hep-ph/9701420
FTUV/97-04
IFIC/97-04
IC/97/7
January 30, 1997

Neutrino Conversions in a Polarized Medium

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Abstract

Electron polarization induced by magnetic fields can modify the potentials relevant for describing neutrino conversions in media with magnetic fields. The magnitudes of polarization potentials are determined for different conditions. We show that variations of the electron polarization along the neutrino trajectory can induce resonant conversions in the active-sterile neutrino system, but cannot lead to level crossing in the active-active neutrino system. For neutrino flavour conversions the polarisation leads only to a shift of the standard MSW resonance. For polarizations $\lambda \lesssim 0.04$ the direct modifications of the potential (density) due to the magnetic field pressure are smaller than the modifications due to the polarization effect. We estimate that indeed the typical magnitude of the polarization in the sun or in a supernova are not expected to exceed 10^{-2}. However even such a small polarization may lead to interesting consequences for supernova physics and for properties of neutrino signals from collapsing stars.

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1 Introduction

Neutrino propagation in magnetized media has attracted considerable attention recently, both from the point of view of the early universe cosmology as well as astrophysics. The presence of magnetic fields in the Universe [1] as well as in various astrophysical objects [2] can affect neutrino conversion rates and this could have important implications.

It has been shown that the neutrino dispersion relations in media with non-zero magnetic fields are modified with respect to those in vacuo and that the effect of the magnetic field can be equivalently described as a correction to the neutrino self-energy [3, 4]. An alternative equivalent approach to this problem has been given in ref. [5] where the matrix element of the axial vector current has been calculated for an electron-positron plasma in the presence of a magnetic field. The basic result is that the potential relevant for neutrino propagation acquires a component (axial potential) which is proportional to the scalar product of the neutrino momentum and the magnetic field vector. In particular, it has been claimed that the axial potential can be larger than vector current term, thus inducing the possibility of a new type of resonant neutrino conversions [6, 7].

Several papers have considered the implications of the axial potential in the propagation of neutrinos in media which may be magnetized either by regular or random magnetic fields. In the latter case neutrino conversions become aperiodic [5]. The effect of such axial potentials on active-sterile supernova and early universe neutrino conversions has been discussed [8]. In addition, the corresponding effect of strong random magnetic field upon neutrino transitions induced by a transition magnetic moment in the early universe hot plasma or in a supernova was discussed in ref. [9]. It has also been realized recently [10] that even small polarization effects may have very remarkable implications. For example, they may lead to an explanation for the birth velocities of pulsars as resulting from asymmetries due to neutrino conversions in the cooling protoneutron star.

In this paper we consider some general features of the neutrino propagation in polarized media. We generalize the results of an early computation of the polarization effect in ref. [11]. In particular, we show that the previously considered magnetization effects can be equivalently treated as the effect of *polarization* of the electrons induced by the magnetic field.

We find the potentials in terms of the averaged polarization of the medium (sect. 2) and then calculate the averaged polarizations in the magnetic field for various physical conditions (sect. 3.). This approach gives a more transparent physical interpretation of the results and allows one to obtain an important restriction which was missed in ref. [6, 7], a fact which led to some incorrect statements. In sect. 4 we study the influence of the polarization on neutrino conversions. We show that electron polarization can induce resonant conversions in the active-sterile neutrino system, but can lead only to a shift of the usual MSW resonance [12, 13] for neutrino flavour conversions. In sect.5 we consider possible implications of the polarization effects for solar and supernova neutrinos. We have explored quantitatively the expected magnitude of the polarization which is consistent with realistic density, Y_e and temperature profiles found in the sun or a supernova. We have estimated that for polarizations $\lambda \lesssim 0.04$ the modifications in the potential are smaller than the corresponding direct modifications of the potential due to field pressure. We estimate the typical magnitudes of the polarization for various physical situations. Even though we find that the expected values of the polarization is small, it can lead to interesting consequences for supernova physics and for properties of neutrino signals from collapsing stars.

2 Effective potentials in a polarized medium

In what follows we will consider mainly polarization effects of electrons in a medium. In most cases the polarization of nuclei and nucleons is much weaker. Moreover, in the standard model only electrons are relevant for conversions involving only active neutrinos. We will consider nucleons polarization in section 3.3 and 3.5.

The effect of the medium is described by the potential

$$V = \langle \Psi | H_{int} | \Psi \rangle \ , \tag{1}$$

where Ψ is the wave function of the system neutrino-medium, and H is the standard Hamiltonian of the weak interaction at low energies

$$H_{int} = \frac{G_F}{\sqrt{2}} \nu \gamma^{\mu} (1 - \gamma_5) \nu \bar{e} \gamma_{\mu} (g_V + g_A \gamma_5) e \ . \tag{2}$$

Here G_F is the Fermi constant and g_V and g_A are the effective vector and axial-vector coupling constants in the standard model.

Let us consider the propagation of ultra-relativistic neutrinos with helicity - 1 in medium with free electrons having the distribution (density)

$$\frac{f(\vec{\lambda}_e, \vec{p}_e)}{(2\pi)^3}$$

over the momentum, $\vec{p}_e$, and the polarization $\vec{\lambda}_e$. The vector of polarization is determined as

$$\vec{\lambda}_e = \omega_e^{\dagger} \vec{\sigma} \omega_e, \tag{3}$$

and ω_e denotes the two-component spinor of the electrons. The total number density of electrons, n_e, equals

$$n_e = \sum_{\vec{\lambda}} \int \frac{d^3 p_e}{(2\pi)^3} \ f(\vec{\lambda}_e, \vec{p}_e). \tag{4}$$

In an *unpolarized* medium, $\vec{\lambda}_e = 0$, the potential is determined by the vector component of the electron current:

$$V = V^V(\vec{p}_e) = \sqrt{2} G_F \, g_V \frac{f_e(\vec{p}_e)}{(2\pi)^3} \left(1 - \frac{\vec{p}_e \cdot \widehat{k}_{\nu}}{E_e} \right), \tag{5}$$

where $\widehat{k}_{\nu} \equiv \vec{p}_{\nu}/|\vec{p}_{\nu}|$ with $\vec{p}_{\nu}$ being the neutrino momentum, E_e is the energy of electrons. The expression in eq. (5) should be integrated over the $\vec{p}_e$ distribution of electrons. In an isotropic medium the second term is averaged out and we get the usual formula [13]

$$V = \sqrt{2} G_F \, g_V n_e \ , \tag{6}$$

where the total concentration n_e is determined in eq. (4). Eq. (6) also holds for the anisotropic case when the fluxes of electrons moving in opposite directions are equal. Such a situation is realized in a magnetized medium (see sect.3).

In the case of a *polarized* medium the axial vector current also contributes [11]. Performing a straightforward calculation we get the general expression

$$V^A(\vec{\lambda}_e, \vec{p}_e) = \sqrt{2} G_F\, g_A \frac{f(\vec{\lambda}_e, \vec{p}_e)}{(2\pi)^3} \left[\frac{(\vec{p}_e \cdot \vec{\lambda}_e)}{E_e} - \frac{m_e}{E_e} (\widehat{k}_\nu \cdot \vec{\lambda}_e) - \frac{(\vec{p}_e \cdot \vec{\lambda}_e)(\vec{p}_e \cdot \widehat{k}_\nu)}{E_e(E_e + m_e)} \right], \quad (7)$$

Note that $V^A \propto \lambda_e$, and therefore in an unpolarized medium $V^A = 0$.

Let us consider some special cases of eq. (7).

- For a *non-relativistic electron medium*, $\vec{p}_e \approx 0$, we get from eq. (7)

$$V^A \approx -\sqrt{2} G_F\, g_A n_e \left(\widehat{k}_\nu \cdot \left\langle \vec{\lambda}_e \right\rangle \right) , \quad (8)$$

  where the average polarization of electrons is defined as

$$\left\langle \vec{\lambda}_e \right\rangle = \frac{1}{n_e} \sum_{\vec{\lambda}} \int \frac{d^3 p_e}{(2\pi)^3} \vec{\lambda}\, f(\vec{\lambda}_e, \vec{p}_e), \quad (9)$$

  The total potential is

$$V = \sqrt{2} G_F\, n_e [g_V - g_A \left(\widehat{k}_\nu \cdot \left\langle \vec{\lambda}_e \right\rangle \right)]. \quad (10)$$

- In the case of *ultra-relativistic electrons* we get from eq. (7)

$$V^A \approx \sqrt{2} G_F\, g_A \frac{f(\vec{\lambda}_e, \vec{p}_e)}{(2\pi)^3} (\widehat{k}_e \cdot \vec{\lambda}_e) \left[1 - (\widehat{k}_e \cdot \widehat{k}_\nu) \right] . \quad (11)$$

  Here $\widehat{k}_e \equiv \vec{p}_e / |\vec{p}_e|$ is the unit vector in the direction of electrons momentum. If electrons are polarized in the transverse plane the potential is zero for any momenta of neutrinos. The potential is suppressed if neutrinos and electrons are moving in the same direction.

