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CP Violation with Three Oscillating Neutrino Flavours

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Abstract

We explore the prospects of observing leptonic CP violation in a neutrino factory in the context of a scenario with three strongly oscillating neutrinos able to account for the solar, the atmospheric and the LSND results. We address also the problems related with the fake asymmetries induced by the experimental device and by the presence of matter.

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1 Measuring leptonic CP violation with neutrino beams from a muon collider

Muon storage rings, muon colliders, and their physics potential are being studied intensively at FNAL and at CERN [1]. In particular, a muon storage ring at some 20 GeV, being the first step in these projects, would serve to produce intense neutrino beams of unique quality. This possibility has received much attention [2, 3, 4, 5] recently. The straight sections of a muon collider would serve as sources of $\overline{\nu}_\mu$ and of ν_e with energy spectra perfectly calculable from muon decay, when positive muons are stored, and, similarly, of ν_μ and $\overline{\nu}_e$ with well-known energy spectra when negative muons are stored.

In this work we explore the possibilities of studying CP violation in the leptonic sector, at a neutrino factory of this kind, by comparing the oscillation probabilities of CP-conjugate channels $\nu_i \to \nu_j$ and $\bar{\nu}_i \to \bar{\nu}_j$ with $(i \neq j)$. The most suitable channels for studying CP violation are $\nu_e \to \nu_\mu$ and $\bar{\nu}_e \to \bar{\nu}_\mu$, as well as their T conjugate partners $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$. In these channels, unlike the $\nu_\mu \to \nu_\tau$ channels, the CP violating part of the oscillation probability is not hidden by the CP conserving part [6], so that large asymmetries between CP-conjugate channels may arise, provided the leptonic CKM matrix allows for large violation of CP. Among these, the channels $\nu_e \to \nu_\mu$ and $\bar{\nu}_e \to \bar{\nu}_\mu$ seem to be the most promising because it may be easier to disentangle negative from positive muons, in a large detector of high density, than to disentangle electrons from positrons.

As a matter of example, we study the case of neutrino beams from stored muons with energy $E_\mu = 20$ GeV, and an experimental arrangement where they travel over a distance of some 730 km, i.e. from CERN to the Gran Sasso laboratory, or, likewise, from FNAL to the Soudan mine. But we also give the scaling behaviour of the effects with either energy or distance. Within the range of squared mass differences and mixing angles, matter effects are important, see also [3]. However, unlike the case where neutrinos traverse the Earth from the antipode [7], they are easy to cope with because the density in the Earth's crust is essentially constant.

The $\nu_e \to \nu_\mu$ oscillation probability can be measured as follows: Suppose the electron neutrinos are produced by the decay of a number N_{μ^+} of positive muons in the straight section of the storage ring pointing to the detector. The ν_μ which appear when there is oscillation of ν_e into ν_μ, are detected by their charged current interaction in the detector. The number of observed muon neutrinos, n_{ν_μ} is given by,

$$n_{\nu_\mu} = N_{kT}\ 10^9\ N_A \int F_{\nu_e}\ \sigma_{\nu_\mu}\ P(\nu_e \to \nu_\mu)\ dE \tag{1}$$

where F_{ν_e} is the forward flux of electron neutrinos from a number N_{μ^+} of positive muon decays, σ_{ν_μ} is the charged current cross section per nucleon and $P(\nu_e \to \nu_\mu)$ is the oscillation probability for neutrinos traveling inside the Earth taking into account matter effects. N_{kT} is the size of the detector in kilotons. An analogous formula holds in the case of anti-neutrinos.

Adopting the sample design configuration for muon production, cooling, acceleration and storage described by Geer [2], the number of available muons of either sign is approximately $8 \cdot 10^{20}$ per year, for muons stored at an energy of 20 GeV. Of these, one fourth decay in a straight section directed towards the neutrino detector with a 10 kT target some $\sim$ 730 km downstream, yielding about $2 \cdot 10^{20}$ neutrinos per year and an identical number of anti-neutrinos. We use these numbers in what follows, and refer the reader to [2] for details of the design of neutrino beams.

