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# Resonant Conversion of Massless Neutrinos in Supernovae

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## Abstract

It has been noted for a long time that, in some circumstances, *massless* neutrinos may be *mixed* in the leptonic charged current. Conventional neutrino oscillation searches in vacuum are insensitive to this mixing. We discuss the effects of resonant massless-neutrino conversions in the dense medium of a supernova. In particular, we show how the detected  $\bar{\nu}_e$  energy spectra from SN1987a and the supernova *r*-process nucleosynthesis may be used to provide very stringent constraints on the mixing of *massless* neutrinos.

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# 1 Introduction

In the original scenario developed by Mikheyev and Smirnov [1] the resonant neutrino conversion in matter requires non-degenerate neutrino masses and non-vanishing mixing angles in vacuum. In the basis of two neutrino flavour eigenstates, the evolution Hamiltonian describing the neutrino propagation in matter is given by

$$\begin{aligned}\mathcal{H} &= \begin{pmatrix} H_e & H_{e\alpha} \\ H_{e\alpha} & H_\alpha \end{pmatrix}, \quad \alpha = \mu (\tau), \\ H_e &= V_e - \frac{\delta m^2}{4E_\nu} \cos 2\theta, \quad H_\alpha = V_\alpha + \frac{\delta m^2}{4E_\nu} \cos 2\theta, \\ H_{e\alpha} &= H_{\alpha e} = \frac{\delta m^2}{4E_\nu} \sin 2\theta,\end{aligned}\tag{1}$$

where  $V_e$  and  $V_\alpha$  are the well-known Wolfenstein *diagonal* matter potentials arising from coherent neutrino scatterings off matter particles [2]. In Eq. (1),  $E_\nu$  is the neutrino energy, and  $\delta m^2$  and  $\theta$  are the neutrino mass-squared difference and the mixing angle in vacuum, respectively. One can see that the *effective* mixing in matter between  $\nu_e$  and  $\nu_\alpha$  states is induced by the “vacuum” term  $(\delta m^2/4E_\nu) \sin 2\theta$ .

It has been noticed for a long time that the presence of  $SU(2) \otimes U(1)$  isosinglet neutral heavy leptons [3] in general leads to flavour-changing neutral-current (FCNC) interactions of neutrinos [4]. As a result, there can be non-trivial leptonic mixing (and CP violation) [5] involving the conventional isodoublet neutrinos even in models where these neutrinos remain strictly massless, as in the Standard Model, due to an exactly conserved lepton number [6]. The non-vanishing massless-neutrino mixing angles arise due to the presence of extra heavy gauge singlet neutral states. In this scenario the interaction of massless neutrinos with matter constituents gives rise to a non-trivial neutrino evolution Hamiltonian, analogous to Eq. (1) [8, 9], which can mix the neutrino identities [8, 9]. This Hamiltonian is characterized by a new type of weak potentials whose diagonal and off-diagonal matrix elements will be discussed below.

The implications of both *standard* and *non-standard* neutrino interactions for the neutrino propagation in dense media have been extensively studied [10, 11]. In particular, the birth of neutrino astronomy, with the detection of neutrinos from the Sun [12] and SN1987a [13, 14], has offered the opportunity to probe various neutrino

properties, such as neutrino masses and mixings, neutrino lifetimes, neutrino magnetic moments, and generically, any *non-standard* interactions of neutrinos.

In this paper we focus on the particular scenario of massless-neutrino mixing suggested in Ref. [8]. This scenario can be relevant only for the neutrino propagation in strongly-neutronized media. Such media exist perhaps only in supernovae. We show how to probe the mixing in the light neutrino sector by considering two different aspects of the supernova process. We examine how the massless-neutrino conversion of the type  $\bar{\nu}_e \leftrightarrow \bar{\nu}_\alpha$  can affect the detected  $\bar{\nu}_e$  energy spectra [15, 16, 17] from SN1987a. We also consider the implications of such conversions for the supernova  $r$ -process nucleosynthesis, following the same lines of reasoning adopted in Ref. [18]. Rather stringent limits on the mixing of massless neutrinos may be derived from both considerations. These limits are very remarkable because the mixing of massless neutrinos cannot be sharply constrained through neutrino oscillation searches. Being strictly massless, these neutrinos cannot develop any phase difference *in vacuum*, and as a result, neutrino oscillations can not occur.

In Sec. 2 we give a quick reminder on the theoretical framework of Ref. [8]. In Sec. 3 we present the general features of the resonant massless-neutrino conversion in matter. Sec. 4 discusses resonant conversions of massless-neutrinos in supernovae and the implications of such conversions for supernova neutrino detection and  $r$ -process nucleosynthesis. We summarize our results and conclude in Sec. 5.

## 2 The Massless and Mixed Neutrino Model

In the Standard Model the absence of right-handed neutrino states naturally implies that neutrinos stay massless to all orders of perturbation even after the gauge symmetry breaking and there are no Cabibbo-Kobayashi-Maskawa-like [19] mixing matrices in the weak leptonic charged current. In this case the total lepton number  $L$  comes out as an *accidental* symmetry [20] due to the gauge structure and renormalizability of the theory.

On the other hand, any number of gauge singlet neutral leptons can be introduced

since they do not carry triangle anomaly [4]. These extra states can arise in left-right symmetric, grand-unified or superstring-inspired models [6, 7, 21, 22]. In this case the lepton number is no longer an *accidental* symmetry and it may be imposed *by hand*. The simplest such scheme [5, 6, 7] contains three two-component gauge singlet neutral leptons  $S$  added to the three right-handed neutrino components  $\nu^c$  present in  $\text{SO}(10)$ . For definiteness we consider this model at the  $SU(2)\otimes U(1)$  level. The assumed conservation of lepton number leads to a neutral mass matrix with the following texture in the basis  $(\nu, \nu^c, S)$ :

$$\begin{pmatrix} 0 & D & 0 \\ D^T & 0 & M \\ 0 & M^T & 0 \end{pmatrix}, \quad (2)$$

where the Dirac matrix  $D$  describes the coupling between the weak doublet  $\nu$  and the singlet  $\nu^c$ , and where the other Dirac matrix  $M$  connects the singlet states  $\nu^c$  and  $S$ . It is easy to see that, as expected, the three conventional neutrinos remain massless, while the other six neutral 2-component leptons combine into three heavy Dirac fermions [5, 7].

