NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM

No. 1162

TUNNEL CORRECTION FOR COMPRESSIBLE SUBSONIC FLOW

By A. v. Baranoff

Translation

"Zur Frage der Kanalkorrektur bei kompressibler Unterschallströmung"

Deutsche Luftfahrtforschung, Forschungsbericht Nr. 1272

Washington
July 1947

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM NO. 1162

TUNNEL CORRECTION FOR COMPRESSIBLE SUBSONIC FLOW*

By A. V. Baranoff

SUMMARY

This report presents a treatment of the effects of the tunnel walls on the flow velocity and direction in a compressible medium at subsonic speed by an approximate method. Solutions with numerical calculations are given for the rotationally symmetric and two-dimensional problems of the flow past bodies, as well as for the downwash effect in the tunnel with circular cross section.

1. SYMBOLS[1]

b	wing span of the model wing
Γ	circulation
h	half of the tunnel height, two-dimensional case
j	profile volume of the model, two-dimensional case
μ	Mach number squared in the undisturbed flow
q	variable of integration
R	tunnel radius
σ	variable of integration
τ	volume of the model, case of rotational symmetry

*"Zur Frage der Kanalkorrektur bei kompressibler Unterschall-strömung," FB 1272, Zentrale für wissenschaftliches Berichtswesen über Luftfahrtforschung (ZWB) Berlin-Adlershof, July 5, 1940.

[1]This list only contains symbols appearing in the final results (equations (17), (25), (31), (32), (41), and (42)). Symbols used in intermediate calculations are explained at the point of their introduction.

U flow velocity

$\bar{u}$ increased velocity at the tunnel wall

u* additional axial velocity (due to the constriction of the stream)

w* additional upwash velocity (due to the constriction of the stream)

ξ, ρ, or ξ, η are the coordinates for the case of rotational symmetry or two dimensions, rendered dimensionless by division by R or h.

2. GENERAL STATEMENT OF THE PROBLEM

The effect of the tunnel walls on the flow around a body acquires increased significance at high velocities as much through compressibility as through the often unfavorable ratio of model dimensions to tunnel diameter. In this, the question concerns the effects on the flow speed and direction, the first case of which is, possibly, that of a model, symmetrically suspended, in a flow where there is zero lift, while the second case is that of a circulatory flow past a thin profile. The differential equation for compressible subsonic flow should be taken as a basis, here, in the approximation form named after Prandtl. If Φ represents the velocity potential, in cylindrical coordinates this equation, then, reads:

$$\frac{\partial^2\Phi}{\partial r^2} + \frac{1}{r}\frac{\partial\Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\vartheta^2} + (1 - \mu)\frac{\partial^2\Phi}{\partial x^2} = 0 \tag{1}$$

This holds for a so-called, near parallel flow, that is a uniform principal flow in the direction of the x-axis on which is superimposed a flow of ordinarily small velocity.

Now let Φ be the potential of the flow in the medium, unconfined, and Φ^* the potential of the additional flow appearing because of the effects of the tunnel walls. Φ^* certainly satisfies the differential equation (1) in the entire range of the interior of the tunnel as a good approximation. The same cannot be said of Φ because in the vicinity of the body the deviations from the principal flow can be of the same order of magnitude as the principal flow, itself. The quantity, $\bar{\Phi}$, will, however, certainly do at a distance from the body, that is, in the neighborhood of the tunnel wall, possibly, just as well as Φ^* of equation (1).

Now since the connection between Φ and Φ^* consists of the fact that

$$\frac{\partial}{\partial r}(\Phi + \Phi^*) = 0 \tag{2}$$

at the tunnel wall, the Φ^* desired is touched only slightly by the uncertainty in the potential Φ in the vicinity of the body as far as it succeeds, that is, in giving solutions of equation (1) of such a kind which describe the action of the body, which the flow moves past at a great distance from it with sufficient accuracy. First of all, in the following the rotationally symmetrical and the two-dimensional problem for the flow past a model will be treated, for which ascertaining a correction factor for the flow velocity or its Mach number is the object of this investigation. In the conclusion, the problem of downwash correction factor is handled in connection with that. In a formal sense the method in all three cases depends on the same artifice (compare reference 1), namely, in that the condition at the edge (reference 2) is satisfied, first of all, within a finite longitudinal section $2l$ of the tunnel cylinder, and the limit $l \to \infty$ is taken only then. The solutions all appear, therefore, in the form of Fourier integrals. It should be mentioned that in the two-dimensional case the method of reflection of the singularities (reference 2) leads to a solution that is more convenient for the purpose of numerical calculation.

3. EFFECT OF THE TUNNEL WALL ON THE FLOW PAST BODIES

(CASE OF ROTATIONAL SYMMETRY)

It is logical to describe the disturbances that a body past which there is flow, causes at some distance from itself by a superposition of sources and sinks in which the source and sink potentials satisfying equation (1) are readily expressible. The discussion is limited, at this point, to the case where the body is small enough in comparison to the tunnel radius so that its action can be replaced accurately enough by that of a single dipole. The potential of such a dipole with the x-axis as its axis of symmetry, figure 1, reads:

$$\Phi = \frac{m_r}{4\pi} \frac{x}{\sqrt{x^2 + (1 - \mu)r^2}^{\,3}}$$

Regarding the meaning of the dipole moment m_r, arguments will not be presented till section 5.

