Sodankylä Geophysical Observatory Publications (2003) 92:121–123

Absolute Optical Calibrations Using a Simple Tungsten Bulb: Theory

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Received: 7.12.2001 – Accepted: 14.6.2002

Abstract. The absolute spectral intensity calibration of optical detectors has always been difficult. In the past, this was only possible using expensive sources, which had been cross-calibrated against national standards. We describe a simple theoretical approach to absolute optical calibrations using any ordinary Tungsten light bulb. A key element of the theory is transforming Tungsten into an equivalent black body radiator. This permits direct application of Stefan-Boltzmann's and Planck's formulas of radiation.

1 Units and definitions

Aurora, airglow and nebula are extended sources and their brightness is expressed as luminance B. As the light from aurora and airglow is a result of molecular and atomic processes emitting photons, the resulting luminance B_{ph} is most conveniently given in photon units:

$$B_{ph} = \text{photons m}^{-2}\text{ s}^{-1}\text{ sr}^{-1} \tag{1}$$

For calibration, Tungsten filament bulbs are to be used and their filament emissivity B_w are given in energy units:

$$B_w = \text{W m}^{-2}\text{ sr}^{-1} \tag{2}$$

The relationship between B_{ph} and B_w is:

$$B_w = B_{ph} h\nu = B_{ph}\frac{hc}{\lambda} \tag{3}$$

(W m^{-2} sr^{-1})

where h is Planck's constant, ν frequency, c the speed of light and λ wavelength. The photon intensity I_R is given by the unit Rayleigh (Hunten et al., 1956). By definition:

$$I_R = 1\text{ Rayleigh (R)} = 10^6\text{photons cm}^{-2}\text{ s}^{-1} \tag{4}$$

which is the number of Mega-photons emitted per second in 4π steradian and in one square centimeter column integrated through the emitting region. The Rayleigh is not a true luminance or SI unit. To convert this to true luminance SI units we use the following expression:

$$B_{ph} = I_R\frac{1}{4\pi}10^{10} \tag{5}$$

(photons m^{-2} s^{-1} sr^{-1})

An optical instrument can be visualized as a box with an input area S (m^{-2}) and a solid acceptance angle ω (sr). Filling the acceptance angle with an extended source of luminance B_{ph}, and assuming no losses, the detector collects photons at the following rate n_{ph}:

$$n_{ph} = B_{ph} S w \tag{6}$$

(photons s^{-1})

2 Creating a luminance surface for laboratory calibration

To simulate an airglow/auroral emitting surface in the laboratory, we illuminate a Lambertian surface of known albedo α from a distant source of known photon output rate P_{ph} (photons s^{-1}). In this exercise, the photon source is any ordinary Tungsten bulb and the Lambertian surface is a white cardboard sheet. Assuming that the photon output rate P_{ph} emitted from the source is constant and omni-directional, the photon intensity I_{ph} onto the Lambertian surface is:

$$I_{ph} = P_{ph}\cos(\theta)\frac{1}{L^2}\frac{1}{4\pi} \tag{7}$$

(photons m^{-2} s^{-1})

where L is the distance between source and reflecting surface and θ is the angle between the surface normal and the source direction. Consider a Lambertian surface of area S and albedo α illuminated by a photon flux I_{ph}: By definition, the luminance of the surface B_{ph} is independent of the look

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angle ϕ. The incremental number of photons ∂n_{ph} emitted from the surface for an incremental look angle $\partial\phi$ is:

$$\partial n_{ph} = B_{ph}(S\cos(\phi))(2\pi\sin(\phi))\partial\phi \quad (8)$$

(photons s^{-1})

Re-arranging Equ. 8 and using:

$$\cos(\phi) = \frac{\partial(\sin(\phi))}{\partial\phi} \quad (9)$$

the incremental number of photons becomes:

$$\partial n_{ph} = 2\pi B_{ph} S\sin(\phi)\partial(\sin(\phi)) \quad (10)$$

(photons s^{-1})

The total photon flux emitted from the Lambertian surface is:

$$n_{ph} = 2\pi S B_{ph}\int_0^1 x\,dx = \pi S B_{ph} \quad (11)$$

(photons s^{-1})

Using Equ. 7, for a Lambertian surface of albedo α the total photon flux emitted is:

$$n_{ph} = \alpha I_{ph} S \quad (12)$$

(photons s^{-1})

Hence, equating Equ. 11 and 12, the luminance B_{ph} of the Lambertian surface is:

$$B_{ph} = I_{ph}\frac{\alpha}{\pi} \quad (13)$$

(photons m^{-2} s^{-1} sr^{-1})

which may be easily converted to Rayleighs using Equ. 5.

3 The emissivity of a Tungsten bulb

The spectral emittance $E_{\lambda W}$ of a Tungsten filament surface is expressed by the Planck formula:

$$E_{\lambda W}(\lambda,T) = \varepsilon(\lambda,T)\frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1} \quad (14)$$

(W m^{-2} $(m)^{-1}$)

Since Tungsten is not a blackbody radiator, we have to correct the emission formula with the spectral emissivity factor for Tungsten $\varepsilon(\lambda,T)$ which is a function of wavelength and temperature. The emissivity factor is reasonably constant within the visible range from 400 to 600 nm, having a value of about 0.47 ± 0.01 (Handbook of Physics and Chemistry, CRC Press). Integrating $E_{\lambda W}(\lambda,T)$ over the whole wavelength region gives the Stefan-Boltzmann equation:

$$E_W(T) = \int E_{\lambda W}(\lambda,T)d\lambda = \varepsilon_m(T)\sigma T^4 \quad (15)$$

(W m^{-2})

where σ is the Stefan-Boltzmann constant given by:

$$\sigma = \frac{2\pi^5 k^4}{15c^2h^3} = 5.67\times10^{-8} \quad (16)$$

(W m^{-2} K^{-4})

and $\varepsilon_m(T)$ is the total emissivity for a non-black body substance. For Tungsten, $\varepsilon_m(T)$ is a rather smooth function of temperature (Handbook of Physics and Chemistry, CRC Press) and can be expressed by two second order polynomials for the two temperature ranges used in the calibration.

