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Spontaneous CP-violation in the left-right model and the kaon system

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Abstract

A left-right model with spontaneous CP breakdown, consistent with the particle physics phenomenology, is presented. Constraints on free parameters of the model: mass of the new right handed gauge boson M_2 and ratio r of the two vacuum expectation values of the bidoublet, are found from the measurement of ϵ in the kaon system. For most of the parameter space, M_2 is restricted to be below 10 TeV. Higher masses can be achieved only by fine tuning of Kobayashi-Maskawa matrix elements, quark masses, r and the phase α which is the unique source of CP-violation in the model. Large number of combinations of signs of quark masses, which are observables of the model, are found to be not allowed since they contradict with data. The range of ϵ'/ϵ the model predicts is around 10^{-4} in magnitude.

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1 Introduction

The origin of CP-violation [1] remains a mystery despite of the spectacular progress made during the last twenty years in understanding the weak and electromagnetic interactions in the framework of spontaneously broken gauge theories. The Standard Model (SM) allows, in its six-quark version, for the appearance of a phase in the effective quark-quark-vector boson vertex [2]. This phase, the Kobayashi-Maskawa (KM) phase, can be used to parametrize the amount of CP-violation in the SM. More precisely, this phase is responsible in the kaon system for a non-vanishing value of the ϵ paramete,r which measures the amount of $\Delta S = 2$ CP-violation. In this model, the parameter ϵ', which measures the $\Delta S = 1$ amount of CP-violation, turns out to be naturally small [3], in agreement with the present experimental result [4].

However, the KM mechanism for incorporating CP-violation to the SM cannot be fully satisfactory since it does not explain where CP-violation comes from. Moreover, there are indications that the amount of CP-violation in the SM is not sufficient for generating baryon asymmetry of the Universe [5]. Therefore one has to look for possible sources of CP-violation beyond the SM.

One of the most attractive extensions of the SM is the model based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [6]. In addition to the original idea of providing an explanation to the observed parity violation of the weak interaction at low energies it also turned out to be capable to explain the lightness of the ordinary neutrinos via the so-called see-saw mechanism [7]. In this model the Lagrangian is left-right symmetric but the vacuum is not invariant under the parity transformation. The left-right symmetry is spontaneously broken. At high energies the new particle degrees of freedom like the new right handed gauge bosons W_2, will appear.

The same argumentation can be applyed in the case of CP-violation. One can assume that the original Lagrangian is symmetric under CP transformation but the vacuum breaks CP spontaneously. Despite of the fact that the processes with spontaneous CP-violation in the left-right model have been studied previously in several works [8, 9, 10], a careful analysis of left-right models indicated that it could be impossible to construct a phenomenologically acceptable model with spontaneously broken CP [11] because of the flavour changing neutral currents (FCNC) occurring in those models. However, it turned out that not all the solutions had been taken into account in these analysis. It was shown in Ref.[12] that in two-doublet models CP-violation can occur spontaneously without violating FCNC restrictions. Recently, a similar result was shown for the left-right models [13]. Namely, even with the minimal Higgs sector containing a bidoublet ϕ and two triplets $\Delta_{L,R}$ it is possible to obtain spontaneous breakdown of CP and satisfy FCNC constraints. For this issue the β terms of the Higgs potential, the non diagonal quartic couplings between the two scalar triplets and the bidoublet, which were taken to be zero in previous works, play a crucial role.

Motivated by these results we re-analyse the K^0-$\bar{K}^0$ system assuming a phenomenologically consistent left-right model with a discrete CP symmetry at the Lagrangian level, i.e. CP is violated only spontaneously. All CP-violating observables in the SM are proportional to λ^6 in the Wolfenstein parametrization. However, since CP-violation in the left-right model can occur even with two quark generations the dominant left-right contribution in the kaon system is proportional to λ^2 only. This makes the kaon system very sensitive to searches for the left-right symmetry. In our analysis we take into account new measurements of the quark masses, KM matrix elements and strong coupling constant α_s as well as the recent developments in understanding of hadronic matrix elements and QCD corrections in the left-handed [14] and right-handed [10] sectors of the K system. We show that with the present data the measurements of ϵ and ϵ'/ϵ allow us to restrict the parameter space of the model considerably. In particular, without fine tuning, the ratio r of vacuum expectation values (vevs) of the bidoublet and the mass of the new right handed gauge boson M_2 should be limited to a quite narrow range in order to explain the observed CP-violation.

The outline of the paper is the following. In Section 2 we present our model in detail. In Section 3 we parametrize the most general KM matrix in terms of a single CP-violating phase arising from the spontaneos symmetry breaking (SSB). In order to do that we study the quark mass matrices in the model. In Section 4 we carry out the analysis of ϵ and ϵ'/ϵ in terms of our model and find restrictions on the model parameters. A summary is given in Section 5.

2 Left-right symmetric model with spontaneous CP-violation

We begin with presenting the minimal $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model with a left-right discrete symmetry. In the left-right symmetric models each generation of quarks and leptons are assigned to the multiplets

$$Q = \begin{pmatrix} u \\ d \end{pmatrix} \quad , \quad L = \begin{pmatrix} \nu \\ e \end{pmatrix} \quad , \tag{1}$$

with the quantum numbers $(T_L, T_R, B-L)$

$$\begin{aligned} Q_L \ &: \ \left(\frac{1}{2}, 0, \frac{1}{3}\right) \quad , \quad L_L \ : \ \left(\frac{1}{2}, 0, -1\right), \\ Q_R \ &: \ \left(0, \frac{1}{2}, \frac{1}{3}\right) \quad , \quad L_R \ : \ \left(0, \frac{1}{2}, -1\right). \end{aligned} \tag{2}$$

CP-violation in the model will arise from the Higgs sector and so we must spend a bit of time for a more detailed description of this sector.

The Higgs sector consists of a bidoublet

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \tag{3}$$

and the triplets

$$\Delta_{L,R} = \begin{pmatrix} \frac{\Delta^+_{L,R}}{\sqrt{2}} & \Delta^{++}_{L,R} \\ \Delta^0_{L,R} & \frac{-\Delta^+_{L,R}}{\sqrt{2}} \end{pmatrix} \tag{4}$$

with the quantum numbers $\phi \ : \ (\frac{1}{2}, \frac{1}{2}^*, 0) \ , \ \Delta_L \ : \ (1, 0, 2) \ , \ \Delta_R \ : \ (0, 1, 2)$, respectively.

Only the fields ϕ_1^0, ϕ_2^0, Δ_L^0 and Δ_R^0 can acquire vevs without violating electric charge. If Δ_L or Δ_R acquire a vev, then $B-L$ is necessarily broken. Further, if $\langle\Delta_R\rangle \neq \langle\Delta_L\rangle$, then parity breakdown is also ensured. The new feature that we have analysed, and we want to discuss in this work, is the phenomenological consequence of supposing that the vevs of the neutral fields are not real. In this case CP is spontaneously broken and we arrive at a unified picture of parity, time-reversal and $B-L$ violation.

