VERTEX FUNCTIONS IN K-MESON-

NUCLEON SCATTERING

APPROVED:

Major Professor

Minor Professor

Director of the Department of Physics

Dean of the Graduate School

VERTEX FUNCTIONS IN K-MESON-
NUCLEON SCATTERING

THESIS

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By

Hsu Hsiung Kang, B. S.
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CHAPTER I

INTRODUCTION

The purpose of this study was to investigate some theoretical approaches to the scattering of positive k-mesons by nucleons in an attempt to explain the experimental data. There are several methods for making field theoretical calculations such as the weak-coupling approximation, dispersion relations, expansion in inverse coupling constants, and the method of functional averaging. In this work the problem has been investigated by the technique of the weak coupling approximation. In order to carry out such a study one uses Feynman's viewpoint, that the proper or most useful approach is to try to calculate the probability amplitudes directly from some space-time diagrams rather than to start from some Schroedinger-like differential equation and try to solve it by some approximate technique. With this viewpoint one makes certain basic assumptions about the type of interaction involved and then proceeds to calculate physically meaningful quantities which describe the processes.

In the following, two approaches have been taken. In one it has been assumed that scattering takes place through a "direct interaction" analogous to Compton scattering of photons by electrons. For this approach only elastic scattering of positive k-mesons by neutrons is considered. The possible Feynman diagrams, the associated matrix elements, and the scattering cross-sections are the subject of chapters II and III. In the second approach the scattering process is assumed to occur by an "exchange interaction", analogous to Yukawa's original hypothesis of an exchange meson. In this treatment all three of the following processes have been considered:

$$k^+ + p \longrightarrow k^+ + p$$
$$k^+ + n \longrightarrow k^+ + n$$
$$k^+ + n \longrightarrow k^o + p$$

An isotopic spin analysis relates the probability amplitudes of these three processes and in this treatment this idea has been used to calculate one of these from the other two. This analysis and its results are the subject of chapters IV and V. An appendix includes a computer program for several useful relativistic kinematic expressions and the quantities evaluated in this thesis.

The S-matrix element, the probablity amplitude, for the process can be written from the rules for Feynman graphs. The amplitude corresponding to a particular graph is built up by associating factors with the elements of the graph. Those factors independent of specific details of the interaction are

1. For an incoming fermion line in the graph a factor, $U(p,s)$, the spinor wave function.

2. For an outgoing fermion line a factor, $\overline{U}(p,s)$, the

adjoint spinor wave function.

3. For each intermediate fermion line with momentum q a factor, $1/(\not{q}-M)$, the propagator, where M is the mass of the intermediate state particle.

4. For each intermediate meson line of spin zero with momentum q a factor, $1/(q^2-M^2)$, the propagator.

5. For each direct interaction vertex a coupling factor:

a. g for scalar interactions.

b. $\mp(\frac{g}{\mu})\not{k}$ for vector interactions.

c. $g\gamma_5$ for pseudoscalar interactions.

d. $\mp(\frac{g}{\mu})\gamma_5\not{k}$ for pseudovector interactions.

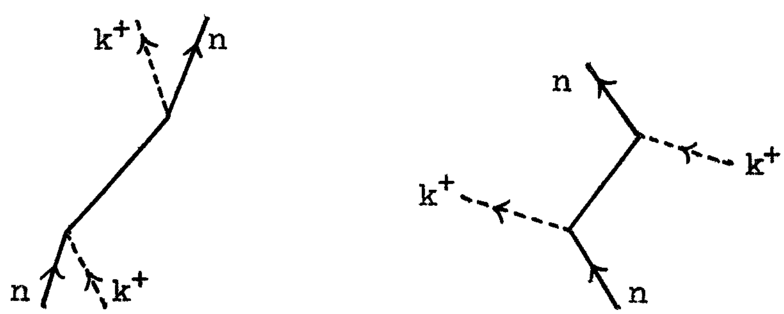
6. For each strongly interacting vertex some functions (in general unknown) of the scattering parameters.

Part of the effort in the second approach in this work was in trying different vertex functions to obtain a good qualitative description of the scattering.

CHAPTER II

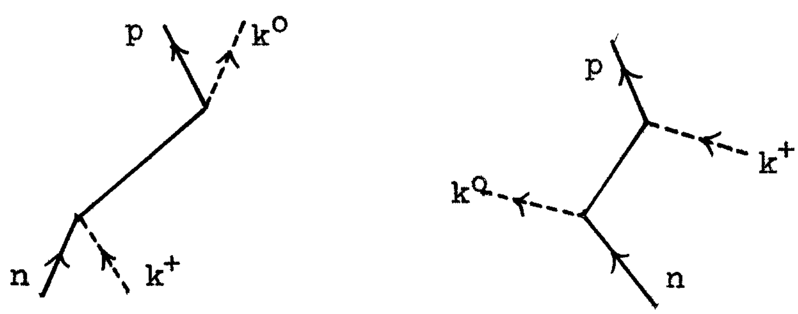
DIRECT INTERACTION

In this section the mechanism of scattering between the positive k-meson and neutron is considered to be a direct interaction. It is analogous to Compton scattering in quantum electrodynamics. The differential scattering cross-section is calculated using the familiar field theoretic methods of the S-matrix. The possible Feynman diagrams for the processes are the following:



(a) (b)

for non-charge-exchange scattering



(c) (d)

and for charge-exchange scattering.

Fig. 1

The charge, z-component of isotopic spin (I_z), and strangeness (S) conservation laws will yield the properties of the intermediate-state particle for each type of interaction.

First consider the diagram of Fig.1(a); the conservation laws require that the sum of the strangeness numbers of both incoming particles give the strangeness of the intermediate particle, so $S=1+0=1$. Likewise the z-component of isotopic spin, I_z, for the intermediate particle is $I_z=\frac{1}{2}-\frac{1}{2}=0$. Therefore a positively charged particle with S=1 and $I_z=0$ is indicated. As is known, there is no such elementary particle.

A similar analysis of Fig. 1(b) requires a negatively charged intermediate particle with $I_z=-1$ and $S=-1$. Of the known elementary particles, only the negative sigma hyperon, Σ^-, has these properties. Thus the non-charge-exchange reaction is assumed to have a Σ^- as the intermediate-state particle.

Fig. 1(c) requires a positive intermediate particle with $I_z=0$ and $S=-1$. As before, no such particle is known, so this diagram cannot represent charge-exchange scattering.

Fig. 1(d) requires a neutral intermediate particle with $I_z=0$ and $S=-1$. Of the known elementary particles, there are two which fulfill these requirements: the Σ^0 and Λ^0 hyperons.

An interesting consequence of this interaction is that two intermediate particle states contribute to the charge-exchange interaction while only one intermediate particle state is involved in the non-charge-exchange interaction.

The differential cross-section in the center-of-mass system for these two cases may now be calculated and the values will be compared with the experimental values. Fig. 1(b) represents the following process: a real neutron of four-momentum p_1 and spin s emits a real k^+-meson of momentum k_2, and then propagates as a virtual intermediate fermion until it absorbs an incoming k^+-meson, of momentum k_1, to become a real neutron of momentum p_2 and spin one-half. For charge-exchange scattering Fig. 1(d) represents the initial neutron emitting a k^o-meson, with the uncharged virtual intermediate particle later absorbing a k^+-meson and going off as a proton of momentum p_2 with spin one-half.

The k-meson is assumed to be a pseudoscalar particle, which means that the wave function representing the k-meson is a pseudoscalar under Lorentz transformation, i.e. $\psi_k \longrightarrow \psi_k$ for proper Lorentz transformation, but $\psi_k \longrightarrow -\psi_k$ for improper Lorentz transformation. There are two possible types of coupling, either by pseudoscalar or by pseudovector interaction. Both will be considered in this work.

Considering non-charge-exchange scattering first, the pseudoscalar coupling matrix element is

$$
\begin{aligned}
\mathfrak{M} &= \bar{U}_2(g\gamma_5)(\not{q}-M)^{-1}(g\gamma_5)U_1 \\
&= g^2\bar{U}_2\gamma_5(\not{p}_1-\not{k}_2-M)^{-1}\gamma_5 U_1 \\
&= g^2\frac{\bar{u}_2\gamma_5(\not{p}_1-\not{k}_2+M)\gamma_5 u_1}{(\not{p}_1-\not{k}_2-M)(\not{p}_1-\not{k}_2+M)} \\
&= g^2\frac{\bar{u}_2\gamma_5(\not{p}_1-\not{k}_2+M)\gamma_5 u_1}{\sum_{\mu\nu}(p_1-k_2)(p_1-k_2)\gamma_\mu\gamma_\nu-M^2}
\end{aligned}
$$

$$= g^2 \frac{\overline{U}_2 \gamma^2_5(-\not p_1+\not k_2+M)\ U_1}{\sum_\mu [p_{1\mu}^2+k_{2\mu}^2-2(p_{1\mu}k_{2\mu})] - M^2}$$

$$= g^2 \frac{\overline{U}_2 \gamma^2_5(-\not p_1+\not k_2+M)\ U_1}{m^2+\ {}^2-2(p_1k_2)-M^2}$$

$$=-g^2 \frac{\overline{U}_2 \gamma^2_5(-\not p_1+\not k_2+M)\ U_1}{2(p_1k_2)+(M^2-m^2-\mu^2)}$$

$$= g^2 \frac{\overline{U}_2(-\not p_1-\not k_2+M)U_1}{2(p_1k_2)+C^2},$$

where $C^2=M^2-m^2-\mu^2$

$\gamma_5^2=-1.$

The Dirac equation can be written,

$m\psi=(\gamma_o E-\vec{p}\cdot\vec{\gamma})$

$\not p\psi=m\psi.$

Therefore,

$$=g^2 \frac{\overline{U}_2(-m+\not k_2+M)U_1}{2(p_1k_2)\ +C^2}$$

$$=g^2 \frac{\overline{U}_2(\not k_2+\alpha)U_1}{2(p_1k_2)\ +C^2},$$

with $\alpha = M - m.$

Now from the conservation of four-momenta:

$$\not p_1+\not k_1 = \not p_2+\not k_2$$

$$\overline{U}_2(\not p_1+\not k_1)U_1 = \overline{U}_2(\not p_2+\not k_2)U_1$$

$$\overline{U}_2(m+\not k_1)U_1 = \overline{U}_2(m+\not k_2)U_1$$

$$\overline{U}_2\not k_1 U_1 = \overline{U}_2\not k_2 U_1 \qquad (1)$$

so that $\mathcal{M}$ becomes

$$\mathcal{M} = \frac{g^2 \bar{u}_2 (\not{k}_1 + \alpha) u_1}{2(p_1 k_2) + C^2} .$$

The complex-conjugate matrix element is

$$\mathcal{M}^* = \frac{g^2 u_1^* (\not{k}_1^* + \alpha) \bar{u}_2^*}{2(p_1 k_2) + C^2} .$$

Since $\bar{U} = U^* \beta$, $\bar{U}\beta = U^*$, $\bar{U}^* = \beta U^* = \beta U$

$$k_1 = (\gamma_0 \omega_k - \vec{\gamma} \cdot \vec{k}_1)^* = \gamma_0 \omega_k + \vec{\gamma} \cdot \vec{k}_1 ,$$

$$\mathcal{M}^* = g^2 \frac{\bar{u}_1 \gamma_0 (\gamma_0 \omega_k + \vec{\gamma} \cdot \vec{k}_1 + \alpha) \gamma_0 u_2}{2(p_1 k_2) + C^2}$$

$$= g^2 \frac{\bar{u}_1 (\gamma_0^3 \omega_k - \gamma_0^2 \vec{\gamma} \cdot \vec{k} + \alpha) u_2}{2(p_1 k_2) + C^2}$$

$$= g^2 \frac{\bar{u}_1 (\not{k}_1 + \alpha) u_2}{2(p_1 k_2) + C^2} ,$$

and $|\mathcal{M}|^2 = \mathcal{M}^* \mathcal{M} = g^4 [2(p_1 k_2) + C^2][\bar{U}_1 (\not{k}_1 + \alpha) U_2][\bar{U}_2 \not{k}_1 + \alpha) U_1]$.

The average over initial and sum over final spins is

$$\frac{1}{2} \sum_{r,s=1}^{2} |\mathcal{M}|^2 = \frac{1}{2} g^4 [2(p_1 k_2) + C^2]^{-2} [\bar{U}_1^{(r)} (\not{k}_1 + \alpha) U_2^{(s)}][\bar{U}_2^{(s)} (\not{k}_1 + \alpha) U_1^{(r)}]$$

The projection operator is (1, p. 33)

$$\sum_{r,s=1}^{2} U_2^{(s)} \bar{U}_2^{(s)} = \overset{+}{\Lambda}(\vec{p}_2) = \frac{\not{p}_2 + m}{2m} .$$

Further, the sum over r may be extended from 1 to 4 provided the projection operator is inserted, i.e.

$$\frac{1}{2} \sum_{r,s=1}^{2} |\mathcal{M}|^2 = \frac{1}{2} g^4 [2(p_1 k_2) + C^2]^{-2} \sum_{r=1}^{4} \bar{U}_1^{(r)} (\not{k}_1 + \alpha) \overset{+}{\Lambda}(\vec{p}_2) (\not{k}_1 + \alpha) \overset{+}{\Lambda}(\vec{p}_1) U_1^{(r)}$$

$$\frac{1}{2}\sum_{r,s=1}^{2}|\mathcal{M}|^2=\frac{g^4}{8m}\left[2(p_1k_2)+C^2\right]^{-2}\sum_{r=1}^{4}\bar{U}_1^{(r)}(\not{k}_1+\alpha)(\not{p}_2+m)(\not{k}_1+\alpha)(\not{p}_1+m)U_1^{(r)}$$
$$=\frac{g^4}{8m}\left[2(p_1k_2)+C^2\right]^{-2}\ \mathrm{Tr}\ (\not{k}_1+\alpha)(\not{p}_2+m)(\not{k}_1+\alpha)(\not{p}_1+m)$$

where the fact $\sum_{r=1}^{4}\bar{U}^{r}\,Q\,U^{r}=\mathrm{Tr}\,Q$ has been used (2, p. 195).

To evaluate the trace:

$$\mathrm{Tr}(\not{k}_1+\alpha)(\not{p}_2+m)(\not{k}_1+\alpha)(\not{p}_1+m)$$
$$=\mathrm{Tr}\ \not{k}_1\not{p}_2\not{k}_1\not{p}_1+m^2\not{k}_1\not{k}_1+\not{k}_1\not{p}_2\alpha m+\not{k}_1\not{p}_2\alpha m+\not{p}_2\not{k}_1\alpha m+\not{p}_2\not{p}_1\alpha^2+\not{k}_1\not{p}_1 m+\alpha^2m^2$$

one uses the following results of the trace theorems:

$$\mathrm{Tr}(\not{k}_1\not{p}_2\not{k}_1\not{p}_1)=4\ (k_1p_2)(k_1p_1)-(k_1k_1)(p_2p_1)+(k_1p_1)(p_2k_1)$$
$$\mathrm{Tr}(m^2\not{k}_1\not{k}_1)=4m^2(k_1k_1)$$
$$\mathrm{Tr}(\not{k}_1\not{p}_2\alpha m)=4\alpha m(k_1p_2)$$
$$\mathrm{Tr}(\not{k}_1\not{p}_1 m\alpha)=4\alpha m(k_1p_1)$$
$$\mathrm{Tr}(\not{p}_2\not{k}_1\alpha m)=4\alpha m(p_2k_1)$$
$$\mathrm{Tr}(\not{p}_2\not{p}_1\alpha^2)=4\alpha^2(p_2p_1)$$
$$\mathrm{Tr}(\not{k}_1\not{p}_1\alpha m)=4\alpha m(k_1p_1)$$
$$\mathrm{Tr}(\alpha^2\ m^2)=4\ \ m^2\alpha^2$$

The only problem left is to evaluate the above traces. However, it is necessary to examine first some properties of particle scattering in the center-of-mass coordinate system, which is the one chosen for this work. In this system, the scattering particles have equal and opposite momenta making an angle θ with the initial momenta. Consider Fig. 1 where p_1, k_1, p_2 and k_2 represent the four-momenta of the incident and scattered particles, respectively. In this system before collision:

$$p_1=(E_p,\ \vec{p}_1)=(\ p_1^2+m^2\ ^{\frac{1}{2}},-\vec{p})=(\ [(-p)^2+m^2]^{\frac{1}{2}},-\bar{p})=([\vec{p}^2+m^2]^{\frac{1}{2}},\ -\vec{p})$$

$$k_1=(\ \omega,\ \vec{k}_1)=([\vec{k}_1^2+m^2]^{\frac{1}{2}},\ \vec{p})=([\vec{p}^2+m^2]^{\frac{1}{2}},\ \vec{p})\quad ;$$

after collision:

$$p_2=(E_p',\ \vec{p}_2)=([\vec{p}_2^2+m^2]^{\frac{1}{2}},\ -\vec{p}\,')=(\ [(-\vec{p}\,')+m^2]^{\frac{1}{2}},-\vec{p}\,')=([\vec{p}\,'^2+m^2]^{\frac{1}{2}},-\vec{p}\,')$$

$$k_2=(\omega',\ \vec{k}\ )=([\vec{k}_2^2+\mu^2]^{\frac{1}{2}},\quad \vec{p}\,')=([\vec{p}\,'^2+\mu^2]^{\frac{1}{2}},\ \vec{p}\,').$$

Since the conservation-of-momentum law holds, one obtains

$$p_1 + k_1 = p_2 + k_2$$

$$p_1 - k_2 = p_2 - k_1$$

$$p_1^2+k_2^2-2(p_1k_2)= p_2^2+k_1^2-2(p_2k_1)$$

$$m^2+\mu^2-2(p_1k_2)= m^2+\mu^2-2(p_2k_1)$$

$$(p_1k_2)=(p_2k_1)$$

Assuming that the collision between the k^+-meson and neutron is elastic, then $\vec{p} = \vec{p}\,'$ in the center-of-mass system.

Since the conservation-of-energy law holds,

$$E_p + \omega = E_p' + \omega'$$

$$\omega'=(\ \vec{p}\,'^2-m^2)=(\ \vec{p}^2-m^2)=\omega$$

$$\omega'=\omega$$

$$E_p'= E_p$$

so that

$$(k_1p_2)=(p_1k_2)=E_p'\omega-\vec{p}_2\vec{k}_1=E_p'\omega+|\vec{p}\,'|.|\vec{p}|\cos\ \theta =E_p\omega+|\vec{p}|^2\ \cos\ \theta$$

$$(k_1p_1)=\omega E_p-\vec{k}_1\cdot\vec{p}_1= E_p\omega+\vec{p}\cdot\vec{p}= E_p\omega+|\vec{p}|^2$$

$$(k_1k_1)=\omega^2\ -\vec{k}_1\cdot\vec{k}_1=\omega^2-\vec{p}^2=\mu^2$$

$$(p_2p_1)= E_pE_p'-\vec{p}_2\cdot\vec{p}_1= E_pE_p'-|\vec{p}||\vec{p}|\cos\ \theta\ =\ E_p^2-|\vec{p}|^2\ \cos\ \theta$$

$$E_p=(\vec{p}^2+m^2)^{\frac{1}{2}}=(\omega^2-\mu^2+m^2)^{\frac{1}{2}}.$$

Evaluating $|\mathcal{M}|^2$, the square of the invariant amplitude, in the center-of-mass system one has

$$\frac{1}{2}\sum_{r,s=1}^{2}|\mathcal{M}|^2 = \frac{g^4}{2m}\left[2(p_1k_2)+C^2\right]^{-2}\Big[(k_1p_2)(k_1p_1)-(k_1k_1)(p_2p_1)+(k_1p_1)(p_2k_1)+ m^2(k_1k_1)+\alpha m(k_1p_2)+\alpha m(p_2k_1)+\alpha^2(p_1p_2)+\alpha m(k_1p_1)+\alpha^2m^2\Big]$$

$$=\frac{g^4}{2m}\left[2(E_p\omega+|\vec{p}|\cos\theta)+C^2\right]^{-2}\Big[2(E_p\omega+|\vec{p}|^2\cos\theta)(E_p\omega+|\vec{p}|^2)-\mu^2(E_p-|\vec{p}|^2\cos\theta) +m\mu^2+2\alpha m(E_p\omega+|\vec{p}|^2\cos\theta)+2\alpha m(E_p+|\vec{p}|^2)+\alpha^2(E_p^2-|\vec{p}|^2\cos\theta)+\alpha^2m^2\Big]$$

$$=\frac{g^4}{2m}\left[2(E_p\omega+|\vec{p}|^2\cos\theta)+C^2\right]^{-2}\Big\{(2\omega^2-\mu^2+\alpha^2)E_p^2+\left[4\alpha m\omega+2\omega(1+\cos\theta)|\vec{p}|^2\right]E_p +(2\cos\theta|\vec{p}|^2+\mu^2\cos\theta+2\alpha m\cos\theta-2\alpha m-\alpha^2\cos\theta)|\vec{p}|^2+m^2\mu^2+\alpha^2m^2\Big\}$$

$$=\frac{g^4}{2m}\Big\{2\left[\omega(\omega^2-\mu^2+m^2)^{\frac{1}{2}}+(\omega^2-\mu^2)\cos\theta\right]+C^2\Big\}^{-2}\cdot (2\omega^2-\mu^2+\alpha^2)(\omega^2-\mu^2+m^2)+\left[4\alpha m+2\omega(\omega^2-\mu^2)(1+\cos\theta)\right](\omega^2-\mu^2+m^2)^{\frac{1}{2}} +m^2\mu^2+\alpha^2m^2+\left[(2\omega^2-\mu^2+2\alpha m-\alpha^2)\cos\theta+2\alpha m\right](\omega^2-\mu^2)\Big\}.$$

The differential cross-section is given by (1, p. 285)

$$d\sigma=\frac{2\pi\rho_F}{J_{inc}}\left\{\frac{1}{2}\sum_{r,s=1}^{2}|\mathcal{M}|^2\cdot\frac{m}{E_p}\cdot\frac{m}{E_p'}\cdot\frac{2\pi}{\omega}\cdot\frac{2\pi}{\omega'}\right\}, \qquad (2)$$

where

ρ_F = the density of final states per unit energy range

J_{inc} = the density of incident particles times the velocity of incident particles.

The four terms at the end come from the normalization chosen. With this normalization the density of incident particles is one, and the velocity cna be obtained from the following:

$$|\vec{k}_1| = \frac{\mu v_1}{\sqrt{1-v_1^2}}$$

$$|\vec{k}_1|^2-|\vec{k}_1|^2v_1^2=\mu^2\ v_1^2$$

$$v_1^2 = \frac{|\vec{k}_1|^2}{|\vec{k}_1|^2 + \mu^2}$$

$$|\vec{v}_1| = \frac{|\vec{k}_1|}{\omega} = \frac{|\vec{k}_1|}{\sqrt{k_1^2 + \mu^2}} \quad ;$$

therefore,

$$|\vec{v}_2| = \frac{|\vec{p}_1|}{E_p} \quad .$$

The density of particles is unity; thus the velocity is numerically equal to the current, J_{inc}, or for the center-of-mass system,

$$\text{current} = |\vec{v}_1 - \vec{v}_2| = |\vec{v}_1| - |\vec{v}_2| = \frac{\vec{k}_1}{\omega} + \frac{|\vec{p}_1|}{E_p}$$

$$= \bar{p}\,\left(\frac{1}{\omega} + \frac{1}{E_p}\right) .$$

The density of states per unit energy can be obtained from the number of states being equal to

$$_F dE_F = (2\pi)^{-3} |\vec{k}_2|^2 d|\vec{k}_2|\, d\Omega$$

$$= (2\pi)^{-3} |\vec{k}_2|^2 \frac{\partial |\vec{k}_2|}{\partial E_F}\, d\Omega \quad ,$$

and $\frac{\partial |\vec{k}_2|}{\partial E_F}$ can be evaluated as follows:

$$E_F = \omega + E_p = \omega' + E_p'$$

$$= \omega' + (\vec{p}_2^{\,2} + m^2)^{\frac{1}{2}}$$

$$= \omega' + (\bar{k}_2^2 + m^2)^{\frac{1}{2}}$$

Since $\vec{p}_1 + \vec{k}_1 = \vec{p}_2 + \vec{k}_2$

$$-\vec{p} + \vec{p} = \vec{p}_2 + \vec{p}_2 = 0 \text{ , and}$$

therefore $\vec{p}_2 = -\vec{k}_2$. So that

$$E_F = \omega' + (|\vec{k}_2|^2 + m^2)^{\frac{1}{2}}$$

$$\frac{\partial E_F}{\partial |\vec{k}_2|} = \frac{\partial \omega'}{\partial |\vec{k}_2|} + \tfrac{1}{2}(|\vec{k}_2|^2 + m^2)^{-\frac{1}{2}}$$

$$= \frac{\partial \omega'}{\partial |\vec{k}_2|} + |\vec{k}_2|(|\vec{k}_2|^2 + m^2)^{-\frac{1}{2}}$$

$$= \frac{|\vec{k}_2|}{\omega'} + \frac{|\vec{k}_2|}{E_p'}$$

$$= \frac{|\vec{k}_2|(E_p' + \omega')}{\omega' E_p} \quad .$$

Since $\omega'^2 = |\vec{k}_2|^2 + \mu^2$

$$2\omega' \frac{\partial \omega'}{\partial |\vec{k}_2|} = 2|\vec{k}_2| \quad ,$$

therefore $\dfrac{\partial \omega'}{\partial |\vec{k}_2|} = \dfrac{\vec{k}_2}{\omega'}$.

