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Perspectives for the radiative return at meson factories *

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The measurement of the pion form factor and, more generally, of the cross section for electron–positron annihilation into hadrons through the radiative return has become an important task for high luminosity colliders such as the Φ- or B-meson factories. This quantity is crucial for predictions of the hadronic contributions to $(g-2)_\mu$, the anomalous magnetic moment of the muon, and to the running of the electromagnetic coupling. But the radiative return opens the possibility of many other physical applications. The physics potential of this method at high luminosity meson factories is discussed, the last upgraded version of the event generator PHOKHARA is presented, and future developments are highlighted.

1. INTRODUCTION

Electron–positron annihilation into hadrons is one of the basic reactions of particle physics, crucial for the understanding of hadronic interactions. At high energies, around the Z resonance, the measurement of the inclusive cross section and its interpretation within perturbative QCD [1,2] give rise to one of the most precise and theoretically founded determinations of the strong coupling constant α_s [3]. Also, measurements in the intermediate energy region, between 3 GeV and 11 GeV can be used to determine α_s and at the same time give rise to precise measurements of charm and bottom quark masses [4]. The low energy region is crucial for predictions of the hadronic contributions to a_μ, the anomalous magnetic moment of the muon, and to the running of the electromagnetic coupling from its value at low energy up to M_Z.

Last, but not least, the investigation of the exclusive final states at large momenta allows for tests of our theoretical understanding of form factors within the framework of perturbative QCD. Beyond the intrinsic interest in this reaction, these studies may provide important clues for the interpretation of exclusive decays of B- and D-mesons, a topic of evident importance for the extraction of CKM matrix elements.

2. HADRONIC CONTRIBUTIONS TO a_μ AND $\alpha_{\rm QED}$

The main uncertainty to a_μ and $\alpha_{\rm QED}$ is driven by their respective hadronic contributions, which are estimated though dispersion integrals

$$a_\mu^{\rm had,LO} = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^\infty \frac{ds}{s}\, K(s)\, R(s) \ , \qquad (1)$$

$$\Delta\alpha_{\tt had}(m_Z^2) = -\frac{\alpha m_Z^2}{3\pi}\ {\rm Re} \int_{4m_\pi^2}^\infty \frac{ds}{s}\, \frac{R(s)}{s-m_Z^2} \ ,$$

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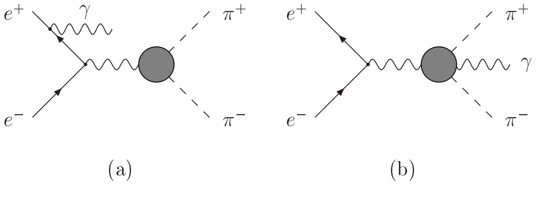
where the spectral function

$$R(s) \propto |\langle 0|J_\mu|\mathrm{had},(\gamma)\rangle|^2 \ , \qquad (2)$$

is obtained from experimental data of the reaction $e^+e^- \to$ hadrons. The most recent experimental result for a_μ [5] shows a 1.9σ discrepancy with respect to the SM prediction for this quantity [6,7,8]. Alternatively, one can also use current conservation (CVC) and isospin symmetry to obtain $R(s)$ from τ decays. In the latter, a 0.7σ discrepancy is found [6], which however is incompatible with the e^+e^- based result. Unaccounted isospin breaking corrections due to the difference of the mass and width of the neutral to the charged ρ-meson could explain this discrepancy [8], leaving the e^+e^- based analysis as the most reliable.

3. RADIATIVE RETURN AT MESON FACTORIES

The recent advent of Φ- and B-meson factories allows us to exploit the radiative return to explore the hadronic cross section in the whole energy region from threshold up to the nominal energy of the collider in one homogeneous data sample [9,10]. The radiative suppression factor $\mathcal{O}(\alpha/\pi)$ is easily compensated at these factories by their enormous luminosity.



(a) (b)

Figure 1. Leading order contributions to the reaction $e^+e^- \to \pi^+\pi^-\gamma$ from ISR (a) and FSR (b).

