CERN-TH/95-33
FTUV/95-10,IFIC/95-10
Lund-MPh-95/03
hep-ph/9503228

NON-DECOUPLING OF HEAVY NEUTRINOS AND LEPTON FLAVOUR VIOLATION

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Abstract

We consider a class of models predicting new heavy neutral fermionic states, whose mixing with the light neutrinos can be naturally significant and produce observable effects below the threshold for their production. We update the indirect limits on the flavour non-diagonal mixing parameters that can be derived from unitarity, and show that significant rates are in general expected for one-loop-induced rare processes due to the exchange of virtual heavy neutrinos, involving the violation of the muon and electron lepton numbers. In particular, the amplitudes for μ–e conversion in nuclei and for $\mu \to e e^+ e^-$ show a non-decoupling quadratic dependence on the heavy neutrino mass M, while $\mu \to e\gamma$ is almost independent of the heavy scale above the electroweak scale. These three processes are then used to set stringent constraints on the flavour-violating mixing angles. In all the cases considered, we point out explicitly that the non-decoupling behaviour is strictly related to the spontaneous breaking of the SU(2) symmetry.

CERN-TH/95-33
3 March 1995

1 Introduction

New heavy neutral fermions could affect low-energy measurements, made well below their production thresholds. In general, these new states mix with the three light neutrinos, thus modifying their neutral current (NC) and charged current (CC) couplings. Such effects depend on the light–heavy neutrino mixing angles, which can then be constrained by the set of NC and CC precision data at "low" energy [1]–[3]. If the explanation for the smallness of the known neutrino masses is given by the see-saw mechanism [4], these mixing angles are proportional to the square roots of the ratios of the light and the heavy mass eigenvalues, so that their effects on the light neutrino couplings to the standard gauge bosons vanish when the heavy neutrino masses go to infinity. In this limit, the heavy neutrinos completely decouple from the low-energy physics. In fact, not only the tree-level light–heavy mixing effects, but also the loop diagrams involving the heavy neutrinos, contributing to physical observables, turn out to be suppressed by inverse powers of the heavy mass scale [5, 6].

However, the see-saw mechanism is not the only possible scenario to explain the lightness of the known neutrinos. In particular, a viable alternative involving heavy neutral states has been considered [7]–[10], where vanishingly small masses for the known neutrinos are predicted by a symmetry argument, and at the same time large light–heavy mixing angles are allowed. In this case, due to the mixing effects in the light neutrino interactions, the new neutral fermions affect the low-energy physics even if their masses are very large.

In the present paper, we discuss the conditions under which the mixing angles between the standard and new neutral fermions heavier than M_Z can be large, keeping at the same time the masses of the known neutrinos below the laboratory limits. We will then study the limits that can be set on the mixing parameters, concentrating on the ones which would induce Lepton Flavour Violation (LFV). In particular, we will consider a class of models that allow total lepton number conservation, and we will show that loop contributions involving virtual heavy neutrinos to the decays $\mu \to e\gamma$, $\mu \to ee^+e^-$, and to the process of μ–e conversion in nuclei, can be large and then provide significant constraints on the mixing parameters. In particular for a heavy mass scale around the electroweak scale the induced $\mu \to ee^+e^-$ and μ–e conversion rates *increase* with the masses of the heavy neutrinos, showing a non-decoupling behaviour, and the present limits on the processes put significant constraints in the space of the light–heavy mixing parameters and new neutrino masses. The planned experiments looking for muon–electron conversion are specially suitable for finding signals arising from this kind of physics.

The paper is organized as follows. In section 2, we discuss the class of models for neutrino mass that predict significant light-heavy mixing. In section 3, we review the direct constraints on the flavour-diagonal, and up-date the resulting indirect limits on the flavour-changing, mixing parameters. In section 4, we compute the non-decoupling contributions to the decays $\mu \to e\gamma$, $\mu \to ee^+e^-$, and to the process of μ–e conversion in nuclei. We discuss their importance and their interplay in constraining the models, and we show how the non-decoupling behaviour is strictly related to the breaking of the electroweak symmetry. Finally, section 5 summarizes our conclusions.

2 Models for light–heavy neutrino mixing

Let us first consider a generic extension of the Standard Model (SM), including right-handed neutrinos (singlets under the SM group) as the only new neutral fermions. We can arrange all the independent neutrino degrees of freedom in two vectors of *left-handed* fields, ν and $N \equiv C\bar{\nu}_R^T$, where C is the charge-conjugation matrix and a family index is understood. In the basis (ν, N) the mass matrix can be written in a block form as

$$\mathbf{M} = \begin{pmatrix} m_{\nu\nu} & m_{\nu N} \\ m_{\nu N}^T & M_{NN} \end{pmatrix}. \tag{1}$$

The entry $m_{\nu\nu}$ can be due to a possible lepton-number-violating vacuum expectation value (VEV) of a triplet Higgs field, as in left–right models. Although in principle the singlet neutrinos can be light, we are interested in the case when all the new (i.e. non-SM) states are heavier than $\sim M_Z$. This is also the theoretical expectation in most models, which generally predict large masses for the heavy states. In the one family case, as far as the entry $m_{\nu\nu}$ can be neglected, we have the usual see-saw mechanism [4] for the generation of a small neutrino mass. In this case, the light–heavy mixing angle θ depends on the ratio of the light and heavy mass scales, as $\sin^2\theta \sim m/M$ ($m \sim m_{\nu N}^2/M_{NN}$, $M \sim M_{NN}$). For $M \gtrsim M_Z$, taking the laboratory limits on the ν_e, ν_μ and ν_τ masses [11], we get the bounds $\sin^2\theta_{\nu_e} \lesssim 10^{-10}$, $\sin^2\theta_{\nu_\mu} \lesssim 10^{-6}$, $\sin^2\theta_{\nu_\tau} \lesssim 10^{-3}$, which are too small to have any phenomenological interest. To have significant light–heavy mixing in the one family case, we are left with only one solution, requiring that the entry $m_{\nu\nu}$ be non vanishing, and satisfying the relation $m_{\nu\nu} = m_{\nu N}^2/M_{NN}$. This would ensure that the mass matrix of Eq. (1) be singular, so that the mixing angle $\sin\theta \sim m_{\nu N}/M_{NN}$ would no longer be related to the ratio of the light to the heavy eigenvalues, and would be allowed to be as large as $O(1)$. However, it seems hard to find a reasonable motivation for the underlying fine tuning of the parameters in the mass matrix. The model with three families of left- and right- handed neutrinos ($\nu \equiv (\nu_e, \nu_\mu, \nu_\tau)$, $N \equiv (N_e, N_\mu, N_\tau)$) allows a solution even in the case $m_{\nu\nu} = 0$ [12]. In this case, the fine-tuning conditions, which allow for finite light–heavy mixing and vanishing mass of the known neutrinos, are the following: 1) the Dirac mass matrix $m_{\nu N}$ is of rank 1, that is all the three lines (rows) are proportional; 2) the trace $Tr(m_{\nu N}^T M_{NN}^{-1} m_{\nu N}) = 0$ (assuming that M_{NN} is not singular).

