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# Lepton asymmetries and primordial hypermagnetic helicity evolution

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**Abstract.** The hypermagnetic helicity density at the electroweak phase transition (EWPT) exceeds many orders of magnitude the galactic magnetic helicity density. Together with previous magnetic helicity evolution calculations after the EWPT and hypermagnetic helicity conversion to the magnetic one at the EWPT, the present calculation completes the description of the evolution of this important topological feature of cosmological magnetic fields. It suggests that if the magnetic field seeding the galactic dynamo has a primordial origin, it should be substantially helical. This should be taken into account in scenarios of galactic magnetic field evolution with a cosmological seed.

**Keywords:** Chern-Simons anomaly, hypermagnetic field, hypermagnetic helicity, lepton asymmetry, electroweak phase transition

## Contents



## 1 Introduction

The magnetic helicity is a relatively new and very attractive point of interest in cosmic magnetohydrodynamics (MHD) and dynamo theory. The point is that the magnetic helicity $H = \int \mathbf{AB} \mathrm{d}^3 x$ where $\mathbf{B}$ is the magnetic field and $\mathbf{A}$ is the vector potential is an integral of motion in MHD in the absence of viscosity (inviscid case). It looks natural to base an understanding of cosmic MHD on balance equations for conserved quantities such as magnetic field energy and magnetic helicity. It has been noted that the magnetic helicity conservation is much more restrictive in astrophysical objects than the energy conservation [1, 2]. The point is that usually there is a huge supply of kinetic energy in the form of a general rotation of a celestial body and it is quite easy to imagine a spectral energy flux which influences energy balance including magnetic field energy.

The nature of the initial fields (and corresponding magnetic helicities) that seed subsequent dynamo or turbulent amplifications is largely unknown [3, 4]. It might be that the seed fields are produced during epoch of galaxy formation from frozen-in magnetic fields of protogalaxy experiencing gravitational collapse, or ejected by the first supernovae or active galactic nuclei. Let us call this as the standard astrophysical scenario A. Alternatively the seed fields might originate from much earlier epochs of the Universe expansion, down to the cosmological inflation phase transition epoch [5]. Let us call that as the cosmological scenario B. The standard view in scenario A is that the magnetic field evolution in the early Universe (or an astrophysical object) starts from a state with almost vanishing magnetic helicity. The dynamo amplification of a seed field in astrophysical scenarios produces, however, a large-scale magnetic field with substantial magnetic helicity. In such case one must compensate it providing a contribution of magnetic helicity of small-scale magnetic fields. Then one faces with a severe problem of how to redistribute magnetic helicity over the desired scales in order to keep it small. Moreover, it is difficult to prevent a disastrous dynamo suppression of large-scale magnetic helicity by helical small-scale magnetic field. Alternatively in scenario B, if the seed magnetic field contained a lot of magnetic helicity one can use that as the helicity required to obtain the desired large-scale field at later epochs. This makes the dynamo generation of galactic magnetic fields much less constrained than in the standard scenario A. If the galactic dynamo exploits the primordial magnetic helicity then one must expect that a large-scale galactic magnetic field has also a preferable sign of helicity. Observations show indeed some hint that one of the possible helicity signs seems preferable [6].

Relying on scenario B with the hypermagnetic field evolution passing through the electroweak phase transition (EWPT) epoch, we explore here the magnetic helicity generation in the early universe. This suggests as a new alternative the possibility that the magnetic field starts from a state with a substantial supply of primordial magnetic helicity. Indeed, a very small fluctuation of a seed hypermagnetic field at very early epochs before the EWPT woul be exponentially amplified due to the presence of a large right electron asymmetry, $\xi_{eR} = \mu_{eR}/T \neq 0$ [7–9] and hence acquire a huge initial magnetic helicity by the EWPT epoch and subsequently. This is possible only for hypermagnetic fields having a non-trivial topological structure with non-vanishing linkage number $n \neq 0$ [10]. The hypermagnetic helicity density given by the product $h_Y = \mathbf{Y}\cdot\mathbf{B}_Y$, hence proportional to a large hypermagnetic field squared value $h_Y \sim B_Y^2\Lambda$. This gets transformed to the magnetic helicity, which can therefore be much larger than the helicity density associated to galactic magnetic fields, $h_Y \gg h_{\rm gal}$ where $h_{\rm gal} \sim B_{\rm gal}^2 l_{\rm gal}$. For $B_{\rm gal} \simeq 10^{-6}\ G$, $l_{\rm gal} = 100\ kpc = 3\times 10^{23}\ cm$, this is estimated as $h_{\rm gal} = 3\times 10^{11}\ G^2{\rm cm}$. Thus, the primordial magnetic helicity can be the main supply for magnetic helicity of galaxies. The goal of this paper is to provide a complete description of magnetic helicity evolution passing through the EWPT epoch (we also comment on later epochs within the causal picture). In Section 2 we calculate the hypermagnetic helicity in the symmetric phase of the hot primordial plasma, using the corresponding solution of the Faraday equation. In Section 3 we review the evolution of the hypermagnetic helicity passing through the EWPT and in Section 4 we make the final comments on our results and on the perspectives for the cosmological origin of helical galactic magnetic fields.

