Small neutrino masses and gauge coupling unification

Sofiane M. Boucenna,[1,*] Renato M. Fonseca,[1,†] Félix González-Canales,[1,‡] and José W.F. Valle[1,§]
[1] *Instituto de Física Corpuscular (CSIC-Universitat de València), Apdo. 22085, E-46071 Valencia, Spain.*
(Dated: February 6, 2015)

The physics responsible for gauge coupling unification may also induce small neutrino masses. We propose a novel gauge mediated radiative seesaw mechanism for calculable neutrino masses. These arise from quantum corrections mediated by new $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1) gauge bosons and the physics driving gauge coupling unification. Gauge couplings unify for a 3-3-1 scale in the TeV range, making the model directly testable at the LHC.

PACS numbers: 14.60.Pq, 12.60.Cn, 14.60.St, 14.70.Pw, 12.15.Ff

Preliminaries

The fact that gauge coupling unification is a "near-miss" within the Standard Model (SM) provides an indication in favor of the idea of unification [1]. Likewise, the existence of neutrino masses, required to account for neutrino oscillation data [2], also provides another motivation towards unified or GUT (Grand Unified Theory)-like extensions of the SM. However, the most characteristic feature of GUT-type unification, namely matter instability, has so far defied experimental confirmation [3]. On the other hand, neither the generation of neutrino masses nor the tilting in the evolution of the gauge couplings require unification in the conventional sense. For instance, it is well known that the gauge couplings merge in the minimal supersymmetric extension the SM, provided that supersymmetric states lie around the TeV scale [3, 4]. So far, though, there has been no trace of such states in the LHC data [5].

Here we consider an alternative approach in which new physics at the TeV scale realizes an extended electroweak gauge structure with perturbatively conserved baryon-number. For definiteness we consider the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ [6, 7] (3-3-1) framework, which implies that the number of generations equals the number of colors, in order to cancel anomalies. This scheme has attracted attention recently also in connection with B physics [8–10], or flavor symmetries [11–13]. In this letter we present a model in which the gauge couplings can naturally unify at some accessible energy, and where small calculable neutrino masses are induced by new gauge bosons exchange, in the absence of supersymmetry. Neutrino masses arise at the TeV scale [14] instead of the conventional high-scale seesaw mechanism [15]. We first recall that, by adding three gauge singlet fermions S_i, the light neutrinos acquire mass only at one-loop order [16]. Unfortunately, however, unification does not occur, as can be seen in Fig. 1. This is mostly due to fact that the new gauge bosons make α_L weaker at high energies, while the new colored particles strengthen α_C. Hence we contemplate the possibility of unifying the gauge couplings in such a scheme [1] by promoting the three fermion singlets to three octets of the enlarged electroweak symmetry. The new variant not only opens the possibility of reconciling neutrino mass generation with gauge coupling unification but also provides a novel radiative seesaw mechanism. The $SU(3)_L \otimes U(1)_X$ gauge group is broken down to the standard $SU(2)_L \otimes U(1)_Y$ model at some scale M_{331} characterizing the new gauge boson masses. This scale is found to lie in the $1-10$ TeV range, with a plethora of new states expected to be directly accessible to LHC searches.

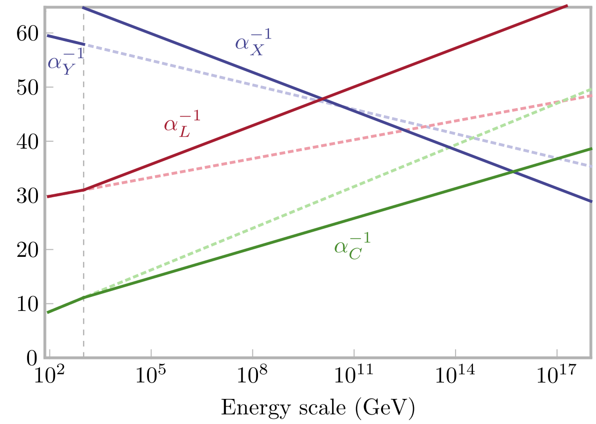

FIG. 1: Running of the gauge couplings in the SM (dashed lines) and in the model in Ref. [16] (solid lines). Here the M_{331} scale is set to 1 TeV.

