

FTUV/05-0624  
IFIC/05-27  
SNUTP/05-012  
APCTP/05-008

## A QCD sum rule analysis of the pentaquark

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### Abstract

We perform a QCD sum rule calculation to determine the mass and the parity of the lowest lying pentaquark state. We include operators up to dimension  $d = 13$  in the OPE and the direct instanton contributions. We find evidence for a positive parity state. The contribution from operators of dimension  $d > 5$  is instrumental in determining the parity of the state and achieving the convergence of the sum rule.

Pacs: 12.38.Aw, 12.38.Lg, 12.39.Ba, 12.39.-x

Keywords: quarks, instanton, hadrons, pentaquark

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# 1 Introduction

The experimental and theoretical status of  $\Theta^+$ -pentaquark remains controversial [1], [2]. The QCD sum rules (SRs) have shown to be a very powerful tool for the investigation of the properties of conventional [3] and exotic multiquark hadronic states [4]. Several attempts to describe the properties of  $\Theta^+$  pentaquark using SRs have appeared [5, 6, 7, 8, 9, 10]. However, these calculations were either restricted to low dimension operators [5, 6, 7, 8] or they used interpolating currents which did not have the most suitable quantum numbers to project onto the  $\Theta^+$  [9, 10].

The dynamics associated with the instanton, the 't Hooft interaction, has been successful in understanding the spectroscopy of four-quark [11] and H-dibaryon [12, 13] states and it was important for the spectroscopy of the pentaquark [14]. Moreover, instantons are crucial for understanding chiral symmetry breaking in the strong interactions [15, 16] and lie at the basis of chiral soliton model for baryons, which has predicted the pentaquarks and their peculiar properties, e.g. small widths and masses [17].

In the SR calculations thus far, the contribution from so-called direct instantons, has not been investigated <sup>1</sup>. It is well known that direct instantons play an important role in the SRs calculations to determine the properties of the pseudoscalar mesons and the nucleon octet baryons [19, 20]. We showed, in a mixed model-SR calculation, that they might be also important for the pentaquarks [21].

We perform a calculation for the pentaquark SRs which takes into account operators up to dimension  $d = 13$  and direct instanton contributions, and leads to evidence for a positive parity state whose mass is close to the observed mass.

## 2 The standard OPE contribution to the sum rules

The QCD sum rule approach starts from the correlator of some relevant current,

$$\Pi(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T \eta_\Theta(x) \bar{\eta}_\Theta(0) | 0 \rangle = \hat{q} \Pi_1(q^2) + \Pi_2(q^2) . \quad (1)$$

Here  $\eta_\Theta$  represents a current with non vanishing projection onto the pentaquark state. We use the conventional notation  $\hat{q} = \gamma \cdot q$ .

The use of the narrow resonance approximation,

$$\text{Im}\Pi(q^2) = \pi \lambda_\Theta^2 (\hat{q} + M_\Theta) \delta(q^2 - M_\Theta^2) + \theta(q^2 - s_0^2) [ \hat{q} \text{Im}\Pi_1(q^2) + \text{Im}\Pi_2(q^2) ] , \quad (2)$$

where  $M_\Theta$  is the mass of the pentaquark,  $\lambda_\Theta$  its residue,  $s_0$  the threshold, and the appropriate dispersion relations lead to the so-called chirality even

$$\frac{1}{\pi} \int_0^{s_0^2} ds^2 e^{-s^2/M^2} \text{Im}\Pi_1^{OPE}(s^2) = \lambda_\Theta^2 e^{-M_\Theta^2/M^2} , \quad (3)$$

and chirality odd

$$\frac{1}{\pi} \int_0^{s_0^2} ds^2 e^{-s^2/M^2} \text{Im}\Pi_2^{OPE}(s^2) = \lambda_\Theta^2 M_\Theta e^{-M_\Theta^2/M^2} \quad (4)$$

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<sup>1</sup>The direct instanton contribution of ref. [18] should vanish due to the Pauli principle for the quarks in the instanton field.

sum rules.