- The expression for the potential is simplified if electrons have a certain helicity:

$$V^A(\vec{\lambda}_e = \pm \widehat{k}_e) = \pm \sqrt{2} G_F\, g_A \frac{f(\vec{\lambda}_e, \vec{p}_e)}{(2\pi)^3} \left[\frac{p_e}{E_e} - (\widehat{k}_e \cdot \widehat{k}_\nu) \right] . \quad (12)$$

  In the ultra-relativistic case this expression is reduced to eq. (11) and in non-relativistic case we get eq. (8) .

- The case which is important for a *magnetized medium* (see sect. 3) is when there are two equal electron fluxes moving in opposite directions but heaving the same polarization along the momentum (electrons in the lowest Landau level). Using eq. (12) we find

$$V^A = -\sqrt{2} G_F\, g_A n_e (\widehat{k}_\nu \cdot \vec{\lambda}_e). \quad (13)$$

  Here n_e is the total concentration of electrons in both fluxes. Let us underline that this relativistic expression coincides with the non-relativistic formula eq. (8).

In what follows we will consider the axial vector potential in eq. (8) or eq. (13).

The total effective potential resulting from electrons, protons and neutrons in an electrically neutral medium can be written in the form,

$$V = \sqrt{2} G_F \, n_e \left[g_V - g_A \widehat{k}_\nu \cdot \left\langle \vec{\lambda}_e \right\rangle \right] + \sqrt{2} G_F \, n_n g_V^n , \tag{14}$$

In eq. (14), the second term describes neutrino-nucleon scattering, with n_n being the neutron concentration.

The effect of the medium on neutrino propagation is determined by the difference of potentials. For the case of $\nu_e \to \nu_\mu, \nu_\tau$ flavour conversion only charge current neutrino-electron scattering gives a net contribution. Using $g_V = -g_A = 1$ one finds

$$\begin{aligned} V_{e\mu} &= \sqrt{2} G_F \, n_e \left[1 + \widehat{k}_\nu \cdot \left\langle \vec{\lambda}_e \right\rangle \right] \\ &= \sqrt{2} G_F \, n_e \left[1 + \langle \lambda_e \rangle \cos \alpha \right] , \end{aligned} \tag{15}$$

where α is the angle between the neutrino momentum and the average polarization of electrons. There is no effect of nucleons in this case. Depending on the direction of polarizations the axial term can either enhance or suppress the potential. The maximal effect is obtained in the case of complete polarization in the direction of the neutrino momentum, $\langle \lambda \rangle = 1$. In the case of complete polarization against the neutrino momentum, $\cos \alpha = -1$, $\langle \lambda \rangle = 1$, we get $V_{e\mu} = 0$. Clearly, the axial vector term can not overcome the vector term, $|V_V| \geq |V_A|$. Thus it can not change the sign of $V_{e\mu}$ and therefore it can not induce resonant conversions.

Let us now assume also that some amount of positrons is present in medium. Then the axial part of the potential for $g_A = -1$ is given by,

$$V^A = \sqrt{2} G_F \, \widehat{k}_\nu \cdot \left[n_e \left\langle \vec{\lambda}_e \right\rangle + n_e^+ \left\langle \vec{\lambda}_{e^+} \right\rangle \right] , \tag{16}$$

where n_e^+ is the positron concentrations. Note that electron and positron contributions have the same sign, so that the net effect is determined by the relative polarization of electrons and positrons. Due to the different signs of their electric charges electrons and positrons are polarized in opposite directions in the magnetic field and therefore, $\left\langle \vec{\lambda}_{e^+} \right\rangle = - \left\langle \vec{\lambda}_e \right\rangle$, so that

$$V^A = \sqrt{2} G_F \, \widehat{k}_\nu \cdot \left\langle \vec{\lambda}_e \right\rangle \left[n_e - n_e^+ \right] . \tag{17}$$

Note that the vector part also depends on the difference of electrons and positrons concentrations and thus the total potential is determined by $\Delta n \equiv n_e - n_e^+$.

In the case of conversion into sterile neutrinos, the difference of potentials has also the contribution from neutrino-nucleon scattering. With unpolarized nucleons we find,

$$V_{es} = \sqrt{2} G_F \, n_e \left[\left(1 - \frac{n_n}{2 n_e} \right) + \frac{1}{2} \widehat{k}_\nu \cdot \left\langle \vec{\lambda}_e \right\rangle \right] . \tag{18}$$

Now the polarization term can be bigger than the vector one, thus leading to the possibility of level crossing induced by the axial term.

In next section we will calculate $\left\langle \vec{\lambda}_e \right\rangle$ for different physical conditions.

3 Polarization in a medium with magnetic field

Let us calculate the polarization $\left\langle \vec{\lambda}_e \right\rangle$ in a medium with electrons and positrons in the presence of a magnetic field. Suppose that the magnetic field is in the positive z direction, $\vec{B} = (0, 0, B)$. In this case the energy spectrum of electrons is quantized according to

$$\varepsilon(p_z, n, \lambda_z) = \sqrt{p_z^2 + m_e^2 + eB(2n + 1 + \lambda_z)} \ \ (n = 0, 1, 2, ..., \ \ \lambda_z = \pm 1), \tag{19}$$

where p_z is the neutrino momentum in the z direction, m_e is the electron mass, $e\,(> 0)$ is the absolute value of the electric charge, and λ_z is the z component of $\vec{\lambda}_e$. One can rewrite eq. (9) as

$$\langle \lambda_e \rangle = \frac{1}{n_e} \sum_{n=0}^{\infty} \sum_{\lambda_z} \frac{eB}{(2\pi)^2} \int_{-\infty}^{\infty} dp_z \ \lambda_z f(p_z, n, \lambda_z), \tag{20}$$

with

$$n_e = \sum_{n=0}^{\infty} \sum_{\lambda_z} \frac{eB}{(2\pi)^2} \int_{-\infty}^{\infty} dp_z \ f(p_z, n, \lambda_z). \tag{21}$$

Here, we have replaced the integration over $dp_x dp_y$ by a summation over n, taking into account that $\int dp_x dp_y \to \sum_n 2\pi eB$. We take for the distribution function $f(p_z, n, \lambda_z)$ as the usual Fermi-Dirac form

$$f(p_z, n, \lambda) = \frac{1}{\exp[(\varepsilon(p_z, n, \lambda_z) - \mu)/T] + 1}, \tag{22}$$

where μ is the chemical potential of electrons and T is the temperature. From eq. (19) one sees that the energy spectrum of electrons consists of the lowest Landau level, $n = 0, \lambda_z = -1$, plus pairs of degenerate levels with opposite polarizations. As a result only the lowest level survives in the sum in eq. (20) [5, 4], so that the average polarization of electrons can be expressed as,

$$\langle \lambda_e \rangle = -\frac{n_0}{n_e} \equiv \frac{1}{n_e} \frac{eB}{(2\pi)^2} \int_{-\infty}^{\infty} dp_z \ \frac{-1}{\exp[(\sqrt{p_z^2 + m_e^2} - \mu)/T] + 1} \ , \tag{23}$$

where n_0 is the electron number density in the lowest Landau level. Similarly, for positrons, we obtain

$$\langle \lambda_{e^+} \rangle = \frac{n_0^+}{n_e^+} = \frac{n_0(\mu \to -\mu)}{n_e(\mu \to -\mu)}, \tag{24}$$

where n_0^+ is the positron concentration in the lowest Landau level. From eq. (16) the axial-vector potential induced by the polarization can be written (for $g_A = -1$) as,

$$V^A = -\sqrt{2} G_F \, (n_0 - n_0^+) \cos \alpha_B \ , \tag{25}$$

where α_B is the angle between the neutrino momentum and the direction of the magnetic field. Note that since the electrons in the lowest Landau level are polarized against the field, $\cos \alpha = -\cos \alpha_B$. Substituting the expression for n_0^+ and n_0 given in eq. (23) into eq. (25) we reproduce the same formula for V^A found by other methods in ref. [5] and in ref. [4] ¶.