The model

We consider a simple variant of the model introduced in [16], where the fermion singlet is now promoted to an octet representation of $SU(3)_L$. The model is based on the same $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge symmetry, extended with a global $U(1)_{\mathcal{L}}$, which is necessary in order to consistently define lepton number, and an auxiliary

[1] For other RGE studies in the context of 3-3-1 models see Ref. [17].

parity symmetry whose purpose will be made clear below.

The model contains three generations of lepton $SU(3)_L$ anti-triplets, two generations of quark triplets and one of anti-triplets (quarks and charged leptons are accompanied with their right-handed $SU(3)_L$ singlet partners), three generations of fermion octets, and finally three scalar boson anti-triplets. We summarize the particle content of the model in Table I. The allowed lepton interactions compatible with the quantum number assignments given in Table I are the following:

$$\mathscr{L}_{\text{leptons}} = \left(y^{\ell}_{ij}\right)^* \psi^T_{L,i} C \ell^c_{R,j} \phi^*_1 + \left(y'_{ij}\right)^* \psi^T_{L,i} C \Omega^c_j \phi^*_2 + \frac{1}{2}\left(M_{8ij}\right)^* \left(\Omega^c_i\right)^T C \Omega^c_j + \text{h.c.} \tag{1}$$

The components of ψ_L and the ϕ_j are written as:

$$\psi_{L,i} = \begin{pmatrix} \ell_L \\ -\nu_L \\ N^c \end{pmatrix}_i, \; \phi_1 = \begin{pmatrix} \phi^0_1 \\ -\phi^+_1 \\ \widetilde{\phi}^+_1 \end{pmatrix}, \; \phi_{2,3} = \begin{pmatrix} \phi^-_{2,3} \\ -\phi^0_{2,3} \\ \widetilde{\phi}^0_{2,3} \end{pmatrix}. \tag{2}$$

The scalars take vacuum expectation values (*vevs*) in the directions $\langle\phi_1\rangle = (k_1, 0, 0)$ and $\langle\phi_{2,3}\rangle = (0, -k_{2,3}, n_{2,3})$. As for the octets, one can write

$$\Omega^c_i = \begin{pmatrix} -\frac{1}{\sqrt{2}}T^0 + \frac{1}{\sqrt{6}}\widetilde{N}^c & -T^+ & \overline{\ell}_L \\ -T^- & \frac{1}{\sqrt{2}}T^0 + \frac{1}{\sqrt{6}}\widetilde{N}^c & -\overline{\nu}_L \\ \widetilde{\ell}_L & -\widetilde{\nu}_L & -\frac{2}{\sqrt{6}}\widetilde{N}^c \end{pmatrix}_i, \tag{3}$$

such that Ω^c_i is transformed into $U\Omega^c_i U^\dagger$ under an $SU(3)_L$ gauge transformation, where U is the transformation matrix of the triplet representation.

Under $SU(2)_L \otimes U(1)_Y$, each Ω^c_i breaks into the representations $(\mathbf{3}, 0) \equiv \left(T^+, T^0, T^-\right)_i$, $\left(\mathbf{2}, \frac{1}{2}\right) \equiv \left(\overline{\ell}_L, -\overline{\nu}_L\right)_i$, $\left(\mathbf{2}, -\frac{1}{2}\right) \equiv \left(\widetilde{\nu}_L, \widetilde{\ell}_L\right)_i$, $(\mathbf{1}, 0) \equiv \widetilde{N}^c_i$, so there are four new charged leptons (T^+, T^-, $\widetilde{\ell}_L$, $\overline{\ell}_L$) and four new neutral fermion states (T^0, $\overline{\nu}_L$, $\widetilde{\nu}_L$, $\widetilde{\ell}_L$) in each generation.

	Left-handed Fermions							Scalars		
	ψ_L	ℓ^*_R	Q^{12}_L	Q^3_L	u^*_R	d^*_R	Ω^*	ϕ_1	ϕ_2	ϕ_3
$SU(3)_C$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$SU(3)_L$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{8}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$
$U(1)_X$	$-\frac{1}{3}$	1	0	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
$U(1)_{\mathcal{L}}$	$-\frac{1}{3}$	1	$-\frac{2}{3}$	$\frac{2}{3}$	0	0	0	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$
Z_2	+	−	+	−	−	+	+	−	+	+
multiplicity	3	3	2	1	4	5	3	1	1	1

TABLE I: Field content of the model.