The phenomenological implications of this picture are manifest when considering the resulting charged-current (CC) Lagrangian in the massless-neutrino sector:

$$\mathcal{L}_{\text{CC}} = \frac{ig}{\sqrt{2}} W_\mu \bar{e}_{aL} \gamma_\mu K_{ai} \nu_{iL} + \text{h.c.}, \quad a = e, \mu, \tau, \quad i = 1, 2, 3, \quad (3)$$

where the mixing matrix  $K$  is not unitary, since it is a sub-matrix of the full rectangular matrix including also the heavy states [4]. Therefore, the non-diagonal elements of the matrix  $K$  cannot be rotated away through a redefinition of the massless-neutrino fields. In this way a non-vanishing mixing arises among the massless neutrinos. The corresponding form of the neutral-current (NC) Lagrangian for the massless-neutrino sector is

$$\mathcal{L}_{\text{NC}} = \frac{ig}{2 \cos \theta_W} Z_\mu P_{ij} \bar{\nu}_{iL} \gamma_\mu \nu_{jL}, \quad (4)$$

where  $P = K^\dagger K$ . Unlike in the Standard Model, the matrix  $P$  is diagonal but generation-dependent, signalling the violation of weak universality.

For definiteness, we later on use an explicit parametrization of the matrix  $K$ , confining ourselves to the case of two (massless) neutrinos  $\nu$ . We may write the mixing matrix  $K$  as [4, 8]

$$K = R\mathcal{N}, \quad (5)$$

where  $R$  is a  $2 \times 2$  rotation matrix,

$$R = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \quad (6)$$

and where the diagonal matrix,

$$\mathcal{N} = \begin{pmatrix} \mathcal{N}_1 & 0 \\ 0 & \mathcal{N}_{2,3} \end{pmatrix}, \quad (7)$$

describes the effective non-orthogonality of the two neutrino flavours, i.e.,

$\langle \nu_e | \nu_{\mu,\tau} \rangle \equiv -\sin \theta \cos \theta (\mathcal{N}_1^2 - \mathcal{N}_{2,3}^2)$ . The corresponding NC couplings in Eq. (4) are now expressed through

$$P = \mathcal{N}^2. \quad (8)$$

It is also convenient to define

$$\mathcal{N}_i^2 \equiv (1 + h_i^2)^{-1}, \quad i = 1, 2(3), \quad (9)$$

where the  $h_i$  parameters reflect the deviation from the *standard* neutrino coupling.

Before entering into the discussion of the resonant massless-neutrino conversion in matter, we describe the present upper limits from laboratory experiments on the relevant parameters  $h_i^2$  and  $\theta$ . We first note that the laboratory limits on the leptonic mixing angle  $\theta$  are rather weak since no oscillations between two strictly massless neutrinos can develop in vacuum. However, although not strictly justified *a priori* from the point of view of laboratory constraints, we will assume the small-mixing angle approximation. This will be justified *a posteriori* in view of our results. In this way we have  $\nu_i \sim \nu_a$  [ $a = e, \mu(\tau)$ ], and we can analogously interpret  $h_i^2$  as  $h_a^2$ .

There have been extensive studies of experimental universality tests which restrict the parameters  $h_a^2$ . For the case of  $h_\tau^2$  one can still allow values in the range of a few percent [9, 23], whereas the constraints on  $h_e^2$  and  $h_\mu^2$  are more stringent. Therefore, from now on we focus on the  $(\nu_e, \nu_\tau)$  system, for which the universality limits are the weakest. Moreover, the present experimental situation cannot exclude that the difference  $h_\tau^2 - h_e^2$  can be positive as required later on in our discussion.

### 3 Resonant Massless-Neutrino Conversion

Here we briefly recall the main features of the resonant conversion mechanism of massless neutrinos emerging from the previous scenario. For convenience we choose to write the system of Schroedinger equations, which describe the propagation of the two neutrinos in matter, in the basis defined as [8]

$$\tilde{\nu}_a \equiv [R\mathcal{N}^{-1}R^T]_{ab} \nu_b, \quad a, b = e, \tau. \quad (10)$$

Although this basis is somehow artificial, it almost coincides with the flavour basis for small lepton universality violation or small mixing angle  $\theta$ . In this basis, the Schroedinger equations can be written as

$$i \frac{d}{dr} \begin{pmatrix} \tilde{A}_e \\ \tilde{A}_\tau \end{pmatrix} = \sqrt{2} G_F \frac{\rho}{m_N} \begin{pmatrix} \tilde{H}_e & \tilde{H}_{e\tau} \\ \tilde{H}_{e\tau} & \tilde{H}_\tau \end{pmatrix} \begin{pmatrix} \tilde{A}_e \\ \tilde{A}_\tau \end{pmatrix}, \quad (11)$$

where  $\tilde{A}_{e,\tau}$  are the amplitudes corresponding to the neutrino states in the basis of Eq. (10),  $G_F$  is the Fermi constant,  $\rho$  is the matter density, and  $m_N$  is the nucleon mass. The entries of the evolution Hamiltonian are now given by

$$\begin{aligned} \tilde{H}_e &= Y_e(\mathcal{N}_e c^2 + \mathcal{N}_\tau s^2)^2 - \frac{1}{2} Y_n(\mathcal{N}_e^2 c^2 + \mathcal{N}_\tau^2 s^2), \\ \tilde{H}_{e\tau} &= \tilde{H}_{\tau e} = [Y_e(\mathcal{N}_e c^2 + \mathcal{N}_\tau s^2) - \frac{1}{2} Y_n(\mathcal{N}_e + \mathcal{N}_\tau)](\mathcal{N}_\tau - \mathcal{N}_e) s c, \\ \tilde{H}_\tau &= Y_e s^2 c^2(\mathcal{N}_\tau^2 - \mathcal{N}_e^2) - \frac{1}{2} Y_n(\mathcal{N}_e^2 s^2 + \mathcal{N}_\tau^2 c^2), \end{aligned} \quad (12)$$

where for brevity we have used the short-hand notations  $s = \sin \theta$  and  $c = \cos \theta$ .