In general, our symmetry breaking would be triggered by the vevs

$$\langle\phi\rangle = \begin{pmatrix} \frac{k_1}{\sqrt{2}} & 0 \\ 0 & \frac{k_2}{\sqrt{2}} \end{pmatrix} \quad , \quad \langle\Delta_{L,R}\rangle = \begin{pmatrix} 0 & 0 \\ \frac{v_{L,R}}{\sqrt{2}} & 0 \end{pmatrix}, \tag{5}$$

satisfying the following hierarchy: $|v_R| \gg |k_1|, |k_2| \gg |v_L|$. Here all the vevs can be complex. However, we still have a freedom to absorb two of these phases in such a way that two vevs are real and two complex: $v_R = |v_R|e^{i\theta}$ and $k_2 = |k_2|e^{i\alpha}$.

The Higgs sector contains 20 degrees of freedom of which 14 correspond to physical states, the latter split into four doubly-charged, four singly-charged and six neutral scalar fields. The remaining six degrees of freedom are eaten by the massive gauge bosons $W^{\pm}_{1,2}$ and $Z^0_{1,2}$ during the SSB.

Let us now discuss the form of the scalar field potential. The discrete left-right symmetry requires the potential to be invariant under

$$\Psi_L \longleftrightarrow \Psi_R \quad \Delta_L \longleftrightarrow \Delta_R \quad \phi \longleftrightarrow \phi^\dagger, \tag{6}$$

where Ψ denotes any fermion. We assume that the global phases allowed to appear in the transformations above are absorbed by the proper redefinition of the fields. Further, the most general scalar field potential cannot have trilinear terms: because of the nonzero $B-L$ quantum numbers of the Δ_L and Δ_R triplets, these must always appear in the quadratic combinations $\Delta_L^\dagger\Delta_L$, $\Delta_R^\dagger\Delta_R$, $\Delta_L^\dagger\Delta_R$ or $\Delta_R^\dagger\Delta_L$. These combinations can never be combined with a single bidoublet ϕ in such a way as to form $SU(2)_L$ and $SU(2)_R$ singlets. Nor can three bidoublets be combined so as to yield a singlet. However, quartic combinations of the form $\beta\ \mathrm{Tr}(\Delta_L^\dagger\phi\Delta_R\phi^\dagger)$ are allowed by the left-right symmetry. Following these conditions the most general form of the Higgs potential is

$$V = V_\phi + V_\Delta + V_{\phi\Delta}, \tag{7}$$

where

$$\mathbf{V}_\phi = -\mu^2_{i,j}\mathrm{Tr}(\phi_i^\dagger\phi_j) + \lambda_{i,j,k,l}\,\mathrm{Tr}(\phi_i^\dagger\phi_j)\,\mathrm{Tr}(\phi_k^\dagger\phi_l),$$

$$\mathbf{V}_\Delta = -\mu_i^2\,\mathrm{Tr}(\Delta_i\Delta_i^\dagger) + \rho_{i,j}\,\mathrm{Tr}(\Delta_i\Delta_i)\mathrm{Tr}(\Delta_j^\dagger\Delta_j^\dagger),$$

$$\mathbf{V}_{\phi\Delta} = \alpha_{i,j,k}\,\mathrm{Tr}(\phi_i^\dagger\phi_j)\mathrm{Tr}(\Delta_k\Delta_k^\dagger) + \beta_{i,j,k,l}\,\mathrm{Tr}(\phi_j^\dagger\Delta_k\phi_i\Delta_l^\dagger).$$

Here we have introduced a shorthand notation in which every term in the last equations stands for the generic term of its type. The full potential contains all possible independent combinations of the fields of such type and can be found in Ref.[13].

The presence of the β terms in addition to the complexity of the vevs is going to bring the desired spontaneous CP-violation. In fact, it is due to the β terms that the first derivative equations are no longer homogeneous allowing the phase degrees of freedom to survive. Previous works had eliminated these non-diagonal quartic couplings between the two scalar triplets and the bidoublet in order to avoid the ocurrence of FCNC at the minimum of the potential. However, as was shown in Ref.[13], these FCNC can be kept under control and still retain the terms allowing for spontaneous breakdown of CP. It is important to notice that, in order to have spontaneous CP-violation in the left-right symmetric model, one needs to have complex vevs in both bidoublet and one triplet i.e., one needs two phases. However, the leptonic sector does not concern us here and the consequences of CP-violation in the quark sector arise only from the Higgs field ϕ belonging to the $(\frac{1}{2}, \frac{1}{2}, 0)$ representation. Therefore, only the phase α is going to be relevant.

Besides that, the minimal left-right symmetric models with spontaneous CP-violation possess the useful property that all the CP-violating observables can be expressed in terms of a single phase, a ratio of scalar vevs, quark masses and weak mixing angles, but not on unconstrained quantities such as Yukawa couplings or additional phases. This happens only when the discrete left-right symmetry and CP symmetry are imposed on the Lagrangian as occurs in our model.

3 Parametrization of the KM matrix

An important prelude to the phenomenological study of CP-violation in any model is to identify the number of genuine CP phases. By the geniune CP phases we mean the phases left over when we have used all our freedom to redefine the particle fields. To carry out this procedure, we need to know the structure of the mass matrices in the model. We, therefore, start with the Yukawa Lagrangian.

The most general Yukawa Lagrangian for quarks in the left-right model is given by

$$\mathcal{L}_{\mathcal{Y}} = f\bar{\Psi}_L\phi\Psi_R + h\bar{\Psi}_L\tilde{\phi}\Psi_R + h.c., \tag{8}$$

where f and h are the Yukawa couplings and the summation over families is understood. A direct consequence of imposing CP as spontaneously broken symmetry, togheter with the discrete left-right symmetry, is that the Yukawa couplings matrices f and h in Eq.(8) must be real and symmetric. After the SSB the quark mass matrices generated by $\langle\phi\rangle_0$ are

$$\begin{aligned} M^u &= \frac{1}{\sqrt{2}}\left(fk_1 + hk_2e^{-i\alpha}\right) = \frac{k_1}{\sqrt{2}}\left(f + hre^{-i\alpha}\right), \\ M^d &= \frac{1}{\sqrt{2}}\left(hk_1 + fk_2e^{i\alpha}\right) = \frac{k_1}{\sqrt{2}}\left(h + fre^{i\alpha}\right), \end{aligned} \tag{9}$$

where M^u (M^d) is the up (down) type quark mass matrix and $r \equiv |k_2|/k_1$. The only complex parameter in Eq.(9) is the complex phase in $k_2 = |k_2|e^{i\alpha}$ which is the unique source of CP-violation in the charged fermion mass matrices that appear in our model.

