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Double parton distributions in Light-Front constituent quark models

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Abstract Double parton distribution functions (dPDF), accessible in high energy proton-proton and proton nucleus collisions, encode information on how partons inside a proton are correlated among each other and could represent a tool to explore the 3D proton structure. In recent papers, double parton correlations have been studied in the valence quark region, by means of constituent quark models. This framework allows to understand clearly the dynamical origin of the correlations and to establish which, among the features of the results, are model independent. Recent relevant results, obtained in a relativistic light-front scheme, able to overcome some drawbacks of previous calculations, such as the poor support, will be presented. Peculiar transverse momentum correlations, generated by the correct treatment of the boosts, are obtained. The role of spin correlations will be also shown. In this covariant approach, the symmetries of the dPDFs are unambiguously reproduced. The study of the QCD evolution of the model results has been performed in the valence sector, showing that, in some cases, the effect of evolution does not cancel that of correlations.

Keywords proton structure · relativistic models

1 Introduction

In high energy hadron-hadron collisions, more than one parton in each of the hadron can contribute to the cross section. This is the multiple partonic interactions (MPI) phenomenon which, even if its contribution is suppressed by a power of Λ^2_{QCD}/Q^2 with respect to the single parton interaction, with Q the center-mass energy in the collision, has been already observed (see, *e.g.*, Ref. [1]). In this scenario MPI represent a background for the search of new Physics, *e.g.*, at the LHC. In this work we focus our attention to the double parton scattering (DPS) which can be observed in many channels, *e.g.*, WW with dilepton productions and double Drell-Yan processes (see, *e.g.* Refs. [2; 3; 4; 5] for recent reviews). At the LHC, DPS, whose evidence has been observed [6], represents also a background for

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Higgs production. In this framework the DPS cross section can be written, following [7], in terms of the so called double parton distribution functions (dPDFs), $F_{ij}(x_1, x_2, \mathbf{z}_\perp, \mu)$, which describe the joint probability of finding two partons of flavors $i, j = q, \bar{q}, g$ with longitudinal momentum fractions x_1, x_2 and transverse separation $\mathbf{z}_\perp$ inside the hadron:

$$d\sigma = \frac{1}{S} \sum_{i,j,k,l} \int d\mathbf{z}_\perp \, F_{ij}(x_1, x_2, \mathbf{z}_\perp, \mu) F_{kl}(x_3, x_4, \mathbf{z}_\perp, \mu) \hat{\sigma}_{ik}(x_1 x_3 \sqrt{s}, \mu) \hat{\sigma}_{jl}(x_2 x_4 \sqrt{s}, \mu) \,. \tag{1}$$

The partonic cross sections $\hat{\sigma}$ refer to the hard, short-distance processes, S is a symmetry factor, present if identical particles appear in the final state and μ is the renormalization scale which is taken, for simplicity, to be the same for both partons. For the evaluation of the DPS contributions to proton-proton scattering at LHC kinematics, the following approximation, for the dPDF, is usually made:

$$F_{ij}(x_1, x_2, \mathbf{z}_\perp, \mu) = q_i(x_1, \mu) q_j(x_2, \mu) \, \theta(1 - x_1 - x_2)(1 - x_1 - x_2)^n T(\mathbf{z}_\perp, \mu) \ , \tag{2}$$

