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Compact multigluonic scattering amplitudes with heavy scalars and fermions

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Abstract

Combining the Berends-Giele and on-shell recursion relations we obtain an extremely compact expression for the scattering amplitude of a complex scalar-antiscalar pair and an arbitrary number of positive helicity gluons. This is one of the basic building blocks for constructing other helicity configurations from recursion relations. We also show explicity that the all positive helicity gluons amplitude for heavy fermions is proportional to the scalar one, confirming in this way the recently advocated SUSY-like Ward identities relating both amplitudes.

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1 Motivation

To achieve a successful physics program at LHC there must be a good control over all the possible expected backgrounds. These, among other processes, require the evaluation of multipartonic scattering amplitudes at higher orders in the perturbative expansion. Without this information the identification of any signal of new physics is only partial. Despite their relevance, Yang-Mills scattering amplitudes are very poorly known, mainly because the number of Feynman diagrams increases exponentially with the number of the external fields involved in the processes. This is one of the main reason for elaborating other techniques to obtain the amplitudes. Between them one of the most successful is the use of recursion relations within the helicity amplitude formalism. The helicity amplitude formalism [1, 2, 3] has been proven to be an elegant and efficient tool to calculate multipartonic scattering amplitudes. Recursion relations extensively used in the literature at tree [4, 5] and one-loop level [6, 7] to calculate multipartonic scattering amplitudes.

Based on old insights, [8], in Ref. [9] Witten presents the idea of a weak-weak duality between supersymmetric $\mathcal{N} = 4$ Yang-Mills and topological B string theories in twistor space. Inspired by, but independent of these findings, a new method for the evaluation of scattering amplitudes in gauge theories has been proposed [10], the so called CSW. It is based on the recursive use of off-shell Maximal Helicity Violating amplitudes (MHV) [11] as basic vertices for new amplitudes. Recent works have accomplished interesting progress since the original formulation, and the method has been refined by introducing more efficient recursion relations [12, 13], the so called BCFW, and extending this approach to the one-loop level [14, 15].

Extending the BCFW formalism to massive particles, on-shell recursion relations at tree-level have been introduced in Ref. [16] for massive scalars, and in Ref. [17] for vector boson and fermions. Scattering amplitudes with heavy scalars and up to four gluons of positive helicity were first derived in Ref. [18]. In Ref. [16] all the helicity configurations with up to four gluons have been computed by using the on-shell recursion relations. These results have been extended to amplitudes with an arbitrary number of gluons of identical helicity or one gluon of opposite helicity in Ref. [19]. The approach of Forde and Kosower [19] is based on a basic ansatz for the all positive helicity amplitude which is shown to fullfil the BGKS [16] recursion relations, and which is used to construct the rest of the helicity configurations. Using off-shell recursion relations [4], multigluonic scattering amplitudes with heavy fermions and an arbitrary number of gluons of positive helicity have been calculated in Ref. [20].

We are concern in this note with two kind of multigluonic scattering amplitudes, and more in concrete with their relation: the first involves heavy fermions and are interesting by its own, due the expected rich phenomenology driven by the heavy quarks at LHC. The second of the amplitudes, with complex colored massive scalars, are of use in the unitarity method for computing massless loop amplitudes in nonsupersymmetric gauge theories [14]. In a recent paper [21] it has been demonstrated that a SUSY-like model Ward identities relate both amplitudes with heavy scalars and fermions. The apparent quite different structure of the results presented in Refs. [19] and [20] makes however quite difficult to test explicitly that relationship, apart for amplitudes with a few gluons due to their simplicity.

Is our aim to show explicitly inside QCD, that for a given helicity configuration the multigluonic massive heavy quark and the massive heavy scalar amplitudes are related by a simple overall kinematical factor. For this we construct in Sec. (2) the off-shell massive scalar amplitude. In Sec. (3) we review the equivalent fermionic amplitude and present the relation with the scalar case. Finally Sec. (4) contains our summary. Some notations and definitions issues are gathered in an Appendix.

2 Scalar amplitudes

The colour ordered off-shell current of an on-shell complex scalar of four-momentum p_1 and $(n-2)$-gluons of four-momenta p_2 to p_{n-1} and positive helicity is given in terms of the off-shell scalar current with less gluons, and the off-shell gluonic current J^μ of the rest of the gluons:

$$S(1_s;2^+,\ldots,n-1^+) = -\frac{\sqrt{2}}{y_{1,n-1}} \sum_{k=1}^{n-2} S(1_s;2^+,\ldots,k^+)\, p_{1,k}\cdot J(k+1^+,\ldots,n-1^+)\ , \tag{1}$$