In principle, the reaction $e^+e^- \to \gamma +$ hadrons receives contributions from both initial- and final-state radiation (Fig. 1), ISR and FSR respectively. Only the former is of interest for the radiative return. A variety of methods to disentangle FSR from the ISR contribution have been described in detail in [9,11,12,13,14]. The first possibility is based on the fact that FSR is dominated by photons collinear to π^+ or π^-, ISR by photons collinear to the beam direction. This suggests that one should consider events with photons well separated from the charged pions and preferentially close to the beam. The second option is based on the markedly different angular distributions of ISR and FSR, and the characteristic feature of their interference. Various charge-asymmetric distributions can be used for independent tests of the FSR model amplitude, with the forward–backward asymmetry as simplest example. Note however that at B-factories the $\pi^+\pi^-\gamma$ final state is completely dominated by ISR.

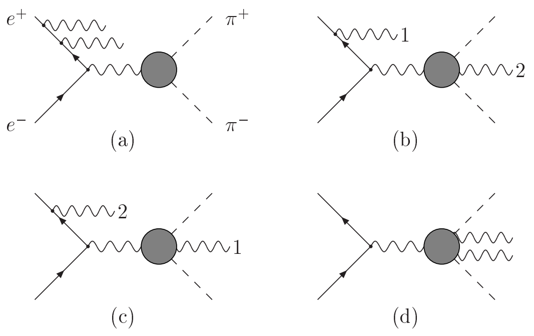


Figure 2. Typical amplitudes contributing to the reaction $e^+e^- \to \pi^+\pi^-\gamma\gamma$. For two photons emitted either from the electron/positron or the hadronic system. Only one representative is displayed.

Higher order radiative corrections (see Figs. 2 and 3) are also important for the precise extraction of the hadronic cross section through the radiative return. Furthermore, simulation of individual exclusive channels: e.g. $\pi^+\pi^-$, K^+K^-, $p\bar{p}$, $\pi^0\pi^+\pi^-$, 4π , 5π, 6π, $\pi\pi\eta$, $K\bar{K}\pi$, $K\bar{K}\pi\pi$, $2K2\bar{K}$, $K\bar{K}\eta$, in the low energy region (< 2-3 GeV) requires a fairly detailed parametrization

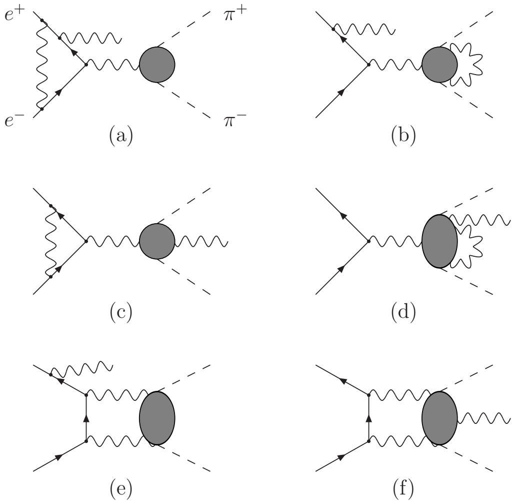


Figure 3. Typical amplitudes describing virtual corrections to the reaction $e^+e^- \to \pi^+\pi^-\gamma$. Permutations are omitted.

of various form factors.

4. MONTE CARLO EVENT GENERATORS

The proper analysis requires necessarily the construction of Monte Carlo event generators. The event generators EVA [9] and EVA4π [15] were based on a leading order treatment of ISR and FSR, supplemented by an approximate inclusion of additional collinear radiation based on structure functions. Subsequently, the event generator PHOKHARA was developed [11,12,13]; it is based on a complete next-to-leading order (NLO) treatment of radiative corrections [16,17]. In its version 2.0 it included ISR at NLO and FSR at LO for $\pi^+\pi^-$ and $\mu^+\mu^-$ final states, and four-pion final states (without FSR) with some improvements with respect to the formulation described in [15].