Since M^u and M^d are symmetric complex matrices, they can be diagonalized by the orthogonal transformations

$$V^u M^u V^{uT} = D^u,$$
$$V^d M^d V^{dT} = D^d, \tag{10}$$

where V^u and V^d are unitary matrices and D^u and D^d the diagonal quark mass matrices. Since the charged current interaction of the theory can be written in the quark mass eigenstate basis as

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} \left(W_L^{\dagger\mu} \, \overline{u}_L \, K_L \, \gamma_\mu \, d_L \; + \; W_R^{\dagger\mu} \, \overline{u}_R \, K_R \, \gamma_\mu \, d_R \right) \; + \; h.c., \tag{11}$$

where K_L and K_R are the left and right KM matices, it follows that by choosing to diagonalize the quark mass matrices in the form (10) we have implicitly fixed our phase convention for quarks in such a way that the relation between K_L and K_R is

$$K_L = V^{u\,\dagger} V^d = K_R^*. \tag{12}$$

Therefore, the KM angles in K_L and K_R are equal and the total number of independent phases in both matrices together is the same as one unitary matrix can contain, which is $\frac{1}{2}N(N+1)$. Performing an appropriate rephasing of the quark fields some of the phases can be shifted from the left sector to the right one and vice versa, but, in general, not all of them can be removed from one matrix to the other.

For a moment we will work in the basis (12). However, for our phenomenological analysis in three generations it is more convenient to choose a basis in which there is only one phase, the KM matrix phase δ of the SM, left in K_L. This allows us to use the SM expressions for the CP-violating observables coming from the left sector. In the two generation analysis all the phases can be shifted to K_R.

Eliminating the matrices f and h in Eq.(9) we arrive at a matrix equation of the form

$$(1-r^2) W D^u W + (r^2 e^{2i\alpha} - 1) D^u = 2ir \sin\alpha \, K D^d K^T, \tag{13}$$

where $W = V^{u\dagger} V^{u*}$ is a unitary symmetric matrix and $K = K_L$ in the representation (12). This equation is exact for any number of generations and cannot be solved exactly. In the first approximation, inspired by the experimental data, in which K is taken to be diagonal one can easily show that Eq.(13) has solutions only if the following requirement is fulfilled

$$\frac{|\, r \sin\alpha \,|}{1-r^2} \leq \frac{m_b}{m_t}. \tag{14}$$

Putting this into another way, in order to give quarks the experimentally observed masses through the Lagrangian (8) the parameters in Eq.(9) must satisfy the condition (14). It has been shown [10] that for three generations already this first approximation gives very good results if compared with the complete numerical calculation.

As will be argued later, avoiding fine tuning of α to extremely small values the expression (14) implies $|\, r \,| \leq \mathcal{O}(m_b/m_t)$. Another important consequence of the condition (14) is that two independent parameters r and α can be reduced to a significant one which satisfies $| r\sin\alpha | \leq m_b/m_t$. As we will see later, in first approximation all the phases in K will appear as linear in $r\sin\alpha$.

Now we calculate explicitly the phases in the KM matrices in terms of r and α. When $r=0$, the mass matrices have the form

$$M_0^u = \frac{k_1 f}{\sqrt{2}}, \quad M_0^d = \frac{k_1 h}{\sqrt{2}}, \tag{15}$$

and Eq.(9) can be rewritten as

$$\begin{aligned} M^u &= M_0^u + r M_0^d e^{-i\alpha}, \\ M^d &= M_0^d + r M_0^u e^{i\alpha}. \end{aligned} \tag{16}$$

As suggested by (14), we will work under the assumption that r is so small that we can use it as a small perturbation parameter and include only terms of lowest order in r. Under this assumption we can treat the second term in Eq.(16) as a small perturbation and solve the unitary matrices which diagonalize M^u and M^d to the lowest nontrivial order in r. The zeroth order mass matrices can be parametrized in terms of the quark masses and mixing angles. By proper choice of the flavour basis we can assume that M_0^u is diagonal without loss of generality. In the same basis, M_0^d can be written in terms of the down type quark masses and mixing angles. Of course, the quark masses and the angles will be modified when r is included, but since we are calculating the complex phases in the diagonalizing matrices $V^{u,d}$ to the lowest order in r, the corrections to these masses and angles are almost negligible.

First we will work assuming only two generations. The generalization to the three generation case will be done afterwards. The reason for such an approach will become clear a bit later. The most general parametrization of K_L and K_R in two generations can be written as follows

$$\begin{aligned} K_L &= e^{-i\frac{\gamma}{2}} \begin{pmatrix} e^{i\frac{\delta_2}{2}}\cos\theta & e^{i\frac{\delta_1}{2}}\sin\theta \\ -e^{-i\frac{\delta_1}{2}}\sin\theta & e^{-i\frac{\delta_2}{2}}\cos\theta \end{pmatrix}, \\ K_R &= e^{i\frac{\gamma}{2}} \begin{pmatrix} e^{-i\frac{\delta_2}{2}}\cos\theta & e^{-i\frac{\delta_1}{2}}\sin\theta \\ -e^{i\frac{\delta_1}{2}}\sin\theta & e^{i\frac{\delta_2}{2}}\cos\theta \end{pmatrix}, \end{aligned} \tag{17}$$

where θ is the Cabbibo angle. After solving Eq.(10) for the complex phases in V^u and V^d we obtain the following equations for δ_1, δ_2 and γ to lowest order in r

$$\begin{aligned} \delta_1 &= r\sin\alpha \left[\frac{1}{2}\left(\frac{A}{m_u} - \frac{B}{m_c} - \frac{C}{m_d} + \frac{D}{m_s}\right) + 2\left(\frac{m_s - m_d}{m_u + m_c} - \frac{m_c - m_u}{m_s + m_d}\right)\cos^2\theta\right], \\ \delta_2 &= r\sin\alpha \left[\frac{1}{2}\left(\frac{A}{m_u} - \frac{B}{m_c} + \frac{C}{m_d} - \frac{D}{m_s}\right) - 2\left(\frac{m_s - m_d}{m_u + m_c} - \frac{m_c - m_u}{m_s + m_d}\right)\sin^2\theta\right], \\ \gamma &= r\sin\alpha \, \frac{1}{4}\left(\frac{A}{m_u} + \frac{B}{m_c} + \frac{C}{m_d} + \frac{D}{m_s}\right), \end{aligned} \tag{18}$$

where

$$\begin{aligned} A &= m_d\cos^2\theta + m_s\sin^2\theta, \\ B &= m_d\sin^2\theta + m_s\cos^2\theta, \\ C &= m_u\cos^2\theta + m_c\sin^2\theta, \\ D &= m_u\sin^2\theta + m_c\cos^2\theta. \end{aligned} \tag{19}$$

In these expressions all the quark masses can be both positive or negative. Strictly speaking, they are the physical masses (defined to be positive) with additional plus or minus signs which arise from the Yukawa couplings. We prefer to keep the signs in the masses instead of absorbing them to the phases of the KM matrices. Unlike in the SM, where the observables do not depend on these signs of masses, in the left-right model the signs themselves are observables. Because of this, it is important to keep track of the signs and to disentangle their physical significance. Just by inspection, we can see from Eq.(18) that for a given value of $r \sin \alpha$, there are as many distinct solutions as there are signs of masses, up to an overall sign which is not observable. That is $2^5 = 32$ solutions. However, we will show later that some of them can be ruled out on a phenomenological basis. The remaining ones can be divided into two groups depending on the relative sign of the SM and left-right contributions to ϵ.

