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COMPARISON OF THE ENERGY METHOD WITH THE ACCELEROMETER

METHOD OF COMPUTING DRAG COEFFICIENTS FROM FLIGHT DATA

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ADVANCE CONFIDENTIAL REPORT

COMPARISON OF THE ENERGY METHOD WITH THE ACCELEROMETER METHOD

OF COMPUTING DRAG COEFFICIENTS FROM FLIGHT DATA

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SUMMARY

A comparison was made between the energy and the accelerometer methods of computing drag coefficients from flight tests of the P-51B and the P-39N airplanes. The energy method was found to be unreliable with the present types of instrumentation except under the practically unattainable conditions of very steady flight at constant indicated airspeed within a vertically stationary air mass.

INTRODUCTION

In the past, many measurements of the drag of airplanes in flight have involved the use of some variation of the so-called "energy method" (described in the appendix) for the reduction of the data. The results obtained by this method are frequently very erratic, occasionally even giving negative drag coefficients.

Since the results from the "accelerometer method" (described in the appendix) appear to be consistent with computed and wind-tunnel data, the flight-test results reported in reference 1 offer an excellent opportunity for comparison of the two methods to determine the cause of the discrepancies. This report presents such a comparison and includes supplemental data obtained from two special dives performed at constant indicated airspeed in the P-39-N-1 airplane.

DESCRIPTION OF EQUIPMENT

Airplanes

The airplanes used for the tests were the North American P-51B-1-NA and the Bell P-39N-1. Complete descriptions of these airplanes are given in references 1 and 2.

Instrumentation

Standard NACA photographically recording instruments, installed in both airplanes, were used to obtain airspeed, pressure altitude, normal and longitudinal accelerations, and (for the P-39N airplane only) angle of attack as functions of time. Values of free-air temperature at different altitudes were obtained during ascent from the pilot's indicating instrument, and were corrected for temperature rise due to impact of the air stream.

The airspeed-measuring systems utilized, as sources of pitot-static and total pressures, freely swiveling pitot-static heads mounted on booms located beneath the wing tips. These booms extended approximately 0.8 of the local wing chord ahead of the leading edge. Each pitot-static head consisted of two static-pressure tubes and one total-pressure tube, which permitted the use of independent sources of static pressure for both the airspeed and the altitude recorders.

The pressure lines from the pitot-static head to the recording instruments were made as short as possible in order to minimize lag, and the lines to the recording airspeed meter were balanced to give equal flow rates in the static and total-pressure tubes during rapid changes in pressure. Ground tests of mock-ups of the airspeed and the altitude pressure lines indicated that the lag in the systems at the maximum rates of descent caused an error in the recorded altitude of about 20 feet at 15,000 feet (negligible) for both airplanes.

A correction for the position error of the pitot-static tube was determined from flights at various airspeeds past a fixed visual reference point of known constant pressure altitude. It was assumed that the measurements of total pressure were correct and that the variation of recorded altitude with airspeed (obtained by a special sensitive altimeter) at the constant pressure altitude resulted from

the position of the static tube only. Since the maximum error in altitude, as determined by this calibration, was less than the accuracy of the recording altimeter used in the dive tests, no attempt was made to correct the altimeter readings for position error.

The accuracy of the swiveling pitot-static head has been investigated at Mach numbers up to 0.80 in the 16-foot windtunnel at the Ames Aeronautical Laboratory. The results show that the effects of compressibility not included in the regular airspeed calibration are negligible over the flight Mach number range investigated.

The correst indicated airspeed was computed by use of the standard formula

$$V_1 = 1703 \left[\left(\frac{H-p}{p_0} + 1 \right)^{0.286} - 1 \right]^{1/2}$$

where

V_1 correct indicated airspeed, miles per hour

H free-stream total pressure

p free-stream static pressure

p_0 standard atmospheric pressure at sea level

METHOD OF ANALYSIS

If accurate drag data are available for an airplane in flight it is possible to analyze, indirectly, the errors involved in the use of the energy method by working backward from the drag data, through the energy method equations, to obtain values of airspeed or altitude. A comparison of these values with the actual values recorded during the flight offers some insight as to the nature of the errors. The following discussion gives a derivation of the necessary equations and a detailed explanation of the process.

The equations for the drag coefficient, based on the energy and the accelerometer methods (excluding propeller thrust) have been derived in the appendix. The equation from the energy method has been shown to be

$$C_D = \frac{W}{qS}\left(\frac{dh/dt}{V_T} - \frac{dV_T/dt}{g}\right) \qquad (1)$$

where

W   weight of airplane, pounds

q   free-stream dynamic pressure, pounds per square foot

S   wing area, square feet

h   true altitude, feet

V_T true airspeed, feet per second

t   time, seconds

g   acceleration due to gravity, 32.2 feet per second per
     second

In this discussion the slope of the altitude curve
dh/dt will be assumed postive when h decreases, and the
slope of the airspeed curve dV_T/dt will be assumed posi-
tive when V_T increases; hence the minus sign in equation
(1). Equation (1) can be transformed as follows:

$$\frac{dV_T}{dt} = \left(\frac{dh/dt}{V_T} - \frac{C_D qS}{W}\right) g \qquad (2)$$

or

$$\frac{dh}{dt} = \left(\frac{C_D qS}{W} + \frac{dV_T/dt}{g}\right) V_T \qquad (3)$$

The following expressions, solvable by graphical in-
tegration, can be derived from equations (2) and (3):

$$V_T = V_{T_0} + \int_0^t \left(\frac{dh/dt}{V_T} - \frac{C_D qS}{W}\right) g \, dt$$

$$h = h_0 - \int_0^t \left(\frac{C_D qS}{W} + \frac{dV_T/dt}{g}\right) V_T \, dt$$

Thus, if the drag data obtained by the accelerometer
method are approximately correct and the recorded values of
altitude are assumed to be correct, the corresponding air-
speed-time curve can be computed by a series of approxi-
mations and integrations. In the same manner, the corre-
sponding altitude-time curve can be computed if the recorded
values of airspeed are assumed to be correct. The dif-
ference between the computed and recorded airspeed or alti-
tude curves, then, is the amount of error which would pro-
duce the corresponding error in the drag-coefficient curve.