Our choice of current in the pentaquark correlator is

$$\eta_{\Theta}^A = \frac{1}{4\sqrt{2}} \epsilon_{afg}\epsilon_{abc}\epsilon_{bde} [(u_d^T C d_e) \gamma_5 C \bar{s}_c^T] [u_f^T C \gamma_5 d_g] , \quad (5)$$

whose structure corresponds to the A–state of refs. [14, 21], which consists of the product of a scalar  $ud\bar{s}$  triquark and a pseudoscalar  $ud$ –diquark. It can be easily seen, that this current has the same structure as that of ref. [6] except for the  $\gamma_5$  in front of the strange quark field. The consequence of this similarity is that for the chirality odd sum rule our results become identical to their results, if we restrict the calculation to low dimension operators, take into account our different normalization and an additional negative sign due to negative intrinsic parity of our current, Eq. (5).

We have also considered the current,

$$\eta_{\Theta}^B = \frac{1}{4\sqrt{6}} \epsilon_{acd} [(u_a^T C \gamma_\mu d_b + u_b^T C \gamma_\mu d_a) \gamma_5 \gamma^\mu C \bar{s}_c^T] [u_c^T C \gamma_5 d_d] , \quad (6)$$

which corresponds to the B–state of refs. [14, 21] and contains a vector  $ud\bar{s}$  triquark and a scalar  $ud$ –diquark. This current has also negative intrinsic parity. However, our analysis has shown that the B current coupling with  $\Theta^+$  is weak and therefore no definitive conclusion about the values of the mass and residue can be drawn from the consideration of its correlator. We will therefore not discuss it further.

Let us proceed with the analysis of the chirality odd SR for the pentaquark, Eq. (4). This SR is directly related to the mass of the state and usually is more stable than the chirality even SR for the ground state baryons [20] and triquark  $ud\bar{s}$  states [21].

After Borel transforming the pentaquark correlator has dimension  $d = 13$ . The calculation of the chirality odd sum rule will be performed taking into account operators up to dimension  $d = 13$ . To this order we obtain good stability in the OPE result. The operators of higher dimension ( $d > 13$ ) appear in the sum rule multiplied by inverse powers of the Borel mass, thus, in the interesting Borel mass region, their contribution is small and can be safely neglected.

With our interpolating current, the relevant trace to the chirality odd sum rule is expressed as

$$\begin{aligned} \text{Tr}\langle 0|T\eta_{\Theta}(x)\bar{\eta}_{\Theta}(0)|0\rangle_{odd} &= -(i)^5 \frac{1}{32} \epsilon_{abc}\epsilon_{bde}\epsilon_{afg}\epsilon_{a'b'c'}\epsilon_{b'd'e'}\epsilon_{a'f'g'} \text{Tr}[\gamma_5 C S_{c'c}^{s,T}(-x)C\gamma_5] \\ &\times \left( \text{Tr}[C S_{dd'}^{u,T} C S_{ee'}^d] \text{Tr}[C S_{gg'}^{d,T} C \gamma_5 S_{ff'}^u \gamma_5] + \text{Tr}[C S_{df'}^{u,T} C S_{eg'}^d \gamma_5] \text{Tr}[C S_{ge'}^{d,T} C \gamma_5 S_{fd'}^u] \right. \\ &\left. - \text{Tr}[C S_{df'}^{u,T} C S_{ee'}^d C S_{fd'}^{u,T} C \gamma_5 S_{gg'}^d \gamma_5] - \text{Tr}[C S_{dd'}^{u,T} C S_{eg'}^d \gamma_5 C S_{ff'}^{u,T} C \gamma_5 S_{ge'}^d] \right) \end{aligned} \quad (7)$$

where the superscripts on the quark propagator mean the quark flavor and  $a, b, c, \dots$ , are the color indices. In Fig. 1 the diagrams which contribute to the chirality odd SR up to  $d = 13$  are shown. In order to calculate the correlator to a certain order we need to consider the quark propagator to the appropriate dimension. In Fig. 2 we show the corresponding OPE diagrams for the quark propagator which lead to

$$\begin{aligned} S_{ab}^q(x) &= -i\langle 0|Tq_a(x)\bar{q}_b(0)|0\rangle \\ &= \delta_{ab}(\hat{x}F_1^q + F_2^q) - i\tilde{g}G_{ab}^{\mu\nu} \frac{1}{x^2}(\hat{x}\sigma_{\mu\nu} + \sigma_{\mu\nu}\hat{x}) \\ &\quad - m_q \tilde{g} G_{ab}^{\mu\nu} \sigma_{\mu\nu} \left( \ln(-x^2\Lambda^2/4) + 2\gamma_{EM} \right), \end{aligned} \quad (8)$$