¶Note that in ref. [4] the result in eq. (25) was obtained through the calculation of one-loop diagrams using electron Green functions in the medium in the presence of magnetic field.

Let us show that the same expression for the potential eq. (25) is true even for the general case of eq. (7). Let us first calculate the contribution to potential V^A from the electrons in the level characterized by p_z, n, λ. For this we should average general expression for $V^A(\vec{p}_e, \vec{\lambda}_e)$ eq. (7) , over the azimuthal angle of the electrons (this angle fixes direction of the electron momentum in the plane orthogonal to the magnetic field). Averaging over two possible directions of momentum $\pm p_z$ we find

$$\bar{V}^A(p_z, n, \lambda_z) = \frac{1}{4\pi} \sum_{\pm p_z} \int d\phi V^A(\vec{p}_e, \vec{\lambda}_e) = -\sqrt{2} G_F \, g_A \lambda_z \cos\alpha_B \left[1 - \frac{E_e^2 - p_z^2 - m_e^2}{E_e(E_e + m_e)} \right] , \tag{26}$$

where $E_e = \varepsilon(p_z, n, \lambda_e)$ is given in eq. (19) . Summing over all the levels we get total axial vector potential:

$$V^A = \sum_{n=0}^{\infty} \sum_{\lambda_z} \frac{eB}{(2\pi)^2} \int_{-\infty}^{\infty} dp_z \; \bar{V}^A(p_z, n, \lambda_z) \; f(p_z, n, \lambda_z). \tag{27}$$

The contributions from the levels with the same energy and opposite λ_z cancel each other, and the effect is determined by the lowest Landau level. For this level $\lambda_z = -1$ and $E_e^2 = p_z^2 + m_e^2$, so that second term in eq. (26) is zero and integration becomes trivial:

$$V^A = -\sqrt{2} G_F \, g_A n_0 \lambda_e \cos\alpha_B \tag{28}$$

which coincides with eq. (25).

One remark is in order. The expressions for the potentials in terms of averaged polarizations have been obtained for free-electron wave functions. These expressions can also be used in the weak magnetic field limit: $eB \ll \langle p_z^2 \rangle$. In general, for strong magnetic fields the use of free-electron wave functions is not justified (if fact, in eq. (26) and eq. (27) we have used the modified dispersion relation eq. (19)). In our case, however, the task is simplified since the polarization potential is determined only by electrons at the lowest Landau level. In this level the electrons are moving along the magnetic field, they obey the vacuum relation between the momentum and energy, and are described by free-electron wave functions. Therefore we can apply immediately the results of sect. 2 even for the case of strong magnetic fields.

3.1 Strongly degenerate electron gas

Let us now determine the expression for the electron concentration at the lowest Landau level n_0 for the case of a strongly degenerate electron gas: $(\mu - m_e)/T \gg 1$. In this case the Fermi-Dirac distribution eq. (22) can be approximated by the step function:

$$[\exp((\sqrt{p_z^2 + m_e^2 + 2eBn} - \mu)/T) + 1]^{-1} \to \theta(\mu - \sqrt{p_z^2 + m_e^2 + 2eBn}) \tag{29}$$

(for the levels with $\lambda = -1$; due to degeneracy it is enough to consider the levels with this polarization). Using approximation eq. (29) we obtain from eq. (23)

$$n_0 = \frac{eBp_F}{2\pi^2}, \tag{30}$$

where $p_F = \sqrt{\mu^2 - m_e^2}$. The Fermi momentum p_F is determined from the expression for the total electron concentration n_e which can be obtained by the explicit integration in eq. (21) with eq. (29):

$$n_e = \frac{eBp_F}{2\pi^2} + \sum_{n=1}^{n_{max}} \frac{2eB\sqrt{p_F^2 - 2eBn}}{2\pi^2}. \tag{31}$$

The factor 2 in the sum takes into account the degeneracy of levels. Note that in general p_F depends on B. In eq. (31) the first term is the contribution from the lowest Landau level and the second one results from the summation over all higher Landau levels. The summation goes up to a maximum value $n_{max} = [p_F^2/(2eB)]$, the integer part of $p_F^2/(2eB)$. If the magnetic field is very strong, $2eB \geq p_{F_e}^2$, the sum vanishes and all electrons are at the main Landau level, which means that the electron gas is fully polarised and all the electron spins are aligned opposite to the magnetic field. From eq. (30) and eq. (31) we find

$$n_0 = \frac{n_e}{1 + \sum_{n=1}^{n_{max}} \sqrt{1 - \frac{2eBn}{p_F^2}}}. \tag{32}$$

Clearly $n_e \geq n_0$.

Let us now consider two extreme cases.

1. Strong magnetic field limit:

$$2eB > p_F^2. \tag{33}$$

The sum in eq. (32) disappears so that $n_e = n_0$, i.e. all the electrons are in the first Landau level: the medium is completely polarized. In this case we get from eq. (31)

$$p_F = \frac{2\pi^2 n_e}{eB} \ , \tag{34}$$

and the condition eq. (33) of complete polarization can be written as

$$B > \frac{1}{e}\left(\sqrt{2}\pi^2 n_e\right)^{2/3} \ . \tag{35}$$

Clearly, the higher the density the larger magnetic field required in order to have complete polarization.

2. Weak field limit:

$$eB \ll p_F^2. \tag{36}$$

The the sum in eq. (31) contains contributions from many Landau levels and dominates over the first term. The sum can be approximated by integration as follows,

$$n_e = \frac{eBp_F}{2\pi^2}\left[1 + \frac{p_F^2}{eB}\int_{2eB/p_F^2}^{1} dx\sqrt{1-x}\ \right] \approx \frac{p_F^3}{3\pi^2}\Big[1 + \frac{3eB}{2p_F^2}\Big] \ . \tag{37}$$

From this we get the usual expression for p_F in a medium without magnetic field:

$$p_F \simeq (3\pi^2 n_e)^{1/3} \ . \tag{38}$$

Inserting this p_F in eq. (30) we have,

$$n_0 = \frac{eB}{2}\left(\frac{3n_e}{\pi^4}\right)^{1/3} , \quad (39)$$

and consequently,

$$\left\langle \vec{\lambda}_e \right\rangle = -\frac{e\vec{B}}{2}\left(\frac{3}{\pi^4}\right)^{1/3} n_e^{-2/3}. \quad (40)$$

The polarization effect increases linearly with B and decreases as $n_e^{-2/3}$. Using eq. (40) and eq. (25) we get for the effective potential of the electrons,

$$V = \sqrt{2}G_F\, n_e - \frac{G_F\, eB}{\sqrt{2}}\left(\frac{3n_e}{\pi^4}\right)^{1/3} \cos\alpha_B. \quad (41)$$

This expression coincides with one used in [7, 10]. Although one might think from eq. (41) [6, 7] that the axial contribution may be dominant, it should be clear from our discussion above that this expression is correct only if the polarization term is small in comparison with vector current contribution.

The general behaviour of the polarization $\langle\lambda_e\rangle = n_0/n_e$ with respect to the magnetic field and density can be obtained from eq. (32) for the case of strong degeneracy. The result is shown in Fig. 1. In this figure the lines of equal polarization basically correspond to the dependence given in eq. (40) ‖.

3.2 Electrons at finite temperature

The polarization effect decreases as the temperature of the medium increases.

- For small finite temperatures, $T \ll \mu - m_e$, temperature effects give only a small negative contribution to the electron density in the first Landau level [14],

$$n_0 \sim \frac{eBp_F}{2\pi^2}\Big[1 - \frac{\pi^2}{6}\Big(\frac{m_e T}{p_F^2}\Big)^2 + ...\Big] . \quad (42)$$

- For the opposite case of weak degeneracy, $(\mu - m_e)/T < 1$, strong polarization is achieved if the first term in eq. (21) dominates over the higher levels. In particular,

$$\frac{1}{\exp[(m_e-\mu)/T]+1} \gg \frac{1}{\exp[(\sqrt{m_e^2+2eB}-\mu)/T]+1} . \quad (43)$$

From this we get,

$$\frac{\sqrt{m_e^2+2eB}-\mu}{T} \gg 1. \quad (44)$$

If this condition is satisfied, the contribution to the total density from the higher Landau levels ($n = 1, 2, 3...$) is strongly suppressed for all momenta p_z, and as a result nearly complete polarization is achieved, $n_e \sim n_0$. Again here we identify two cases:

‖The small jumps in these lines for $\langle\lambda_e\rangle$ arise from discrete changes in the sum eq. (32).