i.e., a complete factorized form of the dPDF is assumed. In particular the $\mathbf{z}_\perp$ and $x_1 - x_2$ dependences are factorized and the standard single parton distribution functions (PDF), $q(x)$, are introduced. This means that possible double parton correlations between the two interacting partons are neglected. dPDFs are non perturbative quantities so that they cannot be easily evaluated in QCD. As it happens for the PDF, a useful procedure for their estimate is a calculation at the hadronic scale, $Q_0 \sim \Lambda_{QCD}$, by means of quark models. In order to compare the obtained results with data taken, *e.g.*, at high energy scales, $Q > Q_0$, it is necessary a perturbative QCD (pQCD) evolution of the model calculations, using the dPDFs evolution equations known since a long time ago [8; 9]. By using this procedure, future data analysis of the DPS processes could be guided, in principle, by model calculations. The first model evaluations of the dPDF have been the ones in Refs.[10; 11]. In the first scenario use has been made of a modified version of the MIT bag model in the cavity-approximation in order to introduce double parton correlations by hand, recovering momentum conservation. In the second case the dPDFs have been calculated in a non relativistic (NR) constituent quark model (CQM) framework, since CQM, in the valence region, predict PDFs, generalized parton distribution functions (GPDs) and transverse momentum dependent parton distributions (TMDs) rather well (see, *e.g.*, Refs. [12; 13; 14]). These expectations motivated the analysis of Refs. [11; 15] and the present one. The main results found in Refs. [10; 11] are that, in the valence quark region, the approximations used to write Eq. (2) are badly violated. In the CQM picture, where the dynamical origin of double correlations is clear, the origin of this violation can be properly understood. One should notice that both the analyses of Refs. [10; 11] have some inconsistencies. First, dPDF do not vanish in the non physical region, $x_1 + x_2 > 1$,*i.e.*, they have a wrong support. Moreover, as already pointed out, in order to obtain some information on the dPDF at small values of x and at high Q^2, where LHC data are taken, the pQCD evolution of the calculated dPDF is necessary. In a recent paper of ours [15], a CQM calculation of the dPDF has been performed including relativity through a fully Poincaré covariant Light-Front (LF) approach. Thanks to this treatment it is possible to study strong interacting systems with a fixed number of on-shell constituents (see Refs. [16; 17] for general reviews). Moreover, being the hyperplane of the LF, *i.e.*, the plane where the initial conditions are defined, tangent to the Light-Cone, the Deep Inelastic Scattering (DIS) phenomenology is automatically included into the scheme. In particular, in this framework, which has been used extensively for hadronic calculations, (see, *e.g.*, Refs. [18; 19; 20], some symmetries of the dPDF are restored and the bad support problem is fixed, so that the pQCD evolution of the dDPDFs is more precise. The results of this analysis will be summarized in the following sections.

2 dPDFs in light-Front CQM

In this section the procedure adopted for the LF calculation of the dPDFs will be presented. The validity of the approximations Eq. (2) in this relativistic scenario will be checked. To this aim, the LF approach has been chosen due to its nice properties, in particular the fact that LF boosts and plus component of the momenta ($a^+ = a_0 + a_z$) are kinematical operators. The Fourier- transform of the dPDF

$$F_{ij}^{\lambda_1,\lambda_2}(x_1, x_2, \mathbf{k}_\perp) = \int d\mathbf{z}_\perp \, e^{i\mathbf{z}_\perp \cdot \mathbf{k}_\perp} F_{ij}^{\lambda_1,\lambda_2}(x_1, x_2, \mathbf{z}_\perp) \ , \tag{3}$$

will be analized. In the above equation, the dPDF is introduced for two quarks of flavors i and j and helicities $\lambda_{i(j)}$, respectively. The full procedure of the calculations of the dPDF in the LF approach, starting from the formal definition of this quantity, and thanks to a proper extension of the proceeding presented in Ref. [20; 21], developed in that case for the GPDs calculations, is shown in details in Ref. [15]. Here the main steps are summarized. In particular, one can start from the expression of a light-cone correlator (see, *e.g.*, Ref. [4]), written in terms of the proton state and of LF quantized fields of the interacting quarks. In order to find a general expression of the dPDF in the valence region, use has been made of the so called "LF wave function" (LFWF) representation [17] to describe the proton state. In this formalism the latter quantity is written as a sum over partonic Fock states with all the correct normalizations preserved. In the LFWF representation, only the first term of this summation, representing the contribution of the valence quarks, has been taken into account. A crucial point of the procedure is the possibility of describing the LF proton state starting from the Instant Form (canonical) one, where most quark models are developed. To this aim the following relation between one particle LF state, $|\mathbf{k},\lambda\rangle_{[l]}$, and the corresponding IF one, $|\mathbf{k},\lambda'\rangle_{[i]}$, has been used:

