ABSOLUTE OPTICAL CALIBRATION USING A SIMPLE TUNGSTEN LIGHT BULB: EXPERIMENT

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ABSTRACT

Absolute spectral intensity calibration of optical detectors has always been difficult. Up to now it was only possible through the use of expensive sources, which are cross-calibrated against national standards. At the 28AM optical meeting, a simple theoretical approach to absolute optical calibrations was described using any ordinary tungsten light bulb [1]. A key element of the theory is transforming tungsten into its equivalent blackbody radiator. This permits direct application of Stefan-Bolzmann's and Planck's formulas of radiation. The theory has been tested by comparing three household tungsten light bulbs with a calibrated source at several wavelengths typically used in auroral research. The results of this experiment are most encouraging.

1. BACKGROUND

At the 28AM optical meeting in Oulu, Finland (2001) Harang and Kosch [1] laid out the theory for absolute optical calibrations using any ordinary clear-glass tungsten light bulb, two multi-meters (voltage, current and resistance), and a Lambertian screen. Using published data for the emissivity of tungsten [2], which is a function of temperature and wavelength, tungsten is transformed into its equivalent blackbody radiator. Using Stefan-Bolzmann's and Planck's formulas of radiation the temperature, emitting area and spectral emittance of the filament can be determined. Knowing the geometry between the bulb and screen (distance and angle) and the spectral albedo of the screen, the spectral luminance (brightness) of the illuminated screen can be determined either in SI units or Rayleighs [3].

Obvious sources of error are: (1) The filament is not pure tungsten. (2) The bulb glass will not transmit all wavelengths equally, in particular, UV wavelengths will be strongly absorbed. (3) Not all the energy supplied to the filament will be radiated as light and the bulb holder becomes hot as proof of this. However, it is expected that at visible and near infrared wavelengths, where much aeronomy is performed, the calibration procedure described in [1] will be accurate. The full derivation will not be repeated here except for a few important points.

Eq. 1 shows the spectral emittance $(E_{ph\lambda f})$ of a blackbody filament in SI units (photons.s⁻¹.m⁻².(m)⁻¹), corrected for the emissivity (ε) of tungsten [4], which is a function of wavelength (λ) and temperature (T):

$$E_{ph\lambda f}(\lambda, T) = \varepsilon(\lambda, T) \cdot \frac{2\pi c}{\lambda^4} \cdot \frac{1}{e^{hc/\lambda kT} - 1}$$
(1)

where c is the speed of light $(3x10^8 \text{ m.s}^{-1})$, h is Planck's constant (6.63x10⁻³⁴ J.s), and k is Boltzmann's constant (1.38x10⁻²³ J.K⁻¹). The experimental arrangement is given in Fig. 1:



Fig. 1. Calibration set up.

Eq. 2 shows the spectral luminance $(B_{ph\lambda})$, or brightness, of the Lambertian screen in SI units (photons.s⁻¹.m⁻².sr⁻¹.(m)⁻¹):

 $B_{ph\lambda}(\lambda, T) = a_{f} \cdot E_{ph\lambda f}(\lambda, T) \cdot \alpha(\lambda) \cdot \frac{1}{L^{2}} \cdot \frac{1}{4\pi^{2}} \cdot \cos(\theta)$ (2)

where a_f is the emitting area of the filament, α is the albedo of the screen, which is wavelength dependent, L is the distance between the filament and screen, and θ is the angle subtended between the filament and detector (D). Eq. 2 can easily be converted into energy units (watt.m⁻².sr⁻¹) using Eq. 3:

$$B_{w} = B_{ph} \cdot \frac{hc}{\lambda}$$
(3)

Rayleighs (I_R) is a non-SI photon intensity unit defined as the number of mega-photons emitted per second in 4π steradian and in one square centimetre column integrated though the emitting region [3].

$$I_{R} = 1 \text{ Rayleigh } (R) = 10^{6} \text{ photons.cm}^{-2}.\text{s}^{-1}$$
(4)

Eq. 5 converts Rayleighs to true luminance SI units (photons.m⁻².s⁻¹.sr⁻¹):

$$\mathbf{B}_{\rm ph} = \mathbf{I}_{\rm R} \cdot \frac{1}{4\pi} \cdot 10^{10} \tag{5}$$

