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SO(10) grand unification model for degenerate neutrino masses

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Abstract

We propose an SO(10) scheme where neutrino masses can simultaneously explain the solar and atmospheric neutrino deficits, together with a hot dark matter component. In our scheme the ν_e, ν_μ, and ν_τ are approximately degenerate with a mass of about 2 eV, which can lead to an observable neutrinoless double beta decay rate. The model is based on a realization of the seesaw mechanism in which the main contribution to the light neutrino masses is universal, due to a suitable SU(2) horizontal symmetry, while the splittings between ν_e and ν_μ explain the solar neutrino deficit and that between ν_μ and ν_τ explain the atmospheric neutrino anomaly.

1. Introduction

Up to now the only hints in favour of massive neutrinos come from astrophysics and cosmology. These involve four different measurements of a reduced solar neutrino flux [1], two or three measurements of a lower flux of ν_μ neutrinos relative to ν_e neutrinos produced in the atmosphere [2], and the indications for the need for a neutrino component in the dark matter of the universe, inferred from the comparison of recent COBE data [3] with data on the amplitude of primordial fluctuations, e.g. from IRAS, on smaller distance scales.

While particle physics strongly suggests that neutrinos are massive, it unfortunately is unable to specify the scale that should characterize the masses of the neutrinos. Therefore the above solar and atmospheric neutrino deficits plus the indications for hot dark matter in the universe constitute our only clues into the pattern of neutrino masses.

It was first noted that the simplest way to reconcile these observations require the existence of four light neutrinos, one of which must be sterile [4–7], in order to comply with the LEP measurements of the invisible Z decay width. The possible patterns of neutrino masses and mixing have already been discussed and theoretically implemented.

However, another pattern with only three light neutrinos is possible if they are almost degenerate in mass [8,9]. This possibility has not attracted as much attention because, in the simplest seesaw model, the neutrino masses are expected to be in the ratios $m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} = m_u^2 : m_c^2 : m_t^2$ and therefore degeneracy is quite unlikely.

It is well known, however, that in the most general seesaw model there is an additional contribution to the light neutrino masses, involving the vacuum expectation value (VEV) of a triplet scalar boson, as introduced in Ref. [10]. Moreover, it has been showed that in left-right symmetric implementations of the seesaw

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mechanism such a nonvanishing triplet VEV is always induced [11].

In this letter we propose an SO(10) model in which the main contribution to the effective light neutrino masses arises from such an induced triplet VEV and their splittings are generated by the usual so-called seesaw contributions which scale with the quark masses as above. The model reconciles all three hints for neutrino masses and is consistent with cosmological as well as particle physics constraints, such as those from the proton decay limits and the measurements of the electroweak mixing parameter $\sin^2 2\theta_W$ and the strong coupling constant α_s derived from LEP and other data. Moreover, it leads to an observable neutrinoless double beta decay ($\beta\beta_{0\nu}$) rate.

The enhanced $\beta\beta_{0\nu}$ decay rate is rather intriguing in view of the present experimental situation. Indeed, as more data from the Heidelberg-Moscow experiment accumulates the limits do not seem to improve, and may be interpreted as a 2σ effect [12]. With current matrix element calculations, this would correspond to an effective Majorana mass of about 2 eV, of the same magnitude as found in our scheme.

2. Preliminaries

Before describing our SO(10) model, we will first review the phenomenologically required parameter values of our proposed solution to the solar, atmospheric and dark matter neutrino mass hints involving almost degenerate Majorana neutrino masses.