TESTS, RESULTS, AND DISCUSSION

Unrestricted Flight Conditions

Figures 1 and 2 show examples of airspeed and altitude
calculated by the foregoing method from the data of a
propeller-off dive of the P-51B airplane (dive No. 2,
reference 1). Figures 3 and 4 show drag curves for the same
flight, calculated by the energy and the accelerometer methods,
from which it is apparent that the energy method yields
extremely erratic results.

The errors in the altitude curve (fig. 1) or in the
airspeed curve (fig. 2) necessary to produce the variation
of drag coefficient shown in figures 3 and 4 are very small,
and it is apparent that a small error in either airspeed or
altitude could cause sufficient change in slope to produce
a very large error in the computed drag coefficient. In
fact, even small errors in fairing these curves could lead
to appreciable errors in drag coefficient. Furthermore, it
should be noted that the curves of figures 1 and 2 represent
the outside limits of the error; that is, the error in each
quantity was computed on the basis of no error in the other
quantity. Actually, the error probably is divided between
airspeed and altitude and thus is less than that shown for
either.

The dives reported in reference 2 did not follow any
definite flight technique; that is, airspeed and altitude
were allowed to vary unrestrained, the sole objective of the
pilot being to attain as high Mach numbers as possible. The
question has been raised, however, as to whether dives at
constant indicated airspeed would yield data amenable to
the energy method. It was believed that this type of dive
would yield the smoothest possible variation of true airspeed

with time, with the resultant increase in accuracy of the
measured slopes dV_T/dt and decrease in any unknown errors
due to rapid pressure changes.

Restricted Flight Conditions

In order to ascertain the effect of a definite flight
technique on the results of the energy method, therefore,
two dives at nearly constant indicated airspeed were per-
formed in the P-39N-1 airplane. A sensitive longitudinal
accelerometer was included in the instrumentation for these
flights, so that a direct comparison could be made between
the doubtful quantities involved in the energy method and
corresponding quantities of established accuracy used in the
accelerometer method.

Time histories of variables measured during the two dives
are presented in figures 5 and 6. The necessary corrections
for free-air temperature were applied to the indicated values
of airspeed and rate of change of pressure dp/dt to obtain
the true values of airspeed and rate of change of altitude
dh/dt used in the computations of the longitudinal force
factor.

Figure 7 shows a comparison of the longitudinal force
factors $(A_Z \sin \alpha - A_X \cos \alpha)$ from the accelerometer method

and $\left(\dfrac{dh/dt}{V_T} - \dfrac{dV_T/dt}{g} \right)$ from the energy method. Theoretically,

during any one dive, these quantities should be identical.
The resultant drag coefficient computed from these data is
presented in figure 8. It will be observed that during the
time interval for each dive in which the conditions were
extremely steady (70 to 145 seconds in dive number 1, and
55 to 150 seconds in dive number 2) the maximum difference
in drag coefficient ΔC_D is approximately 0.0021 and the
average ΔC_D is very small. The agreement in results is
considerably improved over that of the unrestricted dive.
(See figure 3.) It is apparent, however, that the slightest
variation from steady conditions causes wide variation be-
tween the computed longitudinal force factors and renders
the energy method unreliable.

Furthermore, since the derivation of the energy method
does not allow for any extraneous acquisition of energy,

such as that due to vertical air currents or changes of
vertical air currents, considerable error could be intro-
duced by the airplane's transit through a rising or de-
scending air mass.

It is believed that, if extremely steady flight condi-
tions (including constant indicated airspeed) could be main-
tained by the pilot, the energy method of computing drag
coefficient would be of value.[1] In practical flight-research
work, especially at high Mach numbers, however, the required
conditions are seldom achieved and are difficult to recognize
when they are achieved. Thus, it is believed that the
energy method cannot be relied upon for conclusive results.

CONCLUSIONS

From comparison with the accelerometer method, the
energy method of computing drag coefficient from flight data
is believed to be unreliable with the present instrumentation
for the following reasons:

1. The conditions of constant indicated airspeed and
very steady flight, which appear to be essential for accurate
application of the energy method, are impractical for ordi-
nary flight-research work (especially at high Mach numbers)
and are difficult to achieve and to recognize.

2. The energy method is correct only for flights in air
having no vertical velocity.

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[1]It is instructive to note that, since, theoretically,
the energy method should be exact for flight in still air, it
is possible that the altitude and airspeed curves of figures 1
and 2 represent a type of pressure calibration and that the
recording systems are actually in error by various amounts.
The differences shown in figures 1 and 2, however, only can
give a qualitative picture of the error because the relative
amounts of the total error attributable to altitude and to air-
speed are not known and the vertical motion of the air mass
during the dives could not be determined. Further study along
this line might be of considerable interest.