$$|\mathbf{k},\lambda\rangle_{[l]} = (2\pi)^{3/2}\sqrt{m^2+\mathbf{k}^2}\sum_{\lambda\lambda'} D^{1/2}_{\lambda\lambda'}(R_{cf}(\mathbf{k}))|\mathbf{k},\lambda'\rangle_{[i]} \ , \tag{4}$$

where the Melosh rotation, which allows to rotate the canonical helicity, λ, into the LF spin, λ', is introduced:

$$D^{1/2}_{\mu\lambda}(R_{cf}(\mathbf{k})) = \langle\mu\left|\frac{m+x_iM_0-i\boldsymbol{\sigma}_i\cdot(\hat{z}\times\mathbf{k}_\perp)}{\sqrt{(m+x_iM_0)^2+\mathbf{k}_\perp^2}}\right|\lambda\rangle \ . \tag{5}$$

In the above equation, $x_i = \dfrac{k_i^+}{P^+}$ is the longitudinal momentum fraction carried by the i parton, with P^+ the plus component of the proton momentum, $M_0 = \sum_i\sqrt{m^2+\mathbf{k}_i^2}$ the total free energy mass of the partonic system, $k_{iz} = -(m^2+k_{i\perp}^2-k_i^{+2})/(2k_i^+)$, μ and λ generic canonical spins. Actually, we are interested in the proton, a composite system, and the validity of Eq. (4), supposed for free states, is questionable. Nevertheless, in the following, we will use a relativistic mass equation built in accord with the Bakamjian-Thomas Construction of the Poincaré generators, and Eq. (4) can be used (see Ref. [16]). Using a lengthy but straightforward procedure, a final expression of the dPDF is obtained. It reads [15]:

$$\begin{aligned} F^{\lambda_1,\lambda_2}_{q_1q_2}(x_1,x_2,\mathbf{k}_\perp) = 3(\sqrt{3})^3 \int \left[\prod_{i=1}^{3} d\mathbf{k}_i \sum_{\lambda_i^f\tau_i}\right] \delta\left(\sum_{i=1}^{3}\mathbf{k}_i\right) \Psi^*\left(\mathbf{k}_1+\frac{\mathbf{k}_\perp}{2},\mathbf{k}_2-\frac{\mathbf{k}_\perp}{2},\mathbf{k}_3;\{\lambda_i^f,\tau_i\}\right) \\ \times \widehat{P}_{q_1}(1)\widehat{P}_{q_2}(2)\widehat{P}_{\lambda_1}(1)\widehat{P}_{\lambda_2}(2)\,\Psi\left(\mathbf{k}_1-\frac{\mathbf{k}_\perp}{2},\mathbf{k}_2+\frac{\mathbf{k}_\perp}{2},\mathbf{k}_3;\{\lambda_i^f,\tau_i\}\right) \\ \times\,\delta\left(x_1-\frac{k_1^+}{P^+}\right)\delta\left(x_2-\frac{k_2^+}{P^+}\right) \ . \end{aligned} \tag{6}$$

The canonical proton wave function $\psi^{[c]}$ is embedded in the function Ψ here above, which can be written as follows:

$$\Psi(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_2;\{\lambda_i^f,\tau_i\}) = \prod_{i=1}^{3}\left[\sum_{\lambda_i^c} D^{*1/2}_{\lambda_i^c\lambda_i^f}(R_{cf}(\mathbf{k}_i))\right]\psi^{[c]}(\{\mathbf{k}_i,\lambda_i^c,\tau_i\}) \ , \tag{7}$$

where here λ_i^c, τ_i are the canonical partonic helicity and isospin respectively and the short notation $\{\alpha_i\}$ instead of $\alpha_1,\alpha_2,\alpha_3$ is introduced. Moreover isospin and spin projection operators are introduced in order to access dynamical correlations for the unpolarized and longitudinal polarized i quark of a given flavor:

$$\hat{P}_{u(d)}(i) = \frac{1\pm\tau_3(i)}{2} \ , \ \hat{P}_{\lambda_k}(i) = \frac{1+\lambda_k\sigma_3(i)}{2} \ . \tag{8}$$

