The minimal adjoint-$\mathsf{SU(5)} \times \mathsf{Z_4}$ GUT model

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Abstract: An extension of the adjoint $\mathsf{SU(5)}$ model with a flavour symmetry based on the $\mathsf{Z_4}$ group is investigated. The $\mathsf{Z_4}$ symmetry is introduced with the aim of leading the up- and down-quark mass matrices to the Nearest-Neighbour-Interaction form. As a consequence of the discrete symmetry embedded in the $\mathsf{SU(5)}$ gauge group, the charged lepton mass matrix also gets the same form. Within this model, light neutrinos get their masses through type-I, type-III and one-loop radiative seesaw mechanisms, implemented, respectively, via a singlet, a triplet and an octet from the adjoint fermionic **24** fields. It is demonstrated that the neutrino phenomenology forces the introduction of at least three **24** fermionic multiplets. The symmetry $\mathsf{SU(5)} \times \mathsf{Z_4}$ allows only two viable zero textures for the effective neutrino mass matrix. It is showed that one texture is only compatible with normal hierarchy and the other with inverted hierarchy in the light neutrino mass spectrum. Finally, it is also demonstrated that $\mathsf{Z_4}$ freezes out the possibility of proton decay through exchange of colour Higgs triplets at tree-level.

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Contents

1 Introduction

Grand Unified Theories (GUT) are natural extensions of the Standard Model (SM) and provide an appealing framework for the search of the theory of flavour. Most GUT models try to unify the three gauge couplings of SM in a unique coupling within a simple group. This is sustained by the fact that the SM gauge couplings seem to unify at high scale, $\Lambda \approx 10^{15-17}$ GeV, when they evolve through the renormalisation group equations. In such GUT constructions, not only the SM gauge coupling unify, but also the SM fermions are tight in larger multiplets opening the possibility for the implementation of a flavour symmetry. Another important signature of most GUTs is the prediction for proton decay [1], which has not yet been observed and severely constrains these models.

The first GUT model was realisable within the $\mathsf{SU}(5)$ gauge group [2] in 1974. This minimal model fits the fifteen SM fermionic degrees of freedom in two unique representations: $\mathsf{5}^*$ and $\mathsf{10}$, per generation. It is well established that this model is ruled out, since it does not reproduce the correct mass ratios among the charged leptons and down-type quarks and also the particle content does not lead to an accurate gauge coupling unification. During the last decades, many attempts have been proposed in the literature in order to construct consistent GUT models [3–6] based on the $\mathsf{SU}(5)$ group. In particular, the mass mismatch between the charged leptons and down-type quarks in the minimal $\mathsf{SU}(5)$ can be easily corrected if one accepts higher dimension operators in the model without enlarging the field content [3, 5]. Alternatively, one can build a non-renormalisable solution where the mass mismatch is explained by adding an extra $\mathsf{45}$ Higgs multiplet [7, 8].

Although GUT multiplets contain both quark and lepton fields this is not enough to fully determine the properties of their observed masses and mixings. Indeed, GUT models do not solve the "flavour puzzle" present in the SM, however the new GUT relations among quark and lepton Yukawa matrices are an excellent starting point for building a flavour symmetry. There have been many approaches to understand the intricate "flavour puzzle" in the context of GUTs. An attractive possibility is to assume the vanishing of some Yukawa interactions by the requirement of a discrete symmetry, so that new "texture zeroes" appear in the Yukawa matrices [9–14]. Symmetries may predict new relations among fermion masses and their mixings. Nevertheless, the opposite is not true in general, because zeroes in the Yukawa matrices can also be obtained by performing some set of transformations (weak basis transformations) leaving the gauge sector diagonal [15–17].

The Nearest-Neighbour-Interaction (NNI) is an example of a weak basis in which the up- and down-quark mass matrices, M_u and M_d, share the same texture-zero form:

$$M_{u,d} = \begin{pmatrix} 0 & A_{u,d} & 0 \\ A'_{u,d} & 0 & B_{u,d} \\ 0 & B'_{u,d} & C_{u,d} \end{pmatrix} ,$$

where the constants $A_{u,d}$, $A'_{u,d}$, $B_{u,d}$, $B'_{u,d}$ and $C_{u,d}$ are independent and complex. Since this parallel structure is a weak basis, no physical predictions can be made unless further assumptions are considered. This is the case of the Fritzsch Ansatz [18, 19] where, in addition of the NNI structure, one requires Hermiticity for both M_u and M_d, which cannot be obtained through a weak basis transformation. It is well known that the Fritzsch Ansatz can no longer accommodate the current experimental quark mixings. However, it was shown in ref. [20] that deviations from the Hermiticity around 20% were compatible with the experimental data.

It was shown in ref. [20] that it is possible to obtain up- and down-quark mass matrices M_u and M_d with the NNI structure through the implementation of an Abelian discrete flavour symmetry in the context of the two Higgs doublet model (2HDM). In that context, the minimal realisation is the group Z_4. In a general 2HDM, a NNI form for each Yukawa coupling matrices cannot be a weak basis choice. Indeed, the requirement of the Z_4 symmetry does imply restrictions on the scalar couplings to the quarks, although one gets no impact on the quark masses and the Cabibbo-Kobayashi-Maskawa matrix [21, 22].

The goal of this article is to study whether it is possible to implement a Z_4 flavour symmetry, as in refs. [20, 23], that leads to quark mass matrices M_u and M_d with the NNI form in the context of the adjoint-$\mathsf{SU}(5)$ Grand Unification [6]. The requirement of a flavour symmetry that enforces a particular pattern in both up- and down-quark Yukawa couplings has phenomenological implications for the leptonic sector. The adjoint-$\mathsf{SU}(5)$ model strengths the weak points left in ref. [23], namely it improves the unification of the gauge couplings, solves the mass mismatch between the charged leptons and down-type quarks at renormalisable level, alleviates the constraint imposed by the proton lifetime and introduces a richer mechanism to generate light neutrino masses.

The minimal version of the adjoint-$\mathsf{SU}(5)$ model [6] consists in adding to the minimal $\mathsf{SU}(5)$ version a 45 Higgs representation, 45_H, together with an adjoint fermionic field, $\rho(24)$

to generate light neutrino masses. In this minimal setup one $\rho(\mathbf{24})$ is enough to account for the observed low-energy neutrino data compatible with a non-zero lightest neutrino mass [24–27]. Nevertheless, the nature of the Z_4 symmetry requires an extension of the number of $\rho(\mathbf{24})$ fields, $n_{\mathbf{24}}$. The fermionic $\rho(\mathbf{24})$ fields make possible the generation of neutrino masses through the interaction with the $\mathbf{45}_H$ field, since gauge interactions forbid a singlet right-handed neutrino to couple to $\mathbf{45}_H$. Neutrino masses arise from three different types of seesaw mechanisms: type-I [28–31], type-III [32, 33] and radiative seesaw [34–36]. The radiative seesaw is realizable through the octet-doublet $S_{(8,2)}$ of the $\mathbf{45}_H$ multiplet at the one-loop level.

This paper is organised as follows: in section 2 the $\mathsf{SU}(5) \times \mathsf{Z}_4$ model is described in detail. Next, in section 3 we address the issues of the unification of the gauge couplings as well as the phenomenology of proton decay in the model. The successful textures for the effective neutrino mass matrix together with some comments on the generation of the baryon asymmetry of the universe through leptogenesis are discussed in section 4. Then, our numerical results showing the viability of the leptonic textures considered are sketched in section 5. Finally, the conclusions are drawn in section 6.

2   The model

The adjoint-$\mathsf{SU}(5)$ model [6] contains three generations of $\mathbf{5}^*$ and $\mathbf{10}$ fermionic multiplets which accommodate the SM fermion content. In addition one adjoint fermionic multiplet $\rho(\mathbf{24})$ is introduced for the purpose of generating the light neutrino masses and mixings. In this minimal version, three non-vanishing light neutrino masses arise from three different seesaw mechanisms, as it will become clear later. Since our aim is to enlarge the symmetry of the Lagrangian with an extra Z_4 symmetry it forces us to consider n_{24} copies of $\mathbf{24}$ fermionic field.

The Higgs sector is composed by an adjoint multiplet, $\Sigma(\mathbf{24})$, a quintet $\mathbf{5}_H$ and one 45-dimensional representation $\mathbf{45}_H$. Details of the SM fields contained in the GUT representations are given in appendix A.

