The low-scale approach to neutrino masses

Sofiane M. Boucenna,[1, *] Stefano Morisi,[2, †] and José W.F. Valle[1, ‡]

[1] *AHEP Group at IFIC, Instituto de Física Corpuscular – Parque Científico, Universitat de València*
C/Catedrático José Beltrán, 2 E-46980 Paterna, València, Spain.
[2] *DESY, Platanenallee 6, D-15735 Zeuthen, Germany.*

(Dated: September 9, 2014)

In this short review we revisit the broad landscape of low-scale $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ models of neutrino mass generation, with view on their phenomenological potential. This includes signatures associated to direct neutrino mass messenger production at the LHC, as well as messenger-induced lepton flavor violation processes. We also briefly comment on the presence of WIMP cold dark matter candidates.

arXiv:1404.3751v2 [hep-ph] 8 Sep 2014

CONTENTS

* msboucenna@gmail.com
† stefano.morisi@gmail.com
‡ valle@ific.uv.es

1. INTRODUCTION

The flavor problem, namely why we have three families of fermions with the same standard model quantum numbers, but with very hierarchical masses and a puzzling pattern of mixing parameters, constitutes one of the most challenging open problems in particle physics. In this regard neutrinos are probably the most mysterious particles. Indeed, while the discovery of the Higgs boson by the ATLAS and CMS experiments at the Large Hadron Collider (LHC) at CERN [1–3] has clarified to some extent the nature of electroweak symmetry breaking, the origin of neutrino masses remains elusive. With standard model fields one can induce Majorana neutrino masses through the non-renormalizable dimension-5 operator

$$\mathcal{O}_{\text{dim}=5} = \frac{\lambda}{\Lambda} LLHH \tag{1}$$

or higher order ones, e. g. $LLHH(H^\dagger H)^m$ [4–9], where λ is a dimensionless coupling and Λ denotes some unknown effective scale. However, strictly speaking, we still do not know whether neutrinos are Dirac or Majorana fermions, and many issues remain open regarding the nature of the associated mass-giving operator, for example,

- its underlying symmetries, such as total lepton number,

- its flavor structure which should account for the observed oscillation pattern,

- its dimensionality,

- its characteristic scale,

- its underlying mechanism.

This leads to considerable theoretical freedom which makes model building an especially hard task, a difficulty which to a large extent persists despite the tremendous experimental progress of the last fifteen years [10, 11].

Indeed the origin of neutrino mass remains so far a mystery. From oscillation studies we can not know the absolute neutrino mass scale. Still we know for certain that neutrinos are the lightest known fermions. Their mass must be below the few eV scale from tritium beta decay studies at the Katrin experiment [12], with somewhat stronger, though more model dependent limits coming from cosmology [13] and from negative neutrinoless double beta decay searches [14]. Unfortunately this vast body of information is far from sufficient to underpin the nature of the neutrino mass generation mechanism.

Mechanisms inducing neutrino mass may be broadly divided on the basis of whether the associated messengers lie at the high energy scale, related say, to some unification scheme or, in contrast, they involve new physics at the TeV scale, potentially accessible at the LHC.

For simplicity here we tacitly assume neutrino masses to come from Weinberg's operator in Eq. (1). This operator can arise in a rich variety of different pathways [15]. For instance in the case of the standard type-I seesaw mechanism [16–21] the right-handed neutrino messengers have a Majorana mass at some large scale, fitting naturally in Grand Unified Theories (GUTs). There are, however, many alternative realizations of the dimension-5 operator, such as the type-II [19, 22–25] and type-III seesaw [26] constructions, in which the messengers have non-trivial gauge quantum numbers. Such schemes are *bona fide* high-scale seesaw in the sense that, to account for the observed neutrino masses with reasonable strength for the relevant neutrino Yukawa couplings, one needs very large scales for the messenger mass, hence inaccessible to collider experiments. Of course within such scenarios one may artificially take TeV scales for the messenger mass by assuming tiny Yukawas, so as to account for the smallness of neutrino mass [1]. However by doing so one erases a number of potential phenomenological implications. Hence we call such standard seesaw varieties as *high scale seesaw*. It has long ago been realized [19] that, carrying no anomalies,

---

[1] One can avoid this in schemes where *ad hoc* cancellations [27] or symmetries [28, 29] prevent seesaw-produced masses. We do not consider such a special case in this review. Similarly we will not assume any family symmetry restricting the flavor structure of models.
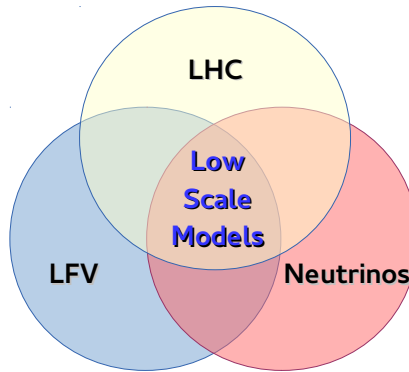
FIG. 1. Low scale neutrino mass models at the crossroad of high and low energy experiments.

singlets can be added in an arbitrary number to any gauge theory. Within the framework of the standard model $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge structure, the models can be labeled by an integer, m, the number of singlets. For example, to account for current neutrino oscillation data, a type-I seesaw model with two right-handed neutrinos is sufficient ($m = 2$). Likewise for models with $m = 1$ in which another mechanism such as radiative corrections (see below) generates the remaining scale. Especially interesting are models with $m > 3$, where one can exploit the extra freedom to realize symmetries, such as lepton number L, so as to avoid seesaw-induced neutrino masses, naturally allowing for TeV-scale messengers. This is the idea behind the inverse [30] and linear seesaw schemes [31–33] described in the next section. We call such schemes as genuine *low scale seesaw* constructions. A phenomenologically attractive alternative to low-scale seesaw are models where neutrino masses arise radiatively [34].

In principle one can assume the presence of supersymmetry in any such scheme, though in most cases it does not play an essential role for neutrino mass generation, *per se*. However we give an example where it could, namely, when the origin of neutrino mass is strictly supersymmetric because R-parity breaks. Indeed, neither gauge invariance nor supersymmetry require R-parity conservation. There are viable models where R-parity is an exact symmetry of the Lagrangian but breaks spontaneously through the Higgs mechanism [35, 36] by an $L = 1$ vacuum expectation value. As we will explain in the next section this scheme is hybrid in the sense that it combines seesaw and radiative contributions. In all of the above one can assume that the neutrino mass messengers lie at the TeV mass scale, hence have potentially detectable consequences.

In this review we consider the low-scale approach to neutrino masses. We choose to map out the possible schemes taking their potential phenomenological implications as guiding criteria, focusing on possible signatures at the LHC and lepton flavor violation (LFV) processes (Fig. 2). The paper is organized as follows: in section 2 we review low energy seesaw schemes, in section 3 we discuss one, two and three-loop radiative models. In section 4 we discuss the supersymmetric mechanism and we sum up in section 5.

2. SEESAW MECHANISM

2.1. High scale seesaw

Within minimal unified models such as SO(10), without gauge singlets, one automatically encounters the presence of new scalar or fermion states that can act as neutrino mass mediators inducing Weinberg's operator in Eq. (1). This leads to different variants of the so-called seesaw mechanism. One possibility is to employ the right-handed neutrinos present in the **16** of SO(10), and are broadly called type-I seesaw schemes [16–21] (see Fig. 2). Similar unified constructions can also be made substituting the right-handed neutrino exchange by that of an exotic hypercharge-

neutral isotriplet lepton [26],

$$\Sigma = (\Sigma^+, \Sigma^0, \Sigma^-),$$

which is called type-III seesaw [26]. An alternative mediator is provided by a hypercharge-carrying isotriplet coming from the **126** of SO(10), and goes by the name type-II seesaw mechanism [19, 22, 23, 25] (see Fig. (2)).

The three options all involve new physics at high scale, typically close to the unification scale. While model-dependent, the expected magnitude of the mass of such messengers is typically expected to be high, say, associated to the breaking of extra gauge symmetries, such as the $B - L$ generator.

Within standard type-I or type-III seesaw mechanism with three right-handed neutrinos the isodoublet neutrinos get mixed with the new messenger fermions by a 6×6 seesaw block diagonalization matrix that can be determined perturbatively using the general method in [21]. For example in the conventional type-I seesaw case the 6×6 matrix U that diagonalizes the neutrino mass is unitary and is given by

$$U = \begin{pmatrix} \left(I - \frac{1}{2}m_D^*(M_R^*)^{-1}M_R^{-1}m_D^T\right)V_1 & m_D^*(M_R^*)^{-1}V_2 \\ -M_R^{-1}m_D^T V_1 & \left(I - \frac{1}{2}M_R^{-1}m_D^T m_D^*(M_R^*)^{-1}\right)V_2 \end{pmatrix} + O(\epsilon^3), \tag{2}$$

where V_1 and V_2 are the unitary matrices that diagonalizes the light and heavy sub-block respectively. From Eq. (2) one sees that the active 3×3 sub-block is no longer unitary and the deviation from unitary is of the order of $\epsilon^2 \sim (m_D/M_R)^2$. The expansion parameter ϵ is very small if the scale of new physics is at the GUT scale so the induced lepton flavor violation processes are suppressed. In this case there are no detectable direct production signatures at colliders nor LFV processes. This follows from the well know type-I seesaw relation

$$m_\nu \sim m_D^2/M_{\text{messenger}}$$

where $M_{\text{messenger}} = M_R$ implying that

$$\epsilon^2 \sim m_\nu/M_R \tag{3}$$

is suppressed by the neutrino mass, hence negligible regardless of whether the messenger scale M_R lies in the TeV scale [2]. As a result there is a decoupling of the effects of the messengers at low energy, other than providing neutrino masses. This includes for example lepton flavor violation effects in both type-I and type-III seesaw mechanism. Regarding direct signatures at collider experiments these require TeV scale messengers which can be artificially implemented in both type-I and type-III cases by assuming the Dirac-type Yukawa couplings to be tiny. This makes messenger production at colliders totally hopeless in type-I seesaw, but does not affect the production rate in type-III seesaw mechanism, since it proceeds with gauge strength [39].
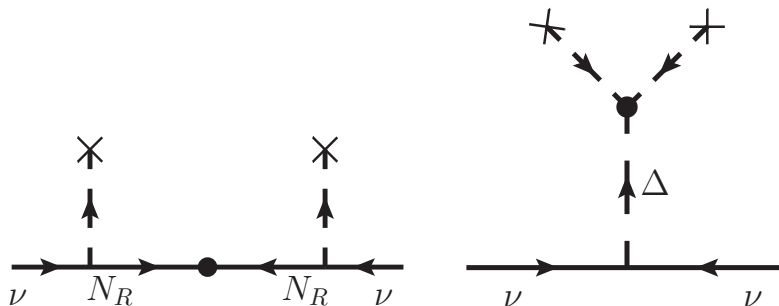