where $p_{1,k} = p_1 + p_2 + \ldots + p_k$ and $S(1_s) = 1$. We also define $y_{1,k} = p_{1,k}^2 - m^2$. For all gluons of positive helicity the gluonic current has the form [4]:

$$J^\mu(i^+,\ldots,j^+) = \frac{\langle\xi|\gamma^\mu \not{p}_{i,j}|\xi\rangle}{\sqrt{2}\langle\xi i\rangle\langle\langle i,j\rangle\rangle\langle j\xi\rangle}\ , \tag{2}$$

where

$$\langle\langle i,j\rangle\rangle = \langle i(i+1)\rangle\langle(i+1)(i+2)\rangle\cdots\langle(j-1)j\rangle\ , \tag{3}$$

with $\langle\langle i,i\rangle\rangle = 1$. The null vector ξ is the reference gauge vector which is assumed to be the same for all the gluons. Then from Eq. (1), we get

$$S(1_s;2^+,\ldots,n-1^+) = -\frac{1}{y_{1,n-1}\,\langle(n-1)\xi\rangle} \sum_{k=1}^{n-2} S(1_s;2^+,\ldots,k^+)\, \frac{\langle\xi|\not{p}_{1,k}\not{p}_{k+1,n-1}|\xi\rangle}{\langle\xi(k+1)\rangle\langle\langle k+1,n-1\rangle\rangle}\ . \tag{4}$$

To obtain the recursion relation in Eq. (1) we apply the Berends-Giele rules [4] and consider the $\phi g \phi^\dagger$ vertex

$$V(p_1,k^\mu,p_2) = \frac{1}{\sqrt{2}}(p_2-p_1)^\mu\ , \tag{5}$$

where p_1, k and p_2 are the four-momenta of the scalar, the gluon and the antiscalar respectively, and the $\sqrt{2}$ comes from the normalization conventions used in colour ordered Feynman rules. Four-point vertices do not contribute to the current with all the gluons of the same helicity, since

$$J(i^+,\ldots,j^+)\cdot J(k^+,\ldots,l^+) = 0\ . \tag{6}$$

Let's anticipate our result for the scalar current with an arbitrary number of gluons:

$$\begin{aligned} S(1_s;2^+,\ldots,n-1^+) &= \frac{\langle(n-2)\xi\rangle}{\langle(n-2)(n-1)\rangle\,\langle(n-1)\xi\rangle}\, S(1_s;2^+,\ldots,n-2^+) \\ &+ \frac{i}{y_{1,n-1}}\, A_n(1_s;2^+,\ldots,n-1^+;n_s)\ , \end{aligned} \tag{7}$$

where

$$\begin{aligned} A_n(1_s;2^+,\ldots,n-1^+;n_s) = i\,\frac{m^2}{y_{12}\,y_{1,3}\,\langle\langle 2,n-1\rangle\rangle}\Bigg\{ [2|\not{p}_1\not{p}_{23}|n-1] \\ + \sum_{j=1}^{n-5} [2|\not{p}_1\not{p}_{23}|w_1]\,\frac{\langle w_1|\not{p}_{1,w_1-1}|w_2]}{-y_{1,w_1}}\cdots\frac{\langle w_j|\not{p}_{1,w_j-1}|n-1]}{-y_{1,w_j}}\Bigg\}\ , \end{aligned} \tag{8}$$

with $w_1 < w_2 < \ldots < w_j$ and $w_k \in [4, \ldots, n-2]$, is the corresponding on-shell amplitude which is obtained from the off-shell current by removing the propagator of the off-shell antiscalar, and imposing momentum conservation: $y_{1,n-1} = 0$. The well known one-, two-, and three-gluon on-shell scattering amplitudes are

$$A_3(1_s; 2^+; 3_s) = i\,\frac{\langle\xi|\not{p}_1|2]}{\langle\xi 2\rangle}\ , \tag{9}$$

$$A_4(1_s; 2^+, 3^+; 4_s) = i\,\frac{m^2[23]}{y_{12}\,\langle 23\rangle}\ , \tag{10}$$

$$A_5(1_s; 2^+, 3^+, 4^+; 5_s) = i\,\frac{m^2[2|\not{p}_1\not{p}_{23}|4]}{y_{12}\,y_{1,3}\langle\langle 2,4\rangle\rangle}\ . \tag{11}$$

To obtain these results we have performed the following transformation in the first term of Eq. (4):

$$\langle\xi|\not{p}_1\not{p}_{2,n-1}|\xi\rangle = \frac{1}{y_{12}}\left(m^2\langle\xi 2\rangle[2|\not{p}_{3,n-1}|\xi\rangle + \langle\xi|\not{p}_1|2]\langle 2|\not{p}_1\not{p}_{2,n-1}|\xi\rangle\right)\ , \tag{12}$$

together with

$$[2|\not{p}_{3n-1}|\xi\rangle = \frac{1}{y_{1,3}}\left([2|\not{p}_1\not{p}_{23}\not{p}_{4,n-1}|\xi\rangle - [32]\langle 3|y_{12} + \not{p}_{12}\not{p}_{3,n-1}|\xi\rangle\right)\ . \tag{13}$$