Let us begin by computing the number of produced muon neutrinos. Experimental cuts needed to eliminate background as well as detecting efficiencies will be included later on. The neutrino fluxes at a neutrino factory have simple analytical forms that follow from the well-known formulae for muon decay. Let $x = E_\nu/E_\mu$ be the fractional neutrino energy. For unpolarized muons of either charge, and neglecting corrections of order m_μ^2/E_μ^2 the normalized fluxes of forward moving electron neutrinos are

$$g_{\nu_e,\bar{\nu}_e}(x) = 12\, x^2\, (1-x) \tag{2}$$

and, for each neutrino type, the flux in the forward direction due to N_μ decaying muons is

$$F \;=\; \left.\frac{d^2 N_\nu}{dx\, d\Omega}\right|_{\theta\simeq 0} \;=\; \frac{E_\mu^2\, N_\mu}{\pi\, m_\mu^2\, L^2}\, g_\nu(x) \tag{3}$$

The above expressions are valid for a detector placed in the forward direction whose transverse dimensions are much smaller than the beam's transverse size $\sim (L\, m_\mu/2\, E_\mu)$.

We assume that the interaction cross sections due to charged current interactions scale linearly with the energy even in the very low energy part of the spectrum

$$\sigma_{\nu_e} = .67 \cdot 10^{-38}\text{cm}^2\ E\ (\text{GeV}) \tag{4}$$

$$\sigma_{\bar{\nu}_e} = .34 \cdot 10^{-38}\text{cm}^2\ E\ (\text{GeV}) \tag{5}$$

Regarding matter effects, let us remind the reader of the fact that of all neutrino species only ν_e and $\bar{\nu}_e$ have elastic scattering amplitudes on electrons due to charged current interaction. This, as is well known, induces effective "masses" $\mu = \pm 2 E_\nu a$, where the upper sign refers to the electron neutrino, the lower sign to the corresponding anti-neutrino, and where $a = \sqrt{2} G_F n_e$, n_e being the electron density.

Matter effects [8] are important provided the interaction term μ,

$$\mu = 7.7 \cdot 10^{-5}\text{eV}^2 \left(\frac{\rho}{\text{gr/cm}^3}\right)\left(\frac{E_\nu}{\text{GeV}}\right) \tag{6}$$

is comparable to, or bigger than, the quantity $\Delta_{m_{ij}^2} = m_i^2 - m_j^2$ for some mass difference and neutrino energy.

CP related observables often involve the comparison between measurements in two charge-conjugate modes of the factory. One example of an asymmetry is [9]

$$a_{CP}^{tot} = \frac{\int P(\nu_e \to \nu_\mu) F_{\nu_e} \sigma_{\nu_e} dE - \int P(\bar{\nu}_e \to \bar{\nu}_\mu) F_{\bar{\nu}_e} \sigma_{\bar{\nu}_e} dE}{\int P(\nu_e \to \nu_\mu) F_{\nu_e} \sigma_{\nu_e} dE + \int P(\bar{\nu}_e \to \bar{\nu}_\mu) F_{\bar{\nu}_e} \sigma_{\bar{\nu}_e} dE} \tag{7}$$

or in other terms,

$$a_{CP}^{tot} = \frac{n_{\nu_\mu}/N_{\mu^+} \ - \ n_{\bar{\nu}_\mu}/N_{\mu^-}}{n_{\nu_\mu}/N_{\mu^+} \ + \ n_{\bar{\nu}_\mu}/N_{\mu^-}} \tag{8}$$

In vacuum this quantity a_{CP}^{tot} would be a pure CP odd observable. The voyage through our CP uneven planet, however, induces a nonzero asymmetry even if CP is conserved, since ν_e and $\bar{\nu}_e$ are affected differently by the electrons in the Earth [10]. Therefore, to obtain the genuine CP odd quantity of interest, the matter effects must be subtracted with sufficient precision.

For this purpose, we compute the matter asymmetry in the absence of CP violation, or fake CP asymmetry, by

$$a_{CP}(\delta = 0) = \frac{\int P(\nu_e \to \nu_\mu) \mid_{\delta=0} \ F_{\nu_e} \sigma_{\nu_e} dE \ - \ \int P(\bar{\nu}_e \to \bar{\nu}_\mu) \mid_{\delta=0} \ F_{\bar{\nu}_e} \sigma_{\bar{\nu}_e} dE}{\int P(\nu_e \to \nu_\mu) \mid_{\delta=0} \ F_{\nu_e} \sigma_{\nu_e} dE \ + \ \int P(\bar{\nu}_e \to \bar{\nu}_\mu) \mid_{\delta=0} \ F_{\bar{\nu}_e} \sigma_{\bar{\nu}_e} dE} \tag{9}$$

where we take into account matter effects but set $\delta = 0$ in the transition probabilities.

The total asymmetry a_{CP}^{tot} that will be found in an experiment of the type described above, is a function of $a_{CP}(\delta = 0)$, eq. (9), and of the asymmetry in vacuum (taking due account of CP violation)

$$a_{CP}^{vac} = \frac{\int P^{vac}(\nu_e \to \nu_\mu) \ F_{\nu_e} \ \sigma_{\nu_e} \ dE \ - \ \int P^{vac}(\bar{\nu}_e \to \bar{\nu}_\mu) \ F_{\bar{\nu}_e} \ \sigma_{\bar{\nu}_e} \ dE}{\int P^{vac}(\nu_e \to \nu_\mu) \ F_{\nu_e} \ \sigma_{\nu_e} \ dE \ + \ \int P^{vac}(\bar{\nu}_e \to \bar{\nu}_\mu) \ F_{\bar{\nu}_e} \ \sigma_{\bar{\nu}_e} \ dE} \tag{10}$$

where $P^{vac}(\nu_e \to \nu_\mu)$ and $P^{vac}(\bar{\nu}_e \to \bar{\nu}_\mu)$ are the oscillation probabilities in vacuum. Provided $a_{CP}(\delta = 0)$ is not too large.

$$a_{CP}^{vac} \approx a_{CP}^{tot} - a_{CP}(\delta = 0) \tag{11}$$

is a good approximation. In any case, the error one makes in calculating a_{CP} by means of eq.(11) is smaller than the uncertainties on $a_{CP}(\delta = 0)$ itself. In addition, the error can be estimated by calculating the T-odd asymmetry [11], for each neutrino energy,

$$a_T(E_\nu, \delta) = \frac{P(\nu_e \to \nu_\mu) \ \ - \ P(\nu_\mu \to \nu_e)}{P(\nu_e \to \nu_\mu) \ \ + \ P(\nu_\mu \to \nu_e)} \tag{12}$$

where a nonzero value cannot be induced by matter effects. This also means that a_T a cleaner quantity in testing T violation than is a_{CP} for CP violation.

An important component of any study of muon appearance due to $\nu_e \to \nu_\mu$ oscillations is the event selection strategy for the μ's produced from charged current interactions of the ν_μ's. For neutrino experiments using a muon storage ring, the detailed prescription for event selection can be formulated only after the detector design is specified. There are, however, some basic issues concerning the signal and the backgrounds which all experiments are likely to be concerned with. On general grounds the background to a wrong sign muon signal is associated with the numerous decay processes that can produce fake muons: pions maskerading as muons, muonic charged currents (here one would also have to miss the right sign muon) or electronic charged currents (here one has also in the decay of the latter right sign muons), to name only a few. Without referring to a specific detector and the corresponding simulation toil we trust the experimental proficiency by setting an overall detection efficiency of 30% and by making a cut $E_\nu > 5$ GeV to eliminate inefficiently observed low energy interactions.

2 Who mixes, two, three, or four flavours?

With growing evidence for non vanishing neutrino masses, experimental studies of neutrino oscillations, and their analysis in terms of three (or more) flavours, have become popular and will continue to be of central significance for lepton physics in the future.

The easiest way to describe any individual case of oscillations is to use a scheme where only two neutrino flavours are allowed to mix. Indeed, much work was done on analyses of neutrino oscillations in terms of two flavours but, as was pointed out by us and by others, the results for the squared mass differences may be misleading when applied to the real lepton world which contains three flavours. We summarize the situation regarding the squared mass differences as follows.

In models involving three oscillating flavours one often relies on squared mass differences which are taken from analyses of individual oscillation experiments in the framework of a two-flavour scheme. For instance, if one assumes the solar, the atmospheric and the LSND oscillations to be governed by just a single oscillation "frequency", $\Delta\mathcal{M}^2$, then the characteristic frequencies of the three oscillations, i.e.

$$\Delta\mathcal{M}^2_{\text{solar}} = 10^{-10}\text{eV}^2 \quad \text{or} \quad 10^{-5}\text{eV}^2$$

$$10^{-3}\text{eV}^2 \leq \Delta\mathcal{M}^2_{\text{atmospheric}} \leq 10^{-2}\text{eV}^2$$

$$10^{-1}\text{eV}^2 \leq \Delta\mathcal{M}^2_{\text{LSND}} \leq 10^{1}\text{eV}^2$$

cannot be reconciled with just three neutrino mass eigenstates. Therefore, in order to simultaneously accommodate all three oscillations as observed, under the assumption stated above, we must introduce (at least) a fourth neutrino. Since we know from the width of the Z^0 boson that only three neutrino species have normal weak interactions, this extra, fourth neutrino must be sterile.

As there is no other, direct evidence for the existence of one or more sterile neutrinos, one is lead to conclude that assuming all observed oscillation phenomena to involve but a single $\Delta\mathcal{M}^2$ is erroneous. Suppose, instead, that there are only three neutrinos, with masses such that

$$m_3^2 - m_2^2 \equiv \Delta M^2 \ll m_2^2 - m_1^2 \equiv \Delta m^2 \tag{13}$$

Then, as was shown in [13], it is possible to explain the LSND result as an oscillation involving ΔM^2, the flavour conversion of solar neutrinos as one involving Δm^2 and the atmospheric neutrino anomaly as a mixture of both frequencies. In contrast to these findings, an analysis of the atmospheric data assuming (erroneously) that only one $\Delta\mathcal{M}^2$ is involved would find a value intermediate between those corresponding to the LSND and solar effects, as observed.

In calculating observable effects of CP violation in neutrino oscillations we assume a scenario with three flavours, where the two squared mass differences obey the inequality (13) and lie in the range

$$10^{-4}\ \mathrm{eV}^2 \leq \Delta m^2 \leq 10^{-3}\ \mathrm{eV}^2\,, \quad \Delta M^2 \approx 0.3\ \mathrm{eV}^2\,. \tag{14}$$

If there are three Dirac neutrino types, then the flavour eigenstates are related to the mass eigenstates by a 3×3 unitary matrix

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}e^{i\delta} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} \tag{15}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. If the neutrinos are Majorana particles, there are two extra phases, but these do not affect oscillations [12].

The transition probability in vacuum for a neutrino changing from ν_i ($\bar{\nu}_i$) to ν_j ($\bar{\nu}_j$) is given by the sum and the difference of CP-even and CP-odd pieces, respectively, [6]

$$\begin{aligned} P(\nu_i \to \nu_j) &= P_{CP}(\nu_i \to \nu_j) + P_{\not{CP}}(\nu_i \to \nu_j) && (16)\\ P(\bar{\nu}_i \to \bar{\nu}_j) &= P_{CP}(\nu_i \to \nu_j) - P_{\not{CP}}(\nu_i \to \nu_j), && (17)\end{aligned}$$

where

$$\begin{aligned} P_{CP}(\nu_i \to \nu_j) &= \delta_{ij} - 4\mathrm{Re}J_{12}^{ji}\sin^2\Delta_{12} - 4\mathrm{Re}J_{23}^{ji}\sin^2\Delta_{23} - 4\mathrm{Re}J_{31}^{ji}\sin^2\Delta_{31}, \\ P_{\not{CP}}(\nu_i \to \nu_j) &= -8\sigma_{ij}J\sin\Delta_{12}\sin\Delta_{23}\sin\Delta_{31}, \end{aligned} \tag{18}$$

with J the Jarlskog invariant and

$$\begin{aligned} J_{kh}^{ij} &\equiv U_{ik}U_{kj}^\dagger U_{jh}U_{hi}^\dagger \\ \Delta_{ij} &\equiv \Delta m_{ij}^2 L/4E \\ \sigma_{ij} &\equiv \sum_k \varepsilon_{ijk} \end{aligned} \tag{19}$$

3 Results for CP asymmetries with three flavours

As stated above we henceforth assume a scenario with three flavours of neutrinos characterized by the squared mass differences (14) and the strong mixing found in [13], solution I, where

$$\theta_{12} \approx 35.5^0 , \quad \theta_{23} \approx 27.3^0 , \quad \theta_{13} \approx 13.1^0 . \tag{20}$$

Although a detailed comparison might need further, refined analysis, this range of squared mass differences and the set of mixing angles (20) describes all observed neutrino anomalies in an overall and satisfactory manner. Here we show that this same set of parameters predicts a CP asymmetry which may well be large enough to be detectable with neutrino beams from a muon storage ring as described in sect. 1.

We organize the discussion of our results as follows: We first present our main result for the expected asymmetry. Next, the role of matter effects is illustrated by some examples, followed by a comparison of CP violating asymmetries with time reversal violating asymmetries. We then turn to a comparison with previous results and show why we find asymmetries which are sizeably larger than the ones estimated previously. Finally, some remarks on efficiencies and statistical uncertainties are added.

Let n be the number of muons, $\overline{n}$ the number of antimuons detected in one year's time in a detector placed at 732 km from the collider. Assuming for a moment an efficiency of 100%, we find the asymmetry $(n - \overline{n})/(n + \overline{n})$ shown in Fig. 1, as a function of the neutrino energy E_ν. Part (a) of the figure refers to the lower limit $\Delta m^2 = 10^{-4}$ eV^2, part (b) refers to the upper limit $\Delta m^2 = 10^{-3}$ eV^2 of (14). The solid line corresponds to setting $\delta = 0$ (no CP violation), the dashed line shows the full asymmetry, assuming $\delta = \pi/2$. As a matter of example Fig. 2 shows the absolute numbers n and $\overline{n}$, obtained in one year of running, under the same assumptions as before and for $\Delta m^2 = 10^{-3}$ eV^2.

The role of matter effects: Clearly, the smaller matter effects the cleaner the measurement of the effects of genuine CP violation will be. As the asymmetry due to charged-current matter interaction grows faster than the CP asymmetry, as a function of the baseline, intermediate distances between collider and neutrino detector are preferred over long distances. We illustrate this observation quantitatively by defining the asymmetry

$$a_{CP}(E_\nu, \delta) = \frac{P(\nu_e \to \nu_\mu) - P(\bar{\nu}_e \to \bar{\nu}_\mu)}{P(\nu_e \to \nu_\mu) + P(\bar{\nu}_e \to \bar{\nu}_\mu)} \tag{21}$$

and by calculating the ratio $a_{CP}(E_\nu, 0)/[a_{CP}(E_\nu, \pi/2) - a_{CP}(E_\nu, 0)]$, as a function of E_ν. Fig. 3 shows this quantity for a baseline of 732 km, part (a), and a baseline of 7332 km, part (b), corresponding to the distance from FNAL to the Gran Sasso. In the case of the very long baseline, the CP asymmetry is completely swamped by matter effects.

CP- versus T-asymmetry: We also computed the T-odd asymmetry (12) for the example $\Delta m^2 = 10^{-4}$ eV^2 and the shorter baseline $L = 732$ km and compared it to the CP asymmetry (21) from which the matter effects were subtracted. We found them to agree within reasonable limits, thus corroborating the approximation (11).

Comparison with previous results: The authors of ref. [3] who use the following sample set of parameters

$$\Delta m^2_{12} = 10^{-4}\ \mathrm{eV}^2,\ \Delta m^2_{23} = 10^{-3}\ \mathrm{eV}^2,\ \sin^2\theta_{12} = .5,\ \theta_{23} = 45^0,\ \theta_{13} = 13^0\,, \tag{22}$$

find CP violating effects which are markedly smaller than the ones we showed above. The explanation for this difference is simple: The most noticeable difference between the set (22) and ours is the value of Δm^2_{23}. Assuming the values (22) both the (12)- and the (13)-channels are strongly affected by matter effects, the effective sines $(\sin\theta_{12})_{matter}$ and $(\sin\theta_{13})_{matter}$ as defined in [3] quickly tend to 1 as the parameter μ, eq. (6), increases. Consequently, the simultaneous interplay of all three flavours and, hence, the visibility of CP violation decrease. In contrast to this situation our values of squared mass differences are such that only $(\sin\theta_{12})_{matter}$ is affected while $(\sin\theta_{13})_{matter}$ and, of course, $(\sin\theta_{23})_{matter}$ remain unaffected. It is convenient to define an effective mixing matrix V_{ik} which is obtained from (15) by replacing the sines and cosines by the matter affected sines and cosines, $(\sin\theta_{ik})_{matter}$, etc.

To illustrate the comparison Fig. 4(a) shows the pertinent, effective matrix elements for our set of parameters, eqs. (14) and (20), as a function of μ, while Fig. 4(b) shows the same matrix elements for the set (22). In the latter case, both V_{11} and V_{12} tend to zero with μ increasing to its value in the Earth's crust.