FIG. 2. Neutrino mass generation in the type-I seesaw (left) and type-II seesaw (right). The black disks show where lepton number violation takes place.

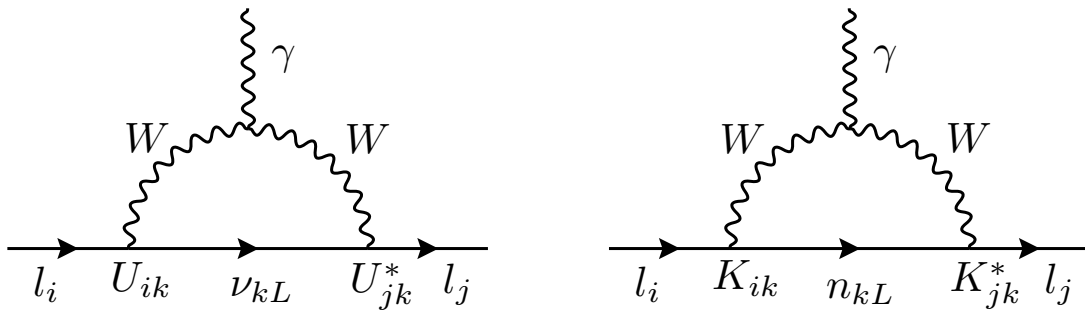[2] Weak universality tests as well as searches at LEP and previous colliders rule out lower messenger mass scales [37, 38].

FIG. 3. Radiative decays $\ell_i \to \ell_j \gamma$ in the standard model with massive light neutrinos (left) and heavy neutrinos (right).

Coming to the type-II scheme, neutrino masses are proportional to the vev of the neutral component of a scalar electroweak triplet Δ^0 and we have

$$m_\nu = y_\nu \, v_T \,, \quad \text{with} \quad v_T = \frac{\mu_T \, v^2}{M_T^2} \,, \tag{4}$$

where v is the vev of the standard model Higgs, M_T is the mass of the scalar triplet Δ, y_ν is the coupling of the neutrino with the scalar triplet and μ_T is the coupling (with mass dimension) of the trilinear term between the standard model Higgs boson and the scalar triplet $H^T \Delta H$. Assuming y_ν of order one, in order to have light neutrino mass there are two possibilities: either M_T is large or μ_T is small. The first case is the standard type-II seesaw where all the parameters of the model are naturally of order one.

In such high scale type-I and type-III seesaw varieties neutrino mass messengers are above the energy reach of any conceivable accelerator, while lepton flavor violation effects arising from messenger exchange are also highly suppressed. Should lepton flavor violation ever be observed in nature, such schemes would suggest the existence of an alternative lepton flavor violation mechanism. A celebrated example of the latter is provided the exchange of scalar leptons in supersymmetric models [40–42].

In contrast, if type-II seesaw schemes are chosen to lie at the TeV scale, then lepton flavor violation effects as well as same-sign di-lepton signatures at colliders remain [43], see below. Obviously supersymmetrized "low-scale" type-II seesaw have an even richer phenomenology [44, 45].

2.2. Low scale type-I seesaw

The most general approach to the seesaw mechanism is that provided by the standard $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge group structure which holds at low energies. Within this framework one can construct seesaw theories with an arbitrary number of right handed neutrinos, m [19], since gauge singlets carry no anomalies. In fact the same trick can be upgraded to other extended gauge groups, such as $SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ or Pati-Salam and also unified groups such as $SO(10)$ [46, 47] or E_6. This opens the door to genuine low-scale realizations of the seesaw mechanism.

Before turning to the description of specific low scale type-I seesaw schemes lets us briefly note their basic phenomenological feature, namely that in genuine low-scale seesaw schemes Eq. (3) does not hold so that, for light enough messengers, one can have lepton flavor violation processes [48–50]. For example, radiative decays $\ell_i \to \ell_j \gamma$ proceed through the exchange of light (left panel) as well as heavy neutrinos (right) in Fig. (3). Clearly expected lepton flavor violation rates such as that for the $\mu \to e\gamma$ process are too small to be of interest. Another important conceptual feature of phenomenological importance is that lepton flavor violation survives even in the limit of strictly massless neutrinos (i.e. $\mu \to 0$, see text below) [51, 52].
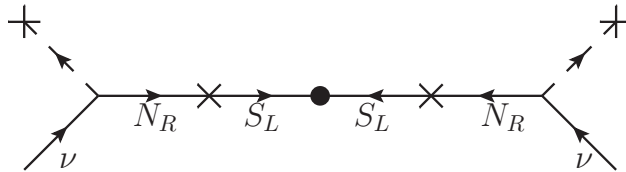
FIG. 4. Neutrino mass generation in the type-I inverse seesaw.

1. Inverse type-I seesaw

In its simplest realization the inverse seesaw extends the standard model by means of two sets of electroweak two-component singlet fermions N_{Ri} and S_{Lj} [30]. The lepton number L of the two sets of fields N_R and S_L can be assigned as $L(N_R) = +1$ and $L(S_L) = +1$. One assumes that the fermion pairs are added sequentially, i.e. $i, j = 1, 2, 3$, though other variants are possible. After electroweak symmetry breaking the Lagrangian is given by

$$\mathcal{L} = m_D \bar{\nu}_L N_R + M \overline{N}_R S_L + \mu \tilde{S}_L S_L + \text{h.c.}, \tag{5}$$

We define $\tilde{S}_L \equiv S_L^T C^{-1}$ where C is the charge conjugation matrix, m_D and M are arbitrary 3×3 Dirac mass matrices and μ is a Majorana 3×3 matrix. We note that the lepton number is violated by the μ mass term here. The full neutrino mass matrix can be written as a 9×9 matrix instead of 6×6 as in the typical type-I seesaw and is given by (in the basis ν_L, N_R and S_L)

$$M_\nu = \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M^T \\ 0 & M & \mu \end{pmatrix} \tag{6}$$

The entry μ may be generated from the spontaneous breaking of lepton number through the vacuum expectation value of a gauge singlet scalar boson carrying $L = 2$ [53].

It is easy to see that in the limit where $\mu \to 0$ the exact U(1) symmetry associated to total lepton number conservation holds, so the light neutrinos are strictly massless. However individual symmetries are broken hence flavor is violated, despite neutrinos being massless [51, 52]. For complex couplings, one can also show that CP is violated despite the fact that light neutrinos are strictly degenerate [54, 55]. The fact that flavor and CP are violated in the massless limit implies that the attainable rates for the corresponding processes are unconstrained by the observed smallness of neutrino masses, and are potentially large.

This feature makes this scenario conceptually and phenomenologically interesting and is a consequence of the fact that the lepton number is conserved. However when $\mu \neq 0$ light neutrino masses are generated, see Fig. (4). In particular in the limit where $\mu, m_D \lesssim M$ [3] the light neutrino 3×3 mass matrix is given by

$$m_\nu \simeq m_D \frac{1}{M} \mu \frac{1}{M^T} m_D^T. \tag{7}$$

It is clear from this formula that for "reasonable" Yukawa strength or m_D values, M of the order of TeV, and suitably small μ values one can account for the required light neutrino mass scale at the eV scale. There are two new physics scales, M and μ, the last of which is very small. Therefore it constitutes an extension of the standard model from below, rather than from above. For this reason, it has been called inverse seesaw: in contrast with the standard type-I seesaw mechanism, neutrino masses are suppressed by a small parameter, instead of the inverse of a large one. The

---

[3] On the other hand, the opposite limit $\mu \gg M$ is called double seesaw. In contrast to the inverse seesaw, the double seesaw brings no qualitative differences with respect to standard seesaw and will not be considered here.
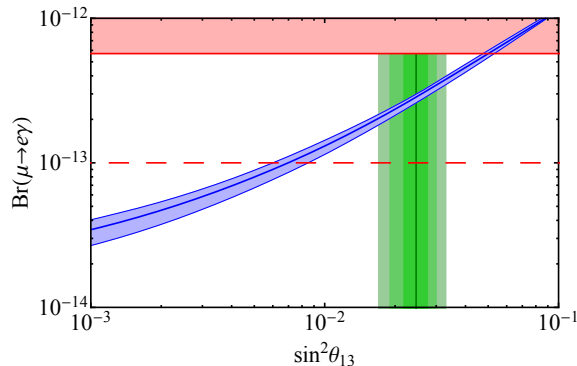
FIG. 5.   Branching ratios $Br(\mu \to e\gamma)$ in the inverse seesaw model of neutrino mass [49].

smallness of the scale μ is *natural* in t'Hooft's sense, namely in the limit $\mu \to 0$, a symmetry the symmetry is enhanced since lepton number is recovered [4].