Substitution of these relations into the differential cross-section, equation (2), gives:

$$\frac{d\sigma}{d\Omega} = \frac{2\pi}{|\vec{P}|\left(\frac{1}{\omega}+\frac{1}{E_p}\right)}(2\pi)^{-3}\,\bar{k}^{\,2}\,\frac{\omega E_p'}{\bar{k}_2\,(E_p + \ ')}\left\{\tfrac{1}{2}\sum_{r,s=1}^{2}|\mathcal{M}|^2\right\}\frac{m}{E_p}\cdot\frac{m}{E_p}\cdot\frac{2\pi}{\omega}\cdot\frac{2\pi}{\omega'}$$

$$= \frac{E_p}{(\omega + E_p)}\cdot\frac{m^2}{(E_p' + \omega')E_p}\left\{\tfrac{1}{2}\sum_{r,s=1}^{2}|\mathcal{M}|^2\right\}$$

$$= \frac{m^2}{(\omega + E_p)^2}\left\{\tfrac{1}{2}\sum_{r,s=1}^{2}|\mathcal{M}|^2\right\}$$

$$= \left\{m^2/[\omega + (\ ^2 - \ ^2 + m^2)^{\frac{1}{2}}]^2\right\}\cdot\left\{\tfrac{1}{2}\sum_{r,s=1}^{2}|\mathcal{M}|^2\right\}$$

Therefore, the differential cross-section for non-charge-exchange pseudoscalar coupling is

$$\frac{d\sigma}{d\Omega} = \frac{m^2}{[\omega + (\omega^2 - \mu^2 + m^2)^{\frac{1}{2}}]^2}\cdot\frac{g^4}{2m^2}\left\{2[\omega(\omega^2 - \mu^2 + m^2)^{\frac{1}{2}} + (\omega^2 - \mu^2)\cos\theta] + C^2\right\}^{-2}$$

$$\left\{(2\omega^2 - \mu^2 + \lambda^2)(\omega^2 - \mu^2 + m^2) + 4\lambda m + 2\omega(\omega^2 - \mu^2)(1 + \cos\theta)\right](\omega^2 - \mu^2 + m^2)^{\frac{1}{2}}$$

$$+ m^2\mu^2 + \lambda^2 m^2 + \left[(2\omega^2 - \mu^2 + 2\lambda m - \lambda^2)\cos\theta + 2\lambda m\right](\omega^2 - \mu^2)\Big\}.$$

The only term that needs to be changed for the case of pseudovector coupling is the $\frac{1}{2}\sum_{r,s=1}^{2}|\mathfrak{M}|^2$ in the differential cross-section. All the rest is good for all the other terms that will be done. Consequently, the matrix element for pseudovector coupling will now be written down and evaluated:

$$\begin{aligned}\mathfrak{M} &= g^2\bar{u}_2(-\gamma_5\frac{\not{k}_2}{\mu})(\not{p}_1-\not{k}_2-M)^{-1}(\ _5\frac{\not{k}_1}{\mu})u_1\\ &=-\frac{g^2}{\mu^2}\frac{\bar{u}_2(\gamma_5\not{k}_2)(\not{p}_1-\not{k}_2+M)(\gamma_5\not{k}_1)u_1}{(\not{p}_1-\not{k}_2-M)(\not{p}_1-\not{k}_2+M)}\\ &=\frac{g^2}{\mu^2}\frac{\bar{u}_2(-\not{k}_2)(\gamma_5^2)\ (-\not{p}_1+\not{k}_2+M)\not{k}_1u_1}{\sum_{\mu\nu}(p_1-k_2)\ (p_1-k_2)\gamma_\mu\gamma_\nu\ -M^2}\\ &=-\frac{g^2}{\mu^2}\frac{\bar{u}_2\not{k}_2(\not{p}_1-\not{k}_2-M)\not{k}_1u_1}{m^2+\mu^2-2(p_1k_2)-M^2}\\ &=-\frac{g^2}{\mu^2}\frac{\bar{u}_2\not{k}_2(\not{p}_1\not{k}_1-\not{k}_2\not{k}_1-M\ \not{k}_1)u_1}{-C^2-2(p_1k_2)}\\ &=\frac{g^2}{\mu^2}\frac{\bar{u}_2(\not{k}_2\not{p}_1\not{k}_1-\not{k}_2\not{k}_2\not{k}_1-M\not{k}_2\not{k}_1)u_1}{C^2+2(p_1k_2)}\quad,\end{aligned}\qquad(4)$$

where $C^2=M^2-m^2-\ ^2$.

Now the terms $\not{p}_1\not{k}_1$, $\not{k}_2\not{k}_2$ and $\not{k}_2\not{k}_1$ must be evaluated.
Considering $\not{k}_2\not{k}_2$ first,

$$\not{k}_2\not{k}_2=\sum_{\mu\nu}k_{2\mu}k_{2\nu}\gamma_\mu\gamma_\nu=\sum_{\mu}k_{2\mu}k_{2\nu}\gamma_\mu^2=\sum_{\mu}k_{2\mu}^2=\mu^2.\qquad(5)$$

Similarly, $\not{p}_1\not{k}_1+\not{k}_1\not{p}_1=2(p_1k_1)$

$$\text{or } \not{p}_1\not{k}_1=2(p_1k_1)-\not{k}_1\not{p}_1.\qquad(6)$$

From the conservation of the momentum, it then follows that

$$\bar{u}_2(\not{p}_1+\not{k}_1)^2u_1=\bar{u}_2(\not{p}_2+\not{k}_2)(\not{p}_1+\not{k}_1)u_1$$

$$\bar{u}_2[m^2+\mu^2+2(p_1k_1)]\,u_1=\bar{u}_2[m^2+m(\not{k}_1+\not{k}_2)+\not{k}_2\not{k}_1]\,u_1.$$

Using equation (1) $\bar{u}_2 \not{k}_1 u_1 = \bar{u}_2 \not{k}_2 u_1$

then $\bar{u}_2 \not{k}_2 \not{k}_1 u_1 = \bar{u}_2 [\mu^2 + 2(p_1 k_1) - 2m\not{k}_1] u_1$. (7)

Combining these three (5), (6) and (7) in (4) gives

$$= \frac{g^2}{\mu^2} \frac{\bar{u}_2 \{\not{k}_2 [2(p_1 k_1) - \not{k}_1 \not{p}_1] - \mu^2 k_1 - M\not{k}_2 \not{k}_1\} u_1}{2(p_1 k_2) + C^2}$$

$$= \frac{g^2}{\mu^2} \frac{\bar{u}_2 \{[2(p_1 k_1) - \mu^2] \not{k}_1 - (M+m)[\mu^2 + 2(p_1 k_1) - 2m\not{k}_1]\} u_1}{2(p_1 k_2) + C^2}$$

$$= \frac{g^2}{\mu^2} \frac{\bar{u}_2 \{[2(p_1 k_1) - 2m\ ' - \ ^2\ \not{k}_1 - \alpha' [\mu^2 + 2(p_1 k_1)]\} u_1}{2(p_1 k_2) + C^2} .$$

Now let

$$\alpha' = M + m$$

$$A = 2(p_1 k_1) - 2m\alpha' - \ ^2$$

$$= 2(E_p + |\vec{p}|^2) - 2m\ ' - \ ^2$$

$$= 2(\omega^2 - \mu^2 + m^2)^{\frac{1}{2}} \omega + 2(\omega^2 - \mu^2) - 2m\alpha' - \mu^2$$

$$= 2(\omega^2 - \mu^2 + m^2)^{\frac{1}{2}} \omega + 2\omega^2 - 2m\alpha' - 3\mu^2$$

$$B = \alpha' [\mu^2 + 2(p_1 k_1)]$$

$$= \alpha' [\mu^2 + 2(\omega^2 - \mu^2 + m^2)^{\frac{1}{2}} \omega + 2(\omega^2 - \mu^2)]$$

$$= \alpha' [2(\omega^2 - \mu^2 + m^2)^{\frac{1}{2}} + 2\omega^2 - \mu^2]$$

and

$$= \frac{g^2}{\mu^2} \frac{\bar{u}_2 (A\not{k}_1 - B) u_1}{2(p_1 k_2) + C^2} .$$

Then the complex-conjugate matrix element is

$$\mathcal{M}^* = \frac{g^2}{\mu^2} \frac{u_1^* (A\not{k}_1^* - B) \bar{u}_2^*}{2(p_1 k_2) + C^2}$$

$$= \frac{g^2}{\mu^2} \frac{\bar{u}_1 (A\not{k}_1 - B) u_2}{2(p_1 k_2) + C^2}$$

$$= \frac{g^2}{\mu^2} \frac{\bar{u}_1 (A\not{k}_1 - B) u_2}{2(p_1 k_2) + C^2} .$$

Therefore:

$$\frac{1}{2}\sum_{r,s=1}^{2}|\mathfrak{M}_b|^2=\frac{g^4}{2\mu^4}[2(p_1k_2)+c^2]^{-2}\sum_{r,s=1}^{2}[\bar{u}_1^{(r)}(A\not{k}_1-B)u_2^{(s)}u_2^{(s)}(A\not{k}_1-B)u_1^{(r)}]$$

$$=\frac{g^4}{2\mu^4}[2(p_1k_2)+c^2]^{-2}\,Tr\left\{\frac{1}{4m^2}(A\not{k}_1-B)(\not{p}_2+m)(A\not{k}_1-B)(\not{p}_1+m)\right\}$$

$$=\frac{g^4}{8m^2}[2(p_1k_2)+c^2]^{-2}\,Tr\{A^2\not{k}_1\not{p}_2\not{k}_1\not{p}_1-ABm\not{k}_1\not{p}_2+A^2m^2\not{k}_1\not{k}_1 -ABm\not{k}_1\not{p}_1-ABm\not{p}_2\not{k}_1+B^2\not{p}_2\not{p}_1-ABm\not{k}_1\not{p}_1+B^2m^2\} \quad (8)$$

where the terms containing odd numbers of daggers have been dropped. As before one has

$$Tr(A^2\not{k}_1\not{p}_2\not{k}_1\not{p}_1)=4A^2[(k_1p_2)(k_1p_1)-(k_1k_1)(p_2p_1)+(k_1p_1)(p_2k_1)]$$

$$Tr(ABm\,\not{k}_1\not{p}_2)=4ABm(k_1p_2)$$

$$Tr(A^2m^2\not{k}_1\not{k}_1)=4A^2m^2(k_1k_1)$$

$$Tr(ABm\not{k}_1\not{p}_1)=4ABm(k_1p_1)$$

$$Tr(B^2\not{p}_2\not{p}_1)=4B^2(p_2p_1)$$

$$Tr(B^2m^2)=4B^2m^2 \quad .$$

Combining all of these in equation (8) gives:

$$\frac{1}{2}\sum_{r,s=1}^{2}|\mathfrak{M}_b|^2=\frac{4g^4}{8\mu^4m^2}[2(p_1k_2)+c^2]^{-2}\{A^2(k_1p_2)(k_1p_1)-A^2(k_1k_1)(p_2p_1)+A^2(k_1p_1)(p_2k_1) -2ABm(k_1p_2)+A^2m^2(k_1k_1)+2ABm(k_1p_1)+B^2(p_2p_1)+B^2m^2\}$$

$$=\frac{g^4}{2\mu^4m^2}\{2[\omega(\omega^2-\mu^2+m^2)^{1/2}+(\omega^2-\mu^2)\cos\theta]+c^2\}^{-2}\cdot \{2A^2(E_p\omega+|\vec{P}|^2\cos\theta)(E_p\omega+|\vec{P}|^2)-A^2\mu^2(E_p^2-|\vec{P}|^2\cos\theta)+A^2m^2\mu^2 -2ABm(E_p\omega+|\vec{P}|^2)+B^2(E_p^2-|\vec{P}|^2\cos\theta)+B^2m^2\}$$

$$=\frac{g^4}{2\mu^4m^2}\{2[\omega(\omega^2-\mu^2+m^2)^{1/2}+(\omega^2-\mu^2)\cos\theta]+c^2\}^{-2}\{(2A^2\omega-A^2\mu^2+B^2)E_p^2 +(2A^2|\vec{P}|^2\omega\cos\theta+2A^2\omega|\vec{P}|^2-4ABm\omega)E_p+(2A^2|\vec{P}|^2\cos\theta+A^2\mu^2\cos\theta -2ABm\cos\theta-2ABm-B^2\cos\theta)|\vec{P}|^2+A^2m^2\mu^2+B^2m^2\}$$

$$=\frac{g^4}{2\mu^4m^2}\{2[\omega(\omega^2-\mu^2+m^2)^{1/2}+(\omega^2-\mu^2)\cos\theta]+c^2\}^{-2}\cdot \{(2A^2\omega^2-A^2\mu^2+B^2)(\omega^2-\mu^2+m^2)+[2A^2\omega(\omega^2-\mu^2)(1+\cos\theta)-4ABm\omega]\cdot (\omega^2-\mu^2+m^2)^{1/2}+[(2A^2\omega^2-A^2\mu^2-2ABm-B^2)\cos\theta-2ABm](\omega^2-\mu^2)+A^2m^2\mu^2+B^2m^2\}.$$

This is the $\frac{1}{2}\sum_{r,s=1}^{2}|\mathcal{M}|^2$ for non-charge-exchange scattering assuming a pseudovector interaction. The differential scattering cross-section is given by

$$\frac{d\sigma}{d\Omega}=\frac{m^2}{[\omega+(\omega^2-\mu^2+m^2)^{\frac{1}{2}}]^2}\left\{\frac{1}{2}\sum_{r,s=1}^{2}|\mathcal{M}|^2\right\},$$

and the total cross-section is given by

$$\sigma=2\pi\int_0^{\pi}\frac{d\sigma}{d\Omega}\sin\theta\, d\theta\quad .$$

The form of the charge-exchange scattering is a consequence of the fact that there exist two known particles which may be the intermidiate-state particle. In the following it is assumed that one is coupled through the pseudoscalar interaction and the other by the pseudovector interaction. The sigma hyperon Σ^0 is assumed to couple through the pseudoscalar interaction, and the lambda hyperon Λ^0 through the pseudovector interaction.

The matrix element for the charge-exchange scattering is

$$\mathcal{M}=g^2\bar{u}_2\gamma_5(\not p_1-\not k_2-M)^{-1}\gamma_5 u_1+g'^2\bar{u}_2(-\gamma_5\frac{\not k_2}{\mu})(\not p_1-\not k_2-M')^{-1}(\gamma_5\frac{\not k_1}{\mu})u_1$$

where g and M refer to the sigma hyperon, and g' and M' refer to the lambda hyperon.

Making the same manipulations on this as on the previous ones gives:

$$\mathcal{M}=g^2\left\{\frac{\bar{u}_2\gamma_5(\not p_1-\not k_2+M)\gamma_5 u_1}{(\not p_1-\not k_2-M)(\not p_1-\not k_2+M)}+\frac{r}{\mu^2}\cdot\frac{\bar{u}_2(-\gamma_5\not k_2)(\not p_1-\not k_2+M')(\gamma_5\not k_1)u_1}{(\not p_1-\not k_2-M')(\not p_1-\not k_2+M')}\right\}$$

where $r=g'^2/g^2$. So that

$$\mathcal{M} = g^2\left\{\frac{\bar{u}_2\gamma_5^2(\not{p}_1+\not{k}_2+M)u_1}{m^2+\mu^2-2(p_1k_2)-M^2} + \frac{\gamma}{\mu^2}\cdot\frac{\bar{u}_2\not{k}_2\gamma_5^2(-\not{p}_1+\not{k}_2+M')\not{k}_1u_1}{m^2+\mu^2-2(p_1k_2)-M'^2}\right\}$$

$$= g^2\left\{\frac{\bar{u}_2(\not{p}_1-\not{k}_2-M)u_1}{-2(p_1k_2)-C^2} - \frac{\gamma}{\mu^2}\cdot\frac{\bar{u}_2\not{k}_2(\not{p}_1-\not{k}_2-M')\not{k}_1u_1}{-2(p_1k_2)-C'^2}\right\}$$

$$= g^2\left\{\frac{\bar{u}_2(\not{k}_1+\alpha)u_1}{2(p_1k_2)+C^2} - \frac{\gamma}{\mu^2}\cdot\frac{\bar{u}_2(A'\not{k}_1-B')u_1}{2(p_1k_2)+C'^2}\right\},$$

with $\alpha = M-m$

$C^2 = -m^2-\mu^2+M^2$

$C'^2 = m^2-\mu^2+M'^2$

$A' = 2(p_1k_1)-2m(M'+m)-\mu^2$

$\quad = 2(\omega^2-\mu^2+m^2)^{\frac{1}{2}}\omega + 2\omega^2-2m(m+M')-3\mu^2$

$B' = (M'+m)[\mu^2+2(p_1k_1)]$

$\quad = (M'+m)[2(\omega^2-\mu^2+m^2)^{\frac{1}{2}}\omega+2\omega^2-\mu^2]$.

The complex-conjugate matrix element is

$$\mathcal{M}^* = g^2\left\{\frac{u_1^*(\not{k}_1^*+\alpha)\bar{u}_2^*}{2(p_1k_1)+C^2} - \frac{\gamma}{\mu^2}\cdot\frac{u_1^*(A'\not{k}_1^*-B')\bar{u}_2^*}{2(p_1k_2)+C'^2}\right\}$$

$$= g^2\left\{\frac{\bar{u}_1\beta(\not{k}_1^*+\alpha)\beta u_2}{2(p_1k_2)+C^2} - \frac{\gamma}{\mu^2}\cdot\frac{\bar{u}_1\beta(A'\not{k}_1^*-B')\beta u_2}{2(p_1k_2)+C'^2}\right\}$$

$$= g^2\left\{\frac{\bar{u}_1(\not{k}_1+\alpha)u_2}{2(p_1k_2)+C^2} - \frac{\gamma}{\mu^2}\cdot\frac{\bar{u}_1(A'\not{k}_1-B')u_2}{2(p_1k_2)+C'^2}\right\}.$$

Then

$$\frac{1}{2}\sum_{r,s=1}^{2}|\mathcal{M}|^2 = \frac{g^4}{2}\sum_{r,s=1}^{2}\left\{\frac{\bar{u}_1^{(r)}(\not{k}_1+\alpha)u_2^{(s)}\bar{u}_2^{(s)}(\not{k}_1+\alpha)u_1^{(r)}}{[2(p_1k_2)+C^2]^2}+\right.$$

$$\frac{\gamma^2}{\mu^4}\cdot\frac{\bar{u}_1^{(r)}(A'\not{k}_1-B')u_2^{(s)}u_2^{(s)}(A'\not{k}_1-B)u_1^{(r)}}{[2(p_1k_2)+C'^2]^2} - \frac{\gamma}{\mu^2}\frac{\bar{u}_1^{(r)}(\not{k}_1+\alpha)u_2^{(s)}u_2^{(s)}(A'\not{k}_1-B')u_1^{(r)}}{[2(p_1k_2)+C^2][2(p_1k_2)+C'^2]}$$

$$\left.-\frac{\gamma}{\mu^2}\cdot\frac{\bar{u}_1^{(r)}(A'\not{k}_1-B')u_2^{(s)}u_2^{(s)}(\not{k}_1+\alpha)u_1^{(r)}}{[2(p_1k_2)+C'^2][2(p_1k_2)+C^2]}\right\}$$

$$\frac{1}{2}\sum_{\gamma,s=1}^{2}|\mathcal{M}|^2 = \frac{g^4}{8m^2}\Big\{ [2(p_1 k_2)+C^2]^{-2}\,\mathrm{Tr}[(\not k_1+\alpha)(\not p_2+m)(\not k_1+\alpha)(\not p_1+m)]$$
$$+[2(p_1 k_2)+C'^2]^{-2}\cdot\mathrm{Tr}[(A'\not k_1-B')(\not p_2+m)(A'\not k_1-B')(\not p_1+m)]\cdot\frac{\gamma^2}{\mu^2}$$
$$-[2(p_1 k_2)+C^2]^{-1}[2(p_1 k_2)+C'^2]^{-1}\cdot\mathrm{Tr}[(\not k_1+\alpha)(\not p_2+m)(A'\not k_1-B')(\not p_1+m)]\cdot\frac{\gamma}{\mu^2}$$
$$-[2(p_1 k_2)+C'^2]^{-1}[2(p_1 k_2)+C^2]^{-1}\cdot\mathrm{Tr}[(A'\not k_1-B')(\not p_2+m)(\not k_1+\alpha)(\not p_1+m)]\cdot\frac{\gamma}{\mu^2}\Big\}.$$

The first two terms in the above are of the same type that were evaluated previously. The only thing new to do is to calculate the trace of the "interference terms". This will now be done. This part can be written as,

$$-[2(p_1k_2)+C^2]^{-1}[2(p_1k_2)+C'^2]^{-1}\frac{\gamma}{\mu^2}\{\mathrm{Tr}[(\not k_1+\alpha)(\not p_2+m)(A'\not k_1-B')(\not p_1+m)]+\mathrm{Tr}[(A'\not k_1-B')(\not p_2+m)(\not k_1+\alpha)(\not p_1+m)]\}$$
$$=-[2(p_1k_2)+C^2]^{-1}[2(p_1k_2)+C'^2]^{-1}\{\mathrm{Tr}[A'\not k_1\not p_2\not k_1\not p_1-B'\alpha\not p_2\not p_1-B'm\not k_1\not p_1-\alpha A'm\not k_1\not p_1-B'm\not k_1\not p_2+\alpha A'm\not p_2\not k_1-A'm^2\not k_1\not k_1-B'\alpha m^2]\}\frac{2\gamma}{\mu^2}$$
$$=-2[(p_1k_2)+C^2]^{-1}[2(p_1k_2)+C'^2]^{-1}\{\mathrm{Tr}[A'\not k_1\not p_2\not k_1\not p_1-B'\alpha\not p_2\not p_1-(B'+\alpha A')m\not k_1\not p_1+(\alpha A'-B)m\not p_2\not k_1-A'm^2\not k_1\not k_1-B'\alpha m^2]\}\cdot\frac{\gamma}{\mu^2},$$

where the terms containing an odd number of daggers have been dropped. Now using the theorems of trace the terms of trace become:

$$\mathrm{Tr}(A'\not k_1\not p_2\not k_1\not p_1)=4A'[(k_1p_2)(k_1p_1)-(k_1k_1)(p_2p_1)+(k_1p_1)(p_2k_1)$$
$$\mathrm{Tr}(B'\alpha\not p_2\not p_1)=4B'\alpha(p_2p_1)$$
$$\mathrm{Tr}\{(B'+\alpha A')m\not k_1\not p_1\}=4(B'+\alpha A')m(k_1p_1)$$
$$\mathrm{Tr}\ (\alpha A'-B)m\,\not p_2\not k_1\ =4(\alpha A'-B)(p_2k_1)$$
$$\mathrm{Tr}(B'\alpha m^2)=4B'\alpha m^2\ .$$

So that the above can be written as

$$=-\{2[\omega E_p+|\vec{P}|^2\cos\theta]+C^2\}^{-1}\cdot\{2[\omega E_p+|\vec{P}|^2\cos\theta]+C'^2\}^{-1}\cdot\frac{\gamma}{\mu^2}\cdot 8\cdot\{2A'(E_p\omega+|\vec{P}|^2)(E_p\omega+|\vec{P}|^2\cos\theta)$$
$$-A'\mu^2(E_p^2-|\vec{P}|^2\cos\theta)-B'\alpha(E_p^2-|\vec{P}|^2\cos\theta)-(B'+\alpha A')m(E_p\omega+|\vec{P}|^2)+(\alpha A'-B)m(E_p\omega+|\vec{P}|^2\cos\theta)$$
$$-A'm^2\mu^2-B'\alpha m^2\}$$

$$=-\left\{2\left[\omega(\omega^2-\mu^2+m^2)^{\frac{1}{2}}+(\omega^2-\mu^2)\cos\theta+c^2\right\}^{-1}\left\{2\left[\omega(\omega^2-\mu^2+m^2)^{\frac{1}{2}}+(\omega^2-\mu^2)\cos\theta\right]+c'^2\right\}^{-1}\cdot$$

$$\frac{8\gamma}{\mu^2}\left\{(2A'\omega^2-A'\mu^2-B'\alpha)(\omega^2-\mu^2+m^2)+\left[2A'\omega(\omega^2-\mu^2)(1+\cos\theta)-2B'm\omega\right]\cdot\right.$$
$$(\omega^2-\mu^2+m^2)^{\frac{1}{2}}+\left[(2A'\omega^2-A\mu^2+B'\alpha+\alpha A'm-B'm)\cos\theta-B'm-\alpha A'm\right](\omega^2-\mu^2)$$
$$\left.-A'm^2\mu^2-B'\alpha m^2\right\} \quad .$$

Therefore,

$$\frac{1}{2}\sum_{\gamma,s=1}^{2}|\mathfrak{M}|^2=\frac{g^4}{\alpha m^2}\Big\{D^{-2}\left\{(2\omega^2-\mu^2+\alpha^2)(\omega^2-\mu^2+m^2)+\left[4\alpha m\omega+2\omega(\omega^2-\mu^2)(1+\cos\theta)\right]\right.$$
$$\left.(\omega^2-\mu^2+m^2)^{\frac{1}{2}}+\left[(2\omega^2-\mu^2+2\alpha m-\alpha^2)\cos\theta+2\alpha m\right](\omega^2-\mu^2)+m^2(\mu^2+\alpha^2)\right\}$$
$$+\frac{\gamma^2}{\mu^4}\cdot D'^{-2}\left\{(2A^2\omega^2-A^2\mu^2+B^2)(\omega^2-\mu^2+m^2)+\left[2A^2\omega(\omega^2-\mu^2)(1+\cos\theta)-4ABm\omega\right]\right.$$
$$\left.(\omega^2-\mu^2+m^2)^{\frac{1}{2}}+\left[(2A^2\omega^2-A^2\mu^2-2ABm-B^2)\cos\theta-2ABm\right](\omega^2-\mu^2)+A^2m^2\mu^2+B^2m^2\right\}$$
$$-\frac{2\gamma}{\mu^2}D^{-1}D'^{-1}\cdot\left\{(2A'\omega^2-A'\mu^2-B'\alpha)(\omega^2-\mu^2+m^2)+\left[2A'\omega(\omega^2-\mu^2)(1+\cos\theta)-2B'm\omega\right](\omega^2+\mu^2+m^2)^{\frac{1}{2}}\right.$$
$$\left.+\left[(2A'\omega^2-A'\mu^2+B'\alpha+\alpha A'm)\cos\theta-B'm-\alpha A'm\right](\omega^2-\mu^2)-A'm^2\mu^2-B'\alpha m^2\right\}\Big\} ,$$

where $D=2\left[\omega(\omega^2-\mu^2+m^2)^{\frac{1}{2}}+(\omega^2-\mu^2)\cos\theta\right]+(M^2-m^2-\mu^2)$

$D'=2\left[\omega(\omega^2-\mu^2+m^2)^{\frac{1}{2}}+(\omega^2-\mu^2)\cos\theta\right]+(M'^2-m^2-\mu^2)$.