These considerations can be generalized. The neutrino mass matrix should be singular with a three times degenerate zero eigenvalue, in the limit in which the masses of the three known neutrinos are neglected. In the see-saw mechanism, this is ensured by letting the heavy scale go to infinity, which implies that the light–heavy mixing angles go to zero. However, if for some reason the mass matrix is (three times) singular even for *finite* values of the heavy scale, the light–heavy mixing angle can be substantial. Any model realizing this idea in a natural way, e.g. due to a symmetry argument, is a viable alternative to the see-saw mechanism to explain the lightness of the known neutrinos.

For instance, let us assume that pairs N, N' of (left-handed) new neutrinos exist, with the lepton-number assignments $L(N) = -L(N') = L(\nu) = 1$, and that L is conserved. We understand a family index, i.e. $\nu \equiv (\nu_e, \nu_\mu, \nu_\tau)$, $N \equiv (N_1, ..., N_{n-3})$, $N' \equiv (N'_1, ..., N'_{n-3})$, where $n-3$ is the number of new pairs of neutral fermions. Then, in the basis (ν, N, N'), the mass matrix is

$$\mathbf{M} = \begin{pmatrix} 0 & 0 & M_{\nu N'} \\ 0 & 0 & M_{NN'} \\ M^T_{\nu N'} & M^T_{NN'} & 0 \end{pmatrix}, \tag{2}$$

which is singular and ensures that three eigenstates form massless Weyl neutrinos. In fact, as in the SM, the light states remain with no chirality partners and hence massless.[1] On the contrary, the heavy states form Dirac neutrinos, whose left-handed components are mainly the N and whose right-handed parts are given by $C\overline{(N')}^T$. Mass matrices of the form (2) have been considered in Ref. [7]. They can arise in generalized E_6 models [8]–[10], as well as in models predicting other kinds of vector multiplets (singlets, triplets, ...) or new mirror multiplets of leptons [13] with neutral components N, N'.

The mass matrix (2) can be put in a "Dirac diagonal" form by an "orthogonal" transformation,

$$\mathbf{U}^T\mathbf{M}\mathbf{U} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & M \\ 0 & M & 0 \end{pmatrix}, \tag{3}$$

where the block M is diagonal. The *unitary* matrix $\mathbf{U}$ in Eq. (3) can be chosen in the form

$$\mathbf{U} = \begin{pmatrix} A & G & 0 \\ F & H & 0 \\ 0 & 0 & K \end{pmatrix}. \tag{4}$$

Several relations amongst the blocks in (4) can also be deduced from the unitarity condition $\mathbf{U}^\dagger\mathbf{U} = \mathbf{U}\mathbf{U}^\dagger = \mathbf{1}$. Equation (4) describes the mixing between the ν and N states, the mixing parameters being the elements of the matrix

$$GH^{-1} = -(FA^{-1})^\dagger = [M_{\nu N'} M^{-1}_{N'N'}]^*. \tag{5}$$

Clearly, if for the relevant matrix elements $M_{\nu N'} \sim M_{NN'}$, the mixings between ν and N can be arbitrarily large. We will consider in the following two particular cases:

i) The new neutrinos N are *ordinary*, i.e. they belong to a weak (left-handed) doublet. Then $M_{\nu N'}$ and $M_{NN'}$ could be generated by vacuum expectation values of Higgs fields transforming in the same way under SU(2) so that the ν–N mixing could be naturally close to maximal. In particular, we can consider the SM with $n > 3$ families, and with $n-3$ right-handed neutrinos. Then N would describe the neutrinos of the new $n-3$ families, appearing with right-handed partners $\overline{N'}$. In this picture, the three known neutrinos remain strictly massless since they have no right-handed component. The same scenario arises in some lepton-number-conserving E_6 models [10], predicting three new ordinary N

[1]Small L-violating Majorana mass terms for the states ν and N could also be allowed [9], and could be relevant for explaining the solar neutrino deficit via neutrino oscillations.

states (one per standard family) and six isosinglets, three of which can play the role of our N' in such a way that the relevant part of the mass matrix assumes the form of Eq. (2).

ii) Another case that has been considered in the literature [8] corresponds to both N and N' singlets. In this case, a significant light–heavy mixing can be expected only if the SU(2)-invariant mass term $M_{NN'}$ is generated not far from the electroweak scale.

In both the above cases, the N' states are assumed to be isosinglets. This implies that the block $M_{\nu N'}$ violates SU(2), and can be generated by the VEV of a doublet Higgs field at the electroweak scale.

3 Constraints from flavour-diagonal processes

The mixing with heavy states would affect the observables at energies below the threshold for their production. Following e.g. Ref. [2], we introduce a vector N to describe all the new independent neutral fermionic degrees of freedom which mix with the three known left-handed neutrinos $\nu \equiv (\nu_e, \nu_\mu, \nu_\tau)$. We will use only left-handed fields, without distinguishing between neutrinos and antineutrinos. Then the light n and heavy $\mathcal{N}$ mass eigenstates can be obtained by a unitary transformation

$$\begin{pmatrix} \nu \\ N \end{pmatrix} = \begin{pmatrix} A & G \\ F & H \end{pmatrix} \begin{pmatrix} n \\ \mathcal{N} \end{pmatrix}. \tag{6}$$

This general formalism covers in particular the cases discussed in the previous section. In the case of Eqs. (2)–(4), this means that we are now focusing on the submatrix involving the light–heavy mixing, given by Eq. (6), which is unitary. The light–heavy mixing is described by the matrix G, and is reflected also in the non-unitarity of the block A ($AA^\dagger + GG^\dagger = 1$). Notice that A also describes the leptonic Cabibbo–Kobayashi–Maskawa mixing, in the basis where the mass matrix for the (light) charged leptons is diagonal.

