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The Proton Spin in the Chiral Bag Model:
Casimir Contributions and Cheshire Cat Principle

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Abstract

The flavor singlet axial charge has been a source of study in the last years due to its relation to the so called *Proton Spin Problem*. The relevant flavor singlet axial current is anomalous, i.e., its divergence contains a piece which is the celebrated $U_A(1)$ anomaly. This anomaly is intimately associated with the η' meson, which gets its mass from it. When the gauge degrees of freedom of QCD are confined within a volume as is presently understood, the $U_A(1)$ anomaly is known to induce color anomaly leading to "leakage" of the color out of the confined volume (or bag). For consistency of the theory, this anomaly should be canceled by a boundary term. This "color boundary term" inherits part or most of the dynamics of the volume (i.e., QCD). In this paper, we exploit this mapping of the volume to the surafce via the color boundary condition to perform a complete analysis of the flavor singlet axial charge in the chiral bag model using the Cheshire Cat Principle. This enables us to obtain the hitherto missing piece in the axial charge associated with the gluon Casimir energies. The result is that the flavor singlet axial charge is small independent of the confinement (bag) size ranging from the skyrmion picture to the MIT bag picture, thereby confirming the (albeit approximate) Cheshire Cat phenomenon. .

1 Introduction

The possibility of formulating a physical theory by means of equivalent field theories defined in terms of different field variables, leads to a construction principle for phenomenologically sensible and conceptually powerful models, referred to as the Chesire Cat Principle (CCP)[1, 2]. In 1+1 dimensions fermionic theories are bosonizable [3] and the CCP can be made exact and transparent. Furthermore in the supersymmetric world, the powerful nonrenormalizable theorems allow a web of exactly equivalent theories to be established. In the real four-dimensional nonsupersymmetric world, bosonization with a finite number of degrees of freedom is not exact. However based on the unproven "theorem" of Weinberg [4], it seems possible to argue that the CCP should hold also in four dimensions, albeit approximately. In view of recent developments on establishing the network of dualities in what is believed to be a fundamental theory (i.e., string theory) where one might say that the exact CCP holds, the notion of a precise CCP in the real world is no longer so preposterous. The aim of the present investigation is to show the full consistency of the CCP in the hadronic world for the case of the *Proton Spin*, which was not fully satisfactorily established in our previous efforts in this direction [5, 6, 7][1]. We complete the program in this paper.

Quantum Chromodynamics (QCD) is the theory of the hadronic phenomena [8]. At sufficiently low energies or long distances and for a large number of colors N_C, it can be described accurately by an effective field theory in terms of meson fields [9]. In this regime, the color fermionic description of the theory is extremely complex due to confinement. However the implementation of the CCP in a two phase scenario called the Chiral Bag Model (CBM) has proven surprisingly powerful [10].

What is the CBM? Let space-time be divided in two regions by a hypertube, that is, the evolving bag. In the interior of the tube, the dynamics is defined in terms of the microscopic QCD degrees of freedom, quarks and gluons. In the exterior, one assumes an equivalent dynamics in terms of meson fields, i.e., one that respects the symmetries of the original theory and the basic postulates of quantum field theory [4]. The two descriptions are matched by defining the appropriate boundary conditions which implement the symmetries and confinement [1, 10]. What this does effectively is to delegate all or part of the principal elements of the dynamics taking place inside (QCD) the bag to the boundary. We will see that this strategy works quite efficiently in the problem at hand.

In this scenario the CCP states that the hadron physics should be approximately independent of the spatial size of the confinement region or the bag [1]. This realization of the principle has been tested in many instances in hadronic physics with fair success [2].

There is one case, however, where the realization of the CCP has not been as successful as in the other cases, namely, the calculation of the flavor singlet axial charge (FSAC) of the nucleon. Indeed in the previous efforts [5, 6, 7], the CCP was realized only partially as it seemed to fail at certain points such as for zero bag radius. It is the leitmotiv of this work to remove this apparent failure.

The observable FSAC has become very relevant in the nucleon structure in recent years, because it is associated with the so called *Proton Spin Problem* [11]. The experimentally observed small value for the FSAC implies a strong violation of the so-called Ellis-Jaffe sum rule [12] and

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[1]Note that in these papers, we have shown that the CCP holds for non-zero bag radii but it failed when the bag radius shrank to a point, implying that in the model studied, the pure skyrmion and the MIT bag did not have the equivalent structure required by the CCP.

therefore implies that the polarization of the proton is not carried exclusively by the valence quarks. It is also very interesting from the formal point of view, because the flavor singlet axial current – which is the origin of the observable – is anomalous, and its anomaly is related in a non-trivial way with the gluonic structure of the theory [13].

In the CBM, the scenario of how the CCP is realized – which is the central issue of our problem – is very intricate. As stated, the flavor singlet axial current is associated with the anomaly and effectively with the η' meson. Thus, besides the pion field of the conventional effective theories which accounts for spontaneously broken chiral symmetry, the correct treatment of the FSAC requires minimally the inclusion of a field describing the η' meson. We shall label it for brevity $\eta(x)$ since no confusion will arise in what follows.