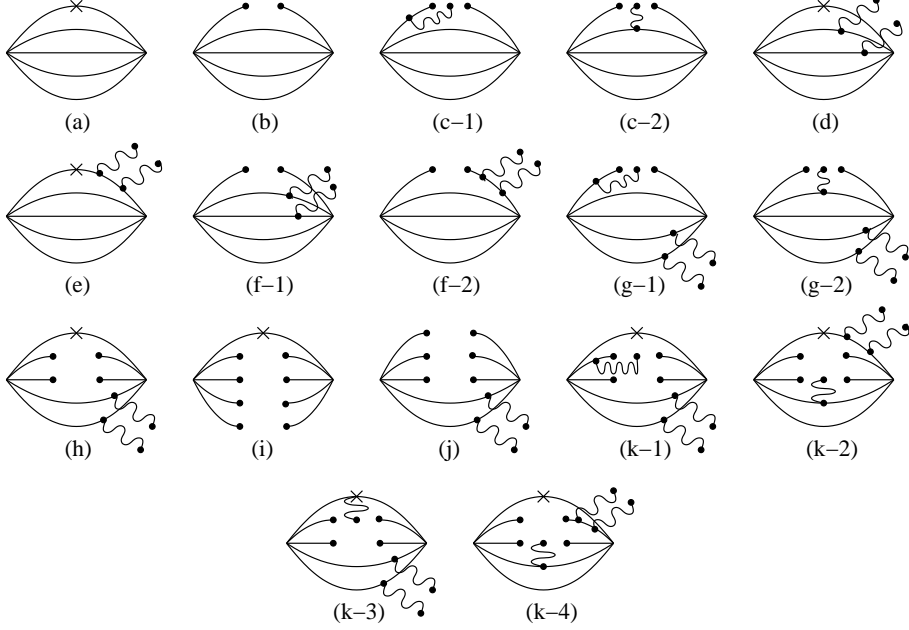


Figure 1: Diagrams contributing to the chirality odd pentaquark sum rule in our calculation. The quark propagator on top corresponds to the strange quark and  $\times$  denotes a quark mass insertion.

$a, b$  are the color indices and  $\tilde{g} = g_c/32\pi^2$ . The two functions are given by

$$\begin{aligned}
F_1^q &= \frac{1}{2\pi^2 x^4} + \frac{m_q \langle \bar{q}q \rangle}{48} + i \frac{m_q x^2}{27 \cdot 3^2} g_c \langle \bar{q}\sigma \cdot Gq \rangle, \\
F_2^q &= i \frac{m_q}{4\pi^2 x^2} + i \frac{\langle \bar{q}q \rangle}{12} - \frac{x^2}{192} g_c \langle \bar{q}\sigma \cdot Gq \rangle + i \frac{g_c^2 x^4}{29 \cdot 3^3} \langle \bar{q}q \rangle \langle G^2 \rangle \\
&\quad + i \frac{m_q g_c^2}{29 \cdot 3\pi^2} \langle G^2 \rangle x^2 \left( \ln(-x^2 \Lambda^2/4) + 2\gamma_{EM} - \frac{2}{3} \right), \tag{9}
\end{aligned}$$

where  $\gamma_{EM}$  is the Euler–Mascheroni constant and we take  $\Lambda = 500$  MeV [22]. Note, that for massless  $u, d$  quarks,  $F_i^u = F_i^d$ .