In this case the seesaw expansion parameter $\epsilon \sim m_D/M$ also characterizes the strength of unitarity and universality violation and can be of order of percent or so [50, 58], leading to sizable lepton flavor violation rates, close to future experimental sensitivities. For example, with $m_D = 30$ GeV, $M = 300$ GeV and $\mu = 10$ eV we have that $\epsilon^2 \sim 10^{-2}$. The deviation from the unitary is typically of order ϵ^2. As mentioned above, typical expected lepton flavor violation rates in the inverse seesaw model can be potentially large. For example, the rates for the classic $\mu \to e\gamma$ process are illustrated in Fig. 2 2.2 1. The figure gives the predicted branching ratios $Br(\mu \to e\gamma)$ in terms of the small neutrino mixing angle θ_{13}, for different values of the remaining oscillation parameters, with the solar mixing parameter $\sin^2\theta_{12}$ within its 3σ allowed range and fixing the inverse seesaw parameters as: $M = 1$ TeV and $\mu = 3$ KeV. The vertical band corresponds to the 3σ allowed θ_{13} range.

Regarding direct production at colliders, although kinematically possible, the associated signatures are not easy to catch given the low rates as the right handed neutrinos are gauge singlets and due to the expected backgrounds (see for instance [59]).

The way out is by embedding the model within an extended gauge structure that can hold at TeV energies, such as an extra U(1) coupled to $B - L$ which may arise from SO(10) [33]. Viable scenarios may also have TeV-scale $SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ or Pati-Salam intermediate symmetries [60]. In this case the right handed messengers can be produced through a new charged [61–63] or neutral gauge boson [64]. In fact one has the fascinating additional possibility of detectable lepton flavor violation taking place at the large energies now accessible at the LHC [64].

2.   Linear type-I seesaw

This variant of low-scale seesaw was first studied in the context of $SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ theories [31, 32] and subsequently demonstrated to arise naturally within the SO(10) framework in the presence of gauge singlets [33]. The lepton number assignment is as follows: $L(\nu_{Li}) = +1$, $L(N_{Ri}) = 1$ and $L(S_{Li}) = +1$ so that, after electroweak symmetry breaking the Lagrangian is given by

$$\mathcal{L} = m_D \bar{\nu}_L N_R + M_R \overline{N}_R S_L + M_L \nu_L \tilde{S}_L + \text{h.c.} \tag{8}$$

———

[4] There are realizations where the low scale of μ is radiatively calculable. As examples see the supersymmetry framework given in [56], or the standard model extension suggested in [57].

Notice that the lepton number is broken by the mass term proportional to M_L. This corresponds to the neutrino mass matrix in the basis ν_L, N_R and S_L given as

$$M_\nu = \begin{pmatrix} 0 & m_D^T & M_L \\ m_D & 0 & M_R \\ M_L & M_R & 0 \end{pmatrix} \tag{9}$$

If $m_D \ll M_{L,R}$ then the effective light neutrino mass matrix is given by

$$m_\nu = m_D\, M_L \frac{1}{M_R} + \text{Transpose.} \tag{10}$$

Note that, in contrast with other seesaw varieties which lead to $m_\nu \propto m_D^2$, this relation is linear in the Dirac mass entry, hence the origin of the name "linear seesaw". Clearly neutrino masses will be suppressed by the small value of M_L irrespective of how low is the M_R scale characterizing the heavy messengers. For example, if one takes the SO(10) unification framework [33], natural in this context, one finds that the scale of M_L, i.e v_L, is related to the scale of M_R, i.e. v_R through

$$v_L \sim \frac{v_R\, v}{M_{\text{GUT}}}, \tag{11}$$

where M_{GUT} is the unification scale of the order of $\mathcal{O}(10^{16}\,\text{GeV})$ and v is the electroweak breaking scale of the order of $\mathcal{O}(100\,\text{GeV})$. Replacing the relation (11) in Eq. (10) the new physics scale drops out and can be very light, of the order of TeV.

Neutrino mass messengers are naturally accessible at colliders, like the LHC, since the right handed neutrinos can be produced through the Z' "portal", as light as few TeV. The scenario has been shown to be fully consistent with the required smallness of neutrino mass as well as with the requirement of gauge coupling unification [33]. Other $\text{SU}(3) \otimes \text{SU}(2)_{\text{L}} \otimes \text{SU}(2)_{\text{R}} \otimes \text{U}(1)_{\text{B}-\text{L}}$ and Pati-Salam implementations also been studied in [60].

Similarly to the inverse type-I seesaw scheme, we also have here potentially large unitarity violation in the effective lepton mixing matrix governing the couplings of the light neutrinos. This gives rise to lepton flavor violation effects similar to the inverse seesaw case. Finally we note that, in general, a left-right symmetric linear seesaw construction also contains the lepton number violating Majorana mass term $\tilde{S}_L S_L$ considered previously.

2.3. Low scale type-III seesaw

Here we consider a variant of the low scale type-III seesaw model introduced in [65] based on the inverse seesaw mechanism [30] but replacing the N_R lepton field with the neutral component Σ^0 of a fermion triplet under $\text{SU}(2)_L$ with hypercharge zero [66],

$$\Sigma = (\Sigma^+, \Sigma^0, \Sigma^-).$$

As in the the inverse type-I seesaw one introduces an extra set of gauge singlet fermions S_L with lepton number $L(S_L) = +1$ and $L(\Sigma^0) = +1$. The mass Lagrangian is given by

$$\mathcal{L} = m_D \bar{\nu}_L \Sigma^0 + M \overline{\Sigma^0} S_L + \mu\, \tilde{S}_L S_L - \frac{1}{2} m_\Sigma \text{Tr}(\overline{\Sigma}\Sigma^c) + \text{h.c.} \tag{12}$$

In the basis (ν, Σ^0, S_L) the neutrino mass matrix is given by

$$M_\nu = \begin{pmatrix} 0 & m_D^T & 0 \\ m_D & m_\Sigma & M^T \\ 0 & M & \mu \end{pmatrix}. \tag{13}$$

As in the inverse seesaw case, in the limit $\mu = 0$ the light neutrinos are massless at tree level even if the mass term m_Σ breaks lepton number. And for a small $\mu \neq 0$ neutrinos get mass. Again, the scale of new physics is naturally small leading to sizable lepton flavor violation rates.

Model	Scalars	Fermions	LFV	LHC
Type-I		$(\mathbf{1},\mathbf{1},0)_{+1}$	✗	✗
Type-II	$(\mathbf{1},\mathbf{3},2)_{+2}$		✓	✓
Type-III		$(\mathbf{1},\mathbf{3},0)_{+1}$	✗	✓
Inverse		$(\mathbf{1},\mathbf{1},0)_{+1}$	✓	✗
Linear		$(\mathbf{1},\mathbf{1},0)_{+1}$	✓	✗
Inverse Type-III		$(\mathbf{1},\mathbf{3},0)_{+1},\ (\mathbf{1},\mathbf{1},0)_{+1}$	✓	✓

TABLE I. Phenomenological implications of low-scale $\mathrm{SU(3)_c} \otimes \mathrm{SU(2)_L} \otimes \mathrm{U(1)_Y}$ seesaw models together with their particle content. The subscript in the representations is lepton number. "✗" would change to "✓" in the presence of new gauge bosons or supersymmetry, as explained in the text.

On the other hand the charged component of the fermion triplet $\Sigma^{\pm}$ gives also a contribution to the charged lepton mass matrix

$$M_{\mathrm{ch.lep}} = \begin{pmatrix} M_l & m_D \\ 0 & m_\Sigma \end{pmatrix}, \tag{14}$$

leading to a violation of the Glashow-Iliopoulos-Maiani mechanism [67] in the charged lepton sector, leading to tree level contributions to $\mu \to eee$ and similar tau decay processes.

As in the standard type-III seesaw mechanism [26], universality violation is also present here. However, in contrast to the standard case, here its amplitude is of the order

$$\epsilon^2 \sim (m_D/m_\Sigma)^2,$$

which need not be neutrino mass suppressed. Indeed, in the inverse type-III seesaw scheme neutrino masses are proportional to the parameter μ. As a result there are sizeable lepton flavor violation processes such as $\mu \to e\gamma$ and $\mu \to eee$, whose attainable branching ratios are shown in Fig. (6).

Finally, to conclude this discussion, we stress that, in contrast with the inverse type-I seesaw mechanism, here the neutrino mass messenger Σ^0, being an isotriplet member, has gauge interactions. Hence, if kinematically allowed it will be copiously produced in collider experiments like the LHC [39].