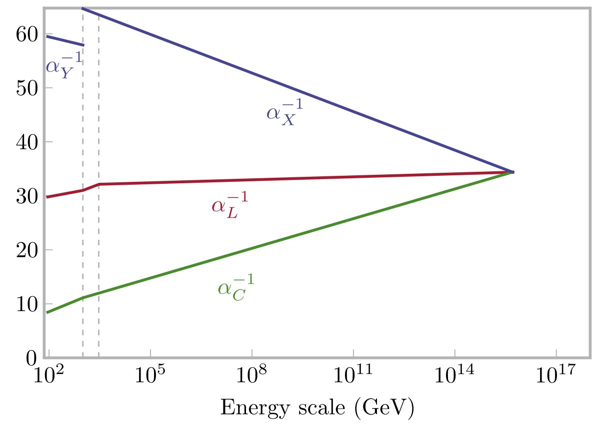

FIG. 2: Gauge coupling unification in the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ model with three fermion octets with a 3 TeV mass.

Note that the $U(1)_{\mathcal{L}}$ charge assignment of Ω_i and ϕ_2 differs from the one of the singlet of Ref. [16] so as to allow for a (vector-like) octet mass term, M_8, in Eq. 1 [2]. On the other hand, the Z_2 symmetry forbids a $\psi_L\psi_L\phi_1$ coupling, which leads to the existence of one massless neutrino state in each generation, at the tree level.

Note also that the electric charge and lepton number assignments of the particles of the model follow from [3]

$$Q = T_3 + \frac{1}{\sqrt{3}}T_8 + X\,, \tag{4}$$

$$L = \frac{4}{\sqrt{3}}T_8 + \mathcal{L}\,, \tag{5}$$

where T_3 and T_8 are the diagonal generators of $SU(3)_L$.

Gauge coupling unification

The one-loop renormalization group equation of the $\alpha_i \equiv g_i^2/4\pi$ is given by [20, 21]:

$$\frac{d\alpha_i^{-1}}{dt} = -\frac{b_i}{2\pi}\,, \tag{6}$$

where t is the logarithm of the energy scale, and the b_i coefficients are functions of the Casimir of the gauge group, $C\left(G_i\right)$, and of the Dynkin index of the scalar and (Weyl) fermion representations, $T\left(S\right)$ and $T\left(F\right)$, respectively:

$$b_i = -\frac{11}{3}C\left(G_i\right) + \frac{2}{3}\sum_F T\left(F\right) + \frac{1}{3}\sum_S T\left(S\right)\,. \tag{7}$$

For the SM, the b_i are $b^{\rm SM} = \left\{-7, -\frac{19}{6}, \frac{41}{10}\right\}$, while in the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ phase they read

[2] This term is required in order to provide an adequately large mass to the new charged leptons.

[3] As T_8 is a gauge generator, there is no physical Goldstone boson associated with spontaneous lepton number violation [18, 19].

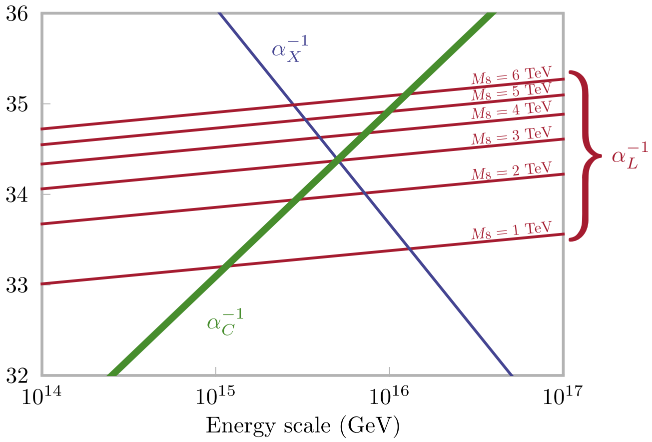

FIG. 3: Values of the gauge coupling constants near unification, assuming that the 3-3-1 scale is 1 TeV. The thickness of the α_C line reflects the 1σ uncertainty in the measurement of the strong coupling constant at the m_Z scale [3], while the octet mass affects the running of α_L. One sees how unification prefers M_8/M_{331} to lie roughly between 1 and 6.