The most recent version of PHOKHARA, version 3.0 [13], allows for the simultaneous emission of one photon from the initial and one photon from the final state. This includes in particular the radiative return to $\pi^+\pi^-(\gamma)$ and thus the measurement of the (one-photon) inclusive $\pi^+\pi^-$ cross section, an issue closely connected to the question of $\pi^+\pi^-(\gamma)$ contributions to a_μ.

Recently, a new Monte Carlo event generator, EKHARA [18], has been constructed to simulate the reaction $e^+e^- \to \pi^+\pi^-e^+e^-$, a potential background of the radiative return in particular at lower energies.

Experimental studies presented in [19,20,21,22, 23] indeed demonstrate the power of the method and seem to indicate that a precision of one per cent or better is within reach. In view of this progress a further improvement of theoretical understanding and Monte Carlo generators is required.

5. PHOTONIC CORRECTIONS TO a_μ

The issue of photon radiation from the final states is closely connected to the question of photonic contributions to a_μ, see Fig. 4 (for related discussions, see e.g. [24,25]). At the present level of precision of hadronic contributions to a_μ, roughly half to one per cent, these corrections start to become relevant.

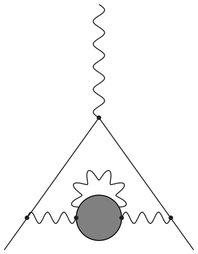


Figure 4. Photonic corrections to a_μ.

Qualitatively the order of magnitude of this effect can be estimated either by using the quark model with $m_u \approx m_d \approx m_s \approx 180$ MeV (adopted to describe the lowest order contribution [26]), or with $m_u \approx m_d \approx m_s \approx 66$ MeV (adopted to describe the lowest order contribution to $\alpha(M_Z)$) or by using $\pi^+\pi^-$ as dominant intermediate hadronic state plus photons coupled according to sQED. The three estimates (see also [24])

$$\begin{aligned} \delta a_\mu(\text{quark}, \gamma, m_q = 180\ \text{MeV}) &= \ 1.880 \times 10^{-10}\ , \\ \delta a_\mu(\text{quark}, \gamma, m_q = 66\ \text{MeV}) &= \ 8.577 \times 10^{-10}\ , \\ \delta a_\mu(\pi^+\pi^-, \gamma) &= \ 4.309 \times 10^{-10}\ , \end{aligned}$$

are comparable in magnitude and begin to be relevant at the present level of precision of $\pm 8{\cdot}10^{-10}$. This order-of-magnitude estimate suggests that a more careful analysis is desirable.

Soft photon emission is clearly described by the point-like pion model. Hard photon emission, with $E_\gamma \geq \mathcal{O}(100\ \text{MeV})$, however, might be sensitive to unknown hadronic physics. Therefore the size of virtual, soft and hard corrections has to be studied separately. Let us now estimate the contributions from hard photon radiation to a_μ. In Fig. 5 we display the integrand

$$\frac{d}{ds} a_\mu^{\text{had},\gamma}(E^{\text{cut}}) = \frac{\alpha^2}{3\pi^2 s} K(s)\ R^{\text{H}}(s, E^{\text{cut}})\ , \quad (3)$$

for $E^{\text{cut}} = 10$, 100 and 200 MeV as a function of $\sqrt{s} = m(\pi^+\pi^-\gamma)$ between the threshold and 2 GeV. The result is compared with the complete sQED contribution, as derived from point-like pions, and the lowest order contribution from $\pi^+\pi^-$. The integrated result, with E_γ between E^{cut} and infinity, is displayed in Fig. 6. Contributions from the hard region, above 100 MeV, are clearly small with respect to the present experimental and theory-induced uncertainty.

A measurement of $\gamma^* \to \pi^+\pi^-\gamma$ for variable $\sqrt{s}$ is desirable for an independent cross check. This is indeed possible with the radiative return, if events with two photons in the final state are investigated [13]. This was in fact one of the motivations for extending the event generator PHOKHARA to events with simultaneous ISR and FSR.