In short this scheme is a very interesting one both from the point of view of the detectability of collider signatures at the LHC as well as lepton flavor violation phenomenology.
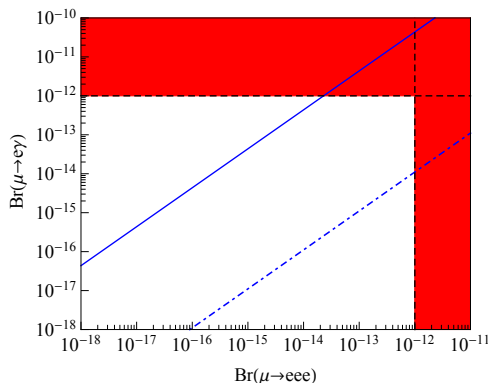

FIG. 6. Branching of μ decay into $3e$ vs the branching of $\mu \to e\gamma$ varying the parameter μ parameter for different values of the mixing between the Σ^0 and S fields, 0.5 (continuous) and 0.1 (dashed) and with M is fixed at 1 TeV.

2.4. Low scale type-II seesaw

We now turn to the so-called type-II seesaw mechanism [19, 22, 23, 25] which, though normally assumed to involve new physics at high energy scales, typically close to the unification scale, may also be considered (perhaps articially) as a low scale construction, provided one adopts a tiny value for the trilinear mass parameter

$$\mu_T \sim 10^{-8} \text{GeV},$$

in the scalar potential, then the triplet mass M_T can be assumed to lie around the TeV scale. Barring naturalness issues, such a scheme could be a possibility giving rise to very interesting phenomenological implications. In fact, in this case, if kinematically allowed, the scalar triplet Δ will be copiously produced at the LHC because it interacts with gauge bosons.

Moreover the couplings y_ν that mediate lepton flavor violation processes are of order one and therefore such processes are not neutrino mass suppressed, as in the standard type-I seesaw. Indeed, from the upper limit $\text{Br}(\mu \to 3e) < 10^{-12}$ it follows that (see Ref. [68])

$$y_\nu^2 < 1.4 \times 10^{-5} \left(\frac{m_\Delta}{1 \,\text{TeV}} \right), \tag{15}$$

implying a sizeable triplet Yukawa coupling. With $y_\nu \sim 10^{-2}$, in order to get adequate neutrino mass values, one needs

$$v_T \sim 10^{-7} \,\text{GeV}, \tag{16}$$

which restricts the scalar triplet vacuum expectation value (vev). For such small value of the vev, the decay of the Δ^{++} is mainly into a pair of leptons with the same charge, while for $v_T > 10^{-4} \,\text{GeV}$, the Δ^{++} decays mainly into a sam-sign WW pair, see Ref. [68].

Note that the tiny parameter μ_T controls the neutrino mass scale but does not enter in the couplings with fermions. This is why the lepton flavor violation rates can be sizable in this case. For detailed phenomenological studies of low energy type-II seesaw see, for example, Ref. [61, 68, 69]

Before reviewing the models based on radiative generation mechanisms for neutrino masses, we summarize the phenomenological implications of low scale seesaw models, together with their particle content, in Tab. (2 2.3).

3. RADIATIVE NEUTRINO MASSES

In the previous sections we reviewed mechanisms ascribing the smallness of neutrino masses to the small coefficient in front of Weinberg's dimension-five operator. This was generated either through tree-level exchange of super-heavy messengers, with mass associated to high-scale symmetry breaking, or conversely, because of symmetry breaking at low scale. In what follows we turn to radiatively induced neutrino masses, a phenomenologically attractive way to account for neutrino masses. In such scenarios the smallness of the neutrino mass follows from loop factor(s) suppression. From a purely phenomenological perspective, radiative models are perhaps quite interesting as they rely on new particles that typically lie around the TeV scale, hence accessible to collider searches.

Unlike seesaw models, radiative mechanisms can go beyond the effective $\Delta L = 2$ dimension-five operator in Eq. (1) and generate the neutrino masses at higher order. This leads to new operators and to further mass suppression. Such an approach has been reviewed in Refs. [7, 70–73]. In what follows we'll survey some representative underlying models up to the third loop level.

3.1. One–loop schemes

A general survey of one-loop neutrino mass operators leading to neutrino mass has been performed in [6]. Neutrino mass models in extensions of the SM with singlet right-handed neutrinos have been systematically analyzed in [74, 75] and for higher representations in [76]. Here we review the most representative model realizations.

1. Zee Model

The Zee Model [77] extends the standard $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ model with the following fields

$$h^+ \sim (\mathbf{1}, \mathbf{1}, +1)_{-2} \quad , \quad \phi_{1,2} \sim (\mathbf{1}, \mathbf{2}, +1/2)_0 \,, \tag{17}$$

where the subscript denotes lepton number. Given this particle content neutrino mass are one-loop calculable. The relevant terms are given by

$$\mathcal{L} = y_i^{ab} \, \overline{L}_a \phi_i \ell_{bR} + f^{ab} \tilde{L}_a \, i\tau_2 \, L_b \, h^+ - \mu \, \phi_1^\dagger \, i\tau_2 \, \phi_2^* \, h^+ + \text{h.c.} \,, \tag{18}$$

where a, b indicate the flavor indices, i.e. $a, b = e, \mu, \tau$, $\tilde{L} \equiv L^T C^{-1}$ and τ_2 is the second Pauli matrix. Notice that the matrix f must be anti-symmetric in generation indices. The violation of lepton number, required to generate a Majorana mass term for neutrinos, resides in the coexistence of the two Higgs doublets in the μ term. The one-loop radiative diagram is shown in Fig. (7). The model has been extensively studied in the literature [78–101], particularly in the Zee-Wolfenstein limit where only ϕ_1 couples to leptons due to a $\mathbb{Z}_2$ symmetry [102].
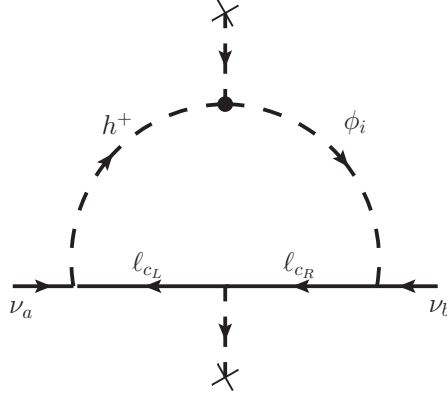

FIG. 7. Neutrino mass generation in the Zee model.

This particular simplification forbids tree-level Higgs-mediated flavor-changing neutral currents (FCNC), although it is now disfavored by neutrino oscillation data [90, 103]. However the general Zee model is still valid phenomenologically [87] and is in testable with FCNC experiments. For instance the exchange of the Higgs bosons leads to tree level decays of the form $\ell_i \to \ell_j \ell_k \bar{\ell}_k$, in particular $\tau \to \mu\mu\bar{\mu}, \mu e \bar{e}$ (see for instance [104]). Collider phenomenology has been studied in [105, 106].

Recently, a variant of the Zee model have been considered in [107] (see also [108]) by imposing a family-dependent $\mathbb{Z}_4$ symmetry acting on the leptons, thereby reducing the number of effective free parameters to four. The model predicts inverse hierarchy spectrum in addition to correlations among the mixing angles.

2. Radiative seesaw model

Another one-loop scenario was suggested by Ma [109]. Besides the standard model fields, three right handed Majorana fermions N_i $(i = 1, 2, 3)$ and a Higgs doublet are added to the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ model,

$$N_i \sim (\mathbf{1}, \mathbf{1}, 0)_{+1} \quad , \quad \eta \sim (\mathbf{1}, \mathbf{2}, +1/2)_0 \,. \tag{19}$$

In addition, a parity symmetry acting only on the new fields is postulated. This $\mathbb{Z}_2$ is imposed in order to forbid Dirac neutrino mass terms. The relevant interactions of this model are given by

$$\mathcal{L} = y_{ab} \, \overline{L}_a \, i\tau_2 \eta^* N_b - \frac{M_{N_i}}{2} \tilde{N}_i N_i + \text{h.c.} \tag{20}$$
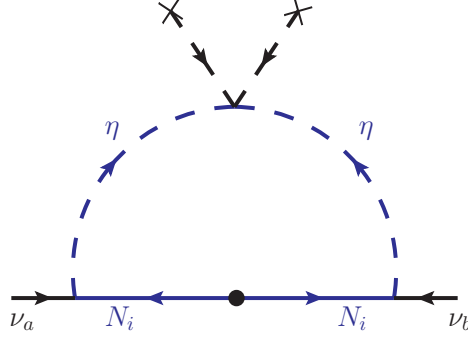
FIG. 8. Neutrino mass generation in the radiative seesaw model. The blue color represents the potential dark matter candidates.

In the scalar potential a quartic scalar term of the form $(H^\dagger \eta)^2$ is allowed. The one-loop radiative diagram is shown in Fig. (8) and generates calculable $\mathcal{M}_\nu$ if $\langle \eta \rangle = 0$, which follows from the assumed symmetry. The neutrino masses are given by

$$(M_\nu)_{ab} = \sum_i \frac{y_{ai} y_{bi} M_{N_i}}{16\pi^2} \left[\frac{m_R^2}{m_R^2 - M_{N_i}^2} \ln \frac{m_R^2}{M_{N_i}^2} - \frac{m_I^2}{m_I^2 - M_{N_i}^2} \ln \frac{m_I^2}{M_i^2} \right], \tag{21}$$

where m_R (m_I) is the mass of the real (imaginary) part of the neutral component of η.

Thanks to its simplicity and rich array of predictions, the model has become very popular and an extensive literature has been devoted to its phenomenological consequences. As is generally the case with multi-Higgs standard model extensions, the induced lepton flavor violation effects such as $\mu \to e\gamma$ provide a way to probe the model parameters. In particular the lepton flavor violation phenomenology has been studied in [110–115]. The effect of corrections induced by renormalization group running have also been considered [116], showing that highly symmetric patterns such as the bimaximal lepton mixing structure can still be valid at high-energy but modified by the running to correctly account for the parameters required by the neutrino oscillation measurements [11]. Collider signatures have also been investigated in [117–120].