1. In the weak field limit, $m_e^2 \gg 2eB$, the condition in eq. (44) becomes,

$$\frac{eB}{m_e T} = \frac{2\mu_B B}{T} \gg 1, \tag{45}$$

where μ_B is the Bohr magneton. According to eq. (45) the interaction energy with the magnetic field (Zeeman energy) should be much bigger than the kinetic energy of the electrons. Polarization itself can be estimated as, in the non-relativistic case,

$$\langle \lambda_e \rangle \sim \frac{\mu_B B}{T} \ . \tag{46}$$

In ultra-relativistic case:

$$\langle \lambda_e \rangle \sim \frac{eB}{6T^2} = \frac{\mu_B B m_e}{3T^2} \ . \tag{47}$$

In non-relativistic case the total potential becomes:

$$V \approx \sqrt{2} G_F n_e \left[1 - \frac{\mu_B B}{T} \cos \alpha_B \right] \tag{48}$$

2. In the strong field limit, $m_e^2 \ll 2eB$, we get from eq. (44),

$$\frac{\sqrt{2eB}}{T} \gg 1. \tag{49}$$

In these cases, there is a strong polarization even if electrons are not degenerate.

The general temperature dependence of the polarization $\langle \lambda_e \rangle$ for different values of the magnetic field is shown in Fig. 2a and 2b. One sees that the depolarization effect due to temperature becomes strong for small values of the degeneracy parameter $(\mu - m_e)/T$. Note that even for very strong magnetic field, e.g. $2eB/p_F^2 = 2$, the polarization is only $\sim 5\%$ for $T \sim 1$ MeV and $\rho Y_e = 10^6$ g/cm^3. For higher density, e.g. $\rho Y_e = 10^8$ g/cm^3 and fixed value of $2eB/p_F^2 = 2$, the depolarization effect is small for $T < 1$ MeV.

3.3 Polarized nucleons

For system of active-sterile neutrinos also the polarization of nucleons gives some effect. Since for sterile neutrinos $V^V = V^A = 0$, the difference of the potentials is determined by the total potential of the active component.

Let us consider an electrically neutral medium which consists of electrons and non-relativistic protons and neutrons with concentrations n_p and n_n correspondingly. The total axial vector potential for the electron neutrino can be written as

$$V^A \approx -\frac{G_F}{\sqrt{2}} \widehat{k}_\nu \cdot \left[-n_e \left(\langle \vec{\lambda}_e \rangle + g_A^N \langle \vec{\lambda}_p \rangle \right) + g_A^N n_n \langle \vec{\lambda}_n \rangle \right] \ , \tag{50}$$

where $g_A^N \approx 1.26$ is the renormalization constant of the axial-vector nucleon current. For the muon (or tau) neutrinos the electron contribution changes, and we get

$$V^A \approx -\frac{G_F}{\sqrt{2}} \widehat{k}_\nu \cdot \left[n_e \left(\langle \vec{\lambda}_e \rangle - g_A^N \langle \vec{\lambda}_p \rangle \right) + g_A^N n_n \langle \vec{\lambda}_n \rangle \right] \ , \tag{51}$$

where $\left\langle \vec{\lambda}_p \right\rangle$ and $\left\langle \vec{\lambda}_n \right\rangle$ are the averaged polarizations of protons and neutrons.

Note that according to eq. (30) and eq. (40) in the limit of a strongly degenerate distribution, the polarization does not depend on the mass of the particle and one would expect the same degree of polarization both for electrons and protons: $\vec{\lambda}_p \approx -\vec{\lambda}_e$. However, in realistic conditions nucleons are typically strongly non-degenerate and their polarization can be estimated as

$$\langle \lambda_{p,n} \rangle \sim \frac{\mu_{p,n} B}{T} \ , \tag{52}$$

where the magnetic moments of proton and neutron equal $\mu_p = 2.79\mu_N$ and $\mu_n = -1.91\mu_N$ with $\mu_N \equiv e/2m_N$ (here m_N is the mass of nucleon).

In the magnetic field due to different signs of the magnetic moments protons and neutrons are polarized in different directions. Therefore according to eq. (50), eq. (51) the contributions of the protons and neutrons to the potentials have the same sign. Moreover, in muon neutrino potential all contributions sum.

If both electrons and neutrons are non-degenerate, then $\lambda_N/\lambda_e \sim 10^{-3}$ and the polarization effect of the nucleons is about three orders of magnitude smaller. However, the degeneracy suppresses polarization so that if the electron distribution is strongly degenerate, whereas at the same time the nucleons are strongly non-degenerate (as realized in central regions of hot neutron stars) the polarization of nucleons might become important.

4 Neutrino transitions in a polarized medium

The effect of the axial vector term in the resonant conversions of active neutrinos was first obtained in ref. [6] and independently in [7]. Here we will confine ourselves to the qualitative effects associated with polarization, and with a rough estimate of the magnitude of the polarization matter potentials for various physical systems. The explicit calculation of the corresponding conversion probabilities will be taken up elsewhere.

For definiteness, we consider a system of two neutrinos ν_e and ν_x, where ν_x is an active neutrino (ν_μ or ν_τ) or a sterile state ν_s. The evolution equation is given by

$$i\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = \begin{pmatrix} V - \Delta\cos 2\theta & \frac{1}{2}\Delta\sin 2\theta \\ \frac{1}{2}\Delta\sin 2\theta & 0 \end{pmatrix}\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix}, \tag{53}$$

where $\Delta \equiv \frac{m_2^2 - m_1^2}{2E}$ with m_2 and m_1 being the neutrino masses, and E is the neutrino energy**.

The effective potential V for $\nu_e - \nu_{\mu,\tau}$ conversions given in previous sections can be written as,

$$V = V_{e;\mu,\tau} = \sqrt{2}G_F\, n_e(r)[1 - b(\vec{r})\cos\alpha(\vec{r})], \tag{54}$$

where

$$b(\vec{r}) \equiv \frac{n_0(\vec{r}) - n_0^+(\vec{r})}{n_e(r) - n_e^+(r)}. \tag{55}$$

characterizes the polarization term. For simplicity we have assumed an isotropic electron density profile. Clearly the axial term changes the matter potential as,

$$V\Big(n_e(r)\Big) \to V = V^V\Big(n_e(r)\Big) + V^A\Big(n_e(r), B(\vec{r}), \alpha(\vec{r})\Big)\ . \tag{56}$$

**For the case of anti-neutrinos the evolution equation is the same but with opposite sign of V.

Polarization can enhance or suppress the potential depending on the direction of the magnetic field. The modification of the potential is of the order 1 if the medium is strongly polarized. If fact, one can envisage an extreme situation in which in some region there is a complete polarization in the direction of neutrino momentum. In this case the polarization term cancels the vector term, leading to $V = 0$. In such a region the evolution of the neutrino system is reduced to that in vacuo.

Let us now consider some possible effects of the polarization.

- Shift of the resonance.

  The resonance condition, $V - \Delta \cos 2\theta = 0$ can be written as,

$$\sqrt{2} G_F \, n_e(r)[1 - b(r)\cos\alpha(r)] - \Delta \cos 2\theta = 0. \tag{57}$$

  There are two ways of interpreting the effects of the polarization.

  1. Shift of the resonance position.
     It is clear that the presence of the polarization term changes the value of r_R at which the resonance condition is fulfilled. In other words, the layer in which the flavour transition takes place is shifted depending on the strength and direction of the magnetic field.
  2. Shift of the neutrino parameters.
     According to eq. (57) the axial vector contribution changes the neutrino parameter (Δ - mass squared difference or energy required for resonance in a certain layer with fixed total density n_e):

$$\Delta \to \frac{\Delta}{(1 - b\cos\alpha)}.$$

     Depending on the polarization direction the parameter Δ can be diminished or increased. This shift may have a number of interesting consequences already discussed in ref. [6, 7]. For example, a system of almost degenerate neutrinos can undergo resonant conversions in a strongly polarized medium in the case of positive (but small) Δ. Note that Δ can not be too small or zero since the mixing of neutrinos is proportional to Δ (see eq. (53)) and with diminishing Δ adiabaticity starts to be broken.