The differential cross-section is given by

$$\frac{d\sigma}{d\Omega}=\frac{m^2}{\left[\omega+(\omega^2-\mu^2+m^2)^{\frac{1}{2}}\right]^2}\left\{\frac{1}{2}\sum_{\gamma,s=1}|\mathfrak{M}|^2\right\}$$

and the total cross-section is

$$\sigma=2\int_0^{\pi}\frac{d\sigma}{d\Omega}\sin\theta\, d\theta \quad .$$

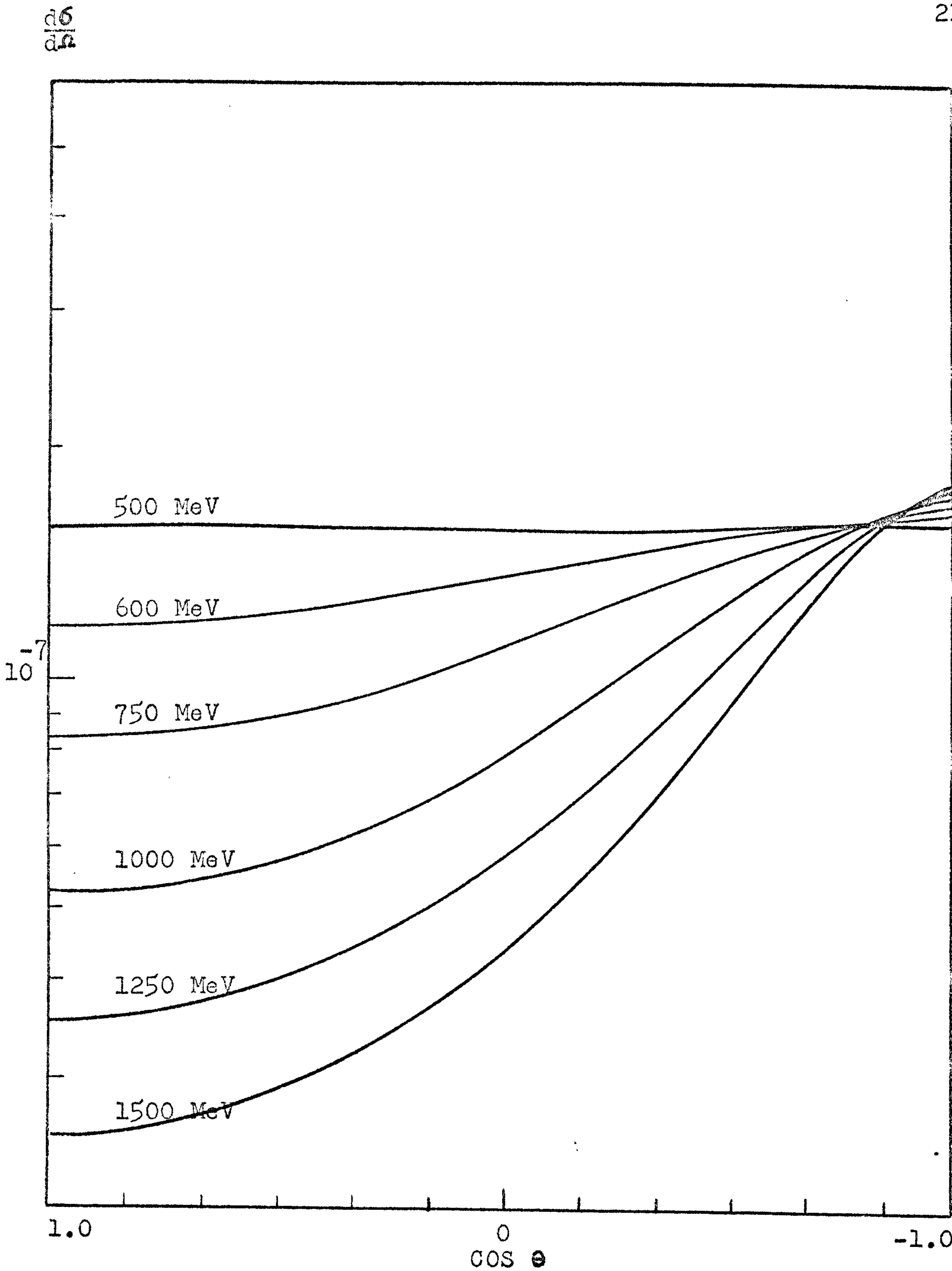


Fig. 2--Angular distribution for non-charge-exchange pseudoscalar coupling.

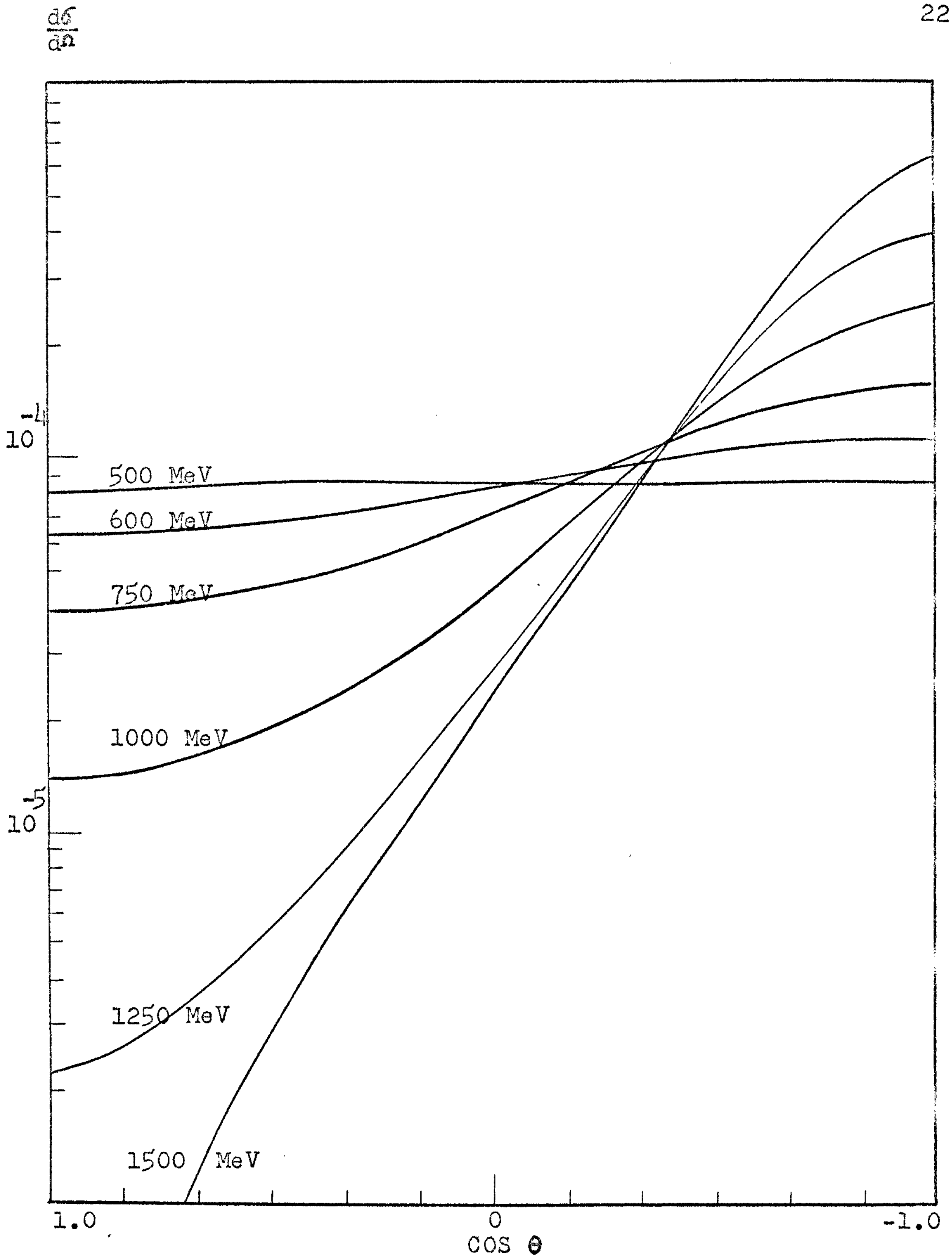


Fig. 3--Angular distribution for non-charge-exchange pseudovector coupling.

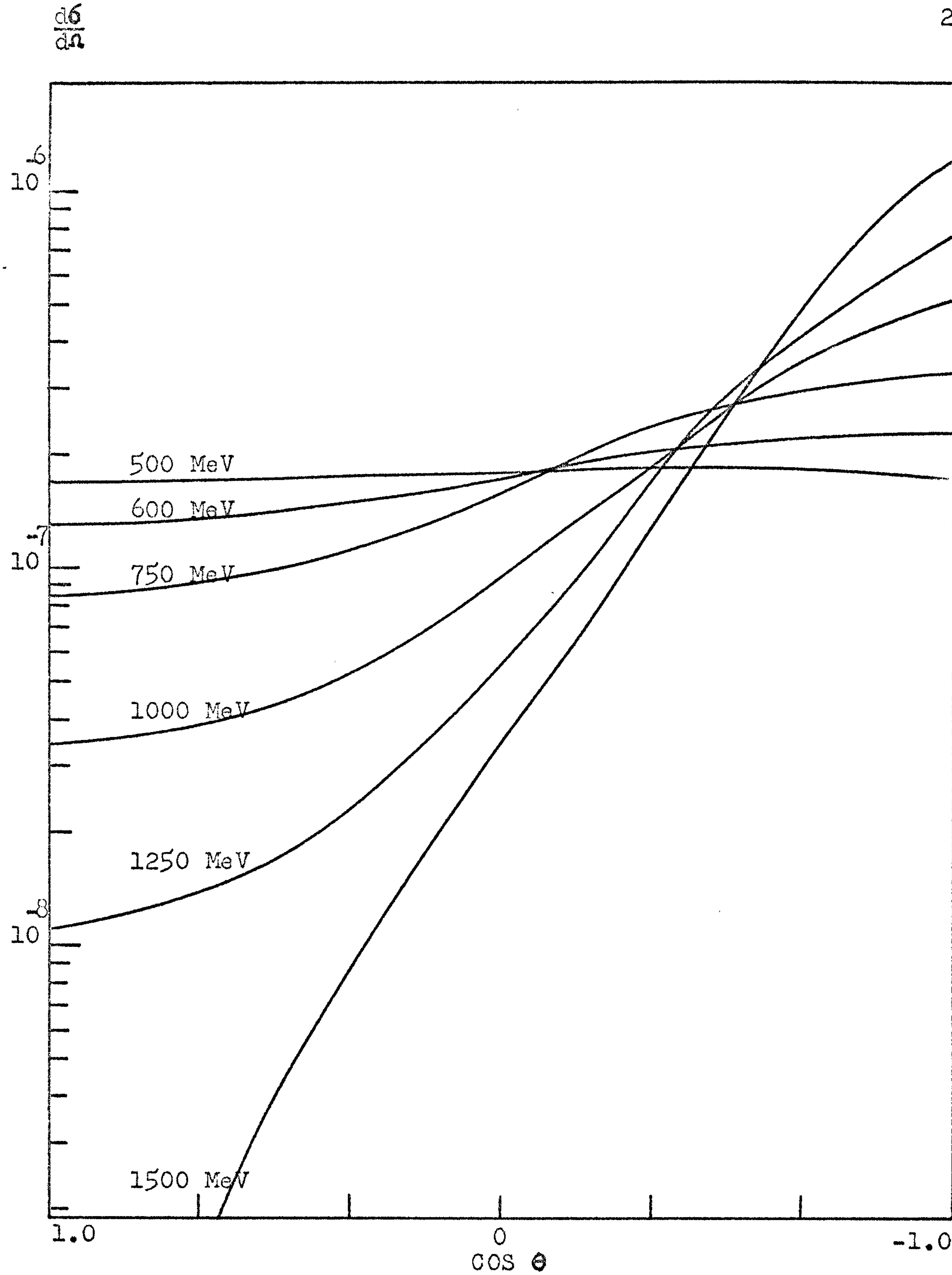


Fig. 4--Angular distribution for charge-exchange r=0.1

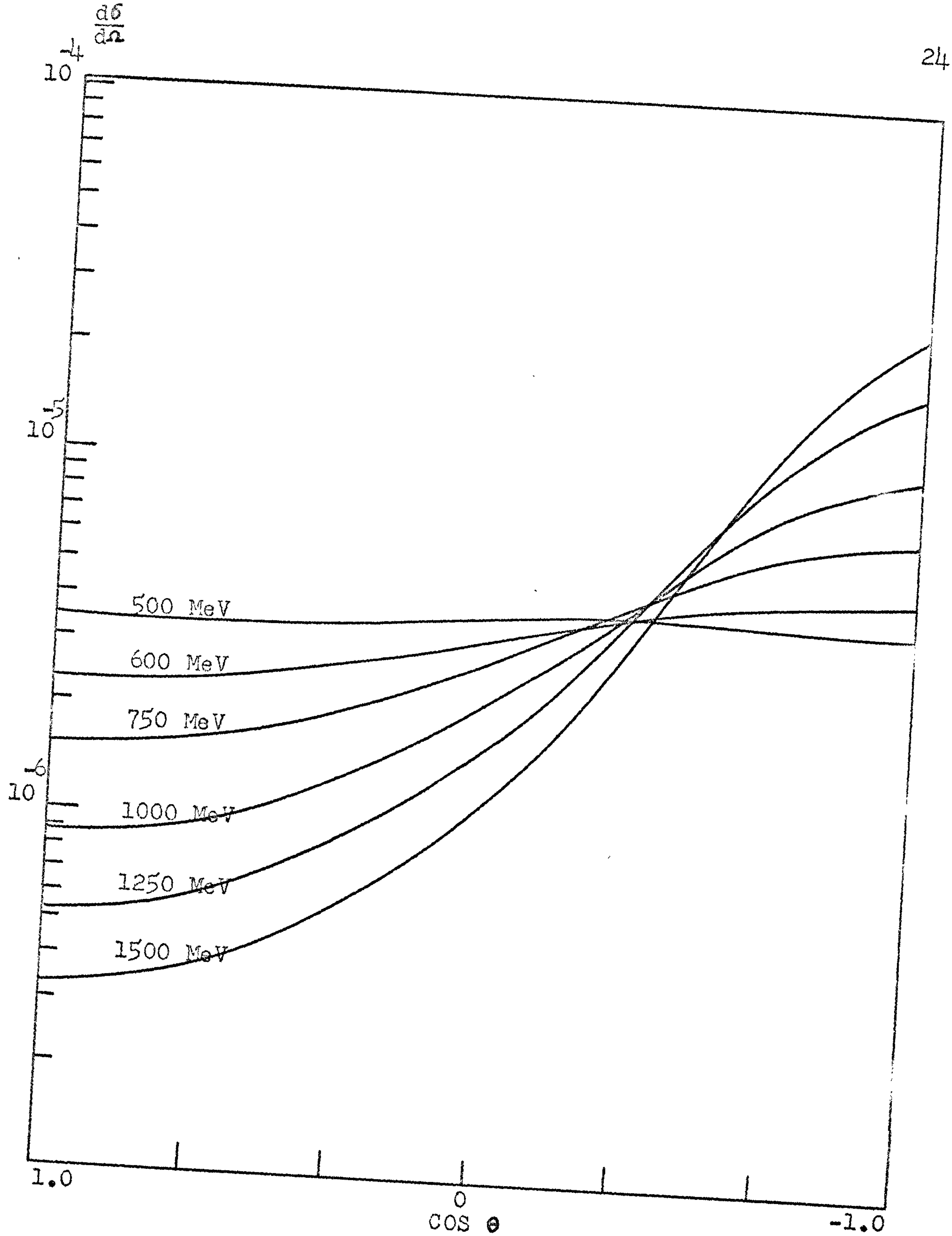


Fig. 5--Angular distribution for charge-exchange r=1

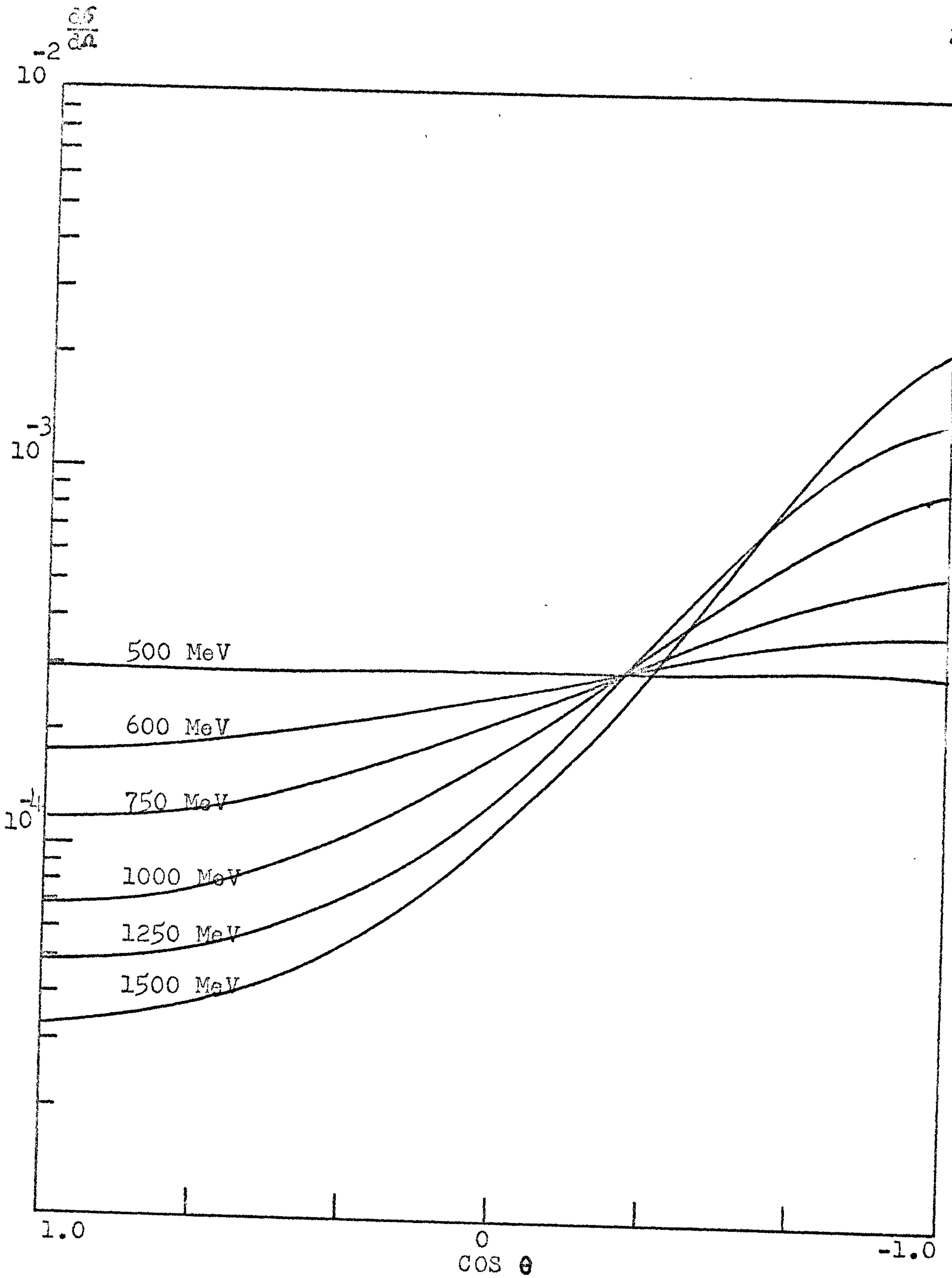


Fig. 6--Angular distribution for charge-exchange r=10

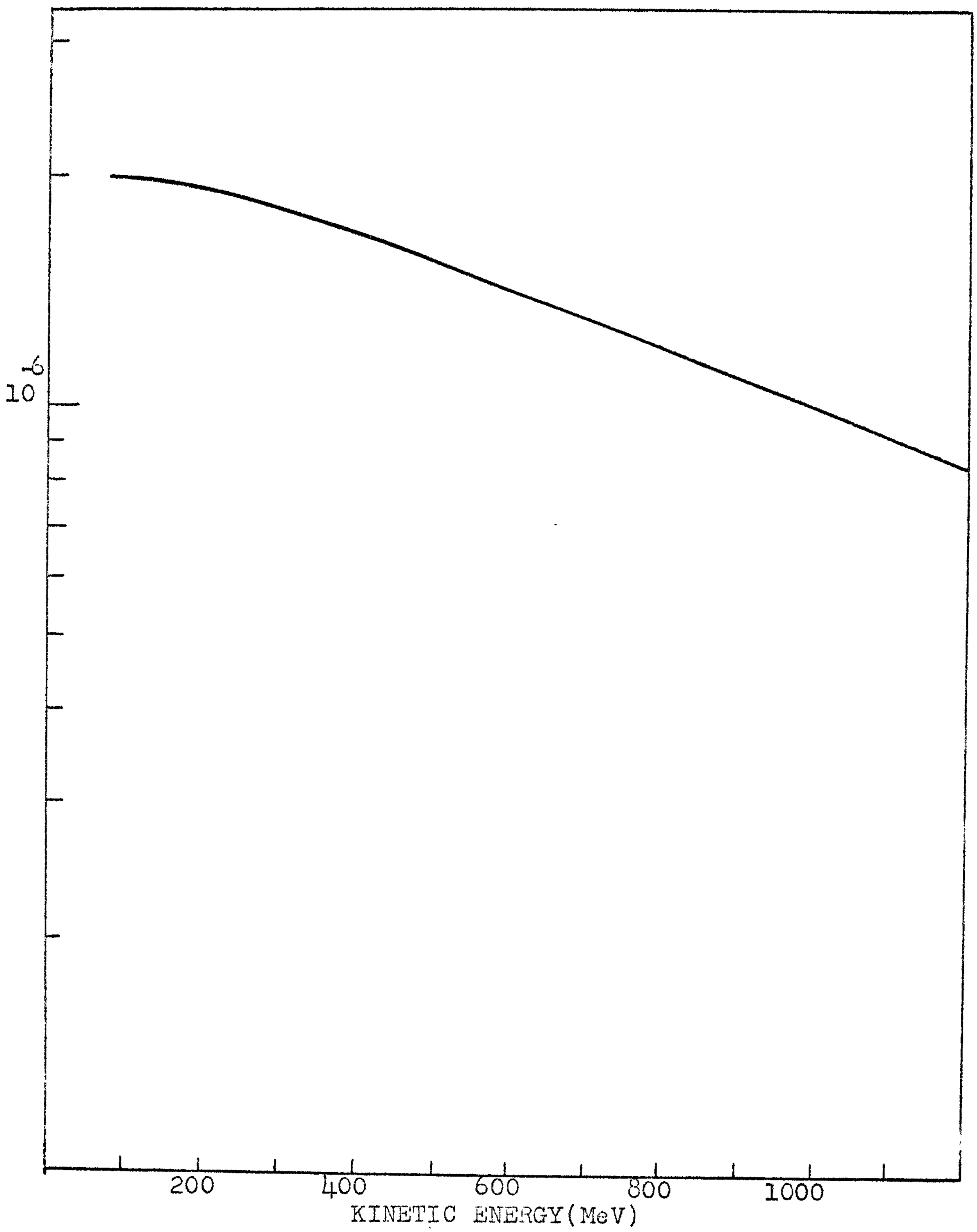


Fig. 7--Total cross-section for non-charge-exchange pseudoscalar coupling scattering.