For our proposed solution the solar neutrino deficit is understood in terms of small-angle (non adiabatic) ν_e to ν_μ matter enhanced MSW conversions [13], and this requires the following ν_e to ν_μ squared mass difference and mixing angle [14],

$$\Delta m^2_{e\mu} \sim 6\times 10^{-6}\text{eV}^2\,,$$
$$\sin^2 2\theta_{e\mu} \sim 7\times 10^{-3}\,. \qquad (2.1)$$

An apparent decrease in the expected flux of atmospheric ν_μ's relative to ν_e's arising from the decays of π's and K's produced in the atmosphere, and from the subsequent secondary muon decays has been observed in three underground experiments, Kamiokande, IMB and possibly Soudan2 [2]. This atmospheric neutrino deficit can be ascribed to neutrino oscillations. Combining these experimental results with observations of upward going muons made by Kamiokande, IMB and Baksan, and with the negative Frejus and NUSEX results [15] leads to the following range of neutrino oscillation parameters [16]

$$\Delta m^2_{\mu\tau} \approx 0.005-0.5\,\text{eV}^2\,, \quad \sin^2 2\theta_{\mu\tau} \approx 0.5\,. \qquad (2.2)$$

Finally, there has been increasing evidence that more than 90% of the mass in the universe is detectable only by its gravitational effects. Recent observations of the large scale structure of the universe by the COBE satellite, when taken together with observations of the amplitude of primordial fluctuations on smaller distance scales, indicate that this dark matter is likely to be a mixture of $\sim$ 30% of hot dark matter (particles which were relativistic at the time of freeze-out from equilibrium in the early universe) and $\sim$ 60% of cold dark matter (particles which were non-relativistic at decoupling) [3,17]. Massive neutrinos provide the most plausible hot dark matter candidate. Choosing the total mass $m_\nu = 93h^2F_\nu\Omega_{\text{total}} = 7$ eV and $h = 0.5$ (the Hubble constant in units of 100 km/s/Mpc) one finds that the fraction of the hot dark matter contributed by neutrinos is $F_\nu = 0.3$ for $\Omega_{\text{total}} = 1$ (the ratio of the total density of the universe to the closure density). In our scheme this dark matter is shared amongst the three types of light neutrinos ν_e, ν_μ and ν_τ. It is interesting to note that such a multi component hot dark matter scenario seems to provide a better fit to the universe structure than a single $\sim$ 7 eV neutrino.

In order to account for the above hints for massive neutrinos coming from solar and atmospheric neutrino deficits as well as hot dark matter we use the most economic pattern of neutrino masses consisting of three almost degenerate neutrinos ν_e, ν_μ and ν_τ. We focus on models of the seesaw type, where there exist some heavy SU(3) $\otimes$ SU(2) $\otimes$ U(1) singlet right-handed neutrinos at some large mass scale M_R. The effective mass matrix for the light neutrinos may be written as

$$m_{ij} - (DM_R^{-1}D^T)_{ij}\,, \qquad (2.3)$$

where the first term corresponds to the term M_1 in Ref. [10] and arises from the triplet VEV, while the second is the one that follows from the block diagonalization of the full 6×6 neutrino mass matrix. Since we wish the three light neutrinos to be closely degenerate, we assume that the leading contribution is the first, and that it is proportional to the identity matrix

$$m_{ij} = m\delta_{ij} . \quad (2.4)$$

The corresponding degeneracy in the light neutrino spectrum is supposed to arise from a suitably chosen horizontal symmetry (see Section 3) which is broken by the quark masses. This happens through the usual seesaw corrections that arise from the second term, whose magnitude we assume to be very small. The lifting of the exact mass degeneracy generates the mass splittings required in order to induce solar neutrino conversions as well as atmospheric neutrino oscillations.

Assuming that CP is conserved one can write the charged current leptonic mixing matrix as a product of three rotations $K_L = \omega_{(e\mu)}\omega_{(\mu\tau)}\omega_{(e\tau)}$, in the corresponding (12), (23) and (13) planes [10], times a diagonal phase matrix which is the square root of neutrino CP signs η_i [18],

$$(K_L)_{ai} = K_{ai}\eta_i^{1/2} , \quad (2.5)$$

where

$$K = \begin{pmatrix} C_{12}C_{13} + S_{12}S_{13}S_{23} & -S_{12}C_{23} & S_{12}C_{13}S_{23} - C_{12}S_{13} \\ S_{12}C_{13} - C_{12}S_{13}S_{23} & C_{12}C_{23} & -(S_{12}S_{13} + C_{12}C_{13}S_{23}) \\ S_{13}C_{23} & S_{23} & C_{13}C_{23} \end{pmatrix} \quad (2.6)$$