A remarkable feature of this model is the natural inclusion of a WIMP (weakly interacting massive particle) dark matter candidate. Indeed, the same parity that makes the neutrino mass calculable, also stabilizes N_i and the neutral component of η. The lightest $\mathbb{Z}_2$-odd particle, either a boson or a fermion, can play the role of WIMP cold dark matter candidate [110, 112, 115, 121–125]. There is also the interesting possibility of the dark matter being warm in this setup [111, 126]. Various extensions of the model have also been considered, for e.g. [127, 128]. For a review on models with one–loop radiative neutrino masses and viable dark matter candidates we refer the reader to the complete classification given in [129, 130].

3.2. Two–loop schemes

As a prototype two–loop scheme we consider the model proposed by Zee [131] and Babu [34] (which first appeared in [22]), that leads to neutrino masses at two-loop level by extending the standard model with two complex singly and doubly [132] charged $SU(2)_L$ singlet scalars

$$h^+ \sim (\mathbf{1}, \mathbf{1}, +1)_{-2} \quad , \quad k^{++} \sim (\mathbf{1}, \mathbf{1}, +2)_{-2}. \tag{22}$$

The relevant terms in the Lagrangian are therefore

$$\mathcal{L} = f_{ab}\, \tilde{L}_a\, i\tau_2\, L_b h^+ + g_{ab} \tilde{\ell}_{aR}\, \ell_{bR} k^{++} - \mu\, h^- h^- k^{++} + \text{h.c.} \tag{23}$$
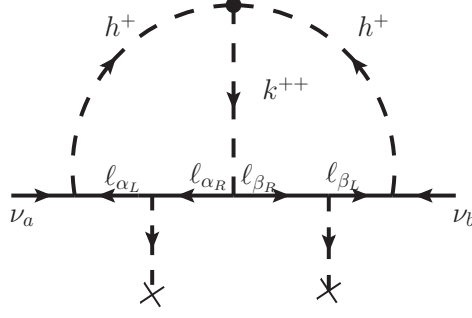
FIG. 9. Neutrino mass generation in the Zee-Babu model.

The trilinear μ term in the scalar potential [5] provides lepton number violation and leads to a calculable Majorana neutrino mass generated at the second loop order, as shown in Fig. (9) and given by

$$(M_\nu)_{ab} \sim \mu \, \frac{1}{(16\pi)^2} \frac{1}{M} \frac{16\pi^2}{3} f_{ac} m_c g_{cd}^* m_d f_{bd} \tag{24}$$

where $M = \max(M_{k^{++}}, M_{h^+})$ and m_a are charged lepton masses [134]. As in the Zee model, the matrix f is anti-symmetric. Therefore the determinant of m_ν vanishes and, as a result, one of the light neutrinos must be massless.

The Zee–Babu model is constrained by a variety of lepton flavor violation processes among which the tree-level lepton flavor violation $\ell_i \to \ell_j \ell_k \bar{\ell}_l$ decays induced by k^{++} exchange and the radiative decays $\ell_i \to \ell_j \gamma$ mediated by the charged scalars h^+ and k^{++}. Weak universality is also violated since the h^+ exchange induces new contributions for muon decay [134–137]. Both lepton flavor violation and weak universality tests constrain the model parameters. Combining lepton flavor violation and universality constraints [135] pushes the mass of h^+ and k^{++} above the TeV scale, for both inverted and normal hierarchies, making it a challenge to probe the model at the LHC. The collider phenomenology of the model have been considered in [134, 135, 138].

3.3. Three–loop schemes

Of the possible three–loop schemes we will focus on the one suggested by Krauss-Nasri-Trodden (KNT) [139]. These authors considered an extension of the standard model with two charged scalar singlets h_1 and h_2 and one right handed neutrino N.

$$h_{1,2}^+ \sim (\mathbf{1}, \mathbf{1}, +1)_{-2} \quad , \quad N \sim (\mathbf{1}, \mathbf{1}, 0)_{+1} \, . \tag{25}$$

As usual in radiative neutrino mass models that include gauge singlet Majorana fermions, an additional $\mathbb{Z}_2$ symmetry is imposed, under which the standard model fields as well as h_1 transform trivially, while N and h_2 are odd. The most general renormalizable terms that may be added to the standard model fermion Lagrangian are

$$\mathcal{L} = f_{ab} \tilde{L}_a \, i\tau_2 L_b \, h_1^+ + g_a N h_2^+ \ell_{a_R} - \frac{M_N}{2} \tilde{N} N + \text{h.c.} \tag{26}$$

Note that the scalar potential contains a term of the form $(h_1 h_2^*)^2$, which makes possible the diagram of Fig. (10) possible. Hence neutrinos acquire Majorana masses induced only at the 3-loop level. Such strong suppression allows for sizable couplings of the TeV scale singlet messenger states.

————

[5] This term can arise spontaneously through the vev of an extra gauge singlet scalar boson [133].
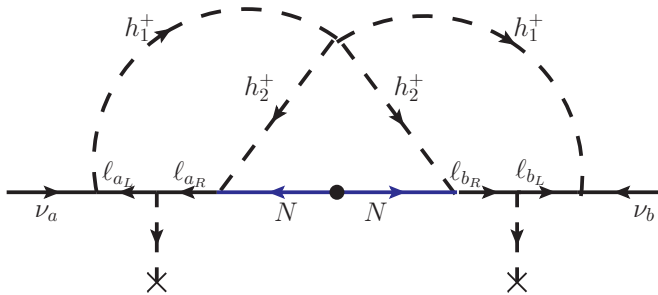
FIG. 10. Neutrino mass generation in the KNT model.

In addition to neutrino masses, the model also includes a WIMP dark matter candidate. Indeed for the choice of parameters $M_{h_2} > M_N$, N is stable and can be thermally produced in the early universe, leading naturally to the correct dark matter abundance.

A very similar model with the same loop topology has been proposed in [140], replacing the neutral gauge singlets by new colored fields and the charged leptons by quarks and in [141] the triplet variant of the model has been introduced. These variations makes the model potentially testable at hadron colliders. Other three loop mass models have also been considered more recently, for instance in [141–144]. A systematic study generalizing the KNT model was presented in [145].

	Model	Scalars	Fermions	LFV	DM	LHC
1–Loop	Zee	$(\mathbf{1}, \mathbf{1}, +1)_{-2}$, $(\mathbf{1}, \mathbf{2}, -1/2)_0$		✓	✗	✓
	Ma	$(\mathbf{1}, \mathbf{2}, +1/2)_0$	$(\mathbf{1}, \mathbf{1}, 0)_{+1}$	✓	✓	✓
2–Loops	Zee-Babu	$(\mathbf{1}, \mathbf{1}, +1)_{-2}$, $(\mathbf{1}, \mathbf{1}, +2)_{-2}$		✓	✗	✓
3–Loops	KNT	$(\mathbf{1}, \mathbf{1}, +1)_{-2}$	$(\mathbf{1}, \mathbf{1}, 0)_{+1}$	✓	✓	✗

TABLE II. Phenomenological implications of radiative $\mathrm{SU}(3)_{\mathrm{c}} \otimes \mathrm{SU}(2)_{\mathrm{L}} \otimes \mathrm{U}(1)_{\mathrm{Y}}$ neutrino mass models discussed in this review. Representations are labelled as in the rest of the paper.

We summarize the models discussed in this section and their phenomenological implications in Tab. (3 3.3).

4. SUPERSYMMETRY AS THE ORIGIN OF NEUTRINO MASSES

The standard formulation of supersymmetry assumes the conservation of a discrete symmetry called R–parity (R_p), under which all the standard model states are R-even, while their superpartners are R-odd [146]. R_p is related to the spin (S), total lepton (L), and baryon (B) number as

$$R_p = (-1)^{(3B+L+2S)}.$$

Hence requiring baryon and lepton number conservation implies R_p conservation. In this case the supersymmetric states must be produced in pairs, while the lightest of them is absolutely stable.

On general grounds, however, neither gauge invariance nor supersymmetry require R_p conservation and many implications can be associated to R-parity violation [147]. The most general supersymmetric standard model extension contains explicit R_p violating interactions. Constraints on the relevant parameters and their possible signals have been analysed [148, 149]. In general, there are too many independent couplings, some of which must be set to zero in order to avoid too fast the proton decay. For these reasons we focus our attention to the possibility that R_p can be an exact symmetry of the Lagrangian, broken spontaneously through the Higgs mechanism [35, 150]. This may occur

via nonzero vacuum expectation values for scalar neutrinos, such as

$$v_R = \langle \tilde{\nu}_{R\tau} \rangle \qquad ; \qquad v_L = \langle \tilde{\nu}_{L\tau} \rangle \ . \tag{27}$$

Here we consider the simplest prototype scheme where supersymmetry seeds neutrino masses in an essential way. The idea is to take the simplest effective description of the above picture, namely, bilinear R-parity violation [151–153]. This is the minimal way to incorporate lepton number and R-parity violation to the Minimal Supersymmetric Standard Model (MSSM), providing a simple way to accommodate neutrino masses in supersymmetry. The superpotential is

$$W = W^{MSSM} + \epsilon_a \widehat{L}_a \widehat{H}_u. \tag{28}$$

The three $\epsilon_a = (\epsilon_e, \epsilon_\mu, \epsilon_\tau)$ parameters have dimensions of mass and explicitly break lepton number by $\Delta L = 1$. Their size and origin can be naturally explained in extended models where the breaking of lepton number is spontaneous [35, 150, 153]. These parameters are constrained to be small ($\epsilon_a \ll m_W$) so as to account for the small neutrino masses. Furthermore, the presence of the new superpotential terms implies new soft supersymmetry breaking terms as well

$$V_{soft} = V_{soft}^{MSSM} + B_a \epsilon_a \tilde{L}_a H_u \,, \tag{29}$$

where the B_a are parameters with units of mass.