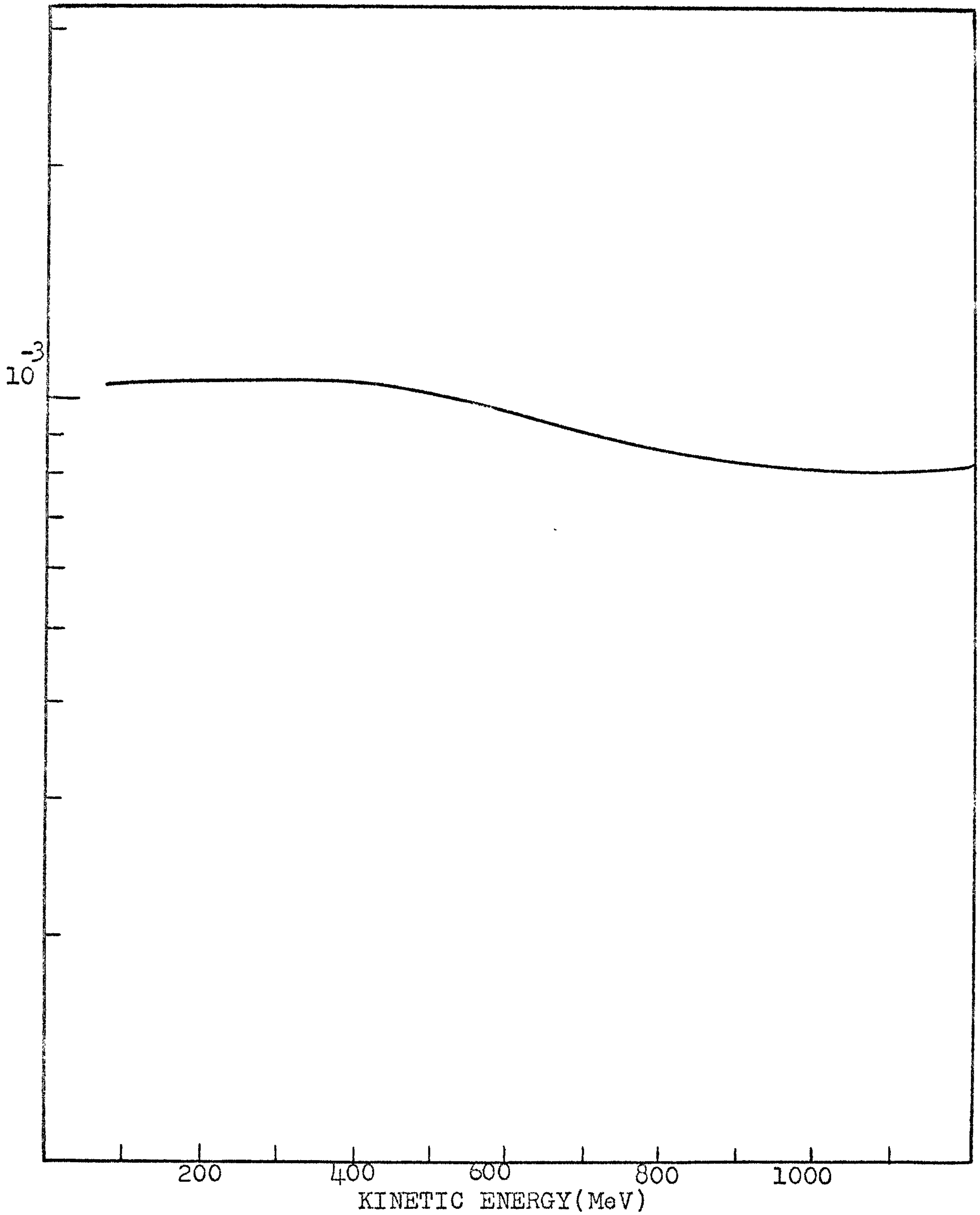


Fig. 8--Total cross-section for non-charge-exchange pseudovector coupling scattering.

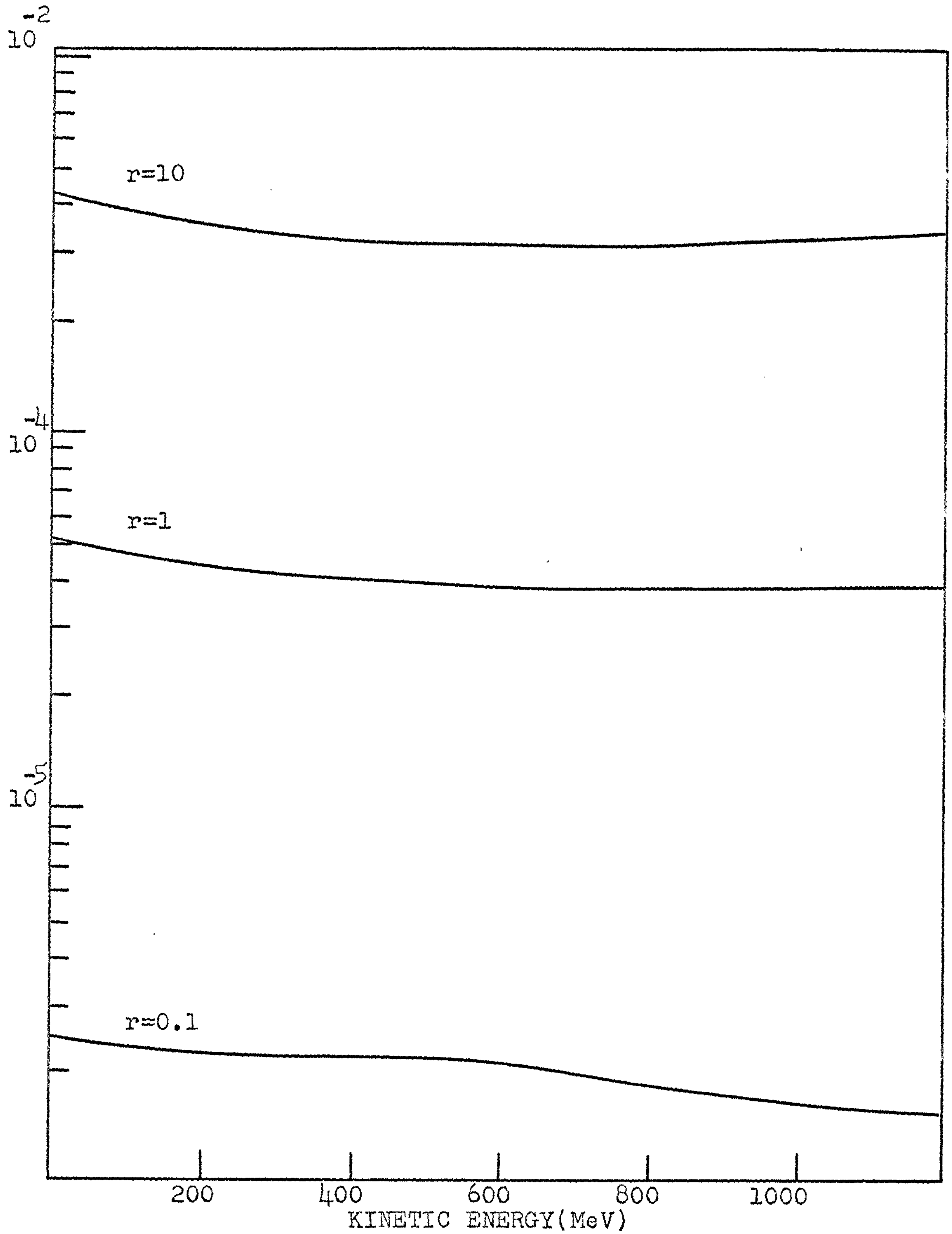


Fig. 9--Total cross-section for charge-exchange scattering

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CHAPTER III

THE RESULTS OF DIRECT INTERACTION

An examination of the calculated value of the total cross-section (Figs. 7-9) shows that it is roughly constant for all cases except for the non-charge-exchange pseudoscalar coupling case, for which it is slightly decreasing with energy. This agrees with the experimental fact that the total cross-section is roughly constant. The calculated angular distribution exhibits back peaking for both charge-exchange and non-charge-exchange cases (Figs. 2-7).

According to an early paper by Ceolin (2, p. 825), the angular distribution for charge exchange is peaked backwards in the range 200 to 305 MeV. However, the recent experiments by Cook (4, p. 2743), Hirsch (5, p. B191), Stenger (7, p. B1111), Chinowsky (3, p. B1411) and Butterworth (1, p. 734) showed that the charge-exchange distribution is peaked forward with zero value for the differential cross-section at zero degrees. The non-charge-exchange distribution is also peaked forward. It is clear that the calculated values are in disagreement with the experimental data, indicating that the assumption of a direct pseudoscalar and pseudovector interaction in this analysis is incorrect. Hooper (6) assumed that the kaon-neutron scattering is a direct scalar interaction and it gave forward peaking for charge-exchange scattering, and backward peaking for non-charge-exchange. Although it was shown that the calculated values for charge-exchange have the property of forward peaking, these results were not in good agreement with experimental values, and were in disagreement with the non-charge-exchange scattering.

Thus some additional investigation is necessary to provide a clearer picture of kaon-neutron scattering. Whether other kinds of interactions should be assumed or higher order processes should be included, will be discussed in the next chapter.

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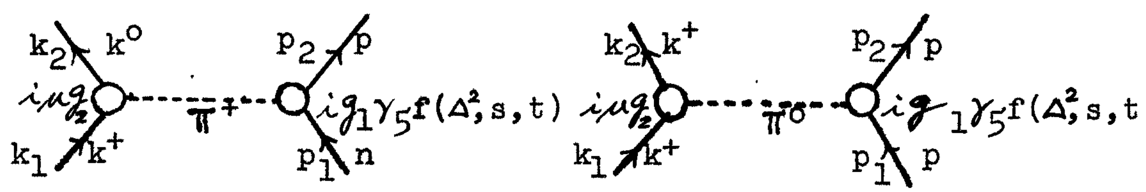
CHAPTER IV

EXCHANGE INTERACTION

In 1935, Yukawa make a model of nuclear forces that was based on a virtual particle being exchanged between nucleons. For a particle of spin zero and mass M one may use the Klein-Gordon propagator in writing the first-order scattering amplitude corresponding to Fig. 10:

$$\frac{1}{(q^2 - M^2)}$$

This expression does not include any factors that correspond to the vertices at which the particle, represented by the dashed line, is absorbed or emitted. The nucleon and kaon, drawn as solid lines, have initial and final momenta p_1, p_2 and k_1, k_2 respectively. The invariant momentum transfer is $q = (p_1 - p_2)$.



(a) Charge-exchange (b) Non-charge-exchange

Fig. 10--Charge-exchange and non-charge-exchange diagrams for kaon-nucleon scattering.

One does not know the form factors corresponding to the vertices, so it will be assumed that for the charge-exchange reaction the vertex function is unity, i.e. $f(\Delta^2)=1$. This is the simplest possible assumption and its usefulness will be determined by the results. The matrix element for this process is

$$\mathcal{M}_I = \bar{u}_2(ig_1\gamma_5)u_1 \frac{1}{(q^2 - M^2)} (i\mu g_2)$$
$$= -\frac{g_1 g_2 \mu}{(q^2 - M^2)} \bar{u}_2\gamma_5 u_1 ,$$

so that its complex-conjuate matrix element is

$$\mathcal{M}_I^* = -\frac{g_1 g_2 \mu}{(q^2 - M^2)} u_1^* \gamma_5^* \bar{u}_2^*$$

$$= -\frac{g_1 g_2 \mu}{(q^2 - M^2)} \bar{u}_1 \gamma_5 u_2 .$$

Therefore, the probability sumed over the final spin states (r) and averaged over the initial spin states (s) is

$$\frac{1}{2}\sum_{r,s=1}^{2} \mathcal{M}_I^* \mathcal{M}_I = \frac{1}{2} \frac{g_1^2 g_2^2 \mu^2}{(q^2 - M^2)} \sum_{r,s=1}^{2} (\bar{u}_2\gamma_5 u_1)(\bar{u}_1\gamma_5 u_1)$$
$$= \frac{1}{2} \frac{g_1^2 g_2^2 \mu^2}{(q^2 - M^2)} \sum_{s=1}^{2} \bar{u}_2\gamma_5 \left(\frac{\not{p}_1 + m}{2m}\right)\gamma_5 u_2$$
$$= \frac{1}{2} \frac{g_1^2 g_2^2 \mu^2}{(q^2 - M^2)} \sum_{s=1}^{4} \bar{u}_2\gamma_5\left(\frac{\not{p}_1+m}{2m}\right)\gamma_5\left(\frac{\not{p}_2+m}{2m}\right)u_2$$
$$= \frac{1}{2} \frac{g_1^2 g_2^2 \mu^2}{(q^2 - M^2)} \mathrm{Tr}\left[\gamma_5\left(\frac{\not{p}_1+m}{2m}\right)\gamma_5\left(\frac{\not{p}_2+m}{2m}\right)\right]$$
$$= \frac{1}{2} \frac{g_1^2 g_2^2 \mu^2}{(q^2 - M^2)} \mathrm{Tr}(\gamma_5\not{p}_1\gamma_5\not{p}_2 + \gamma_5 m\gamma_5 m)$$
$$= \frac{1}{2} \sum_{r,s=1}^{2} |\mathcal{M}|_I^2 ,$$

where the traces of an odd number of 's have beeb dropped. Therefore

$$\mathrm{Tr}(\gamma_5 \not{p}_1 \gamma_5 \not{p}_2) = \mathrm{Tr}(-\gamma_5^2 \not{p}_1 \not{p}_2) = \mathrm{Tr}(\not{p}_1 \not{p}_2) = 4(p_1 \cdot p_2)$$

$$\mathrm{Tr}(\gamma_5 m \gamma_5 m) = \mathrm{Tr}(\gamma_5^2 m^2) = -4m^2, \quad \text{so that}$$

$$\frac{1}{2} \sum_{r,s=1}^{2} |\mathcal{M}_I|^2 = \frac{1}{2} \frac{g_1^2 g_2^2 \mu^2}{(q^2 - M^2)} [(p_1 \cdot p_2) - m^2]$$

$$= \frac{g_1^2 g_2^2 \mu^2 [(p_1 \cdot p_2) - m^2]}{2m^2 [2m^2 - 2(p_1 \cdot p_2) - M^2]}$$

In the center-of-mass system, one has

$$(p_1 \cdot p_2) = E_p - |\vec{p}|^2 \cdot \cos\theta$$

$$= (\omega^2 - \mu^2 + m^2) - (\omega^2 - \mu^2) \cos 0 ,$$

so that $\frac{1}{2} \sum_{r,s=1}^{2} |\mathcal{M}_I|^2$ becomes,

$$\frac{1}{2} \sum_{r,s=1}^{2} |\mathcal{M}_I|^2 = \frac{1}{2} \frac{g_1^2 g_2^2 \mu^2 \; [(\omega^2 + m^2 - \mu^2) - (\omega^2 - \mu^2)\cos\theta - m^2]}{m^2 [2m^2 - 2(\omega^2 - \mu^2 + m^2) - 2(\omega^2 - \mu^2)\cos\theta + M^2]}$$

$$= \frac{1}{2} \frac{g_1^2 g_2^2 \mu^2 \; (\omega^2 - \mu^2)(1 - \cos\theta)}{m^2 [2(\omega^2 - \mu^2)(1 - \cos\theta) + M^2]} .$$

The differential cross-section is

$$\left(\frac{d\sigma}{d\Omega}\right)_I = \frac{m^2}{[\omega + (\omega^2 - \mu^2 + m^2)^{\frac{1}{2}}]^2} \left\{ \frac{1}{2} \sum_{r,s=1}^{2} |\mathcal{M}_I|^2 \right\}$$

$$= \frac{g_1^2 g_2^2 \mu^2 \; (\omega^2 - \mu^2)(1 - \cos\theta)}{2[\omega + (\omega^2 - \mu^2 + m^2)^{\frac{1}{2}}]^2 \; [2(\omega^2 - \mu^2)(1 - \cos\theta) + M^2]^2} .$$

This is the differential cross-section for charge-exchange scattering.

For non-charge-exchange scattering it is necessary to assume some different type of vertex function, otherwise the differential scattering cross-section will be essentially the same form as that above. All other factors in the matrix element corresponding to the Feynman diagram will be the same. Before different possible vertex functions are discussed, it should be pointed out that the Feynman diagram for $k^+ + p \rightarrow k^+ + p$ will also give a matrix element that leads to essentially the same differential cross-section as that for non-charge-exchange scattering. Consequently, it at first appears necessary to assume vertex functions for both reactions

$$k^+ + n \longrightarrow k^+ + n$$

$$k^+ + p \longrightarrow k^+ + p\ ,$$

but an isotopic spin analysis enables one to choose a vertex function to obtain the proper cross-section for one reaction and then calculate the matrix element for the other reaction from the one chosen. In particular, having the matrix elements for $k^+ + n \longrightarrow k^o + p$ and $k^+ + p \longrightarrow k^+ + p$ one can calculate the matrix element for $k^+ + n \longrightarrow k^+ + n$. To relate these matrix elements write them as

$$\langle k^o, p | \hat{S} | k^+, n \rangle \qquad \text{(I)}$$

$$\langle k^+, p | \hat{S} | k^+, p \rangle \qquad \text{(II)} \qquad \langle k^+, n | \hat{S} | k^+, n \rangle \qquad \text{(III)}$$

and now these three reactions can be written in terms of the two total isotopic spin amplitudes, S(1) and S(0). That is, since both the nucleon and kaon are isotopic spin doublets, $s = \frac{1}{2}$, the total possible isotopic spin is +1 or 0. Let $k(\frac{1}{2},\frac{1}{2})$, $k(\frac{1}{2},-\frac{1}{2})$, $v(\frac{1}{2},\frac{1}{2})$, $v(\frac{1}{2},-\frac{1}{2})$ be the isotopic spin eigenvectors of the total and z-component isotopic spin for the proton, neutron, positive kaon, and neutral kaon respectively. What follows is analogous to calculating Clebsch-Gordon coefficients for ordinary angular momentum. The isotopic spin eigenvector for total spin +1 and z-component +1 is

$$t(1,1) = k(\tfrac{1}{2},\tfrac{1}{2})v(\tfrac{1}{2},\tfrac{1}{2}).$$

To obtain other resultant isotopic spin eigenvectors one uses the step-up or step-down operators $\hat{T}_- = \hat{T}_-(1) + \hat{T}_-(2)$ which acting on $t(1,1)$ give (1, p. 33)

$$\hat{T}\, t(1,1) = \sqrt{1(1+1) - 1(1-1)}\; t(1,0)$$

$$= \sqrt{\tfrac{1}{2}(\tfrac{1}{2}+1)-\tfrac{1}{2}(\tfrac{1}{2}-1)}\; k(\tfrac{1}{2},-\tfrac{1}{2})v(\tfrac{1}{2},\tfrac{1}{2})$$

$$+ \sqrt{\tfrac{1}{2}(\tfrac{1}{2}+1)-\tfrac{1}{2}(\tfrac{1}{2}-1)}\; k(\tfrac{1}{2},\tfrac{1}{2})v(\tfrac{1}{2},-\tfrac{1}{2})$$

or, $t(1,0) = \dfrac{1}{\sqrt{2}}\left\{k(\tfrac{1}{2},-\tfrac{1}{2})v(\tfrac{1}{2},\tfrac{1}{2}) + k(\tfrac{1}{2},\tfrac{1}{2})v(\tfrac{1}{2},-\tfrac{1}{2})\right\}$.

Similarly, $t(1,-1) = k(\frac{1}{2},-\frac{1}{2})v(\frac{1}{2},\frac{1}{2})$, and

$$t(0,0) = \frac{1}{\sqrt{2}}\left\{k(\tfrac{1}{2},\tfrac{1}{2})v(\tfrac{1}{2},-\tfrac{1}{2}) - k(\tfrac{1}{2},-\tfrac{1}{2})v(\tfrac{1}{2},\tfrac{1}{2})\right\}.$$

Solving the above for the k v's gives

$k(\frac{1}{2},\frac{1}{2})v(\frac{1}{2},\frac{1}{2}) = t(1,1)$, the k^+, p state

$k(\frac{1}{2},\frac{1}{2})v(\frac{1}{2},-\frac{1}{2}) = \frac{\sqrt{2}}{2}\left\{t(1,0) + t(0,0)\right\}$, the k^+, n state

$k(\frac{1}{2},-\frac{1}{2})v(\frac{1}{2},\frac{1}{2}) = \frac{\sqrt{2}}{2}\left\{t(1,0) - t(0,0)\right\}$, the k^o, p state

$k(\frac{1}{2},-\frac{1}{2})v(\frac{1}{2},-\frac{1}{2}) = t(1,-1)$, the k^o, n state.

The concept of charge independence of the strong forces means that the interactions are independent of the z-component of isotopic spin, but not of the total isotopic spin. Forming the three scattering matrix elements I, II, III and using the orthogonality of the t's gives

$$\mathcal{M}_{I} = \langle k^{0},p|\hat{S}|k^{+},n\rangle = \tfrac{1}{2}\left\{\langle t(1,0)|\hat{S}|t(1,0)\rangle - \langle t(0,0)|\hat{S}|t(0,0)\rangle\right\}$$
$$= \tfrac{1}{2}\left\{S(1)-S(0)\right\}$$

$$\mathcal{M}_{II} = \langle k^{+},p|\hat{S}|k^{+},n\rangle = \langle t(1,1)|\hat{S}|t(1,1)\rangle = S(1)$$

$$\mathcal{M}_{III} = \langle k^{+},n|\hat{S}|k^{+},n\rangle = \tfrac{1}{2}\left\{\langle t(1,0)|\hat{S}|t(1,0)\rangle + \langle t(0,0)|\hat{S}|t(0,0)\rangle\right\}$$
$$= \tfrac{1}{2}\left\{S(1)+S(0)\right\}.$$

These three equations give the desired relations between the scattering matrix elements. In particular it is clear that

$$S(0) = \mathcal{M}_{II} - 2\mathcal{M}_{I},$$

so that

$$\mathcal{M}_{III} = \tfrac{1}{2}\mathcal{M}_{II} + \tfrac{1}{2}\mathcal{M}_{II} - \mathcal{M}_{I}$$
$$= \mathcal{M}_{II} - \mathcal{M}_{I}.$$

Therefore, knowing $\mathcal{M}_{I}$ and $\mathcal{M}_{II}$ one can calculate $\mathcal{M}_{III}$.

