The Cabibbo angle as a universal seed for quark and lepton mixings

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A model-independent ansatz to describe lepton and quark mixing in a unified way is suggested based upon the Cabibbo angle. In our framework neutrinos mix in a "Bi-Large" fashion, while the charged leptons mix as the "down-type" quarks do. In addition to the standard Wolfenstein parameters (λ, A) two other free parameters (ψ, δ) are needed to specify the physical lepton mixing matrix. Through this simple assumption one makes specific predictions for the atmospheric angle as well as leptonic CP violation in good agreement with current observations.

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A striking observation vindicated by recent experimental neutrino data is that the smallest of the lepton mixing angles is surprisingly large, similar to the largest of the quark mixing parameters, namely the Cabibbo angle (θ_c) [1, 2]. An interesting lepton mixing scheme called "Bi-Large" (BL) mixing has been proposed recently [3] and subsequently studied in Refs. [4–6]. This mixing scheme assumes the atmospheric and the solar mixing angles to be equal and proportional to the reactor angle. In contrast to the Bi-maximal (BM) scenario [7, 8], within the BL scheme the atmospheric mixing angle does not need to be strictly "Maximal", but simply "Large" in general. In summary, BL mixing posits, $\sin\theta_{13} \simeq \lambda$, $\sin\theta_{12} = \sin\theta_{23} \sim \lambda$, where $\lambda = \sin\theta_c$.

Such BL mixing ansatz can be motivated in string theories. Indeed, in F-theory motivated Grand Unified Theory (GUT) models, a geometrical unification of charged lepton and neutrino sectors leads to a mild hierarchy in the neutrino mixing matrix in which θ^ν_{12} and θ^ν_{23} become large and comparable while $\theta^\nu_{13} \sim \theta_c \sim \sqrt{\alpha_{GUT}} \sim 0.2$ [9] [1]. Understanding the origin of the above relation from first principles is beyond the scope of this note. We stress however that this ansatz can be associated to specific flavor symmetries as suggested in Ref. [4] or Ref. [10], rather than being a mere "numerical coincidence".

A successful framework for attacking the flavour problem constitutes an important quest in contemporary particle physics. A relevant question arises as to whether attempted solutions to the flavour problem may indicate foot-prints of unification or not. In the present note we look into some possible links between quark and lepton mixing parameters from a phenomenological "bottom-up perspective" [2].

[1] Neglecting the contribution from the charged lepton sector.
[2] An earlier alternative in the literature is "Quark-Lepton complementarity (QLC)" [11–14].

In the quark sector the largest mixing is between the flavor states d and s, and is interpreted in terms of the Cabibbo angle [15] which is approximately 13^0. The matrix V_{CKM} is parametrized in terms of three independent angles and one complex CP phase [16–18]. A clever approximate presentation was proposed by Wolfenstein [19], and is by now standard, namely

$$V_{CKM} = \begin{bmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} \quad (1)$$

up to $\mathcal{O}(\lambda^4)$ where, λ, A, η and ρ are four independent Wolfenstein parameters, with $\lambda = \sin\theta_c \approx 0.22$.

In contrast, the mixing in the lepton sector is very different from quark mixing. While the solar and atmospheric angle are large: $\theta_{12} \approx 35^0$ and $\theta_{23} \approx 49^0$, the 1-3 mixing parameter in the lepton sector is the smallest and was believed to vanish according to the earlier results. However in last few years it has been established [20–22] that this mixing, now precisely measured, is almost as large as the d-s mixing in quark sector, $\theta_{13} \approx 9^0 \sim \mathcal{O}(\theta_c)$. This excludes the simplest proposed schemes of neutrino mixing, which need to be revised in order to be consistent with observation [23]. Up to Majorana phases the Bi-Large mixing factor may be parametrized as follows

$$U_{BL} \approx \begin{bmatrix} c(1-\frac{\lambda^2}{2}) & \psi\lambda(1-\frac{\lambda^2}{2}) & \lambda \\ -c\psi\lambda(1+\lambda) & c^2 - \lambda^3\psi^2 & \psi\lambda(1-\frac{\lambda^2}{2}) \\ \lambda^2\psi^2 - \lambda c^2 & -c\psi\lambda(1+\lambda) & c(1-\frac{\lambda^2}{2}) \end{bmatrix} \quad (2)$$