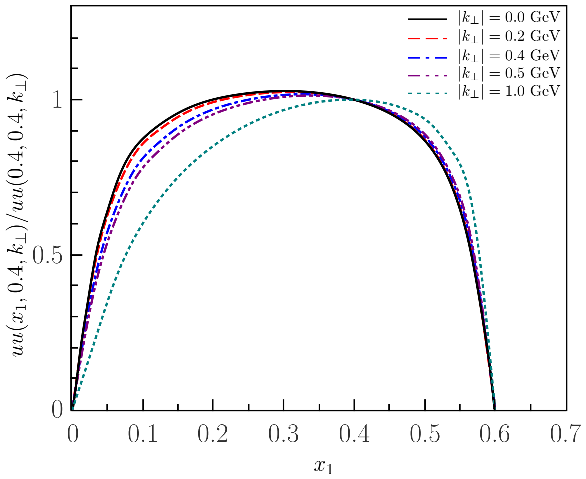

Fig. 1 The ratio r_1, Eq. (11), for five values of $k_\perp$.

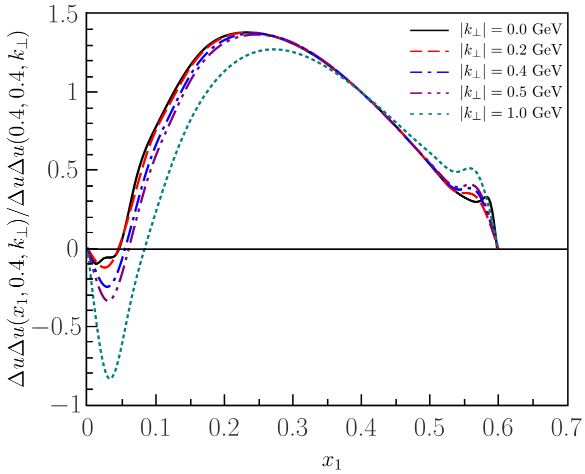

Fig. 2 The ratio r_2, Eq. (11), for five values of $k_\perp$.

A crucial point of this analysis, as already pointed out, is that now the plus components of the momenta are totally kinematical so that one finds:

$$P^+ = \sum_i^3 k_i^+ = M_0 \ , \tag{9}$$

in the intrinsic frame where $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$. Thanks to the relation Eq. (9) the delta function, defining the longitudinal momentum fraction carried by the parton in Eq. (6), can be properly solved without any additional approximation, at variance with what happens in the instant form calculation in Ref. [11]. As a direct consequence, the bad support problem does not show up. The following distributions, different from zero for an unpolarized proton, have been calculated:

$$uu(x_1, x_2, k_\perp) = \sum_{i,j=\uparrow,\downarrow} u_i u_j(x_1, x_2, k_\perp), \quad \Delta u\Delta u(x_1, x_2, k_\perp) = \sum_{i,j=\uparrow,\downarrow} (-1)^{i+j+1} u_i u_j(x_1, x_2, k_\perp) \tag{10}$$