The problem is now to choose a vertex function appropriate to reaction II. In reaction I it was assumed that the vertices were simple points (i.e., not "blobs") and contributed only coupling constants, masses and Dirac γ's to the matrix element. In the present work several different types of vertex functions were considered and their different effects on the cross-sections have been tabulated in Tables VIII-XVII. The vertex functions that gave the best cross section for the $k^+ + p \longrightarrow k^+ + p$ reaction were $i g_1 \gamma_5 f(\Delta^2, s, t) = i g_1 \gamma_5 / \sqrt{\Delta^2}$ for the pion-nucleon vertex and $i \mu g_2$ for the kaon-nucleon vertex. In particular this vertex function removed the zero value of the cross section in the forward direction. So for the calculation of $\mathcal{M}_{II}$ it will be assumed that

$$f(\Delta^2) = 1/\sqrt{\Delta^2} \quad ,$$

where $\sqrt{\Delta^2} = \frac{i}{M}\sqrt{2(\omega^2 - \mu^2)(1-\cos\theta)}$.

The matrix element for non-charge-exchange scattering $k^+ + p \longrightarrow k^+ + p$ is then

$$\mathcal{M}_{II} = -\frac{g_1 g_2 \mu}{(q^2 - M^2)} \frac{1}{\sqrt{\Delta^2}} \bar{U}_2 \gamma_5 U_1 \quad .$$

So the differential cross-section is

$$\left(\frac{d\sigma}{d\Omega}\right)_{II} = \frac{1}{4} \frac{g_1^2 g_2^2 \mu^2 M^2}{[\omega + (\omega^2 - \mu^2 + m^2)^{\frac{1}{2}}]^2 [\, 2(\omega^2 - \mu^2)(1-\cos\theta) + M^2]^2} \quad .$$

From the previous isotopic spin analysis of kaon-nucleon scattering, one has that

(I) $\langle k^0, p | \hat{S} | k^+, n \rangle = \frac{1}{2}\{S(1) - S(0)\} = \mathcal{M}_I$

(II) $\langle k^+, p | \hat{S} | k^+, p \rangle = S(1) = \mathcal{M}_{II}$

(III) $\langle k^+, n | \hat{S} | k^+, n \rangle = \frac{1}{2}\{S(1) + S(0)\} = \mathcal{M}_{III}$

where the matrix elements of $\hat{S}$ are equal to the matrix elements, $\mathcal{M}$, of each scattering. Therefore, for non-charge-exchange scattering $k^+ + n \longrightarrow k^+ + n$ the matrix can be evaluated as follows:

$$\frac{1}{2} S(0) = \frac{1}{2}\mathcal{M}_{II} - \mathcal{M}_I = -\frac{g_1 g_2 \mu}{(q^2 - M^2)} \frac{(1 - \sqrt{\Delta^2})}{2\sqrt{\Delta^2}} \bar{U}_2 \gamma_5 U_1$$

so that

$$_{\text{III}} = \tfrac{1}{2}\, S(1)+S(0) \quad = -\frac{g_1 g_2 \mu}{(q^2 - M^2)} \cdot \frac{(1-\sqrt{\Delta^2})}{\sqrt{\Delta^2}}\, \bar{u}_2 \gamma_5\, u_1,$$

and the differential cross-section is equal to

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{III}} = \tfrac{1}{2}\, \frac{g_1^2\, g_2^2\, \mu^2}{[\omega+(\omega^2-\mu^2+m^2)^{1/2}]^2 \quad [2(\omega^2-\mu^2)(1-\cos\,\theta)+M^2]} \quad .$$

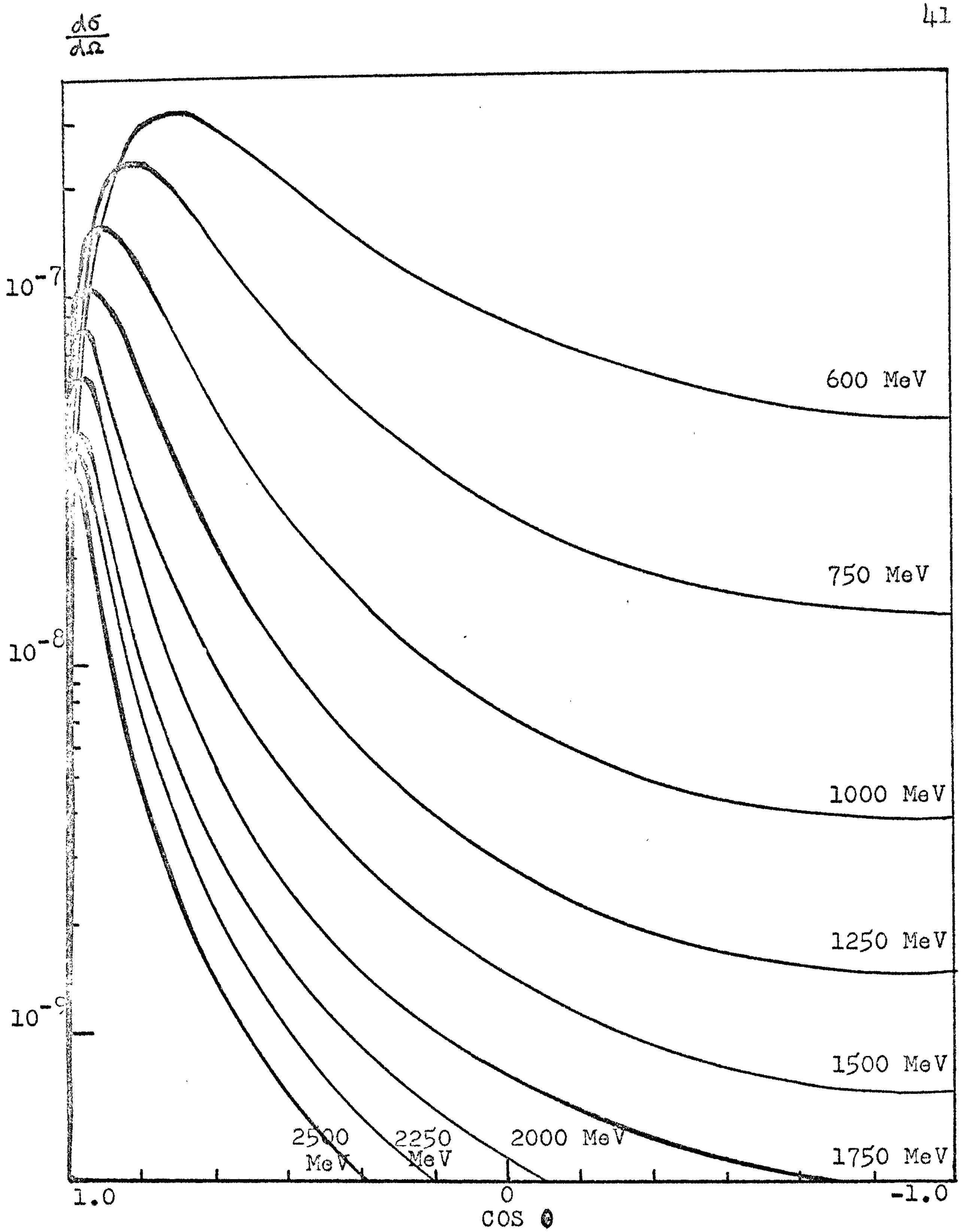


Fig. 11--Angular distribution for charge-exchange scattering $k^+ + n \longrightarrow k^0 + p$.

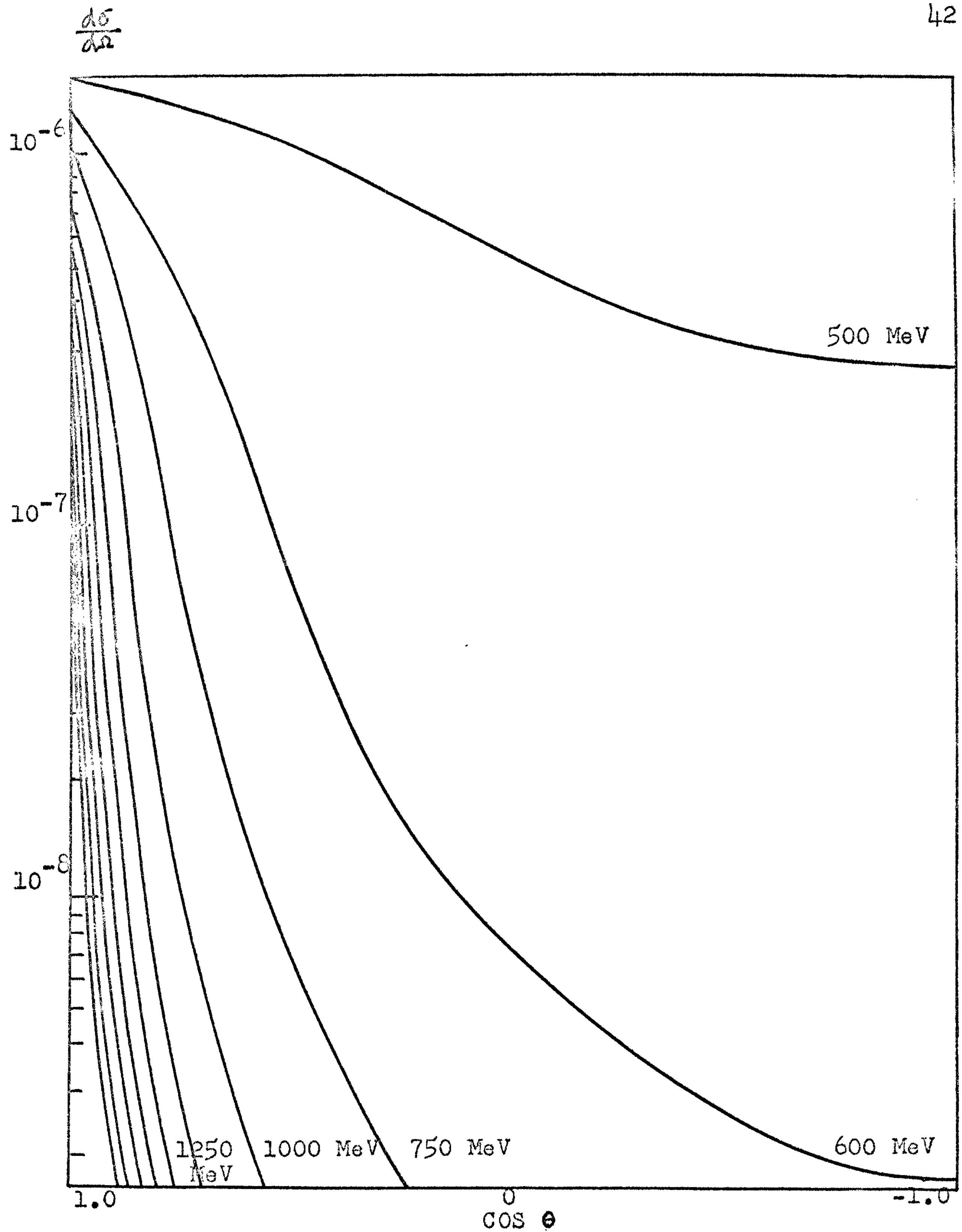


Fig. 12--Angular distribution for non-charge-exchange scattering $k^{+} + p \longrightarrow k^{+} + p$.

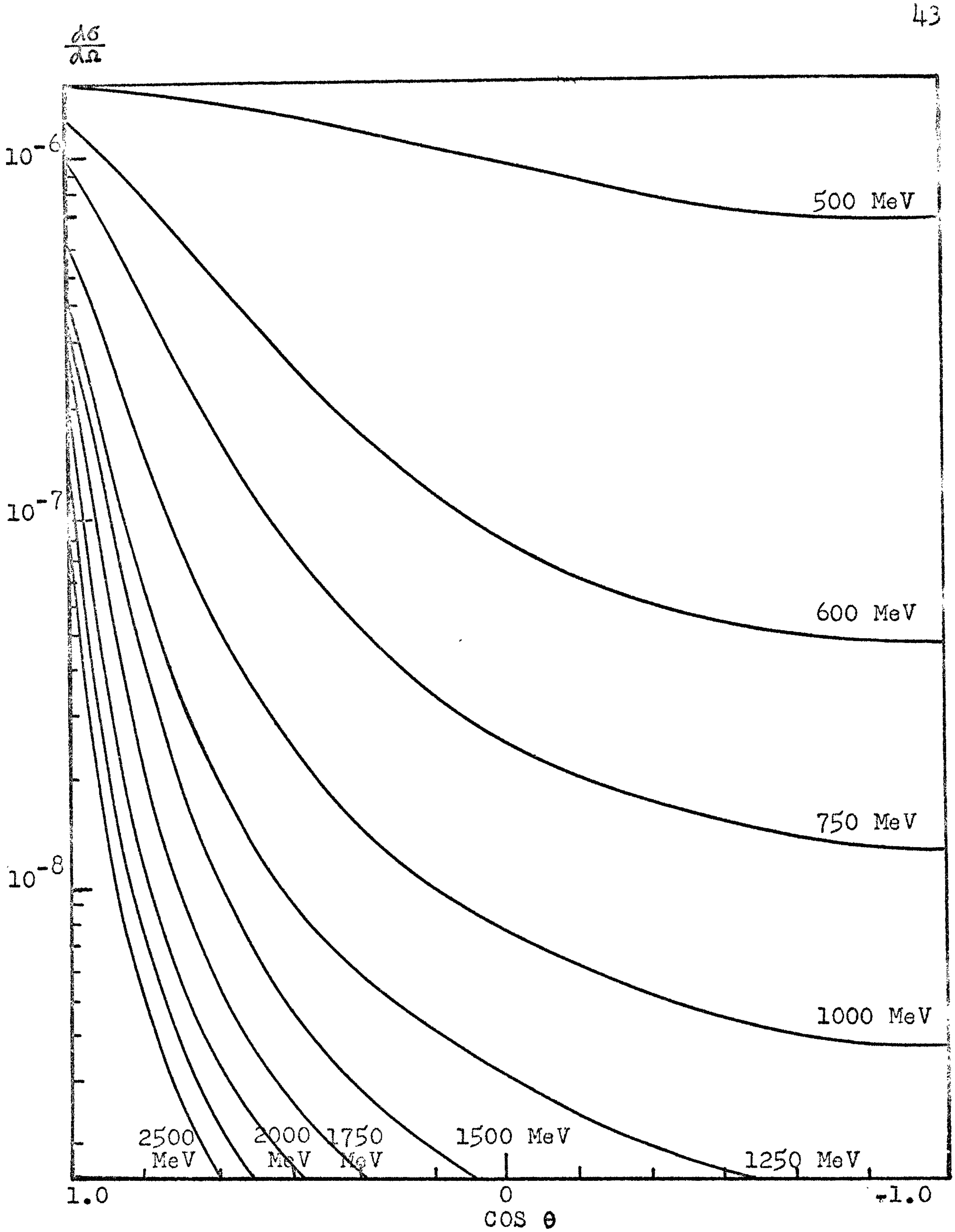


Fig. 13--Angular distribution for non-charge-exchange scattering $k^+ + n \longrightarrow k^+ + n$.

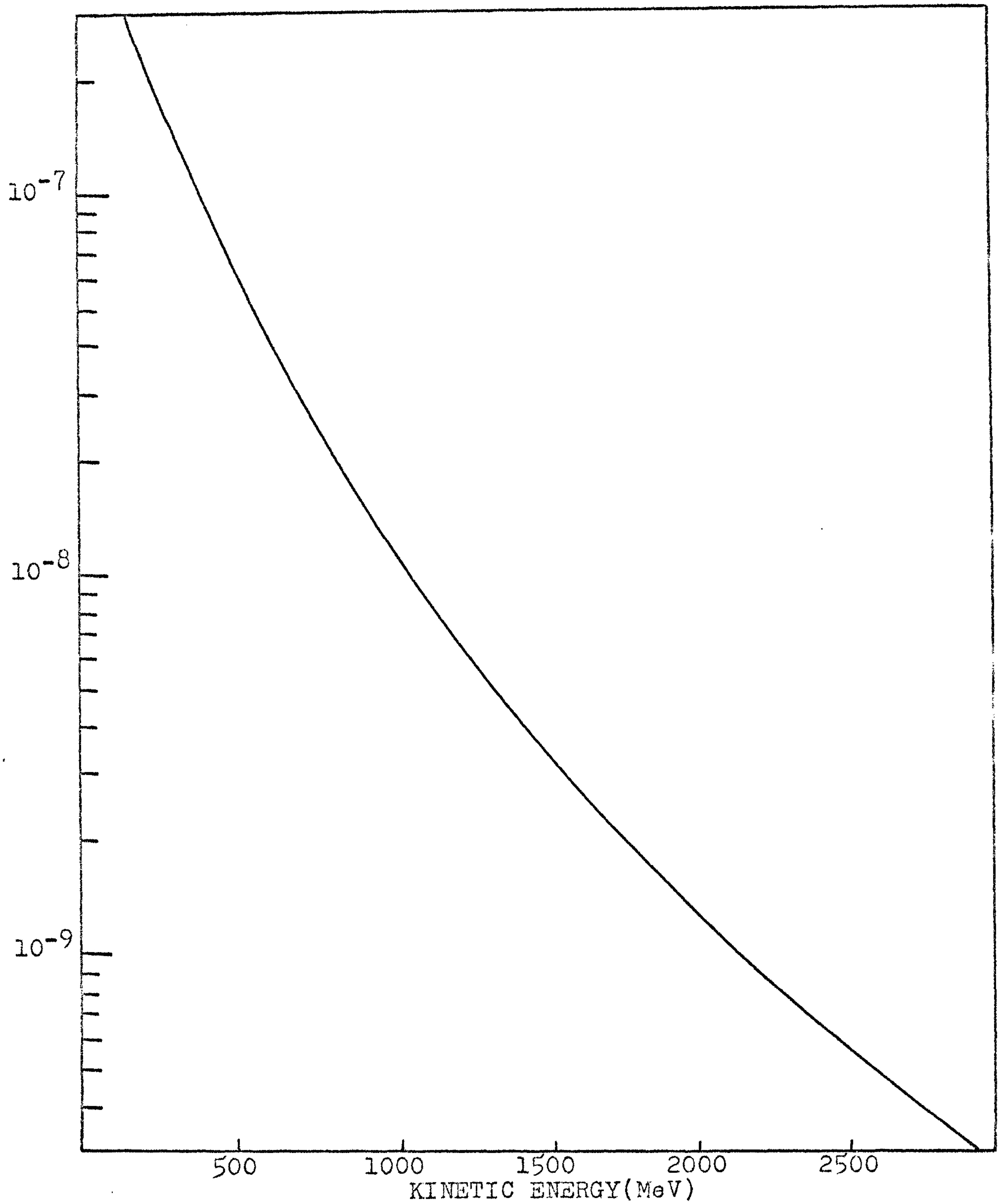


Fig. 14--Total cross-section for charge-exchange scattering $k^+ + n \longrightarrow k^0 + n$.

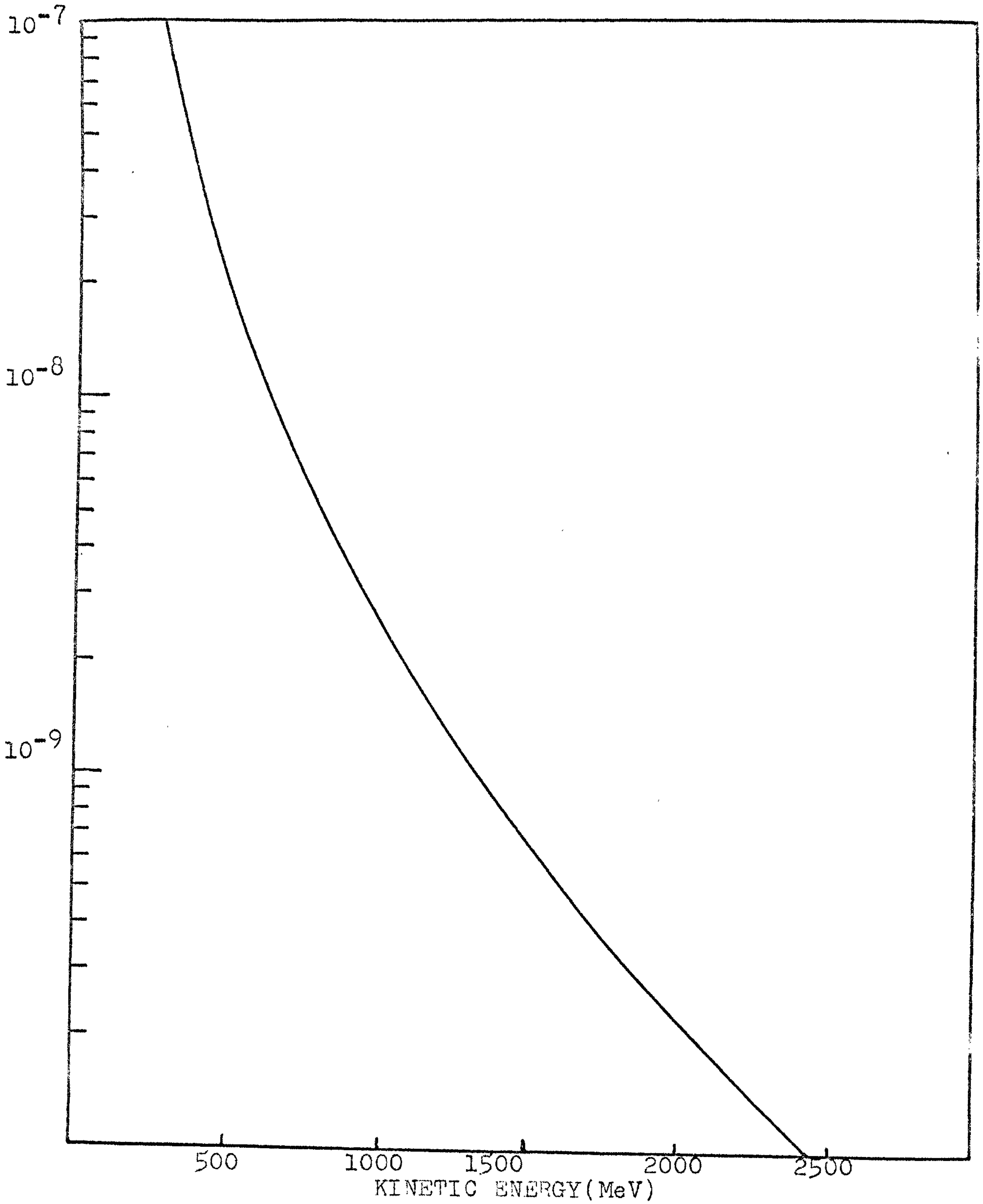


Fig. 15--Total cross-section for non-charge-exchange scattering $k^+ + p \longrightarrow k^+ + p$.