One sees that $\sin\theta_{12} = \sin\theta_{23} \approx \psi\lambda$, with $\sin\theta_{13} = \lambda$. With this parametrization it is evident that the Cabibbo angle is the *seed* for the mixing in both the quark and the lepton sector. Here, $c \approx \cos\sin^{-1}(\psi\lambda)$. In what follows we discuss the possible forms of the charged lepton contribution to the lepton mixing matrix.

As originally proposed the Bi-Large ansatz does not fit current neutrino oscillation data, so that corrections are required. A possibility is that BL arises only in the flavor basis and deviations are induced from the charged lepton sector. Here we consider this case within a GUT inspired framework based upon $SO(10)$ and $SU(5)$.

In simplest SO(10) schemes the charged lepton mass matrix is approximated to that of down type quarks, $M_e \sim M_d$. This leads to the assumption, $U_l \approx V_{CKM}$. In V_{CKM} the dominant parameter is $\theta_{12}^{CKM} = \theta_c$, which is followed by θ_{23}^{CKM}. We classify the parametrization of U_l in two catagories: (i) with 1-2 rotation only: $U_l = U_{12}(\lambda)$ and (ii) with 2-3 rotation in addition to that of 1-2, $U_l = U_{23}(A\,\lambda^2).U_{12}(\lambda)$. As suggested in Ref. [24], we associate a complex phase parameter δ with 1-2 rotation, so that $U_{12} \rightarrow U_{12}(\theta_c, \delta)$. We have,

$$\begin{aligned} U_{l_1} &= \Psi R_{12}^{l}(\theta_{12}^{CKM})\Psi' \\ &\approx \begin{bmatrix} 1-\frac{1}{2}\lambda^2 & \lambda\, e^{-i\delta} & 0 \\ -\lambda\, e^{i\delta} & 1-\frac{1}{2}\lambda^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (\textbf{Type-1}) \end{aligned} \tag{3}$$

$$\begin{aligned} U_{l_2} &= R_{23}^{l}(\,\theta_{23}^{CKM}\,).\Psi.R_{12}^{l}(\,\theta_{12}^{CKM}\,)\Psi' \\ &\approx \begin{bmatrix} 1-\frac{1}{2}\lambda^2 & \lambda\, e^{-i\delta} & 0 \\ -\lambda\, e^{i\delta} & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3 e^{i\delta} & -A\lambda^2 & 1 \end{bmatrix}, \qquad (\textbf{Type-2}) \end{aligned} \tag{4}$$

where, we have $\Psi = diag\{e^{-i\delta/2}, e^{i\delta/2}, 1\}$ and $\Psi' = \Psi^{\dagger}$.

Similar within simplest SU(5) scheme one expects, $M_e \sim M_d^T$. This gives rise to other two possibilities which can be expressed as in the following:

$$\begin{aligned} U_{l_3} &= \Psi R_{12}^{l^T}(\,\theta_{12}^{CKM}\,)\Psi' \\ &\approx \begin{bmatrix} 1-\frac{1}{2}\lambda^2 & -\lambda\, e^{-i\delta} & 0 \\ \lambda\, e^{i\delta} & 1-\frac{1}{2}\lambda^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (\textbf{Type-3}) \end{aligned} \tag{5}$$

$$\begin{aligned} U_{l_4} &= \Psi.R_{12}^{l^T}(\,\theta_{12}^{CKM}\,)\Psi'.R_{23}^{l^T}(\,\theta_{23}^{CKM}\,) \\ &\approx \begin{bmatrix} 1-\frac{1}{2}\lambda^2 & -\lambda\, e^{-i\delta} & A\lambda^3 e^{-i\delta} \\ \lambda\, e^{i\delta} & 1-\frac{1}{2}\lambda^2 & -A\lambda^2 \\ 0 & A\lambda^2 & 1 \end{bmatrix} \quad (\textbf{Type-4}) \end{aligned} \tag{6}$$