The adjoint field Σ has the usual role to break spontaneously the GUT gauge group down to the SM group, i.e. $\mathsf{SU}(3)_{\mathrm{C}} \times \mathsf{SU}(2)_{\mathrm{L}} \times \mathsf{U}(1)_{\mathrm{Y}}$, through its vacuum expectation value (VEV),

$$\langle \Sigma \rangle = \frac{\sigma}{\sqrt{60}} \operatorname{diag}(2, 2, 2, -3, -3). \tag{2.1}$$

The Higgs quintet and the Higgs 45-plet give rise to two doublets, $H_1 \in \mathbf{5}_H$ and $H_2 \in \mathbf{45}_H$, at low-energies and two massive $\mathsf{SU}(3)$ colour triplets, $T_1 \in \mathbf{5}_H$ and $T_2 \in \mathbf{45}_H$. It is essential that the triplets $T_{1,2}$ have masses around the unification scale while the doublets $H_{1,2}$ should remain at the electroweak scale in order to prevent rapid proton decay - the so called doublet-triplet splitting problem. The representations $(\bar{3}, 1, 4/3)$ and $(3, 3, -1/3)$ in $\mathbf{45}_H$ could also induce proton decay. However, it has been shown that the former representation does not contribute at tree level to proton decay, while some states of the latter representation contribute with a constraint milder than the one given by the triplet T_2 [37].

Many mechanisms were proposed in order to avoid the doublet-triplet-splitting problem. One possibility that can be easily invoked within this framework is the missing partner mechanism [38, 39], which consists in having the bosonic representations 50, 50^* and 75 instead of the adjoint Σ to break the GUT group. In the missing partner mechanism the scalar doublets are naturally massless.

The role of the scalar fields 5_H and 45_H is then to break the SM group to $SU(3)_{\mathrm{c}} \times U(1)_{\mathrm{e.m.}}$ through their VEVs

$$\langle 5_H \rangle^{\mathsf{T}} = (0, 0, 0, 0, v_5) \,, \tag{2.2}$$

and

$$\langle 45_H{}^{\alpha 5}_{\beta} \rangle = v_{45} \left(\delta^{\beta}_{\alpha} - 4\, \delta^{\alpha}_4 \delta^4_{\beta} \right) \,, \ \alpha, \beta = 1, \ldots, 4 \,, \tag{2.3}$$

that are related as

$$v^2 \equiv |v_5|^2 + 24\, |v_{45}|^2 = \left(\sqrt{2}\, G_F \right)^{-1} = (246.2\,\mathrm{GeV})^2 \,, \tag{2.4}$$

where G_F is the Fermi constant.

Since all representations are well defined, we can specify the nontrivial transformations of each bosonic/fermionic field R under the Z_4 flavour symmetry as:

$$R \longrightarrow R' = e^{i\, \frac{2\pi}{4} \mathcal{Q}(R)}\, R \,, \quad \mathcal{Q}(R) \in \mathsf{Z}_4 \,. \tag{2.5}$$

The purpose of the discrete symmetry is to obtain the quark mass matrices, M_u, M_d, with the NNI form at low energy scales. In order to preserve the Z_4 symmetry below the unification scale it is required that $\mathcal{Q}(\Sigma) = 0$. In order to implement the missing partner mechanism, the extra bosonic fields, 50, 50^* and 75, must also be trivial under Z_4. Thus, below the unification scale Λ the Z_4 group is preserved in higher orders of perturbation theory, provided that no Nambu-Goldstone boson appears at tree-level due to an accidental global symmetry [40]. At low energies one obtains a two Higgs doublet model with extra fermions invariant under Z_4, which gets broken once the doublets acquire VEVs.

The Z_4-charges are assigned as follows. First we make the choice that the 45_H couples to the bilinear $10_3\, 10_3$, which implies that

$$\mathcal{Q}(45_H) = -2\, \mathcal{Q}(10_3)\,. \tag{2.6}$$

This particular choice does not eliminate any texture on the leptonic sector obtained when varying the fermionic Z_4-charges. Thus, the most general fermionic Z_4-charges that lead to NNI for the quark mass matrices $M_{u,d}$ are

$$\begin{aligned}
\mathcal{Q}(10_i) &= (3q_3 + \phi,\ -q_3 - \phi,\ q_3), \\
\mathcal{Q}(5^*{}_i) &= (q_3 + 2\phi,\ -3q_3,\ -q_3 + \phi)\,,
\end{aligned} \tag{2.7}$$

where $\phi \equiv \mathcal{Q}(5_H)$ and $q_3 \equiv \mathcal{Q}(10_3)$. The charges for the n_{24} adjoint fermions are left free and only some combination of them will lead to realistic effective neutrino mass matrices, as we will see.

In this model, the most general Yukawa interactions are given by the following terms:

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{Y}} = \epsilon_{\alpha\beta\gamma\delta\xi} & \left[\left(\Gamma_u^1\right)_{ij} 10_i^{\alpha\beta} 10_j^{\gamma\delta} \left(5_H\right)^\xi + \left(\Gamma_u^2\right)_{ij} 10_i^{\alpha\beta} 10_j^{\kappa\gamma} \left(45_H\right)_\kappa^{\delta\xi} \right] \\
& + \left(\Gamma_d^1\right)_{ij} 10_i^{\alpha\beta} 5_{j\,\alpha}^* \left(5_H^*\right)_\beta + \left(\Gamma_d^2\right)_{ij} 10_i^{\alpha\beta} 5_{j\,\gamma}^* \left(45_H^*\right)_{\alpha\beta}^\gamma + \mathbf{M}_{kl} \operatorname{Tr}\left(\rho_k \rho_l\right) \\
& + \boldsymbol{\lambda}_{kl} \operatorname{Tr}\left(\rho_k \rho_l \Sigma\right) + \left(\Gamma_\nu^1\right)_{ik} 5_{i\,\alpha}^* \left(\rho_k\right)_\beta^\alpha \left(5_H\right)^\beta + \left(\Gamma_\nu^2\right)_{ik} 5_{i\,\alpha}^* \left(\rho_k\right)_\beta^\gamma \left(45_H\right)_\gamma^{\alpha\beta} + \text{H.c.}\,,
\end{aligned}
\tag{2.8}
$$

where $\alpha, \beta, \dots = 1, \dots, 5$ are $\mathsf{SU(5)}$ indices, i, j are generation indices and $k, l = 1, \dots, n_{24}$. Notice that the Yukawa matrix Γ_u^1 and $\boldsymbol{\lambda}$ as well as the mass matrix $\mathbf{M}$ are symmetric while Γ_u^2 is antisymmetric. Taking into account the charges given in eq. (2.7), the Yukawa coupling matrices $\Gamma_{u,d}^{1,2}$ are given by,

$$
\Gamma_u^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_u \\ 0 & b_u & 0 \end{pmatrix}, \quad
\Gamma_u^2 = \begin{pmatrix} 0 & a_u & 0 \\ a_u' & 0 & 0 \\ 0 & 0 & c_u \end{pmatrix},
\tag{2.9a}
$$

$$
\Gamma_d^1 = \begin{pmatrix} 0 & a_d & 0 \\ a_d' & 0 & 0 \\ 0 & 0 & c_d \end{pmatrix}, \quad
\Gamma_d^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & b_d \\ 0 & b_d' & 0 \end{pmatrix}.
\tag{2.9b}
$$

The up- and down-quark masses as well as the charged lepton masses are given by

$$
\begin{aligned}
M_u &= 4\, v_5\, \Gamma_u^1 + 8\, v_{45}\, \Gamma_u^2\,, \\
M_d &= v_5^*\, \Gamma_d^1 + 2\, v_{45}^*\, \Gamma_d^2\,, \\
M_e &= v_5^*\, \Gamma_d^{1\,\mathsf{T}} - 6\, v_{45}^*\, \Gamma_d^{2\,\mathsf{T}}\,.
\end{aligned}
\tag{2.10}
$$

Substituting the eqs. (2.9) in the mass matrices given in eqs. (2.10) one concludes that all the matrices $M_{u,d,e}$ share the NNI structure. The up-quark mass matrix M_u is no longer symmetric and the mismatch between the down-type and charged lepton matrices is now explained as:

$$
M_d - M_e^T = 8\, v_{45}^*\, \Gamma_d^2\,.
\tag{2.11}
$$

The mass matrices of the fermions ρ_0, ρ_3 and ρ_8 arising from the fermionic-24 fields are given by:

$$
\begin{aligned}
\mathbf{M}_0 &= \frac{1}{4}\left(\mathbf{M} - \frac{\sigma}{\sqrt{30}}\boldsymbol{\lambda}\right), \\
\mathbf{M}_3 &= \frac{1}{4}\left(\mathbf{M} - \frac{3\,\sigma}{\sqrt{30}}\boldsymbol{\lambda}\right), \\
\mathbf{M}_8 &= \frac{1}{4}\left(\mathbf{M} + \frac{2\,\sigma}{\sqrt{30}}\boldsymbol{\lambda}\right).
\end{aligned}
\tag{2.12}
$$