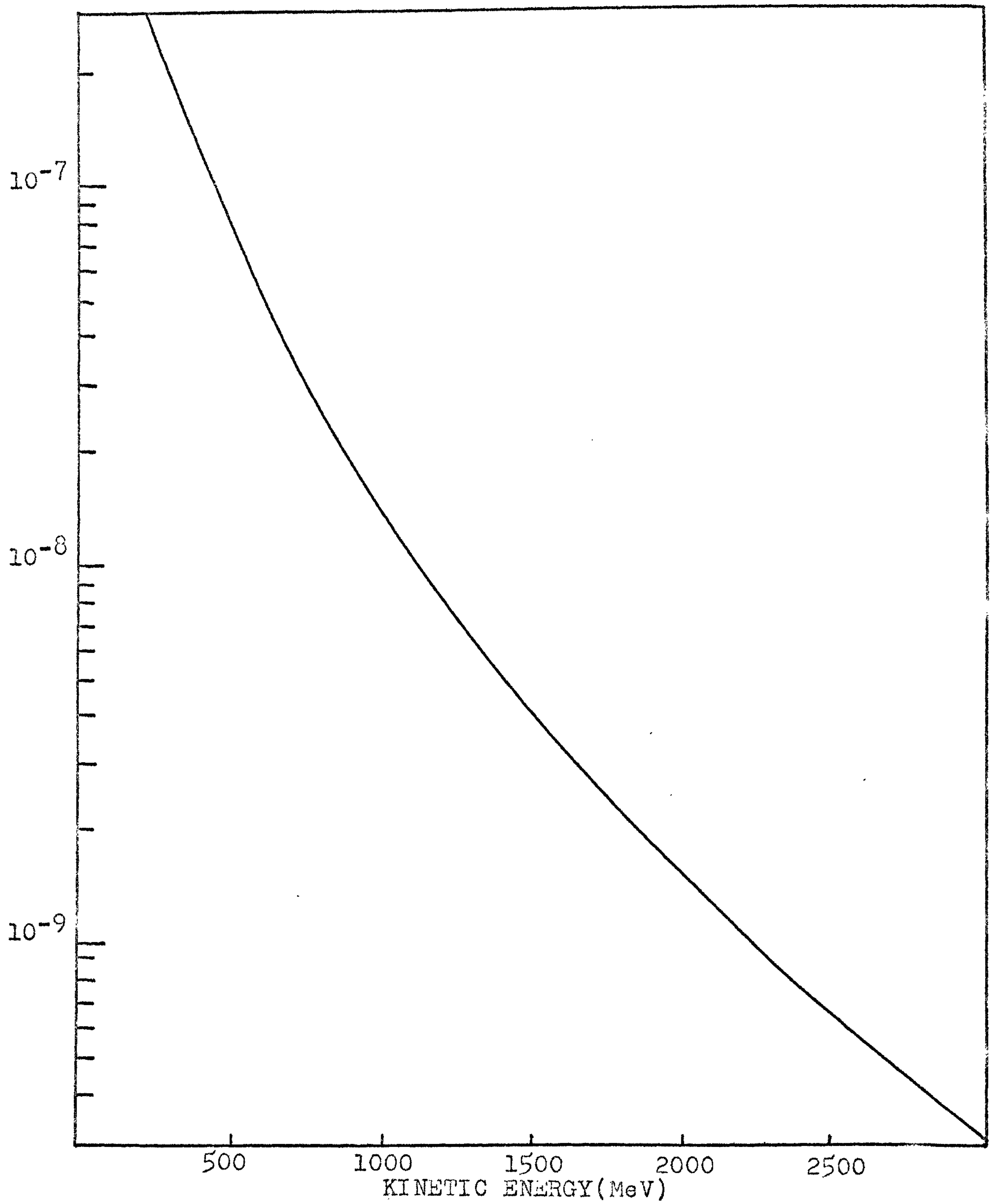


Fig. 16--Total cross-section for non-charge-exchange scattering $k^+ + n \longrightarrow k^+ + n$.

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CHAPTER V

ANALYSIS OF RESULTS OF EXCHANGE INTERACTION

The scattering data available for the three reactions considered here have recently been reviewed (2, p. 168; 1, p. 549; 6, p. 246; 7, p. B1113). The data on $k^+ + p \longrightarrow k^+ + p$ are probably the best because of the difficulty in interpreting kaon-neutron results from kaon-deuterium (or heavier elements) scattering. The primary facts to be explained are

(1) a nearly isotropic angular distribution for $k^+ + p \longrightarrow k^+ + p$ at energies below an energy value corresponding to lab momentum of about 800 MeV/c, i.e., at a lab momentum of 810 MeV/c some anisotropy is observed (4, p. 2743);

(2) a slightly varying, but roughly constant total cross-section for $k^+ + p \longrightarrow k^+ + p$ in the lab kinetic energy range 20 MeV to 20 BeV (6, p. 247);

(3) a differential cross-section for $k^+ + n \longrightarrow k^o + p$ that is peaked forward, that decreases strongly at very small angles, and has a peak that moves toward smaller angles with increasing energy (7, p. 1113);

(4) a total cross-section for $k^+ + n \longrightarrow k^o + p$ that increases fairly strongly with lab kinetic energy from 50 MeV to 500 MeV (6, p. 249);

(5) a roughly constant total cross-section for the

combined processes $k^+ + n \rightarrow k^o + p$ and $k^+ + n \rightarrow k^+ + n$ (6, p. 249).

(6) a differential scattering cross-section for $k^+ + n \rightarrow k^+ + n$ that possibly peaked backward (8, p179). This last fact is open to some question.

The strongest point for the present theory is the $k^+ + n \rightarrow k^o + p$ reaction. The matrix element chosen for this reaction in the previous chapter gives a differential cross-section in excellent agreement with the conditions set forth in number three above. This is to say that a single pion exchange model with a pion-nucleon vertex of the form $ig_2 \mu$ gives the proper angular dependence for the charge-exchange scattering. Since the $k^+ + p \rightarrow k^+ + p$ reaction did not show very small values of the differential cross-section for forward scattering it was assumed that the pion-nucleon vertex was of the form $ig_1 \gamma_5 / \sqrt{\Delta^2}$ and the corresponding differential cross-section calculated. The results are exhibited in Figs. 11-12 and Tables VIII-XVII. It will be observed that for increasing energy the anisotropy becomes more pronounced with very strongforward peaking at energies of the order of 1 BeV. There is, however, some anisotropy even at the lower energies and this must be considered a defect of the assumed model for $k^+ + p \rightarrow k^+ + p$ scattering. Using the matrix elements for $k^+ + p \rightarrow k^+ + p$ and $k^+ + n \rightarrow k^o + p$, the matrix element for $k^+ + n \rightarrow k^+ + n$ was calculated by the isotopic spin analysis and these results are presented in Fig.13 and Tables VIII-XVI. If one takes the results of Ceolin, et al. (3, P. 823) then the calculated differential cross-section is correct. It should be noted that the increasing isotropy (with increasing energy) is not so great for $k^+ + n \longrightarrow k^+ + n$ as for the $k^+ + p \longrightarrow k^+ + p$ reaction. Even the data for $k^+ + n \longrightarrow k^+ + n$ from Helmy, et al. (5, p. 179) are not at variance with the angular distribution found from this model. One might construe what Helmy, et al. report to mean that the non-charge-exchange cross-section is peaked backward. However, their actual statement is that the ratio of $\frac{d\sigma_{ex}}{d\Omega}$ to $\frac{d\sigma_{non}}{d\Omega}$ is decreasing in the back direction. This could be a consequence of a backward peaking of $\frac{d\sigma_{non}}{d\Omega}$, but could also be explained by $\frac{d\sigma_{ex}}{d\Omega}$ falling off move rapidly with increasing scattering angle than $\frac{d\sigma_{non}}{d\Omega}$. This latter is exactly what is observed in this case.

Considering the total cross-sections one can see that for all three reactions the values decrease with increasing energy (Table XVIII and Figures 14-16). This is in agreement with the measured values for the $k^+ + n \longrightarrow k^+ + n$ reaction, but in not good agreement with $k^+ + p \longrightarrow k^+ + p$ and in strong disagreement with $k^+ + n \longrightarrow k^o + p$ (6, p. 247). However, the numerical values are of the right order of magnitude; i.e., the measured total cross-sections have values that range between about 2 and 20 millibarns. The calculated values are, in natural units, on the order of 10^{-6} or 10^{-7} MeV^{-2} for low energies down to 10^{-8} to 10^{-9} MeV^{-2} for high energies, Table XVIII . To go from MeV^{-2} (natural units) to millibarns one multiplies by a factor of 4×10^{-22}, so that a choice of the coupling constants, g_1 and g_2 somewhat smaller than one will give qualitative agreement for the low energies. At the higher energies the calculated total cross-sections require coupling constants too small to be in agreement with the usual values. Thus, despite the number of sucesses of the present model, the decreasing total cross-sections for $k^+ + n \longrightarrow k^0 + p$ and $k^+ + p \longrightarrow k^+ + p$, and the lack of isotropy in $\frac{d\sigma}{d\Omega}$ for low energy $k^+ + p \longrightarrow k^+ + p$ scattering cause one to look for further modifications of the scattering matrix elements. In the present work two other possible types of quantities that might be used as part of trial vertex functions have been evaluated. They are $s = -(p_a + p_b)^2$ and $t = -(p_a - p_c)^2$, where p_a, p_b, and p_c are the four momenta for the reaction $a + b \longrightarrow c + d$. The numerical evaluations of these are presented in Tables XIX-XXX and Figs. 17-19. Of particular use for further investigation is the quantity t. Since it has a behavior very similar to the contribution of $\frac{1}{(q^2 - M^2)}$ to the matrix element (except it is not so strongly changing with increasing energy), adding a term like $\sqrt{t}$ to the vertex function for $k^+ + p \longrightarrow k^+ + p$ will tend to decrease the anisotropy at low energies. This is exactly what is desired, and this modification is recommended for further evaluation. Whether this will remove enough anisotropy, and what effect it will have on the total cross-section for the reaction $k^+ + p \longrightarrow k^+ + p$, the differential cross-section for the reaction $k^+ + n \longrightarrow k^+ + n$, and the total cross-section for the reaction $k^+ + n \longrightarrow k^+ + n$ only an actual evaluation can tell.

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APPENDIX

The following data in each Table were calculated and printed out directly by computor. Tables I-VII are for direct interaction. Tables VIII-XVIII are for exchange interaction. Tables XIX-XXX are the data of relativistic kinematic functions, in which Tables XIX, XXII, XXIII and XXIV are for the reaction $k^+ + p \longrightarrow k^+ + p$; Tables XX, XXV, XXVI and XXVII are for the reaction $k^+ + n \longrightarrow k^o + p$; Tables XXI, XXVIII, XXIX and XXX are for the reaction $k^+ + n \longrightarrow k^+ + n$.

TABLE I

DIFFERENTIAL SCATTERING CROSS SECTION AT W= 500 MEV

ANGLE DEGREES	NONCHARGE EXCHANGE PSEUDOSCALAR	NONCHARGE EXCHANGE PSEUDOVECTOR	CHARGE EXCHANGE r=0.1	CHARGE EXCHANGE r=1	CHARGE EXCHANGE r=10
0	1.5655789E-07	8.0749056E-05	1.7679762E-06	3.5328482E-05	2.9441520E-03
15	1.5660925E-07	8.0789637E-05	1.7687781E-06	3.5352736E-05	2.9463634E-03
30	1.5676006E-07	8.0908837E-05	1.7714278E-06	3.5424018E-05	2.9523629E-03
45	1.5700050E-07	8.1099075E-05	1.7756576E-06	3.5537873E-05	2.9632459E-03
60	1.5731500E-07	8.1348180E-05	1.7811988E-06	3.5687134E-05	2.9768603E-03
75	1.5768281E-07	8.1639960E-05	1.7876927E-06	3.5862229E-05	2.9928343E-03
90	1.5807946E-07	8.1955110E-05	1.7947110E-06	3.6051657E-05	3.0101209E-03
105	1.5847809E-07	8.2272389E-05	1.8017811E-06	3.6242697E-05	3.0275588E-03
120	1.5885137E-07	8.2569974E-05	1.8084166E-06	3.6422175E-05	3.0439456E-03
135	1.5917332E-07	8.2827025E-05	1.8141513E-06	3.6577439E-05	3.0581252E-03
150	1.5942123E-07	8.3025200E-05	1.8185746E-06	3.6697292E-05	3.0690727E-03
165	1.5957750E-07	8.3150219E-05	1.8213660E-06	3.6772964E-05	3.0759855E-03
180	1.5963087E-07	8.3192928E-05	1.8223197E-06	3.6798828E-05	3.0783487E-03
TOT C.S.	1.9751871E-06	1.0240546E-03	2.2425633E-05	4.5049732E-04	3.7614643E-02

TABLE II

DIFFERENTIAL SCATTERING CROSS SECTION AT ω= 600 MEV

ANGLE DEGREES	NONCHARGE EXCHANGE PSEUDOSCALAR	NONCHARGE EXCHANGE PSEUDOVECTOR	CHARGE EXCHANGE r=0.1	CHARGE EXCHANGE r=1	CHARGE EXCHANGE r=10
0	1.1885675E-07	6.3198768E-05	1.3767002E-06	2.3850151E-05	1.8538400E-03
15	1.1940541E-07	6.3693401E-05	1.3870848E-06	2.4071435E-05	1.8726906E-03
30	1.2104358E-07	6.5181181E-05	1.4183552E-06	2.4739506E-05	1.9296597E-03
45	1.2374423E-07	6.7669537E-05	1.4707774E-06	2.5865144E-05	2.0258353E-03
60	1.2745005E-07	7.1156151E-05	1.5444972E-06	2.7459597E-05	2.1624472E-03
75	1.3205539E-07	7.5605151E-05	1.6390423E-06	2.9522941E-05	2.3398408E-03
90	1.3738265E-07	8.0911729E-05	1.7525498E-06	3.2025545E-05	2.5558308E-03
105	1.4315774E-07	8.6858030E-05	1.8807455E-06	3.4882631E-05	2.8034091E-03
120	1.4899406E-07	9.3071549E-05	2.0158900E-06	3.7926733E-05	3.0682296E-03
135	1.5439932E-07	9.9008752E-05	2.1462045E-06	4.0890647E-05	3.3269895E-03
150	1.5882045E-07	1.0399526E-04	2.2565702E-06	4.3421111E-05	3.5485533E-03
165	1.6173164E-07	1.0734261E-04	2.3311374E-06	4.5140667E-05	3.6994278E-03
180	1.6274915E-07	1.0852454E-04	2.3575598E-06	4.5751816E-05	3.7531078E-03
TOT C.S.	1.7304680E-06	1.0310107E-03	2.2359901E-05	4.1133212E-04	3.2931615E-02

TABLE III

DIFFERENTIAL SCATTERING CROSS SECTION AT W= 750 MEV

ANGLE DEGREES	NONCHARGE EXCHANGE PSEUDOSCALAR	NONCHARGE EXCHANGE PSEUDOVECTOR	CHARGE EXCHANGE		
			r=0.1	r=1	r=10
0	8.3834501E-08	3.9515272E-05	8.8123700E-07	1.5623747E-05	1.1855554E-03
15	8.4547404E-08	4.0184085E-05	8.9447961E-07	1.5886897E-05	1.2070848E-03
30	8.6709188E-08	4.2246203E-05	9.3530686E-07	1.6700180E-05	1.2737244E-03
45	9.0385549E-08	4.5870473E-05	1.0070768E-06	1.8136690E-05	1.3917788E-03
60	9.5671906E-08	5.1340850E-05	1.1155263E-06	2.0322337E-05	1.5721594E-03
75	1.0266427E-07	5.9045058E-05	1.2687154E-06	2.3436416E-05	1.8305224E-03
90	1.1140170E-07	6.9420284E-05	1.4762261E-06	2.7696815E-05	2.1861270E-03
105	1.2176465E-07	8.2800682E-05	1.7464429E-06	3.3303945E-05	2.6571551E-03
120	1.3332140E-07	9.9094090E-05	2.0800947E-06	4.0301665E-05	3.2487855E-03
135	1.4515135E-07	1.1726578E-04	2.4587741E-06	4.8323185E-05	3.9310401E-03
150	1.5575622E-07	1.3483540E-04	2.8318798E-06	5.6293577E-05	4.6123901E-03
165	1.6325408E-07	1.4798546E-04	3.1157678E-06	6.2395521E-05	5.1359516E-03
180	1.6597931E-07	1.5291420E-04	3.2232112E-06	6.4712453E-05	5.3351405E-03
TOT C.S.	1.4449356E-06	9.6937713E-04	2.0608361E-05	3.9274082E-04	3.1326959E-02

TABLE IV

DIFFERENTIAL SCATTERING CROSS SECTION AT W= 1000 MEV

ANGLE DEGREES	NONCHARGE EXCHANGE PSEUDOSCALAR	NONCHARGE EXCHANGE PSEUDOVECTOR	CHARGE EXCHANGE r=0.1	CHARGE EXCHANGE r=1	CHARGE EXCHANGE r=10
0	5.2085439E-08	1.3770379E-05	3.6662365E-07	8.9121832E-06	7.1564191E-04
15	5.2706927E-08	1.4281996E-05	3.7631739E-07	9.1280939E-06	7.3348931E-04
30	5.4617608E-08	1.5906038E-05	4.0696652E-07	9.8094059E-06	7.8985478E-04
45	5.7959986E-08	1.8932495E-05	4.6368046E-07	1.1065382E-05	8.9393550E-04
60	6.2983013E-08	2.3924426E-05	5.5643684E-07	1.3108794E-05	1.0636800E-03
75	7.0052884E-08	3.1852440E-05	7.0263937E-07	1.6309467E-05	1.3303855E-03
90	7.9647132E-08	4.4295711E-05	9.3122356E-07	2.1280011E-05	1.7461106E-03
105	9.2286414E-08	6.3643908E-05	1.2876634E-06	2.8979869E-05	2.3928318E-03
120	1.0830163E-07	9.2975937E-05	1.8347226E-06	4.0729073E-05	3.3842184E-03
135	1.2726699E-07	1.3465400E-04	2.6303208E-06	5.7739436E-05	4.8264453E-03
150	1.4702247E-07	1.8604842E-04	3.6435974E-06	7.9340849E-05	6.6664346E-03
165	1.6292032E-07	2.3330133E-04	4.6071632E-06	9.9852658E-05	8.4203336E-03
180	1.6915922E-07	2.5327779E-04	5.0237024E-06	1.0871537E-04	9.1798448E-03
TOT C.S.	1.1060461E-06	8.4127738E-04	1.7152390E-05	3.8340924E-04	3.1807599E-02

TABLE V

DIFFERENTIAL SCATTERING CROSS SECTION AT W= 1250 MEV

ANGLE DEGREES	NONCHARGE EXCHANGE PSEUDOSCALAR	NONCHARGE EXCHANGE PSEUDOVECTOR	CHARGE EXCHANGE r=0.1	CHARGE EXCHANGE r=1	CHARGE EXCHANGE r=10
0	3.5272813E-08	2.2083230E-06	1.1311905E-07	5.3857939E-06	4.8363462E-04
15	3.5756282E-08	2.5326032E-06	1.1893907E-07	5.5446979E-06	4.9740824E-04
30	3.7253334E-08	3.5870809E-06	1.3773062E-07	6.0521304E-06	5.4135497E-04
45	3.9911556E-08	5.6470033E-06	1.7398479E-07	7.0108692E-06	6.2425536E-04
60	4.4003534E-08	9.2868451E-06	2.3709551E-07	8.6328101E-06	7.6419685E-04
75	4.9969609E-08	1.5611813E-05	3.4525495E-07	1.1320893E-05	9.9554516E-04
90	5.8477984E-08	2.6719992E-05	5.3349831E-07	1.5836169E-05	1.3831747E-03
105	7.0478409E-08	4.6555663E-05	8.6947800E-07	2.3620804E-05	2.0500493E-03
120	8.7142492E-08	8.2178537E-05	1.4800911E-06	3.7333553E-05	3.2232000E-03
135	1.0934320E-07	1.4420135E-04	2.5731072E-06	6.1255286E-05	5.2694958E-03
150	1.3591221E-07	2.4024510E-04	4.3418467E-06	9.9242017E-05	8.5229772E-03
165	1.6041526E-07	3.4983216E-04	6.4672058E-06	1.4436185E-04	1.2395671E-02
180	1.7092585E-07	4.0300403E-04	7.5376737E-06	1.6696982E-04	1.4339491E-02
TOT C.S.	8.7394089E-07	7.9788196E-04	1.4937419E-05	3.7687491E-04	3.2594088E-02

TABLE VI

DIFFERENTIAL SCATTERING CROSS SECTION AT W= 1500 MEV

ANGLE DEGREES	NONCHARGE EXCHANGE PSEUDOSCALAR	NONCHARGE EXCHANGE PSEUDOVECTOR	CHARGE EXCHANGE		
			r=0.1	r=1	r=10
0	2.5353702E-08	2.0687977E-07	1.2575599E-08	3.3051585E-06	3.4600196E-04
15	2.5727407E-08	4.4803389E-07	1.6147137E-08	3.4198731E-06	3.5661479E-04
30	2.6889422E-08	1.2420516E-06	2.7904137E-08	3.7892963E-06	3.9068053E-04
45	2.8970972E-08	2.8314635E-06	5.1441751E-08	4.4994469E-06	4.5575803E-04
60	3.2221639E-08	5.7424104E-06	9.4614690E-08	5.7338558E-06	5.6790006E-04
75	3.7064688E-08	1.1048829E-05	1.7364561E-07	7.8607017E-06	7.5912506E-04
90	4.4192819E-08	2.0968649E-05	3.2265549E-07	1.1631294E-05	1.0943868E-03
105	5.4715486E-08	4.0201568E-05	6.1599417E-07	1.8636900E-05	1.7105470E-03
120	7.0317661E-08	7.8780643E-05	1.2194492E-06	3.2338583E-05	2.9041818E-03
135	9.3119248E-08	1.5674939E-04	2.4886898E-06	6.0007847E-05	5.2974523E-03
150	1.2396356E-07	3.0293578E-04	5.0117613E-06	1.1342630E-04	9.9001128E-03
165	1.5656361E-07	5.0821857E-04	8.8165590E-06	1.9258800E-04	1.6716785E-02
180	1.7199079E-07	6.2351408E-04	1.1072980E-05	2.3918854E-04	2.0733084E-02
TOT C.S.	7.0828388E-07	8.8868225E-04	1.4517299E-05	3.7035625E-04	3.3094867E-02

TABLE VII

TOTAL SCATTERING CROSS SECTIONS FOR DIRECT INTERACTION

ENERGY W(C.M.)	KINETIC ENERGY	NONCHARGE EXCHANGE PSEUDOSCALAR	NONCHARGE EXCHANGE PSEUDOVECTOR	CHARGE EXCHANGE		
				r=0.1	r=1	r=10
MeV	MeV	MeV^{-2}	MeV^{-2}	MeV^{-2}		
500	77	1.9759861×10^{-6}	1.0250543×10^{-3}	2.2425633×10^{-5}	4.5049732×10^{-4}	3.7614643×10^{-2}
600	360	1.7304559×10^{-6}	1.0310102×10^{-3}	2.2359901×10^{-5}	4.1133212×10^{-4}	3.2931615×10^{-2}
750	560	1.4440340×10^{-6}	9.6037676×10^{-4}	$2.06\text{-}8361 \times 10^{-5}$	3.9274082×10^{-4}	3.1426959×10^{-2}
1000	870	1.1060455×10^{-6}	8.4127587×10^{-4}	1.7152390×10^{-5}	3.8240924×10^{-4}	3.1807599×10^{-2}
1250	1140	8.7394016×10^{-7}	7.9788100×10^{-4}	1.4937192×10^{-5}	3.7587491×10^{-4}	3.2594988×10^{-2}
1500	1420	7.0828435×10^{-7}	8.8868112×10^{-4}	1.4517299×10^{-5}	3.7025825×10^{-4}	3.3094857×10^{-2}

AFIT FORTRAN PROGRAM FOR

EXCHANGE INTERACTION

```
      DIMENSION TOT(3)
    1 READ,XM,YM,ZM,WM,U,G
    2 FORMAT(/////40X,5HTABLE)
   52 FORMAT(///18X,43HDIFFERENTIAL SCATTERING CROSS SECTION AT W=I5)
    5 FORMAT(67X,3HMEV)
    6 FORMAT(////4X5HANGLE,11X10HFOR CHARGE,13X13HFOR NONCHAGRE)
    3 FORMAT(71X,13HFOR NONCHARGE)
    8 FORMAT(/2X,47H(DEGREES)    EXCHANGE K + --K +P          EXCHANGE)
    9 FORMAT(50X,36HK +P--K +P          EXCHANGE K+   --K +/)
      G=G*G
      DO 15 IW=500,2500,250
      W=IW
      Z=CON(971.1)
      PRINT 2
      PRINT 52,IW
    4 PRINT 5
      PRINT 6
      PRINT 3
      PRINT 8
      PRINT 9
      WW=W*W
      DO7 LL=1,3
    7 TOT(LL)=0.
      UU=U*U
      XMM=XM*XM
      YMM=YM*YM
      ZMM=ZM*ZM
      WMM=WM*WM
      A=WW-UU
      B=SQRTF(A+YMM)
      C=YMM-UU
      S=(W+B)*(W+B)
      Q=(W+P)*(W+P)
      ANSA=(G*G*UU)/(2.*S)
      ANSB=(G*G*UU)/(2.*Q)
   16 NA=0
      NB=5
      NC=1
      DELT=NC
      ND=0
      DO10 IT=NA,NB,NC
      T=IT
      T=T*.017453293
      D=1.-COSF(T)
      DEN=2.*A*E+XMM
      DEF=2.*A*D+ZMM
```

```
      ANS1=(ANSA*A*D)/(DEN*DEN)
      TOT(1)=TOT(1)+ANS1*SIN(T)*DELT*6.293154*0.017453293
      ANS2=(ANSA*ZMM)/(2.*DEF*DEF)
      TOT(2)=TOT(2)+ANS2*SIN(T)*DELT*6.293154*0.017453293
      ANS3=(ANSB)/(2.*DEF)
      TOT(3)=TOT(3)+ANS3*SIN(T)*DELT*6.293154*0.017453293
      ND=ND+1
      IF(ND-6)10,26,25
   25 IF(ND-11)10,27,10
   26 NA=10
      NB=30
      NC=5
      DELT=NC
      GO TO 10
   27 NA=45
      NB=180
      NC=15
      DELT=NC
      PRINT 53,IT,ANS1,ANS2,ANS3
   53 FORMAT(/5XI4,6XE15.7,13XE15.7,13XE15.7)
   54 FORMAT(//6X5HTOTAL)
   55 FORMAT(/2X14HCROSS SECTION=E14.7,13XE15.7,13XE15.7)
      PRINT 54
      PRINT 55, TOT(1),TOT(2),TOT(3)
      IF(W-500.)15,17,15
      W=600
      Z=CON(971.1)
      PRINT 2
      MW=W
      PRINT52,MW
      GO TO 4
      CONTINUE
      GO TO 1
      END
```