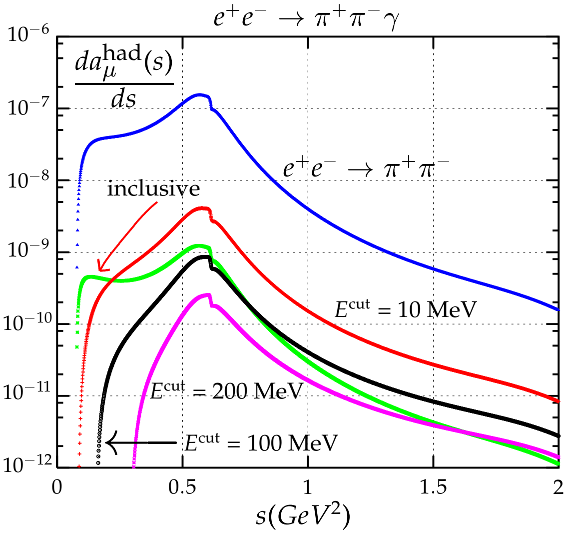


Figure 5. Differential contribution to $a_\mu^{\text{had},\gamma}$ from $\pi^+\pi^-\gamma$ intermediate states for different cutoff values compared with the complete contribution (virtual plus real corrections, labeled ‘inclusive’) evaluated in sQED (FSR), as well as with the contribution from the $\pi^+\pi^-$ intermediate state.

6. PERSPECTIVES FOR THE RADIATIVE RETURN

Let us now describe a number of topics for the development of PHOKHARA [11,12,13], which are of course intimately connected to the future potential of measurements through the radiative return. To large extent this is still a wish list, which will at best be partially fulfilled, depending also on the future development in the experimental analysis. Some of these topics are based on fairly straightforward extensions of the present framework, some would require major calculations. Inclusion of additional hadronic final states is only possible on the basis of parametrizations (not yet available) for the production amplitudes of multi-hadronic states. The radiative return may, finally, give access to completely new phenomena. For example one may scrutinize the $D^0\bar{D}^0$-system, produced in the radiative return to the ψ'', for time-dependent mixing effects, which are not easily accessible in other experiments.

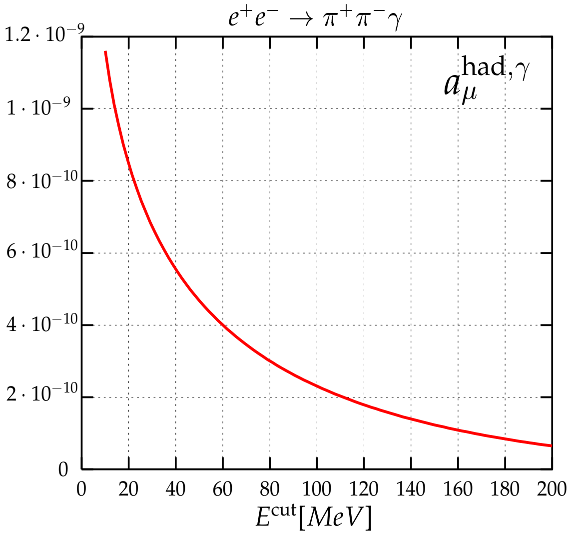


Figure 6. Integrated contribution to $a_\mu^{\mathrm{had},\gamma}$ as a function of the cutoff E^{cut}.

6.1. THE RADIATIVE RETURN TO MUON PAIRS

The present version of PHOKHARA (3.0) includes the amplitudes for

$$e^+e^- \to \mu^+\mu^-\gamma \ , \qquad (4)$$

in Born approximation plus those $\mathcal{O}(\alpha)$ real and virtual corrections which are attached to the electron side [11,12,13]. Important additional corrections are expected from the second order process, where one hard photon is emitted from the electron, another photon is collinear with the muon. These additional terms are presently being included in the simulation [27]. Resonances, narrow ones like J/ψ, wide ones like the ρ-meson and the vacuum polarization from virtual hadronic and leptonic intermediate states affects the μ-pair cross section. In particular the effects of the vacuum polarization should and will be included in a detailed comparison between theory and experiment.