The required mass parameters that characterize the solar neutrino conversions and atmospheric neutrino oscillations are given by

$$\Delta m^2_{e\mu} \simeq 2m\Delta m_{e\mu} , \quad (2.7)$$

$$\Delta m^2_{\mu\tau} \simeq 2m\Delta m_{\mu\tau} , \quad (2.8)$$

where

$$m \simeq 2\text{eV} , \quad (2.9)$$

and the splittings are determined from Eq. (2.1) and Eq. (2.2) to lie in the ranges from 1.5×10^{-6} eV, and 10^{-3} to 10^{-1} eV, respectively. Moreover, the mixing angles 12 and 23 are restricted by Eq. (2.1) and Eq. (2.2) while 13 is a free parameter.

Table 1
Quantum numbers and VEVS of Higgs bosons

$SO(10) \otimes SU(2)_H$	$SU(4) \otimes SU(2)_L \otimes SU(2)_R$	D
(54,1)	(1,1,1)	1
(45,1)	(15,1,1)	-1
$(\overline{126},1)$	$(10,1,3) + (\overline{10},3,1)$	i
$(\overline{126},5)$	(15,2,2)	i
$(10,5)_1$	(1,2,2)	-i
$(10,5)_2$	(1,2,2)	1
(10,1)	(1,2,2)	-1

Table 2
Quantum numbers of matter fields

$SO(10) \otimes SU(2)_H$	D
(16,3)	$\exp -i\pi/4$

Finally we note that [9]

$$\frac{\Delta m^2_{e\mu}}{\Delta m^2_{\mu\tau}} = \mathcal{O}\left(\frac{m_c^2}{m_t^2}\right) , \quad (2.10)$$

where m_c and m_t denote the charm and top quark masses. This relation is the basis of our suggestion of an SO(10) seesaw origin for the solar and atmospheric neutrino mass splittings. On the other hand the mixings are given as in Eq. (2.1) and Eq. (2.2).

3. SO(10) model for degenerate Majorana neutrino masses

In order to account for the required approximate degeneracy in the neutrino spectrum we invoke the existence of a horizontal symmetry, G_H, chosen in such a way that the effective triplet VEV giving rise to the first term in Eq. (2.3) is a G_H singlet, Eq. (2.4).

The particle content of the model is given in Tables 1 and 2. It contains only the known fermions which are assigned to the usual 16-dimensional spinor representation of SO(10). The three families of such fermions form a triplet of the global $G_H = SU(2)_H$ symmetry.

As for the Higgs sector, the SO(10) symmetry is assumed to break down at the GUT scale, M_X, to the left-right symmetric group $SU(2)_L \otimes SU(2)_R \otimes SU(4) \otimes P$ via a nonzero VEV of the singlet Higgs in the 54 (see Table 1). It is important to note that the discrete parity symmetry P implies that the mass of the $SU(2)_L$ and $SU(2)_R$ triplet Higgs bosons are of the same order, which we choose to be determined by v_R. Below we will show that v_R can be determined to be $\sim 10^{14}$ GeV, a value which fits well with neutrino mass estimated from the large scale dark matter observations, $m = 2$ eV. The SU(4) in the intermediate left-right symmetric gauge symmetry group is further broken at a scale $\sim v_R$ via nonzero VEVS of the $SU(3) \otimes SU(2) \otimes U(1)$ singlet Higgs fields Δ_R in the (10,1,3) of the 126 and the corresponding (15,1,1) of the 45 (see Table 1).