     Note that for a system of active neutrinos the possibility of resonant conversion exists only for neutrinos or anti-neutrinos even if a medium is polarized: neutrinos and anti-neutrinos do not simultaneously convert, in contrast with the situation considered in ref. [15]. Moreover, since the polarization term can not overcome the vector potential term it can not induce resonant anti-neutrino flavour conversions if $\Delta > 0$, only for $\Delta < 0$. This conclusion is in conflict with the papers in ref. [6, 7]. Unfortunately, as we will see in the next section, the level of polarization required in order to have strong resonance shift is too large to achieve with reasonable values of the magnetic field, at least for the case of the sun.

- Modification of the adiabaticity.

  The polarization term modifies the dependence of V on r and therefore, influences the adiabaticity of propagation. This in turn changes the transition probability in certain ranges of neutrino parameters θ and Δm^2.

- Noisy media.

  The magnetic field may have a domain structure with different strength and direction in different domains. This leads to a perturbation of $V(r)$ profile which may have a random character. If the typical domain size is smaller than the neutrino oscillation length, this modulation can be considered as density (potential) random fluctuations, similar to those considered in ref. [17, 19].

- Resonant conversion of sterile neutrinos driven by polarization effect.

  Using the effective potential eq. (18) for a system of active-sterile neutrinos we can write the resonance condition as,

$$\sqrt{2}G_F\, n_e(r)\left[1-\frac{n_n(r)}{2n_e(r)}-\frac{1}{2}b(r)\cos\alpha(r)\right]-\Delta\cos 2\theta=0, \tag{58}$$

  where $b(r)$ is defined in eq. (55). For $n_e = n_p < n_n$ the polarization term can be bigger than the vector current contribution,

$$1-\frac{n_n(r)}{2n_e(r)}<\frac{1}{2}b(r)\cos\alpha(r). \tag{59}$$

  When this condition is fulfilled, the resonant conversion can occur even if the Δ term is negligible in eq. (58). In fact the resonant conversion can be driven by the polarization. The resonance condition eq. (58) can be re-written as,

$$\frac{1}{2}b(r)\cos\alpha(r)=1-\frac{n_n(r)}{2n_e(r)}\mp\frac{\Delta\cos 2\theta}{\sqrt{2}G_F\, n_e(r)}, \tag{60}$$

  where the minus sign is for neutrinos and the plus sign is for anti-neutrinos. The ratio $n_n(r)/2n_e(r)$ may have a rather weak dependence on r. Then the level crossing will be due mostly to changes of $b(r)\cos\alpha(r)$ on the way of neutrinos. That is, the change of the strength and the direction of the magnetic field along the neutrino motion will lead to resonant conversion. Note that if Δ is small enough, this condition is satisfied due to the polarization term both for neutrinos and anti-neutrinos. In other words, the polarization can induce resonant conversions in a medium with fixed chemical composition ($n_n/n_e \sim$ const) both in the neutrino and anti-neutrino channels. Unlike the active-active neutrino conversion case discussed above, here there can be simultaneous conversions of neutrinos and anti-neutrinos, similarly to the massless neutrino conversion mechanism [15]. Strictly speaking, we need a finite value of Δ in order to have mixing and adiabaticity, though it may be negligible in the resonance condition, eq. (60).

5 Astrophysics

Although for realistic applications a detailed study of the evolution equation eq. (53) is needed, one can gain some insight by first performing simple estimates of the magnitude of the polarization for different systems.

5.1 Polarization in the Sun

In the sun the degeneracy of electrons is small and the magnitude of the polarization is determined by the interplay of the magnetic field strength and the temperature: $\langle\lambda_e\rangle \sim \mu_B B/T$. Taking for the central region of the sun a maximal possible strength of the magnetic field, $B \sim 10^8$ Gauss, we get $\langle\lambda_e\rangle \sim 4 \cdot 10^{-4}$. In the bottom of the convective zone B could be as large as 10^6 Gauss, leading to $\langle\lambda_e\rangle \sim 3 \cdot 10^{-5}$. Thus in the sun the magnitude of the polarization is too small to expect sizeable effects. It can give only very small perturbations of the density (potential) profile.

5.2 Polarization in supernovae

Let us now estimate the possible allowed magnitude of the polarization $\langle\lambda\rangle_e$ in supernovae. Fig. 3a and 3b illustrate typical density and temperature profiles in a supernova with 20 $M_\odot$ progenitor for two different moments of time [18].

In Figs. 4a and 4b we have plotted required magnetic field profiles for producing a given polarization, $\langle\lambda_e\rangle = 0.01$, 0.1 and 0.99 for the density and temperature profiles shown in Fig. 3a and 3b. The degeneracy parameter, $(\mu - m_e)/T$ is also plotted. Clearly in the earlier epoch (t = 0.15 sec after the core bounce) the degeneracy condition: $(\mu - m_e)/T > 1$ is satisfied in the region $r \lesssim 100$ with $\rho Y_e \sim 10^9$ g/cm^3. The degeneracy parameter increases to 2 or larger at $r \lesssim$ 50 km. In this region we can estimate the magnetic field needed in order to have strong polarization using eq. (35),

$$B \sim 10^{17} (\rho_{12} Y_e)^{2/3} \ \ \text{Gauss}, \tag{61}$$

where ρ_{12} is the matter density in units of 10^{12}g/cm^3. At $r \simeq 100$ km, where $\rho Y_e \sim 10^9$ g/cm^3, one should have $B \gtrsim 3 \cdot 10^{16}$ Gauss for the polarization term to be close to one. For $r \simeq 400$ km ($\rho Y_e \sim 10^8$ g/cm^3) the required field is $\sim 3 \cdot 10^{15}$ Gauss. For external regions ($r > 1000$ km) the degeneracy is rather weak and the depolarization due to temperature becomes important.

As can be seen from Fig. 4b, for the later epoch, strong degeneracy is realized in even more central regions: $r < 12$ km, i.e. practically in the neutrinosphere. For $r > 15$ km the depolarization due to temperature dominates. In Fig. 4b we also show the magnetic field profiles leading to $\langle\lambda_e\rangle = 0.01, 0.1, 0.99$.

Comparing these results with usually accepted large-scale magnetic field profiles:

$$B(r) \sim B_0 \left(\frac{r_c}{r}\right)^n, \quad B_0 = 10^{12} - 10^{14} \ \text{Gauss}, \quad n = 2, 3, \quad r_c = 10 \ \text{km} \tag{62}$$

we conclude that the magnitude of the polarization term does not exceed 1 % level, and they drop below 0.1 % in external regions.

With the profile given in eq. (62) one may conclude that the effect of polarization will give only small modifications of the potentials and their influence on possible neutrino conversion in this region is expected to be weak. Note that in stars of smaller mass the degeneracy parameter is stronger and temperatures are lower. Therefore, the polarization for realistic magnetic fields can be bigger than 1 %.

5.3 Random Magnetic Fields

Besides global large-scale magnetic fields, a star can also have magnetic fields in small domains of size r_D ($r_D \ll r$, where r is the distance from the center to layers with domains). The field in different domains can be randomly oriented. The existence of domains with $r_D \sim 1$ km at $r \gtrsim 10$ km has been considered in ref. [16] in the context of neutron star dynamics. The effect of the axial potential induced by such strong random magnetic fields on active-sterile supernova and early universe neutrino conversions has been discussed in ref. [8, 9]. The strength of the random field inside the domains can be much larger than the strength of a global field, so that the polarization effect could be correspondingly bigger. There can be even stronger fields in filaments, like super-conducting needles.

As we have discussed in section 4, these domains or needles will cause a modulation of the density profile which will have a *noisy* character. If the number of domains along the neutrino path is large enough, even small modulations at the % level can lead to large (of order 1) changes of the transition probabilities [17, 19, 20].

Another effect is that the rotation of the star would induce a time dependence of the neutrino signal, since in different moments of time neutrinos directed to the earth cross different magnetic field domains.