Due to the fact that the Higgs field Σ is trivial under $\mathsf{Z_4}$, the matrices $\mathbf{M}$ and $\boldsymbol{\lambda}$ share the same form and this is also valid for the Majorana matrices $\mathbf{M}_{0,3,8}$. From the Yukawa interactions written in eq. (2.8), one can infer the Yukawa couplings for the ρ_0, ρ_3 and ρ_8
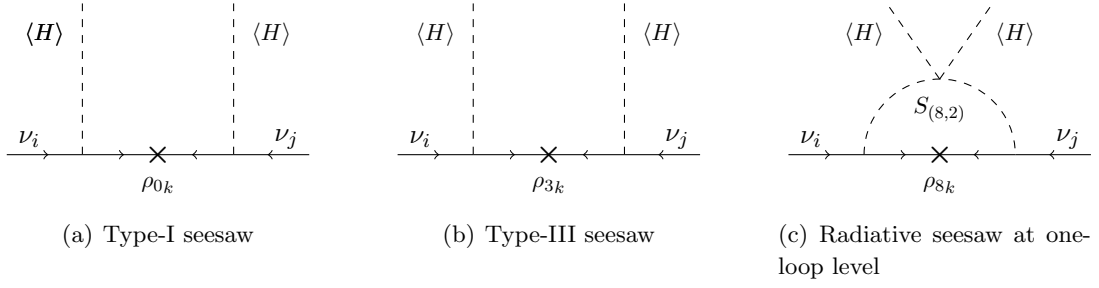
(a) Type-I seesaw   (b) Type-III seesaw   (c) Radiative seesaw at one-loop level

Figure 1. Different seesaw mechanisms present in the model.

fermion fields, which are then given by

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{Y}} ={}& \frac{\sqrt{15}}{2\sqrt{2}} \left[\frac{\cos\alpha}{5} \left(\Gamma_\nu^1\right)_{kl} + \sin\alpha \left(\Gamma_\nu^2\right)_{kl} \right] l_k^\mathsf{T} i\sigma_2 \, \rho_{0l} \, H \\
&+ \frac{\sqrt{15}}{2\sqrt{2}} \left[-\frac{\sin\alpha}{5} \left(\Gamma_\nu^1\right)_{kl} + \cos\alpha \left(\Gamma_\nu^2\right)_{kl} \right] l_k^\mathsf{T} i\sigma_2 \, \rho_{0l} \, H' \\
&+ \frac{1}{\sqrt{2}} \left[\cos\alpha \left(\Gamma_\nu^1\right)_{kl} - 3\sin\alpha \left(\Gamma_\nu^2\right)_{kl} \right] l_k^\mathsf{T} i\sigma_2 \, \rho_{3l} \, H \\
&- \frac{1}{\sqrt{2}} \left[\sin\alpha \left(\Gamma_\nu^1\right)_{kl} + 3\cos\alpha \left(\Gamma_\nu^2\right)_{kl} \right] l_k^\mathsf{T} i\sigma_2 \, \rho_{3l} \, H' \\
&- \frac{\left(\Gamma_\nu^2\right)_{kl}}{\sqrt{2}} l_k^\mathsf{T} i\sigma_2 \, \mathrm{Tr}\left(S_{(8,2)} \rho_{8l} \right) ,
\end{aligned}
\tag{2.13}
$$

where $S_{(8,2)}$ is the scalar octet-doublet belonging to the 45_H representation and the doublet space $(H_1, H_2)^\mathsf{T}$ has also been rotated in terms of new doublets $(H, H')^\mathsf{T}$ such that $\langle H \rangle = v$ and $\langle H' \rangle = 0$ by the appropriate transformation:

$$
\begin{pmatrix} H \\ H' \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} ,
\tag{2.14}
$$

with $\tan\alpha \equiv v_{45}/v_5$.

Taking into account the Majorana mass matrices $\mathbf{M}_{0,3,8}$ and the Yukawa interactions given by eq. (2.13) one can derive, through the seesaw mechanism, the effective neutrino mass matrix m_ν which receives three different contributions, as drawn in figure 1. One has type-I seesaw [28–31] mediated by the fermionic singlets ρ_{0k} and type-III seesaw [32, 33] via the exchange of the $\mathsf{SU}(2)$-triplets ρ_{3k}. There is still the possibility of generating neutrino masses through the radiative seesaw [34, 35], that involves at 1-loop the fermionic ρ_{8k} and the scalar doublet-octet $S_{(8,2)}$ present in the 45_H. The neutrino mass matrix obtained after

integrating out the fields responsible for the seesaw mechanism reads as

$$(m_\nu)_{ij} = - \left(m_0^D \mathbf{M}_0^{-1} m_0^{D\mathsf{T}} \right)_{ij} - \left(m_3^D \mathbf{M}_3^{-1} m_3^{D\mathsf{T}} \right)_{ij}$$

$$- \frac{v^2 \zeta}{8\pi^2} \sum_{k=1}^{n_{24}} \frac{\left(U_8 \, \Gamma_\nu^2 \right)_{ik} \left(U_8 \, \Gamma_\nu^2 \right)_{jk}}{\widetilde{M}_{8\,k}} \, F\left[\frac{M_{S_{(8,2)}}}{\widetilde{M}_{8\,k}} \right], \tag{2.15}$$

where m_0^D, m_3^D are given by

$$m_0^D = \frac{\sqrt{15}v}{2\sqrt{2}} \left(\frac{\cos\alpha}{5} \Gamma_\nu^1 + \sin\alpha \, \Gamma_\nu^2 \right),$$
$$m_3^D = \frac{v}{\sqrt{2}} \left(\cos\alpha \, \Gamma_\nu^1 - 3 \sin\alpha \, \Gamma_\nu^2 \right), \tag{2.16}$$

and $\widetilde{M}_{8\,k}$ are simply the mass-eigenvalues of the Majorana matrix $\mathbf{M}_8$. The unitary matrix U_8 does the rotation of the yukawa matrix Γ_ν^2 to the basis where the matrix $\mathbf{M}_8$ is diagonal. The coefficient ζ is a linear combination of the coefficients in the Higgs potential terms given in eqs. (B.2c) and (B.2f) in the appendix (B). The loop function $F(x)$ is given by

$$F(x) \equiv \frac{x^2 - 1 - \log x}{(1 - x^2)^2}. \tag{2.17}$$

If one assumes $\widetilde{M}_{8\,k} \gg \widetilde{M}_{0\,k} > \widetilde{M}_{3\,k}$ then it suppresses the 1-loop radiative seesaw contribution.

Before closing this section, it is important to comment about the Higgs potential. The most general Higgs potential is given explicitly in the appendix (B). Notice that, terms involving simultaneously the fields 5_H, 24_H and 45_H are forbidden by the Z_4 symmetry. This gives rise to an accidental global continuous symmetry which, upon spontaneous electroweak symmetry breaking, would lead to a massless Nambu-Goldstone boson at tree level [41]. A simple way to cure this problem is by adding a complex $SU(5)$ Higgs singlet S non-trivially charged under Z_4, i.e. $\mathcal{Q}(S) = -2\,q_3 - \phi$, where its potential is given by

$$V_S = \left(\lambda_{\mathrm{SB}} \, 5_\alpha^* \, 24_\beta^\gamma \, 45_\gamma^{\alpha\beta} \, S + \mathrm{H.c.} \right) + \frac{1}{2} \mu_S^2 \, |S|^2 + \lambda_S \, |S|^4 + \lambda_S' \, (S^4 + \mathrm{H.c.}), \tag{2.18}$$

and leads at low-energy to an effective interaction, once the scalar S acquires vacuum expectation value,

$$\lambda_{\mathrm{SB}} \, \sigma \, \langle S \rangle \, H_1^\dagger \, H_2 + \mathrm{H.c.}, \tag{2.19}$$

which softly breaks the symmetry Z_4.

3 Unification and Proton Stability

According to the previous section, between the unification scale and $M_Z = 91.1876 \pm 0.0021\,\mathrm{GeV}$ scale [38] one has a 2HDM with extra fermions, namely ρ_{0k}, ρ_{3k}, and ρ_{8k}, and the two scalars Σ_3 and Σ_8 that can have lower masses. The coulored triplets, T_1 and T_2, and the other scalars contained in the 45_H are set their masses around the GUT scale. We

also assume $M_{\Sigma_3} \simeq M_{\Sigma_8}$. The running of the three gauge coupling constants α_i $(i = 1, 2, 3)$ in the 2HDM with extra particle content can be obtained easily at the one-loop level as

$$\alpha_1^{-1}(\mu) = \alpha_1^{-1}(M_Z) - \frac{b_1}{2\pi} \log\left(\frac{\mu}{M_Z}\right) - \sum_I \frac{b_1^I}{2\pi} \log\left(\frac{\mu}{M_I}\right), \tag{3.1a}$$

$$\alpha_2^{-1}(\mu) = \alpha_2^{-1}(M_Z) - \frac{b_2}{2\pi} \log\left(\frac{\mu}{M_Z}\right) - \sum_I \frac{b_2^I}{2\pi} \log\left(\frac{\mu}{M_I}\right), \tag{3.1b}$$

$$\alpha_3^{-1}(\mu) = \alpha_3^{-1}(M_Z) - \frac{b_3}{2\pi} \log\left(\frac{\mu}{M_Z}\right) - \sum_I \frac{b_3^I}{2\pi} \log\left(\frac{\mu}{M_I}\right), \tag{3.1c}$$

where $\alpha_1 = 5/3\,\alpha_y$, $\alpha_2 = \alpha_w$ and $\alpha_3 = \alpha_s$; the b_i constants are the usual one-loop beta coefficients corresponding to the 2HDM, listed in section A. M_I denotes an intermediate energy scale for extra particle I between the electroweak scale M_Z and the GUT scale Λ, and the coefficients b_i^I account for the new contribution to the one-loop beta functions b_i above the threshold M_I. At the unification scale Λ, the gauge couplings α_i obey to the relation

$$\alpha_U \equiv \alpha_1(\Lambda) = \alpha_2(\Lambda) = \alpha_3(\Lambda). \tag{3.2}$$