## 2 Hypermagnetic helicity

In the comoving frame $\mathbf{V} = 0$ [1] the Faraday induction equation governing the evolution of hypermagnetic fields $\mathbf{B}_Y = \nabla\times\mathbf{Y}$ reads

$$\frac{\partial\mathbf{B}_Y}{\partial t} = \nabla\times\alpha_Y\mathbf{B}_Y + \eta_Y\nabla^2\mathbf{B}_Y, \tag{2.1}$$

where the hypermagnetic helicity coefficient $\alpha_Y$ [9] is given by the right electron chemical potential $\mu_{eR}$ and the hot plasma conductivity $\sigma_{\rm cond}(T)\sim T$ as

$$\alpha_Y(T) = -\frac{g^{'2}\mu_{eR}}{4\pi^2\sigma_{cond}}\ , \tag{2.2}$$

and $\eta_Y = (\sigma_{\rm cond})^{-1}$ is the hypermagnetic diffusion coefficient, $g^{'} = e/\cos\theta_W$ is the Standard Model U(1) gauge coupling. We assume that a certain right electron asymmetry $\sim \mu_{eR}(t_0)$ at a very early cosmological epoch , $t_0 \ll t_{EW}$, has been generated by an unspecified mechanism. This is our starting point. Then in the presence of the hypercharge field $Y_\mu$ this asymmetry $n_{eR} - n_{\bar{e}R} = \mu_{eR}T^2/6$ evolves due to the Abelian anomaly for right electrons,

$$\partial_\mu j^\mu_{eR} = -\frac{g^{'2}y_R^2}{64\pi^2}Y_{\mu\nu}\tilde{Y}^{\mu\nu},\quad y_R = -2, \tag{2.3}$$

which evolves together with the hypermagnetic field in Eq.(2.1) in a self-consistent way. Note that such coupled evolution of $B_Y(t)$ and $\mu_{eR}(t)$ has been recently considered in Ref. [11] for

[1]Note that common expansion can be easily taken into account via conformal coordinates with the change of the cosmological time $t$ to the conformal one $dt \to a(t)d\eta$.

the particular case of the Chern-Simons wave hypermagnetic field configuration, but without considering such important feature as the hypermagnetic helicity which we discuss here.

Our second assumption is the presence of a non-zero initial hypermagnetic field $B_0^Y \neq 0$. This should be a mean field with a small amplitude provided by some stochastic distribution of hypermagnetic fields.

The key parameter in the master Eq. (2.1) is the helicity parameter for the hypermagnetic field, given in eq. (2.2). This can be obtained from the Chern-Simons term in the effective Standard Model Lagrangian density for the hypercharged field $Y_\mu$ :

$$\mathcal{L}_{CS} = -\frac{g^{'2}\mu_{eR}}{4\pi^2}\mathbf{B}_Y\mathbf{Y}. \tag{2.4}$$

An recent interpretation of the Chern-Simons anomaly parameter $\alpha_Y$ as a polarization effect has been given in Ref. [9] using standard statistical averaging of the right electron pseudovector current $< \bar{e}\gamma_j\gamma_5 e >$ in the Standard Model Lagrangian in vacuum (alternative one-loop level calculations in finite temperature field theory were used in [12, 13]).

Multiplying Eq. (2.1) by the corresponding vector potential and adding the analogous expression obtained from the evolution equation governing the vector potential (multiplied by hypermagnetic field) and integrating over space, one gets the evolution equation for the hypermagnetic helicity $\mathrm{H}_Y = \int d^3x \mathbf{Y}\cdot\mathbf{B}_Y$ as

$$\begin{aligned}&\frac{\mathrm{dH}_Y}{\mathrm{dt}} = -2\int_V (\mathbf{E}_Y\cdot\mathbf{B}_Y)d^3x - \oint [Y_0\mathbf{B}_Y + \\ &+\mathbf{E}_Y\times\mathbf{Y}]d^2S = -2\eta_Y(t)\int d^3x(\nabla\times\mathbf{B}_Y)\cdot\mathbf{B}_Y + \\ &+2\alpha_Y(t)\int d^3x B_Y^2(t).\end{aligned} \tag{2.5}$$

Note that we have omitted in the last equality the surface integral $\oint(...)$ since fields vanish at infinity during the symmetric phase. However, such surface integral can be important at the boundaries of different phases at the electroweak phase transition, $T\sim T_{EW}$. In Ref. [10] the authors have studied how the hypermagnetic helicity flux penetrates the surface separating the symmetric and broken phases, and how the hypermagnetic helicity density $h_Y = \mathbf{B}_Y\mathbf{Y}$ converts into the magnetic helicity density $h = \mathbf{BA}$ at the EWPT time, see also discussion in Sec. 3 below.