For the additional potential Φ^*, which gives the action of the tunnel walls on the flow, the following estimate is made:

$$\Phi^* \sim P(r)\, X(x)$$

with which the following equation derives from (1)

$$P'' + \frac{1}{r} P' + (1 - \mu)\, P \frac{X''}{X} = 0 \tag{4}$$

First of all, to satisfy the boundary condition (2) only for $|x| < l$, it is necessary to set

$$X'' + \upsilon^2 X = 0 \tag{5}$$

in which

$$\upsilon = \frac{k\pi}{l} \qquad k = 1,2,3.........$$

It is readily seen, that because of (3) and (2) x appears to an odd power in Φ^* so that only

$$X = \sin \frac{k\pi x}{l} \tag{6}$$

enters in as a solution of (5). On account of (5) equation (4) transforms to

$$P'' + \frac{1}{r} P' - (1 - \mu) \frac{k^2\pi^2}{l^2} P = 0 \tag{7}$$

The solution of this so-called modified Bessel's differential equation is

$$P = I_o\left(\sqrt{1 - \mu}\,\frac{k\pi r}{l}\right) \tag{8}$$

where I_o is the modified Bessel function of the first type and zero order. The corresponding function of the second type does not enter into the question because of the requirement of regularity for Φ^*. The general solution develops from (6) and (8) by summation over all integral values of k. With the use of the dimensionless quantities

$$\xi = \frac{x}{R},\ \rho = \frac{r}{R},\ \lambda = \frac{l}{R} \tag{9}$$

it reads

$$\Phi^* = \sum_k c_k\, I_o\left(\sqrt{1 - \mu}\,\frac{k\pi\rho}{\lambda}\right)\sin\frac{k\pi\xi}{\lambda} \tag{10}$$

The definition of the coefficients c_k follow from the boundary condition (2). To begin with, for $\rho = 1$

$$\sum_k c_k\,\frac{k\pi}{\lambda}\,I_o'\left(\sqrt{1 - \mu}\,\frac{k\pi}{\lambda}\right)\sin\frac{k\pi\xi}{\lambda} = \frac{3\sqrt{1 - \mu}\,m_r}{4\pi R^2}\,\frac{\xi}{\sqrt{\xi^2 + 1 - \mu}^{\,5}} \tag{11}$$

Expanding the right-hand side in a Fourier series in ξ, by comparison of coefficients, after some intermediate calculation, the following is obtained

$$c_k = \frac{\sqrt{1 - \mu}\,m_r}{2\pi^2R^2}\,\frac{\frac{\pi}{\lambda}\displaystyle\int_o^{\lambda}\frac{\cos\frac{k\pi\alpha}{\lambda}\,d\alpha}{\sqrt{\alpha^2 + 1 - \mu}^{\,3}}}{I_o'\left(\sqrt{1 - \mu}\,\frac{k\pi}{\lambda}\right)} \tag{12}$$

Now substituting (12) in (10) and taking the limit $\lambda \rightarrow \infty$ the following is obtained

$$\Phi* = \frac{\sqrt{1 - \mu}\, m_r}{2\pi^2 R^2} \int_0^\infty \frac{\sin (q\xi)\, I_o (\sqrt{1 - \mu}\, q\rho) dq}{I_o' (\sqrt{1 - \mu}\, q)} \int_0^\infty \frac{\cos(q\alpha)\, d\alpha}{\sqrt{\alpha^2 + 1 - \mu}^3} \tag{13}$$

The inner integral in this can be put in a form where it is expressible by a modified Bessel's function of the second type and first order. This is, namely,

$$\int_0^\infty \frac{\cos(q\alpha)\, d\alpha}{\sqrt{\alpha^2 + 1 - \mu}^3} = \frac{qK_1(\sqrt{1 - \mu}\, q)}{\sqrt{1 - \mu}}$$

where K_1 is the function mentioned. Equation (13) reduces, by means of this, to

$$\Phi * = \frac{m_r}{2\pi^2 R^2} \int_0^\infty \frac{\sin(q\xi)\, I_o (\sqrt{1 - \mu}\, q\rho)\, q\, K_1 (\sqrt{1 - \mu}\, q)\, dq}{I_o' (\sqrt{1 - \mu}\, q)} \tag{14}$$

A new variable of integration can be written for $\sqrt{1 - \mu}\, q$ here (represented again by q in the following) and the expression

$$u* = \frac{m_r}{2\pi^2 R^3 \sqrt{1 - \mu}^3} \int_0^\infty \cos\left(\frac{q\xi}{\sqrt{1 - \mu}}\right) I_o(q\rho)\, \frac{q^2 K_1(q)}{I_1(q)}\, dq \tag{15}$$

is obtained for the axial additional velocity. In wind tunnels, there is the possibility of finding the velocity at the tunnel wall by measurement of the static pressure at the tunnel wall. The increased velocity there is computed from the two potentials (3) and (14) for $\xi = 0$ as

$$\bar{u} = \frac{m_r}{2\pi^2 R^3 \sqrt{1 - \mu}^3} \left[\frac{\pi}{2} + \int_0^\infty I_o(q) \frac{q^2 K_1(q)}{I_1(q)}\, dq\right] \tag{16}$$

Eliminating the dipole moment in (15) with the aid of (16), then at the position of the body $(\xi = \rho = 0)$

$$u_o^* = 0.454\bar{u} \tag{17}$$

is obtained for the correction-factor velocity where it can be ascertained by measurement. (See section 7 for the numerical calculation of the factor.) The relationship (17) is independent of Mach number.

4. EFFECT OF THE TUNNEL WALL ON THE FLOW PAST BODIES

(Two-Dimensional Case, see Fig. 2)

In the two-dimensional case the differential equation for the velocity potential reads

$$\frac{\partial^2\Phi}{\partial y^2} + (1 - \mu)\frac{\partial^2\Phi}{\partial x^2} = 0 \tag{18}$$

A solution of this equation, which is associated with the dipole, reads

$$\Phi = \frac{m_e}{2\pi}\frac{x}{x^2 + (1 - \mu)y^2} \tag{19}$$

First of all, the moment m_e is simply regarded as given; later on, its relation to the size of the body and to the Mach number will be discussed further. To fulfill the boundary conditions at the upper and lower tunnel wall $(y = \pm h)$ requires the introduction of an additional potential Φ^* which likewise should satisfy equation (18).

Through the statement

$$\Phi^* \sim Y(y)\,X(x)$$

it is easy to get a general solution. Its form consistent with (19) and the boundary condition reads

$$\varphi^* = \sum_k c_k \cosh\left(\sqrt{1 - \mu}\ \frac{k\pi y}{l}\right) \sin \frac{k\pi x}{l} \tag{20}$$

To satisfy the boundary conditions, exactly the same procedure is to be observed as in the preceding section. After taking the limit as $l \to \infty$ and by applying the dimensionless formulas

$$\xi = \frac{x}{h};\ \eta = \frac{y}{h};\ \lambda = \frac{l}{h} \tag{21}$$

the expression

$$\Phi^* = \frac{m_e \sqrt{1 - \mu}}{\pi^2 h} \int_0^\infty \frac{\sin(q\xi)\ \cosh(\sqrt{1 - \mu}\, q\eta)\,dq}{\sinh(\sqrt{1 - \mu}\ q)} \int_0^\infty \frac{\cos(q\alpha)\,d\alpha}{\alpha^2 + 1 - \mu} \tag{22}$$

is obtained.

On account of

$$\int_0^\infty \frac{\cos(q\alpha)\,d\alpha}{\alpha^2 + 1 - \mu} = \frac{\pi}{2\sqrt{1 - \mu}}\, e^{-q\sqrt{1-\mu}}$$

after the introduction of a new variable of integration finally becomes

$$\Phi^* = \frac{m_e}{\pi h \sqrt{1 - \mu}} \int_0^\infty \sin\left(\frac{q\xi}{\sqrt{1 - \mu}}\right) \frac{\cosh(q\eta)\,dq}{e^{2q} - 1} \tag{23}$$

The axial additional velocity now reads

$$u^* = \frac{m_e}{\pi h^2 (1 - \mu)} \int_0^\infty \cos\left(\frac{q\xi}{\sqrt{1 - \mu}}\right) \cosh(q\eta)\ \frac{q\,dq}{e^{2q} - 1} \tag{24}$$

The increased velocity at the wall $(\xi = 0, \eta = 1)$ is introduced again. The correction velocity at the point $(\xi = \eta = 0)$ is then

$$u_o^* = \frac{1}{3}\bar{u} \tag{25}$$

where $\bar{u}$ is the increased velocity measured at the wall.

5. DEPENDENCY OF THE DIPOLE MOMENT ON BODY VOLUME AND MACH NUMBER

The dipole moments m_r and m_e introduced in equations (3) and (19) should not be set in relation to the volume of the body that the flow passes any more. Since those potentials only contain a single parameter, the volume of the body is the most suitable quantity, in fact, for the definition of this parameter. The flow past the body could be introduced as a series of dipoles that has set to work in its interior. Each individual dipole signifies a certain displacement of the outer flow, which is obtained most easily with the aid of the flow function. Therefore, the relation between the potential and the flow function must be set up, first of all. In its exact form it reads for the case of rotational symmetry

$$\begin{aligned} -r\frac{\rho}{\rho_o}\frac{\partial\Phi}{\partial r} &= \frac{\partial\psi}{\partial x} \\ r\frac{\rho}{\rho_o}\frac{\partial\Phi}{\partial x} &= \frac{\partial\psi}{\partial r} \end{aligned} \tag{26}$$

The equations (26) are not linear on account of the dependency between the density ρ and the velocity, however, they can be linearized into the following form:[a]

$$\begin{aligned} -r\frac{\partial\Phi}{\partial r} &= \frac{\partial\psi}{\partial x} \\ r\frac{\partial\Phi}{\partial x} + r\mu\left(U - \frac{\partial\Phi}{\partial x}\right) &= \frac{\partial\psi}{\partial r} \end{aligned} \tag{26a}$$

---

[a] The author is obliged to Dr. Ing. B. Göthert for pointing this out.

The approximate form (26a) is equivalent to the differential equation for ψ obtained from (1) for Φ the case of rotational symmetry and a corresponding equation for ψ. The flow function of a dipole in a uniform flow reads, therefore, in accord with (26a)

$$\psi = \frac{Ur^2}{2} - \frac{(1 - \mu)m_r}{4\pi} \frac{r^2}{\sqrt{x^2 + (1 - \mu)r^2}^3} \tag{27}$$

By setting ψ equal to 0 the contour of the body past which the stream flows is obtained and from this its volume/τ. It is

$$\tau = \frac{2}{3} \frac{m_r}{U} \tag{28}$$

so that in this case the moment is, therefore, independent of the Mach number. For the two-dimensional case the relations corresponding to the system (26a) may be written down readily. From the flow function satisfying them

$$\psi = Uy - \frac{(1 - \mu)m_e}{2\pi} \frac{y}{x^2 + (1 - \mu)y^2} \tag{29}$$

the volume of the body past which the stream flows (volume within surface of the contour past which the stream flows) is obtained as

$$J = \frac{1}{2} \frac{m_e \sqrt{1 - \mu}}{U} \tag{30}$$

From this is obtained the fact that the dipole moment in the two-dimensional case is dependent on the Mach number. This result is in accord with the so-called Prandtl's rule (reference 3). The objection could be raised against this consideration that it investigates the flow past a body taking as a basis an individual dipole de facto, which does not satisfy the Prandtl condition of slenderness. It might, therefore, have been more acceptable to represent the body possibly by assuming a distribution of sources and sinks along its axis. Now if this is done, then in the extreme case of a very slender body admittedly the same dependency of the product of source strength by source-sink distance on the Mach number is obtained as that for the dipole moments in equations (28) and (30), while on the other hand the numerical factors change; they become equal to 1 in both cases, that is

$$\tau = \frac{m_r}{U} \tag{28a}$$

$$j = \frac{m_\theta \sqrt{1 - \mu}}{U} \tag{30a}$$

The dipole moment for the case of rotational symmetry calculated from (28) would then be, accordingly, 50 percent, and that for the two-dimensional case according to (30) fully 100 percent larger than that from the second consideration. Since the bodies that appear practical as models are slender, as a rule, the advantage belongs rightly to the second consideration in every case.

Therefore, introducing (28a) and (30a) into (15) or (24), now, the following is obtained

$$u^* = \frac{U\tau}{2\pi^2 R^3 \sqrt{1 - \mu}^3} \int_0^\infty \cos\left(\frac{q\xi}{\sqrt{1 - \mu}}\right) I_0(q\rho) \frac{q^2 K_1(q)}{I_1(q)}\, dq \tag{31}$$

for the case of rotational symmetry and

$$u^* = \frac{Uj}{\pi h^2 \sqrt{1 - \mu}^3} \int_0^\infty \cos\left(\frac{q\xi}{\sqrt{1 - \mu}}\right) \cosh(q\eta)\, \frac{q}{e^{2q} - 1}\, dq \tag{32}$$

for the two-dimensional one. At the position of the body, therefore, for $\xi = \rho = 0$ or $\xi = \eta = 0$, the following relations are obtained

$$u_o^* = \frac{0.1268}{\sqrt{1 - \mu}^3} \frac{U\tau}{R^3} \tag{31a}$$

$$U_o^* = \frac{\pi}{24\sqrt{1 - \mu}^3} \frac{Uj}{h^2} = \frac{0.1309}{\sqrt{1 - \mu}^3} \frac{Uj}{h^2} \tag{32a}$$

It is noteworthy that the factor in front of the integral in both cases (equations (31) and (32)) shows the same dependency on the Mach number.

6. DOWNWASH ANGLE CORRECTION IN THE CLOSED TUNNEL AND IN THE OPEN JET

In the two-dimensional case the circulatory flow furnishes no contribution to the angle-of-attack correction factor at the position of the body. On that account only the three-dimensional problem in the tunnel of circular cross section is handled in the following. For this the action of the model can be approximated by a horseshoe vortex of infinitely small span. If instead of the velocity potential Φ of such a vortex its acceleration potential φ is introduced, certain further advantages result, in particular, the possibility of keeping the method of solution applied up till now.

The linearized relation between Φ and φ reads, (reference 4).

$$\Phi = \frac{1}{U}\int_{-\infty}^{x} \varphi dx \tag{33}$$

For that very reason φ is also a solution of the differential equation (1). The horseshoe vortex of infinitely small span corresponds to the acceleration potential of a dipole with its axis in the direction of the z-axis (fig. 3); this potential reads

$$\varphi = \frac{b\Gamma U (1 - \mu) z}{4\pi \sqrt{x^2 + (1 - \mu) (y^2 + z^2)}^3} \tag{34}$$

The appropriate velocity potential can be ascertained from this with (33). It reads

$$\Phi = \frac{b\Gamma z}{4\pi(y^2 + z^2)} \left[\frac{x}{\sqrt{x^2 + (1 - \mu) (y^2 + z^2)}} + 1\right] \tag{35}$$

For given Γ the Mach number exerts no influence on the flow, this holds as much in the plane of the wing $(x = 0)$ as also infinitely far behind the wing $(x \to \infty)$. At the same time it is seen that Φ and all cross components of the velocity at an infinite distance have double the value compared to that at the position of the wing. The tunnel correction factor at an infinite distance is valid, therefore, at the position of the wing, too, if it is multiplied by one-half. The additional potential of the flow coming about through the action of the jet boundary, at an infinite distance, is

$$\Phi_{\infty}^* = \pm \frac{b\Gamma z}{2\pi R^2} \tag{36}$$

in which the upper sign holds for the closed tunnel and the lower sign for the open jet.

Now the general three-dimensional problem is to be treated. At this point an additional potential Φ^* is introduced which allows the boundary conditions at the edge of the jet to be satisfied. It is easily seen that the boundary conditions(2) are also valid for the acceleration potential:

$$\frac{\partial}{\partial r}(\varphi + \varphi^*) = 0 \tag{37}$$

The boundary condition for the open jet is obtained as

$$\varphi + \varphi^* = 0 \tag{38}$$

The courses of calculation for the open jet and closed tunnel run off very much alike. It corresponds, moreover, step by step, to the method described in sections 3 and 4. An abbreviated exposition will do here, therefore, in which only the closed tunnel is taken up, first of all.

On account of (34) the following is applied

$$\varphi^* \sim \cos \vartheta \, P(r) \, X(x)$$

The general solution reads, after making use of (9).

$$\varphi^* = \cos\vartheta\left[c_0\rho + \sum_{k=1}^{\infty} c_k \cos\frac{k\pi\xi}{\lambda} I_1\left(\sqrt{1-\mu}\,\frac{k\pi\rho}{\lambda}\right)\right] \tag{39}$$

where I_1 is the modified Bessel's function of the first type and first order.

The coefficients c_k are determined from the boundary conditions (37), which are satisfied only for $|\xi| < \lambda$. If the limit $\lambda \to \infty$ is taken, the final solution is obtained which after application of the integral representations for the modified Bessel's functions of the second type takes the form

$$\varphi^* = \frac{b\Gamma U \cos\vartheta}{2\pi^2 R^2 \sqrt{1-\mu}} \int_0^{\infty} \cos\frac{q\xi}{1-\mu}\, I_1(q\rho)\, \frac{K_1(q) - qK_2(q)}{I_1'(q)}\, dq \tag{40}$$

From this with the aid of (33) the additional upwash component is obtained

$$W^* = -\frac{b\Gamma}{4\pi^2 R^2} \int_{-\infty}^{\frac{\xi}{\sqrt{1-\mu}}} d\sigma \int_0^{\infty} \cos(q\sigma)\, \frac{q^2K_2(q) - qK_1(q)}{I_1'(q)}\, dq \tag{41}$$

The corresponding upwash component in the open jet is

$$W^* = -\frac{b\Gamma}{4\pi^2 R^2} \int_{-\infty}^{\frac{\xi}{\sqrt{1-\mu}}} d\sigma \int_0^{\infty} \cos(q\sigma)\, \frac{q^2K_1(q)}{I_1(q)}\, dq \tag{42}$$

The results (41) and (42) confirm the observation already made, heretofore, that there is no effect due to compressibility at the position of the wing and at an infinite distance. In the remainder the same additional upwash prevails at a position ξ behind the wing as would be present in incompressible flow at the position $\frac{\xi}{\sqrt{1-\mu}}$. Since the amount of the correction velocity increases monotonically with increasing distance behind the wing in the open jet and likewise increases least in the closed tunnel within a range comparable to the tunnel radius, the compressible flow, therefore, has an absolutely larger correction factor.

7. NUMERICAL RESULTS

In the following the results of several numerical calculations shall be compiled and discussed in detail. The axial velocity u^* for the case of rotational symmetry is best calculated from formula (15) or better still (31). For this purpose the integral

$$F_1 = \frac{1}{2\pi^2}\int_0^{\infty} \cos(q\xi)\ I_0(q\rho)\ \frac{q^2 K_1(q)}{I_1(q)}\ dq \tag{43}$$

is evaluated numerically by Simpson's rule (see table 1).

TABLE 1.- VALUES FOR F. (SEE (43).)

ξ	ρ				
	0	0.25	0.5	0.75	1.0
0	0.1268	0.1298	0.1399	0.1604	0.1996
.25	.1197	------	------	------	.1877
.5	.1056	------	------	------	.1242
.75	.0853	------	------	------	.0710
1.0	.0652	------	------	------	.0409
1.5	.0345	------	------	------	.0199

Next, figure 4 presents the variation of the additional velocity u^* along the tunnel radius in the plane $x = 0$. Since the Mach number in this case only appears in the factor in front of the integral, it is sufficient to plot only $\sqrt{1 - \mu}\ ^3 \frac{R^3}{T} \frac{u^*}{U}$.

It is seen that the additional velocity toward the tunnel edge takes on possibly 60 percent more. The assumption of an additional velocity (compare reference 5), constant over the cross section does not prove correct, therefore.

Figure 5 shows the variation of $\frac{R^3}{T} \frac{u^*}{U}$ along the tunnel axis $(r = 0)$.

For the velocity at the tunnel wall, from a rational point of view, not u^*, but the quantity $\bar{u}$ increased by the displacement flow, is plotted for it is certainly this increased velocity $\bar{u}$ which is accessible for direct measurement. The variation of $\bar{u}$ as a function of x appears in figure 6. For the two-dimensional case (equation (32)) it is necessary to evaluate the integral

$$F_2 = \frac{1}{\pi} \int_0^\infty \cos(q\xi) \cosh(q\eta) \frac{q\,dq}{e^{2q} - 1} \tag{44}$$

The numerical values obtained by Simpson's rule are in table 2.

Figures 7 and 8 show the variation of the additional velocity u^* along the y-axis $(x = 0)$ and along the x-axis $(y = 0)$. Figure 9 gives the induced velocity at the tunnel wall.

The downwash correction factor for the closed tunnel should be represented by means of the upwash w^* according to equation (41). In integrating with respect to σ the unsuitable integral can be avoided by using the following relation in accord with (36)

$$\int_{-\infty}^{0} d\sigma \int_0^\infty \cos(q\sigma) \frac{q^2 K_2(q) - qK_1(q)}{I_1'(q)} dq = \pi$$

Then

$$w^* = \frac{b\Gamma}{4\pi R^2} (1 + K_g) \tag{45}$$

where

$$K_g = \frac{1}{\pi} \int_0^{\frac{\xi}{\sqrt{1-\mu}}} d\sigma \int_0^\infty \cos(q\sigma) \frac{q^2 K_2(q) - qK_1(q)}{I_1'(q)} dq \tag{46}$$

This function has been tabulated for $\frac{\xi}{\sqrt{1 - \mu}} = 0 \ . \ . \ . \ 5$ in which the integral has again been evaluated by Simpson's rule. (See table 3.)

The curve for the variation of k_G is shown in figure 10.

The upwash for the open jet is

$$w^* = - \frac{b\Gamma}{4\pi R^2} \left(1 + K_F \right) \tag{47}$$

by which

$$K_F = \frac{1}{\pi} \int_0^{\frac{\xi}{\sqrt{1-\mu}}} d\sigma \int_0^{\infty} \cos(q\sigma) \frac{q^2 K_1(q)}{I_1(q)} dq \tag{48}$$

Table 4 contains the numerical values.

The curves are presented in figure 10. A comparison with the variation calculated by I. Lotz (reference 1), for $\mu = 0$ and a wing of finite wing span shows good agreement in the case of the open jet, on the other hand somewhat larger deviations for the closed tunnel, without assigning a reason for this different behaviour. On the other hand the variation of both curves of figure 10 agree very well with the results calculated by Tani and Taima, (reference 5) using the Burgers method.

SUMMARY

The problem of the effect of the limitation of the jet on the flow past a model is handled by proceeding from the Prandtl linearization of the differential equation of the compressible medium. The disturbance which the model causes near the wall, at the same time, is represented, approximately, by a dipole or horseshoe vortex. The boundary-value problem arising in this, at the limit of the jet is solved exactly to learn the additional flow due to the effect of the edge of the stream. The solutions are evaluated numerically, to the extent that they are of interest.

Translated by Dave Feingold
National Advisory Committee
for Aeronautics

REFERENCES

1. Lotz, I.: Correction of Downwash in Wind Tunnels of Circular and Elliptic Sections. NACA TM No. 801, 1936.

2. Franke, A., and Weinig, F.: Die Korrektur der Anströmgeschwindigkeit und des Anströmwinkels in einem Hochgeschwindigkeitskanal mit geschlossener Messtecke infolge der Verdrängungsströmung von Tragflügelmodellen. FB 1171, 1939.

3. Prandtl, L.: General Considerations on the Flow of Compressible Fluid. NACA TM No. 805, 1936.

4. Prandtl, L.: Theorie des Flugzeugtragflügels in zusammendrückbaren Medium. Luftf.-Forschg. 13, 1936, p. 313.

5. Lamla, E.: Der Einfluss der Strahlgrenze in Hochgeschwindigkeitskanälen. FB 1007, 1938.

6. Tani, I., and Taima, M.: The Boundary Influence of a Circular Wind Tunnel on the Pitching Moment of the Tailplane. Journ. of the Aeronautical Research Inst., Tokyo Imperial University, Nr. 141, 1936.

With regard to designations, definitions, and tables of modified Bessel's functions see Gray, Mathews, MacRobert, A Treatise on Bessel Functions, London, 1931.

TABLE 2.- VALUES FOR F_2 (SEE (44).)

ξ	η				
	0	0.25	0.5	0.75	1.0
0	0.1309	0.1350	0.1487	0.1771	0.2335
.25	.1283	------	------	------	.2239
.5	.1162	------	------	------	.1696
.75	.1016	------	------	------	.0974
1.0	.0828	------	------	------	.0637
1.5	.0559	------	------	------	.0323

TABLE 3.- VALUES FOR K_g (SEE (46).)

$\dfrac{\xi}{\sqrt{1-\mu}}$	K_g
0	0
.2	.1974
.4	.3829
.8	.6831
1.2	.8726
1.6	.9735
2.0	1.0198
3.0	1.0392
5.0	1.0208

TABLE 4.- VALUES FOR k_F (SEE (48).)

$\dfrac{\xi}{\sqrt{1-\mu}}$	K_F
0	0
.2	.1571
.4	.3057
.8	.5531
1.2	.7186
1.6	.8183
2.0	.8730
4.0	.9677
6.0	.9856

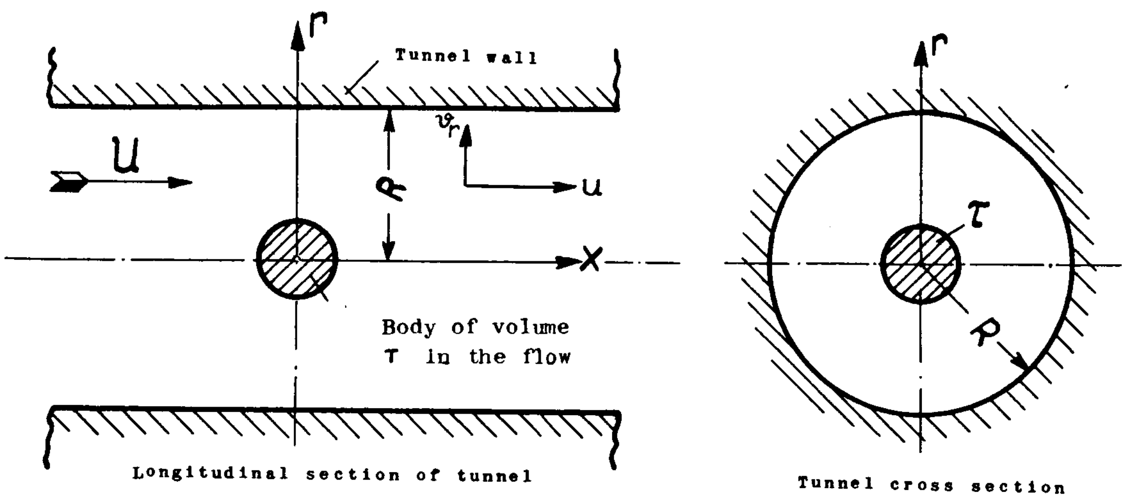

Figure 1.- Designations in the case of rotational symmetry. (x-axis is the axis of symmetry).

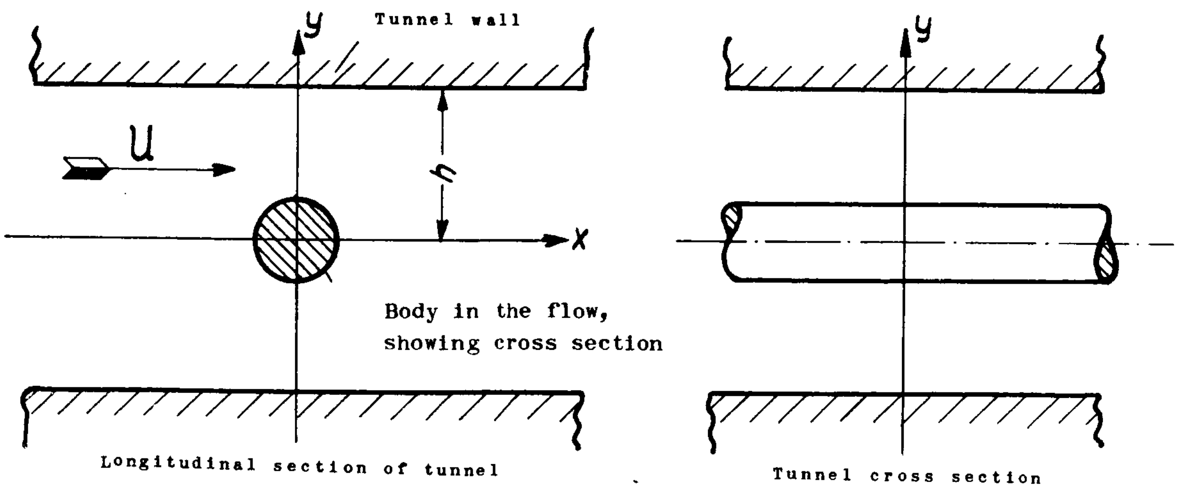

Figure 2.- Designations in the two-dimensional case.

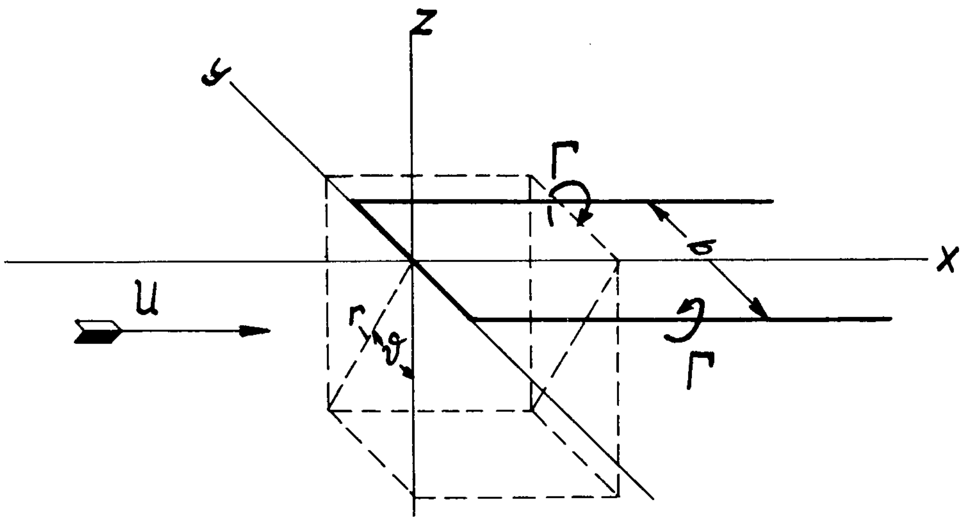

Figure 3.- Horseshoe vortex and axis.

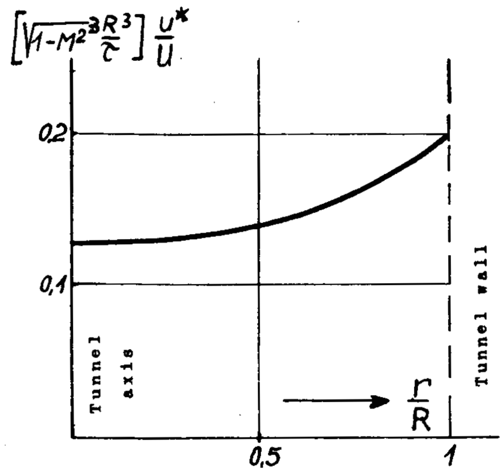

Figure 4.- Axial additional velocity in the plane X = 0. Case of rotational symmetry.

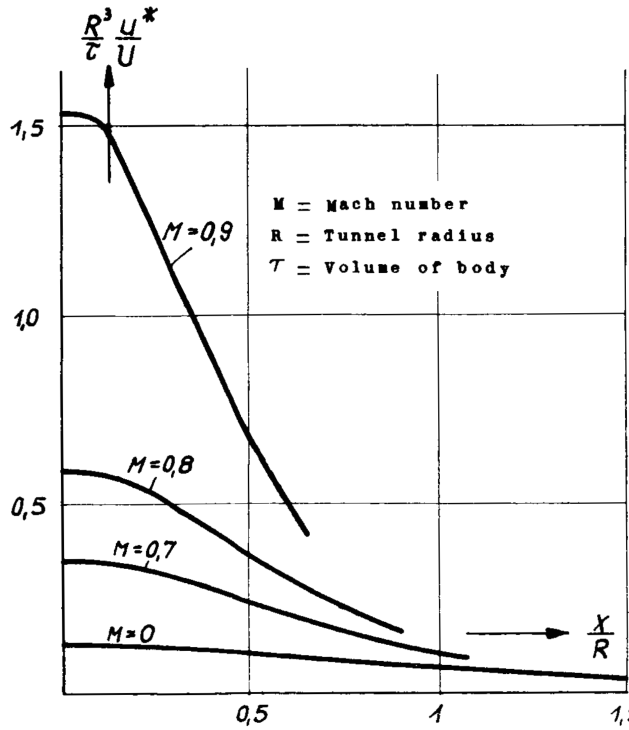

Figure 5.- Axial additional velocity along the tunnel axis. Case of rotational symmetry.

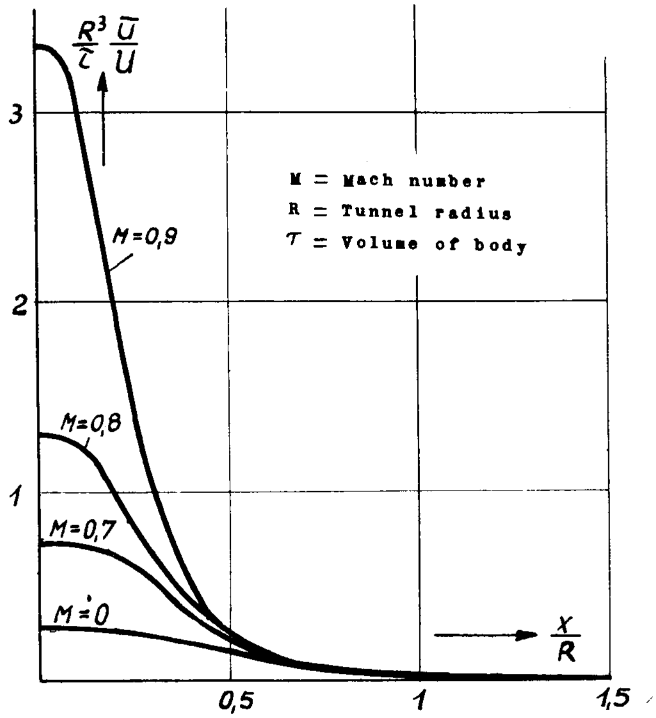

Figure 6.- Induced velocity on the tunnel wall. Case of rotational symmetry.