To get some insight into the unification in the one-loop approximation, let us define the effective beta coefficients B_i [42],

$$B_i \equiv b_i + \sum_I b_i^I r_I, \tag{3.3}$$

where the ratios $0 \leq r_I \leq 1$ that takes into account the intermediate scales are given by

$$r_I = \frac{\ln\left(\Lambda/M_I\right)}{\ln\left(\Lambda/M_Z\right)}. \tag{3.4}$$

It is also convenient to introduce the differences $B_{ij} \equiv B_i - B_j$, define as

$$B_{ij} = B_{ij}^{\text{2HDM}} + \sum_I \Delta_{ij}^I r_I, \tag{3.5}$$

where B_{ij}^{2HDM} corresponds to the 2HDM particle contribution and

$$\Delta_{ij}^I \equiv b_i^I - b_j^I. \tag{3.6}$$

The following B-test is then obtained,

$$B \equiv \frac{B_{23}}{B_{12}} = \frac{\sin^2\theta_W - \dfrac{\alpha}{\alpha_s}}{\dfrac{3}{5} - \dfrac{8}{5}\sin^2\theta_W}, \tag{3.7}$$

together with the GUT scale relation

$$B_{12} \ln\left(\frac{\Lambda}{M_Z}\right) = \frac{2\pi}{5\alpha}\left(3 - 8\sin^2\theta_W\right). \tag{3.8}$$

Notice that the right-hand sides of eqs. (3.7) and (3.8) depend only on low-energy electroweak data. using the following experimental values at M_Z [38]

$$\alpha^{-1} = 127.916 \pm 0.015 \,, \tag{3.9}$$

$$\sin^2 \theta_W = 0.23116 \pm 0.00012 \,, \tag{3.10}$$

$$\alpha_s = 0.1184 \pm 0.0007 \,, \tag{3.11}$$

the above relations read as

$$B = 0.718 \pm 0.003 \,,$$

$$B_{12} \ln\left(\frac{\Lambda}{M_Z}\right) = 185.0 \pm 0.2 \,. \tag{3.12}$$

The coefficients B_{ij} that appear in the left-hand sides of eqs. (3.7) and (3.8) strongly depend on the particle content of the theory. For instance, considering the SM particles with n_H light Higgs doublets, one has $b_1 = 20/5 + n_H/10$, $b_2 = -10/3 + n_H/6$ and $b_3 = -7$, so that these coefficients are given by

$$B_{12} = 36/5 \,, \quad B_{23} = 4 \,. \tag{3.13}$$

where $n_H = 2$ is set and $B = 5/9$ is then not compatible with the calculated value in eq. (3.12) and clearly, the B-test fails badly in the 2HDM case, so that extra particles are needed. In table 1 we present the relevant contributions Δ_{ij} to the B_{ij} coefficients of our setup which include, besides the 2HDM threshold, the fermions ρ_{0k}, ρ_{3k}, and ρ_{8k}, and the two scalars Σ_3 and Σ_8 are considered. For simplicity, we assume the remaining particles at the unification scale and therefore they do not contribute to the gauge coupling running.

Table 1. The relevant Δ_{ij} contributions to the B_{ij} coefficients in the $\mathsf{SU}(5) \times \mathsf{Z}_4$ model.

	2HDM	ρ_3	ρ_8	Σ_3	Σ_8
Δ_{12}	36/5	-4/3	0	-2/3	0
Δ_{23}	4	4/3	-2	2/3	-1

Notice that eqs. (3.12) require $\Delta_{12}^I < 0$ and $\Delta_{23}^I > 0$, it becomes clear from table 1 all extra particles considered in the running improve the unification. We have scanned different ratios r_I and we obtained a large range of solutions that lead to a perfect unification within the experimental errors. The fact that more than one adjoint fermionic field ρ_k is present, it improved the range of intermediate scales M_I consistent with unification. It is now difficult to find strong correlations among the intermediate scales M_I. In addition, the possible values for the unification scale Λ can vary many orders of magnitude. For illustration, in figure 2 we have drawn the mass spectrum of the extra particles included in the running. All the solution obtained are in agreement with a unified gauge coupling $g_U = \sqrt{4\pi/\alpha_U^{-1}} < 1$, where in our numerics we obtained $\alpha_U^{-1} \approx 37$.

Concerning the proton decay some comments are in order. In this model there are mainly two different sources for proton decay, namely via the exchange of the lepto-quark
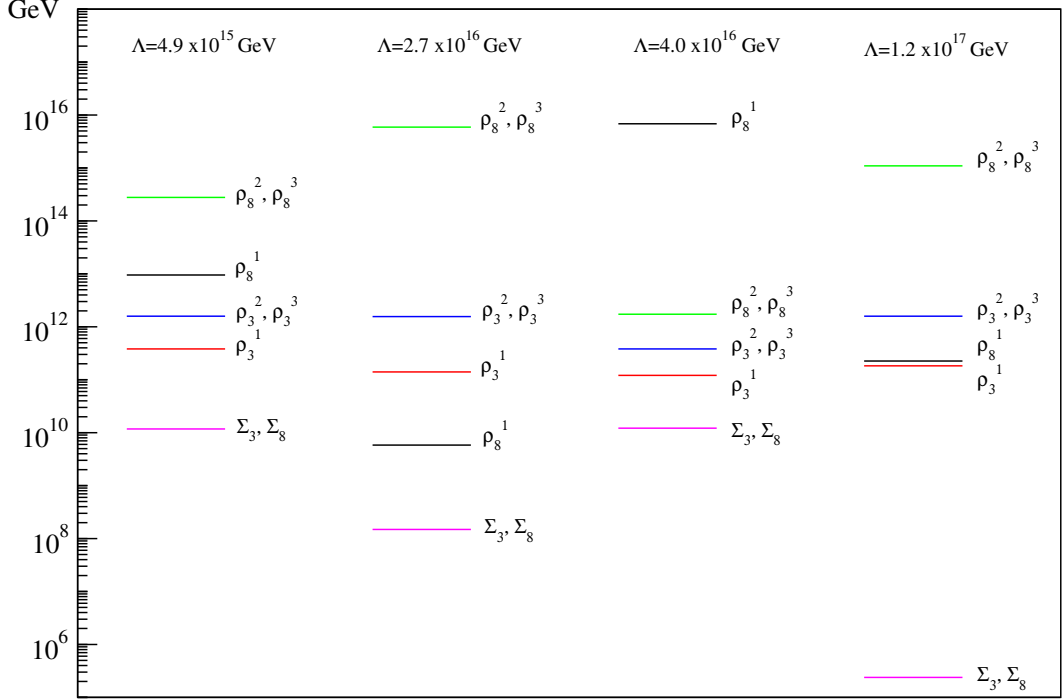
Figure 2. Four illustrative examples showing the mass spectrum of the adjoint fermionic fields, Σ_3 and Σ_8 for different unification scales Λ.

gauge bosons X, Y or via coulored Higgs triplets. Proton decay in both scenarios are mediated four fermion interactions (dimension-six operators).

The gauge bosons X, Y become massive through the Higgs mechanism with a common mass, M_V,

$$M_V = \frac{25}{8} g_U^2 \sigma^2 \,. \tag{3.14}$$

To suppress the X, Y boson proton decay channels, one has necessarily that $M_V \gg m_p$ (the proton mass), that leads to the estimation of the proton decay width as [8]:

$$\Gamma \approx \alpha_U^2 \frac{m_p^5}{M_V^4} \,. \tag{3.15}$$

Making use of the most restrictive constraints on the partial proton lifetime $\tau(p \to \pi^0 e^+) > 8.2 \times 10^{33}$ years [38], one can derive a rough lower bound for the X, Y mass scale M_V,

$$M_V > 4.1 \times 10^{15} \,\text{GeV} \,, \tag{3.16}$$

which corresponds a $\alpha_U^{-1} \approx 37$. Since we assume for the unification scale $\Lambda \sim M_V$, the constraint given by eq. (3.16) determines the scale where the gauge couplings should unify (for a recent review see [1]).