Notice also that the evolution equation in eq. (2.5) is similar to eq. (7) in Ref. [14], which holds after the electroweak phase transition, $T \ll T_{EW}$. There the point-like short-range Fermi neutrino-plasma interaction mediated by heavy $W, Z$-bosons was used, instead of the long-range interaction through the massless hypercharge field $Y_\mu$ in the unbroken phase.

Now using the simplest solution of the Faraday equation (2.1) in the $\alpha^2$-dynamo [15] that corresponds to maximum hypermagnetic field amplification rate [2],

$$B_Y(t) = B_0^Y \exp\left[\int_{t_0}^{t}\frac{\alpha_Y^2(t')}{4\eta_Y(t')}\mathrm{d}t'\right], \tag{2.6}$$

[2]This is the case for the particular hypermagnetic field scale $\Lambda = k^{-1} = \kappa\eta_Y/\alpha_Y$ where $\kappa = 2$, $k$ being the Fourier wave number in $\mathbf{B}_Y(\mathbf{x},t) = \int(d^3k/(2\pi)^3)\mathbf{B}_Y(\mathbf{k},t)e^{i\mathbf{kx}}$.

we obtain from Eq. (2.5) the hypermagnetic helicity :

$$H_Y = H_Y(t_0) + 2(B_0^Y)^2 \int \mathrm{d}^3x \int_{t_0}^{t} \mathrm{d}t^{'} \alpha_Y(t^{'}) \times$$
$$\times \exp\left[2\int_{t_0}^{t^{'}}\left[\frac{\alpha_Y^2(t^{''})}{4\eta(t^{''})}\right]\mathrm{d}t^{''}\right], \tag{2.7}$$

where $H_Y(t_0)$ is the initial helicity value at the moment $t_0$ and we have omitted the hypermagnetic diffusion term. The latter is usually neglected for an ideal Maxwellian plasma $\sigma_{\mathrm{cond}} \to \infty$, $\eta_Y \to 0$, while here we must compare in Eq. (2.5) the second helicity generation term $\alpha_Y \sim (\sigma_{\mathrm{cond}})^{-1}$ given by Eq. (2.2) with the first diffusion term $\eta_Y \sim (\sigma_{\mathrm{cond}})^{-1}$. Notice that in Ref. [14] one could neglect the diffusion term for the Maxwellian plasma in the broken phase, $T \ll T_{EW}$, since for that case the magnetic helicity coefficent $\alpha$ [14, 16] did not depend on the conductivity.

The direct estimate of the relative magnitude of the terms in the r.h.s. of Eq. (2.5) allows us to neglect the diffusion term only for hypermagnetic field inhomogeneity scales obeying the inequality:

$$\Lambda \gg \frac{\eta_Y}{\alpha_Y}\ . \tag{2.8}$$

For an arbitrary (large) scale in our causal scenario, $l_H > \Lambda = \kappa\eta_Y/\alpha_Y \gg \eta_Y/\alpha_Y$, the amplification of hypermagnetic field is given by [9]

$$B_Y(t) = B_0^Y \exp\left[\left(\frac{1}{\kappa} - \frac{1}{\kappa^2}\right)\int_{t_0}^{t}\frac{\alpha_Y^2(t^{'})}{\eta_Y(t^{'})}dt^{'}\right] =$$
$$= B_0^Y \exp\left[83\left(\frac{1}{\kappa} - \frac{1}{\kappa^2}\right)\int_{x}^{x_0}\frac{\mathrm{d}x^{'}}{x^{'2}}\left(\frac{\xi_{eR}(x^{'})}{0.0001}\right)^2\right], \tag{2.9}$$

and that of the hypermagnetic helicity is given by

$$H_Y(t) = H_Y(t_0) + 2(B_0^Y)^2 \int \mathrm{d}^3x \int_{t_0}^{t} \mathrm{d}t^{'} \alpha_Y(t^{'}) \times$$
$$\times \exp\left[\left(\frac{2}{\kappa} - \frac{2}{\kappa^2}\right)\int_{t_0}^{t^{'}}\left(\frac{\alpha_Y^2((t^{''})}{\eta(t^{''})}\right)\mathrm{d}t^{''}\right] =$$
$$= H_Y(t_0) + 2(B_0^Y)^2 \int \mathrm{d}^3x \int_{t_0}^{t} \mathrm{d}t^{'} \alpha_Y(t^{'}) \times$$
$$\times \exp\left[166\left(\frac{1}{\kappa} - \frac{1}{\kappa^2}\right)\int_{x^{'}}^{x_0}\frac{\mathrm{d}x^{''}}{x^{''2}}\left(\frac{\xi_{eR}(x^{''})}{0.0001}\right)^2\right]. \tag{2.10}$$