5.4 Nucleon polarization

Let us finally estimate possible polarization effects of nucleons in a supernova. Everywhere outside the core the nucleon gas is strongly non-degenerate. In the neutrinosphere with density $10^{11} - 10^{12}$ g/cm^3, $Y_e \sim 0.1$ and temperature $T \sim 7$ MeV, we get $E_F - m_p \approx p_F^2/2m_p \lesssim 5 \times 10^{-2}$ MeV and $(E_F - m_p)/T \lesssim (1-5) \times 10^{-2}$, that is, the nucleons are non-degenerate. Thus eq. (52) can be used and for $B = 10^{14}$ Gauss we find $\langle \lambda_p \rangle \sim 10^{-4}$, $\langle \lambda_n \rangle \sim 8 \cdot 10^{-5}$. Even though the electron gas is degenerate, however, the level of polarization is still higher: $\langle \lambda_e \rangle \sim 10^{-3}$. In the neutrino-sphere the ratio of the axial potentials is given by,

$$|V_e^A| : |V_p^A| : |V_n^A| \sim 1 : 0.2 : 1. \tag{63}$$

In the core at densities 10^{14} g/cm^3, $Y_e \sim 0.3$ and temperatures $T \sim 20$ MeV, the degeneracy parameters for nucleons is: $(E_F - m_p)/T \lesssim 1$, that is, nucleons are only weakly degenerate and one can still use eq. (52) for estimations. For $B = 10^{14}$ Gauss we get $\langle \lambda_p \rangle \sim 4 \cdot 10^{-5}$, $\langle \lambda_n \rangle \sim 3 \cdot 10^{-5}$ but the electron polarization is also strongly suppressed: $\langle \lambda_e \rangle = 3 \cdot 10^{-5}$. We can also estimate the ratio of the axial potentials in the core,

$$|V_e^A| : |V_p^A| : |V_n^A| \sim 1 : 2 : 3. \tag{64}$$

Thus in the central regions of the core all components give comparable contribution to the axial-vector potential which can be important for conversion of the active neutrinos into sterile neutrinos.

5.5 Pressure versus polarization

As follows from our discussion strong polarization effects imply a strong magnetic field. Such a magnetic field also produces a pressure,

$$P_B = \frac{B^2}{8\pi}. \tag{65}$$

This pressure by itself modifies the density distribution and therefore, will influence neutrino conversion as well as the dynamics of the star. Let us compare P_B with the pressure of the degenerate electron gas

$$P_{Gas} = \frac{1}{12\pi^2}\mu^4. \tag{66}$$

The effect on the density (potential) profile is determined by ratio, P_B/P_{Gas} which can be written for relativistic case ($\mu \sim p_F$) as

$$\left(\frac{\Delta V}{V}\right) \sim \frac{P_B}{P_{Gas}} = \frac{3}{32\alpha}\left(\frac{2eB}{p_F^2}\right)^2, \tag{67}$$

where $\alpha \equiv e^2/4\pi$. For magnetic fields which satisfy the strong polarization condition eq. (33) we get

$$\frac{P_B}{P_{Gas}} > \frac{3}{32\alpha} \sim 13. \tag{68}$$

Thus the pressure associated with the magnetic field dominates over the matter pressure. The direct impact of the magnetic field on V and on the dynamics of the star will be stronger than its indirect influence through polarization. With diminishing magnetic fields the direct effect of the field ($\sim B^2$) decreases faster than the polarization effect ($\sim B$). The polarization effect in the weak field limit (see eq. (40)) equals:

$$\left(\frac{\Delta V}{V}\right)_{pol} \sim \lambda = \frac{3}{4}\left(\frac{2eB}{p_F^2}\right). \tag{69}$$

Using this expression one can estimate the direct effect as

$$\left(\frac{\Delta V}{V}\right)_{dir} \sim \frac{1}{6\alpha}\lambda^2. \tag{70}$$

Comparing eq. (69) and eq. (70) we find that the polarization effect becomes larger than the direct effect: $(\Delta V/V)_{pol} > (\Delta V/V)_{dir}$ if

$$\lambda < 6\alpha \sim 4 \times 10^{-2}. \tag{71}$$

In other words, this happens when the magnitude of the polarization term is small. Of course, the effects are quite different. The magnetic pressure can only diminish the matter density and the potential, whereas the polarization can also enhance the potential and, for the case of active-sterile neutrinos, it can even change the sign. Moreover, the polarization term, depending on the direction of the magnetic field, leads to anisotropy of the potential. The pressure depends on the absolute value of the field. Of course in very strong magnetic fields one should take into account both effects simultaneously.

5.6 A comment on pulsar velocities

Even though the polarization effects in a protoneutron star are expected to be small, at the $\lesssim 1\,\%$ level, as we have seen above, they may lead to observable consequences.

The most remarkable is an explanation for the birth velocities of pulsars [10]. The polarization of medium by dipole type magnetic field leads to asymmetric shift of the resonance layer inside the star in one hemisphere with respect to the other. This in turn results in anisotropy of properties of emitted neutrinos. This anisotropy is the reason of the pulsar's kicks.

The required value of the neutrino mass-squared parameter for the mechanism is $\gtrsim 10^4$ eV^2. This value is larger than cosmologically allowed (unless neutrinos have new decay or annihilation channels into majorons) and lies, in particular, outside the preferred range where it can play a role of hot dark matter, $\lesssim 10 - 50$ eV^2. One suggestion to diminish Δm^2 is to assume a strong polarization effect. Indeed, almost complete polarization in the direction of neutrino propagation would strongly suppress the effective potential for fixed total density, so that the resonance condition can be satisfied between neutrinospheres ν_e and ν_τ for much smaller masses [21]. Let us comment on this possibility.

The dependence of polarization on the ρY_e parameter for fixed $T = 7$ MeV is shown in Fig. 5. From this figure one finds that 90 % polarization (which allows one to suppress the potential in certain directions in the hot neutron star) requires a field as strong as $B \gtrsim 10^{17}$ Gauss for $\rho Y_e \sim 10^{11}$ g/cm^3. As follows from sect. 5.4 for such a strong fields the magnetic field pressure will dominate and can strongly change the dynamics of collapse.

Note that polarization does not depend on ρY_e (for fixed T) at small densities and is determined by the magnetic field and temperature. The asymptotic value of polarization at small ρY_e are described by eq. (47).

6 Conclusions

1. We have shown that neutrino propagation in a magnetized medium [3, 6, 7, 8, 9] can be equivalently seen as being associated with the scattering of neutrinos on electrons polarized by the magnetic field. This approach gives a more transparent physical interpretation of the effect and allows one to obtain important restrictions.

2. We have shown that, for neutrino flavour conversions the polarization term of the potential can not overcome usual vector current term. The polarization term can lead to lowering or raising (depending on the direction of polarization) of the the resonant density and, therefore, shift the position of the resonance layer. In contrast, in the case of active-sterile neutrino mixing, the polarization term can be bigger than the vector term. Thus the polarization can induce resonant neutrino conversions, even for small values of the parameter Δ in eq. (53). Moreover, in media with fixed chemical composition the resonant conversions can take place both in neutrino and anti-neutrino channels.

3. For realistic magnetic fields in the sun or in a supernova the polarization does not exceed (0.1-1) %. Although small, the shift of the resonance layer can lead to an explanation of observed peculiar velocities of pulsars.

4. Strong magnetic fields may lead via pressure to a direct perturbation of density and therefore, the potential profile. The effect of such direct perturbation becomes stronger than the polarization effect we have studied for $\lambda \gtrsim 0.04$.

Acknowledgment

We thank E. Akhmedov and A. Kusenko for discussions. This work has been supported by DGICYT under Grants PB95-1077 and SAB94-0325, by a joint CICYT-INFN grant, and by the TMR network ERBFMRXCT960090 of the European Union. H. N. has been supported by Ministerio de Educación y Ciencia. V. S. also got support from RFFR under grant N. 95-02-03724.

