Further Comments on a Vanishing Singlet Axial Vector Charge

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Abstract

The recent suggestion of a vanishing flavor-singlet axial charge of nucleon due to a nontrivial vacuum structure is further amplified. A perturbative QCD discussion, applicable for the heavy quark contributions, relates it to the physics of the decoupling theorem. It is also shown that $g_A^0 \simeq 0$ leads to a negative η'-meson-quark coupling, which has been found to be compatible with the chiral quark model phenomenology.

1 Introduction

There are two popular descriptions of a suppressed quark contribution to the nucleon spin. One approach suggests that it is due to the (low energy) non-perturbative QCD depolarization of quarks inside nucleon[1]. Another approach attributes this suppression to a possible gluon polarization, contributing via the axial anomaly, which reflects the high energy regularization of certain quark loop diagram[2].

Understanding the nucleon spin problem is related to understanding the its flavor singlet axial charge , g_A^0. The latter for a spin $\frac{1}{2}$ target is defined

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[1] For a recent review of the non-perturbative spin/flavor study in general, and chiral quark model in particular, see [1].

by the formula

$$< p'|J^0_{\mu 5}|p >= g^0_A(Q^2)\bar{P}\gamma_\mu\gamma_5 P + g^0_P(Q^2)q_\mu\bar{P}\gamma_5 P \quad (1)$$

where singlet axial vector current is not conserved due to the anomaly:

$$\partial_\mu J^0_{\mu 5}(x) = 2i\sum_i m_i\bar{q}\gamma_5 q + 2N_f\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\widetilde{G^a_{\mu\nu}}. \quad (2)$$

In a recent paper by two of us (NIK and VV)[5] it was argued that the properties of the QCD vacuum lead to a vanishing of the flavor-singlet axial vector charge:

$$g^0_A \simeq 0. \quad (3)$$

We also discussed the relation of our result with the above descriptions.

Let us recall the non-perturbative mechanism behind the suppression of g^0_A. Once the anomaly is interpretated as the motion of the Dirac levels of definite chirality [6], the suppression comes about due to the cancellation between the infrared (IR) and ultraviolet (UV) contributions, which arise in order to preserve the axial-charge of the vacuum. Thus the vanishing of the axial-charge holds in the flavor-singlet channel, independent of the target: be it hadron or constituent quark. The possibility of a target-independent suppression of the singlet nucleon axial charge has also been discussed previously using different approaches[7, 8, 9].

We have also shown how this phenomenon is realized in the instanton model of the vacuum. An instanton dominated non-perturbative QCD is a plausible mechanism for the low energy depolarization as the 't Hooft determinantal interaction [3] flips the quark helicities[4]. This interaction was obtained by taking into account the contribution of only the quark zero modes in the instanton field.

The axial anomaly is related to the UV regularization in the perturbation theory and therefore to the non-zero quark modes. Once the zero mode contribution and the non-zero mode contribution to the singlet axial charge are considered, the suppression occurs [5]. This is how the instanton vacuum realizes our general result.

In this note we shall provide more arguments which support the validity of the singlet axial-charge suppression due to the properties of the QCD vacuum. First, in Sec.2 we present a simple version of this cancellation in the case of heavy quark contribution to the axial vector charge, showing how this cancellation comes about in the perturbative approach. This is in accord with the expectation that heavy particles must decouple from low energy physics[10]. In Sec.3 we then show that the vanishing of the flavor singlet axial charge leads to the negative value for the η'-quark coupling, which has been found in an earlier publication[11] to be compatible with chiral quark model phenomenology.

2 The perturbative version for heavy quarks

We nest present a perturbative version of the result in Eq.(3): the decoupling of the heavy quark contribution to the target spin. In accordance with a decoupling theorem one would expect that any heavy quark Q makes a vanishingly small contribution to the axial vector charge since the involved energy is much smaller than the heavy quark rest energy m_Q.