APPENDIX A

Symbols

Symbols used throughout this report are listed as follows:

V_1 correct indicated airspeed, miles per hour

H   free-stream total pressure

p   free-stream static pressure

p_0 standard atmospheric pressure at sea level

W   weight of airplane, pounds

q   free-stream dynamic pressure, pounds per square foot

S   wing data, square feet

h   true altitude of airplane, feet

V_T true airspeed, feet per second

t   time, seconds

g   acceleration due to gravity, 32.2 feet per second per second

E   energy, foot-pounds

D   drag, pounds

A_Z the ratio of the net aerodynamic force along the airplane Z-axis (positive when directed upward) to the weight of the airplane

A_X the ratio of the net aerodynamic force along the airplane X-axis (positive when directed forward) to the weight of the airplane

R   the resultant of A_Z and A_X

θ   flight-path angle from horizontal, degrees

α   angle of attack, degrees

APPENDIX B

DERIVATION OF THE ENERGY AND THE ACCELEROMETER METHODS

OF COMPUTING DRAG COEFFICIENT FROM FLIGHT DATA

Derivation of Energy Method

The total energy (excluding that due to propeller thrust) of an airplane in flight in still air with respect to a point on the earth at sea level may be expressed by the equation

$$E = Wh + \frac{W(V_T)^2}{2g} \qquad (B1)$$

The rate at which energy is expended by an airplane in flight is equal to the drag multiplied by the velocity, or

$$\frac{dE}{dt} = DV_T \qquad (B2)$$

Differentiating equation (B1) with respect to time gives

$$\frac{dE}{dt} = W\left(\frac{dh}{dt} + \frac{V_T}{g}\frac{dV_T}{dt}\right) \qquad (B3)$$

Combining equations (B2) and (B3) yields

$$D\,V_T = -W\left(\frac{dh}{dt} + \frac{V_T}{g}\frac{dV_T}{dt}\right)$$

or

$$D = -W\left(\frac{dh/dt}{V_T} + \frac{dV_T/dt}{g}\right),$$

and thus

$$C_D = -\frac{W}{qS}\left(\frac{dh/dt}{V_T} + \frac{dV_T/dt}{g}\right)$$

Derivation of Accelerometer Method

The diagram of the accelerations (excluding that due to propeller thrust) acting on an airplane in flight is given below:
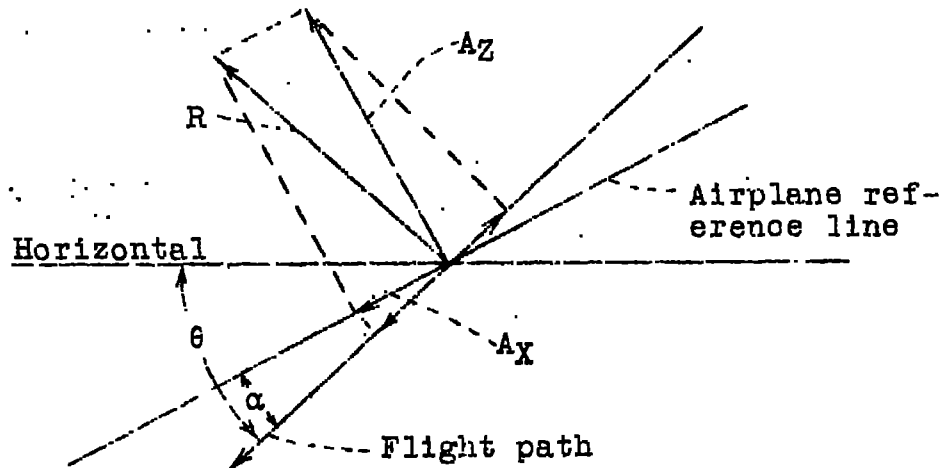
The resultant acceleration acting on the airplane is R. Its components along the airplane axes are A_X and A_Z.

The external (aerodynamic) forces which produce A_X and A_Z are equal to WA_X and WA_Z and act in the same directions as A_X and A_Z.

Positive drag is defined as an external force acting rearward along the flight path. Therefore, since the component of positive WA_X acts forward along the flight path and the component of positive WA_Z acts rearward, for positive values of α, the drag equation becomes

$$D = WA_Z \sin \alpha - WA_X \cos \alpha$$

and

$$C_D = \frac{W}{qS} \left(A_Z \sin \alpha - A_X \cos \alpha \right)$$

REFERENCES

1. Nisson, James M., Gadeberg, Burnett, L., and Hamilton
     William T.: Correlation of the Drag Characteristics
     of a P-51B Airplane Obtained from High-Speed Wind-
     Tunnel and Flight Tests. NACA ACR No. 4K02, 1944.