The proton decay through the exchange of Higgs colour triplets T_1, T_2 is very suppressed, since their suppression is proportional to products of Yukawa couplings, and

therefore they are much smaller than the gauge couplings. Indeed the contribution of these dimension-six operators vanishes at tree-level when the Z_4 symmetry is exact [23]. The dimension-six operators contributing to the proton decay via the colour triplet exchange are given at tree-level by:

$$\sum_{n=1,2} \frac{(\Gamma_u^n)_{ij} (\Gamma_d^n)_{kl}}{M_{T_n}^2} \left[\frac{1}{2}(Q_i Q_j)(Q_k L_l) + (u_i^c e_j^c)(u_k^c d_l^c) \right] . \qquad (3.17)$$

It is then clear from the pattern of the Yukawa coupling matrices Γ_u^1 and Γ_d^2 given in eqs. (2.9) that the only possible non-vanishing contribution of the dimension-six operators given in eq. (3.17) involve necessarily fermions of the third generation. One concludes that at tree-level the proton does not decay through the four-fermion interactions described by the operators given in eq. (3.17).

4 Effective Neutrino Textures

The flavour symmetry present in our model constrains the charges of the fermion fields to be of the form in eq. (2.7). Such charge assignment does not imply any restriction in the neutrino sector. Hence the charges of the 24 dimensional fermionic representations responsible for the neutrino masses are free. In order to analyse the possible patterns for the effective neutrino mass matrices, m_ν, we have considered all the possible values for the Z_4 charges of the adjoint fermionic fields.

Searching for the minimal $SU(5) \times Z_4$ model, i.e. the model with the minimal matter content, we have started by the possibility of having only one extra 24 fermionic representation, as in the $SU(5)$ adjoint original model [6]. However, given the particularities of the Z_4 symmetry, the neutrino mixing pattern that emerges from this picture is now not consistent with the experimental neutrino data [43]. Adding a second 24 fermionic representation does not solve the problem and again the predicted neutrino mixing angles are not in agreement with neutrino oscillations.

The situation changes when we consider three 24 fermionic representations. In this case we obtain different possibilities for the light neutrino mass matrix m_ν that coincide with the matrices found in ref. [23], where a similar Z_4 flavour symmetry was imposed in the context of three right-handed neutrinos. From the various textures for the effective neutrino mass matrix m_ν found in the scan, only two solutions can account successfully for the low-energy neutrino data, namely

$$m_\nu^A = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix} \qquad \text{and} \qquad m_\nu^{A_{(12)}} = \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix} , \qquad (4.1)$$

where the index (12) refers to the fact that texture-$A_{(12)}$ is a permutated form of the texture-A through the permutation matrix $P_{(12)}$,

$$P_{(12)} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \qquad (4.2)$$

isomorphic to the symmetric group S_3. Thus, the effective mass matrices m_ν^A and $m_\nu^{A_{(12)}}$ are related by:

$$m_\nu^{A_{(12)}} = P_{(12)}\, m_\nu^A\, P_{(12)}^\mathsf{T}\,. \tag{4.3}$$

The compatibility of the above textures with the experimental neutrino data has been analyzed in detail in ref. [23] where it was found that just the two-zero textures in A and $A_{(12)}$ are phenomenologically viable. An important result is that texture A is compatible only with normal hierarchy (NH) while texture A_{12} turns to be compatible only with inverted hierarchy (IH) in the light neutrino mass spectrum.

At this point, we would like to remark that the textures given in eq. (4.1) have also been studied in the literature previuosly. According to the standard terminology [44], our allowed matrices A and $A_{(12)}$ would correspond to the ones labelled as $\mathrm{A_2}$ and $\mathrm{D_1}$, respectively. Texture $\mathrm{D_1}$ has been shown to be either disallowed [44–46] or very marginally allowed [47]. However, all of these previous works assume a diagonal charged lepton mass matrix while here we are considering a charged lepton mass matrix with NNI form. Therefore the results in the literature do not strictly apply to our case.

Up to here we have only considered the $\mathsf{Z_4}$ charges for the 24 fermionic fields such as the matrices $\mathbf{M}$ and $\boldsymbol{\lambda}$ are non-singular, i.e., $|\mathbf{M}| \neq 0$ and $|\boldsymbol{\lambda}| \neq 0$ and therefore the Majorana matrices $\mathbf{M}_x$, $x = 0, 3, 8$ are non-singular as well. This condition is mandatory in order to derive the effective neutrino mass matrix using the seesaw formula in eq. (2.15). However, given the flavour symmetry present in our model, among all the possible configurations one can have $|\mathbf{M}_x| = 0$. In these cases $\mathbf{M}_x^{-1}$ is not defined and therefore we can not use the standard seesaw formula, but instead we should consider the singular seesaw mechanism. The singular seesaw, first suggested in the context of a GUT framework [48], has been considered in relation to different anomalies in neutrino physics such as the Simpson neutrino [49–51] or the LSND signal [52–55]. In both cases the modified seesaw scheme has been used to obtain a neutrino mass spectrum with a singlet/sterile neutrino in the energy range between light neutrino masses below the eV and the heavy neutrinos at the seesaw scale.

Here we have analysed the neutrino mass spectrum that emerges from the singular seesaw mechanism with two and three 24 fermionic representations. According to our calculations, it is not possible to obtain an effective neutrino mass matrix compatible with experimental data in any of the cases, since we always get too few light neutrino states.

In summary, our analysis shows that the symmetry requires the presence of at least three fermionic 24 representations and no singular seesaw, i.e., $|\mathbf{M}_x| \neq 0$. Since the parameters of the last matrix depend on the $\mathsf{Z_4}$ neutrino charges $\mathcal{Q}(24_i)$, we can conclude that the neutrino phenomenology has an impact on the $\mathsf{Z_4}$ neutrino charges.

We present in table 2 the $\mathsf{Z_4}$ charge assignment for the fermionic fields that leads to the successful textures for the effective neutrino mass matrix A and $A_{(12)}$, discussed above. The textures for the matrices $\mathbf{M_x}$ and $m_{0,3}^D$ are also shown. The charges for the fermionic fields 10 and 5^* as well as the Higgs fields 5_H and 45_H follow the relations in eqs. (2.6) and (2.7). It is worth pointing out that all Dirac neutrino matrices $m_{0,3}^D$ obtained through the scan have four texture zeroes.

Table 2. The Z_4 fermionic field charges for phenomenologically viable effective neutrino textures.

m_ν	$\mathbf{M}_{0,3,8}$	$\mathcal{Q}(24_i)$	$m_{0,3}^D$	$\mathcal{Q}(5^*)$	$\mathcal{Q}(10_i)$	$\mathcal{Q}(5_H)$	$\mathcal{Q}(45_H)$
$\begin{pmatrix}0&*&0\\ *&*&*\\ 0&*&*\end{pmatrix}$	$\begin{pmatrix}0&0&*\\ 0&*&0\\ *&0&0\end{pmatrix}$	$(1,2,3)$	$\begin{pmatrix}0&0&*\\ *&*&0\\ 0&*&*\end{pmatrix}$	$(3,1,0)$	$(0,2,1)$	1	2
			$\begin{pmatrix}*&0&0\\ 0&*&*\\ *&*&0\end{pmatrix}$	$(1,3,0)$	$(0,2,3)$	3	2
	$\begin{pmatrix}*&0&0\\ 0&0&*\\ 0&*&0\end{pmatrix}$	$(0,1,3)$	$\begin{pmatrix}0&*&0\\ *&0&*\\ *&*&0\end{pmatrix}$	$(1,3,2)$	$(2,0,3)$	1	2
			$\begin{pmatrix}0&0&*\\ *&*&0\\ *&0&*\end{pmatrix}$	$(3,1,2)$	$(2,0,1)$	3	2
$\begin{pmatrix}*&*&*\\ *&0&0\\ *&0&*\end{pmatrix}$	$\begin{pmatrix}0&0&*\\ 0&*&0\\ *&0&0\end{pmatrix}$	$(1,2,3)$	$\begin{pmatrix}*&*&0\\ 0&0&*\\ 0&*&*\end{pmatrix}$	$(2,0,1)$	$(1,3,0)$	1	0
			$\begin{pmatrix}0&*&*\\ *&0&0\\ *&*&0\end{pmatrix}$	$(2,0,3)$	$(3,1,0)$	3	0
	$\begin{pmatrix}*&0&0\\ 0&0&*\\ 0&*&0\end{pmatrix}$	$(0,1,3)$	$\begin{pmatrix}*&0&*\\ 0&*&0\\ *&*&0\end{pmatrix}$	$(0,2,3)$	$(3,1,2)$	1	0
			$\begin{pmatrix}*&*&0\\ 0&0&*\\ *&0&*\end{pmatrix}$	$(0,2,1)$	$(1,3,2)$	3	0

Leptogenesis

In this section we would like to briefly comment about the possibility of having leptogenesis in our model.