The intricacies of the *hedgehog* configuration and its relevance to the fractionation of baryon charge and other observables have been extensively discussed [14] and fairly well understood [15, 16]. They will be implemented in our calculation without much details. Moreover the inclusion of the η' meson carries subtleties of its own. The vacuum fluctuations inside the bag, that induce the baryon number leakage into the *skyrmion* [14], also induce a color leakage if a coupling to a pseudoscalar isoscalar field is allowed [17]. This leakage would break color gauge invariance and confinement in the model unless it is canceled. As suggested in [17], this color leakage can be prevented by introducing into the CBM Lagrangian a counter term of the form

$$\mathcal{L}_{CT} = i\frac{g^2}{32\pi^2} \oint_{\Sigma} d\beta K^\mu n_\mu (\mathrm{Tr}\ln U^\dagger - \mathrm{Tr}\ln U) \tag{1}$$

where N_F is the number of flavors (here taken to be =3), β is a point on a surface Σ, n^μ is the outward normal to the bag surface, U is the $U(N_F)$ matrix-valued field written as $U = e^{i\pi/f}e^{i\eta/f}$ and K^μ the properly regularized Chern-Simons current $K^\mu = \epsilon^{\mu\nu\alpha\beta}(G_\nu^a G_{\alpha\beta}^a - \frac{2}{3}f^{abc}gG_\nu^a G_\alpha^b G_\beta^c)$ given in terms of the color gauge field G_μ^a. Note that (1) manifestly breaks color gauge invariance (both large and small, the latter due to the bag), so the action of the chiral bag model with this term is not gauge invariant at the classical level but as shown in [17], when quantum fluctuations are calculated, there appears an induced anomaly term on the surface which exactly cancels this term. Thus gauge invariance is restored at the quantum level.

The equations of motion for the gluon and quark fields inside and the η' field outside are the same as in [5, 6]. However the boundary conditions on the surface with the inclusion of Eq.(1) read [7]

$$\hat{n} \cdot \vec{E}^a = -\frac{N_F g^2}{8\pi^2 f}\hat{n} \cdot \vec{B}^a \eta \tag{2}$$

$$\hat{n} \times \vec{B}^a = \frac{N_F g^2}{8\pi^2 f}\hat{n} \times \vec{E}^a \eta \tag{3}$$

and

$$\frac{1}{2}\hat{n} \cdot (\bar{\psi}\vec{\gamma}\gamma_5\psi) = f\hat{n} \cdot \partial\eta + \frac{N_F g^2}{16\pi^2}\hat{n} \cdot K \tag{4}$$

where $\vec{E}^a$ and $\vec{B}^a$ are, respectively, the color electric and color magnetic fields. Here ψ is the QCD quark field.

A complete treatment calls for a full Casimir calculation of the gluon modes, which is highly subtle due to the p-wave structure of the η-field. Such a calculation is in progress [18] and will be reported in a later publication. Here we would like to side-step this technically difficult procedure by first assuming the CCP in evaluating the Casimir contribution with the color

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boundary conditions (2), (3) and (4) taken into account and check a posteriori that there is consistency between the assumption and the result.

The next section will define our formulation, recall our old results, and clarify the new contributions. Section 3 will focus on the gluon Casimir contribution to the FSAC, the major contribution of this presentation. Finally section 4 will contain the results, conclusions and the future prospects of ongoing work.

2   The Chiral Bag Formalism

Our aim is to calculate the FSAC in the CBM scenario. In order to do so we need a specific formulation of the model through its equations of motion and boundary conditions. The equations of motion have been shown repeatedly in our previous works [5, 6, 7] and the color boundary conditions were recalled in the introduction. We refer the reader to those references for a detailed discussion on their structure, their resolution and the implementation of gauge invariance and confinement. Our calculation will be carried out in the static spherical cavity approximation, that is, our bag will be a static sphere of radius R dividing two regions of space in which the theory is implemented by QCD for $r < R$, and by an effective meson theory for $r > R$.

2.1   The anomaly and proton spin

The anomalous suppression of the first moment, Γ_1^p, of the polarized proton structure function g_1^p has been the focus of intense theoretical and experimental activity for nearly a decade. While it is now generally accepted that the key to understanding this effect is the existence of the chiral $U(1)$ anomaly in the flavor singlet axial current there are several explanations reflecting different theoretical approaches to proton structure.

The starting point is the sum rule for the first moment, i.e.,

$$\Gamma_1^P(Q^2) \equiv \int_0^1 dx g_1^p(x, Q^2) = \frac{1}{12} C_1^{NS}(\alpha_s(Q^2)) \left(a^3 + \frac{1}{3} a^8 \right)$$

$$+ \frac{1}{9} C_1^S(\alpha_S(Q^2)) a^0(Q^2). \tag{5}$$

Here $C_1(\alpha_s)$ are first moments of the Wilson coefficients of the the singlet (S) and nonsinglet (NS) axial currents and α_s the perturbatively running QCD coupling constant. Moreover a^3, a^8 and $a^0(Q^2)$ are the form factors in the forward proton matrix elements of the renormalized axial current, i.e.,

$$\langle p, s | A_\mu^3 | p, s \rangle = s_\mu \frac{1}{2} a^3, \qquad \langle p, s | A_\mu^8 | p, s \rangle = s_\mu \frac{1}{2\sqrt{3}} a^8,$$

and

$$\langle p, s | A_\mu^0 | p, s \rangle = s_\mu a^0,$$

where p_μ and s_μ are the momentum and the polarization vector of the proton. a^3 and a^8 can be chosen Q^2 independent and may be determined from the $\frac{G_A}{G_B}$ and $\frac{F}{D}$ ratios. $a^0(Q^2)$ evolves due to the anomaly and its evolution can be described in the AB scheme [11] by

$$a^0(Q^2) = \Delta\Sigma - N_F \frac{\alpha_S(Q^2)}{2\pi} \Delta g(Q^2). \tag{6}$$