$b^{331} = \left\{-5, -\frac{13}{2}+2n, \frac{13}{2}\right\}$, for n active fermion octets Ω_i. It should be noted that while we do not speculate here about the possible embedding of $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ into some bigger group (for example E_6), it can be shown on very general grounds that the $U(1)_X$ charge normalization should be $X_{\text{canonical}} = \sqrt{3}/2X$. Given the relation between X and the SM hypercharge indicated by Eq. 4, it follows that $\alpha_Y^{-1} = \frac{1}{5}\left(\alpha_L^{-1} + 4\alpha_X^{-1}\right)$ at the 3-3-1 breaking scale. Figure 2 illustrates the running of the gauge coupling constants in our model, with the 3-3-1 scale fixed at 1 TeV and the three octets Ω_i integrated out at 3 TeV. The exotic scalar states are also integrated out at the 1 TeV scale, although the running of the gauge couplings is not very sensitive to this value. Given that the b_i^{SM} coefficients are not very different from b_i^{331} with the three octets, unification is sensitive mostly to the ratio M_8/M_{331}, and not to the 3-3-1 scale *per se*. The effect is shown in Fig. 3; allowing for threshold and 2-loop effects, one can see that unification constrains $\beta^{-1} \equiv M_8/M_{331}$ to lie between 1 and 6.

Mass matrices

Lepton mass matrices arise from the Lagrangian in Eq. 1 after $SU(3)_L \otimes U(1)_X$ breaks to $U(1)_{\text{em}}$,

$$\mathscr{L}^{\ell,\nu}_{\text{lepton masses}} = \boldsymbol{\ell}^T C \mathcal{M}_\ell^* \boldsymbol{\ell^c} + \frac{1}{2}\boldsymbol{\nu}^T C \mathcal{M}_\nu^* \boldsymbol{\nu} + \text{h.c.} \tag{8}$$

In the basis where $\boldsymbol{\ell} = \left(\ell_L, \widetilde{\ell}_L, T^-\right)^T$ and $\boldsymbol{\ell^c} = \left(\ell_R^c, \overline{\ell}_L, T^+\right)^T$, the charged leptons mass matrix reads

$$\mathcal{M}_\ell = \begin{pmatrix} M^\ell & M'_{331} & M'_W \\ 0 & M_8 & 0 \\ 0 & 0 & M_8 \end{pmatrix}, \tag{9}$$

where the entries are given by

$$(M^\ell)_{ij} \equiv y^\ell_{ij} k_1,\ (M'_W)_{ij} \equiv y'_{ij} k_2,\ \text{and}\ (M'_{331})_{ij} \equiv y'_{ij} n_2\,.$$

Note that the *vev* n_2 (together with n_3) sets the $SU(3)_L \otimes U(1)_X$ breaking scale. In contrast, the *vevs* k_1 and k_2 must lie at the electroweak scale since they belong to $SU(2)_L$ doublets after the breaking of $SU(3)_L$.

From these expressions it follows that amongst the charged leptons, there is only a pair which is light in each generation, namely:

$$\ell_{\text{light}} \propto \ell_L - x\widetilde{\ell}_L - x\alpha T^-\,, \tag{10}$$

$$\ell^c_{\text{light}} \propto \ell^c_R - \frac{M^\ell}{M'_{331}} \frac{x^2}{x^2+1} \overline{\ell}_L\,, \tag{11}$$

where $\alpha \equiv M'_W/M'_{331} = k_2/n_2$ and $x \equiv M'_{331}/M_8$. Here the parameter $x = y'\beta$ is constrained by unification to lie in the range $y'/6 \lesssim x \lesssim y'$. These two 2-component states form the standard Dirac charged lepton which now has a squared mass given by $\left(M^\ell\right)^2 \frac{1}{1+x^2}$, an expression that differs from the SM one. Notice that the presence of states which do not come from the $\left(\mathbf{2}, -\frac{1}{2}\right)$ electroweak representation is α-suppressed. The two remaining pairs of charged leptons are heavy, with octet-scale masses.

Turning now to the neutral fermions, their mass matrix reads:

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & 0 & 0 & M'_{331} & \frac{1}{\sqrt{6}}M'_W & \frac{1}{\sqrt{2}}M'_W \\ 0 & 0 & M'_W & 0 & -\frac{2}{\sqrt{6}}M'_{331} & 0 \\ 0 & {M'_W}^T & 0 & M_8 & 0 & 0 \\ {M'_{331}}^T & 0 & M_8 & 0 & 0 & 0 \\ \frac{1}{\sqrt{6}}{M'_W}^T & -\frac{2}{\sqrt{6}}{M'_{331}}^T & 0 & 0 & M_8 & 0 \\ \frac{1}{\sqrt{2}}{M'_W}^T & 0 & 0 & 0 & 0 & M_8 \end{pmatrix},$$

in the eigenbasis $\boldsymbol{\nu} = \left(\nu_L, N^c, \widetilde{\nu}_L, \overline{\nu}_L, \widetilde{N}^c, T^0\right)^T$. In the one-family approximation, $\mathcal{M}_\nu$ has a null eigenvector

$$\nu_{\text{light}} = \sum_\alpha \omega_\alpha \nu_\alpha\,, \tag{12}$$

with ω given as $\frac{1}{N}\left(1, -\alpha, -x, x\alpha^2, -\sqrt{\frac{3}{2}}x\alpha, -\frac{1}{\sqrt{2}}x\alpha\right)^T$, where N is some normalization factor.Note that the observed neutrinos are mainly a mixture of ν_L and $\widetilde{\nu}_L$ which are both in the $\left(\mathbf{2}, -\frac{1}{2}\right)$ representation of the $SU(2)_L \otimes U(1)_Y$ group. The admixture of the remaining neutrino states are suppressed by at least a factor α, which can be vanishingly small. We have verified that in the multi-generation case $\mathcal{M}_\nu$ has a null eigenvector associated to each of the three generations of leptons. As a result neutrinos are massless in the tree level approximation. This property forms the basis of the radiative mechanism discussed below.

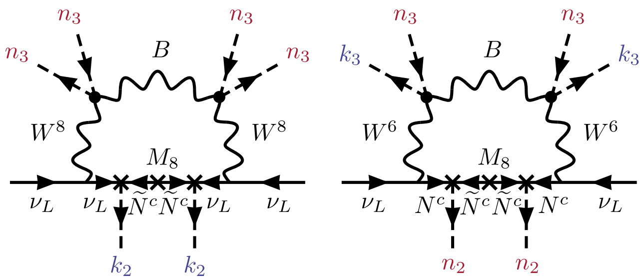

FIG. 4: Diagrams contributing to light neutrino mass.

At the one-loop level, the exchange of gauge bosons will give rise to a dimension-nine operator which, after symmetry breaking, yields a small neutrino mass through diagrams such as those displayed in Fig. 4. In order to understand the result of a detailed exact calculation here we simply focus on a typical contribution illustrated in Fig. 4, which is found to be of the form

$$m_\nu \approx \frac{1}{(4\pi)^2}\frac{1}{36} g_L^4 \left(3g_L^2 + 4g_X^2\right) n_3^2 \left(k_3 n_2 - k_2 n_3\right)^2 M_8 y'^2 \\ \times f\left(\left\langle m_A^2\right\rangle, \left\langle m_\Psi^2\right\rangle\right)\,, \tag{13}$$

where f is some loop function with dimensions of $(\mathrm{mass})^{-6}$, depending on the internal masses. This approximation captures the key features of the exact result such as (i) the gauge nature of the underlying radiative dimension-nine seesaw mechanism, requiring two bosonic and three fermionic mass insertions in the internal lines; as well as (ii) the symmetry structure, both gauge as well as lepton number, as can be seen explicitly. One also sees that no mass is generated in the limit where $k_3n_2 - k_2n_3$ is set to zero [4]. With a *vev* alignment which minimizes this factor, neutrinos get a sub-eV mass even for large y' values. Should y' be significantly smaller than unity, one must suppress the mixing between N^c and ν_L, since in this case one of the neutrino mass eigenstates is mainly N^c, with approximate mass $\frac{2}{3}\left(M'_{331}\right)^2/M_8$. This is readily achieved by setting $\alpha \to 0$.