2. Gasich, Welko E., and Clousing, Lawrence A.: Flight
     Investigation of the Variation of Drag Coefficient
     with Mach Number for the Bell P-39N-1 Airplane.
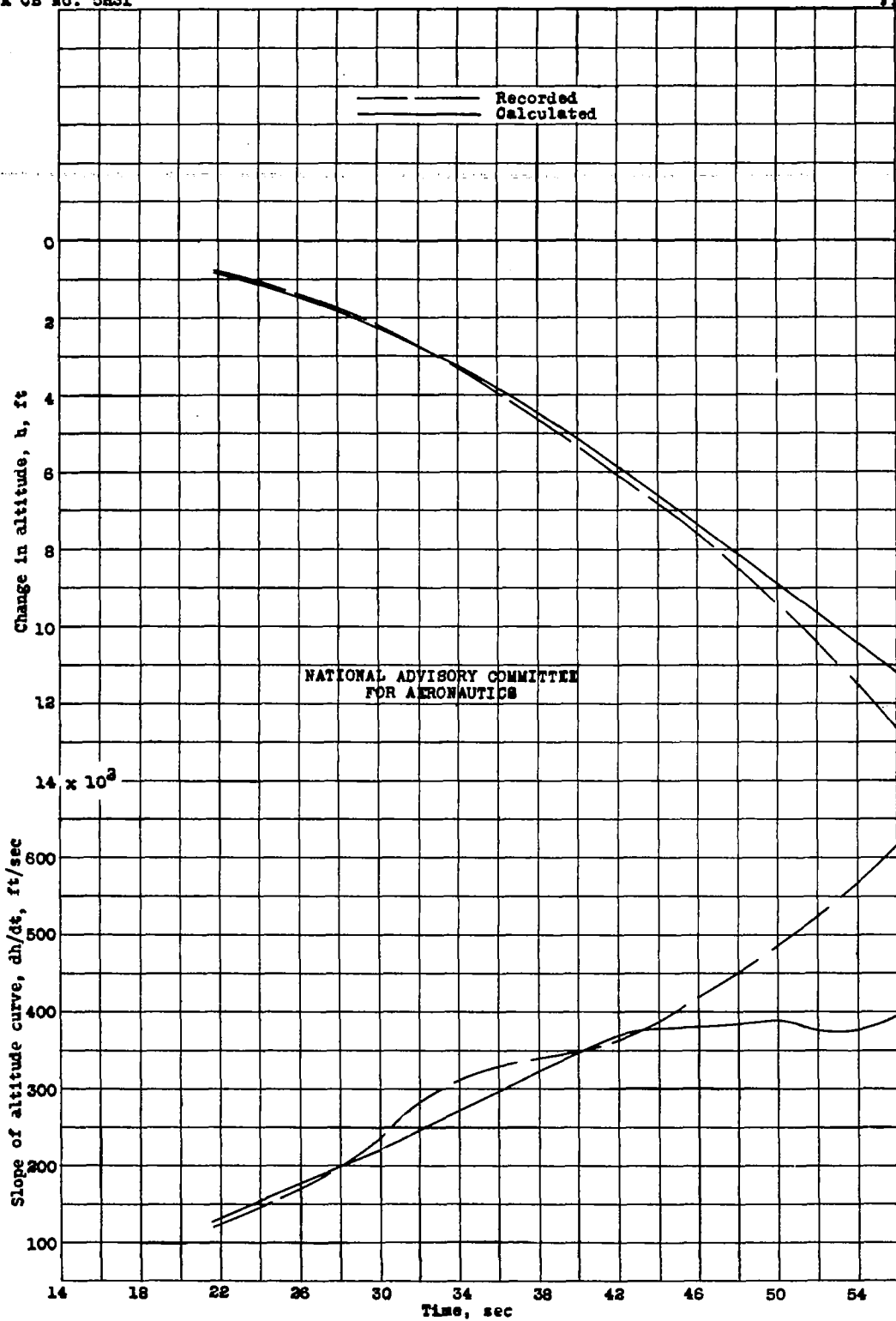   NACA ACR No, 5D04, 1945.