Naive models or the OZI approximation to QCD lead at low energies to

$$a^0 \approx a^8 \approx 0.69 \pm 0.06. \tag{7}$$

Experimentally[19]

$$a^0(\infty) = 0.10 \, ^{+0.17}_{-0.10}. \tag{8}$$

The explanation for this unexpected small value has given rise to many interpretations related to hadron structure, vacuum structure and evolution [20, 21, 22, 23].

2.2 The formalism

To obtain the FSAC, we need to calculate the matrix elements of the flavor singlet axial current. Let us write the current in the CBM as a sum of two terms, one from the interior of the bag and the other from the outside populated by the meson field η' (we will ignore the Goldstone pion fields for the moment; they will be taken into account for the baryon charge leakage)

$$A^\mu = A^\mu_B \Theta_B + A^\mu_M \Theta_M. \tag{9}$$

Since we will be dealing only with the flavor-singlet axial current, we will omit the flavor index in the current. We shall use the short-hand notations $\Theta_B = \theta(R - r)$ and $\Theta_M = \theta(r - R)$ with R being the radius of the bag. We demand that the $U_A(1)$ anomaly be given in this model by

$$\partial_\mu A^\mu = \frac{\alpha_s N_F}{2\pi} \sum_a \vec{E}^a \cdot \vec{B}^a \Theta_B + f m_\eta^2 \eta \Theta_M. \tag{10}$$

Our task is to construct the FSAC in the chiral bag model that is gauge-invariant and consistent with this anomaly equation. Our basic assumption is that in the nonperturbative sector outside of the bag, the only relevant $U_A(1)$ degree of freedom is the massive η' field. This assumption allows us to write

$$A^\mu_M = A^\mu_\eta = f \partial^\mu \eta \tag{11}$$

with the divergence

$$\partial_\mu A^\mu_\eta = f m_\eta^2 \eta. \tag{12}$$

Now the question is: what is the gauge-invariant and regularized A^μ_B such that the anomaly (10) is satisfied? To address this question, we rewrite the current (9) as

$$A^\mu = A^\mu_{B_Q} + A^\mu_{B_G} + A^\mu_\eta \tag{13}$$

such that

$$\partial_\mu (A^\mu_{B_Q} + A^\mu_\eta) = f m_\eta^2 \eta \Theta_M, \tag{14}$$

$$\partial_\mu A^\mu_{B_G} = \frac{\alpha_s N_F}{2\pi} \sum_a \vec{E}^a \cdot \vec{B}^a \Theta_B. \tag{15}$$

The subindices Q and G imply that these currents are written in terms of quark and gluon fields respectively. In writing (14), we have ignored the up and down quark masses. We should stress that since we are dealing with an interacting theory, there is no unique way to separate the different contributions from the gluon, quark and η components. In particular, the separation we adopt, (14) and (15), is non-unique although the sum is without ambiguity . We found however that this separation leads to a natural partition of the contributions in the framework of the bag description for the confinement mechanism that we are using here.
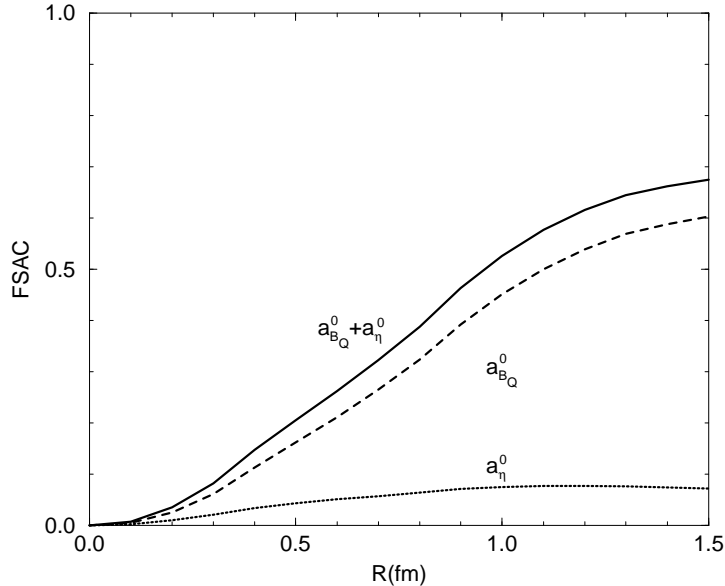
Figure 1: Various contributions to the flavor singlet axial current of the proton as a function of bag radius : (a) quark contribution A_{B_Q}; (b) η' contribution A_η and (c) the sum.