In this scheme, neutrinos get tree-level mass by mixing with the neutralino sector [154–156]. In the basis $(\psi^0)^T = (-i\tilde{B}^0, -i\tilde{W}_3^0, \widetilde{H}_d^0, \widetilde{H}_u^0, \nu_e, \nu_\mu, \nu_\tau)$ the neutral fermion mass matrix M_N this matrix is given by

$$M_{N\chi} = \begin{pmatrix} \mathcal{M}_{\chi^0} & m^T \\ m & 0 \end{pmatrix}, \tag{30}$$

where $\mathcal{M}_{\chi^0}$ is the usual neutralino mass matrix and

$$m = \begin{pmatrix} -\frac{1}{2}g'v_{L_e} & \frac{1}{2}gv_{L_e} & 0 & \epsilon_e \\ -\frac{1}{2}g'v_{L_\mu} & \frac{1}{2}gv_{L_\mu} & 0 & \epsilon_\mu \\ -\frac{1}{2}g'v_{L_\tau} & \frac{1}{2}gv_{L_\tau} & 0 & \epsilon_\tau \end{pmatrix}, \tag{31}$$

is the matrix describing R-parity violation. Here v_{L_a} are the vevs of sneutrinos induced by the presence of ϵ_i and B_i. The smallness of the R-parity violating parameters implies that the components of m are suppressed with respect to those in $\mathcal{M}_{\chi^0}$. Hence the resulting M_N matrix has a type-I seesaw structure so the effective light neutrino mass matrix can be obtained from the usual formula $m_\nu^0 = -m \cdot \mathcal{M}_{\chi^0}^{-1} \cdot m^T$, which can be expanded to give

$$(M_\nu)_{ab} = \alpha \Lambda_a \Lambda_b, \tag{32}$$

where α is a combination of SUSY parameters, while $\Lambda_a = \mu v_{L_a} + v_d \epsilon_a$ are known as the *alignment parameters*. The above matrix is projective and has two zero eigenvalues, therefore only one neutrino is massive at tree level. A natural choice is to ascribe this eigenvalue to the atmospheric scale whereas the solar mass scale, $\Delta m_{sol}^2 \ll \Delta m_{atm}^2$, arises from quantum corrections calculable at the one-loop level of the neutrino mass matrix in Eq. (32). Detailed computations of the one-loop contributions to the neutrino mass matrix are given in Refs. [154, 155]. The corrections are of the type

$$\left(m_\nu^{\mathrm{rad}} \right)_{ab} \approx \alpha^{(\mathrm{rad})} \Lambda_a \Lambda_b + \beta^{(rad)} (\Lambda_a \epsilon_b + \Lambda_a \epsilon_b) + \gamma^{(\mathrm{rad})} \epsilon_a \epsilon_b, \tag{33}$$

where the coefficients $\alpha^{(\mathrm{rad})}, \beta^{(\mathrm{rad})}, \gamma^{(\mathrm{rad})}$ are complicated functions of the SUSY parameters. These corrections generate a second non-zero mass eigenstate associated with the solar scale, and the corresponding mixing angle [6] θ_{12}.

----

[6] The neutrino mixing angles are determined as ratios of R-parity violating parameters ϵ_i and Λ_i.

The bilinear R-parity breaking model offers a hybrid mechanism combining seesaw-type and radiative contributions, thereby providing an explanation for the observed smallness of the solar squared mass splitting with respect to the atmospheric one.

The above scheme is both well motivated and testable at colliders. Indeed in the absence of R-parity, the lightest supersymmetric particle (LSP) is no longer protected and decays to standard model particles. The smallness of the breaking strength, required to account for neutrino masses, makes the lifetime of the LSP long enough so that it may decay within the detector with displaced vertices. Since LSP decays and neutrino masses have a common origin, one can show that ratios of LSP decay branching ratios correlate with the neutrino mixing angles measured at low energies [157]. This provides a remarkable connection which allows one to use neutrino oscillation data to test the model at the LHC see e. g. [158, 159].

5. SUMMARY AND OUTLOOK

We have given a brief overview of the low-scale $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ approach to neutrino mass generation. To chart out directions within such a broad neutrino landscape we used their possible phenomenological potential as a guide. We analyzed signatures associated to direct neutrino mass messenger production at the LHC, as well as messenger-induced lepton flavor violation processes. We have considered seesaw-based schemes as well as those with radiative or supersymmetric origin for the neutrino mass. We summarize our conclusions in table III. We stressed the phenomenological interest on radiative models, low scale seesaw schemes as well as the type-II seesaw "tuned" to lie at the low scale. We also briefly comment on the presence of WIMP cold dark matter candidates.

	Type-I	Type-II	Type-III	Inverse	Linear	Invers Type-III	Radiative
LHC	✗	✓	✓	✗	✗	✓	✓
LFV	✗	✓	✗	✓	✓	✓	✓

TABLE III. Neutrino mass models in terms of their phenomenological potential at the LHC and/or the sizable presence of lepton flavor violation phenomena where we use the same labeling convention as in the text. As we have explained in the text, "✗" could change to "✓" in the presence of new gauge bosons or supersymmetry.

In conclusion if the messengers responsible for the light neutrino masses lie at a very high scale, like in type-I seesaw, it will be very difficult if not impossible to have any detectable signal within the non-supersymmetric $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ seesaw framework. In contrast, within the low scale approach to neutrino mass we can have very interesting phenomenological implications. They can give rise to signatures at high energy collider experiments, as well as lepton flavor violation rates close to the sensitivity of planned experiments. In some of the schemes there is a natural WIMP dark matter candidate. In short, these scenarios may help reconstructing the neutrino mass from a variety of potentially over-constrained set of observables.

ACKNOWLEDGMENTS

[1] 1234547, CERN Cour. **53**, 21 (2013).
[2] G. Aad *et al.* (ATLAS Collaboration), Phys.Lett. **B716**, 1 (2012), arXiv:1207.7214 [hep-ex].
[3] S. Chatrchyan *et al.* (CMS Collaboration), Phys.Lett. **B716**, 30 (2012), arXiv:1207.7235 [hep-ex].
[4] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).

[5] F. Bonnet, M. Hirsch, T. Ota, and W. Winter, JHEP **1303**, 055 (2013), arXiv:1212.3045 [hep-ph].

[6] F. Bonnet, M. Hirsch, T. Ota, and W. Winter, JHEP **1207**, 153 (2012), arXiv:1204.5862 [hep-ph].

[7] A. de Gouvea and J. Jenkins, Phys.Rev. **D77**, 013008 (2008), arXiv:0708.1344 [hep-ph].

[8] F. Bonnet, D. Hernandez, T. Ota, and W. Winter, JHEP **10**, 076 (2009), arXiv:0907.3143 [hep-ph].

[9] M. B. Krauss, D. Meloni, W. Porod, and W. Winter, JHEP **1305**, 121 (2013), arXiv:1301.4221 [hep-ph].

[10] H. Nunokawa, S. J. Parke, and J. W. F. Valle, Prog. Part. Nucl. Phys. **60**, 338 (2008).

[11] D. Forero, M. Tortola, and J. W. F. Valle, Phys.Rev. **D86**, 073012 (2012), arXiv:arXiv:1205.4018 [hep-ph].

[12] A. Osipowicz *et al.* (KATRIN collaboration), (2001), hep-ex/0109033.

[13] J. Lesgourgues, G. Mangano, G. Miele, and S. Pastor, *Neutrino Cosmology* (Cambridge Univ Pr, 2013).

[14] A. Barabash, (2011), 75 years of double beta decay: yesterday, today and tomorrow, arXiv:1101.4502 [nucl-ex].

[15] E. Ma, Phys. Rev. Lett. **81**, 1171 (1998), arXiv:hep-ph/9805219.

[16] P. Minkowski, Phys. Lett. **B67**, 421 (1977).

[17] T. Yanagida, Conf.Proc. **C7902131**, 95 (1979).

[18] M. Gell-Mann, P. Ramond, and R. Slansky, Conf.Proc. **C790927**, 315 (1979), arXiv:1306.4669 [hep-th].

[19] J. Schechter and J. W. F. Valle, Phys. Rev. **D22**, 2227 (1980).

[20] R. N. Mohapatra and G. Senjanovic, Phys.Rev.Lett. **44**, 912 (1980).

[21] J. Schechter and J. W. F. Valle, Phys. Rev. **D25**, 774 (1982).

[22] T. P. Cheng and L.-F. Li, Phys. Rev. **D22**, 2860 (1980).

[23] M. Magg and C. Wetterich, Phys.Lett. **B94**, 61 (1980).

[24] C. Wetterich, Nucl.Phys. **B187**, 343 (1981).

[25] R. N. Mohapatra and G. Senjanovic, Phys. Rev. **D23**, 165 (1981).

[26] R. Foot, H. Lew, X. G. He, and G. C. Joshi, Z. Phys. **C44**, 441 (1989).

[27] J. Kersten and A. Y. Smirnov, Phys. Rev. **D76**, 073005 (2007), arXiv:0705.3221 [hep-ph].