Because of the Schouten identity the rest of the terms can be written as

$$\begin{aligned}\langle\xi|\not{p}_{1,k}\not{p}_{k+1,n-1}|\xi\rangle &= \frac{1}{\langle(n-2)(n-1)\rangle}\Big(\langle\xi(k+1)\rangle\langle k|y_{1,k-1} + \not{p}_{1,k-1}\not{p}_{k,n-1}|\xi\rangle \\ &- \langle\xi k\rangle\langle k+1|y_{1,k} + \not{p}_{1,k}\not{p}_{k+1,n-1}|\xi\rangle\Big)\ ,\end{aligned} \tag{14}$$

with $y_{1,1} = 0$, and

$$\langle n-1|y_{1,n-2} + \not{p}_{1,n-2}\not{p}_{n-1}|\xi\rangle = -y_{1,n-1}\langle\xi(n-1)\rangle\ . \tag{15}$$

The latter generates the first term in Eq. (7) that cancels in the on-shell amplitude. Finally, we use

$$\langle k|\not{p}_{1,k-1}\not{p}_{k,n-1}|\xi\rangle = \sum_{j=k}^{n-1}\langle k|\not{p}_{1,k-1}|j]\langle j\xi\rangle\ , \tag{16}$$

to remove the gauge dependence of the on-shell amplitude.

The number of terms in Eq. (8) grows as 2^{n-5}, but contrary to the Forde-Kosower's ansatz for that amplitude [19] our expression do not contain different powers of the mass, being always proportional to m^2. This fact makes easier the validation of our expression through the on-shell BGKS recursion relations, and allows us to relate the scalar amplitude with the fermionic one in a straightforward way. As in Ref. [19] we perform a shift in the four-momenta of the (2,3) gluons:

$$\begin{aligned}\hat{p}_2^\mu &= p_2^\mu + \frac{z}{2}[2|\gamma^\mu|3\rangle\ , \\ \hat{p}_3^\mu &= p_3^\mu - \frac{z}{2}[2|\gamma^\mu|3\rangle\ .\end{aligned} \tag{17}$$

That shift corresponds to the following shift of the spinors

$$\begin{aligned}|\hat{2}\rangle &= |2\rangle + z|3\rangle\ , \quad |\hat{2}] = |2]\ , \\ |\hat{3}] &= |3] - z|2]\ , \quad |\hat{3}\rangle = |3\rangle\ .\end{aligned} \tag{18}$$

The only term that contributes to the recursion relation is the one where the scalar and the first gluon are factorized in the left side:

$$A_n(1_s^+;2^+,\ldots,n-1^+;n_s) = A_3(1_s^+;\hat{2}^+;-\hat{p}_{12s})\,\frac{i}{y_{12}}\,A_{n-1}(\hat{p}_{12s};\hat{3}^+,\ldots,n-1^+;n_s)\ . \qquad (19)$$

Thus, we have

$$A_n(1_s;2^+,\ldots,n-1^+;n_s) = i\,\frac{m^2\,[2|\not{p}_1\,\hat{\not{p}}_3\,\hat{\not{p}}_{12}\,\hat{\not{p}}_{34}}{y_{12}\,y_{1,3}\,y_{1,4}\,\langle\langle 2,n-1\rangle\rangle}\Bigg\{|n-1] + \sum_{j=1}^{n-6}|w_1]\,\frac{\langle w_1|\not{p}_{1,w_1-1}|w_2]}{-y_{1,w_1}}\cdots\frac{\langle w_j|\not{p}_{1,w_j-1}|n-1]}{-y_{1,w_j}}\Bigg\}\ , \qquad (20)$$

with the gauge choice $\xi = \hat{3}$ in the left amplitude, and $w_k \in [5,\ldots,n-2]$. For the channel under consideration $z = -y_{12}/[2|\not{p}_1|3\rangle$. Using this value for the shifted four-momenta the following relation holds after some algebra

$$[2|\not{p}_1\,\hat{\not{p}}_3\,\hat{\not{p}}_{12}\,\hat{\not{p}}_{34} = [2|\not{p}_1\not{p}_{23}\,(y_{1,4} - \not{p}_4\,\not{p}_{1,3})\ . \qquad (21)$$