As we see, in this model all the CP-violating phases in the hadronic sector can be directly related to $r \sin \alpha$. This feature is independent of how many generations of quarks we have in the model. By choosing the relative phases between quarks fields, in the $N = 2$ case, all the phases in K_L can be removed into K_R. However, in the $N = 3$ case one phase will be left over in K_L. This is nothing but the well known observation by Kobayashi and Maskawa [2]. In the following we will work in the basis where the maximum number of phases are shifted to the right sector.

To extend our model to the three generation case we have to analyse the effect of the KM phase. It is well known that the dominat left-right contributions in the kaon system do not involve the third generation [15], and consequently, only the phases that are present in the two generation K_R will be needed in our analysis. Therefore, the choice of K_R in the form similar to Eq.(17) (all the phases should be multiplied by two due to the shifting them from K_L to K_R) is the most general in our case. To make the phenomenological estimation complete, we will have to use the complete three generation K_L matrix put in the usual SM form. In a suitable convention K_L can be written in the form

$$K_L = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}, \tag{20}$$

where $s_i \equiv \sin \theta_i$, $c_i \equiv \cos \theta_i$ and θ_1, θ_2 and θ_3 are the three KM angles. A lesson we have learned from the Kobayashi-Maskawa $SU(2) \times U(1)$ model is that all the CP-violating quantities in the model are proportional to a single factor $s_1^2 s_2 s_3 \sin \delta$ which is of the order λ^6 in the Wolfenstein parametrization. The dominant left-right contribution to CP-violating observables, however, is of the order of λ^2 since CP-violation in the model can occur with two generations only. Even if the right-handed charged currents are suppressed by the large M_2 and have a very little effect in low energy CP-conserving quantities (we assume this to be always true and use the SM values for the KM matrix entries) in CP-violating observables the left-right part can possibly dominate over the SM one which is strongly suppressed. Therefore the kaon system is very good to search for the left-right symmetry.

Consequently, if in our model with spontanous CP breaking δ itself is of the same order of magnitude as δ_1 or δ_2 (which we have calculated before), then we expect the SM contribution to be somewhat suppressed. To show that, we have to compute the relation between δ and $r \sin \alpha$. Following the same procedure as in the four quark case, after tedious algebra we obtain

$$\delta = r \sin \alpha \, \frac{m_c}{m_s} \left(\frac{s_2 + s_3}{s_3} \right) \left[1 + s_3 \left(s_2 + s_3 \right) \frac{m_t}{m_c} \right]. \tag{21}$$

With this result we can proceed to the phenomenological analysis of our model of spontaneous CP-violation.

4 Constraints on the left-right model parameters from ϵ and ϵ'/ϵ

There exists already an extensive literature on $|\Delta S| = 1, 2$ effective interactions of kaon system in the left-right symmetric models. Since our aim is to perform a phenomenological analysis of our model of spontaneous CP-violation we will adopt the already known expressions of ϵ and ϵ' together with the estimations of hadronic matrix elements, QCD short distance corrections and final state interactions from the most comprehensive works [10, 14]. We note here that for the hadronic matrix elements of the right-handed sector the vacuum saturation approximation is used which is assumed to give precise enough results for our analysis. We will update the previous expressions by using the recent experimental data for quark masses as well as for the modulus of the KM matrix elements. After substituting the KM matrix elements from the previous Section we can use the resulting formulae to constrain the model parameters.

In the left-right model the effective $|\Delta S| = 2$ Hamiltonian gets contributions from the box diagrams presented in Fig.1. There are two charged current gauge boson mass eigenstates W_1, predominantly the left-handed W_L, and W_2, predominantly the right-handed W_R, in the model. Their mixing angle is very small and can be expressed as

$$\zeta = \frac{2r}{1 + r^2} \left(\frac{M_1}{M_2} \right)^2 , \tag{22}$$

where M_1 and M_2 denote masses of the corresponding gauge bosons. The diagram with both W-s being the ordinary left handed W_1 gives the SM contribution. However, in our model the phase δ is not an independent parameter but related to α by Eq.(21). The diagram with two W_2-s is negligible compared with the SM one because of very large mass of the new gauge boson. The couplings of charged Higgs bosons with quarks are suppressed by the factors of m_q/M_H and since their masses are of the order of the right-handed breaking scale one can ignore them. Therefore, the dominant left-right contribution to the effective Hamiltonian of $|\Delta S| = 2$ comes from the diagrams with one W_1 and one W_2. Since the contribution coming from these diagrams is proportional to the mass squared of the up-type quarks which run in the loop, one may expect that diagrams with top quarks are dominant. However, due to the small off-diagonal elements of the KM matrix for the third generation, the contribution from the top quark diagrams relative to the c quark diagrams is of the order $\lambda^8 m_t^2/m_c^2 \sim 10^{-2}$, where $\lambda \sim 0.2$ is the Wolfenstein's expansion parameter. This proves that within our assumption of the CP symmetry of Lagrangian (8), which makes the modulus of the left and right KM matrices to be equal, we can safely neglect the third generation in dealing with the left-right contribution to the parameter ϵ.

Let us now consider the parameter $\epsilon = \epsilon_{SM} + \epsilon_{LR}$, where ϵ_{SM} comes from the left-left and ϵ_{LR} from the left-right box diagram. The expressions we are dealing with can be written as [10, 14]

$$\epsilon_{SM} = e^{i\pi/4}\, 1.34\; s_2 s_3 \sin\delta \left[1 + 860\; S\left(\frac{m_t^2}{M_1^2}\right) s_2 Re V_{ts}\right], \tag{23}$$

where

$$S(x) = x\left[\frac{1}{4} + \frac{9}{4}\frac{1}{(1-x)} - \frac{3}{2}\frac{1}{(1-x)^2}\right] - \frac{3}{2}\left[\frac{x}{1-x}\right]^3 \ln x \tag{24}$$

and

$$\epsilon_{LR} = -e^{i\pi/4}\, 0.36 \sin(\delta_2 - \delta_1) \left[\frac{1.4\, TeV}{M_2}\right]^2 \left(1 + 0.05 \ln\left[\frac{M_2}{1.4\, TeV}\right]\right). \tag{25}$$

Here δ, δ_1 and δ_2 are the KM phases calculated in Section 3 and s_2 and s_3 are defined in Eq.(20). For our computations we have used the numerical values [4] $s_1 = 0.2209 \pm 0.0027$, $s_2 = 0.0430 \pm 0.0258$ and $s_3 = 0.0158 \pm 0.007$, which have been obtained in the SM. We assume that the effect of the new heavy scale in determining the KM matrix elements can be neglected at the first approximation. Since in the expressions of the CP-violating phases the quark masses appear as ratios then the result does not depend on the mass scale at which they are taken. We can therefore choose the running masses at Z_0 scale [16]: $m_u = (1.5 \pm 1.2)$ MeV, $m_d = (4.1 \pm 1.7)$ MeV, $m_s = (83. \pm 30.)$ MeV, $m_c = (0.52 \pm 0.10)$ GeV and $m_t = (180. \pm 13.)$ GeV.