6.2. BENCHMARK TESTS AND COMPARISONS

To arrive at reliable Monte Carlo programs suited for the experimental analysis comparisons between the results of simulations, semianalytical results and comparisons between programs of different authors are of paramount importance. Inclusive semianalytical results of ISR NLO corrections [28,29] are in agreement with PHOKHARA at a per mill level or better [12]. Preliminary studies [30] indicate agreement between PHOKHARA 3.0 [13] and KKMC [31] for the $\mu^+\mu^-\gamma$ mode at the level of two per mill in the region of interest, at least in the mode where FSR had been switched off. The agreement of partial results [32] between PHOKHARA and BABAYAGA [33] event generators in $\pi^+\pi^-\gamma$ modes is encouraging. Semi inclusive analytical results with specific idealized cuts have been published in [34]. However, no comparison with already available numerical results of [11] or with the Monte Carlo program PHOKHARA has been presented. Clearly, all that can only be the starting point for more detailed studies.

6.3. HADRONIC FINAL STATES

i) Two mesons: e.g K^+K^- and $K^0\bar{K^0}$

At present only two-pion and four-pion final states are implemented, and the parametrizations of the form factors are optimized for Q^2 below 3 to 4 GeV^2. Larger values of Q^2, up to $\mathcal{O}(10\ \mathrm{GeV}^2)$ are at reach at B-meson factories. Although the implementation of amplitudes and form factors for arbitrary two meson final states is straightforward once the generator has been coded for $\pi^+\pi^-$, the remaining two-body final states K^+K^- and $K^0\bar{K^0}$ are still missing, as well as a reliable and theoretically well founded formulation for the pion form factor which is applicable both in the ρ and ρ' resonance region and for large Q^2 at the same time.

ii) Three meson: e.g. 3π, $\pi\pi\eta$ and $KK\pi$

Strangeness-, isospin- and CP-conservation alone lead to significant restrictions on the structure of production amplitudes for three meson final states. CP- and G-parity restricts the three-pion state (and all states with an odd number of pions) to the isospin zero configuration, dominated by the ω-resonance and its radial excitations. (The ϕ as well as the J/ψ contributions are suppressed as a consequence of Zweig's rule, but not strictly forbidden.) The $\pi\pi\eta$ (and $\pi\pi\eta'$) mode is restricted to the isospin one channel with a production amplitude modeled according to the process

$$\gamma^* \to \rho \to \eta\rho\ (\to \pi\pi)\ . \tag{5}$$

Here ρ stands for the full set of ρ-meson-like excitations and the amplitude is written as

$$\langle \pi\pi\eta | J_\mu |, 0\rangle \propto F_\rho\left(Q^2\right) F_\rho\left((p_+ + p_-)^2\right) \times$$
$$\epsilon_{\mu\nu\alpha\beta} Q^\alpha (p_+^\beta + p_-^\beta)(p_+^\nu - p_-^\nu)\ . \tag{6}$$

The $KK\pi$ mode may receive contributions from isospin zero and isospin one with a rich structure of ρ- and K^*- resonances in the $K\bar{K}$ and $K\pi$ subchannels respectively. The parametrization of amplitudes for final states with higher multiplicities, involving K, π, and η will quickly lead to enormously complicated resonance structures and only data-driven analysis can lead to reliable parametrization of the corresponding amplitudes.

iii) Baryons:
The situation is simple for the measurement of the proton (and neutron) electric and magnetic form factors,

$$\langle p\bar{p} | J_\mu |, 0\rangle = \gamma_\mu F_1 + \sigma_{\mu\nu}\left(p_+^\nu + p_-^\nu\right) \frac{F_2}{4m}\ ,$$
$$G_M = F_1 + F_2\ , ,$$
$$G_E = F_1 + \tau F_2\ , \tag{7}$$

with $\tau \equiv Q^2/4m_B^2 > 1$. Sizeable event rates can be reached out to fairly high Q^2 (see Fig.7) and the angular distributions allow for a clean separation of the two form factors [35].