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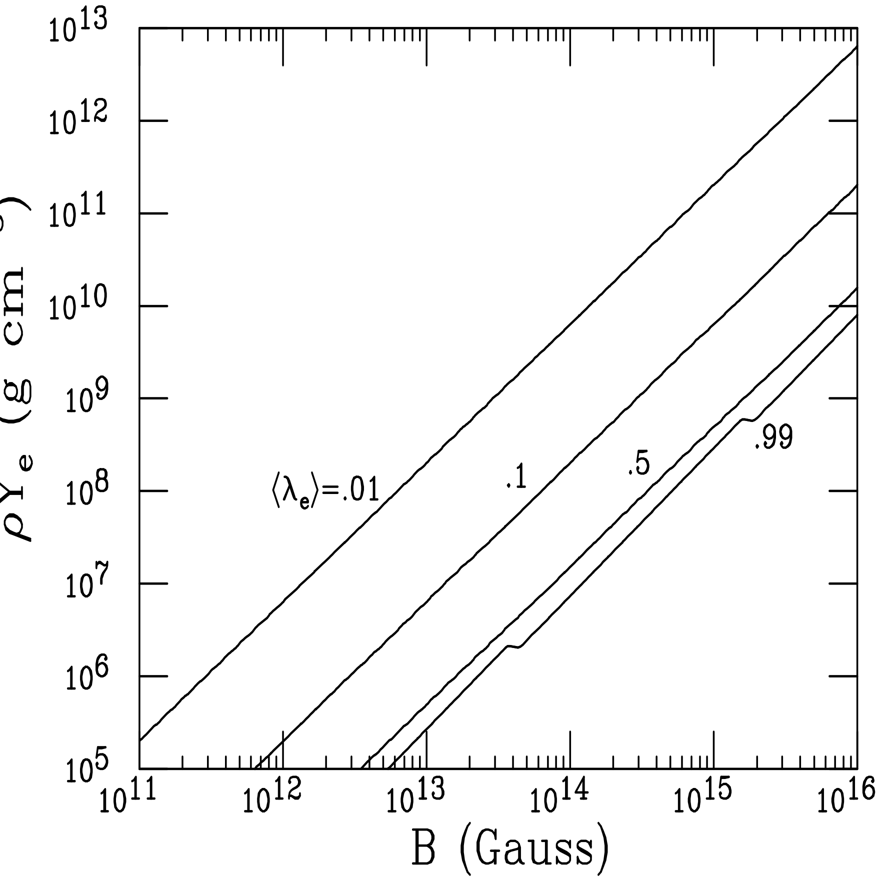

Fig. 1: Contour plot of the magnitude of electron polarization $\langle\lambda_e\rangle$ in the $\rho Y_e - B$ plane. We assumed the strong electron degeneracy and used the relation $p_F^3 = 3\pi^2 n_e$.

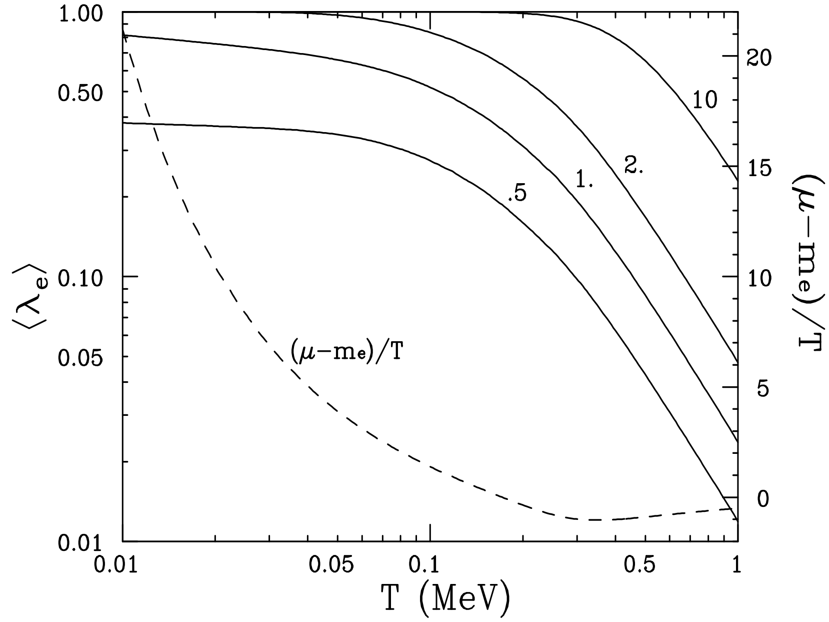

Fig. 2a: Magnitude of electron polarization $\langle \lambda_e \rangle$ as a function of T for different values of $2eB/p_F^2 = 0.5$, 1.0, 2.0, 10, indicated by numbers in the figure. We fixed the density, ρY_e, to be $10^6(\mathrm{g/cm^3})$. The degeneracy parameter $(\mu - m_e)/T$ is also plotted by dashed line.

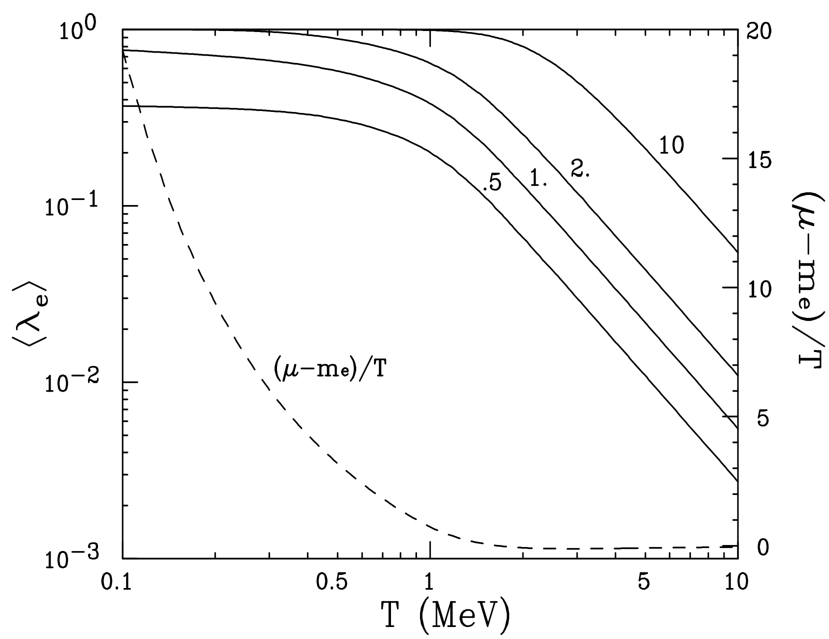

Fig. 2b: Same as the above figure but for $\rho Y_e = 10^8(\mathrm{g/cm^3})$.

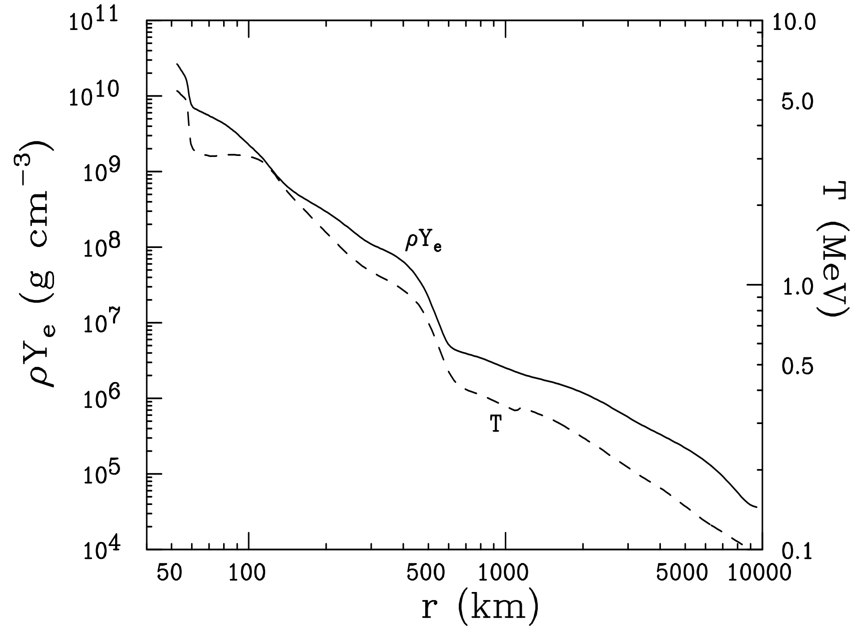

Fig. 3a: The profiles of ρY_e (solid line) and temperature (dashed line) at $t = 0.15$ sec after the bounce from Wilson's model.