Here the ratio $x = T/T_{EW} = (t_{EW}/t)^{1/2}$ is given by the Friedman law and $\xi_{eR} = \mu_{eR}/T$ is the dimensionless right electron asymmetry. Thus, from Eq. (2.9) for the extremum value $\kappa = 2$ we obtain the strongest amplification eq. (2.6) and from Eq. (2.10) one finds the corresponding value of the hypermagnetic helicity in eq. (2.7).

Let us comment on our reference value choice $\xi_{eR} \sim 10^{-4}$ used in eqs. (2.9),(2.10). Taking into account the right electron (positron) asymmetry $n_{eR} - n_{\bar{e}R} = \mu_{eR}T^2/6$ in the presence of chirality flip processes with the rate $\Gamma$, and substituting the hyperelectric field $\mathbf{E}_Y = -\mathbf{V} \times \mathbf{B}_Y + \eta_Y \nabla \times \mathbf{B}_Y - \alpha_Y \mathbf{B}_Y$ [9] into the Abelian anomaly Eq. (2.3) rewritten in uniform medium as $\partial_t(n_{eR} - n_{\bar{e}R}) = -(g'^2/4\pi^2)\mathbf{E}_Y\mathbf{B}_Y$ one finds the kinetic equation for $\mu_{eR}$:

$$\frac{\partial \mu_{eR}}{\partial t} = -\frac{6g'^2(\nabla \times \mathbf{B}_Y)\cdot \mathbf{B}_Y}{4\pi^2 T^2 \sigma_{\rm cond}} - (\Gamma_B + \Gamma)\mu_{eR}. \tag{2.11}$$

Here the rate $\Gamma_B = 6(g'^2/4\pi^2)^2 B_Y^2/T^2\sigma_{cond}$ coming from the helicity term $\sim \alpha_Y$ occurs in strong hypermagnetic fields much bigger than the chirality flip rate $\Gamma$, $\Gamma_B \gg \Gamma$ [9].

Under the assumption of slowly changing hypermagnetic fields $B_Y(t) \approx const$, and choosing the Chern-Simons wave configuration of the hypercharge field as,

$$Y_0 = Y_z = 0, \quad Y_x = Y(t)\sin k_0 z, \quad Y_y = Y(t)\cos k_0 z,$$

for which $(\nabla \times \mathbf{B}_Y)\cdot\mathbf{B}_Y = B_Y^2(t)k_0$, $B_Y(t) = k_0 Y(t)$, we can easily solve the kinetic equation (2.11) getting:

$$\xi_{eR}(t) = \left[\xi_{eR}(t_0) - \frac{Q}{\Gamma_B + \Gamma}\right] e^{-(\Gamma_B+\Gamma)(t-t_0)} + \\ + \frac{Q}{\Gamma_B + \Gamma} \approx \frac{Q}{\Gamma_B + \Gamma} \approx \frac{Q}{\Gamma_B} = -\frac{4\pi^2 k_0}{Tg'^2}. \tag{2.12}$$

Here we used notation $Q = -(6g'^2/4\pi^2T^3\sigma_{cond})(\nabla \times \mathbf{B}_Y)\cdot\mathbf{B}_Y = -(6g'^2/4\pi^2T^3\sigma_{cond})B_Y^2 k_0$. In obtaining (2.12) we neglected the time dependence for times $t \to t_{EW}$ for which $\Gamma_B t_{EW} \gg 1$. On the other hand, retaining the time term for the zero initial asymmetry $\xi_{eR}(t_0) = 0$ we get from (2.12) the asymptotical growth of $\mid \xi_{eR} \mid$ in a strong hypermagnetic field due to the Abelian anomaly:

$$\mid \xi_{eR}(t) \mid = \frac{\mid Q \mid}{\Gamma_B + \Gamma}\left[1 - e^{-(\Gamma_B+\Gamma)(t-t_0)}\right] \approx \\ \approx \frac{4\pi^2 k_0}{Tg'^2}\left[1 - e^{-\Gamma_B(t-t_0)}\right].$$

Taking into account for the survival condition of the Chern-Simons wave versus ohmic diffusion, $k_0 < 10^{-7}T$, substituting weak coupling $g'^2 = 0.12$ we get the estimate of the lepton asymmetry in a strong hypermagnetic field, $\mid \xi_{eR} \mid \sim 3 \times 10^{-5}$, hence we adopted $\xi_{eR} \sim 10^{-4}$ as the reference value in Eqs.(2.9),(2.10) above. For a topologically non-trivial 3D-hypermagnetic field configuration with linkage (Gauss) number $n \gg 1$ for which the pseudoscalar $\mathbf{B_Y} \cdot (\nabla \times \mathbf{B_Y}) \sim n$ one can expect the right electron asymmetry at the level $\xi_{eR} \sim 10^{-4}$, which will be used below as an estimated reference value with respect to which we choose to normalize the right electron asymmetry $\xi_{eR}$.