The physical lepton mixing matrix is simply

$$U_{lep} = U_l^{\dagger}.U_{BL}.I_{\phi}\,, \tag{7}$$

where U_{BL}, represents the Bi-Large neutrino mixing matrix and $I_{\phi} = diag(e^{i\alpha}, e^{i\beta}, 1)$, where α and β are the two additional CP violating phases associated to the Majorana nature of the neutrinos [17] [3]. In what follows we base our discussion upon the above four different choices of the charged lepton diagonalizing matrix choices of U_l in Eqs. (3)-(6).

As an example here we choose the Type-4 charged lepton diagonalizing matrix, U_{l_4}, (see Eq. (6)) and construct the Type-4 BL based scheme,

$$(U_{lep})_4 = U_{l_4}^{\dagger}.U_{BL}.I_{\phi}, \tag{8}$$

In $(U_{lep})_4$, the free parameters are ψ and δ. From $(U_{lep})_4$, the mixing angles are given by

$$s_{13}^2 \;\approx\; \lambda^2\left(s_\theta^2 + 2s_\theta\cos\delta + 1\right), \tag{9}$$

$$s_{12}^2 \;\approx\; s_\theta^2 + \lambda^2\left(c_\theta^4 + s_\theta^4 - s_\theta^2\right) + 2c_\theta^2\lambda s_\theta\cos\delta, \tag{10}$$

$$s_{23}^2 \;\approx\; s_\theta^2 + \lambda^2\left(2Ac_\theta s_\theta + s_\theta^4 + 2s_\theta^3\cos\delta - s_\theta^2 - 2s_\theta\cos\delta\right). \tag{11}$$

In order to obtain the rephasing-invariant CP violation parameter relevant for the description of neutrino oscillations we use the relation $J_{CP} = Im[U_{e1}^{*}.U_{\mu3}^{*}.U_{\mu1}.U_{e3}]$ for the Jarkslog invariant J_{CP} [28], and obtain,

$$J_{cp} \approx -c_\theta^2 s_\theta^3 \lambda\sin\delta. \tag{12}$$

where, $s_\theta = \psi\lambda$. It is evident that the all observables are given in terms of the parameters, λ, A, ψ and the unphysical phase δ, of which λ and A are the standard Wolfenstein parameters with $\lambda \approx 0.2245$, $A = 0.823$ [29] while the two parameters: ψ and δ are free.

How to choose ψ and δ? In fact, this task is not too complicated. One can choose ψ and δ in such a way, that any two of the three observable parameters, solar, reactor and atmospheric mixing angles are consistent with the neutrino oscillation data [1, 2], while the prediction for the remaining one will determine the tenability of the model.

First note that the determination of solar and reactor angles is rather stable irrespective of the neutrino mass spectrum. Hence it seems reasonable to use solar and reactor angles for the parametrization of the two unknowns. Hence we focus upon the predictions for θ_{23} and J_{CP} (or δ_{CP}), given their current indeterminacy from global neutrino oscillation data analysis [2]. Although consistent with maximal mixing, the possibility of θ_{23} lying within the first octant is certainly not excluded for normal ordering of neutrino masses. Moreover, probing for CP violation in the lepton sector is the next challenge for neutrino oscillation experiments. Hence

[3] As shown in [25] these phases are physical and affect lepton number violating processes such as neutrinoless double beta decay [26, 27].