2.2.1 The *quark* current $A_{B_Q}^\mu$

The quark current is given by

$$A_{B_Q}^\mu = \bar{\Psi}\gamma^\mu\gamma_5\Psi \tag{16}$$

where Ψ should be understood to be the *bagged* quark field. Therefore the quark current contribution to the FSAC is given by

$$a_{B_Q}^0 = \langle p| \int_B d^3r \bar{\Psi}\gamma_3\gamma_5\Psi|p\rangle. \tag{17}$$

The calculation of this type of matrix elements in the CBM is nontrivial due to the baryon charge leakage between the interior and the exterior through the Dirac sea. But we know how to do this in an unambiguous way. A complete account of such calculations can be found in [6, 16, 15]. The leakage produces an R dependence which would otherwise not be there in the matrix element of Eq.(17), as shown in Fig. 1. It is significant that as seen in the figure there is no contribution for zero radius, that is in the pure skyrmion scenario for the proton. The contribution grows as a function of R towards the pure MIT result that would technically be reached for infinite radius. The result of this calculation was first presented in refs. [5, 6]. No new ingredient has been added.

2.2.2 The *meson* current A_η^μ

Since we shall not add anything new to our previous result obtained in [5, 6], we will just quote the result. Due to the coupling of the quark and η fields at the surface, we can simply write the

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η contribution in terms of the quark contribution,

$$a_\eta^0 = \frac{1 + y_\eta}{2(1 + y_\eta) + y_\eta^2} \langle p| \int_B d^3r \, \bar{\Psi}\gamma_3\gamma_5\Psi |p\rangle. \tag{18}$$

where $y_\eta = m_\eta R$. In Fig. 1 we show the radial dependence of this contribution, which arises from the charge leakage mechanism, and follows the quark distribution. Since the η field has no topological structure, its contribution also vanishes in the skyrmion limit. This illustrates how the dynamics of the exterior can be mapped to that of the interior by boundary conditions. We may summarize the analysis of these two contributions by stating that no trace of the CCP is apparent in Fig. 1. Thus if the CCP were to emerge, the only possibility would be that the gluons do the miracle!

2.2.3 The *gluon* current $A_{B_G}^\mu$

Understanding the FSAC and its implications in the present framework involves crucially the role of the gluon contribution, in particular its static properties and vacuum fluctuations, i.e., the Casimir effects. The calculation of the Casimir effects constitutes the principal aim of this work.

We begin by dividing the gluon current into two pieces

$$A_{B_G}^\mu = A_{G,stat}^\mu + A_{G,vac}^\mu. \tag{19}$$

The first term arises from the quark and η sources, while the latter is associated with the properties of the vacuum of the model. One might worry that this contribution could not be split in these two terms without double counting. That there is no cause for worry can be seen in several different ways. Technically, it is easy to check it by noticing that the former acts on the quark Fock space and the latter on the gluon vacuum. Thus, one can interpret the former as a one gluon exchane correction to the quanity. One can also show this intuitively by making the analogy to the condensate expansion in QCD [25], where the perturbative terms and the vacuum condensates enter additively to the lowest order.

Let us first describe the static term. We assume initially for simplicity that there is no η coupling. Then the boundary conditions for the gluon field would correspond to the original MIT ones [26]. The quark current is the source term that remains in the equations of motion after performing a perturbative expansion in the QCD coupling constant, i.e., the quark color current

$$g\bar{\Psi}_0\gamma_\mu\lambda^a\Psi_0 \tag{20}$$

where the Ψ_0 fields represent the lowest cavity modes. In this lowest mode approximation, the color electric and magnetic fields are given by

$$\vec{E}^a = g_s \frac{\lambda^a}{4\pi} \frac{\hat{r}}{r^2}\rho(r) \tag{21}$$

$$\vec{B}^a = g_s \frac{\lambda^a}{4\pi} \left(\frac{\mu(r)}{r^3}(3\hat{r}\vec{\sigma}\cdot\hat{r} - \vec{\sigma}) + (\frac{\mu(R)}{R^3} + 2M(r))\vec{\sigma} \right) \tag{22}$$

where ρ is related to the quark density ρ' as[2]

$$\rho(r, \Gamma) = \int_\Gamma^r ds\rho'(s) \tag{23}$$

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[2]Note that the quark density that figures here is associated with the color charge, *not* with the quark number (or rather the baryon charge) that leaks due to the hedgehog pion.

and μ, M to the vector current density

$$\begin{aligned}
\mu(r) &= \int_0^r ds \mu'(s), \\
M(r) &= \int_r^R ds \frac{\mu'(s)}{s^3}.
\end{aligned}$$

The lower limit Γ is taken to be zero in the MIT bag model – in which case the boundary condition is satisfied only *globally*, that is, after averaging – and $\Gamma = R$ in the so called *monopole solution* [6, 7] – in which case, the boundary condition is satisfied *locally*.