## 3 Hypermagnetic helicity evolution

Let us now turn to the evolution of the hypermagnetic (magnetic) helicity through various stages in the evolution of the universe, as illustrated in Fig. 1.

### 3.1 Hypermagnetic helicity evolution through the electroweak phase transition

Let us note that for Higgs masses $m_H > 80\ GeV$ the electroweak phase transition cannot be first order in the minimal standard electroweak theory, so that a smooth cross-over between symmetric and broken phases is more likely [17], given the experimental lower bound on Higgs masses and the recent hints from the LHC [18].

However, in the presence of strong hypermagnetic fields in the primordial plasma the dynamics of the phase transition changes in analogy with the superconductivity in the presence of magnetic fields: the second order phase transition may become first order [7]. We rely here on such scenario assuming the presence of strong hypermagnetic fields for which first order EWPT becomes allowed in the mass region $80\ GeV < m_H < 160\ GeV$ (see Fig. 8 in [7]) indicated by current LHC data.

We first consider what happens with the hypermagnetic helicity passing-through the electroweak phase transition. For that let us mention results from paper [10] where the flow of the hypermagnetic helicity in the embryo of the new (broken) phase was considered. If a single bubble of broken phase appears at $T = T_{EW}$ growing with constant velocity, $R(t) = v(t - t_{EW})$ [3], then a value of the flow of hypermagnetic helicity density through the bubble surface is determined by the surface integral in Eq. (2.5) which we neglected above for the symmetric phase with boundary at infinity. The result (Eq. (17) in [10]) shows that the value of the hypermagnetic helicity density penetrating the surface of a single bubble,

$$\frac{h_Y(t)}{G^2 cm} = \frac{5\times 10^{-3} n}{d(cm)}\left(\frac{B_Y(t_{EW})}{1\ G}\right)^2\left(\frac{t - t_{EW}}{t_{EW}}\right)^2, \tag{3.1}$$

is also large. Here the integer $n = \mp 1, \mp 2, ...$ denotes the number of pairs of linked hypermagnetic field loops (or knot number) for the non-trivial 3D-configuration, as in Eq. (7) in Ref. [10]. Note that $n$ is the pseudoscalar entering the Gauss integral for magnetic helicity

$$H(t) = \int d^3x h(\mathbf{x}, t) = n\Phi_1\Phi_2$$

which changes the sign, $n \to -n$, after one of the overlapping oriented loops in a pair of magnetic closed tubes changes the direction.

In order to avoid screening of the hyperelectric field $\mathbf{E}_Y$ and the time component $Y_0$ over the surface of the bubble the thickness $d$ of the domain wall separating the two phases should be less than the Debye radius, $d < r_D = \sqrt{3T_{EW}/4\pi e^2 n_e} \sim 10/T_{EW}$, which allows us to estimate the factor $d^{-1}$ in eq. (3.1) as $[d(cm)]^{-1} > 10^{15}/2$. Then substituting the value of the hypermagnetic field $B_Y(t_{EW})$ estimated in in the leptogenesis scenario as $B_Y(t_{EW}) \sim 5\times 10^{17}\ G$ [9, 21] , one gets from Eq. (3.1) the helicity density

$$h/G^2 cm = 6.25\times 10^{47} n[(t - t_{EW})/t_{EW}]^2.$$

Such huge value is estimated at the moment of the growth of a bubble of the new phase, e.g. for $R(t)/l_H < [(t - t_{EW}/t_{EW})] \sim 10^{-7}$ [4]. Taking into account the subsequent conservation of the net global helicity, summed over different protogalactic scales, one finds values that are much larger than the helicity density associated to galactic magnetic field strengths,

$$h_{gal} \sim 10^{11}\ G^2 cm.$$

[3]Here time is fixed at $T_{EW}$, $(t - t_{EW})/t_{EW} \ll 1$, $v = 0.1 - 1$ accordingly [19, 20].

[4]Such bubble size is relevant before percolation (collision and following junction) of the two bubbles, see Eq. (21) in Ref. [20].