The CP-violating parameter $\epsilon = |\epsilon| e^{i\phi}$ has been measured with a good accuracy $|\epsilon_{exp}| = (2.26 \pm 0.02) \cdot 10^{-3}$ and $\phi \approx \pi/4$ [4]. We fix it to the experimentally measured value and vary the free parameters of the model, M_2, r and α as well as the signs of the quark masses, in such a way as to get the correct ϵ.

Let us study first how the changes of the signs of quark masses affect the value of ϵ (for a moment we leave the phase $e^{i\pi/4}$ aside). In the case of all positive masses $\epsilon_{SM} > 0$ and $\epsilon_{LR} < 0$. Changing the sign of m_d changes the sign of ϵ_{LR}. It is easy to check that changing the sign of m_s changes the sign of ϵ_{SM}, while changes in signs of the other quarks either cange the signs of the both contributions (m_c) or leave them unchanged (m_t). Since $\epsilon_{SM} + \epsilon_{LR}$ should be positive then the situation where both left-left and left-right contributions are negative cannot be realized. Therefore, just on this basis, we can exclude some of the combinations of quark masses as solutions. In principle, we have two qualitatively different situations: either one of the contributions ϵ_{SM}, ϵ_{LR} is negative and another positive or both of them are positive. Therefore, different models can be classified according to the relative sign of the SM and LR contributions to ϵ: Class I if they are different and Class II if they are the same. In general, this classification can be done also by the relative sign of m_d and m_s but one has to remember that, on the contrary to claims in Ref.[9, 10], not all combinations of the signs are viable solutions.

In Fig. 2. we plot M_2 against $r \sin\alpha$ for two different choices of signs of the quark masses using the central values of all the experimentally measured input parameters. For the curve (a) m_d and m_s are taken to be negative and the rest of the masses positive while for the curve (b) only m_d is taken to be negative. These two choices belong to the two distinct classes of models, Class I and II, respectively. We have checked that choosing different combinations of the signs of quark masses inside the classes the curves in Fig.2 change not more than $\sim 20\%$. To present all of them will not enlight the disscusion at all. Therefore, we plot just one representative curve from each class.

Our complete analysis shows that $r \sin\alpha$ is limited to the region

$$0.0005 \leq r \sin\alpha \leq 0.017, \tag{26}$$

where the upper limit comes from Eq.(14). The use of smaller values of $r \sin\alpha$ would lead to lighter W_2 than tolerated by the lower limit coming from the K_S-K_L mass difference [10, 15] if one wants to get the correct ϵ_{exp}.

With $r \sin\alpha$ in such a range and with the central values of experimental data the SM contribution alone is always smaller than $|\epsilon_{exp}|$ and we need the LR part to agree with the experiment. Therefore, also these combinations of signs of quark masses for which $\epsilon_{LR} < 0$ are not allowed in the present case. In the case of Class I (Fig.2 (a)), $r \sin\alpha$ can vary over all the values of (26) and M_2 is a slightly increasing function of $r \sin\alpha$ with the maximum value $M_2 \approx 5.5$ TeV. However, the behaviour of Class II (Fig.2 (b)) is completely different. In this case $r \sin\alpha$ has a lower limit around 0.0045 and M_2 increases very fast when $r \sin\alpha$ approaches the maximum value. This behaviour can be easily understood. Since in Class II the SM and LR contributions have the same sign and the SM part is getting bigger if $r \sin\alpha$ is increasing then we need just a small additional contribution from the LR sector. Therefore, M_2 should be larger. As suggested by grand unified theories [17] the natural values for r are around 10^{-3}. In light of this prediction Fig.2 clearly prefers very light M_2 and models belonging to Class I.

In order to see the allowed space for r and α, in Fig.3 we plot r against α for fixed $r \sin\alpha = 0.001$. For most of α values r is quite flat and our previous discussion is, indeed, valid for most of the parameter space. r increases fast only for very small or close to π phases. For models of spontaneous CP-violation these extreme values of α are unnatural since the vevs of the bidoublet are almost real without any deeper reason. However, there is a more strict argument to prohibit very small values of $\sin\alpha$. In order to provide quarks with the experimentally measured masses and keep α to be very small at the same time we have to make some of the Yukawa couplings in the Lagrangian (8) to be large and the present perturbative calculation is not valid any more. Therefore, in the framework of the perturbation we have performed, the extreme values of α are not allowed. We will assume this in the following.

So far, we have not taken into account the effects of the experimental errors. Looking at the numerical values of the quark masses and s_2 and s_3 we see that the least accurately determined parameters are m_s and s_2. Indeed, a numerical analysis shows that our results are most sensitive to the changes of s_2 which gives the dominant error. If we tune the input parameters within what the experimental results can tolerate in such a way that the SM contribution to ϵ can be bigger than $|\epsilon_{exp}|$ then we have a qualitatively new situation which should be analysed.

In Fig.4 we plot M_2 against $r \sin\alpha$ for the same class of models as in Fig.2 but taking for $s_2 = 0.0688$ i.e. the extreme value allowed by the 1 standard deviation experimental error (68% C.L). With this s_2, $|\epsilon_{SM}|$ is almost maximized by the experimental errors since they are dominated by s_2. The curve (a) in Fig.4 corresponds to the curve (a) in Fig.2. However, for the Class I model we have now another curve (b) which also gives the correct ϵ. Since $|\epsilon_{SM}|$ can be bigger than $|\epsilon_{exp}|$ we have two possibilities: either ϵ_{LR} is large and ϵ_{SM} reduces it to the correct $|\epsilon_{exp}|$ value (curve (a), $m_d,\ m_s < 0$) or ϵ_{LR} is small and serves to reduce ϵ_{SM} to the needed value (curve (b), $m_d,\ m_s > 0$). The curve (c) in Fig.4 denotes the behaviour of M_2 in the case of Class II model.

As can be seen from the asymptotic behaviour of curves (b) and (c), there is a pole in M_2 corresponding to a $r \sin\alpha$ value for which the SM contribution gives exactly the measured $|\epsilon_{exp}|$. Obviously, for such a $r \sin\alpha$ curves (b) and (c) should go to infinity. Assuming the GUT suggested value for r of 10^{-3}, M_2 must have mass around 2-3 TeV. Since the curve (c) can extend only up to the pole, the parameter space in the case of Class II models is even more restricted than previously which does not favor these models. For large $r \sin\alpha$ there are two values of M_2 possible in Class I models but for the most of the $r \sin\alpha$ space M_2 should be in the range 4-10 TeV. If we want to make M_2 to be heavy, say heavier than 20 TeV, we have to do the following. We have to fix the KM matrix entries and quark masses in a way to ensure $|\epsilon_{SM}| \geq |\epsilon_{exp}|$ at least for some $r \sin\alpha$. Then we have to fine tune the parameters r and α to the very small region where the relation $\epsilon_{SM} \approx \epsilon_{exp}$ holds almost exactly. And even doing so we still have a possibility to explain ϵ_{exp} by the curve (a) in Fig.4. We have to conclude that the left-right model with spontaneous CP-violation clearly prefers light M_2.