In order to calculate now the dPDFs, in particular to check whether the approximation, Eq. (2), holds, a proper CQM has to be chosen. To this aim, in order to have a fully consistent procedure, a relativistic model has been used, in particular the one described in Ref. [19], a hyper-central CQM. This model provides a reasonable description of the light hadronic spectrum, it has been used for the estimate of PDFs and GPDs in Refs. [19; 20; 21; 22; 23] and, for the present analysis, since no data are available for the dPDF, it can be used as laboratory to predict the most relevant features of dPDF. In particular, in Figs. 1 and 2 the following ratios have been shown for five values of $k_\perp$:

$$r_1 = \frac{uu(x_1, 0.4, k_\perp)}{uu(0.4, 0.4, k_\perp)}, \quad r_2 = \frac{\Delta u\Delta u(x_1, 0.4, k_\perp)}{\Delta u\Delta u(0.4, 0.4, k_\perp)} \ . \tag{11}$$

In particular, the value $x_2 = 0.4$ has been chosen for an easy comparison with the results of Refs. [10; 11]. From these results it is clear that the factorization ansatz, in Eq. (2), is violated, as it was found already in Refs. [10; 11]. As one can notice, r_1 and r_2 depend on $k_\perp$ so that a factorized form of the dPDFs for the $k_\perp$ dependence is not supported by this approach. One can easily realize, in fact, that a factorized expression for the dPDF would yield $k_\perp$-independent r_1 and r_2. It is also important to notice that the amount of the violation is directly related to the Melosh rotation contributions, a model independent relativistic effect. In Fig. 3 the following ratios

$$r_3 = \frac{2uu(x_1, x_2, k_\perp = 0)}{u(x_1)u(x_2)} \ , \ r_4 = \frac{C\Delta u\Delta u(x_1, x_2, k_\perp = 0)}{\Delta u(x_1)\Delta u(x_2)} \ , \tag{12}$$

where:

$$C = \frac{[\int dx \Delta u(x)]^2}{\int dx_1 dx_2 \Delta u\Delta u(x_1, x_2, k_\perp = 0)} \ , \tag{13}$$

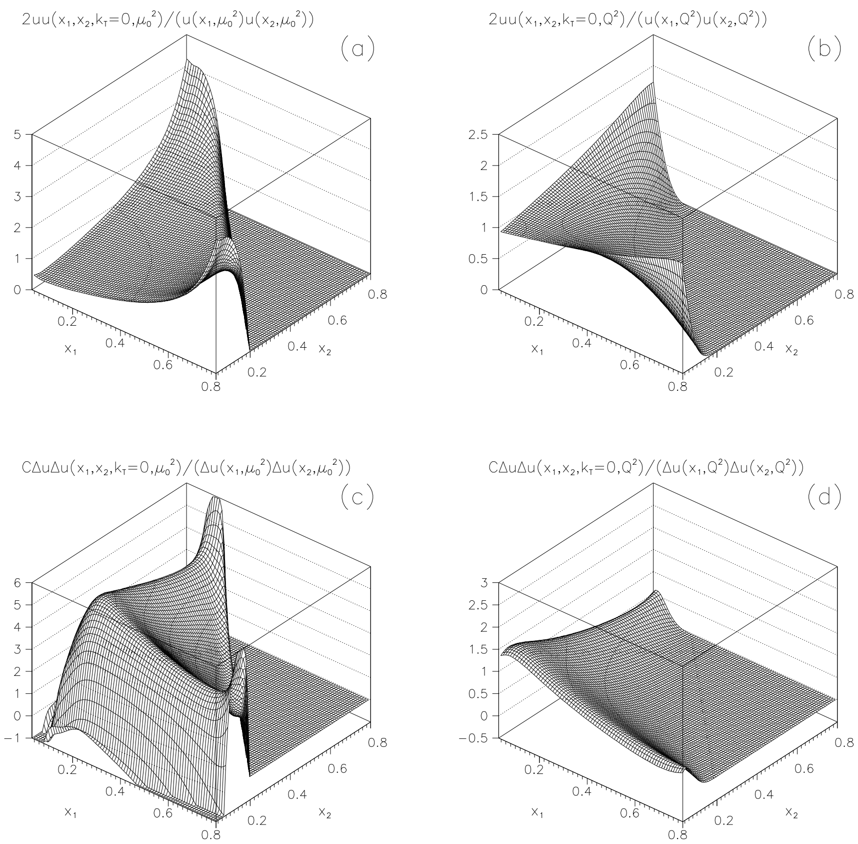

Fig. 3 a) The ratio r_3, Eq. (12), at the hadronic scale; b) the same quantity at a scale $Q^2 = 10$ GeV^2; c) the ratio r_4, Eq. (12), at the hadronic scale μ_0; d) this last quantity at a scale $Q^2 = 10$ GeV^2. The vertical scale of panels (b) and (d) is reduced by a factor of 2 with respect to panels (a) and (c), respectively.