TABLE VIII

DIFFERENTIAL SCATTERING CROSS SECTION AT W= 500 MEV

ANGLE (DEGREES)	FOR CHARGE EXCHANGE $K^+ + n \to K^0 + P$	FOR NONCHARGE EXCHANGE $K^+ + P \to K^+ + P$	FOR NONCHARGE EXCHANGE $K^+ + n \to K^+ + n$
0	.0000000E-99	1.6083301E-06	1.6112896E-06
1	1.3867643E-10	1.6080094E-06	1.6111289E-06
2	5.5403370E-10	1.6070484E-06	1.6106474E-06
3	1.2452402E-09	1.6054487E-06	1.6098456E-06
4	2.2104913E-09	1.6032140E-06	1.6087248E-06
5	3.4473815E-09	1.6003487E-06	1.6072865E-06
10	1.3574631E-08	1.5768173E-06	1.5954261E-06
15	2.9764360E-08	1.5389253E-06	1.5761399E-06
20	5.1069491E-08	1.4885075E-06	1.5501064E-06
25	7.6319763E-08	1.4278491E-06	1.5181936E-06
30	1.0424166E-07	1.3594690E-06	1.4813942E-06
45	1.9201509E-07	1.1325186E-06	1.3520996E-06
60	2.6749939E-07	9.1300764E-07	1.2140126E-06
75	3.2036926E-07	7.2923630E-07	1.0849760E-06
90	3.5173657E-07	5.8755593E-07	9.7389158E-07
105	3.6731624E-07	4.8348517E-07	8.8344108E-07
120	3.7309927E-07	4.0953113E-07	8.1307287E-07
135	3.7373915E-07	3.5877313E-07	7.6101981E-07
150	3.7236767E-07	3.2596122E-07	7.2538568E-07
165	3.7088561E-07	3.0758712E-07	7.0464453E-07
180	3.7028905E-07	3.0167419E-07	6.9783876E-07
TOTAL CROSS SECTION=	3.7734124E-06	8.3214581E-0[illegible]	1.2461842E-05

TABLE IX

DIFFERENTIAL SCATTERING CROSS SECTION AT W= 600 MEV

ANGLE (DEGREES)	FOR CHARGE EXCHANGE $K^+ + n \to K^0 + P$	FOR NONCHARGE EXCHANGE $K^+ + P \to K^+ + P$	FOR NONCHARGE EXCHANGE $K^+ + n \to K^+ + n$
0	.0000000E-99	1.3087140E-06	1.3107626E-06
1	2.1866203E-09	1.3036526E-06	1.3082255E-06
2	8.6471767E-09	1.2886559E-06	1.3006790E-06
3	1.9109939E-08	1.2642313E-06	1.2882938E-06
4	3.3139608E-08	1.2311965E-06	1.2713507E-06
5	5.0176063E-08	1.1906073E-06	1.2502186E-06
10	1.5717416E-07	9.1901049E-07	1.0984035E-06
15	2.4919457E-07	6.3675953E-07	9.1430143E-07
20	2.9572927E-07	4.1893354E-07	7.4160772E-07
25	3.0341800E-07	2.7229548E-07	5.9789083E-07
30	2.8863161E-07	1.7886704E-07	4.8458151E-07
45	2.1147523E-07	5.8562188E-08	2.7727475E-07
60	1.5050353E-07	2.4140479E-08	1.7802238E-07
75	1.1180193E-07	1.2025369E-08	1.2564664E-07
90	8.7436511E-08	6.9465378E-09	9.5496154E-08
105	7.1739485E-08	4.5180512E-09	7.7015341E-08
120	6.1441382E-08	3.2429484E-09	6.5248675E-08
135	5.4692786E-08	2.5343272E-09	5.7681031E-08
150	5.0439345E-08	2.1370285E-09	5.2967177E-08
165	4.8087736E-08	1.9332882E-09	5.0379050E-08
180	4.7334912E-08	1.8704229E-09	4.9553186E-08
TOTAL CROSS SECTION=	1.3672035E-06	5.5529235E-07	1.9647286E-06

TABLE X

DIFFERENTIAL SCATTERING CROSS SECTION AT W= 750 MEV

ANGLE (DEGREES)	FOR CHARGE EXCHANGE $K^+ + n \to K^0 + P$	FOR NONCHARGE EXCHANGE $K^+ + P \to K^+ + P$	FOR NONCHARGE EXCHANGE $K^+ + n \to K^+ + n$
0	.0000000E-99	9.8235981E-07	9.8357418E-07
1	4.4793014E-09	9.7197865E-07	9.7836338E-07
2	1.7387973E-08	9.4182921E-07	9.6307011E-07
3	3.7286221E-08	8.9461055E-07	9.3861791E-07
4	6.2089920E-08	8.3426559E-07	9.0640855E-07
5	8.9463451E-08	7.6527495E-07	8.6812162E-07
10	2.0212340E-07	4.1913470E-07	6.4246377E-07
15	2.2776913E-07	2.0467021E-07	4.4895120E-07
20	2.0434842E-07	1.0172162E-07	3.1650347E-07
25	1.7000321E-07	5.3768504E-08	2.3011016E-07
30	1.3875014E-07	3.0425996E-08	1.7309887E-07
45	7.8589232E-08	7.7811460E-09	8.7537393E-08
60	4.9888509E-08	2.8783767E-09	5.3240932E-08
75	3.4961452E-08	1.3572966E-09	3.6560255E-08
90	2.6455864E-08	7.6018656E-10	2.7360997E-08
105	2.1276813E-08	4.8526134E-10	2.1860478E-08
120	1.7993581E-08	3.4421466E-10	1.8411387E-08
135	1.5887996E-08	2.6697174E-10	1.6214534E-08
150	1.4578845E-08	2.2406391E-10	1.4854491E-08
165	1.3860851E-08	2.0218065E-10	1.4110474E-08
180	1.3631860E-08	1.9544572E-10	1.3873464E-08
TOTAL CROSS SECTION=	5.2713630E-07	1.5176585E-07	6.9109448E-07

TABLE XI

DIFFERENTIAL SCATTERING CROSS SECTION AT W= 1000 MEV

ANGLE (DEGREES)	FOR CHARGE EXCHANGE $K^{+}+n \rightarrow K^{0}+P$	FOR NONCHARGE EXCHANGE $K^{+}+P \rightarrow K^{+}+P$	FOR NONCHARGE EXCHANGE $K^{+}+n \rightarrow K^{+}+n$
0	.0000000E-99	6.4389475E-07	6.4444631E-07
1	6.8759833E-09	6.2791699E-07	6.3640035E-07
2	2.5666910E-08	5.8343281E-07	6.1344372E-07
3	5.1771495E-08	5.1913714E-07	5.7865585E-07
4	7.9727997E-08	4.4559701E-07	5.3610571E-07
5	1.0503604E-07	3.7196860E-07	4.8981541E-07
10	1.4874302E-07	1.2602828E-07	2.8511056E-07
15	1.1953177E-07	4.3969548E-08	1.6840512E-07
20	8.7188251E-08	1.7867842E-08	1.0735329E-07
25	6.3927191E-08	8.3667384E-09	7.3461094E-08
30	4.8161965E-08	4.3874323E-09	5.3196678E-08
45	2.4301005E-08	1.0060921E-09	2.5474060E-08
60	1.4748511E-08	3.5682542E-10	1.5170755E-08
75	1.0115009E-08	1.6490219E-10	1.0313178E-08
90	7.5642114E-09	9.1344046E-11	7.6757252E-09
105	6.0407218E-09	5.7927300E-11	6.1125291E-09
120	5.0861085E-09	4.0921641E-11	5.1375472E-09
135	4.4783659E-09	3.1655899E-11	4.5186283E-09
150	4.1022336E-09	2.6525267E-11	4.1362728E-09
165	3.8965063E-09	2.3913544E-11	3.9273651E-09
180	3.8309764E-09	2.3110450E-11	3.8608551E-09
TOTAL CROSS SECTION=	1.8268046E-07	4.0483914E-08	2.2642731E-07

TABLE XII

DIFFERENTIAL SCATTERING CROSS SECTION AT W= 1250 MEV

ANGLE (DEGREES)	FOR CHARGE EXCHANGE $K^+ + n \to K^0 + P$	FOR NONCHARGE EXCHANGE $K^+ + P \to K^+ + P$	FOR NONCHARGE EXCHANGE $K^+ + n \to K^+ + n$
0	.0000000E-99	4.4796540E-07	4.4824145E-07
1	8.2007614E-09	4.2884457E-07	4.3857082E-07
2	2.9163230E-08	3.7833115E-07	4.1193228E-07
3	5.4751089E-08	3.1197347E-07	3.7406632E-07
4	7.7438028E-08	2.4490440E-07	3.3142714E-07
5	9.3285352E-08	1.8631532E-07	2.8907735E-07
10	9.1886717E-08	4.3797673E-08	1.4015716E-07
15	6.1234310E-08	1.2738530E-08	7.5587407E-08
20	4.0690237E-08	4.7358816E-09	4.6088248E-08
25	2.8366053E-08	2.1142418E-09	3.0794070E-08
30	2.0743931E-08	1.0780685E-09	2.1989379E-08
45	1.0053950E-08	2.3807072E-10	1.0333388E-08
60	6.0121029E-09	8.3275760E-11	6.1115178E-09
75	4.0947009E-09	3.8235990E-11	4.1411975E-09
90	3.0505734E-09	2.1105733E-11	3.0767343E-09
105	2.4307303E-09	1.3356740E-11	2.4475976E-09
120	2.0437559E-09	9.4233982E-12	2.0558600E-09
135	1.7979571E-09	7.2837013E-12	1.8074481E-09
150	1.6460508E-09	6.1000970E-12	1.6540862E-09
165	1.5630352E-09	5.4979459E-12	1.5703266E-09
180	1.5366029E-09	5.3128381E-12	1.5436650E-09
TOTAL CROSS SECTION=	8.2052200E-08	1.5931593E-08	9.9212489E-08

TABLE XIII

DIFFERENTIAL SCATTERING CROSS SECTION AT W= 1500 MEV

ANGLE (DEGREES)	FOR CHARGE EXCHANGE $K^{+}+n \to K^{0}+P$	FOR NONCHARGE EXCHANGE $K^{+}+P \to K^{+}+P$	FOR NONCHARGE EXCHANGE $K^{+}+n \to K^{+}+n$
0	.0000000E-99	3.2698795E-07	3.2713821E-07
1	8.9195688E-09	3.0610487E-07	3.1651953E-07
2	2.9972909E-08	2.5422902E-07	2.8845493E-07
3	5.2016511E-08	1.9298115E-07	2.5131757E-07
4	6.7504299E-08	1.3855144E-07	2.1294671E-07
5	7.4838277E-08	9.6824420E-08	1.7801551E-07
10	5.5980966E-08	1.7325513E-08	7.5302339E-08
15	3.3345875E-08	4.5248653E-09	3.8482938E-08
20	2.1077401E-08	1.6044981E-09	2.2915799E-08
25	1.4321628E-08	6.9924286E-10	1.5127927E-08
30	1.0320810E-08	3.5168341E-10	1.0728557E-08
45	4.9064599E-09	7.6273474E-11	4.9963420E-09
60	2.9136032E-09	2.6507241E-11	2.9454229E-09
75	1.9779503E-09	1.2134016E-11	1.9928182E-09
90	1.4710013E-09	6.6868837E-12	1.4793716E-09
105	1.1708962E-09	4.2277091E-12	1.1762996E-09
120	9.8385374E-10	2.9809283E-12	9.8773665E-10
135	8.6517360E-10	2.3031960E-12	8.6822213E-10
150	7.9187663E-10	1.9284728E-12	7.9446023E-10
165	7.5183618E-10	1.7378871E-12	7.5418202E-10
180	7.3908950E-10	1.6793064E-12	7.4136209E-10
TOTAL CROSS SECTION=	4.2552286E-08	7.6876883E-09	5.0805993E-08

TABLE XIV

DIFFERENTIAL SCATTERING CROSS SECTION AT W= 1750 MEV

ANGLE (DEGREES)	FOR CHARGE EXCHANGE $K^+ + n \to K^0 + P$	FOR NONCHARGE EXCHANGE $K^+ + P \to K^+ + P$	FOR NONCHARGE EXCHANGE $K^+ + n \to K^+ + n$
0	.0000000E-99	2.4801883E-07	2.4810650E-07
1	9.2770092E-09	2.2619232E-07	2.3693802E-07
2	2.9256019E-08	1.7560718E-07	2.0876946E-07
3	4.6767092E-08	1.2232952E-07	1.7424546E-07
4	5.5867693E-08	8.0667716E-08	1.4149650E-07
5	5.7537693E-08	5.2332212E-08	1.1396732E-07
10	3.4952145E-08	7.6365788E-09	4.3535671E-08
15	1.9376979E-08	1.8627136E-09	2.1501505E-08
20	1.1886401E-08	6.4214469E-10	1.2624444E-08
25	7.9570998E-09	2.7596827E-10	8.2760938E-09
30	5.6863267E-09	1.3771361E-10	5.8463425E-09
45	2.6739272E-09	2.9565170E-11	2.7088566E-09
60	1.5816857E-09	1.0237551E-11	1.5940204E-09
75	1.0718156E-09	4.6784868E-12	1.0775775E-09
90	7.9633313E-10	2.5759119E-12	7.9957891E-10
105	6.3350564E-10	1.6277214E-12	6.3560291E-10
120	5.3211725E-10	1.1473134E-12	5.3362585E-10
135	4.6782307E-10	8.8627784E-13	4.6900856E-10
150	4.2812950E-10	7.4198648E-13	4.2913488E-10
165	4.0645054E-10	6.6861055E-13	4.0736380E-10
180	3.9954981E-10	6.4605836E-13	4.0043470E-10
TOTAL CROSS SECTION=	2.4333813E-08	4.2113533E-09	2.8845431E-08

TABLE XV

DIFFERENTIAL SCATTERING CROSS SECTION AT W= 2000 MEV

ANGLE (DEGREES)	FOR CHARGE EXCHANGE $K^+ + n \to K^0 + P$	FOR NONCHARGE EXCHANGE $K^+ + P \to K^+ + P$	FOR NONCHARGE EXCHANGE $K^+ + n \to K^+ + n$
0	.0000000E-99	1.9401872E-07	1.9407290E-07
1	9.4065065E-09	1.7176223E-07	1.8260258E-07
2	2.7684969E-08	1.2395406E-07	1.5512203E-07
3	4.0732560E-08	7.9227638E-08	1.2401706E-07
4	4.5048785E-08	4.8308181E-08	9.6839700E-08
5	4.3533453E-08	2.9409314E-08	7.5558885E-08
10	2.2566848E-08	3.6790943E-09	2.6724744E-08
15	1.1919187E-08	8.5719973E-10	1.2899833E-08
20	7.1713364E-09	2.9018587E-10	7.5055275E-09
25	4.7555168E-09	1.2361251E-10	4.8986264E-09
30	3.3805211E-09	6.1382105E-11	3.4519459E-09
45	1.5788720E-09	1.3094586E-11	1.5943695E-09
60	9.3168442E-10	4.5240901E-12	9.3714914E-10
75	6.3064229E-10	2.0653262E-12	6.3319492E-10
90	4.6827008E-10	1.1365043E-12	4.6970874E-10
105	3.7238996E-10	7.1792124E-13	3.7332020E-10
120	3.1272236E-10	5.0592905E-13	3.1339199E-10
135	2.7489861E-10	3.9076977E-13	2.7542513E-10
150	2.5155246E-10	3.2712386E-13	2.5199922E-10
165	2.3880346E-10	2.9476124E-13	2.3920942E-10
180	2.3474554E-10	2.8481500E-13	2.3513892E-10
TOTAL CROSS SECTION=	1.4957364E-08	2.5138153E-09	1.7647373E-08

TABLE XVI

DIFFERENTIAL SCATTERING CROSS SECTION AT W= 2250 MEV

ANGLE (DEGREES)	FOR CHARGE EXCHANGE $K^{+}+n \to K^{0}+P$	FOR NONCHARGE EXCHANGE $K^{+}+P \to K^{+}+P$	FOR NONCHARGE EXCHANGE $K^{+}+n \to K^{+}+n$
0	.0000000E-99	1.5563896E-07	1.5567408E-07
1	9.3850231E-09	1.3329081E-07	1.4406459E-07
2	2.5668392E-08	8.9036921E-08	1.1774493E-07
3	3.4806441E-08	5.2323652E-08	9.0262254E-08
4	3.5887233E-08	2.9728274E-08	6.8036506E-08
5	3.2849931E-08	1.7153407E-08	5.1681172E-08
10	1.5069356E-08	1.9073235E-09	1.7233335E-08
15	7.6924559E-09	4.3033562E-10	8.1857938E-09
20	4.5673779E-09	1.4388499E-10	4.7333095E-09
25	3.0094702E-09	6.0927120E-11	3.0800834E-09
30	2.1317570E-09	3.0154734E-11	2.1668802E-09
45	9.9112225E-10	6.4057062E-12	9.9871290E-10
60	5.8391708E-10	2.2098200E-12	5.8659128E-10
75	3.9495055E-10	1.0081242E-12	3.9619974E-10
90	2.9314499E-10	5.5454282E-13	2.9384932E-10
105	2.3306750E-10	3.5022375E-13	2.3352315E-10
120	1.9569471E-10	2.4677391E-13	1.9602288E-10
135	1.7200948E-10	1.9058692E-13	1.7226764E-10
150	1.5739232E-10	1.5953693E-13	1.5761145E-10
165	1.4941081E-10	1.4374965E-13	1.4960997E-10
180	1.4687045E-10	1.3889776E-13	1.4706346E-10
TOTAL CROSS SECTION=	9.7212666E-09	1.5965227E-09	1.1429009E-08

TABLE XVII

DIFFERENTIAL SCATTERING CROSS SECTION AT W= 2500 MEV

ANGLE (DEGREES)	FOR CHARGE EXCHANGE $K^+ + n \to K^0 + P$	FOR NONCHARGE EXCHANGE $K^+ + P \to K^+ + P$	FOR NONCHARGE EXCHANGE $K^+ + n \to K^+ + n$
0	.0000000E-99	1.2746806E-07	1.2749174E-07
1	9.2597073E-09	1.0526025E-07	1.1585467E-07
2	2.3462092E-08	6.4893920E-08	9.0966894E-08
3	2.9407134E-08	3.5187976E-08	6.6985145E-08
4	2.8472999E-08	1.8776050E-08	4.8930931E-08
5	2.4895455E-08	1.0357538E-08	3.6342067E-08
10	1.0380556E-08	1.0509497E-09	1.1576368E-08
15	5.1689416E-09	2.3164335E-10	5.4348966E-09
20	3.0401629E-09	7.6769338E-11	3.1287816E-09
25	1.9941391E-09	3.2370547E-11	2.0316850E-09
30	1.4090244E-09	1.5983948E-11	1.4276553E-09
45	6.5301221E-10	3.3853558E-12	6.5702745E-10
60	3.8428742E-10	1.1666460E-12	3.8570114E-10
75	2.5978961E-10	5.3196874E-13	2.6045003E-10
90	1.9277027E-10	2.9254600E-13	1.9314276E-10
105	1.53223[illegible]45E-10	1.8473029E-13	1.5347951E-10
120	1.2865320E-10	1.3015186E-13	1.2882689E-10
135	1.1307476E-10	1.0051201E-13	1.1321145E-10
150	1.0346166E-10	8.4133691E-14	1.0357771E-10
165	9.8212885E-11	7.5806540E-14	9.8318378E-11
180	9.6542348E-11	7.3247425E-14	9.6644587E-11
TOTAL CROSS SECTION=	6.6049693E-09	1.0629159E-09	7.7420039E-09

TABLE XVIII

TOTAL SCATTERING CROSS SECTIONS—EXCANGE INTERACTION

ENERGY W(C.M.)	KINETIC ENERGY	FOR CHARGE EXCHANGE $k^{+}+n \rightarrow k^{o}+p$	FOR NONCHARGE EXCHANGE $k^{+}+p \rightarrow k^{+}+p$	FOR NONCHANGE EXCHANGE $k^{+}+n \rightarrow k^{+}+n$
MeV	MeV	MeV^{-2}	MeV^{-2}	MeV^{-2}
500	77	3.7734124×10^{-6}	8.3214581×10^{-6}	1.2461842×10^{-5}
600	360	1.3572035×10^{-6}	5.5529235×10^{-7}	1.9647286×10^{-6}
750	560	5.2713636×10^{-6}	1.5176585×10^{-7}	6.9109448×10^{-7}
1000	870	1.8268046×10^{-7}	4.0463914×10^{-8}	2.2642731×10^{-7}
1250	1140	8.2052200×10^{-8}	1.5931593×10^{-8}	9.9213489×10^{-7}
1500	1420	4.2552286×10^{-8}	7.6887783×10^{-9}	5.0804993×10^{-8}
1750	1690	2.4333813×10^{-8}	4.2113533×10^{-9}	2.8845431×10^{-8}
2000	1960	1.4957364×10^{-8}	2.5138153×10^{-9}	1.7647373×10^{-8}
2250	2190	9.7212266×10^{-9}	1.5965227×10^{-9}	1.1429009×10^{-8}
2500	2420	6.6049693×10^{-9}	1.0629159×10^{-9}	7.7429939×10^{-9}

AFIT FORTRAN PROGRAM FOR RELATIVISTIC KINEMATIC FUNCTION

S AND $\sqrt{G/F}/16\pi^2 S$

```
1 READ,XM,YM,ZM,WM
2 FORMAT(40X, 5HTABLE)
3 FORMAT(47X,I3)
4 FORMAT(///23X, 37HRELATIVISTIC KINEMATIC FUNCTION A AND)
5 FORMAT(////20X, 1HE, 15X1HS/)
6 FORMAT(/18X, I5, E19.7, E23.7/)
  PRINT 2
  PRINT 3
  PRINT 4
  PRINT 5
  DO 15 IE=500,1500.100
  E=IE
  ANS=XM*XM+YM*YM+2.*X*Y
  S=ANS
  X=XM*XM
  Y=YM*YM
  Z=ZM*ZM
  W=WM*WM
  F=S*S+X*X+Y*Y-2.*S*X-2.*S*Y-2.*X*Y
  G=S*S+Z*Z+W*W-2.*S*Z-2.*S*W-2.*Z*W
  ANS1=SQRTF(G/F)/(16.*3.1416*3.1416*S)
  PRINT 6, IE,ANS,ANS1
  Z=CON(971.1)
  GO TO 1
  END
```

TABLE XIX

RELATIVISTIC KINEMATIC FUNCTION S AND $\sqrt{G/F}/16\pi^2 S$

E	S	$\sqrt{G/F}/16\pi^2 S$
500	2.0624552E+06	3.0703911E-09
600	2.2500952E+06	2.8143450E-09
700	2.4377352E+06	2.5977162E-09
800	2.6253752E+06	2.4120530E-09
900	2.8130152E+06	2.2511589E-09
1000	3.0006552E+06	2.1103871E-09
1100	3.1882952E+06	1.9861850E-09
1200	3.3759352E+06	1.8757896E-09
1300	3.5635752E+06	1.7770199E-09
1400	3.7512152E+06	1.6881314E-09
1500	3.9388552E+06	1.6077118E-09

TABLE XX

RELATIVISTIC KINEMATIC FUNCTION S AND $\sqrt{G/F}\,/16\pi^2 S$

E	S	$\sqrt{G/F}\,/16\pi^2 S$
500	2.0661962E+06	1.7845674E-09
600	2.2540962E+06	2.7593148E-09
700	2.4419962E+06	2.5707045E-09
800	2.6298962E+06	2.3944977E-09
900	2.8177962E+06	2.2382882E-09
1000	3.0056962E+06	2.1002859E-09
1100	3.1935962E+06	1.9778999E-09
1200	3.3814962E+06	1.8687815E-09
1300	3.5693962E+06	1.7709551E-09
1400	3.7572962E+06	1.6827899E-09
1500	3.9451962E+06	1.6029411E-09

