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## The Role of the Dilaton in Dense Skyrmion Matter

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### Abstract

In this note, we report on a remarkable and surprising interplay between the  $\omega$  meson and the dilaton  $\chi$  in the structure of a single skyrmion as well as in the phase structure of dense skyrmion matter which may have a potentially important consequence on the properties of compact stars. In our continuing effort to understand hadronic matter at high density, we have developed a unified field theoretic formalism for dense skyrmion matter using a single Lagrangian to describe simultaneously both matter and meson fluctuations and studied *in-medium* properties of hadrons. The effective theory used is the Skyrme model Lagrangian gauged with the vector mesons  $\rho$  and  $\omega$ , implemented with the dilaton field that describes the spontaneously broken scale symmetry of QCD, in a form consistent with the symmetries of QCD and our expectations regarding the high density limit. We analyze the restoration of scale invariance and chiral symmetry as the density of the system increases. In order to preserve the restoration of scale symmetry and chiral symmetry, signalled in our case by the vanishing of the expectation value of the dilaton, *and* to be consistent with the “vector manifestation” of hidden local symmetry, a density dependent  $\omega$  coupling is introduced. We uncover the crucial role played by both the dilaton and the  $\omega$  meson in the phase structure of dense medium and discover how two different phase transition regimes arise as we “dial” the dilaton mass.

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# 1 Introduction

Hadronic matter at high density is presently poorly understood, and the issue of the equation of state (EOS) in the density regime appropriate for the interior of compact stars remains a wide open problem. In the absence of model-independent theoretical tools such as lattice and of experimental guidance for dense hadronic matter, it is difficult to assess the reliability – perhaps even relevance – of the plethora of scenarios predicted by a variety of models for compact stars available in the literature. It is in this context that the skyrmion approach to dense matter anchored on large  $N_c$  QCD, with density effects simulated on crystal lattice, was put forward in a series of papers [1, 2, 3] as a potentially promising and consistent approach.

Motivated by a recent development on hidden local symmetry (HLS) approach to low-energy effective field theory for hadrons [4] which gave – at least in the chiral limit – an elegant and unambiguous prediction of the behavior of light-quark hadrons at high temperature and/or at high density <sup>1</sup>, both the dilaton field  $\chi$  and the vector meson fields  $\rho$  and  $\omega$  were incorporated into the Skyrme Lagrangian in [3] to construct dense skyrmion matter on crystal lattice. In HLS theory of [4], combining local gauge invariance of the flavor gauge fields  $\rho$  and  $\omega$  and matching of the low-energy effective theory to QCD at a suitable matching scale uncovered a unique fixed point – called “vector manifestation (VM)” fixed point – to which dense matter flows as the density increases toward the critical chiral phase transition point. The unambiguous prediction was that the hidden gauge coupling constant  $g$  and the physical pion decay constant  $f_\pi$  would vanish at that point, with the consequence that the vector meson mass would vanish at the chiral transition. This meant that the vector mesons would be essential degrees of freedom in many-nucleon dynamics under extreme conditions. Now the Lagrangian used in [3] is a gauge fixed one – at unitary gauge – and hence lacks the intrinsic dependence that represents matching to QCD present in hidden local gauge invariant theory. What the scalar dilaton does is, as first described in [5], to simulate, albeit approximately, in the gauged Skyrme Lagrangian the intrinsic dependence that figures importantly in HLS theory.

What was found in [3] was disturbing and, at the same time, highly interesting. In the absence of the vector mesons, the presence of the dilaton provides a reasonable scenario for chiral restoration [2]. However in the presence of the vector mesons, in particular, the  $\omega$  meson, while chiral symmetry is restored at some density with the order parameter  $\langle\sigma\rangle \sim \langle\bar{q}q\rangle \rightarrow 0$  – which is required by symmetry on crystal lattice [6], the pion decay constant  $f_\pi$  which is tagged to the expectation value of the  $\chi$  field  $\langle\chi\rangle^*$  (where the asterisk stands for medium) does not vanish at the point where the condensate is zero. This aspect is related to the fact that  $\omega$  mass cannot vanish in the model – in fact, it increases at higher density – which is at odds with the vector manifestation of HLS theory. This means a strong repulsion developing as density increases, an aspect which is qualitatively important for nuclear physics.