The problem of analysing the Higgs potential has been explicitly considered by Mohapatra and Senjanovic in Ref. [11] where they exhibit the full potential and its minimization. Here we will write only the part of the Higgs potential which is relevant in order to show how we can get the VEV seesaw relation for left and right Higgs triplets in the 126, namely [3]

$$M^2_{\Delta_L} 126\,\overline{126} + a_1 126\,\overline{126}\,126\,\overline{126} + a_2 10\,10\,126\,126 + a_3 10\,10\,\overline{126}\,\overline{126} + \text{h.c.} \quad (3.1)$$

The $SU(3) \otimes SU(2) \otimes U(1)$ gauge symmetry is broken by the doublet VEVS in the 10 and 126. After substituting the values of the corresponding effective v_u and v_d VEVS one finds

$$M^2_{\Delta_L} v_L^2 + a_1 v_L^2 v_R^2 + 2(a_2 v_u^2 + a_3 v_d^2) v_R v_L + \cdots , \quad (3.2)$$

where v_u and v_d are the VEVS of the two standard model doublets in the 10-plet while $v_R = \langle \Delta_R \rangle$ is the VEV of the $SU(2)_R$ triplet and $v_L = \langle \Delta_L \rangle$ is the VEV of the $SU(2)_L$ triplet contained both in the 126. This gives, after extremizing the potential with respect to v_L,

$$v_L = \frac{(a_2 v_u^2 + a_3 v_d^2) v_R}{M^2_{\Delta_L} + a_1 v_R^2} . \quad (3.3)$$

The minimal assumption one can make for the fermion masses is to take just one complex 10-dimensional Higgs representation. In this case one would have $\mathrm{Tr}(m_u) = a v_u + b v_d$ while $\mathrm{Tr}(m_d) = -a v_d - b v_u$ where a and b are constants. In order to have $m_b \ll m_t$ one must assume $b \ll a$. One may forbid the b-term either with a global continuous Peccei-Quinn symmetry or via a discrete symmetry D. In the first case if the Peccei-Quinn symmetry is broken softly at the weak scale that would lead to the existence of many Higgs bosons contributing to the up and down fermion masses at low energies. This would in turn destroy natural flavour conservation in weak interactions. For this reason we assume instead the existence of a discrete symmetry D, defined by Tables 1 and 2.

[3] Other terms arising from 10's or other possible representations like 120 or 126 do not change this argument

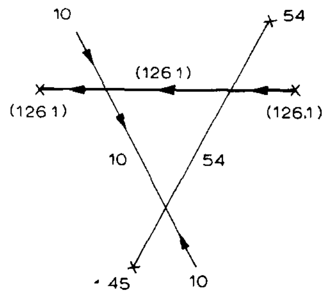

Fig. 1. Typical graph contributing to the mixing between up and down-type Higgs bosons.

We use in total three complex SO(10) 10-plets two transforming as 5-plets and one as a singlet under the $SU(2)_H$ symmetry and, in addition, two 126 representations. The (126,5)-plet is assumed to have a large positive $(\text{mass})^2$ of order the GUT scale, so that it does not break the B-L symmetry and therefore does not contribute to the Majorana mass M_R.

It is easy to see that, owing to our discrete symmetry D, there is no mixing at the tree level between Higgs doublets that contribute to up and down quark mass matrices and we can therefore assume that in the low energy theory there is only one pair of such doublets, one coupled to up and the other to down fermions. Alternatively, if there is only one fine tuning in the theory (corresponding to the usual hierarchy problem) then the down-type fermions such as the b quark can naturally get masses due to Higgs boson mixing, after radiative corrections, as illustrated in Fig. 1 [19].

The corresponding $SO(10) \otimes SU(2)_H \otimes D$ invariant Yukawa couplings have the form

$$\mathcal{L} = (16,3)[\lambda_1(10,5)_1^* + \lambda_2(10,5)_2 + \lambda_3(10,1)^* + \lambda_4(\overline{126},5) + \lambda_5(\overline{126},1)](16,3) + \text{h.c.}, \quad (3.4)$$

where all fields are given in Tables 1 and 2.