involving the standard PDFs in the denominators, calculated within the same CQM, have been presented. In Figs. 3(a), 3(c) these quantities are shown at the hadronic scale obtaining that a factorized approximation of the dPDF, in terms of the single PDFs, is not favored in the valence quark region. Notice that the factors 2 and C, in Eq. (12), are inserted in order to have these ratios equal to 1 in the kinematical region where the approximation, Eq. (2), is valid. Moreover, in Ref. [15] the pQCD evolution at Leading-Order of the calculated dPDFs has been also presented. In particular, for the moment being, the evolution of the dPDF has been addressed for $k_\perp = 0$, taking the same scale for the two acting partons and analyzing only the valence quark contributions. In this case the evolution equations are obtained as a direct generalization of the well known DGLAP ones, see Refs. [8; 9]. Since only the non singlet case has been studied here, one needs to solve the homogeneous part of the generalized DGLAP equations by using the Mellin transformations of the dPDFs, see Ref. [15] for details. If we use their simple ansatz as an input to our calculation, our code reproduce the results of the authors of Ref. [24]. Once the dPDFs have been evaluated at a generic high energy scale, *e.g.*, $Q^2 = 10$ GeV^2, the ratios r_3 and r_4 have been shown again in Figs. 3(b), 3(d). The most important results of this analysis are that, for small values of x, where data are taken, *e.g.*, at the LHC, $r_3 \sim 1$, which means that, in the unpolarized case, dynamical correlations are suppressed after the evolution. Nevertheless, by looking at r_4, in Fig 3(d), it is found that double spin correlations still contribute.

3 Conclusions

In this work, dPDFs contributing to the DPS cross section have been calculated by means of a LF CQM. The main achievement is the fully Poincaré covariance of the description, which allows to restore the expected symmetries, and the vanishing of the dPDF in the forbidden kinematical region, $x_1 + x_2 > 1$. In the analysis of the dPDFs at the hadronic scale we found that the approximations of these quantities with a complete factorized ansatz, in the $x_1 - x_2$ and $(x_1, x_2) - k_\perp$ dependences, is violated, in agreement with previous results [10; 11]. Moreover, a pQCD analysis of the dPDFs, necessary to evaluate these quantities at higher energy scales with respect to the hadronic one where the CQM predictions are valid, has been also performed. For the unpolarized quarks case the dynamical correlations are suppressed in the small x region, while double spin correlations are found to be still important. Further analysis, including into the scheme non perturbative sea quarks and gluons and the evolution of the singlet sector, important to describe the dPDF at low x, are in progress, as well as the study of correlations in pA scattering, along the line of Ref. [25].