The objective of this paper is to precisely identify the problem involved and suggest possible resolutions to the problem.

The content of this paper is as follows. In Section 2, we pinpoint what the problem is. In Section 3 is given our proposed solution to the problem. In Section 4, the properties of a single skyrmion are studied. We pay special attention to the role of the dilaton field and the spatial structure of the realization of scale and chiral symmetry within the skyrmion. Section 5 is devoted to the study of the phase transitions in dense skyrmion matter and the fundamental role of the dilaton field in their realizations. Some concluding remarks are given in Section 6.

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<sup>1</sup>In this note, we will be mainly considering density effects. However many of the arguments used for density hold also for temperature.

## 2 The Problem

In order to clarify the problem involved, we first discuss the Lagrangian used in [3]:

$$\mathcal{L} = \mathbf{L}_n + \mathbf{L}_{an} \quad (1)$$

where

$$\begin{aligned} \mathcal{L}_n &= \frac{f_\pi^2}{4} \left( \frac{\chi}{f_\chi} \right)^2 \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{f_\pi^2 m_\pi^2}{4} \left( \frac{\chi}{f_\chi} \right)^3 \text{Tr}(U + U^\dagger - 2) \\ &\quad - \frac{f_\pi^2}{4} a \left( \frac{\chi}{f_\chi} \right)^2 \text{Tr}[\ell_\mu + r_\mu + i(g/2)(\vec{\tau} \cdot \vec{\rho}_\mu + \omega_\mu)]^2 - \frac{1}{4} \vec{\rho}_{\mu\nu} \cdot \vec{\rho}^{\mu\nu} - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} \\ &\quad + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{m_\chi^2 f_\chi^2}{4} \left[ (\chi/f_\chi)^4 (\ln(\chi/f_\chi) - \frac{1}{4}) + \frac{1}{4} \right], \end{aligned} \quad (2)$$

$$\mathbf{L}_{an} = \frac{3}{2} g \omega_\mu B^\mu \quad (3)$$

where

$$U = \exp(i\vec{\tau} \cdot \vec{\pi}/f_\pi) \equiv \xi^2, \quad (1.a)$$

$$\ell_\mu = \xi^\dagger \partial_\mu \xi, \text{ and } r_\mu = \xi \partial_\mu \xi^\dagger, \quad (1.b)$$

$$\vec{\rho}_{\mu\nu} = \partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu + g \vec{\rho}_\mu \times \vec{\rho}_\nu, \quad (1.c)$$

$$\omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \quad (1.d)$$

$$B^\mu = \frac{1}{24\pi^2} \varepsilon^{\mu\nu\alpha\beta} \text{Tr}(U^\dagger \partial_\nu U U^\dagger \partial_\alpha U U^\dagger \partial_\beta U). \quad (1.e)$$

The Lagrangian (2) is the normal part of the flavor  $U(2)$  chiral Lagrangian that includes the vector mesons  $\rho$  and  $\omega$  and the dilaton field  $\chi$  as relevant degrees of freedom in addition to the pion field  $\pi$ , and (3) is the anomalous part of the Lagrangian known as Wess-Zumino term. Unlike HLS theory which has manifest local gauge invariance, (2) is gauge fixed to unitary gauge with the density-independent parameters fixed at a given scale. As such, it lacks the intrinsic density dependence present in HLS theory with the VM fixed point. Now what the dilaton does is to mimic this intrinsic dependence by multiplying the parameters with the expectation value  $\langle \chi \rangle^*$  of the dilaton field with respect to the vacuum modified by the density. The dilaton  $\chi$  that figures here is to capture the physics of the spontaneous breaking of scale invariance by the QCD vacuum. There is a subtlety in this matter in that scale symmetry is broken both explicitly by the trace anomaly of QCD and spontaneously by the vacuum. Unlike internal symmetries which are broken both explicitly and spontaneously, e.g., chiral symmetry, however, scale symmetry can be broken spontaneously *only if* it is broken explicitly [7]. This makes its implementation a delicate task. This subtlety can be easily managed for the normal component of the Lagrangian, and the resulting Lagrangian is (2) using the method of [8].