iv) Multi-particle continuum:
Up to now we have restricted the discussion to exclusive final states. The situation may become even more complicated for the multi-particle continuum at larger Q^2, say above $\mathcal{O}(10\ \mathrm{GeV}^2)$. In the parton model this reaction would be modeled by $e^+e^- \to \gamma q\bar{q}$. Hadronisation of the $q\bar{q}$ state could be simulated in a phenomenological way. However, final state radiation will be an important background, and elaborate methods, based e.g. on the markedly different angular distributions of ISR versus FSR will have to be developed to analyze and separate the two contributions [see e.g. [13] Eq.(3)]. In principle this method could also be employed to measure the R-ratio above charm threshold. Making use of the large boost of the D-mesons and their displaced decay vertices, the separation of charm and non-charm contributions might be feasible, thus providing an important input for the determination of m_c through sum rules [36]. From the purely technical viewpoint PHOKHARA can already now be used for the simulation of this reaction, by simply replacing muon mass and charge by the corresponding values for the charm quarks (Note, that a sizeable forward-backward asymmetry is predicted from ISR-FSR interference also in this case.).

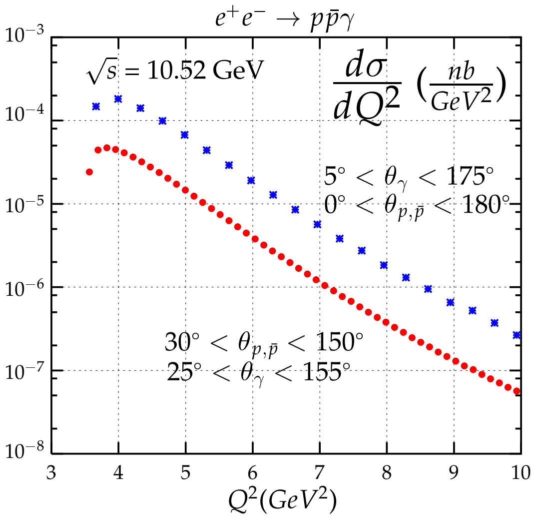


Figure 7. Differential in Q^2 cross section of the process $e^+e^- \to p\bar{p}\gamma$ for two different sets of cuts.

6.4. NARROW RESONANCES

The production of narrow bottonium states through the radiative return has been discussed in some detail in [37] (see also [38,39]). Here we shall concentrate on the J/ψ-resonance and its radial excitations. Neglecting for the moment radiative corrections and interference between the resonance and the continuum amplitude, the cross section is given by

$$\begin{aligned}\sigma_R &= \frac{\Gamma_e}{M}\frac{12\pi\alpha}{s}\left\{\frac{1+(M^2/s)^2}{1-(M^2/s)^2}\log\left(\frac{1+\cos\theta_m}{1-\cos\theta_m}\right)\right.\\ &- \left.\left(1-\frac{M^2}{s}\right)\cos\theta_m\right\} . \qquad (8)\end{aligned}$$

The angular region of the photon is restricted by $|\cos\theta| \leq \cos\theta_m$ and Γ_e denotes the resonance partial decay rate into e^+e^-. For an integrated luminosity of 100fb^{-1} this corresponds to $34{\cdot}10^4$ J/ψ events, out of which 20000 events will be found in the $\mu^+\mu^-$ channel. It is remarkable, that the results form the BABAR-collaboration for this reaction (see [21,22]) have lead to a value for $\Gamma_e\Gamma_\mu/\Gamma_{tot}$ which already now is more precise than the world average as compiled in [40]. The enormous statistics gives access to a multitude of final states. In particular it is possible to measure simultaneously the cross section on- and off-resonance. For the double ratio