Our result for the SR including operators up to dimension  $d = 13$  has the form

$$\begin{aligned}
&\frac{1}{4} \left[ -\frac{1}{15} m_s M^{12} E_5 \Big|_{(a)} - \frac{2}{15} f_s a M^{10} E_4 \Big|_{(b)} + \frac{1}{6} f_s m_0^2 a M^8 E_3 \Big|_{(c)} \right. \\
&- \frac{1}{12} b m_s M^8 E_3 \Big|_{(d)} - \frac{1}{12} b m_s M^8 W_3 \Big|_{(e)} - \frac{4}{27} b f_s a M^6 E_2 \Big|_{(f)} \\
&+ \frac{1}{12} f_s m_0^2 a b M^4 E_1 \Big|_{(g)} - \frac{1}{12} m_s b a^2 M^2 E_0 \Big|_{(h)} + \frac{8}{27} m_s a^4 \Big|_{(i)} \\
&\left. - \frac{1}{72} b f_s a^3 \Big|_{(j)} + \frac{1}{48} m_s m_0^2 b a^2 \Big|_{(k)} \right] = \tilde{\lambda}_\Theta^2 M_\Theta e^{-M_\Theta^2/M^2}. \tag{10}
\end{aligned}$$

Each term corresponds to a diagram in Fig. 1. The residue is defined by  $\tilde{\lambda}_\Theta = (4\pi)^4 \lambda_\Theta$ . The contributions from the continuum are given by the following functions :

$$E_n(M) = \frac{1}{\Gamma(n+1)M^{2n+2}} \int_0^{s_0^2} dx e^{-x/M^2} x^n,$$

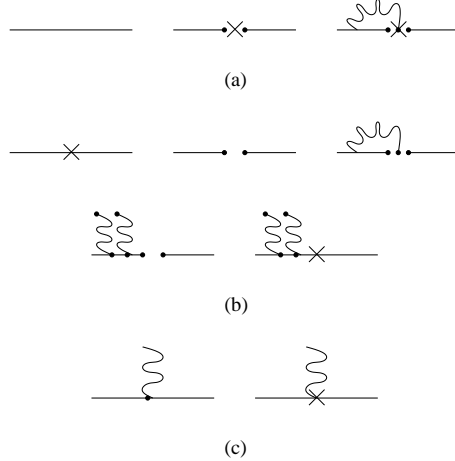


Figure 2: (a) The terms  $F_1^q$  in the quark propagator, (b) the terms  $F_2^q$  in the propagator, (c) two last terms in Eq. 8.

$$\begin{aligned}
W_n(M) = & \frac{1}{\Gamma(n+1)M^{2n+2}} \int_0^{s_0^2} dx e^{-x/M^2} x^n \left( -2 \ln(x/\Lambda^2) + \ln \pi \right. \\
& \left. + \psi(n+1) + \psi(n+2) + 2\gamma_{EM} - \frac{2}{3} \right)
\end{aligned} \tag{11}$$

with  $\psi(n) = 1 + 1/2 + \dots + 1/(n-1) - \gamma_{EM}$ . The numerical values of various quantities in the sum rule will be given in below.

We have checked that our result Eq. (10) taking into account only contributions of operators with dimensions  $d \leq 9$  is in agreement with the previous calculations [6], [8]. We should mention that due to specific spin and color structure of our current Eq. (5) the potentially important contributions from operators  $\langle \bar{\psi}\psi \rangle^3$ ,  $\langle \bar{\psi}\psi \rangle^2 \langle \bar{\psi}g_c\sigma \cdot G\psi \rangle$ ,  $m_s \langle g_c \bar{q}\sigma \cdot Gq \rangle^2$ , and  $\langle \bar{s}s \rangle \langle g_c \bar{q}\sigma \cdot Gq \rangle^2$  can not appear in chirality odd sum rules. The absence of the contribution of these operators can be seen by direct analysis of the terms in Eq. (7). We would like to mention that their appearance depends strongly on the structures of the interpolating current. For example, in the recent paper [23], where another pentaquark current has been used, it was shown that these operators give non-vanishing contribution. We do not include the contributions proportional to  $\langle g_c^3 GGG \rangle$  and  $\langle g_c^2 GG \rangle^2$ , which in the OPE are related to terms of higher orders in the expansion in the strong coupling constant, and therefore their contributions are expected to be very small for the light quark systems (see for example the discussion in ref. [8] on the three-gluon condensate contribution). This statement is in the agreement with a general observation that pure gluonic operators are not very important in QCD SRs for hadrons consisting of light  $u$ -,  $d$ -, and  $s$ - quarks [24], [25].