It is the anomalous part (3) – referred to as “Wess-Zumino term” – that poses the problem, both conceptually and in practice. In general, there are three independent terms (excluding external fields) in the Wess-Zumino term with the coefficients undetermined by the symmetry. The simple form given in (3) is arrived at by choosing the arbitrary coefficients in a special, ad hoc, way and by using the equation of motion for the  $\rho$  field [9]. Whether or not this is a reasonable procedure will be discussed below. What is important for our discussion is that as it stands, it is scale-invariant and hence does not couple to  $\chi$ . To see that this is the main

culprit of the problem we have in this model, consider the energy per baryon contributed by this term [3]:

$$\left(\frac{E}{B}\right)_{WZ} = \frac{1}{4}\left(\frac{3g}{2}\right)^2 \int_{Box} d^3x \int d^3x' B_0(\vec{x}) \frac{\exp(-m_\omega^*|\vec{x} - \vec{x}'|)}{4\pi|\vec{x} - \vec{x}'|} B_0(\vec{x}') \quad (4)$$

where ‘‘Box’’ corresponds to a single FCC cell. This quantity diverges unless it is screened by  $m_\omega^*$ . Therefore if the  $\omega$  mass were to go down as required by the vector manifestation, the skyrmion-skyrmion interactions would become strongly repulsive with increasing density. This forces the  $\omega$  mass  $m_\omega^*$ , and hence  $\langle\chi\rangle^*$ , to increase. Note that in HLS theory with the vector manifestation,  $g^2$  in (4) drops to zero as the density approaches the critical, so the problem is avoided.

The upshot is that although symmetry consideration requires that there be change-over from skyrmions to half-skyrmions at some high density [6, 1, 2], there is a strong repulsion in the interactions which would make the putative phase transition occur at very high density. If this were a reality, then there would be an important ramification on the structure of compact stars. For instance, such a strong repulsion would rule out possible kaon condensation at density low enough to be relevant for compact stars [10], increase the maximum stable neutron star mass above 2 times the solar mass [11] and hence prevent the formation of low-mass black holes in the Universe [12]. If the HLS scenario with the vector manifestation is correct, then such a repulsion will be suppressed giving a totally different picture. At present, the situation is unclear and which scenario is the viable one remains to be seen.

### 3 A Resolution

Assuming that there is nothing wrong with (2), we focus on the Wess-Zumino term in the Lagrangian. Our objective is to find an alternative to (3) that leads to a behavior consistent with the VM of HLS theory.

As mentioned, (3) has no justification other than its simplicity.<sup>2</sup> For instance, the arbitrary coefficients generally present in the Wess-Zumino terms are adjusted to give the single term with the constant fixed so as to give a correct decay rate for  $\omega \rightarrow 3\pi$ . However there are compelling indications that the  $\omega \rightarrow 3\pi$  decay does not go through the direct coupling. Indeed, both HLS theory of [4] and holographic dual QCD that incorporates an infinite tower of vector mesons [14] show that the decay is totally vector-dominated, i.e.  $\omega \rightarrow \rho\pi \rightarrow 3\pi$ . This means that a term of the form (3) will not figure in  $L_{an}$ . It should be replaced by terms of the form  $\sim \text{Tr}(\pi dA dA)$  with  $A$  the gauge field. It is possible that such terms could change the interaction structure. Also the way the topological baryon current  $B_\mu$  is derived à la anomaly [15, 16] suggests that implementing the trace anomaly of QCD in hidden local symmetric theory is perhaps subtler than what is presently known.