During the electroweak phase transition the hypermagnetic helicity density converted from the symmetric phase to the Maxwellian one is redistributed within a bubble over its volume in correspondence with the ratio of volumes

$$V_{surface}/V_{ball} = 3d/R(t).$$

This is because we calculated the helicity flux density in eq. (3.1) only within a thin spherical layer $V_{surface} = 4\pi R^2(t)d$. Then assuming that all bubbles are tight-fitting each other within the horizon size $l_H(t_{EW}) = 1.44\ cm$ or the mean magnetic helicity density coincides with that within one bubble, and using the ratio $3d/R(t)$ we obtain from (3.1) at $T \simeq T_{EW}$ the magnetic helicity density in the new (broken) phase

$$\begin{aligned}\frac{h(x\sim 1)}{G^2\ cm} &= \frac{1.5\times 10^{-2}n}{(l_H(t_{EW})/1\ cm)}\left(\frac{B_Y(t_{EW})}{1\ G}\right)^2 \times \\ &\times \left(\frac{R(t)}{l_H(t_{EW})}\right) = \frac{1.5\times 10^{-9}n}{1.44}\left(\frac{B_0^Y}{1\ G}\right)^2 \times \\ &\times \exp\left[\frac{166(\kappa-1)}{\kappa^2}\int_1^\infty \frac{\mathrm{d}x}{x^2}\left(\frac{\xi_{eR}(x)}{0.0001}\right)^2\right],\end{aligned} \tag{3.2}$$

where we have substituted $R/l_H = 10^{-7}$ as an estimate of the beginning of percolation (junction) of bubbles (see Eq. (21) in Ref. [20]).

We now turn to a brief discussion of the bounds on the topological linkage number $n = \pm 1, \pm 2, ...$ ($|n| > 1$) in eq. (3.2).

Starting from Gauss formula,

$$H = n\Phi^2 = nB^2\pi^2\Lambda^4\ ,$$

and substituting the corresponding helicity density $h = 3H/4\pi R^3$, and using the maximum helicity density $h_{\max} = B^2\Lambda$ one can find a bound on "n" from the requirement that $h < h_{\max}$:

$$n < \frac{4}{3\pi}\left(\frac{R}{\Lambda}\right)^3 = \frac{4\times 10^6}{\kappa^3}\left(\frac{\xi_{eR}(T_{EW})}{0.0001}\right)^3, \tag{3.3}$$

where we have substituted $R = 10^{-7}l_H(T_{EW})$ and $\Lambda = \kappa\eta_Y/\alpha_Y = 3.3{\times}10^6\kappa/[T_{EW}(\xi_{eR}/0.0001)]$.

On the other hand, from the same bound $h < h_{\max}$ using Eq. (3.2) and cancelling $B^2(t_{EW})$ one finds:

$$\frac{h(x=1)}{B^2\Lambda} = \frac{1.5\times 10^{-9}n(\xi_{eR}(T_{EW})/0.0001)}{2.88\times 3.3\times 10^6\kappa\times 10^{-16}} < 1,$$

or

$$\kappa > 1.6n\left(\frac{\xi_{eR}}{0.0001}\right). \tag{3.4}$$

Combining (3.3) and (3.4) from the chain of inequalities we get,

$$n < \frac{4\times 10^6}{\kappa^3}\left(\frac{\xi_{eR}(T_{EW})}{0.0001}\right)^3 < \frac{4\times 10^6}{(1.6)^3n^3},$$

hence we find an upper bound on the linkage number

$$\mid n \mid < 33\ . \tag{3.5}$$

Note that it does not depend on a right electron chemical potential nor on the bubble size before percolation, chosen in Eq. (3.3) as $R = 10^{-7} l_H(T_{EW})$ [19]. Indeed, the scale $\Lambda \sim \kappa$ is proportional to the bubble size $R$, $\Lambda \sim R$, as seen from Eq. (3.4) where $\kappa \sim R$ through the helicity $h(x = 1)$ in Eq. (3.2).

### 3.2 Hypermagnetic helicity evolution before the electroweak phase transition

For an arbitrary scale $\Lambda = \kappa\eta_Y/\alpha_Y$, $\kappa > 1$, we obtain from eq. (2.10) the hypermagnetic helicity in the unbroken phase ($x \geq 1$) as

$$\begin{aligned}\frac{h_Y(x)}{G^2\ cm} =& \\ &-0.88\times 10^{-8}\left(\frac{B_0^Y}{1\ G}\right)^2\int_x^\infty \frac{\mathrm{d}x'}{x'^3}\left(\frac{\xi_{eR}(x')}{0.0001}\right)\times \\ &\times \exp\left[\frac{166(\kappa-1)}{\kappa^2}\int_{x'}^\infty \frac{\mathrm{d}x''}{x''^2}\left(\frac{\xi_{eR}(x'')}{0.0001}\right)^2\right],\end{aligned} \tag{3.6}$$

where we put $x_0 = \infty$, substituted $\alpha_Y(t)$ from eq. (2.2), and used the expansion time $t = (M_{Pl}/1,66\sqrt{g^*})/2T^2 = M_0/2T^2$, omitting the initial helicity value $H_Y(t_0)$.