If the calibration is to be done in Rayleighs, then substituting Eq. 1 into Eq. 2, substituting Eq. 2 into Eq. 5, re-arranging Eq. 5, and recalling that Rayleighs must be specified for a chosen wavelength, gives Eq. 6 (Rayleighs.nm⁻¹):

$$I_{R}(R) = \int_{\lambda}^{\lambda+1} \frac{1}{10^{10}} \cdot \varepsilon(\lambda, T) \cdot \frac{c}{\lambda^{4}} \cdot \frac{1}{e^{hc/\lambda kT} - 1} \cdot a_{f} \cdot \alpha(\lambda) \cdot \frac{1}{\pi} \cdot \frac{1}{L^{2}} \cdot \cos(\theta) d\lambda$$
(6)

The unknowns in Eq. 6 are $\alpha(\lambda)$, which is described in Section 2, a_f and T. a_f (m²) is obtained from Eq. 7:

$$a_{f} = \frac{V_{f} \cdot I_{f}}{\varepsilon_{m}(T_{f}) \cdot \sigma \cdot T_{f}^{4}}$$
(7)

where V_f is voltage applied to the filament, I_f is the current flowing through the filament, ε_m is the temperature-dependent wavelength-integrated total emissivity of tungsten [4], and σ is Stefan-Boltzmann's constant (5.67x10⁻⁸ W.m⁻².K⁻⁴). Clearly, the photon emitting area is obtained from measuring the power entering the filament and the filament temperature. T_f is obtained from Eq. 8:

$$\mathbf{r}(\mathbf{T}_{\mathrm{f}}) = \mathbf{r}(\mathbf{T}_{0}) \cdot \frac{\mathbf{R}(\mathbf{T}_{\mathrm{f}})}{\mathbf{R}(\mathbf{T}_{0})}$$
(8)

where r is the temperature-dependent Ohmic resistivity of the tungsten filament (μ ohm.cm) [4], R is the Ohmic resistance of the filament (R = V/I), T₀ is ambient temperature, and T_f is the filament operating temperature. By measuring the filament resistance when the bulb is both cold and hot, and knowing the resistivity of tungsten for the ambient cold temperature [4], the resistivity and temperature of the filament can be deduced when it is glowing hot.

Stars seem an obvious candidate to use as standard candles as they are numerous and have well known optical spectra. In addition, the variable absorption of the atmosphere is automatically compensated for. However, the fact that they are point sources makes them unsuitable for optical calibration where the brightness of a finite surface is required, i.e. in auroral and airglow applications.

2. LAMBERTIAN SCREEN

A Lambertian screen is needed to convert the tungsten bulb, which is a point light source, into a diffuse emitting surface, which is the typical target of optical observations in aeronomy. This also facilitates the use of Rayleighs [3], which is a surface unit. A Lambertian screen may be manufactured out of ordinary white card. Harang and Kosch [1] did not describe fully how to determine the albedo of a home-made screen: This is done below.

Ideally, the Lambertian screen should have an albedo (α) equal to one. However, this is generally not the case. There may also be some wavelength dependence. A check on the albedo coefficient can be done by the experiment shown in Fig. 2:



Fig. 2. Measuring albedo.

We let the lamp illuminate a Lambertian screen of area S_1 at a distance L_1 . Assuming that the photon output rate (P_{ph}) emitted from the source is constant and omnidirectional, the intensity I_1 on the area S_1 is then:

$$I_1 = \frac{P_{ph}}{4\pi} \cdot S_1 \cdot \cos(\theta_1) \cdot \frac{1}{L_1^2}$$
(9)

assuming that $S_1/L_1^2 \ll 1$. The luminance B_1 is given by:

$$B_1 = \frac{\alpha \cdot I_1}{\pi} \tag{10}$$

At a distance L_2 from the screen we place another identical Lambertian screen, which is illuminated by the first screen. The intensity I_2 on the second screen is:

$$I_{2} = B_{1} \cdot S_{1} \cdot \cos(\theta_{3}) \cdot \frac{S_{2}}{L_{2}^{2}} \cdot \frac{\alpha}{S_{2}} =$$

$$\frac{\alpha}{L_{2}^{2}} \cdot B_{1} \cdot S_{1} \cdot \cos(\theta_{2}) \cdot \cos(\theta_{3})$$
(11)

and its luminance is:

$$B_2 = \frac{I_2}{\pi} = \frac{\alpha}{\pi \cdot L_2^2} \cdot B_1 \cdot S_1 \cdot \cos(\theta_2) \cdot \cos(\theta_3) \quad (12)$$

Again, we assume that $S_2/L_2^2 \ll 1$, as well as $\theta_1 \approx 0$ and $\theta_2 \approx 0$. By measuring the luminance of the two screens, the (identical) albedo can be deduced:

$$\alpha = \frac{B_1}{B_2} \cdot \frac{\pi L_2^2}{S_1}$$
(13)

3. EXPERIMENT

Three ordinary clear-glass tungsten bulbs were used to test the theory with nominal power ratings of 25, 40 and 60 W. The ambient temperature was 15 $^{\circ}$ C and the normal 240 V_{ac} operating voltage was used.