In an electrically neutral medium,  $Y_e$  and  $Y_n$  are defined as

$$Y_e \equiv \frac{n_e}{n_e + n_n}, \quad Y_n = 1 - Y_e, \quad (13)$$

where  $n_e$  and  $n_n$  are the net electron and the neutron number densities in matter, respectively. Note that the evolution matrix has no energy dependence, which implies that for the corresponding antineutrino system  $(\bar{\nu}_e, \bar{\nu}_\tau)$  this matrix just changes its overall sign. Clearly, in this scenario, resonant neutrino conversion can also occur provided the condition  $\tilde{H}_e = \tilde{H}_\tau$  is fulfilled [8]. In fact, the same resonance condition holds for both  $\nu_e \leftrightarrow \nu_\tau$  and  $\bar{\nu}_e \leftrightarrow \bar{\nu}_\tau$  channels. As a result, in the thermal phase of supernova neutrino emission, *both* neutrinos *and* antineutrinos can simultaneously undergo this resonance. This will be very important for our subsequent discussion in Sec. 4.

In order to simplify Eq. (11), we take advantage of the small parameters  $h_a^2$  expected from the universality constraints. With the previous assumption of small  $\theta$ , we obtain

$$i \frac{d}{dr} \begin{pmatrix} \tilde{A}_e \\ \tilde{A}_\tau \end{pmatrix} = \sqrt{2} G_F \frac{\rho}{m_N} \begin{pmatrix} Y_e - \frac{1}{2} Y_n (1 - h_e^2) & \frac{1}{2} \eta (Y_n - Y_e) \sin 2\theta \\ \frac{1}{2} \eta (Y_n - Y_e) \sin 2\theta & -\frac{1}{2} Y_n (1 - h_\tau^2) \end{pmatrix} \begin{pmatrix} \tilde{A}_e \\ \tilde{A}_\tau \end{pmatrix}, \quad (14)$$

where the parameter  $\eta$  is defined as

$$\eta \equiv \frac{1}{2} (h_\tau^2 - h_e^2). \quad (15)$$

The mixing angle  $\theta_m$  and the neutrino oscillation length  $L_m$  in matter are given by

$$\sin^2 2\theta_m = \frac{\eta^2 (Y_n - Y_e)^2 \sin^2 2\theta}{(Y_e - \eta Y_n)^2 + \eta^2 (Y_n - Y_e)^2 \sin^2 2\theta}, \quad (16)$$

$$L_m = \frac{2\pi}{\sqrt{2} G_F (\rho/m_N) [(Y_e - \eta Y_n)^2 + \eta^2 (Y_n - Y_e)^2 \sin^2 2\theta]^{1/2}}, \quad (17)$$

respectively.

The resonance condition now reads

$$Y_e = \eta Y_n. \quad (18)$$

Here we should stress that a positive value of  $\eta$  is necessary for the above equation to hold. Moreover, due to the bounds on the lepton universality violation,  $\eta \lesssim O(10^{-2})$ , the condition in Eq. (18) can be fulfilled only in a strongly-neutronized medium. This is why the present mechanism cannot work in the matter background of the Sun ( $Y_n \leq 0.33$ ) [8, 9] or Earth ( $Y_n \sim 0.5$ ). On the other hand, the material composition just above the neutrinosphere in type II supernovae ( $Y_e \ll Y_n$ ) can satisfy Eq. (18), as shown later.

In our subsequent discussion, we will employ the simple Landau-Zener approximation [24, 25] to estimate the conversion probability after the neutrinos cross the resonance. Under this approximation, the probability for  $\nu_e \leftrightarrow \nu_\tau$  and  $\bar{\nu}_e \leftrightarrow \bar{\nu}_\tau$  conversions is given by

$$\begin{aligned} P &= 1 - \exp\left(-\frac{\pi^2}{2} \frac{\delta r}{L_m^{\text{res}}}\right) \\ &\approx \exp\left[-32 \times \left(\frac{\rho_{\text{res}}}{10^{12} \text{g/cm}^3}\right) \left(\frac{\eta}{10^{-2}}\right) \left(\frac{H}{\text{cm}}\right) \sin^2 2\theta\right], \\ \delta r &= 2H \sin 2\theta, \quad H \equiv \left|\frac{d \ln Y_e}{dr}\right|_{\text{res}}^{-1}, \end{aligned} \quad (19)$$

where  $L_m^{\text{res}}$  is the neutrino oscillation length at resonance and  $\rho_{\text{res}}$  are the corresponding matter density. In deriving the above equation, we have used  $Y_n \approx 1$  for the neutron abundance near resonance. Notice that for  $\delta r/L_m^{\text{res}} > 1$  resonant neutrino conversion will be adiabatic [1]. It is also important to note that the conversion probability does not depend on the neutrino energy [cf. Eqs. (11) and (12)].