Various alternative ways-out explored so far [17], however, do not offer a clear-cut solution. Here in the absence of any reliable clue, we try the simplest, admittedly *ad hoc*, modification of the Lagrangian (3) that allows a reasonable and appealing way-out. Given our ignorance as to how spontaneously broken scale invariance manifests in matter, we shall simply forego the requirement that the anomalous term be scale invariant and multiply the  $\omega \cdot B$  term by  $(\chi/f_\chi)^n$  for  $n \gtrsim 2$ . We have verified that it matters little whether we pick  $n = 2$  or  $n = 3$  [17]. We

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<sup>2</sup>The  $\omega \cdot B$  term of the form (3) was used by Adkins and Nappi to stabilize the Skyrme soliton [13]. However in the presence of a large number of vector mesons as in holographic dual QCD [14], this coupling does not exist, and the  $\omega$  plays no role in the stabilization.

therefore take  $n = 3$ :

$$L'_{an} = \frac{3}{2}g(\chi/f_\chi)^3\omega_\mu B^\mu \quad (5)$$

This additional factor has two virtues:

- i) It leaves meson dynamics in free space (i.e.  $\chi/f_\chi = 1$ ) unaffected, since chiral symmetry is realized à la sigma model as required by QCD.
- ii) It plays the role of an effective density-dependent coupling constant so that at high density, when scale symmetry is restored and  $\chi/f_\chi \rightarrow 0$ , there will be no coupling between the  $\omega$  and the baryon density as required by hidden local symmetry with the vector manifestation.

The properties of this Lagrangian for the meson ( $B = 0$ ) sector are the same as in our old description. The parameters of the Lagrangian are determined by meson physics and given in Table 1 in [3].

## 4 The B=1 Skyrmion : Hedgehog Ansatz

The solitons of this effective theory are skyrmions. From Eq.(1), the spherically symmetric hedgehog Ansatz for the  $B = 1$  soliton solution of the standard Skyrme model can be generalized to

$$U^{B=1} = \exp(i\vec{\tau} \cdot \hat{r}F(r)), \quad (6)$$

$$\rho_{\mu=i}^{a,B=1} = \varepsilon^{ika}\hat{r}^k \frac{G(r)}{gr}, \quad \rho_{\mu=0}^{a,B=1} = 0, \quad (7)$$

$$\omega_{\mu=i}^{B=1} = 0, \quad \omega_{\mu=0}^{B=1} = f_\pi W(r), \quad (8)$$

and

$$\chi^{B=1} = f_\chi C(r). \quad (9)$$

The boundary conditions that the profile functions satisfy at infinity are

$$F(\infty) = G(\infty) = W(\infty) = 0, \quad C(\infty) = 1 \quad (10)$$

while near the origin,

$$F(0) = \pi, \quad G(0) = -2, \quad W'(0) = 0, \quad C'(0) = 0. \quad (11)$$

The equations of motion for the various profile functions  $F(r)$ ,  $G(r)$ ,  $W(r)$  and  $C(r)$  follow from the minimization of the soliton mass. The numerical results on the properties of the  $B = 1$  hedgehog skyrmion, the mean-square baryon number radius, the mean-square energy radius, the mass and the contributions to the mass from the different meson terms are reproduced in Table 1 for different values of the dilaton mass.

Two regimes can be distinguished in the table as a function of dilaton mass chosen in a wide range. Let us call them the long-range dilaton (LRD) regime, which occurs for “small” dilaton mass, say,  $m_\chi \leq 1200$  MeV, and the short-range dilaton (SRD) regime, which occurs for “large” mass, say,  $m_\chi \geq 1300$  MeV. They show very different properties which we describe, and whose origin we shall discuss.

In the LRD regime, the baryon density is concentrated in a small volume ( $\sqrt{\langle r^2 \rangle_B} < 0.15$  fm), and the skyrmion is light ( $M_{sol} < 1000$  MeV). However, note that the radius associated

Table 1: Baryonic mean square radius, energy mean square radius and mass for the skyrmion. We also show the different contributions to the mass from the various terms.