The first step is to measure the filament resistance at ambient temperature: It is very important to ensure that the Ohm-meter does not heat up the filament during the measurement. This is probable due to the very low mass of tungsten. A modern meter, which uses only a very small probing current, is essential. Otherwise, the resistance must be measured at regular time intervals until it has stabilised (resistance varies with temperature) and extrapolation back to zero time will give the filament cold resistance. Likewise, it is essential that the bulb has sufficient time to cool down to ambient temperature if it has been used. This can take more than 15 minutes!



Fig. 3. Measuring the filament characteristics.

Using the experimental arrangement shown in Fig. 3, the resistance of the filament and the power going into the bulb are easily measured when the voltage source is switched on. The various filament parameters for each bulb are given in the Table 1 where subscript 0 is for ambient temperature and subscript f is for operating temperature.

	25 W	40 W	60 W
$T_0(K)$	288	288	288
$T_{f}(K)$	2.55×10^3	2.60×10^3	2.61×10^3
$R_0(\Omega)$	158.8	99.6	65.1
$R_{f}(\Omega)$	2243.0	1445.8	948.6
$r_0 (\mu \Omega cm)$	5.33	5.33	5.33
$r_{f}(\mu\Omega cm)$	75.28	77.37	77.67
ϵ_0	3.04×10^{-2}	3.04×10^{-2}	3.04×10^{-2}
ε _f	3.07×10^{-1}	3.1×10^{-1}	3.1×10^{-1}
$a_{f}(m^{-2})$	3.49×10^{-5}	4.96×10^{-5}	7.44×10^{-5}

Table 1. Measured parameters of 3 tungsten bulbs.

In order to work at selected wavelengths, interference filters were used for 843, 732, 630 and 557.7 nm. All the filters had a nominal bandwidth of \sim 1 nm and a transmission maximum of \sim 50%. The results, presented in Section 4, were normalised in order to remove the effects of differing filter performance.

The experiment was set up according to Fig. 1 with L = 2 m and $\theta = 0^{\circ}$. A commercial Lambertian screen (owned by UNIS, Norway) was used, which had an albedo of 99% for all the wavelengths used. Table 2 shows the luminance (photons.s⁻¹.m⁻².sr⁻¹.(m)⁻¹) of the reflecting surface computed from Eq. 2:

	25 W	40 W	60 W
557.7 nm	7.57×10^{22}	1.31×10^{23}	2.04×10^{23}
630.0 nm	1.47×10^{23}	2.47×10^{23}	3.84×10^{23}
732.0 nm	2.72×10^{23}	4.49×10^{23}	6.93×10^{23}
843.0 nm	3.97×10^{23}	5.99×10^{23}	9.89×10^{23}

Table 2. Luminance in photons.s⁻¹.m⁻².sr⁻¹.(m)⁻¹.

Table 3 shows the luminance (watt.m⁻².sr⁻¹) of the reflecting surface computed from Eq. 3:

	25 W	40 W	60 W
557.7 nm	2.70×10^4	$4.67 \text{x} 10^4$	7.28×10^4
630.0 nm	4.64×10^4	$7.80 \mathrm{x} 10^4$	1.21×10^{5}
732.0 nm	7.39x10 ⁴	1.22×10^5	1.88×10^5
843.0 nm	9.37×10^4	1.41×10^5	2.33×10^5

Table 3. Luminance in watt.m⁻².sr⁻¹.

4. RESULTS

The experimental set up is described in Section 3. The geometry was kept constant throughout the experiment. A CCD camera was used to take images of the Lambertian screen. Four images of 60 s integration were taken through each of the 4 filters using each of the 3 ordinary tungsten bulbs, plus a known calibration lamp. The calibration lamp belongs to the Finnish Meteorological Institute, which has calibration data from the manufacturer. The calibration lamp was recorded twice, once at the beginning and once at the end of the experiment. The groups of 4 images were averaged together to reduce noise. For each average of 4 images, the average pixel value within a square in the centre of the image was recorded. This process was repeated exactly for all the data, giving 3x4=12 tungsten bulb values (Pix_t) and 2x4=8 calibration lamp values (Pix_c).