References

1. Akesson, T., et al., [Axial Field Spectrometer Collaboration]: Double parton scattering in p-p collisions at $\sqrt{S} = 63$-GeV. Z. Phys. C **34**, 163 (1987)
2. Gaunt, J.R. and Stirling, W.J.: Double Parton Distributions Incorporating Perturbative QCD Evolution and Momentum and Quark Number Sum Rules. JHEP **1003**, 005 (2010)
3. Diehl, M., Ostermeier, D. and Schafer, A.: Elements of a theory for multiparton interactions in QCD. JHEP **1203**, 089 (2012)
4. Manohar, A. V. and Waalewijn, W. J.: A QCD Analysis of Double Parton Scattering: Color Correlations, Interference Effects and Evolution. Phys. Rev. D **85**, 114009 (2012)
5. Bartalini, P., (ed.) and Fanò, L. (ed.): Multiple partonic interactions at the LHC. Proceedings, 1st International Workshop, MPI'08, Perugia, Italy, October 27-31, 2008. arXiv:1003.4220 [hep-ex]
6. Aad, G., et al. [ATLAS Collaboration]: Measurement of hard double-parton interactions in $W- > lv + 2$ jet events at $\sqrt{s} = 7$ TeV with the ATLAS detector. New J. Phys. **15**, 033038 (2013)
7. Paver, N. and Treleani, D.: Multi - Quark Scattering And Large P(t) Jet Production In Hadronic Collisions", Nuovo Cim. A **70** 215 (1982)
8. Kirschner, R.: Generalized Lipatov-Altarelli-Parisi Equations And Jet Calculus Rules. Phys. Lett. B **84**, 266 (1979)
9. Shelest, V. P., Snigirev, A. M. and Zinovev, G. M.: The Multiparton Distribution Equations In QCD. Phys. Lett. B **113**, 325 (1982)
10. Chang, H. M., Manohar, A. V. and Waalewijn, W. J.: Double Parton Correlations in the Bag Model. Phys. Rev. D 87, **034009** (2013)
11. Rinaldi, M., Scopetta, S. and Vento, V.: Double parton correlations in constituent quark models. Phys. Rev. D **87**, 11, 114021 (2013)
12. Traini, M., Mair, A., Zambarda, A., and Vento, V.: Constituent quarks and parton distributions. Nucl. Phys. A **614**, 472 (1997)
13. Scopetta, S. and Vento, V.: Generalized parton distributions in constituent quark models. Eur. Phys. J. A **16**, 527 (2003)
14. Courtoy, A., Fratini, F., Scopetta, S. and Vento, V.: A quark model analysis of the Sivers function. Phys. Rev. D **78**, 034002 (2008)
15. Rinaldi, M., Scopetta, S., Traini, M. and Vento, V.: Double parton correlations and constituent quark models: a Light Front approach to the valence sector. arXiv:1409.1500 [hep-ph]. JHEP in press.
16. Keister, B.D. and Polyzou, W. N.: Relativistic Hamiltonian dynamics in nuclear and particle physics. Adv. Nucl. Phys. **20**, 225 (1991)
17. Brodsky, S.J., Pauli, H. C. and Pinsky, S.S.: Quantum chromodynamics and other field theories on the light cone. Phys. Rept. **301**, 299 (1998)
18. Cardarelli, F., Pace, E., Salmè, G. and Simula, S.: Nucleon and pion electromagnetic form-factors in a light front constituent quark model. Phys. Lett. B **357**, 267 (1995)
19. Faccioli, P., Traini, M. and Vento, V.: Polarized parton distributions and light front dynamics. Nucl. Phys. A **656**, 400 (1999)
20. Boffi, S., Pasquini, B. and Traini, M.: Linking generalized parton distributions to constituent quark models. Nucl. Phys. B **649**, 243 (2003)
21. Boffi, S., Pasquini, B. and Traini, M.: Helicity dependent generalized parton distributions in constituent quark models. Nucl. Phys. B **680**, 147 (2004)
22. Traini, M.: Charge symmetry violation: A NNLO study of partonic observables. Phys. Lett. B **707**, 523 (2012)
23. Traini, M.: NNLO nucleon parton distributions from a light-cone quark model dressed with its virtual meson cloud. Phys. Rev. D **89**, 034021 (2014)
24. Broniowski, W. and Arriola, E. R.: Valence double parton distributions of the nucleon in a simple model. Few Body Syst. **55**, 381 (2014)
25. Salvini, S., Treleani, D. and Calucci, G.: Double Parton Scatterings in High-Energy Proton-Nucleus Collisions and Partonic Correlations. Phys. Rev. D **89**, 016020 (2014)