For $200 < T < 500$ K:

$$\varepsilon_m(T) = -2.5\times10^{-8}T^2 + 1.27\times10^{-4}T - 0.00415 \quad (17)$$

For $1200 < T < 2500$ K:

$$\varepsilon_m(T) = -5.0\times10^{-8}T^2 + 3.11\times10^{-4}T - 0.161 \quad (18)$$

4 The resistivity of Tungsten

We use resistance measurements of the Tungsten filament and resistivity tables of Tungsten (Forsythe and Worthing., 1925) to infer the temperature of the filament. The value of Ohmic resistivity ρ for Tungsten is low at room temperature but increases smoothly as the temperature increases. The resistivity can be expressed by two second order polynomials for the two temperature ranges used in the calibration.

For $200 < T < 400$ K:

$$\rho(T) = 1.25\times10^{-6}T^2 + 0.0236T - 1.57 \quad (19)$$

(μohm cm)

For $1200 < T < 2500$ K

$$\rho(T) = 1.79\times10^{-6}T^2 + 0.0264T - 3.25 \quad (20)$$

(μohm cm)

5 Filament temperature and area

Assuming any normal Tungsten bulb is used, by measuring the filament voltage V_f and current I_f we can deduce the filament resistance $R_f = V_f/I_f$ and the power emitted by the filament $P_W = V_f I_f$, where V_f and I_f can be measured using any multimeter. It is necessary to start the procedure at a known filament temperature: This done by measuring the filament resistance R_0 at room temperature T_0. With the filament power turned on, the resistance ratio:

$$R(T_f)/R(T_0) = \rho(T_f)/\rho(T_0) \quad (21)$$

is easily calculated. Re-arranging:

$$\rho(T_f) = \rho(T_0)\times R(T_f)/R(T_0) \quad (22)$$

(μohm cm)

By using Equ. 19 and 20, the temperature T_f of the filament can be deduced. The power the filament receives is radiated following Stefan-Boltzmann's law:

$$P_W = V_f I_f = a_f \varepsilon_m(T_f)\sigma T_f^4 \quad (23)$$

(W)

From Equ. 23 we deduce the effective filament area a_f:

$$a_f = \frac{V_f I_f}{\varepsilon_m(T_f)\sigma T_f^4} \quad (24)$$

(m^2)

where $\varepsilon_m(T_f)$ is obtained from Equ. 18 and T_f.

6 The Lambertian screen

From Equ. 14 and 3, the spectral emittance $E_{ph\lambda f}$ of a black-body filament, corrected for Tungsten $\varepsilon(\lambda, T)$, is:

$$E_{ph\lambda f}(\lambda, T) = \varepsilon(\lambda, T)\frac{2\pi c}{\lambda^4}\frac{1}{e^{hc/\lambda kT} - 1} \quad (25)$$

(photons s^{-1} m^{-2} $(m)^{-1}$)

Assuming the source to be omni-directional, from Equ. 7 the spectral photon flux $I_{ph\lambda f}$ received on the Lambertian surface is:

$$I_{ph\lambda f}(\lambda, T) = a_f E_{ph\lambda f}(\lambda, T)\frac{1}{L^2}\frac{1}{4\pi}\cos(\theta) \quad (26)$$

(photons s^{-1} m^{-2} $(m)^{-1}$)

where the calibration source, of emitting area a_f, is a distance L from the Lambertian surface at an angle θ to the surface normal. Hence, from Equ. 13, the spectral luminance $B_{ph\lambda}$ of the Lambertian surface is:

$$B_{ph\lambda}(\lambda, T) = \alpha(\lambda)\frac{I_{ph\lambda f}(\lambda, T)}{\pi}$$

or

$$B_{ph\lambda}(\lambda, T) = a_f E_{ph\lambda f}(\lambda, T)\alpha(\lambda)\frac{1}{L^2}\frac{1}{4\pi^2}\cos(\theta) \quad (27)$$

(photons s^{-1} m^{-2} sr^{-1} $(m)^{-1}$)

For a selected wavelength the equivalent Rayleighs are obtained by substituting Equ. 25 into Equ. 27, and Equ. 27 into Equ. 5:

$$I_R(R) = \int_{\lambda}^{\lambda+1\ nm} \frac{2}{10^{10}}\varepsilon(\lambda, T)\frac{c}{\lambda^4}\frac{1}{e^{hc/\lambda kT} - 1} a_f \alpha(\lambda)\frac{1}{\pi}\frac{1}{L^2}\cos(\theta)d\lambda \quad (28)$$

(Rayleighs nm^{-1})

where $\varepsilon(\lambda, T)$ comes from tables (Handbook of Physics and Chemistry, CRC Press), a_f comes from Equ. 24 and T comes from Equ. 22 and 20. Hence, the Lambertian surface constitues an extended source of known luminance for any wavelength.

7 Conclusions

All the terms of Equ. 28 are easily measured giving a precise description of the Rayleigh output of the calibration system for any wavelength. The only measurement not described above is that of the albedo $\alpha(\lambda)$ of the Lambertian surface. This can be simply done keeping the output of the Tungsten bulb constant and first measuring the luminance of the single Lambertian surface. Then measure the luminance of a second Lambertian surface made of the identical material, which is illuminated by the first one. After compensating for geometrical effects, and assuming the two Lambertian surfaces are identical, the albedo can be computed from the ratio of the luminances.

References

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