For completeness we will now analyse ϵ'/ϵ in our model. The effective $\Delta S = 1$ Hamiltonian, giving rise to ϵ', gets contributions from the penguin as well as the tree level diagrams that are depicted in Fig.5. The important left-right contributions come from W_2 exchange and also from the W_1-W_2 mixing. The top quark contribution to the right sector is small [10] and the two family parametrization should work well. Again, we adopt formulae for ϵ' from the previous works and actualize them by using more precise values for the running α_s and quark masses. One has $\epsilon' = \epsilon'_{SM} + \epsilon'_{LR}$, where [10, 14]

$$\epsilon'_{SM} = e^{i(\pi/4+\delta_2-\delta_0)}\, 3.2 \cdot 10^{-2} s_2 s_3 \sin\delta\; H(m_t) \tag{27}$$

and

$$\begin{aligned}\epsilon'_{LR} = e^{i(\pi/4+\delta_2-\delta_0)} 10^{-2} \Bigg\{ & \left[6.8 \left[\frac{\alpha_s(\mu)}{\alpha_s(M_2)}\right]^{-2/b} - 0.30 \left[\frac{\alpha_s(\mu)}{\alpha_s(M_2)}\right]^{4/b} \right] \\ & \frac{M_1^2}{M_2^2} \sin(\delta_2 - \delta_1) + 102\zeta[\sin(\gamma - \delta_1) + \sin(\gamma - \delta_2)] \\ & -9.6\zeta[\sin(\gamma + \delta_1) + \sin(\gamma + \delta_2)] \Bigg\}\,, \end{aligned} \tag{28}$$

where

$$\left[\frac{\alpha_s(\mu)}{\alpha_s(M_2)}\right]^{a/b} = \left[\frac{\alpha_s(\mu)}{\alpha_s(m_b)}\right]^{3a/25} \left[\frac{\alpha_s(m_b)}{\alpha_s(m_t)}\right]^{3a/23} \left[\frac{\alpha_s(m_t)}{\alpha_s(M_2)}\right]^{a/7} . \tag{29}$$

Here $H(m_t) = 0.04$, $b = 11 - 2/3\; n_{flavours}$ and the phases $\delta_2 - \delta_1 \approx 40^0$ in the exponential are the the strong interaction $\pi\pi$ phase shifts (do not mix up with the phases of the KM matrix). As the input value for the running α_s we use $\alpha_s(M_Z) = 0.118$ and evaluate it with the one-loop equation

$$\alpha_s^{-1}(m) = \alpha_s^{-1}(M_Z) + \frac{b}{2\pi} \ln(\frac{M_Z}{m}). \tag{30}$$

There are two contradicting measurements of ϵ'/ϵ. NA31 experiment at CERN claims the result $\mathrm{Re}\,(\epsilon'/\epsilon) = (2.3 \pm 0.7) \cdot 10^{-3}$, while E731 result at Fermilab is compatible with zero $\mathrm{Re}\,(\epsilon'/\epsilon) = (0.60 \pm 0.69) \cdot 10^{-3}$. Therefore, one can conclude that $\mathrm{Re}\,(\epsilon'/\epsilon)$ should be smaller than a few times 10^{-3}.

In Fig.6 we plot $\mathrm{Re}\,(\epsilon'/\epsilon)$ of the models of Class I (m_d and m_s negative) and II (only m_d negative) against $r \sin\alpha$ for two different values of $\alpha = \frac{\pi}{2}$ and $\alpha = \frac{\pi}{30}$. Note that for every value of $r \sin\alpha$ there corresponds a different M_2 which can be determined from Fig.2. An interesting result is that in Class I models $\mathrm{Re}\,(\epsilon'/\epsilon)$ is always negative and in Class II models positive. The absolut values of ϵ'/ϵ are in the range of 10^{-5} - 10^{-3} being notably smaller for the maximum $\sin\alpha$ than for the small values of $\sin\alpha$. This is an effect of having larger r in the latter case. As was argued before, $\sin\alpha$ cannot be too small since we want our expansion in powers of r to remain valid. In the case of Class II model, in which W_2 should be sufficiently heavy, we see the suppression of ϵ'/ϵ due to the large M_2 at large $r \sin\alpha$.

ϵ'/ϵ is rather sensitive to the change of sign of top quark mass, for some parameters it can be modified almost by a factor of two. In principle, with sufficient accuracy, this dependence may shed light on the sign in ϵ'/ϵ experiments. The dependence of ϵ'/ϵ on changes of any input parameters inside the allowed errors is typically of the order of $\sim 20\%$. But even with these changes our conclusions remain the same. In fact, we could not find any allowed region of the parameter space in which ϵ'/ϵ violates the experimental limit. In general, however, the left-right model with spontaneous CP-violation seems to prefer values of $\mathrm{Re}\,(\epsilon'/\epsilon)$ around 10^{-4} in magnitude.

5 Conclusions

Motivated by the possibility of constructing phenomenologically consistent left-right symmetric models with spontaneous breakdown of CP symmetry [13] we carry out the analysis of the model using CP-violating observables in the K system. We parametrize the general KM phases in terms of a single phase α which comes from the vevs of the bidoublet and is the only source of CP-violation in our model. Due to this fact, and also due to the heavy top quark mass which forces the ratio r of two bidoublet vevs to be very small, we find the parameter space of the model to be rather restricted. We adopt the expressions for ϵ and ϵ' together with hadronic matrix elements and QCD corrections from Ref.[10, 14] and actualize them by updating the values of quark masses, KM matrix elements and strong coupling constant.

Using the measurement of ϵ_{exp} we find that for most of the parameter space the mass of the new right-handed gauge boson W_2 should be below 10 TeV. Considerably higher masses of W_2 can be achieved only by fine tuning the KM matrix elements, quark masses and parameters α and r. But even in this case there is another, small value of M_2 which also gives the correct ϵ. This happens because there are two different classes of modes which can be classified according to the relative sign of left-left and left-right contributions to ϵ. The signs of quark masses are observables in left-right models and, unlike in the previous works, we found that many combinations of the signs are not allowed by the data. In the context of grand unified theories which predict the value of r to be 10^{-3} [17] our analysis seems to prefer Class I models in which m_d and m_s have the same sign.

All the predicted values of $\mathrm{Re}\,(\epsilon'/\epsilon)$ for the allowed parameter space (keeping ϵ fixed to the experimental value) are below the accuracy of the present experiments. The most favoured range of $\mathrm{Re}\,(\epsilon'/\epsilon)$ is around 10^{-4} in magnitude being positive for Class II and negative for Class I models. In the light of grand unification for very small r this means that $\mathrm{Re}\,(\epsilon'/\epsilon)$ in the left-right models with spontaneous CP-violation is negative.

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Figure captions

Fig.1. Feynman diagrams contributing to $|\Delta S| = 2$ transition.