Efficiencies and statistical uncertainties: The discussion of statistical uncertainties is straightforward. In order to exclude the possibility that a measurement of a non vanishing genuine CP violation is due to a statistical fluctuation, the measured value must be larger than $n \cdot \delta(a_{CP})_{\mathrm{stat}}$, where $\delta(a_{CP})_{\mathrm{stat}}$ is the 1σ statistical error on a_{CP} in the absence of CP violation, and n is the number of standard deviations we require in order to be happy with our result. Since in absence of CP violation the expectations of n_{ν_μ}/N_{μ^+} and $n_{\bar{\nu}}/N_{\mu^-}$ are equal, we get

$$\delta(a_{CP})_{\mathrm{stat}} = \left(\frac{1}{4\langle n_{\nu_\mu}\rangle} + \frac{1}{4\langle n_{\bar{\nu}_\mu}\rangle}\right)^{1/2} \tag{23}$$

where $\langle n_{\nu_\mu}\rangle$ ($\langle n_{\bar{\nu}_\mu}\rangle$) is the expected number of ν_μ ($\bar{\nu}_\mu$) interactions seen in the detector.

Regarding the contribution of the background to the statistical error, and according to the current estimates, the main source of background will be due to charm production in the charged current neutrino interactions in the detector [14]

$$\begin{array}{rcl} \mu^- \to \nu_\mu \to & \text{CC interaction} & \to \mu^- \mid_{\text{lost}} \\ c \to & \text{c decay} & \to \mu^+ \mid_{\text{found}} \end{array}$$

Clearly, this background only affects the signal corresponding to the $\bar{\nu}_e \to \bar{\nu}_\mu$ oscillations (because we expect there μ^+ appearance), and therefore, this background "noise" should be subtracted appropriately in the counting of $n_{\bar{\nu}_\mu}$ in Eq.(8). Such a subtraction introduces a further source of statistical error. Using the estimate [14] that

$$n_{\bar{\nu}_\mu} \mid_{\rm back} \simeq 10^{-5} n_{\bar{\nu}_\mu} \mid_{P=1} \tag{24}$$

where $n_{\bar{\nu}_\mu} \mid_{P=1}$ is the number of antimuon neutrino interactions that would be seen if all the initial antielectron neutrinos oscillated into antimuon neutrinos, we find

$$\delta(a_{CP})_{\rm stat} = \left(\frac{1}{4\langle n_{\nu_\mu} \rangle} + \frac{1}{4\langle n_{\bar{\nu}_\mu} \rangle} + \frac{10^{-5}\langle n_{\bar{\nu}_\mu} \mid_{P=1} \rangle}{4\langle n_{\bar{\nu}_\mu} \rangle^2} \right)^{1/2} \tag{25}$$

as the complete expression for the statistical error. It is important to notice that even with only one year of data taking and a modest 30% detecting efficiency, the statistical error will be small enough to rule out the possibility of attributing to a statistical fluctuation a measurement of a non vanishing CP violation.

4 Conclusions

In summary, the two rather different "frequencies" (13), together with the strong mixing of all three flavours that describe the solar neutrino deficit, the atmospheric oscillations, and the LSND anomaly, lead to relatively large CP and T violating asymmetries in neutrino oscillations. With the set of parameters (14) and (20) the full interference of all three flavours is well developed and is only moderately damped by matter effects. We have also tried the other solutions to the neutrino anomalies we had found in [13] but find no more than 20% changes in the asymmetries. Among these the CP asymmetry seems large enough to be measurable with neutrino beams from a 20 GeV muon storage ring and with a detector at some 730 km from the source, corresponding to the distance of the Gran Sasso laboratory from CERN or, likewise, of the Soudan mine from FNAL.