## 4 Massless-Neutrino Conversion in Supernovae

### 4.1 Neutrino Emission and $Y_e$ Profile in Supernovae

A supernova occurs when the core of a massive star collapses into a compact neutron star. Almost all of the gravitational binding energy of the final neutron star is radiated in  $\nu_e$ ,  $\bar{\nu}_e$ ,  $\nu_\mu$ ,  $\bar{\nu}_\mu$ ,  $\nu_\tau$ , and  $\bar{\nu}_\tau$ . The last four neutrino species are created by thermal pair production processes inside the neutron star. On the other hand, although most of the  $\nu_e$  and  $\bar{\nu}_e$  are produced in pairs, there is a net excess of  $\nu_e$  over  $\bar{\nu}_e$  due to the neutronization and deleptonization of the core through  $e^- + p \rightarrow n + \nu_e$ . Because all these neutrinos have intense neutral-current scatterings on the free nucleons inside the neutron star, the net lepton number carried by  $\nu_e$  can escape from the neutron star only through diffusion. Therefore, we expect to see the strongest deleptonization effect near the neutrinosphere, where neutrinos stop diffusing and begin free-streaming.

We can estimate the electron fraction near the neutrinosphere as follows. From the approximate chemical equilibrium for  $e^-$ ,  $p$ ,  $n$ , and  $\nu_e$ , we have

$$\mu_{e^-} + \mu_p \sim \mu_n, \quad (20)$$

where for example,  $\mu_{e^-}$  is the electron chemical potential, and where we have set  $\mu_{\nu_e} \sim 0$ . For non-relativistic nucleons, we can write

$$\frac{n_n}{n_p} \sim \exp\left(\frac{\mu_n - \mu_p}{T}\right), \quad (21)$$

where  $T$  is the temperature, and where we have neglected the neutron-proton mass difference. The electron fraction is then given by

$$Y_e \equiv \frac{n_p}{n_p + n_n} \sim \frac{1}{\exp(\mu_{e^-}/T) + 1}. \quad (22)$$



The chemical potential for relativistic and degenerate electrons near the neutrinosphere is approximately given by

$$\mu_{e^-} \approx (3\pi^2 n_e)^{1/3} \approx 51.6(Y_e \rho_{12})^{1/3} \text{ MeV}, \quad (23)$$

where  $\rho_{12}$  is the matter density in units of  $10^{12} \text{ g cm}^{-3}$ . For typical conditions near the neutrinosphere,  $T \sim 4 \text{ MeV}$  and  $\rho_{12} \sim 10$ , by solving Eqs. (22) and (23), we find  $Y_e \sim 6 \times 10^{-3}$ , in good agreement with the numerical supernova models. Therefore, we can expect resonant massless-neutrino conversions to occur above the neutrinosphere as long as the lepton non-universality parameter  $\eta \gtrsim 6 \times 10^{-3}$  [cf. Eq. (18)].

Above the neutrinosphere, the approximate chemical equilibrium between  $\nu_e$  and matter no longer holds. The electron fraction is determined by the following reactions:

$$\nu_e + n \rightleftharpoons p + e^-, \quad (24)$$

$$\bar{\nu}_e + p \rightleftharpoons n + e^+. \quad (25)$$

In fact, Qian *et al.* [18] have shown that  $Y_e$  above the neutrinosphere is given by

$$Y_e \approx \frac{\lambda_{e^+n} + \lambda_{\nu_e n}}{\lambda_{e^-p} + \lambda_{e^+n} + \lambda_{\bar{\nu}_e p} + \lambda_{\nu_e n}}, \quad (26)$$

where for example,  $\lambda_{\nu_e n}$  is the rate for the forward reaction in Eq. (24). In particular, because  $\lambda_{e^-p}$  and  $\lambda_{e^+n}$  quickly decrease with the temperature, the asymptotic value of  $Y_e$  at large radii is approximately given by

$$Y_e \approx \frac{\lambda_{\nu_e n}}{\lambda_{\bar{\nu}_e p} + \lambda_{\nu_e n}}. \quad (27)$$

Therefore, the asymptotic electron fraction above the neutrinosphere is essentially determined by the characteristics of the  $\nu_e$  and  $\bar{\nu}_e$  fluxes, such as their luminosities and energy distributions.

The individual neutrino luminosities in supernovae are approximately the same:

$$L_{\nu_e} \approx L_{\bar{\nu}_e} \approx L_{\nu_{\tau(\mu)}} \approx L_{\bar{\nu}_{\tau(\mu)}}. \quad (28)$$

However, the individual neutrino energy distributions are very different. This is because these neutrinos have different abilities to exchange energy with the neutron star material, and thermally decouple at different temperatures inside the neutron star. Unlike  $\nu_e$  and  $\bar{\nu}_e$ ,  $\nu_{\tau(\mu)}$  and  $\bar{\nu}_{\tau(\mu)}$  are not energetic enough to have charged-current absorptions on the free nucleons inside the neutron star. Furthermore, between  $\nu_e$  and  $\bar{\nu}_e$ ,

$\nu_e$  have more frequent absorptions due to the high neutron abundance in the neutron star matter. As a result,  $\nu_{\tau(\mu)}$  and  $\bar{\nu}_{\tau(\mu)}$  thermally decouple at the highest temperature, and  $\nu_e$  decouple at the lowest temperature. Correspondingly, the average neutrino energies satisfy the following hierarchy:

$$\langle E_{\nu_e} \rangle < \langle E_{\bar{\nu}_e} \rangle < \langle E_{\nu_{\tau(\mu)}} \rangle \approx \langle E_{\bar{\nu}_{\tau(\mu)}} \rangle. \quad (29)$$

Typically, the average supernova neutrino energies are:

$$\langle E_{\nu_e} \rangle \approx 11 \text{ MeV}, \quad \langle E_{\bar{\nu}_e} \rangle \approx 16 \text{ MeV}, \quad \langle E_{\nu_{\tau(\mu)}} \rangle \approx \langle E_{\bar{\nu}_{\tau(\mu)}} \rangle \approx 25 \text{ MeV}. \quad (30)$$

Now we can understand the electron fraction profile in supernovae as illustrated in Fig. 1. In this figure, we plot the typical electron fraction and density profiles in Wilson's supernova model at time  $t > 1$  s after the explosion. The solid line is for the density, and the dotted line is for  $Y_e$ . As we can see, the minimum value of  $Y_e$  occurs near the neutrinosphere. Above the neutrinosphere, the electron fraction is set by the reactions in Eqs. (24) and (25). At large radii, it reaches an asymptotic value much larger than the minimum  $Y_e$ .