Figure 1.- Time history of change in altitude and rate of change in altitude showing comparison between values recorded and calculated by the energy method. North American P-51B-1 airplane with propeller removed.
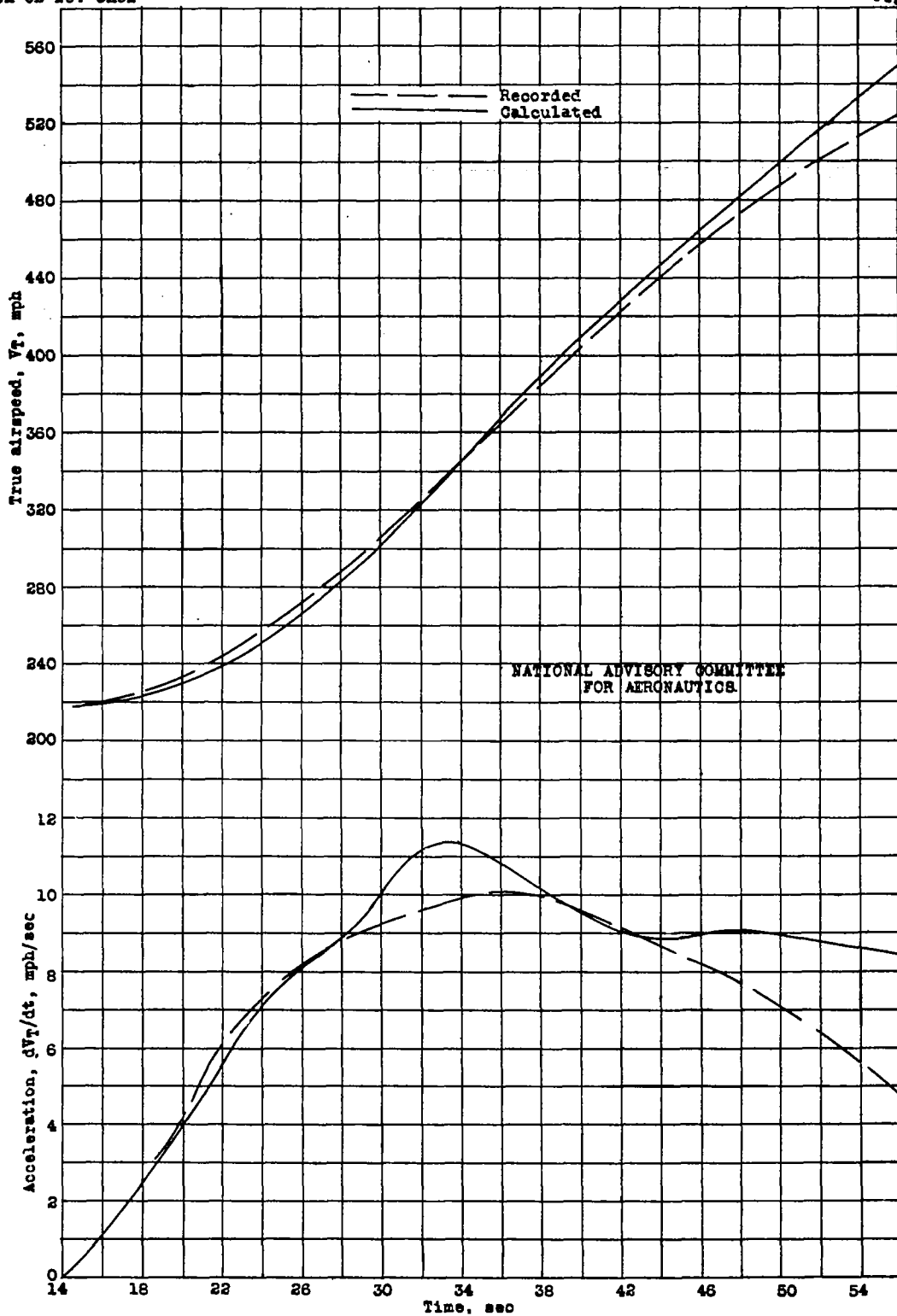
Figure 2.- Time history of airspeed and acceleration along the flight path showing comparison of values recorded and calculated by the energy method. North American P-51B-1 airplane with propeller removed.
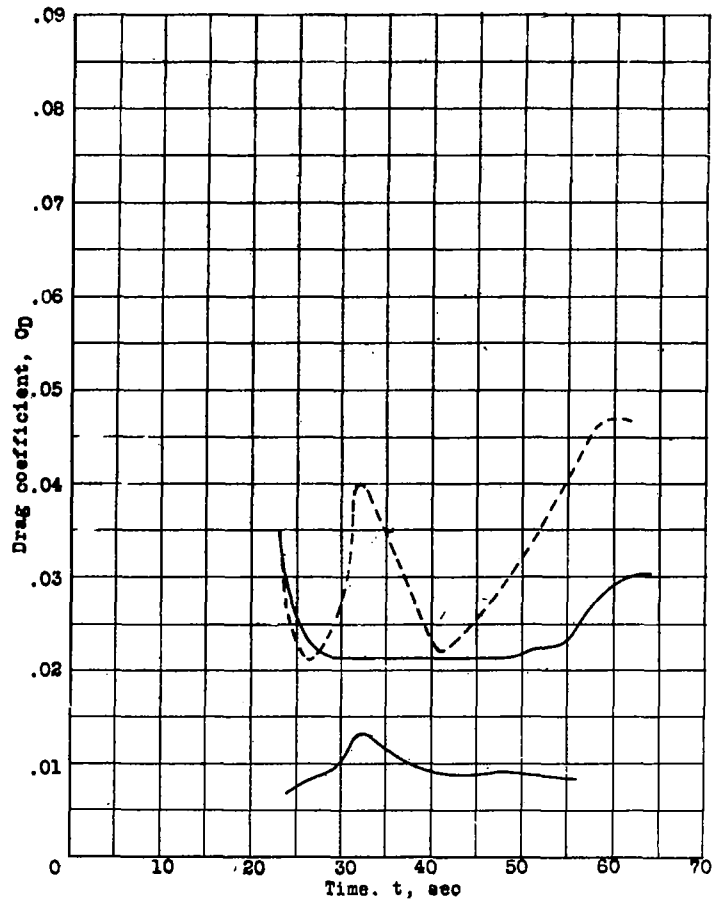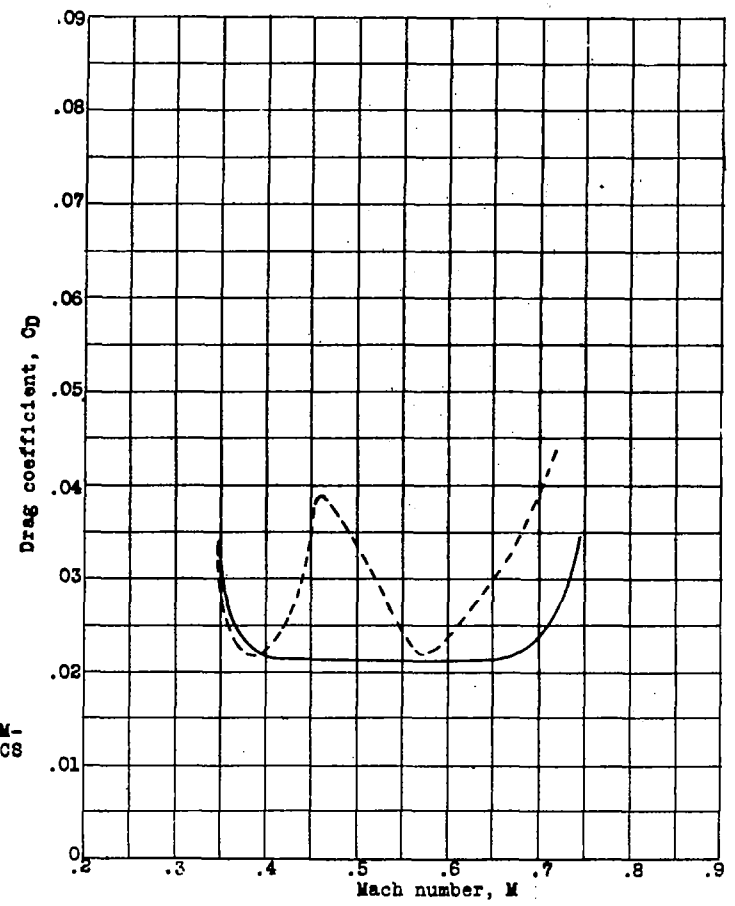
Figure 3.- Time history of drag coefficient as calculated by the accelerometer and energy methods. North American P-51B-1 airplane with propeller removed.