This can be demonstrated, for example[12], by following the heavy quark expansion idea of Witten[13]. When we integrate out the heavy quark field Q, because the presence of the quark mass m_Q, the pseudoscalar density $im_Q\bar{Q}\gamma_5 Q$ in the current divergence does not vanish, but gives rise to a dimension-four operator:

$$im_Q\bar{Q}\gamma_5 Q \longrightarrow -\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\widetilde{G^a_{\mu\nu}} + O\left(m_Q^{-2}\right) \tag{4}$$

The relevant diagram is just the pseudoscalar triangle diagram with two gluon legs. This minus sign, which leads to the above-mentioned cancellation by the anomaly term in Eq.(2), can be understood this way. The anomaly term results from regulating the UV divergence. This can be implemented by introducing into the theory a regulating fermion R with a large mass M_R. This breaks the chiral symmetry and gives rise to a mass term in the divergence of the axial vector current $iM_R\bar{R}\gamma_5 R$. In the limit of $M_R \to \infty$, this mass term is then transformed (when integrating out the R fermion) into the anomaly term:

$$iM_R\bar{R}\gamma_5 R \longrightarrow \frac{\alpha_s}{8\pi}G^a_{\mu\nu}\widetilde{G^a_{\mu\nu}} \tag{5}$$

These two cases of the heavy quark and regulating fermion are exactly the same, the only difference being that the regulating fermion has the opposite metric (its loop integral has an extra minus sign). This accounts for the opposite signs on the RHS in Eqs.(4) and (5), and a vanishing contribution to the axial current divergence.

Then by using Eq.(1) and the absence of the massless pole in the form factor g_P^0 we come to the result in Eq.(3) which can be regarded as an analog of the Goldberger-Treiman relation for the flavor-singlet axial vector channel.

For the light quarks, perturbative QCD is not applicable. The low energy non-perturbative QCD physics has to be modeled, *e.g.,* by the instanton dominance. What is argued in Ref.[5] is that in this case the cancellation survives in the flavor singlet channel, even for the light quarks, but only for the matrix elements of the quark and gluon operators in the right-hand side of Eq.(2), and not for the operators in Eq.(4).

3 Singlet coupling in the chiral quark model

Since the U(1) symmetry is broken (by instantons), there must be mixing among the pseudoscalar states,

$$\begin{aligned} |\eta'\rangle &= cos\Theta\left|\eta^0\right\rangle + sin\Theta\left|\eta^8\right\rangle \\ |\eta\rangle &= -sin\Theta\left|\eta^0\right\rangle + cos\Theta\left|\eta^8\right\rangle \end{aligned} \tag{6}$$

where $\Theta \approx -18^\circ$ is the $\eta - \eta'$ mixing angle. In the pole approximation for the matrix element of the right-hand side of Eq.(2) the vanishing of the singlet axial coupling in (3) becomes

$$-\frac{g_{\eta qq}}{{m_\eta}^2} sin\Theta + \frac{g_{\eta' qq}}{{m'_\eta}^2} cos\Theta \simeq 0 \tag{7}$$

or,

$$g_{\eta' qq} \simeq tan\Theta \frac{{m'_\eta}^2}{{m_\eta}^2} g_{qq}. \tag{8}$$

By substituting in (8) the experimental values for Θ, m_η and $m_{\eta'}$ one can find

$$g_{\eta' qq} \approx -g_{\eta qq}. \tag{9}$$

The negative sign reflects the negative mixing angle, which is related to the resolution, via the instanton physics, of the axial U(1) problem, ${m_{\eta'}}^2 \gg {m_\eta}^2$. The related nucleon result for $g_{\eta' NN}$ has also been discussed by other authors[8, 14]. The specific situation of a negative singlet-meson coupling in an instanton dominated QCD (only for the zero-mode determinantal interaction) has been discussed in Ref.[15].

In an earlier publication[11], it has been noted that a fit of the nucleon spin and flavor structure data in the two parameter broken-U(3) chiral quark model favors the singlet to octet meson-quark coupling ratio to be in the neighborhood of minus one. Thus we can interpret the result in (9) as to have some phenomenological support. This in turn buttresses the idea of a vanishing singlet axial charge of (3).

4 Concluding remarks

In this note we have provided further arguments to support the target-independent suppression of the singlet axial vector charge. A perturbative version of this result reflects the reasonable result of heavy quark decoupling. In the pole dominance approximation, it leads to a singlet meson-quark coupling with the opposite sign of the octet coupling, a result which coincides with that of the chiral quark model phenomenology.

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