From the above discussion of neutrino emission and  $Y_e$  profile in supernovae, we find that it is interesting to study massless-neutrino conversion in supernovae. First of all, the resonance condition for such conversion, Eq. (18) can be fulfilled above the neutrinosphere for  $\eta \sim 0.01$ . Furthermore, conversion between  $\nu_\tau$  ( $\bar{\nu}_\tau$ ) and  $\nu_e$  ( $\bar{\nu}_e$ ) can alter the supernova neutrino characteristics, especially the average neutrino energies in Eq. (30). We can gauge the potential to use supernovae as a sensitive probe of the mixing between massless neutrinos by estimating the adiabatic condition for resonant massless-neutrino conversion. For  $\eta \sim 10^{-2}$ , the resonances occur at densities  $\rho \sim 10^{12}$ – $10^{13} \text{ g cm}^{-3}$ , just above the neutrinosphere. The corresponding scale height for  $Y_e$  is  $H \sim 1$ – $10$  km. From Eq. (19), we see that massless neutrinos can be adiabatically converted for  $\sin^2 2\theta > 10^{-7}$ – $10^{-6}$ . In the following subsections, we discuss two possible ways to probe the mixing between massless neutrinos in supernovae.

## 4.2 Detection of $\bar{\nu}_e$ from SN1987a

The Kamiokande II and IMB detectors observed 11 and 8  $\bar{\nu}_e$  events, respectively, from SN1987a [9,10]. An estimate of the average supernova  $\bar{\nu}_e$  energy can be made from the detection data, although the obtained estimate should be taken with caution, considering the poor statistics and the marginal agreement between the two sets of data. Nevertheless, if we adopt the standard average neutrino energies predicted by the numerical supernova models, then a significant amount of conversion between  $\bar{\nu}_\tau$  and  $\bar{\nu}_e$  can probably be ruled out. This is because the average  $\bar{\nu}_e$  energy inferred from the detection data is much smaller than the average  $\bar{\nu}_\tau$  energy predicted by the numerical supernova models. Specifically, with  $\bar{\nu}_e \leftrightarrow \bar{\nu}_\tau$  conversion, the  $\bar{\nu}_e$  flux at the detectors would be given by

$$\phi_{\bar{\nu}_e} = \phi_{\bar{\nu}_e}^0(1 - P) + \phi_{\bar{\nu}_\tau}^0 P, \quad (31)$$

where  $\phi_{\bar{\nu}_e}^0$  and  $\phi_{\bar{\nu}_\tau}^0$  are the  $\bar{\nu}_e$  and  $\bar{\nu}_\tau$  fluxes in the absence of neutrino conversions, respectively, and  $P$  is the conversion probability. For large  $P$ , based on predictions from numerical supernova models, the  $\bar{\nu}_e$  energy spectra at the detectors would have been significantly harder than detected in the case of SN1987a. From the detection data, Smirnov *et al.* [16] argued that the probability for  $\bar{\nu}_e \leftrightarrow \bar{\nu}_{\tau(\mu)}$  conversion should be less than 0.35.

We can apply the same argument to constrain the mixing between massless neutrinos. Using the density and  $Y_e$  profiles from Wilson's supernova model in Fig. 1, we plot in Fig. 2 two contours of the conversion probability in the  $(\eta, \sin^2 2\theta)$  parameter space. The solid line is for a conversion probability of  $P \approx 0.5$ , and the dashed line is for  $P \approx 0.35$ . We can conclude that mixing between massless  $\bar{\nu}_e$  ( $\nu_e$ ) and  $\bar{\nu}_\tau$  ( $\nu_\tau$ ) at a level of  $\sin^2 2\theta \gtrsim 10^{-6}$  is ruled out for  $\eta \gtrsim 10^{-2}$  due to the non-observation of unexpectedly hard  $\bar{\nu}_e$  energy spectra from SN1987a. Such a stringent upper limit on the mixing angle  $\theta$  justifies the approximation we have made in deriving Eq. (14).

### 4.3 Supernova $r$ -process Nucleosynthesis

Now we consider the effect of massless-neutrino conversions on the the supernova  $r$ -process nucleosynthesis. The  $r$ -process is responsible for synthesizing about a half

of the heavy elements with mass number  $A > 70$  in nature. It has been proposed that the  $r$ -process occurs in the region above the neutrinosphere in supernovae when significant neutrino fluxes are still coming from the neutron star [26]. A necessary condition required for the  $r$ -process is  $Y_e < 0.5$  in the nucleosynthesis region. As we have discussed previously, the  $Y_e$  value at large radii above the neutrinosphere, where the  $r$ -process nucleosynthesis takes place, is determined by the neutrino absorption rates  $\lambda_{\nu_e n}$  and  $\lambda_{\bar{\nu}_e p}$ . In turn, these rates depend on the  $\nu_e$  and  $\bar{\nu}_e$  luminosities and energy distributions.

Qualitatively, we can argue that these rates are proportional to the product of the neutrino luminosity and average neutrino energy. This is because the neutrino absorption rate is given by

$$\lambda_{\nu N} \approx \phi_\nu \langle \sigma_{\nu N} \rangle \propto \frac{L_\nu}{\langle E_\nu \rangle} \langle E_\nu^2 \rangle \propto L_\nu \langle E_\nu \rangle, \quad (32)$$

where  $\phi_\nu$  is the neutrino flux,  $\sigma_{\nu N} \propto E_\nu^2$  is the neutrino absorption cross section, and  $\langle \rangle$  denotes the averaging over the neutrino energy distribution. Therefore, the  $Y_e$  in the nucleosynthesis region is approximately given by

$$Y_e \approx \frac{\lambda_{\nu_e n}}{\lambda_{\bar{\nu}_e p} + \lambda_{\nu_e n}} \approx \frac{1}{1 + \langle E_{\bar{\nu}_e} \rangle / \langle E_{\nu_e} \rangle}. \quad (33)$$

Using the average energies in Eq. (30), we obtain  $Y_e \approx 0.41$ , in good agreement with the numerical supernova models.