TABLE XXI

RELATIVISTIC KINEMATIC FUNCTION S AND $\sqrt{G/F}\,/16\pi^2 S$

E	S	$\sqrt{G/F}\,/16\pi^2 S$
500	2.0661962E+06	3.0648320E-09
600	2.2540962E+06	2.8093496E-09
700	2.4419962E+06	2.5931835E-09
800	2.6298962E+06	2.4079065E-09
900	2.8177962E+06	2.2473393E-09
1000	3.0056962E+06	2.1068477E-09
1100	3.1935962E+06	1.9828882E-09
1200	3.3814962E+06	1.8727048E-09
1300	3.5693962E+06	1.7741220E-09
1400	3.7572962E+06	1.6853992E-09
1500	3.9451962E+06	1.6051278E-09

AFIT FORTRAN PROGRAM FOR RELATIVISTIC KINEMATIC FUNCTION T

```
      DIMENSION  ANS1 (3,200)
    2 FORMAT(37X,5HTABLE)
    4 FORMAT(///23X,33HRELATIVISTIC KINEMATIC FUNCTION T)
    5 FORMAT(////2X,12HANGLE       E)
   52 FORMAT(20X,I5,19XI5,19XI5//)
    6 FORMAT(/9H(DEGREES))
    1 NA = 500
      NB = 700
      READ,XM,YM,ZM,WM,N
      DO 15 I = 1,3
      N = 0
      NC = NA
      DO 10  IE = NA ,NB, 100
      E=IE
      N = N + 1
      ANS=XM*XM+YM*YM+2.*E*YM
      S=ANS
      X=XM*XM
      Y=YM*YM
      Z=ZM*ZM
      W=WM*WM
      F=S*S+X*X+Y*Y-2.*S*X-2.*S*Y-2.*X*Y
      G=S*S+Z*Z+W*W-2.*S*Z-2.*S*W-2.*Z*W
      DO 10 IT = 1, 181, 15
      T = IT - 1
      T=T*.017453293
      D=COS(T)
   10 ANS1(N,IT)=(D*SQRTF(F*G)-S*S-(X-Y)*(Z-W)+S*(X+Y+Z+W))/(2.*S)
      IE2 = NC + 100
      IE3 = NC + 200
      ZQ=CON(971.1)
      PRINT 2
      PRINT 4
      PRINT 5
      PRINT 6
      PRINT 52, NC, IE2, IE3
      DO 11 IT = 1, 181, 15
      ITT = IT - 1
   11 PRINT 54,ITT, ANS1(1,IT), ANS1(2,IT), ANS1(3,IT)
   54 FORMAT(/ 3XI4,10XE14.7,10XE14.7,10XE14.7/)
      NA = NC + 300
      NB = NC + 500
   15 CONTINUE
      GO TO 1
      END
```

TABLE XXII

RELATIVISTIC KINEMATIC FUNCTION T

ANGLE (DEGREES) \ E	500	600	700
0	-4.3981786E+02	-1.0048373E+01	-2.6617568E+00
15	-5.4092152E+02	-3.0495149E+03	-6.0087931E+03
30	-8.3731157E+02	-1.1960714E+04	-2.3617850E+04
45	-1.3088302E+03	-2.6136417E+04	-5.1629830E+04
60	-1.9232926E+03	-4.4610562E+04	-8.8135743E+04
75	-2.6388829E+03	-6.6124129E+04	-1.3064778E+05
90	-3.4067674E+03	-8.9211032E+04	-1.7626880E+05
105	-4.1746519E+03	-1.1229793E+05	-2.2188984E+05
120	-4.8902422E+03	-1.3381152E+05	-2.6440188E+05
135	-5.5047047E+03	-1.5228564E+05	-3.0090779E+05
150	-5.9762233E+03	-1.6646135E+05	-3.2891975E+05
165	-6.2726134E+03	-1.7537255E+05	-3.4652881E+05
180	-6.3737170E+03	-1.7841201E+05	-3.5253494E+05

TABLE XXIII

RELATIVISTIC KINEMATIC FUNCTION T

ANGLE (DEGREES) \ E	800	900	1000
0	-3.9925530E-01	6.0330835E-01	1.1145504E+00
15	-9.0065721E+03	-1.2032435E+04	-1.5080582E+04
30	-3.5411378E+04	-4.7311530E+04	-5.9297875E+04
45	-7.7415336E+04	-1.0343246E+05	-1.2963743E+05
60	-1.3215595E+05	-1.7657070E+05	-2.2130574E+05
75	-1.9590275E+05	-2.6174197E+05	-3.2805574E+05
90	-2.6431147E+05	-3.5314198E+05	-4.4261258E+05
105	-3.3272022E+05	-4.4454201E+05	-5.5716941E+05
120	-3.9646703E+05	-5.2971327E+05	-6.6391939E+05
135	-4.5120763E+05	-6.0285149E+05	-7.5558767E+05
150	-4.9321159E+05	-6.5897242E+05	-8.2592711E+05
165	-5.1961638E+05	-6.9425159E+05	-8.7014449E+05
180	-5.2862255E+05	-7.0628457E+05	-8.8522619E+05

TABLE XXIV

RELATIVISTIC KINEMATIC FUNCTION T

ANGLE (DEGREES) \ E	1100	1200	1300
0	1.3151318E+00	1.4934217E+00	1.4988529E+00
15	-1.8147034E+04	-2.1228457E+04	-2.4322503E+04
30	-7.1355232E+04	-8.3471319E+04	-9.5637085E+04
45	-1.5599711E+05	-1.8248533E+05	-2.0908204E+05
60	-2.6630458E+05	-3.1152303E+05	-3.5692633E+05
75	-3.9476039E+05	-4.6179055E+05	-5.2909466E+05
90	-5.3261038E+05	-6.2304727E+05	-7.1385392E+05
105	-6.7046031E+05	-7.8430414E+05	-8.9861332E+05
120	-7.9891612E+05	-9.3457165E+05	-1.0707816E+06
135	-9.0922358E+05	-1.0636092E+06	-1.2186259E+06
150	-9.9386563E+05	-1.1626233E+06	-1.3320709E+06
165	-1.0470736E+06	-1.2248660E+06	-1.4033853E+06
180	-1.0652220E+06	-1.2460960E+06	-1.4277093E+06

TABLE XXV

RELATIVISTIC KINEMATIC FUNCTION T

ANGLE (DEGREES) \ E	500	600	700
0	-4.8485901E-02	-6.6663846E-02	-8.2043365E-02
15	-1.7350679E+02	-3.0915580E+03	-6.0525236E+03
30	-6.8207542E+02	-1.2155352E+04	-2.3797396E+04
45	-1.4910626E+03	-2.6573786E+04	-5.2025441E+04
60	-2.5453643E+03	-4.5364236E+04	-8.8812927E+04
75	-3.7731486E+03	-6.7246176E+04	-1.3165285E+05
90	-5.0906802E+03	-9.0728383E+04	-1.7762575E+05
105	-6.4082119E+03	-1.1421058E+05	-2.2359867E+05
120	-7.6359961E+03	-1.3609255E+05	-2.6643859E+05
135	-8.6902978E+03	-1.5488300E+05	-3.0322608E+05
150	-9.4992851E+03	-1.6930141E+05	-3.3145410E+05
165	-1.0007853E+04	-1.7836520E+05	-3.4919898E+05
180	-1.0181312E+04	-1.8145670E+05	-3.5525142E+05

TABLE XXVI

RELATIVISTIC KINEMATIC FUNCTION T

ANGLE (DEGREES) \ E	800	900	1000
0	-3.8089793E-02	-7.1098087E-02	-4.9989082E-02
15	-9.0471640E+03	-1.2068829E+04	-1.5112366E+04
30	-3.5571982E+04	-4.7452676E+04	-5.9419456E+04
45	-7.7766884E+04	-1.0374021E+05	-1.2990184E+05
60	-1.3275635E+05	-1.7709559E+05	-2.2175630E+05
75	-1.9679293E+05	-2.6251971E+05	-3.2872307E+05
90	-2.6551263E+05	-3.5419108E+05	-4.4351253E+05
105	-3.3423236E+05	-4.4586245E+05	-5.5830206E+05
120	-3.9826895E+05	-5.3128658E+05	-6.6526870E+05
135	-4.5325841E+05	-6.0464195E+05	-7.5712314E+05
150	-4.9545331E+05	-6.6092959E+05	-8.2760558E+05
165	-5.2197811E+05	-6.9631333E+05	-8.7191257E+05
180	-5.3102524E+05	-7.0838205E+05	-8.8702494E+05

TABLE XXVII

RELATIVISTIC KINEMATIC FUNCTION T

ANGLE (DEGREES) \ E	1100	1200	1300
0	-1.8818834E-01	-2.5178208E-01	-1.6837023E-01
15	-1.8174164E+04	-2.1250807E+04	-2.4340050E+04
30	-7.1457341E+04	-8.3554284E+04	-9.5700828E+04
45	-1.5621878E+05	-1.8266474E+05	-2.0921940E+05
60	-2.6668189E+05	-3.1182791E+05	-3.5715985E+05
75	-3.9531897E+05	-4.6224184E+05	-5.2944006E+05
90	-5.3336356E+05	-6.2365563E+05	-7.1431942E+05
105	-6.7140803E+05	-7.8506957E+05	-8.9919878E+05
120	-8.0004511E+05	-9.3548329E+05	-1.0714790E+06
135	-9.1050838E+05	-1.0646466E+06	-1.2194193E+06
150	-9.9526966E+05	-1.1637570E+06	-1.3329380E+06
165	-1.0485528E+06	-1.2260604E+06	-1.4042988E+06
180	-1.0667268E+06	-1.2473109E+06	-1.4286385E+06

TABLE XXVIII

RELATIVISTIC KINEMATIC FUNCTION T

ANGLE (DEGREES) \ E	500	600	700
0	-7.2597171E-02	-6.6545518E-02	-6.1425157E-02
15	-1.7370083E+02	-3.0946327E+03	-6.0587113E+03
30	-6.8272800E+02	-1.2167426E+04	-2.3821761E+04
45	-1.4925010E+03	-2.6600173E+04	-5.2078684E+04
60	-2.5478219E+03	-4.5409308E+04	-8.8903803E+04
75	-3.7767468E+03	-6.7312987E+04	-1.3178759E+05
90	-5.0955470E+03	-9.0818528E+04	-1.7780752E+05
105	-6.4143472E+03	-1.1432406E+05	-2.2382747E+05
120	-7.6432722E+03	-1.3622777E+05	-2.6671126E+05
135	-8.6985930E+03	-1.5503688E+05	-3.0353638E+05
150	-9.5083661E+03	-1.6946963E+05	-3.3179330E+05
165	-1.0017393E+04	-1.7854242E+05	-3.4955633E+05
180	-1.0191021E+04	-1.8163699E+05	-3.5561498E+05

TABLE XXIX

RELATIVISTIC KINEMATIC FUNCTION T

ANGLE (DEGREES) \ E	800	900	1000
0	-5.7036471E-02	-5.3233090E-02	-4.9905243E-02
15	-9.0566692E+03	-1.2081746E+04	-1.5128857E+04
30	-3.5609333E+04	-4.7503506E+04	-5.9484305E+04
45	-7.7848509E+04	-1.0385138E+05	-1.3004363E+05
60	-1.3289568E+05	-1.7728539E+05	-2.2199831E+05
75	-1.9699944E+05	-2.6280106E+05	-3.2908182E+05
90	-2.6579125E+05	-3.5457069E+05	-4.4399656E+05
105	-3.3458309E+05	-4.4634031E+05	-5.5891127E+05
120	-3.9868685E+05	-5.3185601E+05	-6.6599462E+05
135	-4.5373401E+05	-6.0528999E+05	-7.5794935E+05
150	-4.9597318E+05	-6.6163780E+05	-8.2850854E+05
165	-5.2252583E+05	-6.9705963E+05	-8.7286399E+05
180	-5.3158244E+05	-7.0914124E+05	-8.8799277E+05

TABLE XXX

RELATIVISTIC KINEMATIC FUNCTION T

ANGLE (DEGREES) \ E	1100	1200	1300
0	-7.8281656E-02	-1.4786354E-02	-8.4047828E-02
15	-1.8194144E+04	-2.1274473E+04	-2.4367650E+04
30	-7.1536517E+04	-8.3648046E+04	-9.5809705E+04
45	-1.5639179E+05	-1.8286995E+05	-2.0945749E+05
60	-2.6697747E+05	-3.1217839E+05	-3.5756607E+05
75	-3.9575721E+05	-4.6276142E+05	-5.3004230E+05
90	-5.3395479E+05	-6.2435675E+05	-7.1513201E+05
105	-6.7215244E+05	-7.8595223E+05	-9.0022186E+05
120	-8.0093219E+05	-9.3653513E+05	-1.0726980E+06
135	-9.1151786E+05	-1.0658437E+06	-1.2208067E+06
150	-9.9637330E+05	-1.1650656E+06	-1.3344545E+06
165	-1.0497155E+06	-1.2274390E+06	-1.4058964E+06
180	-1.0679095E+06	-1.2487135E+06	-1.4302640E+06

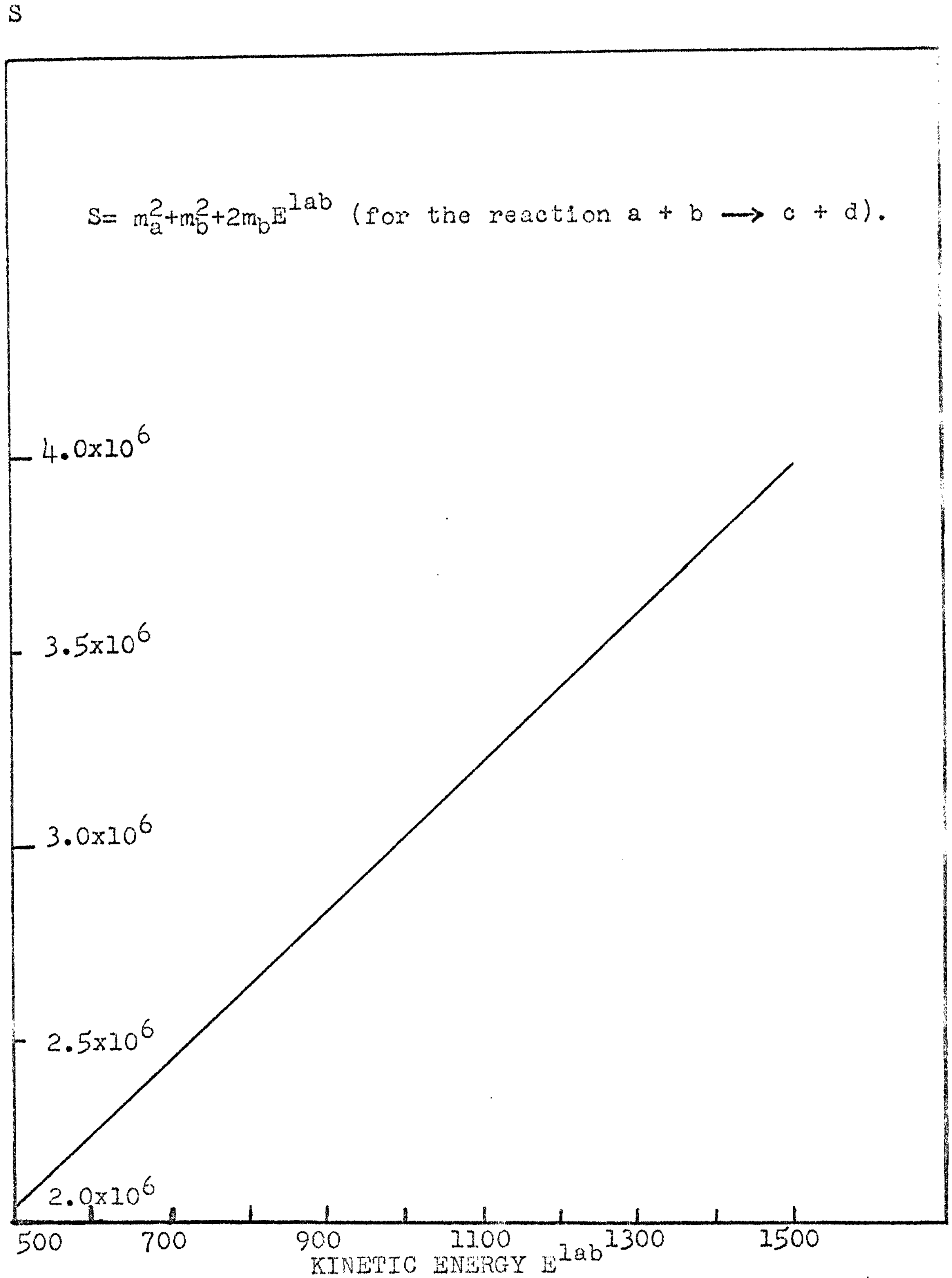

Fig. 17--The relativistic function S for the reaction $k^+ + p \longrightarrow k^+ + p$.

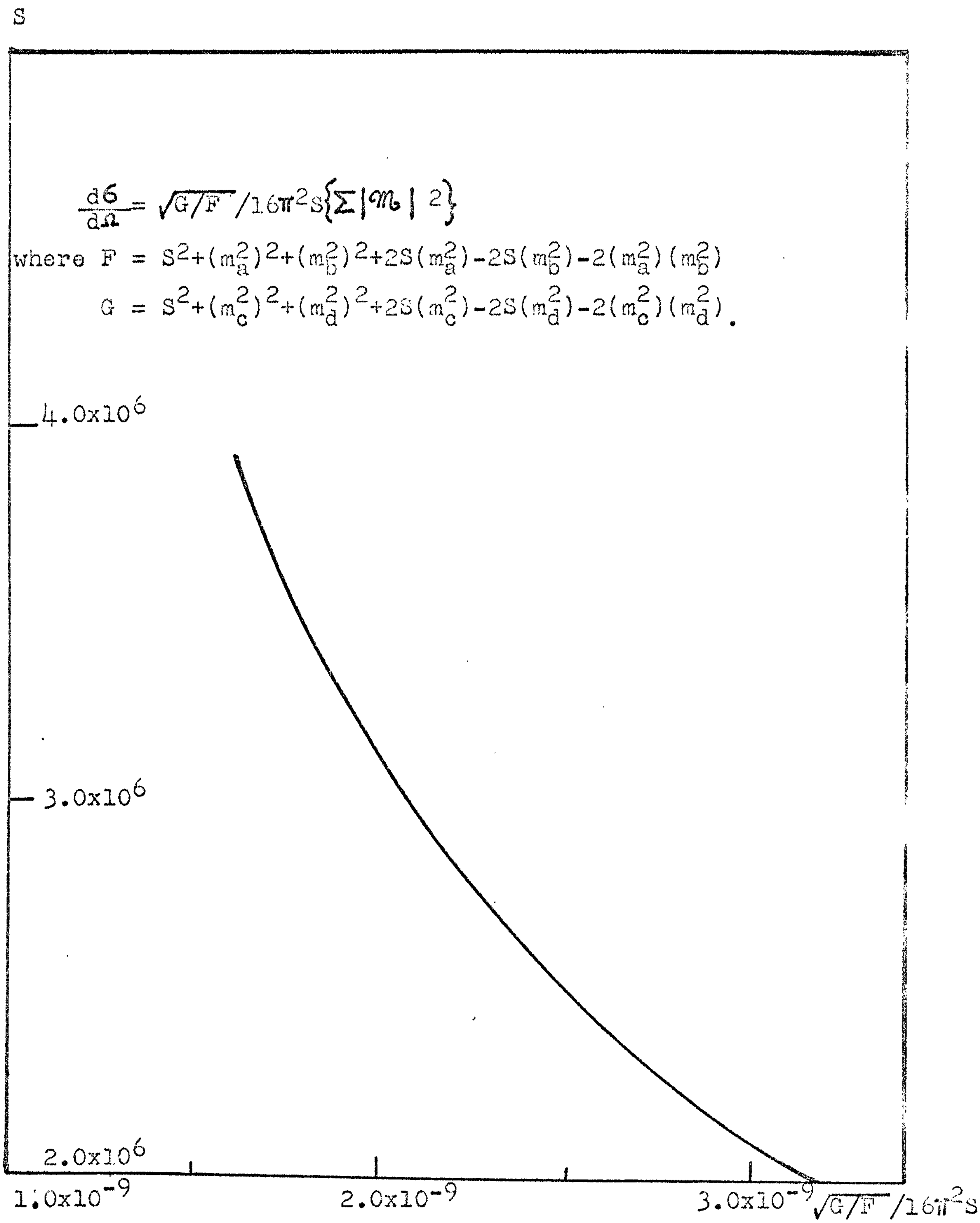

Fig. 18--The function of general equation of differential scattering cross-section for the reaction $k^+ + p \longrightarrow k^+ + p$.

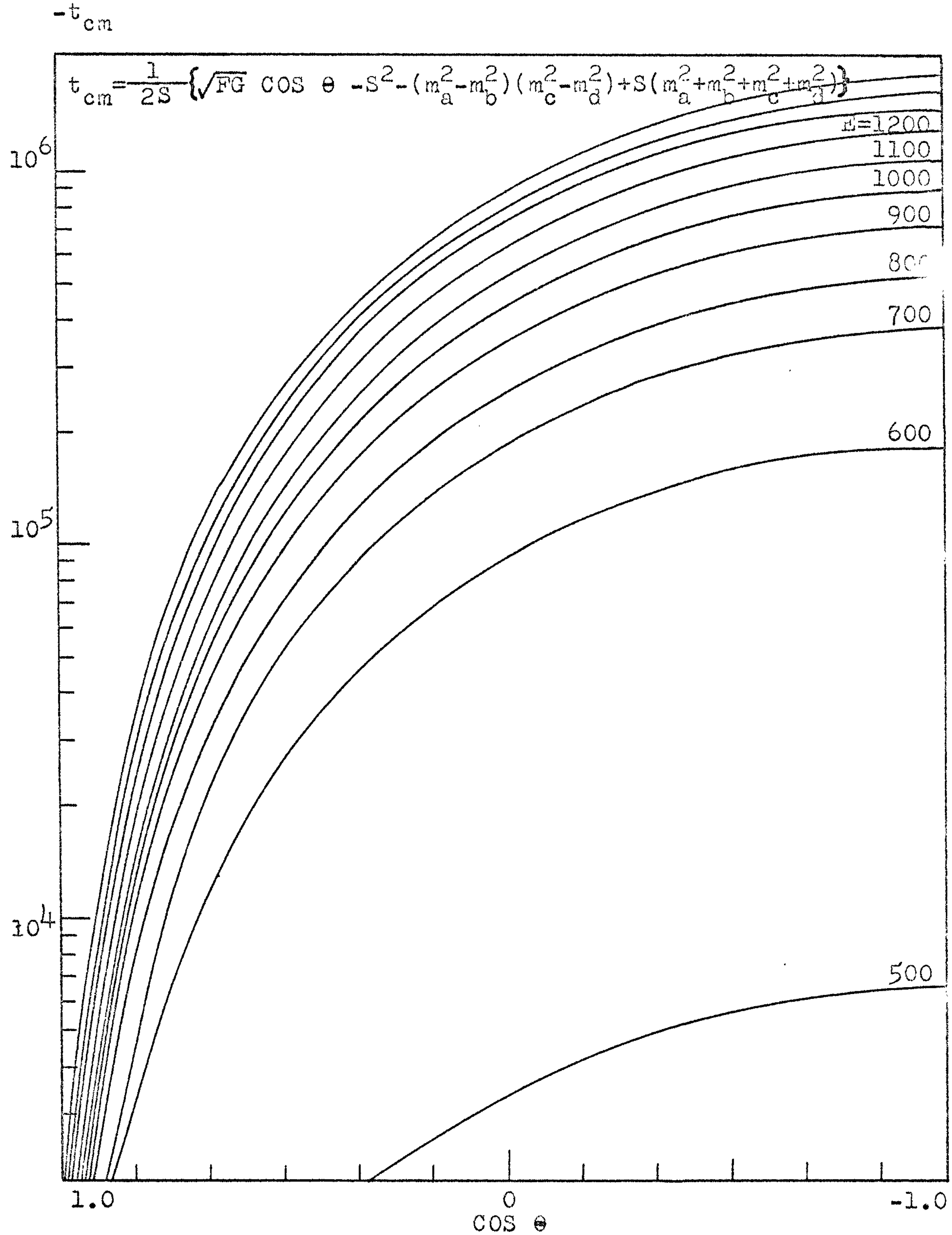

Fig. 19--The relativistic function t_{cm} for the reaction $k^+ + p \longrightarrow k^+ + p$.

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