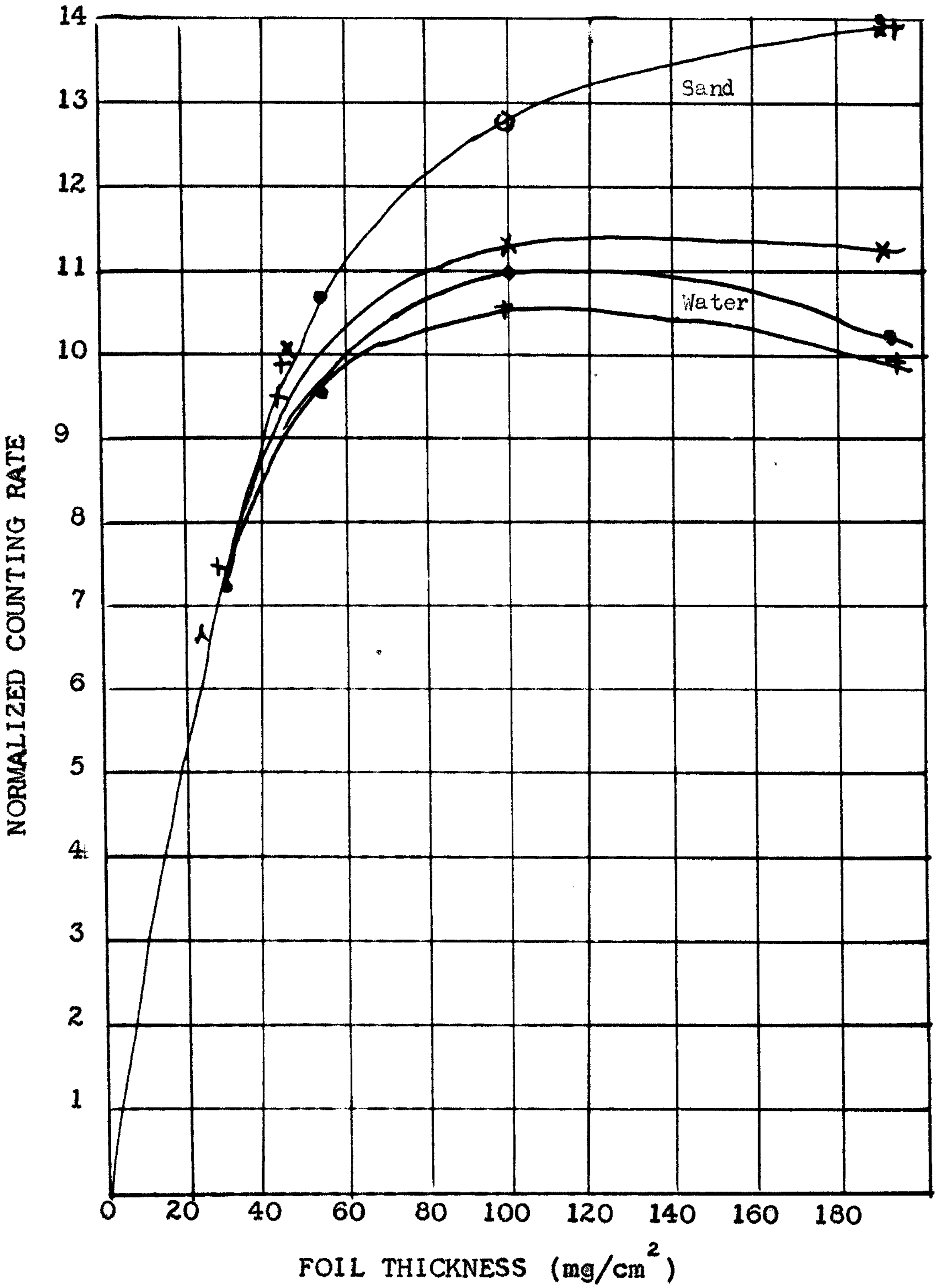

Fig. 8--Normalized counting rate vs foil thickness.

TABLE 3

THE CALCULATED AND EXPERIMENTAL DEPRESSION FACTOR AND THE PROBABILITY OF ABSORPTION

Foil		f (calculated)	f (experimental)
L	0.0386	0.985	
K	0.0847	0.965	0.970
D	0.150	0.939	0.886
E + F	0.260	0.898	0.815
8	0.0511	0.970	
6	0.0873	0.970	0.885
Z	0.150	0.915	0.863
A + B	0.261	0.870	0.747
P	0.0563	0.957	
O	0.0710	0.948	0.900
H	0.151	0.896	0.814
G + I	0.262	0.837	0.725

(170 barns) was taken from Goodman's[2] value of 190 barns for 0.025 ev neutrons. The 170 barns for thermal (0.032 ev) neutrons was found by assuming the 1/v law and multiplying the 190 barns by 0.025/0.032.

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[2] Clark Goodman, The Science and Engineering of Nuclear Power, p. 458.

CHAPTER IV

DISCUSSION OF RESULTS

The depression factors found by experiment indicate about 45 per cent greater depression than do the depression factors found by the Bothe formula. This is not readily explained. The value picked for the ratio of the neutron fluxes determines whether the depression factor is large or small. The value chosen was the one that seemed to make the data consistent in the region where the depression factor was close to unity.

The data taken in the sand shows that the sand has a transport mean free path which is large enough compared with the foil radii for <u>f</u> to aproach unity. The sand is then a good medium for this purpose.

There is one other thing that could explain the difference between the measurements and the theory. Bothe's formula could be in error. However, this is not thought to be the explanation.

For highly accurate determinations of effects as small as the Bothe factor, a better counting arrangement is necessary. The counter drift was about two per cent during the experiment.

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