In processes occurring at energies below the threshold for the production of the heavy states, the standard gauge eigenstate ν_a ($a = e, \mu, \tau$) is effectively replaced by its (normalized) projection $|\nu_a^{light}\rangle$ onto the subspace of the light neutrinos $|n_i\rangle$ ($i = 1, 2, 3$),

$$|\nu_a^{light}\rangle \equiv \frac{1}{c_{\nu_a}} \sum_{i=1}^{3} A^\dagger_{ia} |n_i\rangle, \tag{7}$$

where $c^2_{\nu_a} \equiv \cos^2\theta_{\nu_a} \equiv (AA^\dagger)_{aa}$. The state $|\nu_a^{light}\rangle$ has non-trivial projections on the subspace of the standard neutrinos $|\nu_b\rangle$ as well as on the subspaces of the new neutrinos $|N_B\rangle$. In fact we have

$$\begin{aligned} \sum_b |\langle \nu_b | \nu_a^{light}\rangle|^2 &= \frac{(AA^\dagger)^2_{aa}}{c^2_{\nu_a}} = c^2_{\nu_a}, \\ \sum_B |\langle N_B | \nu_a^{light}\rangle|^2 &= \frac{(AF^\dagger F A^\dagger)_{aa}}{c^2_{\nu_a}} = s^2_{\nu_a}, \end{aligned} \tag{8}$$

with $s^2_{\nu_a} \equiv 1 - c^2_{\nu_a} = \sin^2\theta_{\nu_a}$. The parameter θ_{ν_a} measures the total amount of mixing of the known state of flavour $a = e, \mu, \tau$ with the new states. These three mixing angles are sufficient to describe the *tree-level* effects of the light–heavy mixing in the CC and NC processes at energies below the threshold for the production of the heavy states [2].

The entries of the matrix $GG^\dagger$, describing the mixing with the new neutrinos, are limited by the constraints on CC universality and, if the heavy states do not belong to SU(2) doublets, by the measurement of the Z boson invisible width at LEP [1]–[3]. For the diagonal elements $(GG^\dagger)_{aa} \equiv s^2_{\nu_a}$, the 90% C.L. bounds are [2, 3]

$$s^2_{\nu_e} < 0.007(0.005), \qquad s^2_{\nu_\mu} < 0.002, \qquad s^2_{\nu_\tau} < 0.03(0.01), \tag{9}$$

where the more conservative limits are due to the CC constraints and apply to any kind of heavy neutrinos, while the limits in parentheses correspond to the mixing with SU(2) singlets and take into account also the LEP [14] and SLC [15] data. For a complete discussion, we refer to [2, 3]. These limits can be somewhat relaxed if the cancellations with the effects due to different fermion-mixing parameters which might be present in some extended models are taken into account [1]–[3]. Nevertheless, we will not try to allow for the corresponding fine-tunings and we will consider the stringent bounds of Eq. (9) to be reliable. For the off-diagonal elements of the matrix $GG^\dagger$, indirect limits can be obtained from Eq. (9) and the relation [9]

$$|(GG^\dagger)_{ab}| < |s_{\nu_a} s_{\nu_b}|, \tag{10}$$

which can be deduced from the unitarity of the full mixing matrix by applying the Schwartz inequality. Using the bounds of Eq. (9), we find that all the elements of the matrix $GG^\dagger$ can be constrained as in Table 1.

Loop diagrams involving virtual heavy neutrinos can contribute to flavour-diagonal observables [5, 16], such as the leptonic widths $Z \to l\bar{l}$ or the polarization asymmetries measured at the Z peak [14, 15]. However, taking into account the updated limits of Table 1, the predictions for these observables turn out to be below the attainable experimental limits.

Heavy neutrinos in general also affect the electroweak radiative corrections which are tested e.g. in the LEP experiments. For instance, if the new neutral states N belong to SU(2) doublets $\begin{pmatrix} N \\ E \end{pmatrix}$, they will then contribute to the ρ parameter [17, 18] through loop diagrams, resulting in a (top-like) non-decoupling dependence,

$$\delta\rho \simeq \sum \frac{G_F}{8\sqrt{2}\pi^2} \Delta M^2, \tag{11}$$

where $\Delta M^2 \equiv M_E^2 + M_N^2 - \frac{4M_E^2 M_N^2}{M_E^2 - M_N^2} \ln \frac{M_E}{M_N} \geq (M_N - M_E)^2$; the sum runs over all the new doublets and we have neglected the effects of the light–heavy neutrino mixing. The value of the ρ parameter is constrained by the electroweak data. For $m_t = 174 \pm 16$ GeV, as suggested by the CDF measurement [19], the result is $\delta\rho = 0.0004 \pm 0.0022 \pm 0.002$ [18] (the second error is from the uncertainty in the Higgs mass m_H) for the (m_t-independent) corrections due to possible new physics. From Eq. (11) and for $m_H < 1$ TeV, we then find that any new lepton doublet should be degenerate within $|M_N - M_E| \lesssim 220$ GeV at 90% C.L.. This constraint is significant if the new neutrinos are close to the perturbative limit $M_N \lesssim 1$ TeV, that holds for N belonging to an SU(2) doublet, but in any case within this region it does not require an important fine-tuning.

A second phenomenological constraint on heavy ordinary neutrinos, belonging to weak doublets, comes from the limit on the S Peskin–Takeuchi [21] parameter. The contribution from a multiplet of heavy degenerate fermions is $\Delta S = C\sum_f (t_{3L}(f) - t_{3R}(f))^2/3\pi$, where $t_{3L,R}(f)$ is the weak isospin of the left- (right-) handed component of fermion f, and C is the number of colours [20]. Then the contribution from the set of all the particles in each new ordinary family of heavy fermions is $\Delta S \simeq 2/3\pi > 0$. From the analysis of the electroweak data, one gets the 95% C.L. bound $S < 0.2$, which can be relaxed to $S < 0.4$ if only positive contributions to S [20] are allowed. As a consequence, only a single, or very marginally two, new families are allowed to exist by the present data. This is the maximum number of pairs N, N', for N belonging to a new family and N' isosinglet, and in this case $M_{\nu N'}$ and $M_{NN'}$ can be 3×1 (much less likely 3×2) matrices. However, in the following we will retain the general notation, allowing for an arbitrary number of N, N' pairs. In fact, this number is not important for our discussion (provided it is non-zero), and in addition one can also consider new pairs N, N' from (respectively) an isodoublet and an isosinglet which do not belong to new ordinary families. An example of the latter situation is given by E_6 models themselves [10] that contain new leptonic doublets and isosinglets in the **27** representation, so that in the three-family models there are also three such pairs. In this case, the contribution to the S parameter is zero, since the non-isosinglet new states appear in vector doublets ($t_{3L}(f) = t_{3R}(f)$).

In the case when both N and N' are isosinglets, as in the model [8], the S parameter is not affected, while the contribution to $\delta\rho$ is suppressed by the fourth power in the mixing angles and is negligible [6, 16].

4 Constraints from Flavour-Changing (FC) processes

The indirect limits presented in Table 1 are more stringent than any direct bound on the tree-level effects of the off-diagonal mixings, such as the constraints from the search for neutrino oscillations [9]. However, loop diagrams involving heavy neutrinos give rise to unobserved rare processes such as $\mu \to e\gamma$, $\mu \to ee^+e^-$, $\tau \to l_a l_b^+ l_c^-$ $(a, b, c = e, \mu)$, $Z \to l_a^- l_b^+$ $(a, b = e, \mu, \tau)$, etc. [5, 22]. Taking into account the stringent constraints in Table 1, the rates for all the processes involving the violation of the τ lepton number turn out to be below the experimental sensitivity even for extreme values of the heavy neutrino masses [22]. In other words, it is not possible to improve the limits in Table 1 on the parameters $(GG^\dagger)_{a\tau}$ $(a = e, \mu)$. However the extraordinary sensitivity of the experiments looking for FC processes involving the first two families implies that the constraints from $\mu \to e\gamma$, from $\mu \to ee^+e^-$, or from μ–e conversion in nuclei, are significant and turn out to be stronger than those from Table 1. The diagrams contributing to these processes are proportional to factors involving the light–heavy mixing angles, and in the cases of the processes $\mu \to ee^+e^-$ and μ–e conversion in nuclei, they depend up to quadratically in the heavy neutrino mass scale M. In see-saw models, where the mixing angles are suppressed by inverse powers of the heavy masses, the resulting dependence is $\sim M^{-2}$ [5, 6], in agreement with the decoupling theorem [24]. However, the models discussed in section 2 predict a finite light–heavy mixing independent of the light-to-heavy mass ratio, so that, assuming that the N' states are isosinglets, a genuine $\sim M^2$ dependence is obtained. This non-decoupling behaviour is comparable to the top mass dependence of the ρ parameter [17] and of the $Z \to b\bar{b}$ vertex [25]. Since the effects of any SU(2)-invariant mass term should decouple when the mass term goes to infinity [24], in all these cases the relevant combinations of the mass and mixing parameters entering the graphs are connected to the electroweak breaking scale and cannot exceed ~ 1 TeV. This consideration is obvious for the top-dependent non-decoupling effects, and will be explicitly verified in the following in the case of the heavy-neutrino contributions.

4.1 $\mu \to e\gamma$

Let us first consider the decay $\mu \to e\gamma$, induced by one-loop graphs involving virtual heavy neutrinos [26, 9]. The corresponding branching ratio is given by

$$B(\mu \to e\gamma) = \frac{3\alpha}{8\pi} \left| \sum_i G_{ei} G^\dagger_{i\mu} \phi\left(\frac{M_i^2}{M_W^2}\right) \right|^2, \tag{12}$$

where M_i is the mass of the heavy eigenstate $\mathcal{N}_i$, and the function

$$\phi(x) = \frac{x(1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x)}{2(1-x)^4} \tag{13}$$

varies slowly from 0 to 1 as x ranges from 0 to ∞. Taking into account the 90% C.L. limit $B(\mu \to e\gamma) < 4.9 \times 10^{-11}$ [27], and assuming $M_i \gtrsim M_W$, one gets the estimate $|(GG^\dagger)_{e\mu}| \lesssim 0.95 \times 10^{-3}$ [9]. This bound holds under the assumption that no important fine-tuning works in the sum $\sum_i G_{ei} G^\dagger_{i\mu} \phi\left(\frac{M_i^2}{M_W^2}\right)$, and is independent of the weak isospin of the new states. We will be interested in the case of very heavy neutrinos, $M_i \gg M_W$. In this case $\phi \simeq 1$ and we get the stringent bound

$$|(GG^\dagger)_{e\mu}| \lesssim 0.24 \times 10^{-3}. \tag{14}$$

In the models discussed in Section 2, allowing for large light–heavy mixings not suppressed by see-saw relations, the loop contribution of Eq. (12) does not vanish for large heavy-neutrino masses; however, it does not have a hard non-decoupling dependence on the heavy mass scale, as the $Ze\mu$ vertex and the box diagrams that we will discuss in the next paragraphs.

4.2 $Ze\mu$ vertex

Let us consider now the FC $Z\bar{e}\mu$ current, parameterized in the form

$$J^\mu_{Z\bar{e}\mu} = g\bar{e}\gamma^\mu(k_L P_L + k_R P_R)\mu = \frac{g}{2}\bar{e}\gamma^\mu(k_V - k_A\gamma^5)\mu, \tag{15}$$

where $g = (4\sqrt{2}G_F M_Z^2)^{1/2}$ is the weak coupling constant to the Z boson, and $P_{R,L} = (1 \pm \gamma_5)/2$. The leading contribution from heavy neutrinos $\mathcal{N}_i$ of mass $M_i \gg M_W$ arises from the (convergent) loop diagram in Fig. 1 involving the exchange of *longitudinal* W bosons (more precisely of the would-be Goldstone bosons in the Feynman gauge). Notice that, as far as we are interested only on the quadratic term in the mass of the heavy states, it is consistent to ignore all the other loop contributions to the vertex, since their leading quadratic terms sum to zero when $M_i \gg M_W$. We also remark that the corrections to our approximation, which would become the main contribution for $M_i \lesssim 100$ GeV, would give a phenomenologically small result due to the constraint coming from $\mu \to e\gamma$, Eq. (12), which would be dominant for such relatively light new states. A more formal justification for considering only the graphs in Fig. 1 can be given in the effective lagrangian approach [28].

Neglecting the external momenta, and for $M_i \gg M_W$, the contribution in Fig. 1 reads

$$k_L = k_V = k_A = -\frac{g^2}{128\pi^2 M_Z^2}\mathcal{F}_{e\mu}, \tag{16}$$

where the dependence from the new physics parameters is given by the factor

$$\mathcal{F}_{e\mu} \equiv \sum_{i,j=heavy} S_{ij} M_i M_j f(M_i, M_j), \qquad f(M_i, M_j) = \frac{M_i M_j \ln(M_i^2/M_j^2)}{M_i^2 - M_j^2}. \tag{17}$$

The dependence on the light–heavy mixing angles is contained in the term

$$S_{ij} \equiv G_{\mu i}^* G_{ej}[(G^\dagger G)_{ji} + 2t_3^N (H^\dagger H)_{ji}] = G_{\mu i}^* G_{ej}[(G^\dagger G)_{ji}(1 - 2t_3^N) + 2t_3^N \delta_{ji}], \tag{18}$$

where t_3^N is the weak isospin of the (left-handed) N field. The dependence is quadratic in the light–heavy mixing matrix G, unless the new neutrinos N are weak isosinglets, in which case it is quartic.

The best limit on the FC current $J^\mu_{Z\bar{e}\mu}$ arises from the search for μ–e conversion in nuclei [29, 30]. For general FC couplings k_V and k_A and for nuclei with atomic number $A \lesssim 100$, the induced branching ratio with the total nuclear muon capture rate is [31]

$$R \simeq \frac{G_F^2 \alpha^3}{\pi^2} m_\mu^3 p_e E_e \frac{Z_{eff}^4}{Z} |F(q)|^2 \frac{1}{\Gamma_{capture}} (k_V^2 + k_A^2) Q_W^2, \tag{19}$$

where p_e (E_e) is the electron momentum (energy), $E_e \simeq p_e \simeq m_\mu$ for this process, and $F(q)$ is the nuclear form factor, as measured for example from electron scattering [32]. Here $Q_W = (2Z + N)v_u + (Z + 2N)v_d$ is the coherent nuclear charge associated with the vector current of the nucleon, as a function of the quark couplings to the Z boson and of the nucleon charge and atomic ($A = Z + N$) numbers, and Z_{eff} has been determined in the literature [33]. For $\Gamma_{capture}$ in $^{48}_{22}$Ti we will use the experimental determinations $\Gamma_{capture} \simeq (2.590 \pm 0.012) \times 10^6 \mathrm{s}^{-1}$ [29], $F(q^2 \simeq -m_\mu^2) \simeq 0.54$ and $Z_{eff} \simeq 17.6$. The resulting limit for the FC couplings is then [31]

$$(k_V^2 + k_A^2) < 5.2 \times 10^{-13} \left(\frac{B}{4 \times 10^{-12}}\right), \tag{20}$$

where B is the value of the experimental bound to R, $B = 4 \times 10^{-12}$ at present [29].

Comparing with the prediction of our model, Eqs. (16)–(18), we find that the non observation of the process of μ–e conversion in nuclei results in the bound

$$\frac{|\mathcal{F}_{e\mu}|}{(100\mathrm{GeV})^2} \lesssim 0.97 \times 10^{-3} \left(\frac{B}{4 \times 10^{-12}}\right)^{1/2}. \tag{21}$$

In order to find out the impact of this constraint, we have to specify the value of the weak isospin of the new states involved in the mixing.

Let us consider first the case i) of Section 2, when the new states N are *ordinary*, that is $t_3^N = 1/2$. In this case, a substantial light–heavy mixing can be expected, since

$M_{\nu N'}$ and $M_{NN'}$ can be generated both by the VEVs of SU(2)-doublet Higgs fields (we are assuming that N' are singlets). Then $S_{ij} = G^*_{\mu i} G_{ei} \delta_{ij}$, and only the diagonal terms contribute in Eq. (18). It is easy to show that $f(M, M) = 1$, then Eq. (17) simplifies to

$$\mathcal{F}_{e\mu} = (GM^2G^\dagger)_{e\mu} = (M_{\nu N'} M^\dagger_{\nu N'})_{e\mu}, \tag{22}$$

where M is the diagonal Dirac mass matrix for the heavy states appearing in Eq. (3), and we have used Eqs. (2)–(4). Since $M_{\nu N'}$ and $M_{NN'}$ arise from the breaking of the SU(2) symmetry, their entries are expected to be generated at the electroweak scale. In particular, the heavy masses should be $M_i \lesssim 1$ TeV, assuming perturbation theory not to be spoiled. We see explicitly in this case that the non-decoupling behaviour of the $Ze\mu$ vertex is due to the breaking of the SU(2) symmetry. On the other hand, since $\mathcal{F}_{e\mu} = (M_{\nu N'} M^\dagger_{\nu N'})_{e\mu}$ is naturally expected to be $\sim$ (100 GeV)^2, we see that Eq. (21) indeed represents a strong constraint on the model, like the bounds on the mixing matrix $GG^\dagger$ discussed in the previous section and the limit from $\mu \to e\gamma$ of the previous paragraph.

To compare these different bounds, let us assume for simplicity that the mass differences amongst the heavy states are smaller than their common scale M, namely $|M_i^2 - M_j^2|/(M_i^2 + M_j^2) \ll 1$. The allowed regions in the LFV mixing parameter $S \equiv (GG^\dagger)^{1/2}_{e\mu}$ and the heavy mass scale M is then given in Fig. 2. For 200 GeV $\lesssim M$, the constraint on S from μ–e conversion in nuclei, given by Eq. (21) and represented by the full line in Fig. 2[2], is more stringent than the bound from $\mu \to e\gamma$ (dashed line), resulting from Eqs. (12), (13) and Ref. [27]. If the new states are lighter than $\sim$ 200 GeV, the constraint from $\mu \to e\gamma$ is the most stringent one. The indirect limit from Table 1, $S = (GG^\dagger)^{1/2}_{e\mu} < \sqrt{0.004} = 0.063$, is much worse and would be represented by a horizontal line above the figure.

Let us now consider the case ii) of Section 2, in which the new states N mixing with the light neutrinos are singlets under SU(2). Then from Eq. (18) we see that the mixing factor depends of the fourth power of the light–heavy mixings. One could expect that in this case no significant constraint can be obtained from Eq. (21); however the mass eigenvalues M_i are no longer limited by 1 TeV. In fact, since the states N' are also assumed to be isosinglets, the entries $M_{NN'}$ in Eq. (2) are not related to the electroweak scale and can be expected to be generated at a higher scale. To have an idea of the impact of the constraint of Eq. (21) in this case, let us assume again that the mass differences amongst the heavy states are smaller than their common scale M. In this case, $f(M_i, M_j) \simeq 1$, and using the identity $MG^\dagger = K^T M^T_{\nu N'}$ (which can be deduced from Eqs. (3) and (4)), we find that $\mathcal{F}_{e\mu} \simeq (GMG^\dagger)^2_{e\mu} = (G_{ej} G^\dagger_{i\mu})(K^\dagger M^\dagger_{\nu N'} M_{\nu N'} K)_{ij}$. Since K is unitary, we can expect that $|(K^\dagger M^\dagger_{\nu N'} M_{\nu N'} K)_{ij}| \sim \bar{M}^2_{\nu N'}$, where $\bar{M}_{\nu N'}$ is an average scale for the entries of the SU(2)-breaking mass matrix $M_{\nu N'}$, which is generated at the electroweak scale. Again, we see explicitly that the non-decoupling behaviour is due to the spontaneous breaking of the SU(2) symmetry.

A more drastic approximation, $\mathcal{F}_{e\mu} \sim (GG^\dagger)_{e\mu} \bar{M}^2_{\nu N'}$, gives an expression that is similar to Eq. (22), corresponding to the mixing with doublet neutrinos. In fact, $\bar{M}_{\nu N'} \lesssim 1$ TeV

[2]For $M \to 100$ GeV, towards the left margin of the figure, the non-decoupling contribution to the $Ze\mu$ vertex is not more important than the others we have neglected [28], but in this regime the bound from $\mu \to e\gamma$ dominates [5, 22].

since it breaks SU(2), so that the considerations given in the previous case of doublet new neutrinos can be repeated here. The constraint from μ–e conversion in nuclei can be represented again by the full line in Fig. 2, after the substitution $M \to \bar{M}_{\nu N'}$. In particular, the constraint of Eq. (21) is more stringent than the constraint from $\mu \to e\gamma$ if $\bar{M}_{\nu N'} \gtrsim 200$ GeV.

If $\bar{M}_{\nu N'}$ and $\bar{M}_{N'N'} \simeq M$ are the typical orders of magnitude of the entries of the matrices $M_{\nu N'}$ and $M_{N'N'}$, from Eq. (5) the typical value of the mixing angles is

$$S \equiv (GG^\dagger)^{1/2}_{e\mu} \sim \bar{M}_{\nu N'}/M, \tag{23}$$

which is of course similar to the see-saw formula [4, 6], though no longer related to the ratio of the physical mass eigenstates. Then the range 200 GeV $\lesssim \bar{M}_{\nu N'} \lesssim 1$ TeV and the bound of Eq. (21) correspond to a heavy scale $M \gtrsim (10-300)$ TeV. This last constraint is not problematic, since in the present case we are considering singlet new states which can be originated by SU(2)-invariant VEVs. In fact, the model *predicts* observable LFV effects if the latter inequality is (almost) an equality, corresponding to the existence of an intermediate scale $\lesssim 300$ TeV for the heavy states. However, for the mixing with singlet neutrinos this assumption on the heavy scale would be somewhat arbitrary (unless a justification is given by fully specifying the model), so that in general significant LFV effects are not necessarily *predicted* in this case, contrary to the case of the mixing with new *ordinary* states that we have considered above.

Moreover, Eq. (23) allows us to express the dependence of the leading contribution to the μ–e vertex in terms of the mass parameters alone. In fact, for the mixing with singlet neutrinos we get $\mathcal{F}_{e\mu} \sim (\bar{M}^2_{\nu N'}/M)^2$, which becomes vanishingly small when the invariant mass term $M \to \infty$, since $\bar{M}_{\nu N'} \lesssim 1$ TeV. This can be considered as a generalization of the decoupling theorem [24, 6] to the case of the mixing with singlet neutrinos N in the class of models characterized by Eq. (2).

4.3 $\mu \to e e^+ e^-$

The leading contribution from heavy neutrinos $\mathcal{N}_i$ of mass $M_i \gg M_W$ arises from the (convergent) loop diagrams in Fig. 3 involving the exchange of *longitudinal* W bosons (more precisely of the would-be Goldstone bosons in the Feynman gauge). Again, since we are interested only in the coefficient of the quadratic term in the masses M_i of the heavy states, it is consistent to ignore all the other loop contributions to the process [5, 6, 22]. Neglecting the external momenta, and for $M_i \gg M_W$, we obtain for the branching ratio, compared to the main channel $\mu \to e\nu\bar{\nu}$:

$$\frac{B(\mu \to e e^- e^+)}{B(\mu \to e\nu\bar{\nu})} = 8\left(\frac{g^2}{16^2\pi^2 M_Z^2}\right)^2 \left(|\mathcal{B}_{e\mu} - 2\epsilon_L \mathcal{F}_{e\mu}|^2 + \frac{1}{2}|2\epsilon_R \mathcal{F}_{e\mu}|^2\right), \tag{24}$$

where $\epsilon_L = -1/2 + s_w^2 \simeq -0.27$ and $\epsilon_R = s_w^2 \simeq 0.23$ are the SM left- and right- handed current couplings of the electron to the Z. The mixing factor entering the box diagram is

$$\mathcal{B}_{e\mu} \equiv \sum_{i,j=heavy} G^*_{\mu i} G_{ei} G^*_{ej} G_{ej} M_i M_j f(M_i, M_j), \tag{25}$$

while $\mathcal{F}_{e\mu}$ and the loop integral $f(M_i, M_j)$, entering also in the $Ze\mu$ vertex, are given by Eqs. (17) and (18).

The 90% C.L. experimental bound, $B(\mu \to ee^-e^+) < 1.0 \times 10^{-12}$ [34], then results in the limit

$$\frac{\left(|\mathcal{B}_{e\mu} + 0.54\mathcal{F}_{e\mu}|^2 + \frac{1}{2}|0.46\mathcal{F}_{e\mu}|^2\right)^{1/2}}{(100\,\mathrm{GeV})^2} < 1.4 \times 10^{-3} \left(\frac{B}{10^{-12}}\right)^{1/2} . \tag{26}$$

This constraint is complementary to the limits from $\mu \to e\gamma$ and from μ–e conversion in nuclei, Eqs. (14) and (21), since it affects a different combination of the mixing parameters, namely $\mathcal{B}_{e\mu}$. As a general result, we see that the contribution to the amplitude for $\mu \to ee^+e^-$ presents a leading quadratic dependence on the heavy mass scale, similar to that of the $Ze\mu$ vertex.