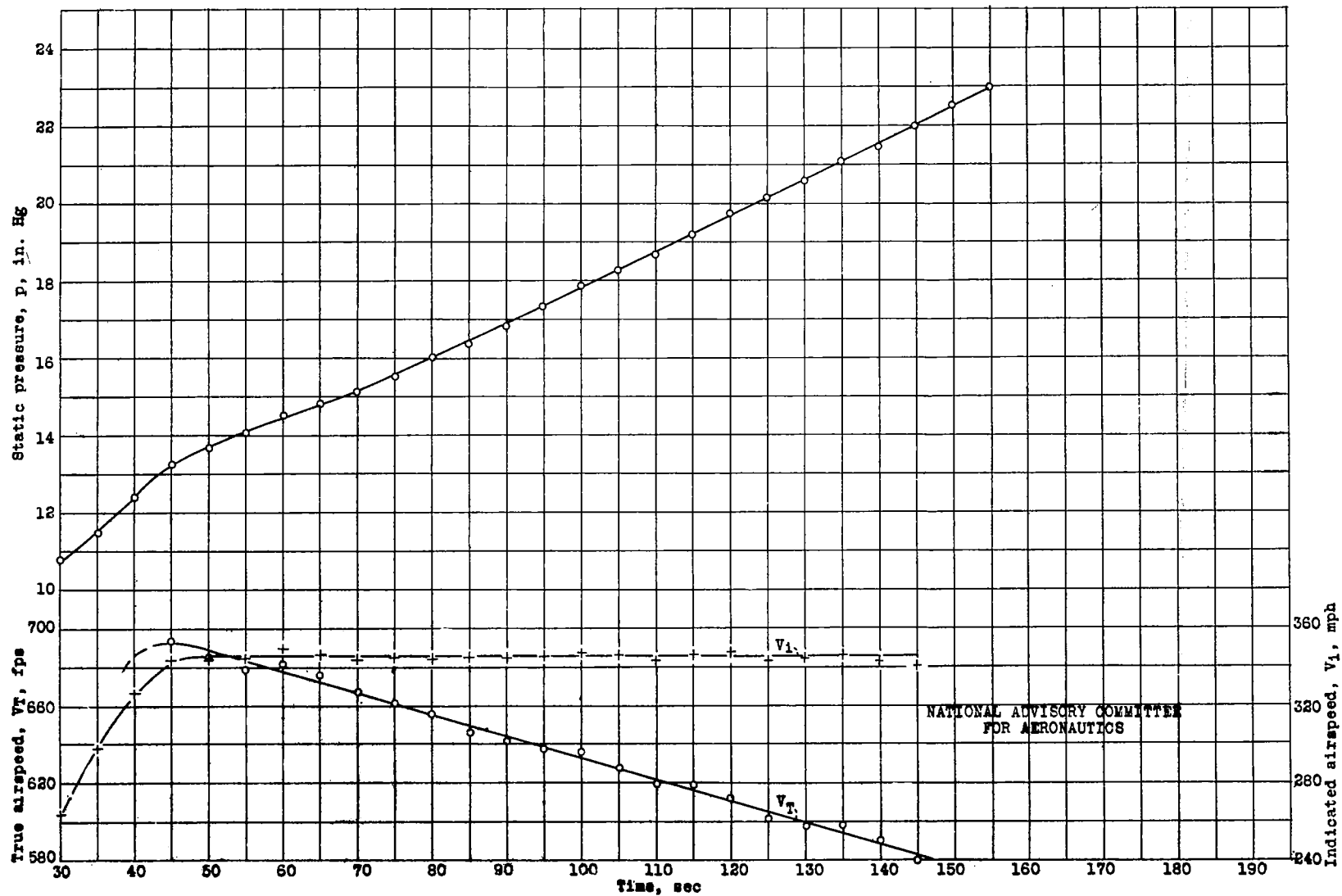Figure 4.- Variation of drag coefficient with Mach number as calculated by the accelerometer and energy methods. North American P-51B-1 airplane with propeller removed.