Dividing the helicity density (3.6) by its value at $T_{EW}$ (3.2) we get the ratio valid at $x \geq 1$:

$$\begin{aligned}\frac{h_Y(x)}{h(x=1)} =& -\frac{8.5}{n}\int_x^\infty \frac{dx'}{x'^3}\left(\frac{\xi_{eR}(x')}{0.0001}\right)\times \\ &\times \exp\left[\frac{166(\kappa-1)}{\kappa^2}\int_{x'}^1 \left(\frac{\xi_{eR}(x'')}{0.0001}\right)^2\frac{\mathrm{d}x''}{x''^2}\right],\end{aligned} \tag{3.7}$$

that for a constant value of the right electron asymmetry $\xi_{eR}/0.0001 = \beta = \mathrm{const}$ equals to

$$\begin{aligned}\frac{h_Y(x)}{h(x=1)} =& \frac{8.5\mid\beta\mid}{n}\left(\frac{1}{a\beta^2}\right)\Big[\frac{\exp[-a\beta^2(1-1/x)]}{x} - \\ &-\frac{\exp[-a\beta^2(1-1/x)]-\exp(-a\beta^2)}{a\beta^2}\Big],\end{aligned} \tag{3.8}$$

where $a = 166(\kappa-1)/\kappa^2$ and we took into account the negative sign of the right electron asymmetry, $\beta < 0$, as seen, e.g., from Eq. (2.12). For a small parameter $a\beta^2 \ll 1$, or $\kappa \gg 166\beta^2$ that is allowed for larger bubbles $R \gg 10^{-7} l_H$ one gets from (3.8) (using relation $t/t_{EW} = (T_{EW}/T)^2 = x^{-2}$) the temporal dependence

$$\begin{aligned}\frac{h_Y(t<t_{EW})}{h(t_{EW})} =& \frac{4.25\mid\beta\mid}{n}\Big[(1-a\beta^2)\frac{t}{t_{EW}} - \\ &-\frac{a\beta^2}{3}\left(\frac{t}{t_{EW}}\right)^{3/2} + O((a\beta^2)^2)\Big],\end{aligned} \tag{3.9}$$

which is almost linear in the region $t < t_{EW}$. Thus, we see from (3.9) that like for EWPT of the first order there is the *jump* of helicity density (here for large scales $\kappa \gg 166\beta^2$):

$$\frac{h_Y(t \lesssim t_{EW})}{h(t_{EW})} = \frac{4.25\mid\beta\mid}{n}, \tag{3.10}$$

where the linkage number $\mid n \mid > 1$ has the upper bound given by Eq. (3.5). Note that the straight lines in Fig. 1 shown for some particular parameters: $a$ given by $\kappa = 2,\ 10^3$; $\mid \beta \mid = 0.1,\ 1$; $n = 1,\ 30$ correspond at $t < t_{EW}$ to the limiting case presented in Eq. (3.9).

Let us stress that our assumption of a constant right electron asymmetry $\xi_{eR}/0.0001 = \beta = \text{const}$ is provided by the adiabatic condition $\partial_t \xi_{eR} \approx 0$ which we tacitly assume.

## 4 Discussion

The lepton asymmetry plays a crucial role in the amplification of hypermagnetic fields, which become helical and supply magnetic helicity to the cosmological and subsequently to the galactic magnetic fields. The lepton asymmetry itself evolves due to the Abelian anomaly for the right electron current given by Eq. (2.3) in the unbroken phase of the Standard Model. It starts from an initial $\xi_{eR}(t_0)$ generated by an unspecified mechanism and then rises driven by $\alpha_Y$-helicity parameter (2.2) arising from the Chern-Simons term (2.4). One should note that the absence of the Chern-Simons term in the broken phase [12] does not mean that parity violation in electroweak interactions disappears. Indeed the polarization effect leading to the helicity $\alpha$ parameter governing Maxwellian field evolution exists due to paramagnetism of fermions populating the main Landau level [16].

The magnetic helicity parameter $\alpha$ which governs the evolution of the magnetic helicity after the electroweak phase transition is shown in Fig. 1 by short horizontal lines for $t > t_{EW}$. This change of the magnetic helicity density profile $h(t)$ is explained by a negligible value of the helicity parameter $\alpha$ for weak interactions in broken phase at $T \ll T_{EW}$, $\alpha \sim G_F$ [14, 16], comparing with $\alpha_Y$ given by Eq. (2.2) for symmetric phase, $\alpha \ll \alpha_Y$ .