[28] P.-H. Gu *et al.*, Phys. Rev. **D79**, 033010 (2009), arXiv:0811.0953 [hep-ph].

[29] P. S. B. Dev, A. Pilaftsis, and U.-k. Yang, Phys.Rev.Lett. **112**, 081801 (2014), arXiv:1308.2209 [hep-ph].

[30] R. N. Mohapatra and J. W. F. Valle, Phys. Rev. **D34**, 1642 (1986).

[31] E. Akhmedov *et al.*, Phys. Lett. **B368**, 270 (1996), hep-ph/9507275.

[32] E. Akhmedov *et al.*, Phys. Rev. **D53**, 2752 (1996), hep-ph/9509255.

[33] M. Malinsky, J. C. Romao, and J. W. F. Valle, Phys. Rev. Lett. **95**, 161801 (2005).

[34] K. S. Babu, Phys. Lett. **B203**, 132 (1988).

[35] A. Masiero and J. Valle, Phys.Lett. **B251**, 273 (1990).

[36] J. C. Romao, C. A. Santos, and J. W. F. Valle, Phys. Lett. **B288**, 311 (1992).

[37] P. Abreu *et al.* (DELPHI), Z. Phys. **C74**, 57 (1997).

[38] J. Beringer *et al.* (Particle Data Group), Phys.Rev. **D86**, 010001 (2012).

[39] R. Franceschini, T. Hambye, and A. Strumia, Phys. Rev. **D78**, 033002 (2008), arXiv:0805.1613 [hep-ph].

[40] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57**, 961 (1986).

[41] M. Hirsch *et al.*, Phys. Rev. **D78**, 013006 (2008), arXiv:0804.4072 [hep-ph].

[42] J. Esteves, J. Romao, M. Hirsch, A. Vicente, W. Porod, *et al.*, JHEP **1012**, 077 (2010), arXiv:1011.0348 [hep-ph].

[43] P. Fileviez Perez, T. Han, and T. Li, (2009), arXiv:0907.4186 [hep-ph].

[44] D. Aristizabal Sierra *et al.*, Phys. Rev. **D68**, 033006 (2003), hep-ph/0304141.

[45] J. N. Esteves *et al.*, JHEP **05**, 003 (2009).

[46] P. Dev and R. Mohapatra, Phys.Rev. **D81**, 013001 (2010), arXiv:0910.3924 [hep-ph].

[47] C.-H. Lee, P. Bhupal Dev, and R. Mohapatra, Phys.Rev. **D88**, 093010 (2013), arXiv:1309.0774 [hep-ph].

[48] F. Deppisch, T. S. Kosmas, and J. W. F. Valle, Nucl. Phys. **B752**, 80 (2006), hep-ph/0512360.

[49] F. Deppisch and J. W. F. Valle, Phys. Rev. **D72**, 036001 (2005), hep-ph/0406040.

[50] D. Forero, S. Morisi, M. Tortola, and J. W. F. Valle, JHEP **1109**, 142 (2011), arXiv:1107.6009 [hep-ph].

[51] J. Bernabeu *et al.*, Phys. Lett. **B187**, 303 (1987).

[52] M. C. Gonzalez-Garcia and J. W. F. Valle, Mod. Phys. Lett. **A7**, 477 (1992).

[53] M. Gonzalez-Garcia and J. Valle, Phys.Lett. **B216**, 360 (1989).

[54] G. C. Branco, M. N. Rebelo, and J. W. F. Valle, Phys. Lett. **B225**, 385 (1989).

[55] N. Rius and J. W. F. Valle, Phys. Lett. **B246**, 249 (1990).

[56] F. Bazzocchi, D. Cerdeno, C. Munoz, and J. W. F. Valle, Phys.Rev. **D81**, 051701 (2010), arXiv:0907.1262 [hep-ph].

[57] F. Bazzocchi, Phys.Rev. **D83**, 093009 (2011), arXiv:1011.6299 [hep-ph].

[58] H. Hettmansperger, M. Lindner, and W. Rodejohann, JHEP **1104**, 123 (2011), arXiv:1102.3432 [hep-ph].

[59] A. Das and N. Okada, Phys.Rev. **D88**, 113001 (2013), arXiv:1207.3734 [hep-ph].

[60] V. De Romeri, M. Hirsch, and M. Malinsky, Phys.Rev. **D84**, 053012 (2011), arXiv:1107.3412 [hep-ph].

[61] P. Nath, B. D. Nelson, H. Davoudiasl, B. Dutta, D. Feldman, *et al.*, Nucl.Phys.Proc.Suppl. **200-202**, 185 (2010), arXiv:1001.2693 [hep-ph].

[62] S. Das, F. Deppisch, O. Kittel, and J. Valle, Phys.Rev. **D86**, 055006 (2012), arXiv:1206.0256 [hep-ph].

[63] J. Aguilar-Saavedra, F. Deppisch, O. Kittel, and J. Valle, Phys.Rev. **D85**, 091301 (2012), arXiv:1203.5998 [hep-ph].

[64] F. F. Deppisch, N. Desai, and J. W. F. Valle, (2013), arXiv:1308.6789 [hep-ph].

[65] E. Ma, (2009), arXiv:0905.2972 [hep-ph].

[66] D. Ibanez, S. Morisi, and J. W. F. Valle, Phys. Rev. **D80**, 053015 (2009), arXiv:0907.3109 [hep-ph].

[67] S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. **D2**, 1285 (1970).

[68] P. Fileviez Perez, T. Han, G.-y. Huang, T. Li, and K. Wang, Phys. Rev. **D78**, 015018 (2008), arXiv:0805.3536 [hep-ph].

[69] F. del Aguila and J. A. Aguilar-Saavedra, Nucl. Phys. **B813**, 22 (2009), arXiv:0808.2468 [hep-ph].

[70] K. Babu and C. N. Leung, Nucl.Phys. **B619**, 667 (2001), arXiv:hep-ph/0106054 [hep-ph].

[71] P. W. Angel, N. L. Rodd, and R. R. Volkas, Phys.Rev. **D87**, 073007 (2013), arXiv:1212.6111 [hep-ph].

[72] Y. Farzan, S. Pascoli, and M. A. Schmidt, JHEP **1303**, 107 (2013), arXiv:1208.2732 [hep-ph].

[73] S. S. Law and K. L. McDonald, Int.J.Mod.Phys. **A29**, 1450064 (2014), arXiv:1303.6384 [hep-ph].

[74] A. Pilaftsis, Z.Phys. **C55**, 275 (1992), arXiv:hep-ph/9901206 [hep-ph].

[75] P. B. Dev and A. Pilaftsis, Phys.Rev. **D86**, 113001 (2012), arXiv:1209.4051 [hep-ph].

[76] P. Fileviez Perez and M. B. Wise, Phys. Rev. **D80**, 053006 (2009), arXiv:0906.2950 [hep-ph].

[77] A. Zee, Phys. Lett. **B93**, 389 (1980).

[78] D. Aristizabal Sierra and D. Restrepo, JHEP **08**, 036 (2006), arXiv:hep-ph/0604012.

[79] A. Y. Smirnov and M. Tanimoto, Phys.Rev. **D55**, 1665 (1997), arXiv:hep-ph/9604370 [hep-ph].

[80] C. Jarlskog, M. Matsuda, S. Skadhauge, and M. Tanimoto, Phys.Lett. **B449**, 240 (1999), arXiv:hep-ph/9812282 [hep-ph].

[81] P. H. Frampton and S. L. Glashow, Phys.Lett. **B461**, 95 (1999), arXiv:hep-ph/9906375 [hep-ph].

[82] A. S. Joshipura and S. D. Rindani, Phys.Lett. **B464**, 239 (1999), arXiv:hep-ph/9907390 [hep-ph].

[83] G. McLaughlin and J. Ng, Phys.Lett. **B464**, 232 (1999), arXiv:hep-ph/9907449 [hep-ph].

[84] K.-m. Cheung and O. C. Kong, Phys.Rev. **D61**, 113012 (2000), arXiv:hep-ph/9912238 [hep-ph].

[85] D. Chang and A. Zee, Phys.Rev. **D61**, 071303 (2000), arXiv:hep-ph/9912380 [hep-ph].

[86] D. A. Dicus, H.-J. He, and J. N. Ng, Phys.Rev.Lett. **87**, 111803 (2001), arXiv:hep-ph/0103126 [hep-ph].

[87] K. Balaji, W. Grimus, and T. Schwetz, Phys.Lett. **B508**, 301 (2001), arXiv:hep-ph/0104035 [hep-ph].

[88] E. Mitsuda and K. Sasaki, Phys.Lett. **B516**, 47 (2001).

[89] A. Ghosal, Y. Koide, and H. Fusaoka, Phys.Rev. **D64**, 053012 (2001), arXiv:hep-ph/0104104 [hep-ph].

[90] Y. Koide, Phys.Rev. **D64**, 077301 (2001), arXiv:hep-ph/0104226 [hep-ph].

[91] B. Brahmachari and S. Choubey, Phys.Lett. **B531**, 99 (2002), arXiv:hep-ph/0111133 [hep-ph].

[92] T. Kitabayashi and M. Yasue, Int.J.Mod.Phys. **A17**, 2519 (2002), arXiv:hep-ph/0112287 [hep-ph].

[93] Y. Koide, Nucl.Phys.Proc.Suppl. **111**, 294 (2002), arXiv:hep-ph/0201250 [hep-ph].

[94] M.-Y. Cheng and K.-m. Cheung, (2002), arXiv:hep-ph/0203051 [hep-ph].