However, in the presence of massless-neutrino conversion, average energies of both  $\bar{\nu}_e$  and  $\nu_e$  can be affected. The corresponding  $Y_e$  in the nucleosynthesis region is given by

$$Y_e \approx \frac{1}{1 + \langle E_{\bar{\nu}_e} \rangle_{\text{eff}} / \langle E_{\nu_e} \rangle_{\text{eff}}}, \quad (34)$$

where

$$\begin{aligned} \langle E_{\bar{\nu}_e} \rangle_{\text{eff}} &\equiv \langle E_{\bar{\nu}_e} \rangle (1 - P) + \langle E_{\bar{\nu}_\tau} \rangle P, \\ \langle E_{\nu_e} \rangle_{\text{eff}} &\equiv \langle E_{\nu_e} \rangle (1 - P) + \langle E_{\nu_\tau} \rangle P. \end{aligned} \quad (35)$$

Due to the the *simultaneous* occurrence of resonant  $\nu_e \leftrightarrow \nu_\tau$  and  $\bar{\nu}_e \leftrightarrow \bar{\nu}_\tau$  conversions, there is a trend to equalize the average  $\nu_e$  and  $\bar{\nu}_e$  energies, and as a result, to increase  $Y_e$  with respect to the case with no neutrino or antineutrino conversions. For conversion probabilities of  $P \approx 0.15, 0.3, \text{ and } 0.8$ , we obtain  $Y_e \approx 0.43, 0.45, \text{ and } 0.49$ . In Fig. 3,

we present the contour lines corresponding to these  $Y_e$  values using the density and  $Y_e$  profiles in Wilson's supernova model. The dotted, dashed, and solid lines in this figure are for  $Y_e \approx 0.43, 0.45,$  and  $0.49,$  respectively.

In order for any  $r$ -process nucleosynthesis to occur, the  $Y_e$  in the nucleosynthesis region must be less than 0.5. However, in the most recent  $r$ -process model by Woosley *et al.* [26], many of the  $r$ -process nuclei are produced only for  $Y_e < 0.45$ . If we take  $Y_e < 0.45$  as a criterion for a successful  $r$ -process, then mixing between  $\nu_e$  ( $\bar{\nu}_e$ ) and  $\nu_\tau$  ( $\bar{\nu}_\tau$ ) at a level of  $\sin^2 2\theta > 10^{-6}$  is excluded for  $\eta \gtrsim 10^{-2}$ . This excluded region is similar to the previous one from considering the detection of  $\bar{\nu}_e$  from SN1987a, because the limits on the conversion probability are about the same in both cases. However, we note that if the  $r$ -process indeed occurs in supernovae, then the consequent limits on the mixing between massless neutrinos are much less dependent on the predicted average neutrino energies than the previous limits obtained by considering the  $\bar{\nu}_e$  energy spectra from SN1987a. This is because the  $r$ -process argument relies only on the ratio of the average neutrino energies [cf. Eq. (33)].

#### 4.4 Comparison of MSW and Massless-Neutrino Conversion Mechanisms in Supernovae

It is instructive at this stage to compare the effects of resonant massless-neutrino conversions with those of the standard MSW mechanism in supernovae. To simplify this comparison, we will assume small vacuum mixing angles ( $\theta \ll 1$ ) in both cases. We first note that in the MSW scenario [1], for a given sign of  $\delta m^2$  (e.g.,  $\delta m^2 > 0$  for  $m_{\nu_\tau} > m_{\nu_e}$ ), only one kind of resonant conversion, either  $\nu_e \leftrightarrow \nu_\tau$  (for  $\delta m^2 > 0$ ), or  $\bar{\nu}_e \leftrightarrow \bar{\nu}_\tau$  (for  $\delta m^2 < 0$ ), can occur. If  $\delta m^2 > 0$  the MSW mechanism would not alter the  $\bar{\nu}_e$  energy spectra from SN1987a, and therefore, no constraints on neutrino masses and mixings can be obtained for this mechanism from the detection data (assuming all the events were due to  $\bar{\nu}_e$ ). In contrast, severe constraints on massive-neutrino mixing can be obtained in this case by requiring  $Y_e < 0.5$  in the nucleosynthesis region to allow a successful  $r$ -process [18]. On the other hand, if  $\delta m^2 < 0$  the MSW mechanism could significantly modify the  $\bar{\nu}_e$  energy spectra and generate an excess of energetic

$\bar{\nu}_e$ . As a result, the parameter region which would give large probabilities for  $\bar{\nu}_e \leftrightarrow \bar{\nu}_\tau$  conversion can be possibly excluded by combining the predicted average supernova neutrino energies and the SN1987a detection data. On the contrary, significant  $\bar{\nu}_e \leftrightarrow \bar{\nu}_\tau$  conversion would tend to decrease  $Y_e$  in the nucleosynthesis region (see eq. (34)) and therefore, would not conflict with the supernova  $r$ -process nucleosynthesis scenario [25]. As we can see, one can only use either the SN1987a detection data (for  $\delta m^2 < 0$ ), or the supernova  $r$ -process nucleosynthesis (for  $\delta m^2 > 0$ ) to constrain neutrino masses and mixings in the MSW mechanism.