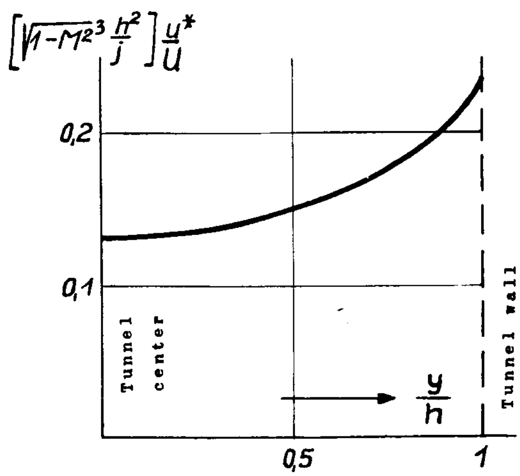

Figure 7.- Axial additional velocity in the plane x = 0. Two-dimensional case.

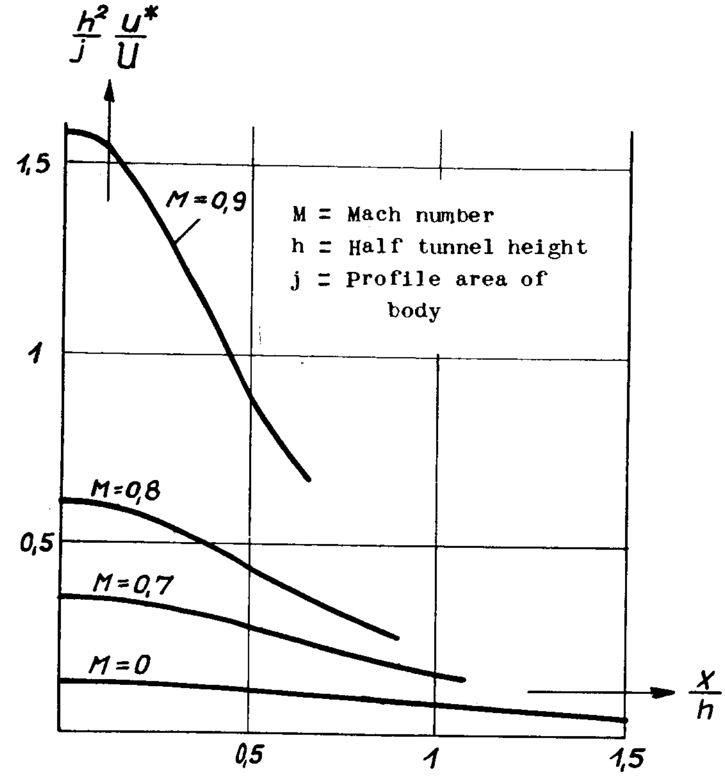

Figure 8.- Axial additional velocity in center of tunnel for two-dimensional case.

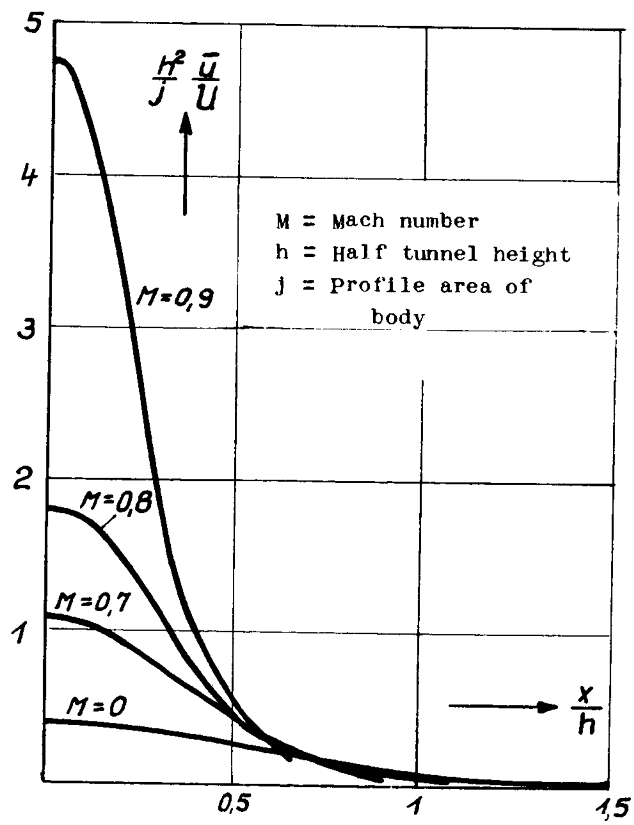

Figure 9.- Induced velocity on the tunnel wall for the two-dimensional case.

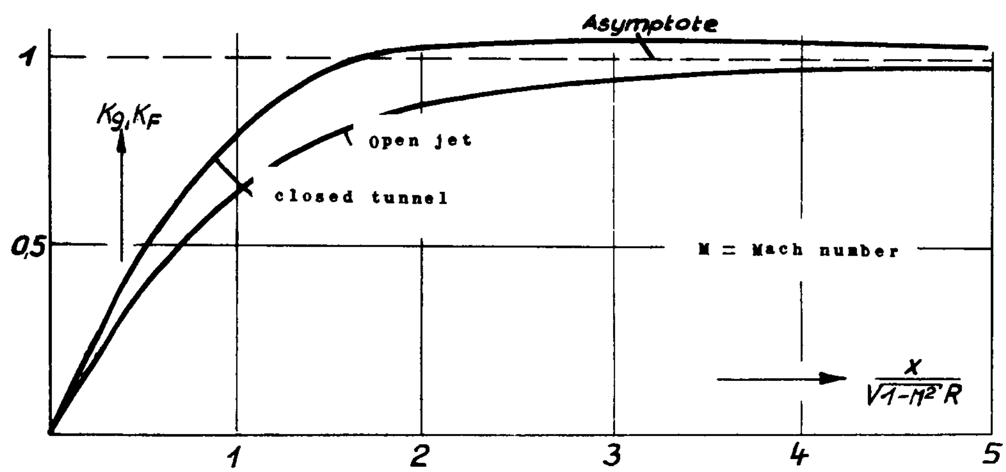

Figure 10.- Downwash correction factors for closed and open tunnels.