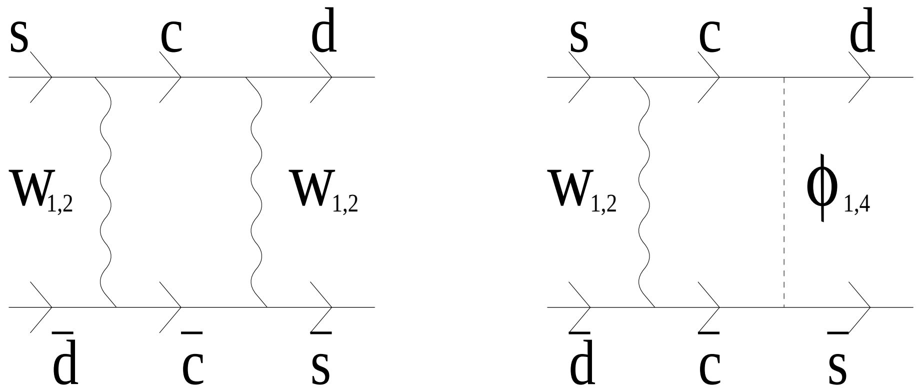
Fig.2. M_2 as a function of $r \sin\alpha$ for Class I (curve a, m_d and m_s negative) and Class II (curve b, m_d is negative) models. Central values of all experimental data are used.

Fig.3. r as a function of α for fixed $r \sin\alpha = 0.001$. α values close to 0 or π are not allowed in order to keep our perturbative results valid.

Fig.4. The same as in Fig.2 with experimental data tuned to give the largest ϵ_{SM}. Curves a and b correspond to the possible values of M_2 in Class I and curve c in Class II, respectively.

Fig.5. Feynman diagrams contributing to $|\Delta S| = 1$ transition.

Fig.6. $\mathrm{Re}\,(\epsilon'/\epsilon)$ as a function of $r \sin\alpha$ in the case of two values of α. The curves in the positive side belong to Class II and in the negative side to Class I.



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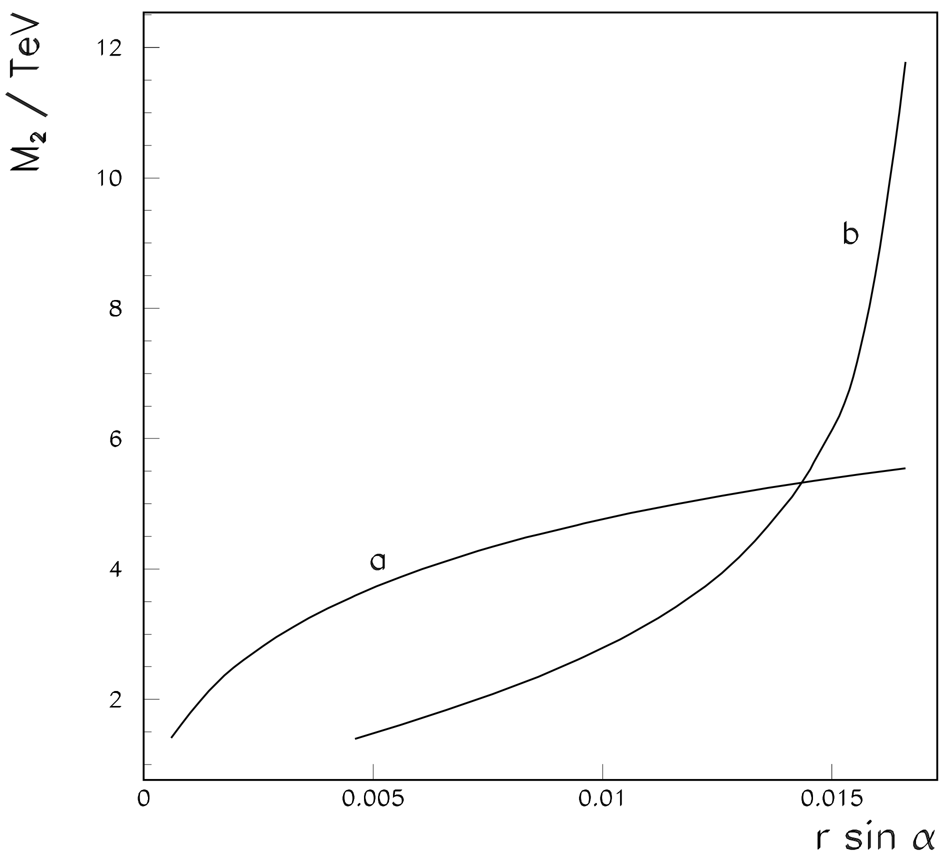
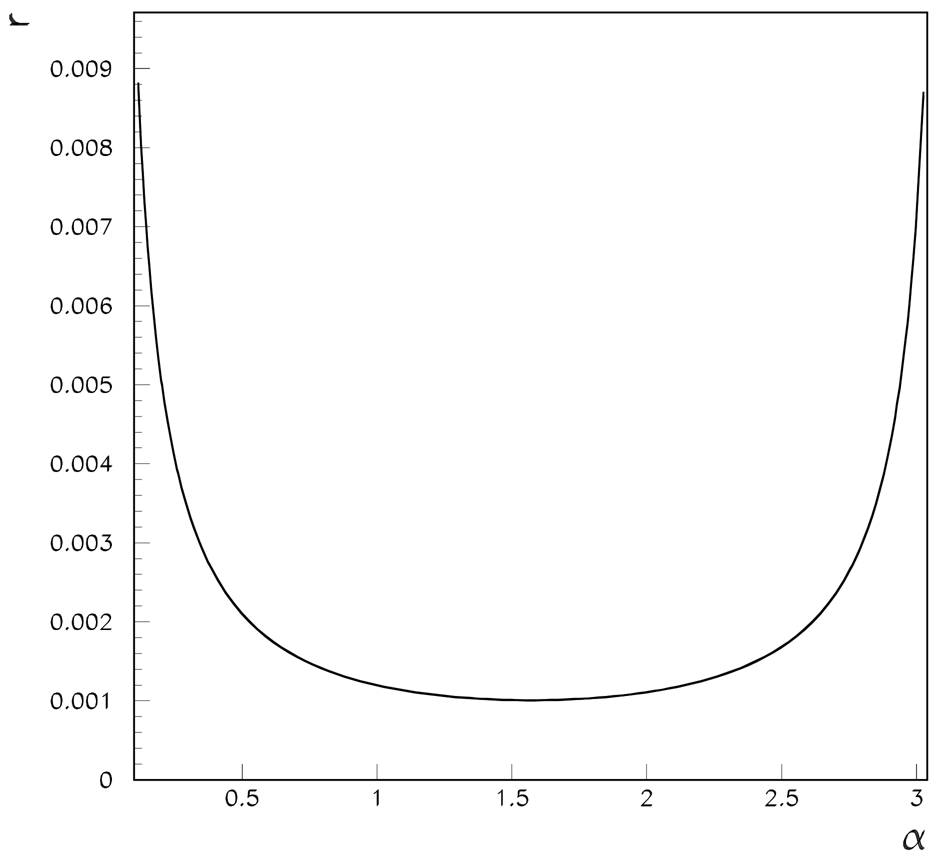