In the case i) of section 2, when the mixing is with new ordinary neutrinos, the vertex contribution of Fig. 3.a depends quadratically on the light–heavy mixing angles, so that we can neglect the the box diagram 3.b, which depends on the fourth power in the mixings. Then the constraint of Eq. (26) becomes

$$\frac{|\mathcal{F}_{e\mu}|}{(100\,\mathrm{GeV})^2} \lesssim 2.3 \times 10^{-3} \left(\frac{B}{10^{-12}}\right)^{1/2} . \tag{27}$$

We find that in this case the limit of Eq. (21) from μ–e conversion in nuclei is stronger by a factor ~ 2, as could be expected from Ref. [31].

For this reason, the limit from $\mu \to ee^+e^-$ can be important only in the case ii) of section 2, when the new states N involved in the mixing are singlets, since in the opposite case the contribution to the $Ze\mu$ vertex is quadratic in the mixing angles and the bound from μ–e conversion in nuclei results in a stronger constraint. When the mixing is with new singlets N, the two general constraints, Eqs. (21) and (26), appear to be of similar strength. For a more quantitative confrontation, let us consider again a particular case, when the mass differences amongst the heavy states are smaller than their common scale, so that $f(M_i, M_j) \simeq 1$. In this case, we have $\mathcal{B}_{e\mu} \simeq (GMG^\dagger)_{ee}(GMG^\dagger)_{e\mu}$, while $\mathcal{F}_{e\mu} \simeq (GMG^\dagger)^2_{e\mu} = (GMG^\dagger)_{ee}(GMG^\dagger)_{e\mu} + (GMG^\dagger)_{e\mu}(GMG^\dagger)_{\mu\mu} + (GMG^\dagger)_{e\tau}(GMG^\dagger)_{\tau\mu}$. If $(GMG^\dagger)_{ee}(GMG^\dagger)_{e\mu}$ is the largest contribution to $\mathcal{F}_{e\mu}$, then $\mathcal{F}_{e\mu} \simeq \mathcal{B}_{e\mu} \simeq (GMG^\dagger)_{ee}(GMG^\dagger)_{e\mu}$, and the constraint of Eq. (26) becomes

$$\frac{|\mathcal{F}_{e\mu}|}{(100\,\mathrm{GeV})^2} \lesssim 0.93 \times 10^{-3} \left(\frac{B}{10^{-12}}\right)^{1/2} , \tag{28}$$

which is as stringent as the bound from μ–e conversion in nuclei, Eq. (21). On the other hand, if $(GMG^\dagger)_{ee}(GMG^\dagger)_{e\mu}$ is not the main part of $\mathcal{F}_{e\mu}$, e.g. $(GMG^\dagger)_{ee}(GMG^\dagger)_{e\mu} < (GMG^\dagger)_{e\tau}(GMG^\dagger)_{\tau\mu}$, then $\mathcal{F}_{e\mu} \gtrsim \mathcal{B}_{e\mu}$. If $\mathcal{B}_{e\mu}$ can be neglected, the rate for $\mu \to ee^+e^-$ is given mainly by the $Ze\mu$ vertex contribution, and the constraint of Eq. (26) is given again by Eq. (27) and is less important by a factor ~ 2 than the limit of Eq. (21) from μ–e conversion in nuclei.

5 Conclusions

We have considered a class of models predicting new heavy neutral fermionic states, whose mixing with the light neutrinos can be naturally significant. In contrast with the see-saw models, the known neutrino masses are predicted to vanish due to a symmetry, such as lepton number. Possible non-vanishing masses for the light neutrinos could then be attributed to small violations of such a symmetry. We have then reviewed the bounds on the flavour-diagonal light–heavy mixing parameters, arising mainly from the constraints on Charged Current Universality and the LEP data, and we have updated and collected in Table 1 the indirect limits on the flavour non-diagonal mixing parameters.

In spite of these stringent constraints, the one-loop-induced rare processes due to the exchange of virtual heavy neutrinos, involving the violation of the muon and electron lepton numbers, which is tested with an impressive experimental precision, have then been shown to be potentially significant. In particular, the $Ze\mu$ vertex, constrained by the non-observation of μ–e conversion in nuclei, and the amplitude for $\mu \to ee^+e^-$, show a non-decoupling quadratic dependence on the heavy neutrino mass M, while $\mu \to e\gamma$ is almost independent of the heavy mass above the electroweak scale. These three processes are then used to set constraints on the LFV parameters entering the corresponding loop diagrams, which turn out to be stronger than the indirect limits of Table 1.

If the mass scale M of the heavy states is in the range $M_Z \lesssim M \lesssim 200$ GeV, the best constraint on the light–heavy LFV mixing comes from $\mu \to e\gamma$. If $M \gtrsim 200$ GeV, and the heavy neutrinos involved in the mixing are not singlets under SU(2), the best constraint is given in most cases by the present data on the search for μ–e conversion in nuclei, and the planned experiments looking for this process have an opportunity to find signals from this kind of physics. In fact, we have pointed out that in this case a significant rate is naturally *expected* in the class of models considered here.

In contrast, if the heavy neutrinos mixing with the light states are singlets under SU(2), then the leading contribution to the $Ze\mu$ vertex is suppressed by two more powers of the light–heavy mixing angles, and the constraint from μ–e conversion in nuclei is comparable to that from $\mu \to ee^+e^-$. These two bounds turn out to be important in a region of the parameter space corresponding to SU(2)-breaking mass entries in the range $200\,\mathrm{GeV} \lesssim M_{\nu N'} \lesssim 1\,\mathrm{TeV}$. In contrast to the model with non-singlet heavy states involved in the mixing, in this latter case of the mixing with isosinglet neutrinos no general prediction can be made on the heavy mass scale, and our constraints are significant only if it lies at an 'intermediate' scale $M \lesssim 300$ TeV. Moreover, in this particular case the prediction for the LFV observables decreases for increasing values of the heavy mass scale $M \to \infty$, resulting in a generalization of the decoupling theorem to this class of 'non-see-saw' models.