$$RD_f \equiv \frac{\sigma_f}{\sigma_{\mu+\mu-}}(\text{on res.})/\frac{\sigma_f}{\sigma_{\mu+\mu-}}(\text{off res.}) \ , \quad (9)$$

one evidently expects $\text{RD}_f = 1$, if the decay $J/\psi \to f$ can proceed through the virtual photon only. This is expected for final states with isospin 1, like 2π, 4π, 6π or $\pi\pi\eta$. Final states with isospin zero, like 3π, 5π, $K\bar{K}$, $K\bar{K}\pi$ are dominated by hadronic J/ψ decays and, in the framework of perturbative QCD mediated by the three gluon intermediate state.

6.5. RADIATIVE PRODUCTION OF $\psi(3770)$

A wealth of physics results has bee obtained from $B\bar{B}$ production at asymmetric B factories. The $B\bar{B}$ state is produced in a 1^{--} configuration, which leads to nontrivial correlations between the decay products of B and $\bar{B}$ mesons. The Lorentz boost allows to resolve the correlated time dependence of the two decays, which in turn is the basis for the measurement of the CP asymmetry. Similar studies can, in principle be performed for the $D\bar{D}$ system with the help of the radiative production of $\psi(3770)$ and its subsequent decay into $D\bar{D}$. From Eq.(8), using $\Gamma_e = 0.26$ keV and assuming an integrated luminosity of 100 fb^{-1} one expects a total sample of 15000 $\psi(3770)$ events. Assuming an integrated luminosity of order 10 ab^{-1}, as suggested for future upgrades of B factories, more than a million of $\psi(3770)$ could be produced. Let us discuss a number of potentially interesting studies. The relative amount of charged versus neutral final states in the decays of $\phi \to K\bar{K}$, $\psi(3770) \to D\bar{D}$ and $\Upsilon(4S) \to B\bar{B}$ and its energy dependence around the resonance is sensitive to phase space effects, Coulomb attraction and hadronic final state interactions. The measurement of $\psi(3770)$ in the radiative return and its separation into charged and neutral modes automatically involves the full energy range.

Let us now speculate about the potential of such a measurement, once millions of $\psi(3770)$ are available [41]. Tagging the flavour of one D meson through the leptonic decay of the other meson, all those studies are feasible, which can also be investigated in the decay of neutral D mesons, which are tagged through the charge of the pion in the $D^* \to \pi D$ decay. Mixing would manifest itself through like sign dilepton events from $D\bar{D}$ decays, and the time dependence of this phenomenom could be studied in the present setup. Another option would be the study of the time dependence of doubly Cabbibo suppressed decays. A different subject, specific to $\psi(3770)$ decay, is the investigation of CP eigenstates in the decay of both D^0 and $\bar{D}^0$ and the corresponding time evolution. Assuming CP-conservation the combination $D^0(\to \pi^+\pi^-)\bar{D}^0(\to \pi^+\pi^-)$ is for example strictly forbidden, independent of the question of $D\bar{D}$ mixing. In contrast to $\psi(3770)$ production at a symmetric charm factory it would be possible to study not only the rate for such a process but also its time dependence, which will carry additional information on direct and indirect CP-violation. Last not least one may use lepton tags on one side and study the time dependence of decays into CP eigenstates at the other side, thus exploring combinations quite similar to those investigated in the $B\bar{B}$ system at asymmetric B-meson factories. Predictions for $D\bar{D}$-mixing and CP violation within the Standard Model [42] lead to fairly small effects which would be difficult to observe. Nevertheless these measurements could lead to important limits and perhaps give access to physics beyond the Standard Model.

7. CONCLUSIONS

Measurements of the pion form factor through the radiative return offer the unique possibility for improved predictions for the muon magnetic moment and the electromagnetic coupling $\alpha(M_Z)$ but also gives access to many new phenomena. The Monte Carlo event generator PHOKHARA is being developed further to include new features.

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