After having clarified the procedure by recalling our old calculation and pointing out the main difference of the present with respect to that one, we proceed to introduce the η field. We perform the same calculation with however the color boundary conditions Eqs.(2) and (3) taken into account. In the approximation of keeping the lowest non-trivial term, the boundary conditions become

$$\hat{r} \cdot \vec{E}_{stat}^a = -\frac{N_F g^2}{8\pi^2 f} \hat{r} \cdot \vec{B}_g^a \eta(R) \tag{24}$$

$$\hat{r} \times \vec{B}_{stat}^a = \frac{N_F g^2}{8\pi^2 f} \hat{r} \times \vec{E}_g^a \eta(R). \tag{25}$$

Here $\vec{E}_g^a$ and $\vec{B}_g^a$ are the lowest order fields [6, 7] given by (21) and (22) and $\eta(R)$ is the meson field at the boundary. The η field is given by

$$\eta(\vec{r}) = -\frac{g_{NN\eta}}{4\pi M} \vec{S} \cdot \hat{r} \frac{1 + m_\eta r}{r^2} e^{-m_\eta r} \tag{26}$$

where the coupling constant is determined from the surface conditions [6, 7].

Note that the magnetic field is not affected by the new boundary conditions, since $\vec{E}_g^a$ points into the radial direction. The effect on the electric field is just a change in the charge, i.e.,

$$\rho_{stat}(r) = \rho(r, \Gamma) + \rho_\eta(R) \tag{27}$$

where

$$\rho_\eta(R) = \frac{N_F g^2}{64\pi^3 M} \frac{g_{NN\eta}}{f} (1 + y_\eta) e^{-y_\eta}. \tag{28}$$

The contribution to the FSAC arising from these fields is determined from the expectation value of the anomaly

$$a_{G,stat}^0 = \langle p | - \frac{N_F \alpha_s}{\pi} \int_B d^3 r x_3 \vec{E}_{stat}^a \cdot \vec{B}_{stat}^a | p \rangle. \tag{29}$$

The result of this contribution is shown in Fig. 2, where we show the MIT solution, the *monopole* one and the correction associated to both due to the color coupling[3]. One sees that including the η contribution in $\rho_{stat}(r)$ brings a non-negligible modification to the FSAC but does not modify the result qualitatively. What is most striking is the drastic difference between the effect of the MIT-like electric field and that of the monopole-like electric field: The former is totally incompatible with the Cheshire Cat property whereas the latter remains consistent independently of whether or not the η contribution is included in ρ_{stat}.

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[3]We have also investigated electric fields of the form $(\frac{A}{r^2} + Br)\hat{r}$, but the results do not change much with respect to the ones shown since the B term tends to be small.
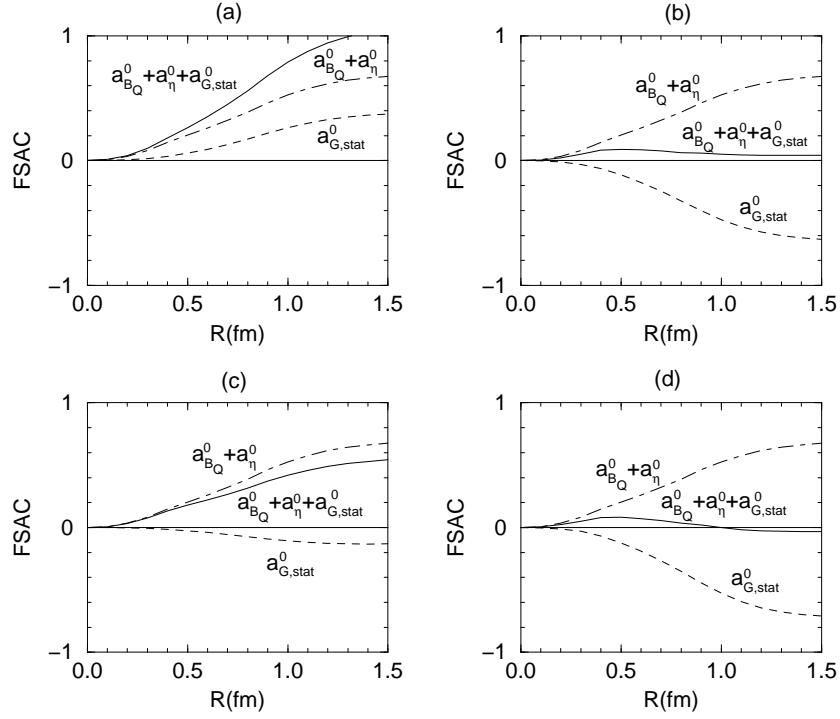
Figure 2: Dependence of $a^0_{G,stat}$ on the choice of Γ and the boundary conditions as a function of bag radius : (a) with an MIT-like electric field without η coupling, (b) with a *monopole*-like electric field without η coupling, (c) with an MIT-like electric field *with* η coupling, and (d) with a *monopole*-like electric field *with* η coupling.

Finally the $A_{G,vac}$ term arises from the so called Casimir effect of the anomaly term. The vacuum in the cavity and the perturbative vacuum are different due to the geometry of the cavity. This effect has been considered for many other observables and also for the quarks in this calculation [16, 15, 6], but never for the gluons. We proceed in the next section to describe this calculation.