$m_\chi$	$\sqrt{\langle r^2 \rangle_B}$	$\sqrt{\langle r^2 \rangle_E}$	$M_{sol}$	$E_\pi$	$E_{m_\pi}$	$E_{\pi\rho}$	$E_\rho$	$E_\omega$	$E_{WZ}$	$E_\chi$
720	0.12	0.39	916	27	0	178	503	-1	1	206
1000	0.12	0.37	956	35	1	184	524	-1	2	211
1200	0.13	0.37	982	53	3	177	526	-1	3	222
1300	0.49	0.59	1410	719	42	34	375	-238	474	5
1500	0.49	0.59	1439	748	42	33	370	-246	489	2
2000	0.50	0.60	1459	757	44	32	367	-248	496	1

with the energy of the skyrmion is appreciable ( $\sqrt{\langle r^2 \rangle_E} \sim 0.4\text{fm}$ ). We can understand these two sizes as follows. Although the dilaton squeezes the skyrmion, so the baryon number gets concentrated in a small core, a large  $\rho$  meson cloud, carrying most of the energy, surrounds this skyrmion core. The  $\omega$  meson, the Wess-Zumino ( $\omega - B$ ) term and the  $\pi$  mass terms contribute negligibly to the mass. Even the pion dynamic term is small. Most of the mass comes, therefore, from the  $\rho$  couplings and the *dilaton* terms.

Figure 1 (left) in which the profile function of all fields are shown proves to be very illuminating for understanding what is going on. The profile function of the dilaton is particularly clarifying. For large values of the skyrmion radius, the dilaton field is as in free space, but for small values, it drops to zero. Thus we see that scale symmetry is realized differently in the interior than in the exterior of the skyrmion. While in the exterior it is spontaneously broken, it is partially restored in the interior. If we look at the profiles of other mesons, we see that those of  $\pi$  and  $\omega$  are large, changing only in the small interior region, while the  $\rho$  meson profile extends further out, confirming our previous discussion. Since both scale symmetry and chiral symmetry are partially restored in the inside region, the pion mass term and the pion dynamic term become also small (i.e. Wigner mode). Finally the  $\omega \cdot B$  coupling and the  $\omega$  term also become small, perhaps reflecting that the vector manifestation [4] is effective. The long-range dilaton makes the skyrmion size small and the baryon number density large. Thus a sort of phase transition takes place to a symmetry restored phase in its interior with vanishing  $\omega$ -couplings, resembling the chiral bag model. Note that most of the mass comes from the  $\rho$  and the dilaton. The  $\rho$  is not squeezed, so the energy radius is large due to its cloud. Thus, this regime is dominated by vector mesons ( $\rho$ ) and partial restored scale symmetry, the dilaton.

We find a completely different structure in the SRD regime. Here the skyrmion is large ( $\sqrt{\langle r^2 \rangle_B} \sim 0.5\text{ fm}$ ), with the baryon density radius nearly coinciding with the energy radius ( $\sqrt{\langle r^2 \rangle_E} \sim 0.6\text{ fm}$ ). It is also heavy ( $M_{sol} \sim 1450\text{ MeV}$ ). The contribution to the mass from the dilaton comes out to be very small, while the pion contribution from the dynamic term is huge. The mass term gives a sizeable contribution. The contribution from the  $\omega$  is large and attractive, and that from the  $\omega \cdot B$  term even larger and repulsive. The  $\rho$  contribution is sizeable, but smaller than in the LRD regime, with its cloud less important. Figure 1 (right panel) shows that the dilaton profile is practically constant and non-zero, indicating that chiral symmetry is realized in Goldstone mode. Here the pion and the  $\omega$  are indispensable for the description of the strong force.

What is remarkable in the numerical results is that the transition between the two regimes is abrupt. This is easy to understand mathematically. For a fixed  $f_\chi$ , the dilaton potential changes sign at a particular value of  $m_\chi$  which induces an abrupt change from Goldstone to Wigner.