The results have been normalised in order to remove the effects of differing filter performance, different tungsten and calibration lamp powers, as well as the spectral response of the CCD detector, since these are not all well known. In order to compare the ordinary tungsten bulbs with the calibration lamp, a dimensionless normalised ratio (Eq. 14) was formed:

$$\frac{\operatorname{Pix}_{t} \times \operatorname{B}_{c}}{\operatorname{B}_{t} \times \operatorname{Pix}_{c}}$$
(14)

where B_t comes from Table 2 and B_c from the calibration lamp manufacturer's data sheet. Eq. 14 should be a constant independent of lamp power,

spectral response of the CCD, filter transmission characteristics and if the tungsten bulb output agrees with that of the commercial calibration lamp. Note that this ratio is not necessarily unity. Fig. 4 shows the result of forming the normalised ratio in Eq. 14.



Fig 4. The normalised ratio between the tungsten and calibration lamps.

Fig. 4. shows that the normalised ratio is rather constant for 630, 732 and 843 nm but not for 557.7 nm. Closer inspection of the 557.7 nm images shows that a light leak had contaminated the images. The spread in the data points is about 15%. Besides noise in the recording system, the most obvious source of uncertainty is power supply fluctuations to the tungsten bulb.

5. CONCLUSIONS

We believe that we have developed a viable alternative to optical calibrations that is not only simple and cheap but also easily arranged by virtually any laboratory. An accuracy of ~15% seems realistic. In the experiment we show that our relative comparison to the Finnish Meteorological Institute calibration lamp is good for 630, 732 and 843 nm. Furthermore, we believe that the anomalous result for 557.7 nm resulted from a light leak. This experiment will be repeated in order to check this discrepancy.

6. **REFERENCES**

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2. Forsythe and Worthing, The Properties of Tungsten and the Characteristics of Tungsten Lamps, *Astrophysical Journal*, Vol. 61, 146, 1925. 3. Hunten, Roach and Chamberlain, A photometer Unit for the Airglow and Aurora, *J. Atmos. Terr. Phys.*, Vol. 8, 345 – 346, 1956.

4. Handbook of Physics and Chemistry, Chap. E-208, CRC.

7. APPENDIX 1

Tungsten emissivity factors ($\times 1000$) as a function of wavelength and temperature:

nm	1600	2000	2400	2800	T(K)
250	440	126	400	411	
250	448	436	422	411	
275	472	459	449	439	
300	482	473	465	455	
325	479	474	465	458	
350	477	473	466	460	
375	480	474	468	461	
400	481	474	468	460	
425	479	472	466	458	
450	477	469	463	455	
500	468	462	455	447	
600	456	448	442	434	
700	445	438	430	420	
800	430	419	408	399	
900	415	404	393	383	
1000	392	382	373	366	
1100	367	361	355	351	
1200	345	342	339	336	
1300	322	323	324	325	
1400	296	306	311	314	
1500	281	290	297	304	
1600	263	275	285	293	
1700	247	261	274	283	
1800	233	249	263	276	
1900	216	233	250	264	
2000	211	229	246	261	
2200	193	212	230	247	
2400	177	197	217	236	
2500	170	191	212	231	
3000	120	150	170	190	
3500	80	110	140	160	

8. APPENDIX 2

Tungsten resistivity and wavelength-integrated total emissivity as a function of temperature:

Temperature	Resistivity		Emissivity
Κ	µohm cm µohm		cm
200	3.20	3.20	.020
300	5.65	5.64	.032
400	8.06	8.06	.042
500	10.56	10.74	.053
600	13.23	13.54	.064
700	16.09	16.46	.076
800	19.00	19.47	.088
900	21.94	22.58	.101
1000	24.93	25.70	.114
1100	27.94	28.89	.128
1200	30.98	32.02	.143
1300	34.08	35.24	.158
1400	37.19	38.53	.175
1500	40.36	41.85	.192
1600	43.55	45.22	.207
1700	46.78	48.63	.222
1800	50.05	52.08	.236
1900	53.35	55.57	.249
2000	56.67	59.10	.260
2100	60.06	62.65	.270
2200	63.48	66.26	.279
2300	66.91	69.90	.288
2400	70.39	73.55	.296
2500	73.91	77.25	.303
2600	77.49	81.00	.311
2700	81.04	84.70	.318
2800	84.70	88.50	.323
2900	88.33	92.30	.329
3000	92.04	96.02	.334
	(a)	(b)	(c)

a, c: Data taken from "Handbook of Physics and Chemistry"

b: Data taken from Forsythe et al. (1925)