As already discussed in refs. [26, 27], the out-of-equilibrium decays of the fermionic fields ρ_0 and ρ_3 in the 24 fermionic representation may generate an asymmetry in the leptonic content of the universe. In the presence of sphaleron processes this leptonic asymmetry would be partially converted into a baryon asymmetry, explaining the observed matter-antimatter asymmetry of the universe. Depending on the mass hierarchy among ρ_0 and ρ_3, the main contribution to the leptonic asymmtery will be dominated by the decays of one of them. In principle, our model has enough freedom to have different mass spectra

for the fermionic fields and therefore, in contrast to the results shown in refs. [26, 27], in our case the leptonic asymmetry may be generated by the decay of ρ_0 (ϵ_0) or ρ_3 (ϵ_3). In both cases, the expression for the generated CP asymmetry would be proportional to:

$$\epsilon_3 \propto \sum_{j \neq 1} \text{Im} \left[\left(m_3^{D\dagger} m_3^D \right)_{1j}^2 \right] , \tag{4.4a}$$

$$\epsilon_0 \propto \sum_{j \neq 1} \text{Im} \left[\left(m_0^{D\dagger} m_0^D \right)_{1j}^2 \right] . \tag{4.4b}$$

Despite the specific flavour structure of the m_i^D matrices, induced by the Z_4 symmetry (see eq. (2.16)), we have checked that in principle there are no cancelations in the terms above and therefore the leptonic asymmetry generated by ρ_0 and ρ_3 can be different from zero. It is clear that more accurate predictions about leptogenesis would require further calculations, considering the effect of the washout over the initial leptonic asymmetry as well as the dynamical evolution of the asymmetry with the solution of Boltzmann equations. For the moment, however, our goal is just to show that the model presented here has enough freedom in the choice of masses and couplings so in principle it is possible to accommodate the CP asymmetry. Further considerations as, for instance, the constraints on the model coming from the requirement of a baryon asymmetry consistent with the observations will be discussed elsewhere.

5 Numerical Results

In this section we analyse the phenomenological implications of the effective neutrino mass matrices m_ν^A and $m_\nu^{A_{(12)}}$ in eq. (4.1). Since the flavour symmetry is valid under perturbative corrections until the breaking of the electroweak gauge symmetry, the form of the Yukawa matrices $\Gamma_d^{1,2}$, $\Gamma_\nu^{1,2}$ and the Majorana mass matrices $\mathbf{M}_{0,3,8}$ remains unchanged. Thus, one can extract the predictions for M_e and m_ν and confront them with the observed neutrino data at M_Z energy scale. The effective neutrino mass matrices obtained $m_\nu^{A/A_{(12)}}$, as already mentioned, are the same as those analysed in ref. [23]. However, the new measurements of the reactor mixing angle θ_{13} [56] have changed the theoretical picture of the light neutrino mixings since then. Therefore, it is worth to revisit the previous analysis in ref. [23] to take into account these new bounds.

Without loss of generality one can write the charged lepton mass matrix, M_e, and the effective neutrino mass matrices, $m_\nu^{(g)}$ as:

$$M_e = K_e \begin{pmatrix} 0 & A_e & 0 \\ A'_e & 0 & B_e \\ 0 & B'_e & C_e \end{pmatrix} , \tag{5.1a}$$

$$m_\nu^g = P_g \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & B_\nu & C_\nu \\ 0 & C_\nu & D_\nu \, e^{i\varphi} \end{pmatrix} P_g^{\mathsf{T}} , \tag{5.1b}$$

where the permutation $g = e$ or (12) according to the table 2 and the constants $A_{e,\nu}$, $B_{e,\nu}$, A'_e, B'_e, $C_{e,\nu}$, D_ν are taken real and positive. The diagonal phase matrix K_e can be parameterised as

$$K_e = \mathrm{diag}(e^{i\kappa_1}, e^{i\kappa_2}, 1) \,, \tag{5.2}$$

and the phase φ in eq. (5.1b) cannot be removed by any field redefinition.

Although the number of the parameters encoded in the pair M_e, m_ν is 12, as the number of independent physical parameters experimentally observed at low energy, the zero pattern exhibited in eqs. (5.1) does imply new constraints among the independent physical parameters, as it will be shown. The PMNS matrix U is given by

$$U = O_e^{\mathsf{T}} K_e^{\dagger} P_g U_\nu \,, \tag{5.3}$$

where the orthogonal matrix O_e is the one that diagonalises $M_e M_e^{\dagger}$ as

$$(K_e \, O_e)^{\dagger} \, M_e \, M_e^{\dagger} \, (K_e \, O_e) = \mathrm{diag}(m_e \,, m_\mu \,, m_\tau) \,, \tag{5.4}$$

while the unitary matrix U_ν diagonalises m_ν as

$$(P_g \, U_\nu)^{\mathsf{T}} \, m_\nu^g \, (P_g \, U_\nu) = \mathrm{diag}(m_1 \,, m_2 \,, m_3) \,. \tag{5.5}$$

The knowledge of the low-energy neutrino mixings appears in the literature in terms of the parameters θ_{12}, θ_{13}, θ_{23} and δ of the Standard Parametrisation (SP) [38], defined in terms of PMNS matrix U invariants as

$$
\begin{aligned}
\sin \theta_{12} &\equiv \frac{|U_{e2}|^2}{\sqrt{1 - |U_{e3}|^2}} \,, \\
\sin \theta_{13} &\equiv |U_{e3}| \,, \\
\sin \theta_{23} &\equiv \frac{|U_{\mu3}|^2}{\sqrt{1 - |U_{e3}|^2}} \,,
\end{aligned}
\tag{5.6}
$$

and the phase δ is given by the Dirac-phase invariant, I,

$$I \equiv \mathrm{Im}\left(U_{\mu3} U_{e3}^* U_{e2} U_{\mu2}^*\right) = \frac{1}{8} \cos\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \sin\delta \,. \tag{5.7}$$

Due to the fact that the PMNS matrix is not rephasing invariant on the right, one defines the Majorana-type phases, φ_{ij}^α, free of any kind of parametrisation as [57]:

$$\varphi_{ij}^\alpha \equiv \arg\left(U_{\alpha i} \, U_{\alpha j}^*\right) \,. \tag{5.8}$$

It has been shown in ref. [57] that the PMNS matrix can be fully reconstructed by six independent Majorana-type phases from eq. (5.8) taking into account that U is a unitary matrix. The Dirac-type phase δ can therefore be expressed as the difference of two Majorana-type phases:

$$I = |U_{\mu3}| \, |U_{\mu2}| \, |U_{e3}| \, |U_{e2}| \sin\left(\varphi_{23}^e - \varphi_{23}^\mu\right) \,. \tag{5.9}$$

In what follows we will use the three Majorana-type phases φ_{23}^e, φ_{23}^μ and φ_{23}^τ.

Table 3. Neutrino oscillation parameter summary from ref. [43]. For Δm_{31}^2, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$, and δ the upper (lower) row corresponds to normal (inverted) neutrino mass hierarchy.

parameter	best fit	3σ range
$\Delta m_{21}^2 \left[10^{-5}\ \text{eV}\right]$	7.62	$7.12 - 8.20$
$|\Delta m_{31}^2| \left[10^{-3}\ \text{eV}\right]$	2.55 2.43	$2.31 - 2.74$ $2.21 - 2.64$
$\sin^2\theta_{12}$	0.320	$0.27 - 0.37$
$\sin^2\theta_{23}$	0.613 (0.427) 0.600	$0.36 - 0.68$ $0.37 - 0.67$
$\sin^2\theta_{13}$	0.0246 0.0250	$0.017 - 0.033$
δ	0.80π -0.03π	$0 - 2\pi$

In our analysis we have calculated O_e numerically using the charged lepton masses given at M_Z scale in the $\overline{MS}$ scheme at 1-loop [58, 59] as

$$m_e = 0.486661305 \pm 0.000000056\ \text{MeV}\,, \tag{5.10a}$$

$$m_\mu = 102.728989 \pm 0.000013\ \text{MeV}\,, \tag{5.10b}$$

$$m_\tau = 1746.28 \pm 0.16\ \text{MeV}\,. \tag{5.10c}$$

Concerning the neutrino sector we have used the most recent three neutrino data from the global fit of neutrino oscillations in ref. [43]. The best fit values and 3σ ranges for the neutrino parameters are presented in table 3.

As in ref. [23], here we have varied all the experimental charged lepton masses and neutrino mass differences within their allowed range given in eq. (5.10) and Table 3, respectively. The mass of the lightest neutrino (m_1 in NH or m_3 in IH) was scanned for different magnitudes below 1 eV. To reconstruct the PMNS matrix, we have also scanned the free parameters A_e, B_e, D_ν and the phases κ_1, κ_2, φ, defined in eq. (5.1). All the remaining parameters are calculated in terms of the former ones. The restriction in this scan was to accept only the input values which correspond to a reconstructed PMNS matrix U that naturally leads to the mixing angles θ_{12}, θ_{23} and θ_{13} within their experimental bounds presented in Table 3.