[95] X. G. He and A. Zee, Phys.Lett. **B560**, 87 (2003), arXiv:hep-ph/0301092 [hep-ph].

[96] K. Hasegawa, C. Lim, and K. Ogure, Phys.Rev. **D68**, 053006 (2003), arXiv:hep-ph/0303252 [hep-ph].

[97] S. Kanemura, T. Ota, and K. Tsumura, Phys.Rev. **D73**, 016006 (2006), arXiv:hep-ph/0505191 [hep-ph].

[98] B. Brahmachari and S. Choubey, Phys.Lett. **B642**, 495 (2006), arXiv:hep-ph/0608089 [hep-ph].

[99] N. Sahu and U. Sarkar, Phys.Rev. **D78**, 115013 (2008), arXiv:0804.2072 [hep-ph].

[100] T. Fukuyama, H. Sugiyama, and K. Tsumura, Phys.Rev. **D83**, 056016 (2011), arXiv:1012.4886 [hep-ph].

[101] F. del Aguila, A. Aparici, S. Bhattacharya, A. Santamaria, and J. Wudka, JHEP **1206**, 146 (2012), arXiv:1204.5986 [hep-ph].

[102] L. Wolfenstein, Nucl.Phys. **B175**, 93 (1980).

[103] P. H. Frampton, M. C. Oh, and T. Yoshikawa, Phys.Rev. **D65**, 073014 (2002), arXiv:hep-ph/0110300 [hep-ph].

[104] X.-G. He, Eur.Phys.J. **C34**, 371 (2004), arXiv:hep-ph/0307172 [hep-ph].

[105] S. Kanemura, T. Kasai, G.-L. Lin, Y. Okada, J.-J. Tseng, *et al.*, Phys.Rev. **D64**, 053007 (2001), arXiv:hep-ph/0011357 [hep-ph].

[106] K. A. Assamagan, A. Deandrea, and P.-A. Delsart, Phys.Rev. **D67**, 035001 (2003), arXiv:hep-ph/0207302 [hep-ph].

[107] K. Babu and J. Julio, (2013), arXiv:1310.0303 [hep-ph].

[108] A. Aranda, C. Bonilla, and A. D. Rojas, Phys.Rev. **D85**, 036004 (2012), arXiv:1110.1182 [hep-ph].

[109] E. Ma, Phys.Rev. **D73**, 077301 (2006), arXiv:hep-ph/0601225 [hep-ph].

[110] J. Kubo, E. Ma, and D. Suematsu, Phys.Lett. **B642**, 18 (2006), arXiv:hep-ph/0604114 [hep-ph].

[111] D. Aristizabal Sierra, J. Kubo, D. Restrepo, D. Suematsu, and O. Zapata, Phys.Rev. **D79**, 013011 (2009), arXiv:0808.3340 [hep-ph].

[112] D. Suematsu, T. Toma, and T. Yoshida, Phys.Rev. **D79**, 093004 (2009), arXiv:0903.0287 [hep-ph].

[113] A. Adulpravitchai, M. Lindner, and A. Merle, Phys.Rev. **D80**, 055031 (2009), arXiv:0907.2147 [hep-ph].

[114] T. Toma and A. Vicente, JHEP **1401**, 160 (2014), arXiv:1312.2840.

[115] D. Schmidt, T. Schwetz, and T. Toma, Phys.Rev. **D85**, 073009 (2012), arXiv:1201.0906 [hep-ph].

[116] R. Bouchand and A. Merle, JHEP **1207**, 084 (2012), arXiv:1205.0008 [hep-ph].

[117] M. Aoki and S. Kanemura, Phys.Lett. **B689**, 28 (2010), arXiv:1001.0092 [hep-ph].

[118] M. Aoki, S. Kanemura, and H. Yokoya, Phys.Lett. **B725**, 302 (2013), arXiv:1303.6191 [hep-ph].

[119] S.-Y. Ho and J. Tandean, Phys.Rev. **D89**, 114025 (2014), arXiv:1312.0931 [hep-ph].

[120] S.-Y. Ho and J. Tandean, Phys.Rev. **D87**, 095015 (2013), arXiv:1303.5700 [hep-ph].

[121] Y. Kajiyama, J. Kubo, and H. Okada, Phys.Rev. **D75**, 033001 (2007), arXiv:hep-ph/0610072 [hep-ph].

[122] D. Suematsu, T. Toma, and T. Yoshida, Phys.Rev. **D82**, 013012 (2010), arXiv:1002.3225 [hep-ph].

[123] Y. Kajiyama, H. Okada, and T. Toma, Eur.Phys.J. **C71**, 1688 (2011), arXiv:1104.0367 [hep-ph].

[124] Y. Kajiyama, H. Okada, and T. Toma, (2011), arXiv:1109.2722 [hep-ph].

[125] M. Klasen, C. E. Yaguna, J. D. Ruiz-Alvarez, D. Restrepo, and O. Zapata, JCAP **1304**, 044 (2013), arXiv:1302.5298 [hep-ph].

[126] P.-K. Hu, (2012), arXiv:1208.2613 [hep-ph].

[127] M. Hirsch, R. Lineros, S. Morisi, J. Palacio, N. Rojas, *et al.*, JHEP **1310**, 149 (2013), arXiv:1307.8134 [hep-ph].

[128] V. Brdar, I. Picek, and B. Radovcic, Phys.Lett. **B728**, 198 (2014), arXiv:1310.3183 [hep-ph].

[129] D. Restrepo, O. Zapata, and C. E. Yaguna, JHEP **1311**, 011 (2013), arXiv:1308.3655 [hep-ph].

[130] S. S. Law and K. L. McDonald, JHEP **1309**, 092 (2013), arXiv:1305.6467 [hep-ph].

[131] A. Zee, Nucl. Phys. **B264**, 99 (1986).

[132] J. Schechter and J. W. F. Valle, Phys. Rev. **D25**, 2951 (1982).

[133] J. T. Peltoniemi and J. W. F. Valle, Phys. Lett. **B304**, 147 (1993), hep-ph/9301231.

[134] M. Nebot, J. F. Oliver, D. Palao, and A. Santamaria, (2007), arXiv:0711.0483 [hep-ph].

[135] D. Schmidt, T. Schwetz, and H. Zhang, (2014), arXiv:1402.2251 [hep-ph].

[136] K. S. Babu and C. Macesanu, Phys. Rev. **D67**, 073010 (2003), hep-ph/0212058.

[137] D. Aristizabal Sierra and M. Hirsch, JHEP **12**, 052 (2006), hep-ph/0609307.

[138] J. Herrero-Garcia, M. Nebot, N. Rius, and A. Santamaria, (2014), arXiv:1402.4491 [hep-ph].

[139] L. M. Krauss, S. Nasri, and M. Trodden, Phys.Rev. **D67**, 085002 (2003), arXiv:hep-ph/0210389 [hep-ph].

[140] J. N. Ng and A. de la Puente, Phys.Lett. **B727**, 204 (2013), arXiv:1307.2606.

[141] A. Ahriche, C.-S. Chen, K. L. McDonald, and S. Nasri, (2014), arXiv:1404.2696 [hep-ph].

[142] M. Aoki, S. Kanemura, and O. Seto, Phys.Rev.Lett. **102**, 051805 (2009), arXiv:0807.0361 [hep-ph].

[143] M. Aoki, S. Kanemura, and O. Seto, Phys.Rev. **D80**, 033007 (2009), arXiv:0904.3829 [hep-ph].

[144] M. Gustafsson, J. M. No, and M. A. Rivera, Phys.Rev.Lett. **110**, 211802 (2013), arXiv:1212.4806 [hep-ph].

[145] C.-S. Chen, K. L. McDonald, and S. Nasri, (2014), arXiv:1404.6033 [hep-ph].

[146] S. P. Martin, (1997), hep-ph/9709356.

[147] R. Barbier *et al.*, Phys. Rept. **420**, 1 (2005), arXiv:hep-ph/0406039.

[148] V. D. Barger, G. F. Giudice, and T. Han, Phys. Rev. **D40**, 2987 (1989).

[149] H. Baer and X. Tata, *Weak scale supersymmetry: From superfields to scattering events* (Cambridge University Press, UK, 2006).

[150] P. Nogueira, J. Romao, and J. Valle, Phys.Lett. **B251**, 142 (1990).

[151] L. J. Hall and M. Suzuki, Nucl. Phys. **B231**, 419 (1984).

[152] M. A. Diaz, J. C. Romao, and J. W. F. Valle, Nucl. Phys. **B524**, 23 (1998).

[153] J. C. Romao, A. Ioannisian, and J. W. F. Valle, Phys. Rev. **D55**, 427 (1997), hep-ph/9607401.

[154] M. Hirsch *et al.*, Phys. Rev. **D62**, 113008 (2000), err-ibid. **D65**:119901,2002, hep-ph/0004115.

[155] M. A. Diaz *et al.*, Phys. Rev. **D68**, 013009 (2003), hep-ph/0302021.

[156] E. J. Chun and S. K. Kang, Phys. Rev. **D61**, 075012 (2000), hep-ph/9909429.

[157] B. Mukhopadhyaya, S. Roy, and F. Vissani, Phys. Lett. **B443**, 191 (1998).

[158] F. de Campos, O. Eboli, M. Magro, W. Porod, D. Restrepo, *et al.*, Phys.Rev. **D86**, 075001 (2012), arXiv:1206.3605 [hep-ph].

[159] F. De Campos *et al.*, Phys. Rev. **D82**, 075002 (2010), arXiv:1006.5075 [hep-ph].