### 3 The direct instanton contribution to the sum rule

In addition to contributions of power type, arising from the OPE expansion, there are *exponential* contributions coming from direct instantons contributions to the correlators [19, 20]. They can be calculated by using the following formula for the quark propa-

gator in the instanton background in the regular gauge

$$S_{ab}^{q.inst}(x, y) = A_q(x, y) \gamma_\mu \gamma_\nu (1 + \gamma_5) (U \tau_\mu^- \tau_\nu^+ U^\dagger)_{ab}, \quad (12)$$

where

$$A_q(x, y) = -i \frac{\rho^2}{16\pi^2 m_q^*} \phi(x - z_0) \phi(y - z_0)$$

and

$$\phi(x - z_0) = \frac{1}{[(x - z_0)^2 + \rho^2]^{3/2}}.$$

Note that  $\rho$  stands for the instanton size and  $z_0$  the center of the instanton ;  $U$  represents the color orientation matrix of the instanton in  $SU(3)_c$  and  $\tau_{\mu,\nu}$  are  $SU(2)_c$  matrices ;  $m_q^* = m_{cur}^q - 2\pi^2 \rho_c^2 \langle \bar{q}q \rangle / 3$  is the effective quark mass in the instanton vacuum and  $m_{cur}^q$  the current quark mass. The final result should be multiplied by a factor of two to take into account the anti-instanton contribution, and has to be integrated over the instanton density.

To leading order in the instanton density, the direct instanton contributions arise from two body  $ud$ ,  $u\bar{s}$ ,  $d\bar{s}$  and three body  $ud\bar{s}$  quark zero mode propagators in the correlator Eq.(1), as shown in Fig. 3.

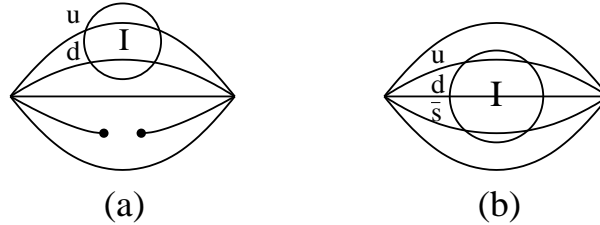


Figure 3: An example of instanton two- and three-body contributions to the correlator of the pentaquark currents.

The final result for two body instanton contribution is

$$\Pi_2(M) = -\frac{n_{eff} \rho_c^4 \langle \bar{q}q \rangle}{2^6 \cdot 3\pi^8 m_q^* m_s^*} \hat{B}[f_6(Q)], \quad (13)$$

where Shuryak's instanton liquid model for QCD vacuum with density  $n(\rho) = n_{eff} \delta(\rho - \rho_c)$  [15] has been used and  $\hat{B}[f_6(Q)]$  is the Borel transform of  $f_6(Q)$  which is defined by

$$f_6(Q) = \int d^4 z_0 \int d^4 x \frac{e^{-iq \cdot x}}{x^6 [z_0^2 + \rho_c^2]^3 [(x - z_0)^2 + \rho_c^2]^3}, \quad (14)$$

where  $\rho_c$  is the average instanton size. There are two types of singularities in Eq.(14). One of them is related to the pole at the origin  $x^2 = 0$ , the other is due to the pole at finite distance from origin  $x^2 \sim -\rho_c^2$ . The pole at  $x^2 = 0$  produces, after Fourier transforming, power terms in  $1/Q^n$  in addition to the exponential type direct instanton contributions  $\exp(-Q\rho_c)$ , arising from finite distances. One should carefully subtract that contribution to avoid double counting with the standard OPE terms. We follow the

procedure suggested in ref. [26] for the analysis of direct instanton contributions to heavy quark decay. More specifically, for a general integral

$$\Pi_{ins} = \int d^4x d^4z_0 e^{iqx} \frac{S(x)}{x^{2n}((x-z_0)^2 + \rho_c^2)^\alpha (z_0^2 + \rho_c^2)^\beta}, \quad (15)$$

where  $S(x)$  contains no singularities for complex  $x_\mu$ , we use Feynman's parameterization