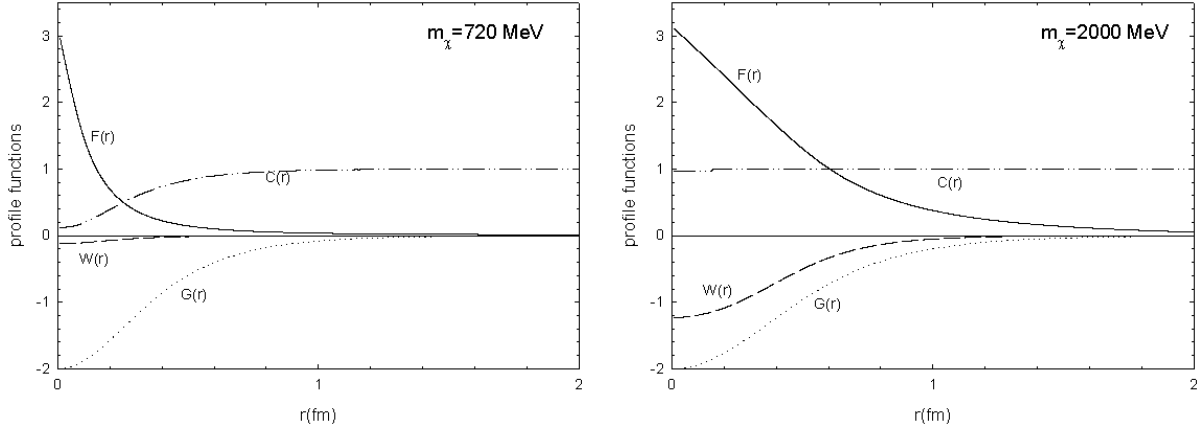


Figure 1: Small skyrmion solution obtained with  $m_\chi = 720$  MeV and large skyrmion with  $m_\chi = 2000$  MeV.

To summarize, we note that with the scale factor multiplied to the Wess-Zumino term, the role of the  $\omega$  field is tightly locked to the behavior of the dilaton field that controls the realization of the two regimes appearing in the single skyrmion system.

## 5 Phase Transitions in Dense Skyrmion Matter

The procedure to describe skyrmion matter follows closely what was done in [3]. It suffices to multiply the scaling factor to the  $\omega - B$  coupling term. At low densities, skyrmion matter is described by an FCC crystal where the nearest neighbor interactions are arranged to have attractive relative orientations. In order to exemplify the phase transitions, we consider the  $m_\pi = 0$  case for which the effects are more dramatic.<sup>3</sup>

We show in Fig. 2 the numerical results on  $\langle\chi\rangle$  and  $\langle\sigma\rangle$  for an exemplary “low” dilaton mass and a “high” dilaton mass as a function of the FCC parameter  $L_F$ , which is related to the baryon density by  $\rho_B = 1/2L_F^3$ <sup>4</sup>. As in the single skyrmion case discussed in the previous section, there are again two regimes, the LRD regime shown on the left panel of Fig. 2 and the SRD regime shown on the right panel.

- In the LSD regime, the symmetry-favored half-skyrmion phase with  $f_\pi \propto \langle\chi\rangle^* \neq 0$  and  $\langle\sigma\rangle \propto \langle\bar{q}q\rangle = 0$  shrinks and the phase transition is characterized by the vanishing  $\langle\chi\rangle$ . The phase transition occurs at about three times the nuclear matter density  $n_0$ , for the set parameters given [3]. Up to but below the critical point, say,  $n = n_c - \epsilon$  where  $n_c$  is the critical density, the skyrmion matter is of an FCC crystal. One cannot say what it is precisely at  $n = n_c$ , but symmetry consideration suggests that it could be in the half-skyrmion phase.
- The situation is completely different in the SRD regime. Here the Goldstone phase in an FCC crystal changes over at  $n = n_p < n_c$  to a half-skyrmion phase in BCC crystal with  $f_\pi \propto \langle\chi\rangle^* \neq 0$  and  $\langle\sigma\rangle = 0$  and then to the Wigner phase with  $f_\pi = \sigma = 0$ . As discussed

<sup>3</sup>In Table 1, a non-vanishing pion mass is put in as a cut-off to regularize divergences in evaluating the rms radii,  $\sqrt{\langle r^2 \rangle_B}$  and  $\sqrt{\langle r^2 \rangle_B}$ .