In contrast, in the case of massless-neutrino conversions, we have seen that for  $\eta \gtrsim 10^{-2}$ , both  $\nu_e \leftrightarrow \nu_\tau$  and  $\bar{\nu}_e \leftrightarrow \bar{\nu}_\tau$  conversions can occur in supernovae. Therefore, both the SN1987a detection data and the supernova  $r$ -process nucleosynthesis should be considered in order to constrain the mixing of massless neutrinos. Of course, if  $\eta < 0$  or  $\eta \ll 10^{-2}$ , then no resonant massless neutrino conversions would occur in supernovae. The constraints on massless-neutrino mixing in this case are perhaps hard to obtain by any means.

It is interesting to note that simultaneous  $\nu_e \leftrightarrow \nu_\tau$  and  $\bar{\nu}_e \leftrightarrow \bar{\nu}_\tau$  conversions would give rise to distinctive supernova neutrino signals in large volume detectors, such as super-Kamiokande [26], SNO [27], and LVD [28]. For example, in the super-Kamiokande detector, the energy distributions for both the isotropic  $\bar{\nu}_e$  events and the forward-peaked  $\nu_e$  events would be altered. With enough statistics, such detectors may be able to distinguish the massless-neutrino conversion scenario from the standard MSW mechanism, should any neutrino conversion indeed occur in supernovae.

## 5 Conclusions

We have discussed the possibility of probing the mixing between massless neutrinos described in the theoretical scheme in Sec. 2. Due to the relatively stringent laboratory bounds on the weak universality violation, the supernova matter background seems to be the unique site where resonant conversions of massless neutrinos can take place. By considering the detection of  $\bar{\nu}_e$  from SN1987a and the supernova  $r$ -process nucleosynthesis, we have obtained stringent limits on the mixing between massless  $\nu_e$

( $\bar{\nu}_e$ ) and  $\nu_\tau$  ( $\bar{\nu}_\tau$ ) presented in Figs. 2 and 3. These limits, at a level of  $\sin^2 2\theta \lesssim 10^{-6}$ , are rather remarkable, because the usual laboratory methods to constrain neutrino mixing through vacuum neutrino oscillation searches are totally insensitive to the mixing between massless neutrinos. Indeed, the supernova limits we have obtained for the mixing between massless  $\nu_e$  ( $\bar{\nu}_e$ ) and  $\nu_\tau$  ( $\bar{\nu}_\tau$ ) are orders of magnitude more stringent than the typical limits on massive-neutrino mixing from laboratory neutrino oscillation searches.

Finally, we hope that our discussions of massless-neutrino conversions in supernovae serve to highlight the interest in sharpening the laboratory limits on universality violation and/or pinning down more accurate supernova models.

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Fig. 1

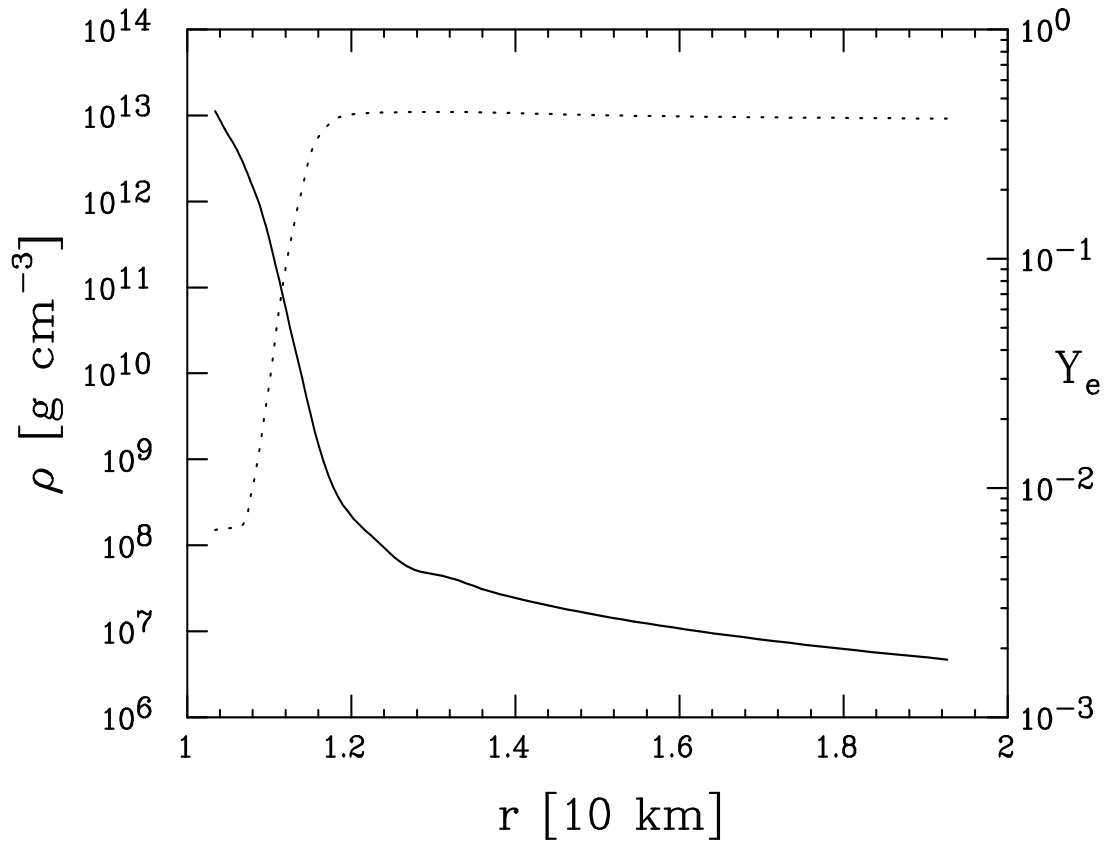


Figure 1: Typical matter density (solid line) and  $Y_e$  (dotted line) profiles in Wilson's numerical supernova model at  $t > 1$  s after the explosion.