The jump of helicity density at $t = t_{EW}$ is the topological effect of a difference between the volume helicity density entering the first line in Eq. (2.5) and the surface helicity term in the same equation. While the volume term gives the smooth function (3.6) in the numerator of the ratio in Eq. (3.7) the surface term leads to the helicity density at EWPT $\sim n$ given by (3.2) in the denominator of Eq. (3.7).

Note also that, following standard practice, we have neglected turbulence due to a non-zero plasma vorticity arising e. g. through bubble collisions during the electroweak phase transition. Such simplification is justified in the treatment of hypermagnetic helicity since the fluid velocity $\mathbf{V}(\mathbf{x}, t)$ does not contribute to helicity evolution. As a result here we have confined our attention only to the $\alpha^2$-dynamo mechanism, avoiding $\alpha\Omega$ -dynamo scenario.

In summary, we have found that the magnetic helicity which becomes an inviscid invariant after the EWPT, varies dramatically before this phase transition. Due to this, the cosmological magnetic field becomes helical before the phase transition and remains helical after it, at least within the range of applicability of our causal scenario. It means that the seed magnetic field for the galactic dynamo if provided by a primordial cosmological magnetic field should be substantially helical. This seed magnetic helicity must be taken into account in scenarios of galactic magnetic field evolution with a cosmological seed. In particular, the intergalactic magnetic field suggested in [22] is expected to be substantially helical.

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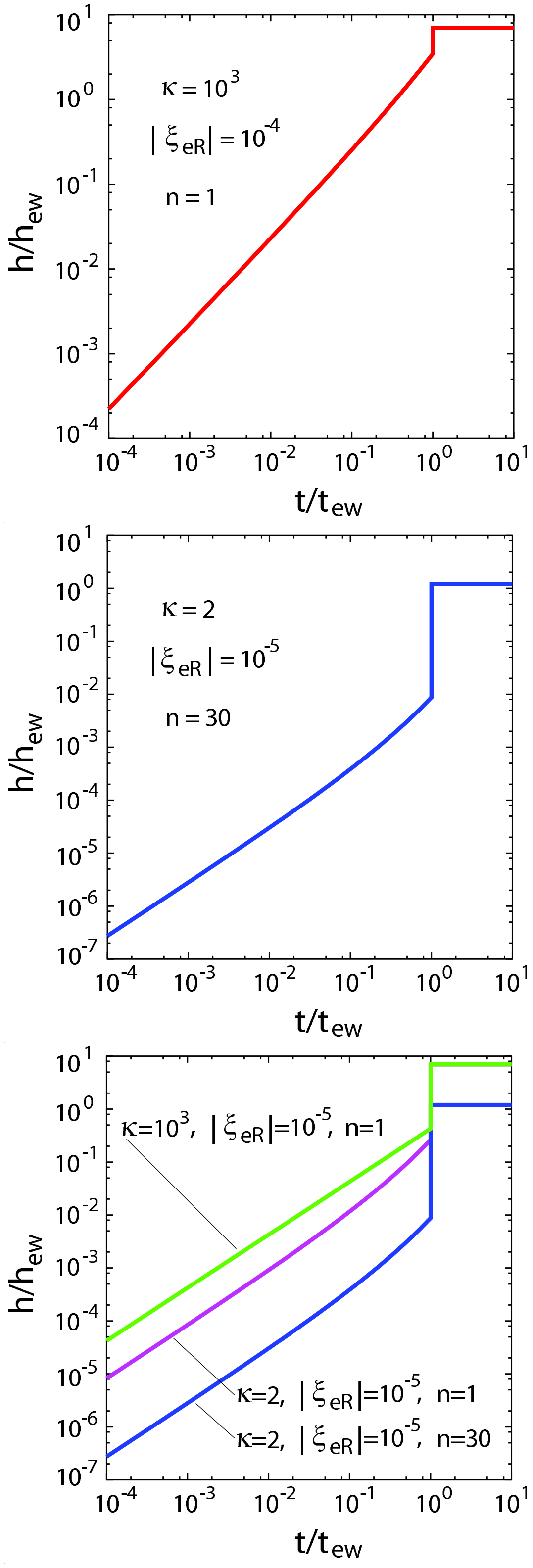


**Figure 1**. Magnetic helicity evolution in the EWPT epoch for various parameter choices: top: $a = 0.166$, $\mid \beta \mid= 1$, $n = 1$; middle: $a = 41.375$ (i.e. $\kappa = 2$), $\mid \beta \mid= 0.1$, $n = 30$; bottom: red line $a = 41.375$ , $\mid \beta \mid= 0.1$, $n = 1$, blue $a = 41.375$, $\mid \beta \mid= 0.1$, $n = 30$, green: $a = 0.166$ (i.e. $\kappa = 10^3$), $\mid \beta \mid= 0.1$, $n = 1$; for $t > t_{EW}$ red and green lines are identical.