After electroweak breaking the fermions acquire Dirac mass matrices which may be written as

$$M_{u,ab} = \lambda_1 u_{1ab} + \lambda_2 u_0 \delta_{ab} + \lambda_3 u_{2ab} + \lambda_4 \omega_{uab}, \quad (3.5)$$

$$M_{d,ab} = \lambda_1 d_{1ab} + \lambda_2 d_0 \delta_{ab} + \lambda_3 d_{2ab} + \lambda_4 \omega_{dab}, \quad (3.6)$$

$$M_{\ell,ab} = \lambda_1 d_{1ab} + \lambda_2 d_0 \delta_{ab} + \lambda_3 d_{2ab} - 3\lambda_4 \omega_{dab}, \quad (3.7)$$

$$M_{D,ab} = \lambda_1 u_{1ab} + \lambda_2 u_0 \delta_{ab} + \lambda_3 u_{2ab} - 3\lambda_4 \omega_{uab}, \quad (3.8)$$

where here we have called u_0 and d_0 the VEVS of the (10,1) along the 5 and $\bar{5}$ and the other VEVS have been defined as

$$u_{1ab} = \langle (10,5)_1 \rangle, \quad u_{2ab} = \langle (10,5)_2 \rangle, \quad (3.9)$$

while ω_{uab} is the VEV of the colour singlet part of the (15,2,2) of (126,5).

Note that all quark masses and mixings, as well as charged lepton masses can be correctly reproduced. In addition the induced VEV of the (126,5) plays an important role in giving an acceptable pattern of Dirac mass matrices for the fermions. On the other hand the (126,1) acquires a large VEV v_R which violates B-L and gives a large mass M_R to right-handed neutrinos proportional to λ_5, and thus to the identity matrix.

In the case where there is only one light Higgs doublet, one may write

$$M_{u,ab} = S_{ab} + \frac{m_b}{m_t} S_{1ab}, \quad (3.10)$$

$$M_{d,ab} = S_{ab} \frac{m_b}{m_t} + S_{1ab}, \quad (3.11)$$

$$M_{\ell,ab} = S_{ab} \frac{m_b}{m_t} + S_{2ab}, \quad (3.12)$$

$$M_{D,ab} = S_{ab} + \frac{m_b}{m_t} S_{2ab} \quad (3.13)$$

where $\mathrm{Tr}(S) = m_t$, $\mathrm{Tr}(S_1) = \mathrm{Tr}(S_2) = 0$.

Let us now turn to the discussion of the seesaw formula for neutrino masses, obtained from our model. Since $v_R = \langle \Delta_R \rangle$ breaks the B-L symmetry, it gives rise to the heavy Majorana mass of the right-handed neutrinos. The modified ν_L-ν_R seesaw mass matrix takes the form

$$\begin{pmatrix} \lambda v_L & M_D \\ M_D^T & \lambda v_R \end{pmatrix}, \quad (3.14)$$

where λ and M_D are 3×3 matrices and v_L is given in Eq. (3.3). Since v_R arises from an $SU(2)_H$ singlet it follows that the λ matrix in Eq. (3.14) is proportional to the identity matrix, since $\lambda \equiv \lambda_5$. The light neutrino mass matrix that follows from diagonalizing Eq. (3.14) is given as

$$m_\nu \approx \lambda v_L - (\lambda v_R)^{-1} M_D M_D^T. \quad (3.15)$$

Using the two-loop renormalization group equations and last measurements of the electroweak mixing parameter $\sin^2 \theta_W$ and the strong coupling constant α_s derived from LEP and other data we determine the mass of the right-handed bosons and, as a result, find $v_R \simeq 0.8 - 1.1 \times 10^{14}$ GeV. Taking $\lambda \simeq 1.5$, $a_1 \simeq .2$ (note that $M^2_{\Delta_L} \sim M^2_{\Delta_R} \simeq .3 v_R^2$) $a_2 \simeq 1$ in Eq. (3.3) we get for the neutrino masses $m_{\nu_e} \simeq m_{\nu_\mu} \simeq m_{\nu_\tau} \simeq 2$ eV, the scale needed for the hot dark matter. On the other hand we find for the ν_e - ν_μ and ν_μ - ν_τ mass splittings

$$\Delta m_{e\mu} = \kappa_c \frac{m_c^2}{\lambda v_R}, \quad (3.16)$$

$$\Delta m_{\mu\tau} = \kappa_t \frac{m_t^2}{\lambda v_R}, \quad (3.17)$$

where we have determined $\kappa_c \simeq .05 - .07$ and $\kappa_t \simeq .2$ from the renormalization group equations. Now using $m_c \simeq 1.5$ GeV and $m_t \simeq 170$ GeV one finds just the right narrow range allowed for the MSW conversions. For the atmospheric neutrino oscillations we find $\Delta m_{\mu\tau} \simeq 4.5 \times 10^{-2}$ eV, corresponding to 0.18 eV^2 for the squared mass difference. Taking into account uncertainties such as in the charm and top masses, one sees that the solution is quite consistently obtained in our model. Note that the induced triplet VEV v_L gives the main contribution to the light neutrino masses $m = \lambda v_L$ and, to a good approximation, the Dirac neutrino mass matrix *is determined by the up quark masses, leading to the successful prediction Eq. (2.10) for the required mass splittings in Eq. (2.1) and Eq. (2.2).*

These mass differences are just right to explain the solar neutrino deficit and the atmospheric neutrino anomalies, while the absolute light neutrino mass scale 2 eV roughly gives the value required for neutrinos to

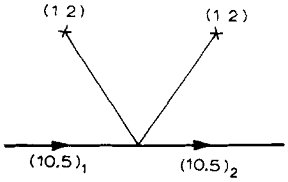

Fig. 2. Typical graphs breaking the SU(2) family symmetry in the Higgs sector.

provide the hot dark matter. Notice that in our model the ν_τ does not play any role in solar neutrino conversions, since the mass difference between ν_e and ν_τ is larger than required for the resonant enhancement to take place. The 12 mixing angle is then chosen in the range of Eq. (2.1), while the 23 angle is chosen to fit the atmospheric neutrino deficit, Eq. (2.2). This angle is quite large and, in our SO(10) model it arises from the diagonalization of the charged lepton mass matrices, and is therefore unrelated with the corresponding angle of the Kobayashi-Maskawa matrix of the quark sector. The 13 angle is basically free and may give rise, in a three parameter fit of the data, to somewhat wider allowed ranges for our solution than given by Eq. (2.1) and Eq. (2.2). However, we may take it to be zero and, as a result, in this case the ν_e does not play any role in atmospheric neutrino oscillations. Therefore in this case the solar and atmospheric neutrino conversions decouple, so that Eq. (2.1) and Eq. (2.2) correctly specify the necessary condition for reconciling the solar and atmospheric neutrino deficits with the hint for hot dark matter.

On the other hand, as illustrated in Fig. 2, the spontaneous breaking of the global $SU(2)_H$ family symmetry with SO(10) singlet, $SU(2)_H$ doublets Higgs fields can be achieved in such a way that there is no mixing between the (126,1) and (126,5) multiplets. As a result, the degenerate neutrino spectrum prediction is unaffected.

4. Discussion

We have proposed a simple SO(10) scheme where quasi-degenerate 2 eV neutrino masses naturally arise. The degenerate ν_e, ν_μ and ν_τ neutrinos fit the hot dark matter inferred from large scale structure simulations. On the other hand the splittings between ν_e and ν_μ explain the solar neutrino deficit and that between ν_μ and ν_τ fits well the atmospheric neutrino anomaly. Therefore all present hints in favour of massive neutrinos are nicely reconciled. The 2 eV mass of the majorana neutrinos can lead to a neutrinoless double beta decay rate that could be observable in enriched germanium experiments.

The model is based on a realization of the seesaw mechanism in which the leading contribution to the light neutrino masses is universal, due to a suitable SU(2) horizontal symmetry, broken only in the other sectors of the theory. The model is in good agreement with all relevant observations such as the proton decay limits and the measurements of α_s and $\sin^2\theta_W$ that come from LEP and other experiments.

Acknowledgements

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Note added

As we finished this work we became aware of two recent papers described in Ref. [20]. The second deals mostly with the phenomenology of degenerate neutrinos at the $SU(3)\otimes SU(2)\otimes U(1)$ level, while the first also gives an SO(10) realization.

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