Figure 5.- Time history of dive No. 1 in which the pilot attempted to hold constant indicated airspeed. Bell P-39N-1 airplane with low engine power and propeller in high pitch.
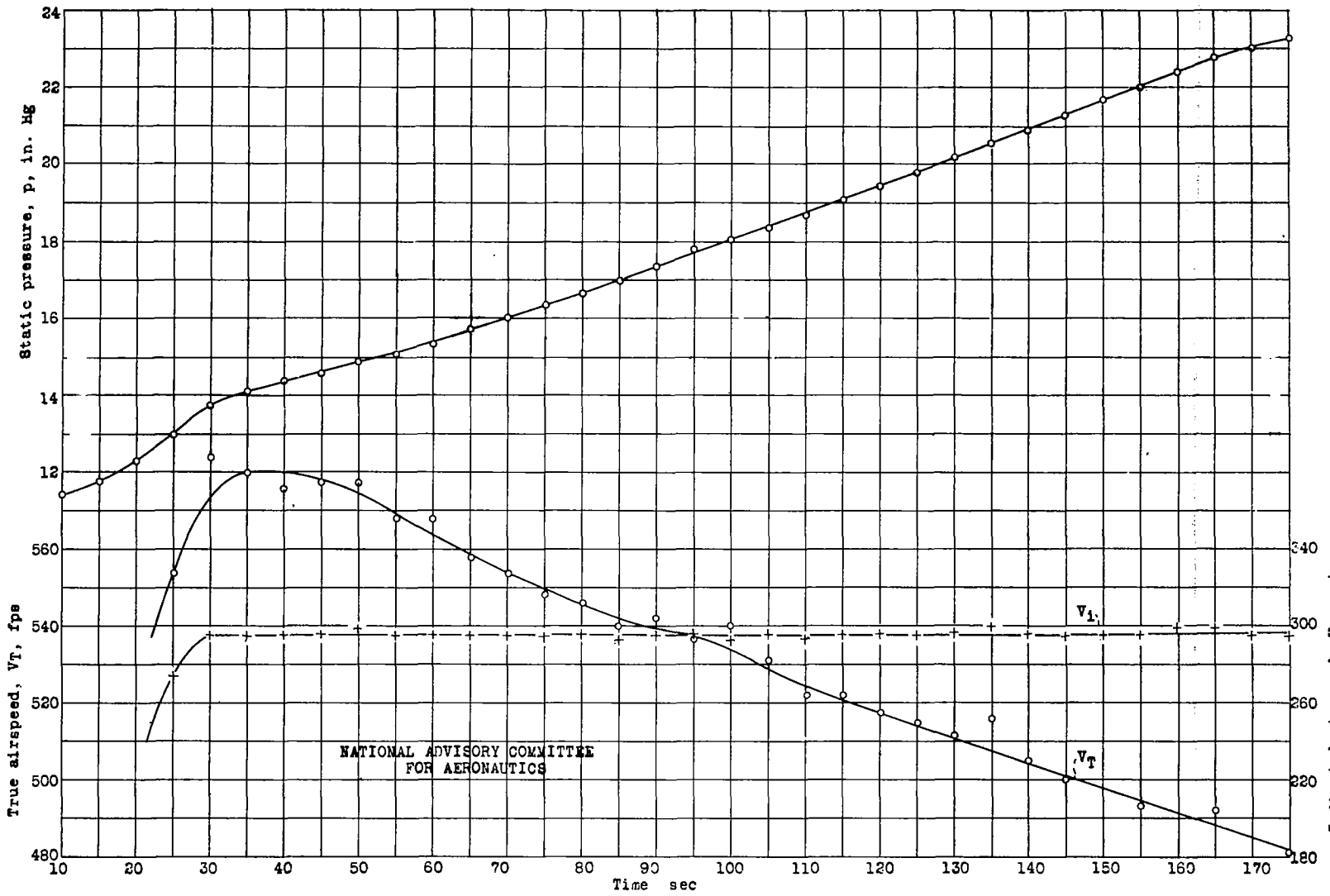
Fig. 5

Figure 6.- Time history of dive No. 2 in which the pilot attempted to hold constant indicated airspeed Bell P-39N-1 airplane, engine throttled, propeller in high pitch.
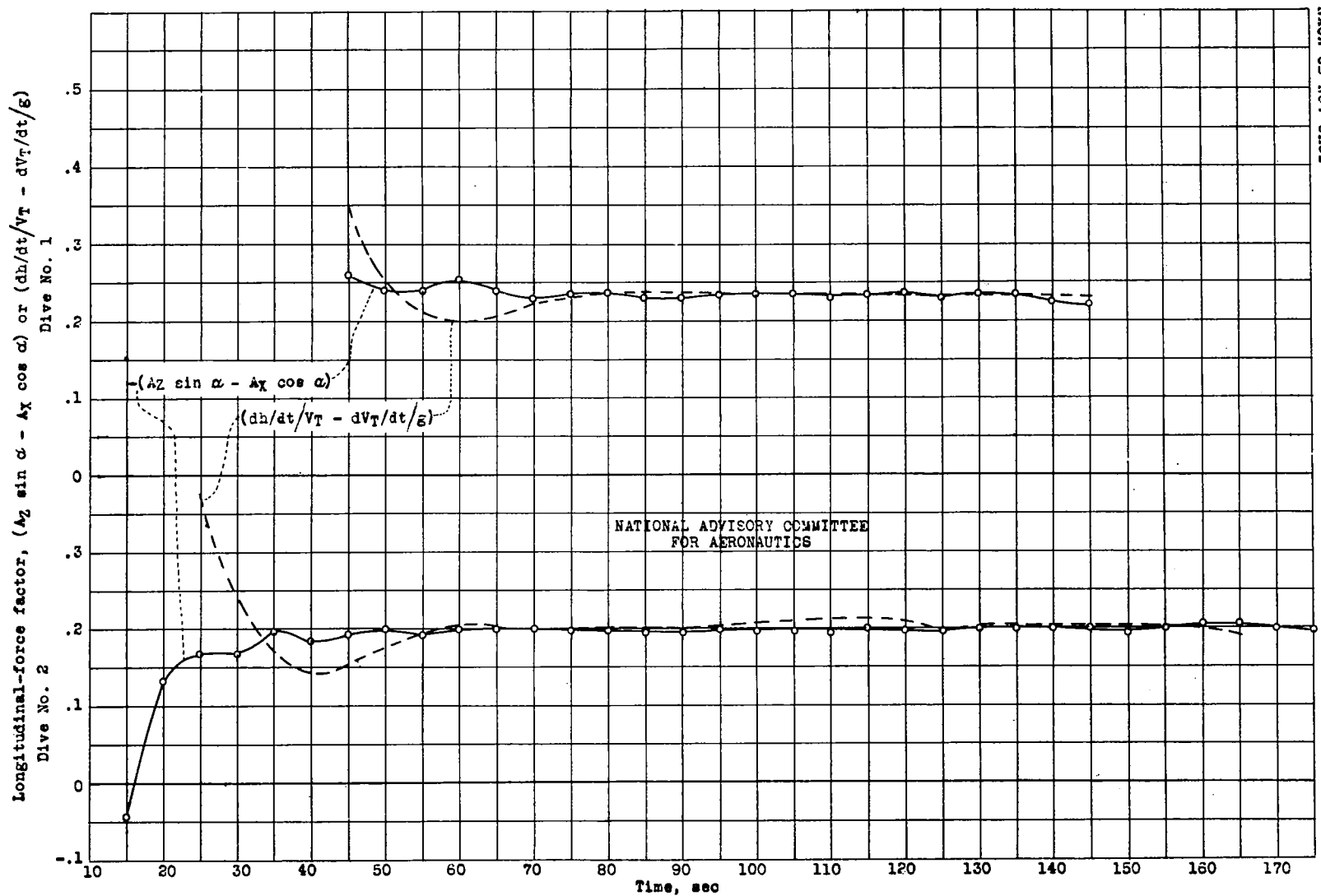
Fig. 6