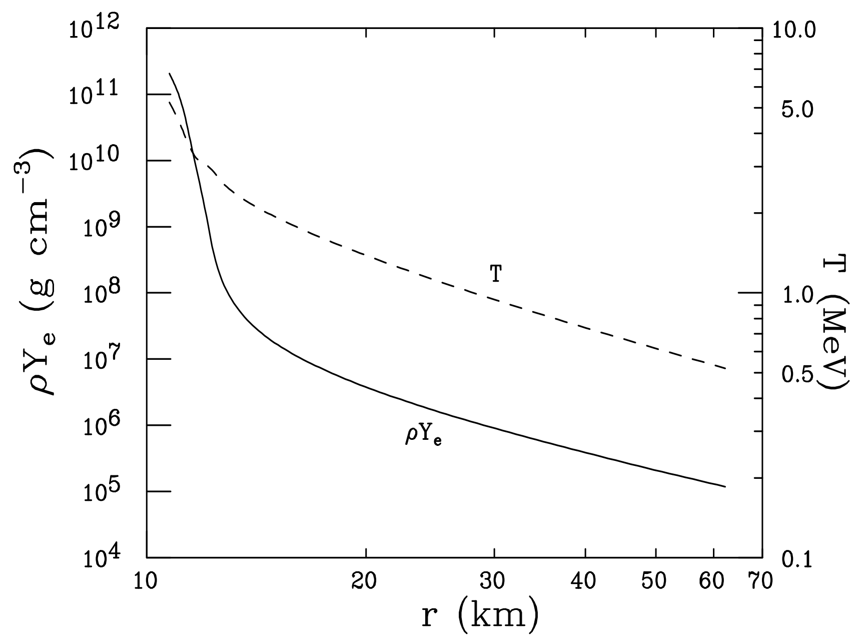

Fig. 3b: Same as Fig. 3a but for the epoch at $t \sim 6$ sec after the bounce from Wilson's model.

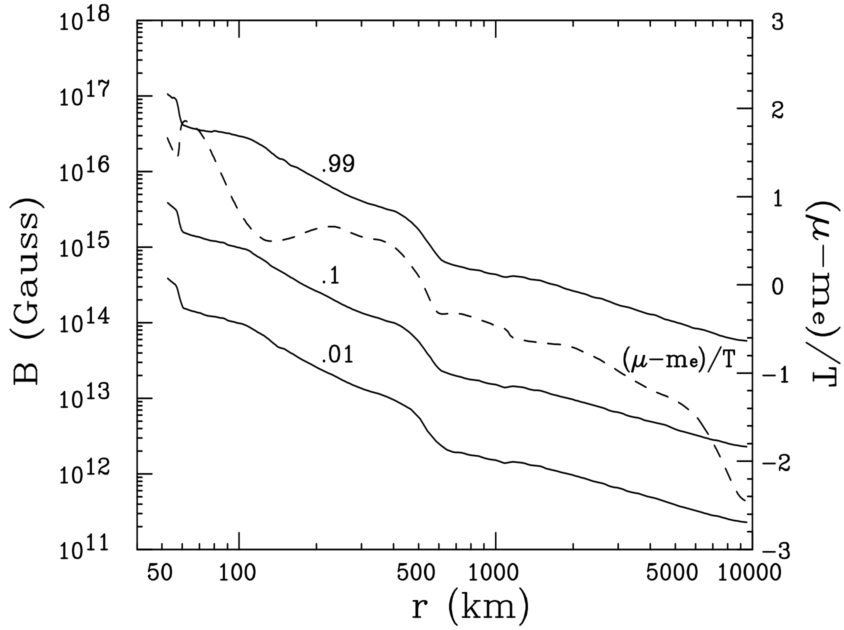

Fig. 4a: Required magnetic field profiles for given polarization, $\langle \lambda_e \rangle = 0.01$, 0.1 and 0.99 for density and temperature profiles shown in Fig. 3a ($t = 0.15$ sec after the bounce). The degeneracy parameter, $(\mu - m_e)/T$ is also plotted by dashed line.

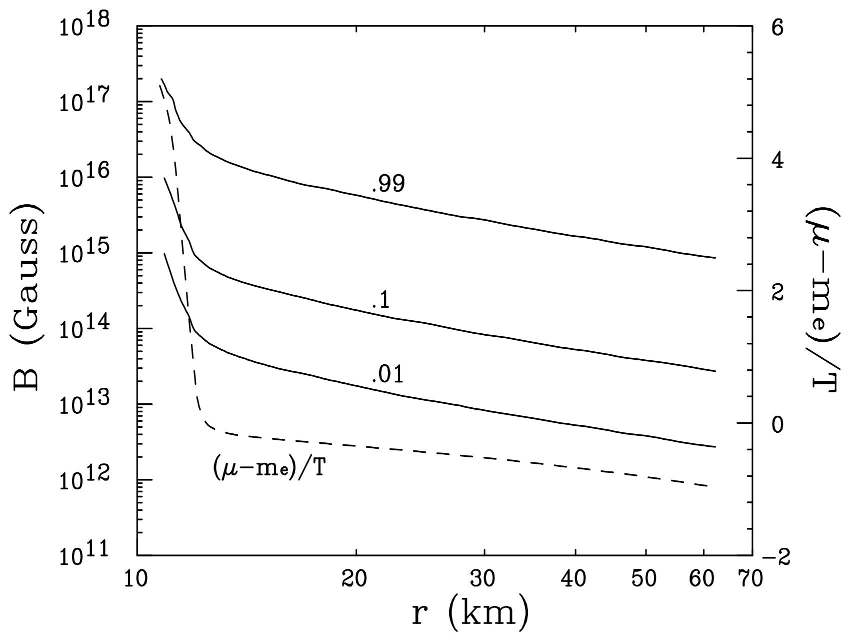

Fig. 4b: Same as 4a but for later epoch ($t \sim 6$) sec of Wilson's model.

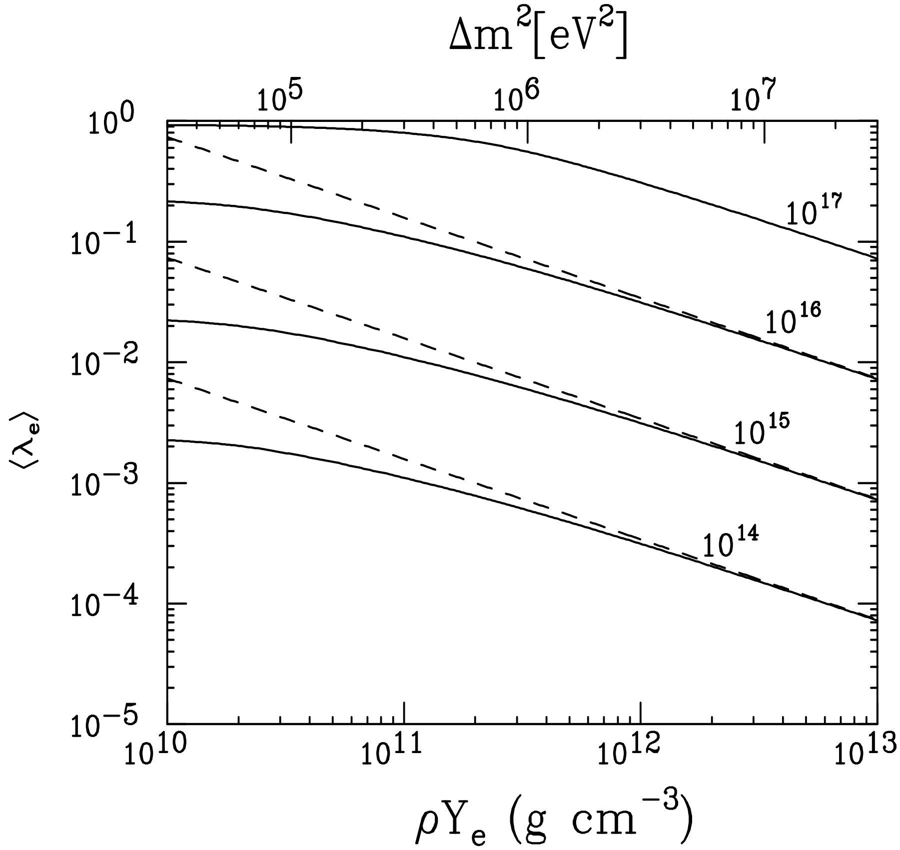

Fig. 5: Magnitude of electron polarization $\langle\lambda_e\rangle$ as a function of ρY_e for fixed temperature, $T = 7$ MeV, and for different values of magnetic field strength, $B = 10^{14}, 10^{15}, 10^{16}$ and 10^{17} Gauss, indicated by the numbers in the figure. We also plot, except for $B = 10^{17}$ Gauss, $\langle\lambda_e\rangle$ calculated by eq.(40) by dashed lines. In the top, we indicate the corresponding Δm^2 values for which $E = 20$ MeV neutrino undergo resonance for the case where no polarization effect exists.