Figure 2: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0.007
0.006
0.005
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0
0.5
1
1.5
2
2.5
3
α

Figure 3:

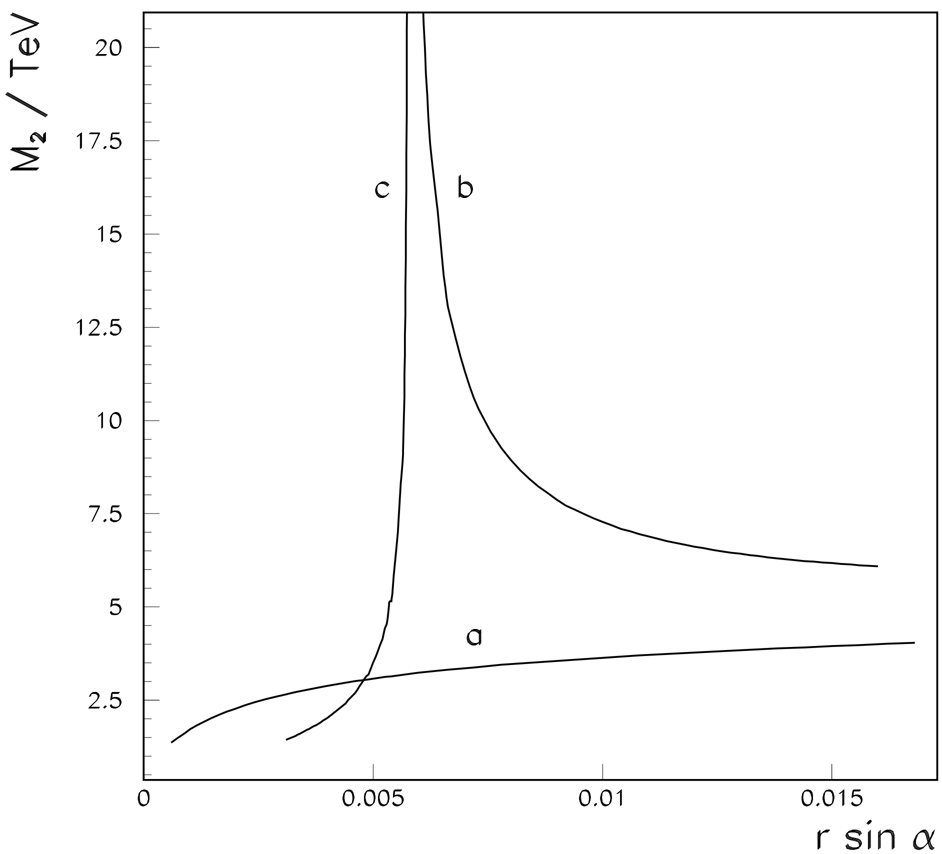


Figure 4:

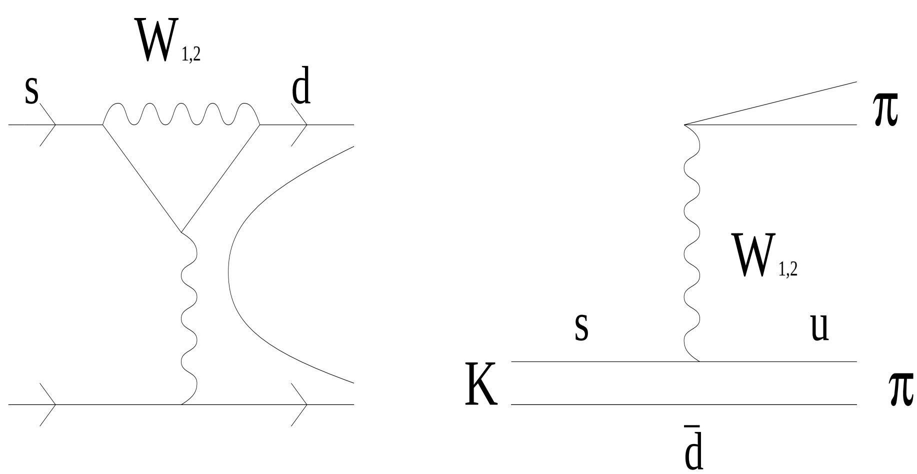
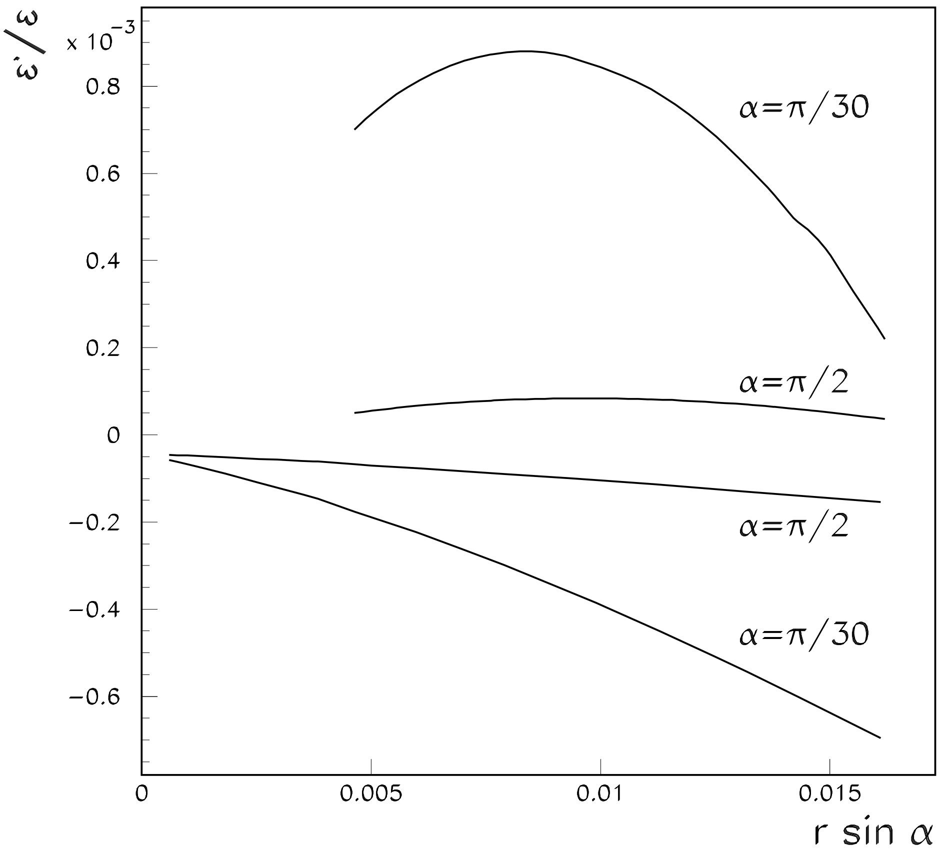


Figure 5:



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x 10^-3
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0.6
0.4
0.2
0
−0.2
−0.4
−0.6
0
0.005
0.01
0.015
r sin α
α=π/30
α=π/2
α=π/2
α=π/30

Figure 6: