Two Meson Scattering Amplitudes and their Resonances from Chiral Symmetry and the N/D Method*

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We study the vector and scalar meson-meson amplitudes up to $\sqrt{s} \lesssim 1.4$ GeV and their associated spectroscopy. The study has been done considering jointly the N/D method, Chiral Symmetry and implications from large N_c QCD [1]. The N/D method provides us with the way to unitarize the tree level amplitudes constructed in agreement with Chiral Symmetry and its breaking (explicit and spontaneous). These amplitudes are calculated making use of the lowest order Chiral Perturbation Theory (χPT) Lagrangians [2] and the exchanges of resonances compatible with Chiral Symmetry as given in [3]. On the other hand the large N_c considerations allow us to distinguish between elementary (as elementary as the pions, for instance) and compound (meson-meson) states. Making use of this formalism one observes that the σ, κ and $a_0(980)$ resonances are meson-meson states originating from the unitarization of the lowest order χPT amplitudes. On the other hand, the $f_0(980)$ is a combination of a strong S-wave meson-meson unitarity effect and of a preexisting singlet resonance with a mass around 1 GeV.

For the vector resonances we reproduce the well known features of Vector Meson Dominance (VMD) and the KSFR relation [4]. The much more important role that unitarity plays in the scalar sector as compared with the vector one is also stressed. In particular, it is clear from our study that the σ and the ρ resonances are states completely different in nature. The σ is a two pion resonance originated from the interaction of the pions whereas the ρ is a state as elementary as the pions themselves whose origin has nothing to do with the interaction between pions.

1. Introduction

χPT [2] can be supplied with the exchange of explicit resonance fields [3]. In doing this, a resummation up to an infinite order in the chiral expansion can be achieved from the expansion of the bare propagator of a resonance. In fact, at $\mathcal{O}(p^4)$, it is seen [3] that the L_i counterterms of χPT are saturated by the exchange of the resonances. However, the amplitudes that can be built directly from χPT at $\mathcal{O}(p^4)$ plus resonance exchanges as in [5], need a unitarization procedure in order to compare directly with experimental

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data (phase shifts, inelasticities...) for the different energy regions, in particular, around the resonance masses. This is one of the aims of the present study.

On the other hand, it is well known that the scalar sector is much more controversial than the vector or tensor ones. In the latter case, the associated spectroscopy can be understood in terms of first principles coming directly from QCD as Chiral Symmetry and Large N_c plus unitarity, once we admit VMD as dictated by phenomenlogy. We try to make use in this work of the same principles than before, that is, Chiral Symmetry, Large N_c and unitarity in coupled channels, in order to study the scalar resonant channels.

2. Formalism

We consider the influence of the unphysical cuts perturbatively. In this we take the zero order approach, that is, we neglect it. In [1] we make estimations of the unphysical cuts contribution and find them to be only a few per cent of our final amplitudes. In [6] it is discussed how to include them up to one loop calculated at $\mathcal{O}(p^4)$.

When taking into account the N/D method [7] the most general structure that an elastic partial wave amplitude, T_L, has when the unphysical cuts are neglected, is [1]:

$$\begin{aligned} \mathrm{T}'_L(s) &= \frac{1}{\mathrm{D}_L(s)} \\ \mathrm{D}_L(s) &= -\frac{(s-s_0)^{L+1}}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\nu(s')^L \rho(s')}{(s'-s)(s'-s_0)^{L+1}} + \sum_{m=0}^{L} a_m s^m + \sum_i^{M_L} \frac{R_i}{s-s_i} \end{aligned} \tag{1}$$

where $\mathrm{T}'_L(s) = \mathrm{T}_L(s)/\nu^L$, $\nu^L = p^{2L}$ with p the center mass three momentum of the particles and s_0 is the subtraction point.

In Large N_c counting rules $\mathrm{T}_L \to 1/N_c$ and hence $\mathrm{D}_L \to N_c$. We then split the D_L in two parts:

$$\begin{aligned} \mathrm{D}_L^\infty &= \sum_{m=0}^{L} a_m^L s^m + \sum_i^{M_L} \frac{R_i}{s-s_i} \to \mathcal{O}(N_c) \\ g(s) &= -\frac{(s-s_0)^{L+1}}{\pi}\int_{s_{th}}^{\infty} ds' \frac{\nu(s')^L \rho(s')}{(s'-s)(s'-s_0)^{L+1}} + \sum_{m=0}^{L} a_m^{SL} s^m \to \mathcal{O}(1) \end{aligned} \tag{2}$$

where $a_m^{L(SL)}$ is the $\mathcal{O}(N_c)(\mathcal{O}(1))$ of the coefficient a_m.

In the large N_c limit a partial wave amplitude is given [8] by local plus pole terms. The local terms are taken from the lowest order χPT amplitudes and the resonant ones from the exchange of resonances as given in [3]. In doing this, we are assuming the concept of Meson Resonance Dominance, in the sense that all the local terms of order higher than two in the χPT expansion are saturated by the resonance exchanges. This was shown to be case at $\mathcal{O}(p^4)$ in [3]. In this way

$$\mathrm{T}_L^\infty = \frac{1}{\mathrm{D}_L^\infty} = T^{(2)} + T^R \tag{3}$$

where $T^{(2)}$ refers to the lowest order χPT amplitude and T^R to the resonance exchange amplitudes. In [1] it is proved that D_L^∞ has enough room to accommodate $T^{(2)}+T^R$ as given above. Thus, our final formula will be:

$$\mathrm{T}_L = p^L\left(\left[T^{(2)}+T^R\right]^{-1}+g(s)\right)^{-1}p^L \tag{4}$$

The extension of the former formula to the coupled channel case is straightforward in a matrix formalism. Then, we have: $T^{(2)}$, $T^R \to T_{ij}^{(2)}$, T_{ij}^R and $g(s)$ and p are now diagonal matrices [1].

3. Vectors

Making use of the former formalism we study simultaneously the ρ and K^* resonances [1]. We obtain the expected leading behaviours of their bare poles and the KSFR [4] value of the coupling with a relative deviation of only a 6%. The former conclusions are obtained after fitting the elastic P-wave $\pi\pi$ and $K\pi$ phase shifts in terms of two parameters: the subtraction constant a^{SL} present in $g(s)$ and the coupling of the vector octet of resonances.

4. Scalars

We study the $SU(3)$ connected meson-meson S-waves with $I=0$, 1 and 2, where I refers to the isospin. Contrary with the former vector channels, which are essentially elastic, for the proposed scalar channels one has to take into account the effect of coupling channels. We include the following channels:

$$\begin{array}{ll} I=0 & \pi\pi(1),\ K\bar{K}(2),\ \eta\eta(3) \\ I=1 & \pi\eta(1),\ K\bar{K}(2) \\ I=1/2 & K\pi(1),\ K\eta(2) \end{array} \tag{5}$$

These are the most relevant channels up to $\sqrt{s}\approx 1.2$ GeV. For energies higher than these other channels become increasingly more important as four pions for $I=0$ or $K\eta'$ $I=1/2$. Thus, a proper study of the set of resonances that we find around 1.4 GeV would require the inclusion of those relevant channels [6].

We fit, up to $\sqrt{s}\lesssim 1.4$ GeV, the following data: $I=0$ elastic $\pi\pi$ phase shifts, $I=0$ $K\bar{K}\to\pi\pi$ phase shifts, $\frac{1-\eta_{00}^2}{4}$ where η_{00} is the inelasticity with $I=0$, $I=1/2$ elastic $K\pi$ phase shifts and a mass distribution for $I=1$ around the $a_0(980)$ resonance.

In a first glance at the PDG [9] one can think that there should be at least two scalar nonets, one with a mass below 1 GeV and another around 1.4 GeV. We first include two scalar nonets but then the fit gives a remarkable feature. The octet around 1 GeV has vanishing coupling constants and the same occurs with the singlet around 1.4 GeV. As a consequence, one can reproduce the scalar data with only a singlet around 1 GeV and an octet around 1.4 GeV. The χ^2 per degree of freedom obtained is almost 1 with 188 experimental points.

From the point of view of the resonance content of our amplitudes, the explicitly included octet with a mass about 1.4 GeV evolves to give rise to poles with masses very close to those of the $K_0^*(1430)$, $a_0(1450)$ and $f_0(1500)$ resonances. In turn, the singlet around 1 GeV evolves to the physical pole of the $f_0(980)$. However, together with the former poles we also find other ones which do not originate from any preexisting resonance ($T^R = 0$). They are meson-meson resonances originating from the unitarization of the lowest order χPT amplitudes. On the other hand, since loops are suppressed in large N_c these resonances disappear for $N_c \to \infty$. They correspond to the $\sigma(500)$, $a_0(980)$, κ and a strong contribution to the physical $f_0(980)$ resonance. Thus, the $f_0(980)$ resonance results from two effects: a preexisting resonance around 1 GeV and a strong $K\bar{K}$ threshold effect.

5. Conclusions

We have presented a systematic procedure to unitarize the tree level amplitudes coming from lowest order χPT and the explicit exchange of resonance fields. We have used this method to study the vector and scalar resonances. For the vectors, we reproduce the well known features of VMD and the KSFR value for the coupling of the vector resonances ρ and K^*. For the controversial scalar channel, the situation is more complicated. After reproducing a large amount of experimental data, we have observed two sets of resonances. Those resonances preexisting to the unitarization: one octet around 1.4 GeV and a singlet around 1 GeV that evolves to the physical $f_0(980)$ resonance. The other set corresponds to meson-meson resonances with a mass $\lesssim$1 GeV: σ, $a_0(980)$, κ and a strong contribution to the $f_0(980)$ from the $K\bar{K}$ threshold. This set of resonances forms a nonet and in fact when we go in our formalism to the $SU(3)$ limit they form an octet of degenerate resonances plus a singlet.

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