<sup>4</sup>Nuclear matter density  $\rho_0 = .17/fm^3$  corresponds to  $L_F \sim 1.43$  fm.

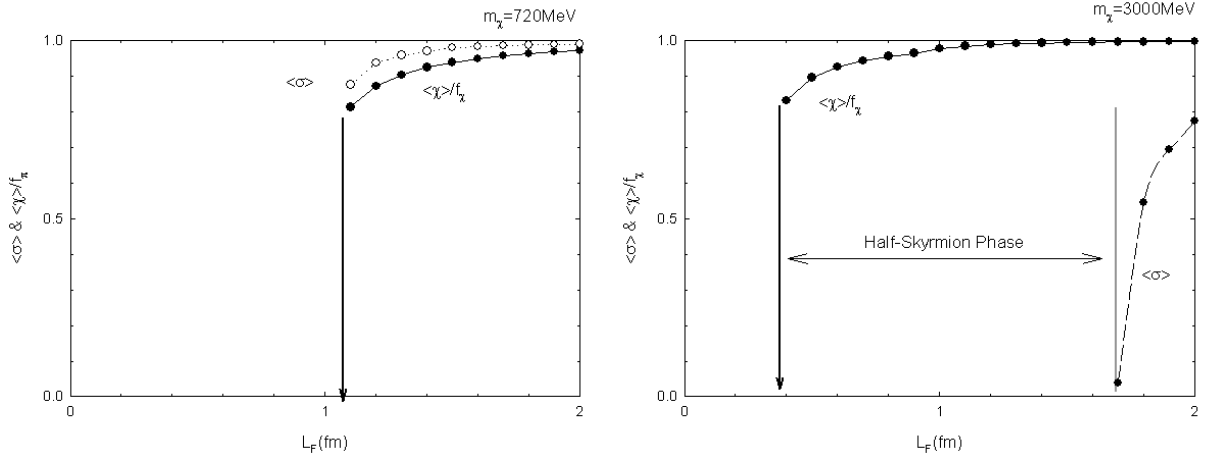


Figure 2: Behavior of  $\langle\chi\rangle$  and  $\langle\sigma\rangle$  as a function of lattice size  $L_F$  in the LRD regime for  $m_\chi = 720$  MeV (left figure) and in the SRD regime for  $m_\chi = 3000$  MeV (right figure).

in [18], when interpreted in HLS theory, the half-skyrmion phase corresponds to the phase in which  $f_\pi = f_\sigma \neq 0$  (where the subscript  $\sigma$  represents the longitudinal component of the  $\rho$  meson) and  $g \simeq 0$ . When  $g = 0$  with  $f_\pi = f_\sigma \neq 0$ , there is an enhancement of symmetry  $SU(2)^2 \rightarrow SU(2)^4$  as pointed out by Georgi [19].

We show in Fig. 3 the phase diagram as a function of  $L_F$  and dilaton mass. This diagram illustrates an intricate nature of the phase structure. As the dilaton mass increases, the critical density at which  $\langle\chi\rangle^* = 0$  sharply increases to a very large value: This occurs before  $\langle\sigma\rangle$  vanishes. For some values of the dilaton mass, we observe an abrupt change leading to a different phase transition scenario in which  $\langle\sigma\rangle$  vanishes at low density, giving rise to a pseudogap phase [1, 2], which remains until  $\langle\chi\rangle$  vanishes at high density.

Since we have changed only the  $\omega \cdot B$  coupling term, the scaling behavior of the in-medium physical quantities remains the same as those in [3], which emphasize the importance of the quantities shown in Fig.2.

## 6 Concluding Remarks

In our effort to find a unified approach to dense matter, i.e. matter at densities higher than that of the normal nuclear matter, which has remained ill-understood up to date, we have uncovered a hitherto unsuspected role that the  $\omega$  meson and the dilaton – associated with the trace anomaly of QCD – play in the structure of dense skyrmion matter. A straightforward implementation of the dilaton field to a gauged skyrmion Lagrangian exposed a serious difficulty in the description of the properties of dense skyrmion matter. We identified the source of the problem in an ill behavior of the  $\omega$  meson in nuclear matter, due primarily to its coupling to the baryon current in the Wess-Zumino term in the effective Lagrangian. In this article, we present a simple and elegant solution to the problem, which exposes a remarkable interplay between the light-quark vector mesons of hidden local symmetry and the dilaton of scale symmetry. Given the poorly understood intricacy involved in the way the dilaton figures in the scale-symmetry breaking, our solution does not yet receive a justification from QCD. An effort is being made to find – if any – such a connection to QCD [17].