From our scan we have found that the mass matrix A in eq. (4.1) is compatible with a neutrino mass spectrum with normal hierarchy while the texture $A_{(12)}$ is compatible with inverted hierarchy. The allowed ranges for the lightest neutrino masses are $m_1 = [0.353, 20.884] \times 10^{-3}$ eV for NH and $m_3 = [2.575, 15.335] \times 10^{-3}$ eV for IH. The presence of a massless neutrino as well as a quasi-degenerate neutrino mass spectrum are excluded in both cases. For the texture A we have found no significant correlations between $\sin^2\theta_{13}$
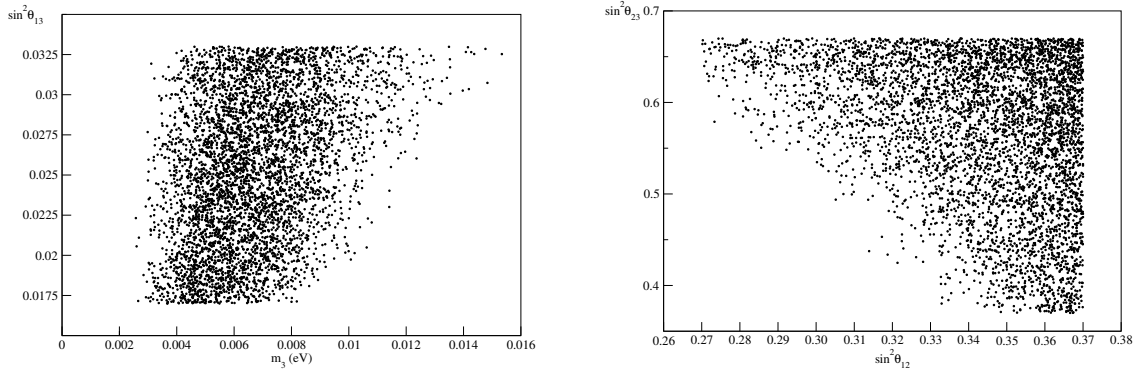
Figure 3. Plot of $\sin^2\theta_{13}$ as a function of m_3 (left panel) and $\sin^2\theta_{23}$ as a function of $\sin^2\theta_{12}$ (right) in the case of texture $m_\nu^{A_{(12)}}$ and inverted hierarchy.

and $\sin^2\theta_{23}$ as a function of m_1, while in the case of texture $A_{(12)}$ some correlations are found, as shown in figure 3. In fact, this correlation is behind the narrower m_3 allowed range for IH in comparison with the allowed m_1 range for NH. We have also verified that textures A and $A_{(12)}$ are not compatible with inverted and normal hierarchies, respectively, even when the new limits on $\sin^2\theta_{13}$ are considered.

In figure 4 we plot the effective Majorana neutrino mass characterizing the neutrinoless double beta decay amplitude m_{ee} with respect to the lightest neutrino mass, m_1 in the case of m_ν^A and normal hierarchy or m_3 in the case of $m_\nu^{A_{(12)}}$ and inverted hierarchy. The shadowed bands correspond to the generic predictions for m_{ee} according to the experimental neutrino data at 3σ, without any further assumption concerning the origin of neutrino masses. If we now restrict our calculations to the adjoint $\mathsf{SU}(5) \times \mathsf{Z}_4$ model presented in this article, the allowed regions are reduced to the darker pointed regions. The horizontal lines in figure 4 correspond to the m_{ee} sensitivity that will be reached by the next generation of neutrinoless double beta decay experiments (see for instance ref. [60]). There we see that, even if a part of the inverse hierarchy band will be experimentally covered in the next years, a sensitivity of around 10-30 meV will be needed in order to probe the effective Majorana mass predicted by our model. Accessing to the predicted region for normal hierarchy will be even tougher, since a sensitivity of the order of 1 meV would be required, far away from the more optimistic scenarios.

6 Conclusions

In this work we have studied the adjoint-$\mathsf{SU}(5) \times \mathsf{Z}_4$ model. The flavour symmetry imposed in the Lagrangian has the purpose to force the quark mass matrices M_u, M_d to have the NNI form after the spontaneous electroweak symmetry breaking [20, 23]. Due to the fermion content of the adjoint-$\mathsf{SU}(5)$, the charged lepton mass matrix M_e has automatically NNI form. In this model the light neutrinos get their masses through type-I, type-III and one-loop radiative seesaw mechanisms, implemented, respectively, via a singlet, a triplet and
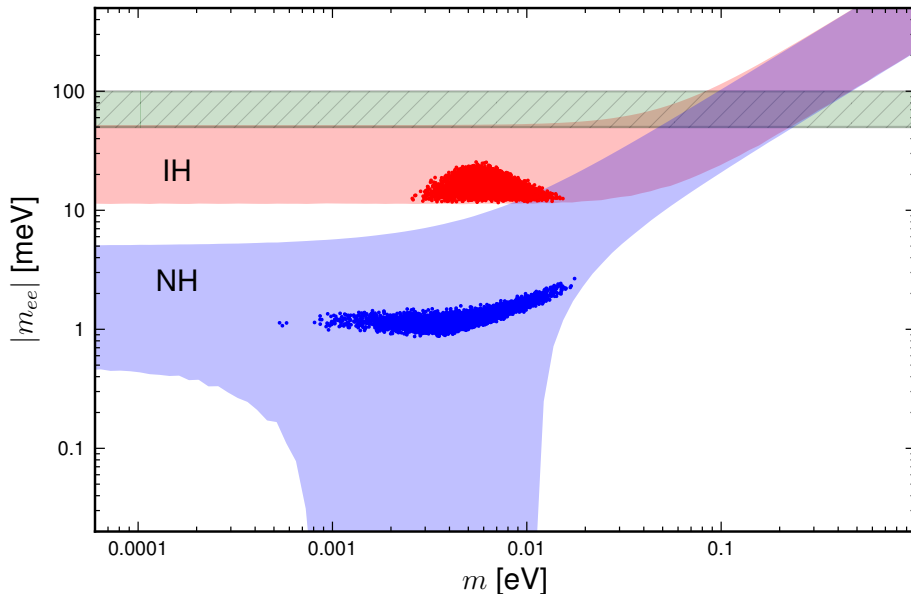
Figure 4.   Effective Majorana neutrino mass m_{ee} as a function of the lightest neutrino mass m for normal and inverted neutrino hierarchy, as indicated. The upper band shows the experimental sensitivity to be achieved in the next years.

an octet from the adjoint fermionic fields. The $SU(5) \times Z_4$ symmetry does not impose any constraint on the adjoint fermionic fields.

We have shown that the model proposed is in agreement with the current experimental data. Neutrino mixings and mass splittings as well as the masses of the charged leptons have been used to constrain the possible textures of the effective light neutrino mass matrix. We have demonstrated that at least three copies of the 24 are needed in order to fully implement the Z_4 flavour symmetry and simultaneously account for the experimental neutrino data. As shown in table 2 only two zero-textures persist: A and $A_{(12)}$, which are compatible with normal and inverted hierarchies, respectively.

One of the main phenomenological implications of the model studied is the prediction of a hierarchical neutrino mass spectrum not compatible with a massless neutrino. This result is particularly important since the neutrino mass spectrum predicted can be used to prove or disprove the model in the near future. At present our results are in agreement with the constraints coming from neutrinoless double beta decay [60] and tritium β decay searches [61] as well as with the cosmological bound on the sum of light neutrino masses [62]. However, a positive signal of neutrinoless double beta decay in the next years as well as a cosmological measurement of the sum of neutrino masses of the order of 0.1 eV would certainly rule out this type of model. Therefore, future experimental improvements in the neutrino physics will be decisive for testing the viability of the $SU(5) \times Z_4$ model.

Acknowledgments

A Matter and Higgs representations

Fermionic representations

The fermionic fields in the model decompose in terms of the SM gauge quantum numbers as:

$$
\begin{aligned}
\mathbf{5}^* &= d^c \oplus L \,, \\
\mathbf{10} &= Q \oplus u^c \oplus e^c \,, \\
\mathbf{24} &= \rho_8 \oplus \rho_3 \oplus \rho_{(3,2)} \oplus \rho_{(\bar{3},2)} \oplus \rho_0 \,.
\end{aligned}
\tag{A.1}
$$

The fermionic representations $\mathbf{5}^*$ and $\mathbf{10}$ can be written as

$$
\mathbf{5}^*_\alpha = (d^c)_\alpha \,, \quad \mathbf{5}^*_i = \varepsilon_{ij} l^j \,,
\tag{A.2}
$$

and

$$
\mathbf{10}^{\alpha\beta} = \frac{1}{\sqrt{2}} \varepsilon^{\alpha\beta\gamma} (u^c)_\gamma, \quad \mathbf{10}^{\alpha i} = -\frac{1}{\sqrt{2}} q^{\alpha i}, \quad \mathbf{10}^{ij} = \frac{1}{\sqrt{2}} \varepsilon^{ij} e^c \,,
\tag{A.3}
$$

where α, β, $\gamma = 1, 2, 3$ and $i, j = 4, 5$. The fermionic field ρ_3, triplet of $\mathsf{SU}(2)$, belonging to the adjoint representation can be written as,

$$
\rho_3 = \frac{1}{2} \begin{pmatrix} \rho_3{}^0 & \sqrt{2}\rho_3{}^+ \\ \sqrt{2}\rho_3{}^- & -\rho_3{}^0 \end{pmatrix} \,,
\tag{A.4}
$$

where

$$
\rho_3{}^\pm = \frac{\rho_3^1 \mp i\rho_3^2}{\sqrt{2}} \,, \qquad \rho_3{}^0 = \rho_3^3 \,.
\tag{A.5}
$$