Fig. 2

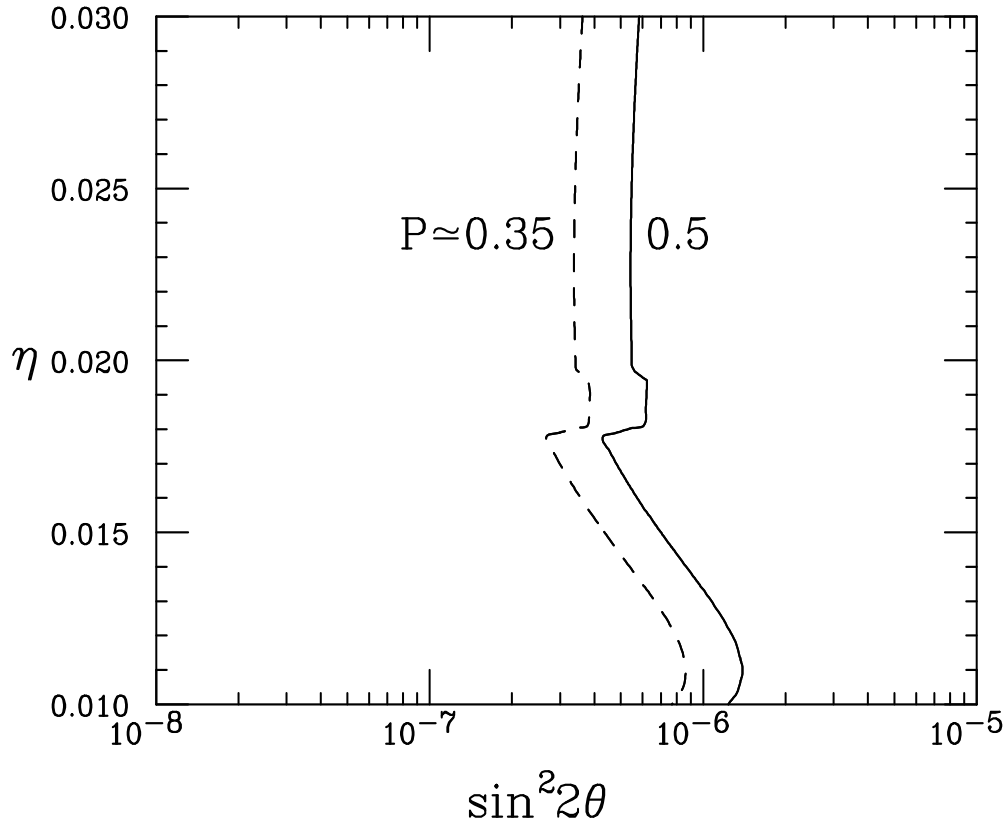


Figure 2: Constraints on massless-neutrino mixing from the detected SN1987a  $\bar{\nu}_e$  energy spectra. The region to the right of the dashed (solid) lines are excluded by the detection data for an allowed conversion probability of  $P < 0.35$  (0.5).

Fig. 3

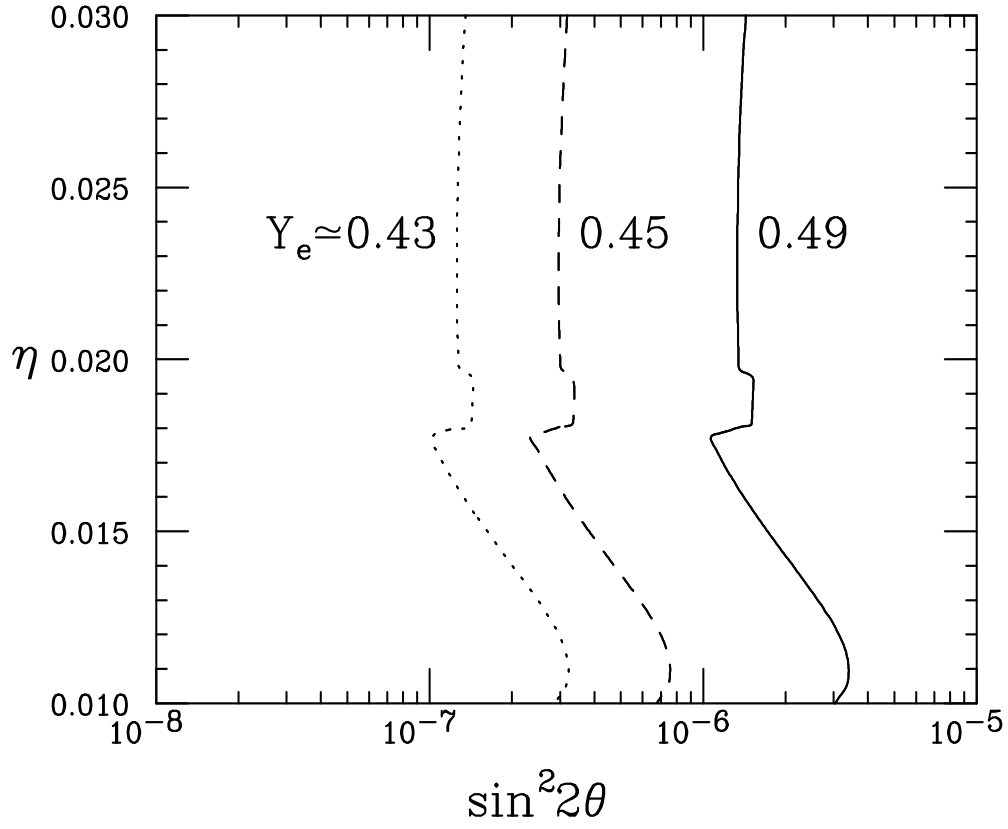


Figure 3: Constraints on massless-neutrino mixing from the supernova  $r$ -process nucleosynthesis. The region to the right of the dotted, dashed and solid lines are excluded for the required values of  $Y_e < 0.43$ ,  $0.45$ , and  $0.49$ , respectively, in the  $r$ -process.