$$\Pi_{ins} = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \int d^4x d^4z_1 e^{iqx} \frac{S(x)}{x^{2n}} \int_0^1 dt \frac{t^{\alpha-1}(1-t)^{\beta-1}}{(t(1-t)x^2 + z_1^2 + \rho_c^2)^{\alpha+\beta}}, \quad (16)$$

where  $z_1 = z_0 - tx$ , but consider only the contribution from the pole at

$$x^2 = -(z_1^2 + \rho_c^2)[t(1-t)]^{-1}. \quad (17)$$

The Borel transform of the function  $f_6$  is given by

$$\begin{aligned} \hat{B}[f_6(Q)] = & -\frac{\pi^4 M^{12}}{2^{13}} \int_0^1 dt \int_0^1 dy \frac{e^{-M^2 \rho_c^2 / (4ty(1-y))}}{y^2(1-y)^2} \left( X^2 + 5X^3 + 10X^4 \right. \\ & \left. + 10X^5 + 5X^6 + X^7 \right), \end{aligned} \quad (18)$$

where  $X = (1-t)/t$ . Note that only the contribution from the pole at finite quark separation has been considered.

We have also performed the calculation of the three body contributions induced by instantons, Fig. 3b, and found that they vanish as the result of the cancellation between diagrams with different  $ud\bar{s}$  combinations in the pentaquark current. The two body  $ud$  instanton contribution also cancels. The only non vanishing contributions arise from the two body  $u\bar{s}$  and  $d\bar{s}$  terms of Eq. (13). The behaviour of the different contributions is associated to the Dirac structure of our current Eq. (5) which includes both scalar and pseudoscalar  $ud$  diquarks with equal weights. The instanton induced contribution is very sensitive to the parity of the state [27] and it flips sign when the parity of the state changes. Of course, the three-body instanton induced forces might give non-zero contribution for other choices of pentaquark currents, for example, for currents with derivatives.

We should mention that three body instanton terms induce forces which give non zero contributions to the mass of some specific triquark  $ud\bar{s}$  pentaquark clusters [14, 21] and furthermore, they give non vanishing contribution to the  $\Theta$  mass within the bag model [28].

## 4 Numerical analysis

We use the following values for parameters at the normalization point 2 GeV [8] (see also recent discussion about uncertainties in values of various condensates in [29])

$$\begin{aligned} \langle \bar{u}u \rangle &= -(243 \text{ MeV})^3 \equiv -\frac{a}{(2\pi)^2}, \\ b = \langle g_c^2 G^2 \rangle &= 0.88 \text{ GeV}^4, \\ ig_c \langle \bar{u}\sigma \cdot Gu \rangle &= m_0^2 \langle \bar{u}u \rangle = 0.8 \text{ GeV}^2 \langle \bar{u}u \rangle, \\ \frac{\langle \bar{s}s \rangle}{\langle \bar{u}u \rangle} &= \frac{\langle \bar{s}\sigma \cdot Gs \rangle}{\langle \bar{u}\sigma \cdot Gu \rangle} = f_s = 0.8, \\ m_s &= 111 \text{ MeV}, \end{aligned} \quad (19)$$

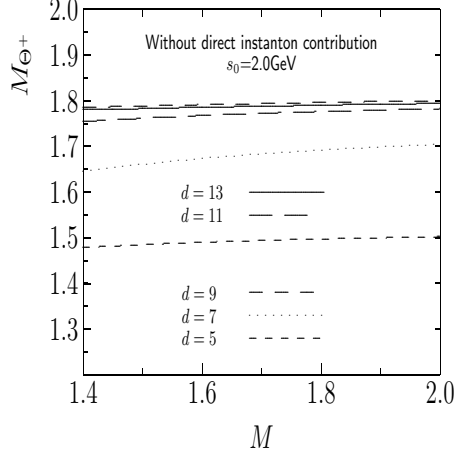


Figure 4: The mass of the pentaquark obtained without direct instanton contributions as a function of the Borel parameter in different orders of the OPE expansion.