Summary and outlook

In this letter we have proposed an electroweak $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge extension of the SM in which neutrinos are massless at tree-level. Even though the neutral fermion mass matrix has a seesaw structure, the messengers only provide mass at the loop level, thanks to the symmetry protection. Gauge mediated radiative corrections generate small calculable neutrino masses. The physics responsible for providing small neutrino masses is also responsible for gauge coupling unification, which can be achieved at a characteristic scale of order TeV in the absence of supersymmetry and of GUT-like interactions. A plethora of new states such as new gauge bosons and fermions makes the model directly testable at the LHC, with a non-trivial interplay between the quark sector and the lepton sector. The presence of such new features is presently under investigation.

We thank Martin Hirsch for valuable discussions. This work was supported by Spanish grants FPA2011-22975 and Multidark CSD2009-00064 (*MINECO*), and PROMETEOII/2014/084 (*Generalitat Valenciana*). FGC acknowledges support from *CONACYT* under grant 208055. RF was supported by *Fundação para a Ciência e a Tecnologia* through grants CERN/FP/123580/2011 and EXPL/FIS-NUC/0460/2013.

[4] As usual in mass insertion methods, they are not accurate enough for reliable estimates. In our case this is manifest as uncertainties in the choice of the "average" mass parameters $\left\langle m_A^2\right\rangle, \left\langle m_\Psi^2\right\rangle$.

* Electronic address: boucenna@ific.uv.es
† Electronic address: renato.fonseca@ific.uv.es
‡ Electronic address: felix.gonzalez@ific.uv.es
§ Electronic address: valle@ific.uv.es

[1] H. Georgi, H. R. Quinn, and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).
[2] D. Forero, M. Tortola, and J. Valle, (2014), arXiv:1405.7540 [hep-ph] .
[3] K. Olive and P. D. Group, Chinese Physics C **38**, 090001 (2014).
[4] U. Amaldi, W. de Boer, and H. Furstenau, Phys. Lett. **B260**, 447 (1991).
[5] G. Aad, T. Abajyan, B. Abbott, J. Abdallah, S. A. Khalek, A. Abdelalim, O. Abdinov, R. Aben, B. Abi, M. Abolins, et al., Physical Review D **87**, 012008 (2013).
[6] M. Singer, J. Valle, and J. Schechter, Phys.Rev. **D22**, 738 (1980).
[7] J. W. F. Valle and M. Singer, Phys. Rev. **D28**, 540 (1983).
[8] A. J. Buras, F. De Fazio, and J. Girrbach-Noe, JHEP **1408**, 039 (2014), arXiv:1405.3850 [hep-ph] .
[9] A. J. Buras, F. De Fazio, and J. Girrbach, JHEP **1402**, 112 (2014), arXiv:1311.6729 [hep-ph] .
[10] A. J. Buras, F. De Fazio, J. Girrbach, and M. Carlucci, JHEP **1302**, 023 (2013), arXiv:1211.1237 [hep-ph] .
[11] P. V. Dong, L. T. Hue, H. N. Long, and D. V. Soa, Phys. Rev. D **81**, 053004 (2010).
[12] V. Vien and H. Long, JHEP **1404**, 133 (2014), arXiv:1402.1256 [hep-ph] .
[13] A. E. Carcamo-Hernandez, E. Catano-Mur, and R. Martinez, Phys.Rev. **D90**, 073001 (2014), arXiv:1407.5217 [hep-ph] .
[14] S. M. Boucenna, S. Morisi, and J. W. Valle, Adv.High Energy Phys. **2014**, 831598 (2014), arXiv:1404.3751 [hep-ph] .
[15] J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980).
[16] S. M. Boucenna, S. Morisi, and J. W. F. Valle, Phys. Rev. D **90**, 013005 (2014).

[17] R. A. Diaz, D. Gallego, and R. Martinez, Int.J.Mod.Phys. **A22**, 1849 (2007), arXiv:hep-ph/0505096 [hep-ph] .
[18] Y. Chikashige, R. Mohapatra, and R. Peccei, Physics Letters B **98**, 265 (1981).
[19] J. Schechter and J. W. F. Valle, Phys. Rev. D **25**, 774 (1982).
[20] D. J. Gross and F. Wilczek, Phys.Rev.Lett. **30**, 1343 (1973).
[21] H. D. Politzer, Phys.Rev.Lett. **30**, 1346 (1973).