Type	ψ	δ/π	$\sin^2\theta_{23}$	δ_{CP}/π	J_{cp}
1	$2.9521^{+0.2087}_{-0.2043}$	$1.764^{+0.0476}_{-0.0428}$	$0.4585^{+0.08543}_{-0.08646}$	$1.2308^{+0.0692}_{-0.0717}$,	$0.0250^{+0.0137}_{-0.0105}$
2	$2.9521^{+0.2087}_{-0.2043}$	$1.764^{+0.0476}_{-0.0428}$	$0.4174^{+0.0921}_{-0.0937}$	$1.2159^{+0.0754}_{-0.0733}$,	$0.0250^{+0.0137}_{-0.0105}$
4	$2.9522^{+0.2087}_{-0.2201}$	$0.7644^{+0.0476}_{-0.0427}$	$0.4585^{+0.0855}_{-0.08641}$	$1.2303^{+0.0717}_{-0.0713}$	$0.0250^{+0.0137}_{-0.0105}$
4	$2.9522^{+0.2087}_{-0.2201}$	$0.7644^{+0.0476}_{-0.0427}$	$0.4996^{+0.0927}_{-0.0935}$	$1.2303^{+0.0717}_{-0.0713}$	$0.0250^{+0.0137}_{-0.0105}$

TABLE I: Summary of the results corresponding to four BL models. ψ and δ corresponds to the central $\pm 3\sigma$ range of s^2_{12}, s^2_{13}, λ and A. We have taken, $s^2_{12} = [0.278, 0.375]$, $s^2_{13} = [0.0177, 0.0297]$, $\lambda = [0.22551 - 0.001, 0.22551 + 0.001]$ and $A = [0.813 - 0.029, 0.813 - 0.040]$. The other observables s^2_{23}, δ_{cp} (the Dirac type CP phase) and J_{cp} (Jarkslog invariant parameter) are the theoretical predictions for each model. This is to be noted that the best result of the Type-4 BL model is consistent with the maximal mixing prediction.

in addition to the prediction for the atmospheric angle, we use the prediction of our ansatz for J_{CP} (or δ_{CP}) in order to scrutinize the viability of our ansatz, in any of the above forms. The results are summarized in Table I. For definiteness we discuss here in more detail only the result for the type-4 BL scheme, see Fig. (1), similar results can be found for the other cases in the Table. In Fig. (1) we plot the free parameters δ and ψ. In the left panel we show the contour plot for s_{13} (horizontal band) and s_{12} (vertical band). The best fit value $s^2_{12} \approx 0.323$ and $s^2_{13} \approx 0.023$ [2] correspond to choosing $\psi \approx 2.967$ and $\delta \approx 0.757\,\pi$. We note that, with above choice of the two parameters, θ_{23} is consistent with maximal. The CP-invariant J_{cp} is approximately 0.02.

The corresponding lepton mixing matrix corresponding to the Type-4 BL scheme is the following,

$$U_4 \approx \begin{bmatrix} -u^*(1+\lambda)\{u(\lambda-1)+\psi\lambda^2\}c & (\psi - u^*c^2)\lambda + \psi\lambda^3 & \lambda - (\frac{\lambda}{2} + u^*\psi)\lambda^2 \\ \frac{c\lambda}{2}\{(\lambda^2-2)(u+\psi) - 2\lambda(\psi + cA\lambda)\} & c^2(1-\frac{\lambda^2}{2}) - \psi\lambda^2\{u + (\psi + cA)\lambda\} & \psi\lambda(1-\lambda^2) + (cA-u)\lambda^2 \\ \{\psi^2 + cA(u+\psi)\lambda\}\lambda^2 - c^2 & -c\{\psi + (\psi + cA)\lambda\}\lambda & -\psi A\lambda^3 + c(1-\frac{\lambda^2}{2}) \end{bmatrix}, \qquad (13)$$

where $u = e^{i\delta}$ and $c = \cos\sin^{-1}(\psi\lambda)$.

In Table. I, we gather the results for all the four BL schemes discussed above.