3   The Gluon Casimir Calculation

The quantity that we wish to calculate is the gluon vacuum contribution to the flavor singlet axial current of the proton. It can be done by evaluating the expectation value

$$\langle 0_B| - \frac{N_F \alpha_s}{\pi} \int_V d^3 r x_3 (\vec{E}^a \cdot \vec{B}^a)|0_B\rangle \tag{30}$$

where $|0_B\rangle$ denotes the vacuum in the bag. The standard way to evaluate this expectation value would be to expand the field operators in terms of the classical eigenmodes that satisfy the equations of motion and the boundary conditions. Although well-defined, this approach is technically involved. We have not yet obtained any quantitative results to report. In this paper, we shall proceed in the opposite direction. Instead of arriving at the CCP as in the standard approach, we shall *assume* the CCP and evaluate the Casimir contribution with the expression that follows from the assumption. The idea goes as follows.

The CCP states that at low energy, hadronic phenomena do not discriminate between QCD degrees of freedom (quarks and gluons) on the one hand and meson degrees of freedom (pions, etas,...) on the other, provided that all necessary quantum effects (e.g., quantum anomalies) are properly taken into account. If we consider the limit where the η excitation is a long wavelength oscillation of zero frequency, the CCP asserts that it does not matter whether we choose to describe the η, in the interior of the infinitesimal bag, in terms of quarks and gluons or in terms of mesonic degrees of freedom. This statement, together with the color boundary conditions, leads to an extremely simple and useful *local* formula [27],

$$\vec{E}^a \cdot \vec{B}^a \approx -\frac{N_F g^2}{8\pi^2}\frac{\eta}{f}\frac{1}{2}G^2, \tag{31}$$

where only the term up to the first order in η is retained in the right-hand side. Here we adapt this formula to the CBM. This means that the couplings are to be understood as the average bag couplings and the gluon fields are to be expressed in the cavity vacuum through a mode expansion. In fact, by comparing the expression for the η' mass derived in [27] using Eq.(31) with that obtained by Novikov et al [28] in QCD sum-rule method, we note that the matrix element of the G^2 in (31) should be evaluated in the *absence* of light quarks. This means, in the bag model, the cavity vacuum. That the surface boundary condition can be interpreted as a local operator is a rather strong CCP assumption which while justifiable for small bag radius, can only be validated a posteriori by the consistency of the result. This procedure is the substitute to the condensates in the conventional discussion.

Substituting Eq.(31) into Eq.(30) we obtain

$$\langle 0_B| - \frac{N_F \alpha_s}{\pi} \int_V d^3 r x_3 (\vec{E}^a \cdot \vec{B}^a)|0_B\rangle$$

$$\approx \left(-\frac{N_F \alpha_s}{\pi}\right)\left(-\frac{N_F g^2}{8\pi^2}\right)\frac{y(R)}{f_0}\langle p|S_3|p\rangle\langle 0_B|\int_V d^3r \frac{1}{2}G^2 x_3 \hat{x}_3|0_B\rangle$$

$$\approx \left(-\frac{N_F \alpha_s}{\pi}\right)\left(-\frac{N_F g^2}{8\pi^2}\right)\frac{y(R)}{f_0}\langle p|S_3|p\rangle(N_c^2-1)$$

$$\sum_n \int_V d^3r(\vec{B}_n^* \cdot \vec{B}_n - \vec{E}_n^* \cdot \vec{E}_n)x_3 \hat{x}_3, \tag{32}$$

where we have used that η has a structure of $(\vec{S}\cdot\hat{r})y(R)$. Since we are interested only in the first order perturbation, the field operator can be expanded by using MIT bag eigenmodes (the zeroth order solution). Thus, the summation runs over all the classical MIT bag eigenmodes. The factor (N_c^2-1) comes from the sum over the abelianized gluons.

The next steps are the numerical calculations to evaluate the mode sum appearing in Eq.(32): (i) introduction of the heat kernel regularization factor to classify the divergences appearing in the sum and (ii) subtraction of the ultraviolet divergences.

3.1   Normalization of the eigenmodes

The classical eigenmode of the (abelianized) gluons confined in the MIT bag can be classified by the total spin quantum numbers (J, M) given by the vector sum of the orbital angular momentum $\vec{L}$ and the spin $\vec{S}$,

$$\vec{J} \equiv \vec{L} + \vec{S}, \tag{33}$$

and the radial quantum number n. There are two kinds of classical eigenmodes according to the relations between the parity and the total spin; (i) M-mode with the parity $\pi = -(-1)^J$ and (ii) E-mode with the parity $\pi = -(-1)^{J+1}$. Here, the extra minus sign is due to the negative intrinsic parity of gluon.

We will work with the vector fields with the gauge choice,

$$G_0 = 0, \text{ and } \vec{\nabla}\cdot\vec{G} = 0. \tag{34}$$

Then the electric field and the magnetic field are obtained through the relations

$$\vec{E} = -\frac{\partial\vec{G}}{\partial t}, \tag{35}$$

$$\vec{B} = \vec{\nabla}\times\vec{G}. \tag{36}$$

Explicitly, the solutions are obtained as

(i) M-modes :

$$\vec{G}^{(M)}_{(n,J,M)}(\vec{r}) = \mathcal{N}_M j_J(\omega_n r)\vec{Y}_{J,J,M}(\hat{r}), \tag{37}$$