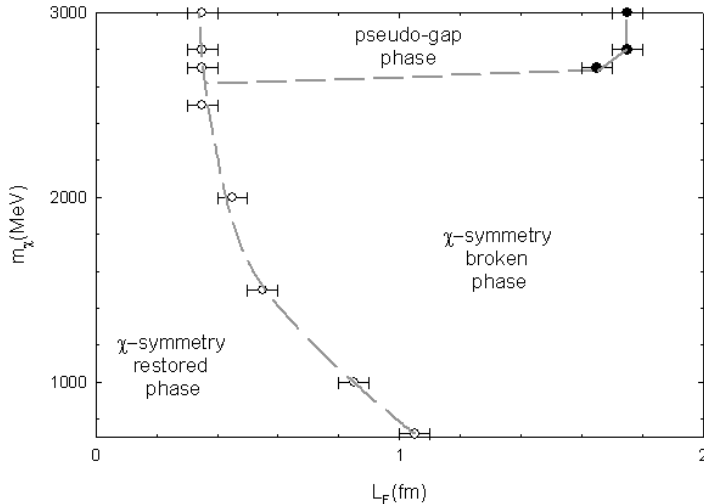


Figure 3: Phase diagram as a function of lattice size  $L_F$  and dilaton mass  $m_\chi$ .

As in our previous work without vector mesons, the skyrmion matter possesses two phases: one which is described by an FCC crystal and another which is described by a half-skyrmion BCC crystal. The main result of the present investigation is the discovery that the dilaton wholly governs the phase transition. We have seen that two regimes are present: A first in which the dilaton dynamics is long-ranged and which leads to what may correspond to a conventional chiral phase transition in which the spontaneously broken scale invariance and chiral symmetry are restored simultaneously; a second in which the dilaton dynamics is short-ranged and which leads to a two-step transition. Initially, chiral symmetry changes at relatively large densities from a spontaneously broken phase to a pseudo-gap phase, and ultimately scale invariance is restored and the system goes into a symmetric phase. The change from one regime to the other is extremely sensitive to a particular value of the dilaton mass, which is certainly dependent on the meson parameters used, and which is associated with a change in sign in the potential of the dilaton which moves abruptly from a double well scenario to a single well scenario. The dilaton mass which delineates the two different scenarios is fairly high, a few times the chiral scale. Nature is most likely in the LRD, so the model favors the phase transition without going through the half-skyrmion phase (or equivalently Georgi’s vector symmetry phase [19]), but since we do not know what the correct structure of the Wess-Zumino term in medium is with respect to scale symmetry, it remains to be seen whether this is what Nature adopts. Even so, what is truly remarkable is that a mild change of the scale structure of the Wess-Zumino term makes such a dramatic change in the properties of dense matter. In particular, the effect of the dilaton mass depending on whether it is of the order of the vector meson mass or several times the vector meson mass is quite striking and unintuitive.

It is interesting to note that the same mechanism operates quite similarly in the single skyrmion description. Thus in the SRD regime, the skyrmion is squeezed to a small size, while the full size of the skyrmion is obtained by a large  $\rho$  meson cloud, while in the LRD regime, the skyrmion is large, and the  $\rho$  cloud smaller. However the mass at which we move from one regime to another is different for the single skyrmion and the skyrmion matter. This can be simply understood by that the potential for the latter is nonperturbatively dressed by the medium as shown in [1]. Finally we see that in the SRD regime, the pion and  $\omega$ -meson fields play

insignificant role, while they are of fundamental importance in the LRD regime. The finding here could offer a way to understand how confinement works in chiral models.

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