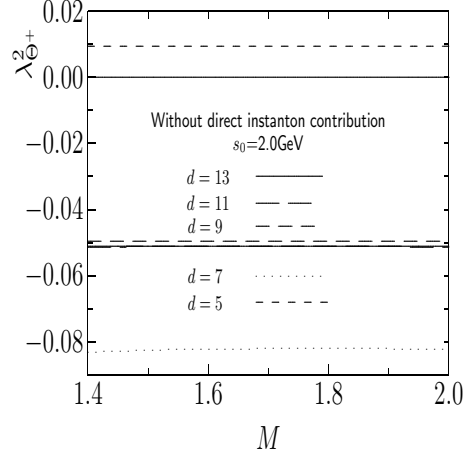


Figure 5: The residue of the pentaquark obtained without direct instanton contribution as a function of the Borel parameter in different orders of OPE expansion.

with  $\rho_c = 1.6 \text{ GeV}^{-1}$  for the average instanton size in the QCD vacuum. In the numerical estimate of the direct instanton contribution the relations of the instanton liquid model [30]

$$\frac{n_{eff}}{m_q^{*2}} = \frac{3}{4\pi^2\rho_c^2}, \quad \frac{m_q^*}{m_s^*} = \frac{1}{f_s - \frac{3m_s}{2\pi^2\rho_c^2\langle\bar{q}q\rangle}}, \quad (20)$$

are used.

In Figs. 4 and 5 the result of the calculation of the pentaquark mass and residue within the standard OPE expansion for the different orders in operator dimensions is shown. In Figs. 6 and 7 the mass and residue of the pentaquark as a function of the value of the Borel parameter with direct instanton contributions are shown. In Fig. 8 we present the results of the calculation of the OPE and the direct instanton contributions to the left-hand side of the SR, Eq.(4). All curves are given for a value of the threshold  $s_0 = 2 \text{ GeV}$ . We chose this value of threshold because the stability was best. From the fit of the sum rules we arrive at the following values for pentaquark mass:  $M_{\Theta^+} = 1.66 \text{ GeV}$  for  $d = 7$ ,  $M_{\Theta^+} = 1.75 \text{ GeV}$  for  $d = 9$ ,  $M_{\Theta^+} = 1.73 \text{ GeV}$  for  $d = 11$ , and  $M_{\Theta^+} = 1.75 \text{ GeV}$  for  $d = 13$  <sup>2</sup>.

One important result of our calculation is in the change of the sign of the squared of the residue when increasing the dimension of the operators which contribute to the OPE. Thus, for  $d = 5$  the sign is positive, while it becomes negative for higher dimensions. In particular, the contribution from the dimension  $d = 7$  operators is crucial for inverting the sign. Due to the negative intrinsic parity of our current Eq. (5), the negative (positive) sign of the squared of the residue implies positive (negative) parity for the state. Therefore,

<sup>2</sup>Once the instanton contribution is included the stability in the Borel parameter for the SR up to  $d = 5$  operators disappears (see Fig. 6). Therefore it is not possible to extract the value of pentaquark mass from the SR with only up to  $d = 5$  operators.



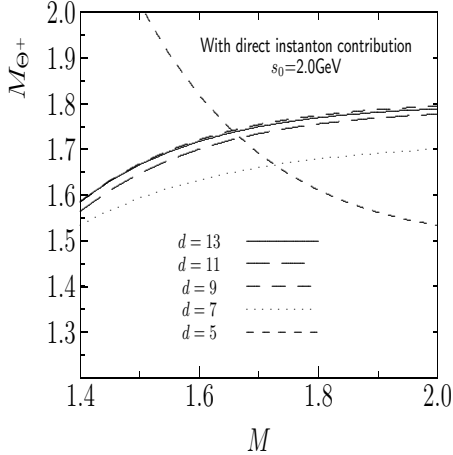


Figure 6: The mass of the pentaquark obtained with direct instanton contributions as a function of the Borel parameter for different orders of OPE expansion.