In summary we proposed a generalized fermion mixing ansatz where the neutrino mixing is Bi-Large, while the charged lepton mixing matrix is CKM-like. Inspired by SO(10) and SU(5) unification, we select four CKM-like charged lepton diagonalizing matrices, U_l's (Type-1,2,3,4) and discuss the phenomenological viability of the resulting schemes. All the four models are congruous with best-fit solar and reactor angles, making definite predictions for the atmospheric angle and CP phase, which may be further tested in upcoming neutrino experiments. In particular the Type-4 BL model appears interesting in the sense that it extends the original BL model to encompass maximal atmospheric mixing. Ours is a "theory-inspired" bottom-up approach to the flavour problem, that highlights the role of θ_c as the universal seed of quark and lepton mixings and incorporates the main characteristic features of unification models. We have shown how this generalizes the original Bi-Large ansatz [3] to make it fully realistic. Further investigation on the physics underlying this ansatz may bring new insights on both fermion mixing and unification.

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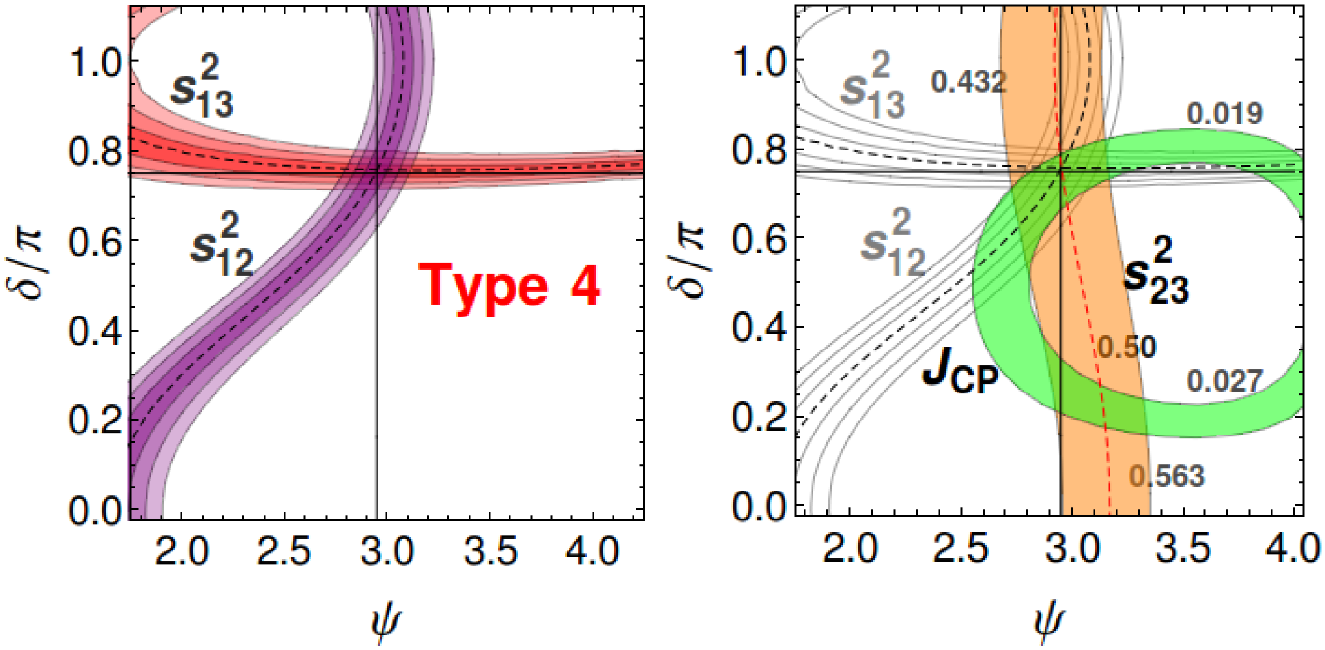

FIG. 1: The parametrization of ψ and δ, and prediction on s^2_{23} and J_{cp} are illustrated for Type-4 BL. For all the cases, ψ and δ are first parametrized with respect to best -fit, 1σ, 2σ and 3σ ranges of s^2_{12} and s^2_{13} which are then are used to predict s^2_{23} and J_{cp}. In the above illustration we fix λ and A at their central values: $\lambda = 0.22551$ and $A = 0.813$