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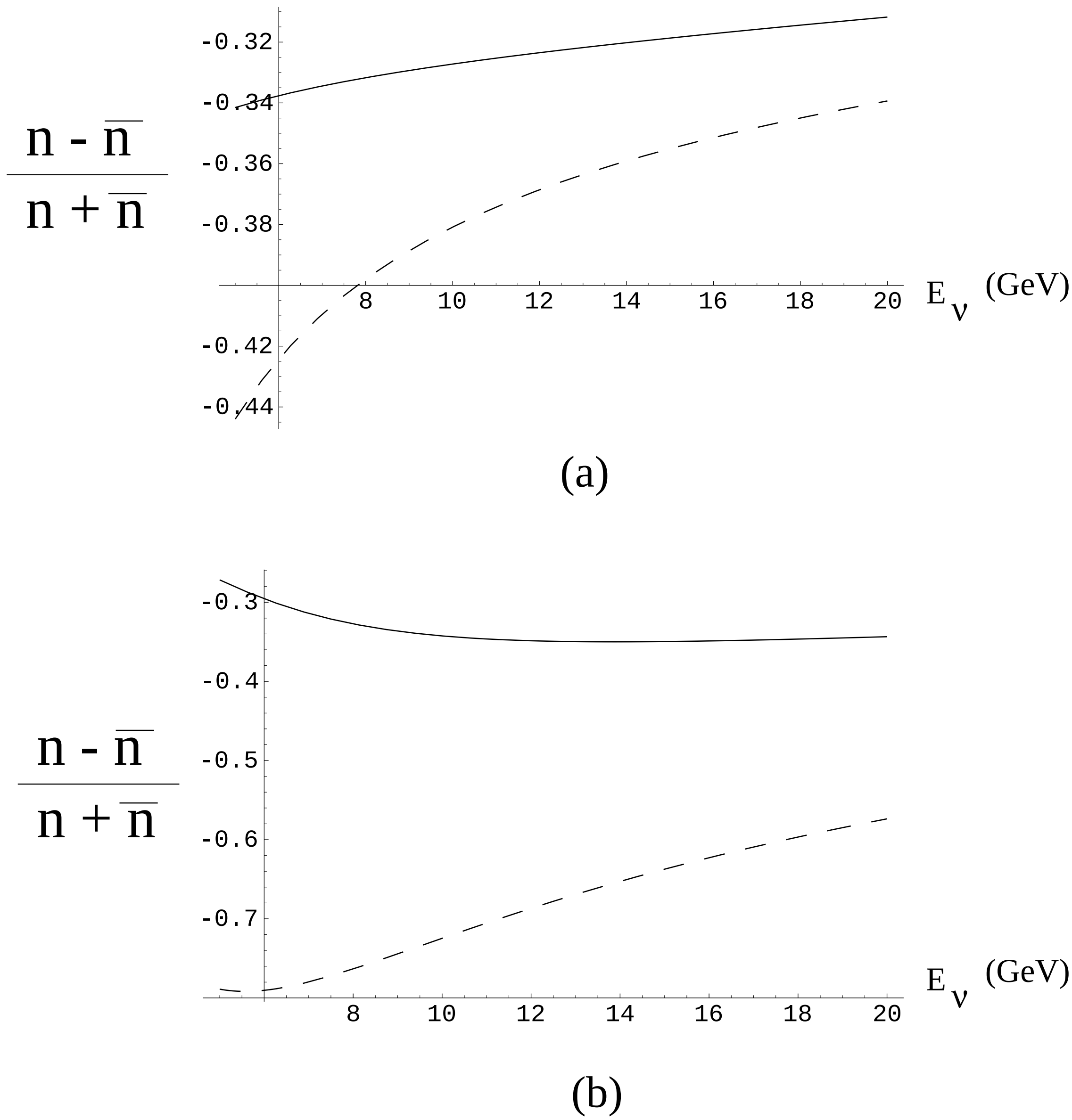


Figure 1: $(n - \overline{n})/(n + \overline{n})$ as a function of the neutrino energy for $\Delta m^2 = 10^{-3}$ eV^2 (a) and $\Delta m^2 = 10^{-4}$ eV^2 (b). The solid line correspond to $\delta = 0$ while the dashed line correspond to $\delta = \pi/2$

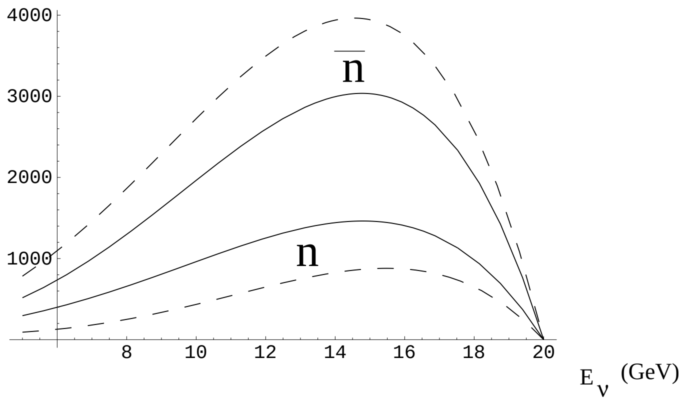


Figure 2: Absolute number of muons and antimuons detected in one year's time in a 732 km baseline for $\Delta m^2 = 10^{-3}$ eV^2 and assuming 100% detecting efficiency. dashed and solid lines as before.