(ii) E-modes :

$$\vec{G}^{(E)}_{(n,J,M)}(\vec{r}) = \mathcal{N}_E\left[-\sqrt{\frac{J}{2J+1}}j_{J+1}(\omega_n r)\vec{Y}_{J,J+1,M}(\hat{r})\right. \tag{38}$$

$$\left.+\sqrt{\frac{J+1}{2J+1}}j_{J-1}(\omega_n r)\vec{Y}_{J,J-1,M}(\hat{r})\right], \tag{39}$$

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where $\vec{Y}_{J,\ell,M}$ is the vector spherical harmonics of the total spin J composed of the angular momentum ℓ and $j_\ell(x)$ is the spherical Bessel functions. The energy eigenvalues are determined to satisfy the MIT boundary conditions as

(i) M-modes :

$$X_n j'_J(X_n) + j_J(X_n) = 0, \tag{40}$$

(ii) E-modes :

$$j_J(X_n) = 0 \tag{41}$$

where we have defined $X_n = \omega_n R$. The normalization constants $\mathcal{N}_{M,E}$ will be specified below.

The field operator $\vec{G}(\vec{r}, t)$ is expanded in terms of the classical eigenmodes as

$$\vec{G}(\vec{r}, t) = \sum_{\{\nu\}} \left(a_{\{\nu\}} \vec{G}_{\{\nu\}}(\vec{r}) e^{-i\omega_n t} + a^\dagger_{\{\nu\}} \vec{G}^*_{\{\nu\}}(\vec{r}) e^{+i\omega_n t} \right), \tag{42}$$

where $\{\nu\}$ denotes the quantum number set $(n, J, M, \lambda = \text{E or M})$.

We determine the normalization constants $\mathcal{N}_{M,E}$ in such a way that the free gluon Hamiltonian operator

$$H = \frac{1}{2} \int_B d^3r (\vec{E} \cdot \vec{E} + \vec{B} \cdot \vec{B}) \tag{43}$$

becomes

$$H = \sum_{\{\nu\}} \omega_{\{\nu\}} a^\dagger_{\{\nu\}} a_{\{\nu\}} \tag{44}$$

when Eq.(42) is substituted into Eq. (43). It leads to a normalization condition for the classical eigenmodes as

$$\int_B d^3r \vec{G}^*_{\{\nu\}} \cdot \vec{G}_{\{\mu\}} = \frac{1}{2\omega_{\{\nu\}}} \delta_{\{\nu\}\{\mu\}}. \tag{45}$$

Then the normalization constants are determined explicitly as

$$\mathcal{N}_M = \left[X_n R^2 \left(j_J^2(X_n) - j_{J-1}(X_n) j_{J+1}(X_n) \right) \right]^{-1/2}, \tag{46}$$

$$\mathcal{N}_E = \left[X_n R^2 j_{J-1}^2(X_n) \right]^{-1/2}. \tag{47}$$

3.2 Matrix elements

The first step is to calculate the matrix elements

$$Q_{\{\nu\}} \equiv \int_B d^3r (\vec{B}^*_{\{\nu\}} \cdot \vec{B}_{\{\nu\}} - \vec{E}^*_{\{\nu\}} \cdot \vec{E}_{\{\nu\}}) x_3 \hat{x}_3. \tag{48}$$

¿From Eq.(37), we obtain

$$\vec{E}_{\{\nu\}}(\vec{r}) = (+i\omega_n) \mathcal{N}_M j_J(\omega_n r) \vec{Y}_{J,J,M}(\hat{r}), \tag{49}$$

$$\vec{B}_{\{\nu\}}(\vec{r}) = (+i\omega_n) \mathcal{N}_M \left[-\sqrt{\frac{J}{2J+1}} j_{J+1}(\omega_n r) \vec{Y}_{J,J+1,M}(\hat{r}) \right. \tag{50}$$

$$\left. + \sqrt{\frac{J+1}{2J+1}} j_{J-1}(\omega_n r) \vec{Y}_{J,J-1,M}(\hat{r}) \right], \tag{51}$$
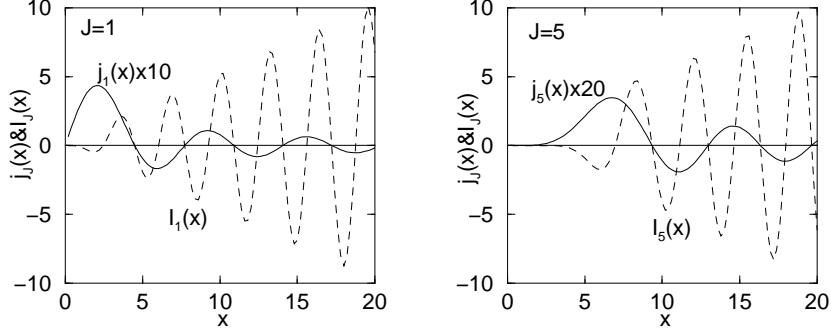
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Figure 3: $j_J(x)$ and $I(x)$ as a function of x.

for the M-modes and the similar equations with $\vec{E}$ and $\vec{B}$ being interchanged for the E-modes.