Figure 7.— Variation of longitudinal force factors from accelerometer and energy methods for two dives at approximately constant indicated airspeed. Bell P-39N-1 airplane with propeller in high pitch.
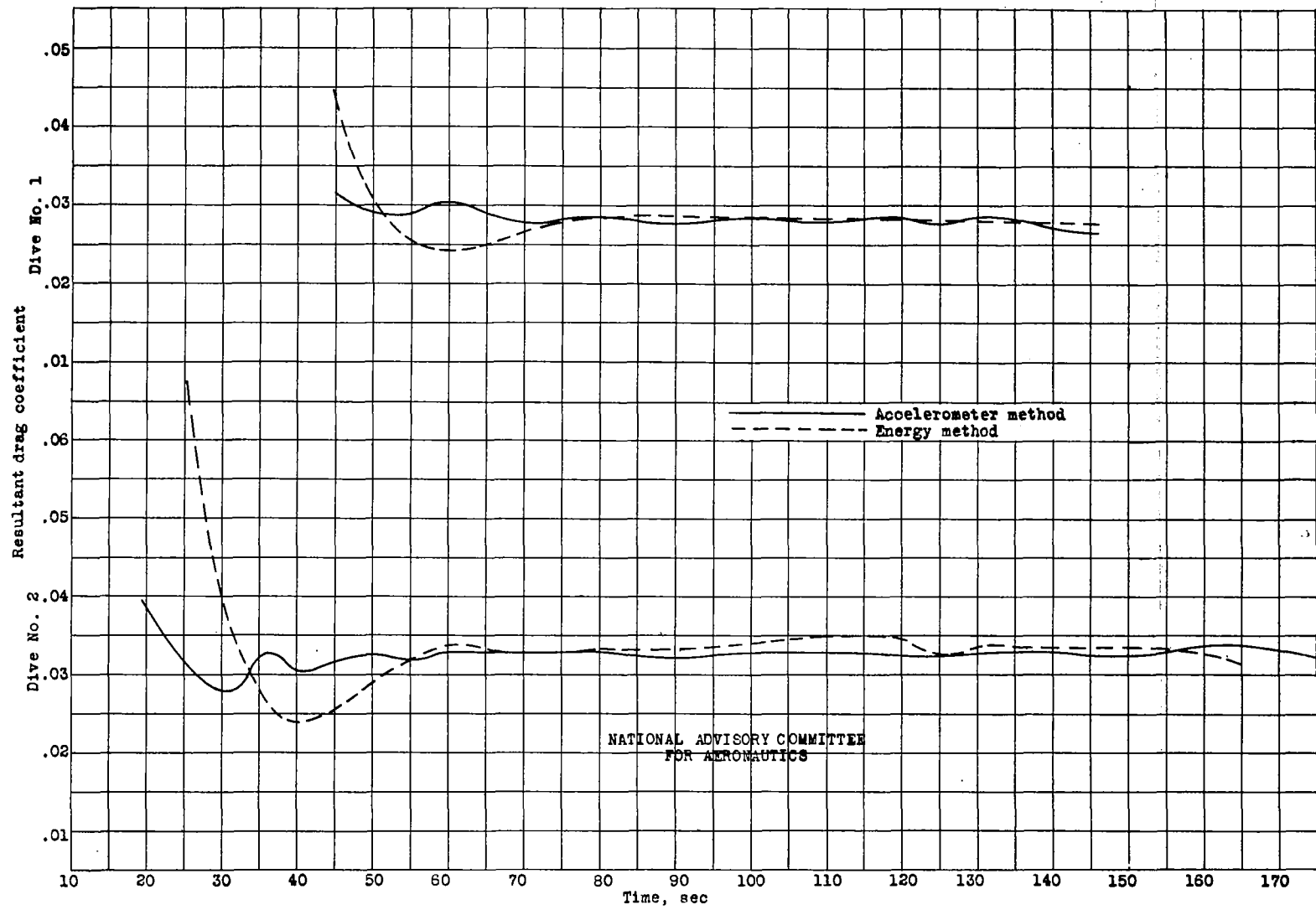
Fig. 7

Fig. 8

Figure 8.- Variation of the resultant drag coefficient calculated by the energy and accelerometer methods for two dives at approximately constant indicated airspeed. Bell P-39N-1 airplane with propeller in high pitch.