Higgs representations

The Higgs content of the model decomposes as

$$
\begin{aligned}
\mathbf{5}_H &= T_1 \oplus H_1 \,, \\
\mathbf{24}_H &= \Sigma_8 \oplus \Sigma_3 \oplus \Sigma_{(3,2)} \oplus \Sigma_{(\bar{3},2)} \oplus \Sigma_0 \,, \\
\mathbf{45}_H &= S_{(8,2)_{\frac{3}{10}}} \oplus S_{(\bar{6},1)_{-\frac{1}{5}}} \oplus S_{(3,3)_{-\frac{1}{5}}} \oplus S_{(\bar{3},2)_{-\frac{7}{10}}} \oplus S_{(\bar{3},1)_{\frac{4}{5}}} \oplus T_2 \oplus H_2 \,,
\end{aligned}
\tag{A.6}
$$

where we have included for completeness the hypercharged properly normalised. The H_1 and H_2 are the usual Higgs doublets and T_1 and T_2 are the colour triplets. The **45** Higgs representation, which the explicit decomposition is given in ref. [27], obeys to the following relations,

$$45_k^{ij} = -45_k^{ji} \quad \text{and} \quad \sum_{j=1}^{5} 45_j^{ij} = 0 \,, \tag{A.7}$$

The different contributions for the beta coefficients b_i of each extra particle besides the 2HDM content are given in table 4.

Table 4. Summary of the b_i constants for relevant particles in the model.

	2HDM	ρ_3	ρ_8	$\rho_{(3,2)}$	$T_{1,2}$	Σ_3	Σ_8	$S_{(8,2)}$
b_1	21/5	0	0	$\frac{5}{3}$	$\frac{1}{15}$	0	0	$\frac{4}{5}$
b_2	-3	$\frac{4}{3}$	0	1	0	$\frac{2}{3}$	0	$\frac{4}{3}$
b_3	-7	0	2	$\frac{2}{3}$	$\frac{1}{6}$	0	1	2

B   The Potential

In this section we give explicitly the terms of the Higgs potential. Notice that index H on the Higgs fields is dropped in the following expressions. The potential V is divided into six parts as follows:

$$
\begin{aligned}
V\,(\mathbf{5}, \mathbf{24}, \mathbf{45}) =& V_1\,(\mathbf{5}) \, + \, V_2\,(\mathbf{24}) \, + \, V_3\,(\mathbf{45}) \, + \, V_4\,(\mathbf{24}, \mathbf{45}) \\
& + V_5\,(\mathbf{5}, \mathbf{24}) \, + \, V_6\,(\mathbf{5}, \mathbf{45}) \,,
\end{aligned}
\tag{B.1}
$$

where each parcell are given by:

$$V_1\,(\mathbf{5}) = -\frac{\mu_\mathbf{5}^2}{2} \, \mathbf{5}^\alpha \, \mathbf{5}_\alpha^* \, + \, \frac{\lambda_1}{4} \, (\mathbf{5}^\alpha \, \mathbf{5}_\alpha^*)^2 \,, \tag{B.2a}$$

$$
\begin{aligned}
V_2\,(\mathbf{24}) =& -\frac{\mu_\mathbf{24}^2}{2} \, \mathbf{24}_\beta^\alpha \, \mathbf{24}_\alpha^\beta + \frac{\lambda_2}{2} \left(\mathbf{24}_\beta^\alpha \, \mathbf{24}_\alpha^\beta \right)^2 \, + \, \frac{a_1}{3} \, \mathbf{24}_\beta^\alpha \, \mathbf{24}_\gamma^\beta \, \mathbf{24}_\alpha^\gamma \\
& + \frac{\lambda_3}{2} \, \mathbf{24}_\beta^\alpha \, \mathbf{24}_\gamma^\beta \, \mathbf{24}_\delta^\gamma \, \mathbf{24}_\alpha^\delta \,,
\end{aligned}
\tag{B.2b}
$$

$$V_3\left(45\right) = -\frac{\mu_{45}^2}{2}\,45^{\alpha\beta}_{\gamma}45^{*\,\gamma}_{\alpha\beta} \;+\; \lambda_4\left(45^{\alpha\beta}_{\gamma}45^{*\,\gamma}_{\alpha\beta}\right)^2$$

$$+\,\lambda_5 45^{\alpha\beta}_{\gamma}45^{*\,\delta}_{\alpha\beta}45^{\kappa\lambda}_{\delta}45^{*\,\gamma}_{\kappa\lambda} \;+\; \lambda_6 45^{\alpha\beta}_{\gamma}45^{*\,\delta}_{\alpha\beta}45^{\kappa\gamma}_{\lambda}45^{*\,\lambda}_{\kappa\delta}$$

$$+\,\lambda_7 45^{\alpha\delta}_{\beta}45^{*\,\beta}_{\alpha\gamma}45^{\kappa\gamma}_{\lambda}45^{*\,\lambda}_{\kappa\delta} \;+\; \lambda_8 45^{\alpha\gamma}_{\delta}45^{*\,\beta}_{\gamma\epsilon}45^{\kappa\delta}_{\alpha}45^{*\,\epsilon}_{\kappa\beta} \qquad\text{(B.2c)}$$

$$+\,\lambda_9 45^{\alpha\gamma}_{\delta}45^{*\,\beta}_{\gamma\epsilon}45^{\kappa\epsilon}_{\alpha}45^{*\,\delta}_{\kappa\beta} \;+\; \lambda_{10} 45^{\alpha\gamma}_{\delta}45^{*\,\beta}_{\gamma\epsilon}45^{\kappa\delta}_{\beta}45^{*\,\epsilon}_{\kappa\alpha}$$

$$+\,\lambda_{11} 45^{\alpha\gamma}_{\delta}45^{*\,\beta}_{\gamma\epsilon}45^{\kappa\epsilon}_{\beta}45^{*\,\delta}_{\kappa\alpha}\,,$$

$$V_4\left(24,45\right) = a_2\,45^{\alpha\beta}_{\gamma}24^{\gamma}_{\delta}45^{*\,\delta}_{\alpha\beta} \;+\; \lambda_{12}\,45^{\alpha\beta}_{\gamma}45^{*\,\gamma}_{\alpha\beta}24^{\delta}_{\epsilon}24^{\epsilon}_{\delta}$$

$$+\,\lambda_{13}\,45^{\alpha\beta}_{\gamma}24^{\delta}_{\alpha}24^{\epsilon}_{\beta}45^{*\,\gamma}_{\delta\epsilon} \;+\; \lambda_{14}\,45^{\alpha\beta}_{\gamma}24^{\gamma}_{\beta}24^{\delta}_{\epsilon}45^{*\,\epsilon}_{\alpha\delta}$$

$$+\,\lambda_{15}\,45^{\alpha\beta}_{\gamma}24^{\gamma}_{\epsilon}24^{\delta}_{\beta}45^{*\,\epsilon}_{\alpha\delta} \;+\; \lambda_{16}45^{\alpha\beta}_{\gamma}24^{\kappa}_{\alpha}24^{\lambda}_{\kappa}45^{*\,\gamma}_{\lambda\beta} \qquad\text{(B.2d)}$$

$$+\,\lambda_{17}\,45^{\alpha\beta}_{\gamma}24^{\gamma}_{\kappa}24^{\kappa}_{\lambda}45^{*\,\lambda}_{\alpha\beta}\,,$$

$$V_5\left(5,24\right) = a_3\,5^{*}_{\alpha}24^{\alpha}_{\beta}5^{\beta} \;+\; \lambda_{18}\,5^{*}_{\alpha}5^{\alpha}24^{\beta}_{\gamma}24^{\gamma}_{\beta} \;+\; \lambda_{19}\,5^{*}_{\alpha}24^{\alpha}_{\beta}24^{\beta}_{\gamma}5^{\gamma}\,, \qquad\text{(B.2e)}$$

and

$$V_6\left(5,45\right) = \lambda_{20}\,45^{\alpha\beta}_{\gamma}45^{*\,\gamma}_{\alpha\beta}5^{*}_{\delta}5^{\delta} \;+\; \lambda_{21}\,45^{\alpha\beta}_{\delta}5^{*}_{\gamma}45^{*\,\gamma}_{\alpha\beta}5^{\delta} \;+\; \lambda_{22}\,45^{\alpha\beta}_{\gamma}45^{*\,\gamma}_{\alpha\delta}5^{*}_{\beta}5^{\delta}\,. \qquad\text{(B.2f)}$$

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