Then, with the help of Eq. (21) it becomes almost trivial to demonstrate that Eq. (8) fullfils the on-shell recursion relation in Eq. (19). The first term in the rhs of Eq. (21) generates all the terms that do not contain the $1/y_{1,4}$ propagator, the second term instead initiates the spinorial chains for which $w_1 = 4$. This fact also explains why the number of terms contributing to the amplitude doubles each time that we add one extra gluon. On the other hand, it is worth to notice that we can bring the lhs of the rhs of Eq. (21) into the form

$$[2|\not{p}_1\not{p}_{23} = [2|(y_{1,3} - \not{p}_3\not{p}_{12})\ . \qquad (22)$$

This suggest that we can either extend the sum in Eq. (8) down to $w_k = 3$, or even better, we can regroup all the terms in the sum into a single one by going upwards. Our final result for the amplitude with all gluons of positive helicity becomes in this way extremely compact:

$$A_n(1_s;2^+,\ldots,n-1^+;n_s) = i\,m^2\,\frac{[2|\prod_{k=3}^{n-2}(y_{1,k} - \not{p}_k\not{p}_{1,k-1})|n-1]}{y_{12}\,y_{1,3}\cdots y_{1,n-2}\,\langle\langle 2,n-1\rangle\rangle}\ . \qquad (23)$$

3 Fermionic amplitudes

The all positive helicity gluon amplitudes with a heavy fermion-antifermion pair have been calculated in Ref. [20]. We have worked out further these expressions with the help of Eq. (14) and Eq. (16) in order to obtain a more compact formulae to compare with the heavy scalar amplitude. With our spinor choice the on-shell helicity conserving amplitude vanishes, and for the helicity flip amplitude we find in a straightforward way the following relationship to the scalar amplitude

$$A_n(1_q^+;2^+,\ldots,n-1^+;n_{\bar{q}}^+) \;=\; \frac{m}{\beta_+\langle 1n\rangle}\,A_n(1_s;2^+,\ldots,n-1^+;n_s)\ , \qquad (24)$$

with β_+ as given in the Appendix. This represents an explicit and independent confirmation of the SUSY-like Ward identities found recently in Ref. [21], that relate several multigluonic amplitudes of heavy scalars and fermions. Since we have obtained a very compact expression for the scalar amplitude, the same simple result holds for the case of heavy fermions.

4 Summary

Combining off-shell and on-shell recursion relations we have obtained an extremely compact expression for the scattering amplitude of a colored scalar-antiscalar pair and an arbitrary number of gluons of positive helicity at tree-level. We think that Eq. (23) is the most reduced expression one can obtain for such process. This result is the main input to obtain other helicity configurations from recursion relations. Due to its simplicity, we expect also that these other amplitudes can be calculated more efficiently and will be written in a more compact way than previously published. SUSY-like Ward identities might also help to extend these simple results to amplitudes with heavy fermions, or viceversa. In particular, we have tested explicity the validity of these identities relating scalar and fermionic amplitudes with an arbitrary number of positive helicity gluons. Eventhough these kind of relations are so far valid just at tree level.

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A Spinors and heavy four-momenta

We follow the conventions of Ref. [20], and denote by p_1^μ and p_n^μ, with $p_1^2 = p_n^2 = m^2$, the four-momenta of the heavy particles. In terms of two light-like vectors ($\bar{p}_1^2 = \bar{p}_n^2 = 0$) these four-momenta can be written as

$$\begin{aligned} p_1^\mu &= \beta_+ \bar{p}_1^\mu + \beta_- \bar{p}_n^\mu \, , \\ p_n^\mu &= \beta_- \bar{p}_1^\mu + \beta_+ \bar{p}_2^\mu \, , \end{aligned} \tag{25}$$

where $\beta_\pm = (1 \pm \beta)/2$ with $\beta = \sqrt{1 - 4m^2/s_{1n}}$ the velocity of the heavy particles, and $s_{1n} = (p_1 + p_n)^2$. Among other advantages, this transformation preserves momentum conservation such that $p_1 + p_n = \bar{p}_1 + \bar{p}_n$. Furthermore, in the massless limit we have: $p_1 \to \bar{p}_1$ and $p_2 \to \bar{p}_2$.

If the heavy particles are fermions, we use the following choice of spinors

$$\bar{u}_\pm(p_1, m) = \frac{\beta_+^{-1/2}}{\langle n^\mp 1^\pm \rangle} \langle n^\mp | \, (\not{p}_1 + m) \, , \qquad v_\pm(p_n, m) = \frac{\beta_+^{-1/2}}{\langle n^\mp 1^\pm \rangle} (\not{p}_n - m) \, |1^\pm\rangle \, , \tag{26}$$

where $|i^\pm\rangle = |\bar{p}_i^\pm\rangle$ are the Weyl spinors of the light-like vectors.

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