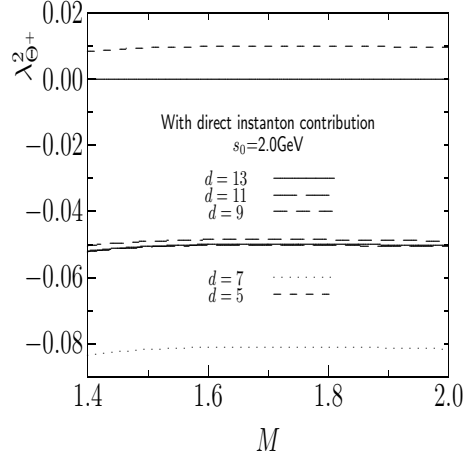


Figure 7: The residue of the pentaquark obtained with direct instanton contribution as a function of the Borel parameter in different orders of the OPE expansion.

our final result for the residue presented in Fig. 7 shows that one can arrive to the wrong conclusion about the parity of the pentaquark state [6], if one takes the decision based on only the contributions of low dimension ( $d = 5$ ) operators. We also stress the necessity to include high dimension operators to get good convergence for the sum rule. It is evident that this effect is directly related to the high dimension of the pentaquark current.

Once we include the contributions from the high dimension operators and the instantons, our result for the  $\Theta^+$  pentaquark mass,  $M_{\Theta^+} \approx 1.75$  GeV, is higher than was given by previous SR calculations [6, 5, 7] but still in rough agreement with the available experimental data, if one admits about 10% accuracy in the predictions of the SR approach due to uncertainties in the values of the various condensates, the mass of the strange quark, the contribution from higher dimension operators  $d > 13$ , higher order pQCD corrections, etc. Furthermore, some additional effects such as the mixing between various pentaquark states [14, 21], which are beyond the scope of the present paper, might give some additional contribution to the mass of the  $\Theta^+$ .

We also note that in our calculation the pentaquark has positive parity in agreement with the soliton model prediction [17]. Our estimate for direct instanton contribution is done within Shuryak's instanton liquid model. We have found that the instanton contribution for the full SR is rather small, but can give a large contribution to it when one considers operators only up to dimension  $d = 5$  (see Figs. 4,6). The smallness of the instanton contribution to the full SR is mainly related to the large mass of the Borel parameter  $M \approx 1.7$  GeV, where we obtain the plateau of stability (Fig. 6). In this region the instanton contribution is small in comparison with the contribution from the high dimension operators in the OPE (Fig. 8).

There is a significant dependence of our results on the value of threshold. This is a common feature in all the studies about the properties of the pentaquark within the QCD sum rule approach. In our case, we have chosen  $s_0 = 2$  GeV to satisfy the physical requirement of having a large stability plateau.

In summary, we have shown the analysis of the QCD sum rules for the  $\Theta^+$  pentaquark current including high dimension operators in the OPE and direct instanton contribu-

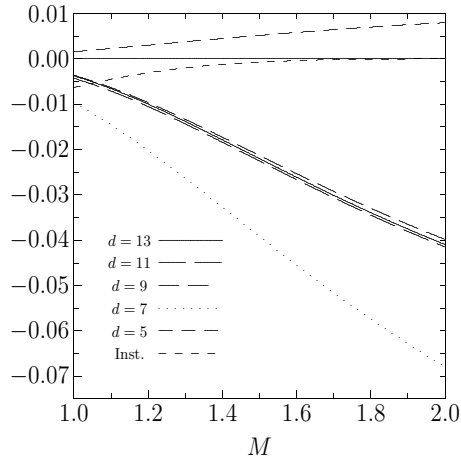


Figure 8: The different OPE orders and direct instanton contributions to the left-hand side of chirality odd SR .

tions. Our results conclude that the role of the high dimension operators is important for obtaining a positive parity for pentaquark state. Our calculation though produces a bound state whose mass is higher than the experimental observation. More sophisticated models and probably states mixing [14, 21] might reduce the obtained value to the observed one.

## 5 Acknowledgments

We are grateful to A.E. Dorokhov, S.V. Esaibegian, S.V. Mikhailov, A.G. Oganesian, and A.A. Pivovarov for very useful discussions. This work was supported by grants by MCyT-FIS2004-05616-C02-01 and GV-GRUPOS03/094 (VV), by Ministerio de Educacion y Ciencia of Spain (SEEU-SB2002-0009) (HJL), and by Brain Pool program of Korea Research Foundation through KOFST, grant 042T-1-1 and in part by RFBR-03-02-17291, RFBR-04-02-16445 (NIK). NIK is very grateful to the School of Physics, SNU for their warm hospitality during the final stage of this work.

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