In all the cases considered, we have explicitly discussed how the non-decoupling behaviour is strictly related to the spontaneous breaking of the SU(2) symmetry. This result could be expected, since the Appelquist–Carazzone theorem [24] applies to the unbroken gauge theory.

ACKNOWLEDGMENTS

We are grateful to A. Santamaria, M.C. Gonzalez-Garcia, E. Nardi, S. Peris, N. Rius, E. Roulet and J. Peltoniemi for several very useful discussions and for critically reading the preliminary versions of the paper.

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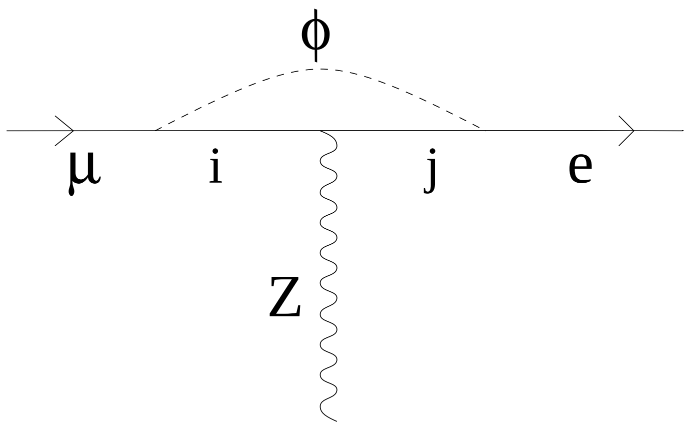
Figure 1: One-loop diagram for the $Ze\mu$ vertex due to virtual heavy neutrinos $\mathcal{N}_{i,j}$ and would-be Goldstone boson $\phi^{\pm}$, representing the Landau gauge leading contribution in the limit $M_{i,j} \gg M_Z$.

Figure 2: 90 % C.L. allowed regions for the LFV mixing $S \equiv (GG^{\dagger})^{1/2}_{e\mu}$ versus the heavy neutrino mass scale M, from the limits on $\mu \to e\gamma$ (dashed line) and on μ–e conversion in nuclei (full line).

Figure 3: One-loop diagrams for $\mu \to ee^{+}e^{-}$ due to virtual heavy neutrinos $\mathcal{N}_{i,j}$ and would-be Goldstone boson $\phi^{\pm}$, representing the Landau gauge leading contributions in the limit $M_{i,j} \gg M_Z$.

	e	μ	τ
e	0.007(0.005)	0.004(0.003)	0.015(0.004)
μ	0.004(0.003)	0.002	0.007(0.004)
τ	0.015(0.004)	0.007(0.004)	0.03(0.01)

Table 1: The 90% C.L. upper bound on the entries of the matrix $GG^{\dagger}$ describing the effects of the light–heavy neutrino mixing. The stronger limits in the parentheses correspond to the mixing with isosinglet neutrinos.



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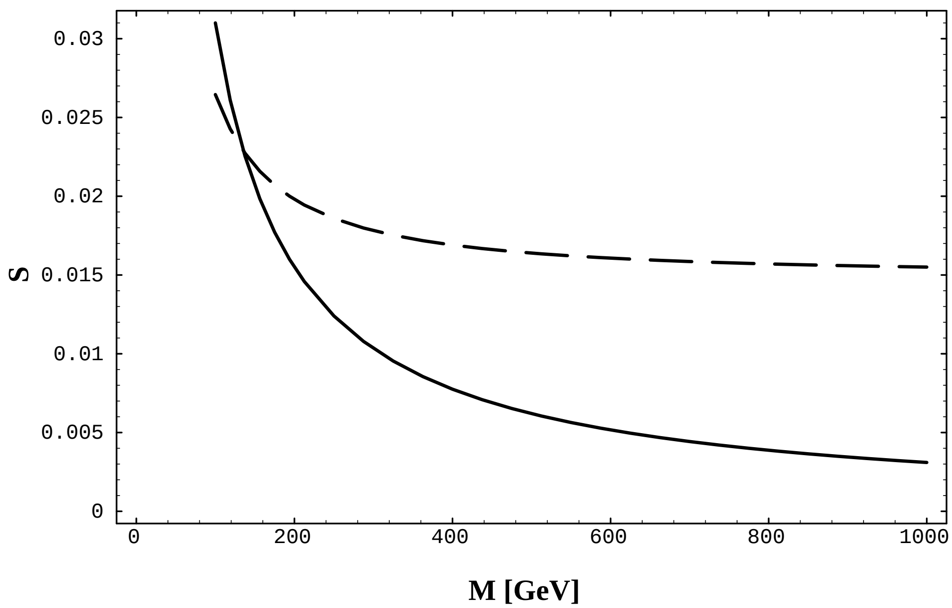
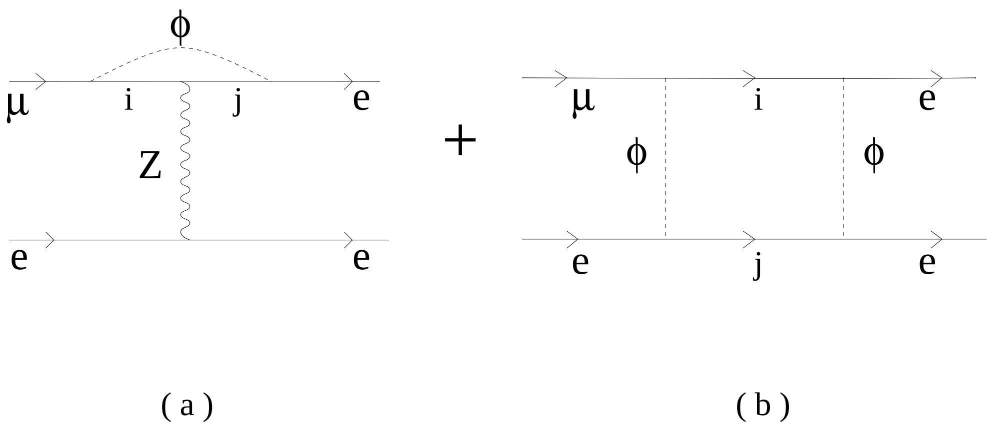


Figure 2: 90 % C.L. allowed regions for the LFV mixing $S \equiv (GG^\dagger)^{1/2}_{e\mu}$ versus the heavy neutrino mass scale M, from the limits on $\mu \to e\gamma$ (dashed line) and on μ–e conversion in nuclei (full line).



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