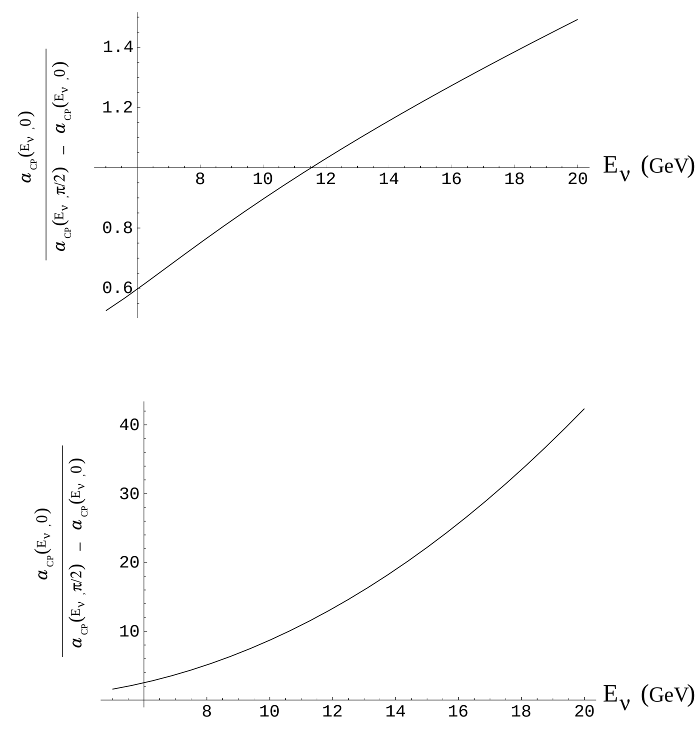
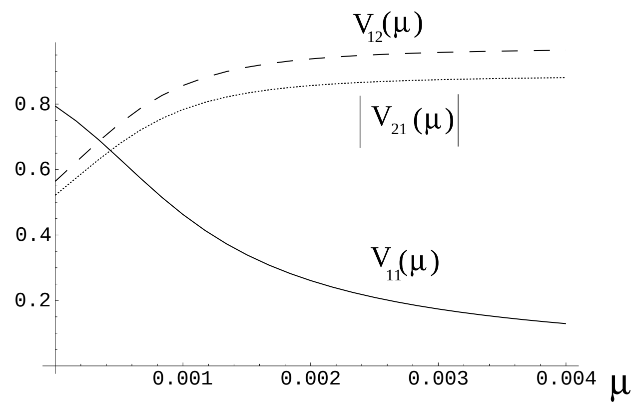
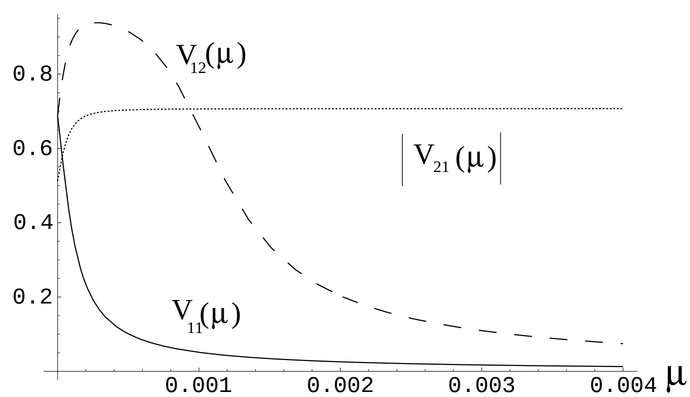


Figure 3: "Noise over signal" ratio as a function of the neutrino energy for $\Delta m^2 = 10^{-3}$ eV^2 and a detector placed at 732 km (a) and 7332 km (b) from the collider.



(a)



(b)

Figure 4: Effective matrix elements as a function of the effective mass for the set of parameters used through this work (a) and those of ref. [3]