We encounter in the calculation the following angular integrals

$$\int d\Omega \vec{Y}^*_{J,\ell,M} \cdot \vec{Y}_{J,\ell,M} \hat{x}_3^2. \tag{52}$$

By using that $\hat{x}_3^2 = (4/3)\sqrt{\pi/5}Y_{20} + 1/3$ and the Wigner-Eckart theorem, we obtain

$$\int d\Omega \vec{Y}^*_{J,\ell,M} \cdot \vec{Y}_{J,\ell,M} \hat{x}_3^2 = c_{J,\ell}(J(J+1) - 3M^2) + \frac{1}{3}, \tag{53}$$

where $c_{J,\ell}$ is a constant that depends only on J and ℓ. We have to perform the summation over M, which runs from $-J$ to J, which cancels the contribution of the first term, therefore we can take *effectively* $1/3$ as the result of the integral.

Finally, we obtain the matrix elements for the M-modes as

$$Q_n^{(M)} = \frac{1}{3} \frac{\int_0^{X_n} x^3 dx \left[j_J^2(x) - \frac{J}{2J+1} j_{J+1}^2(x) - \frac{J+1}{2J+1} j_{J-1}^2(x) \right]}{X_n^3 \left[j_J^2(X_n) - j_{J-1}(X_n) j_{J+1}(X_n) \right]}. \tag{54}$$

In the case of the E-mode, we obtain exactly the same formula except the minus sign in front of it. (Note that the formulas for the electric field and the magnetic field are interchanged.)

We have found that the matrix elements for the E-mode vanish up to our numerical accuracy as shown in Fig. 3. Here, the solid line is the spherical Bessel function $j_J(x)$ and the dashed line is the integral

$$I(x) \equiv \int_0^x y^3 dy \left[j_J^2(y) - \frac{J}{2J+1} j_{J+1}^2(y) - \frac{J+1}{2J+1} j_{J-1}^2(y) \right] \tag{55}$$

We see that the zeroes of $I(x)$ and $j(x)$ coincide, thus showing that $Q_n^{(E)}(X_n) = 0$. We have been unable however to prove this result analytically, except the trivial case of $J = 0$.

3.3 The mode sum

In order to regularize the mode sum, we introduce a heat kernel factor $\exp(-\tau X_n)$;

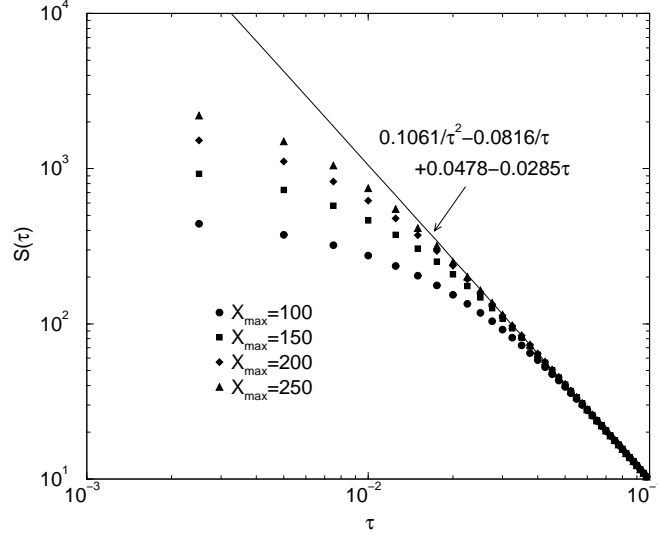$$S(\tau) \equiv \sum_{n,J} (2J+1) Q_{n,J}^{(M)} e^{-\tau X_n}, \tag{56}$$

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Figure 4: Diverging properties of $S(\tau)$ as a function of the heat kernel regularization parameter τ. All the magnetic modes up to $\omega_n R (\equiv X_n)=100$(solid circle), 150(solid square), 200(solid diamond) and 250(solid triangle) are included in the sum.

where we have carried out the trivial sum over M and the vanishing E-mode contribution is excluded.

Fig. 4 shows the numerical results of the sum up to $X_{max}=100$, 150, 200, 250 for the 40 values of τ from 0.0025 to 0.1 with the step 0.0025. We can see that below $\tau < 0.06$ the convergence is poor. However, it is enough to see the presence of an $1/\tau^2$ divergence. If we fit the data above $\tau > 0.06$, we obtain

$$S(\tau) = \frac{0.1061}{\tau^2} - \frac{0.0816}{\tau} + 0.0478 - 0.0285\tau. \tag{57}$$

Apart from a possible logarithmic divergence, there are quadratic and linear divergences as we set τ equal to zero . We shall remove these divergences following a procedure commonly used in Casimir problems [29]. Caveat on this procedure will be highlighted in the discussion section. Now if we neglect logarithmic divergences that might be present , the best way to get rid of the quadratic and linear divergences is to evaluate

$$S(\tau) + 2\tau S'(\tau) + \frac{1}{2}\tau^2 S''(\tau) = \sum_{n,J}(2J+1)Q_{n,J}(1 - 2\tau X_n + 0.5\tau^2 X_n^2)e^{-\tau X_n}. \tag{58}$$

Fig. 5 show the results on this quantity for 80 values of τ ranging from 0.0025 to 1. We see that no serious divergences appear anymore. By fitting the convergent data with the above expressions for τ, we obtain for the finite part of the sum 0.0478, from the cubic function fit, and 0.0456, from the quadratic one. These results are comparable to the finite term of the above naive fitting procedure (57), which yielded 0.0478.

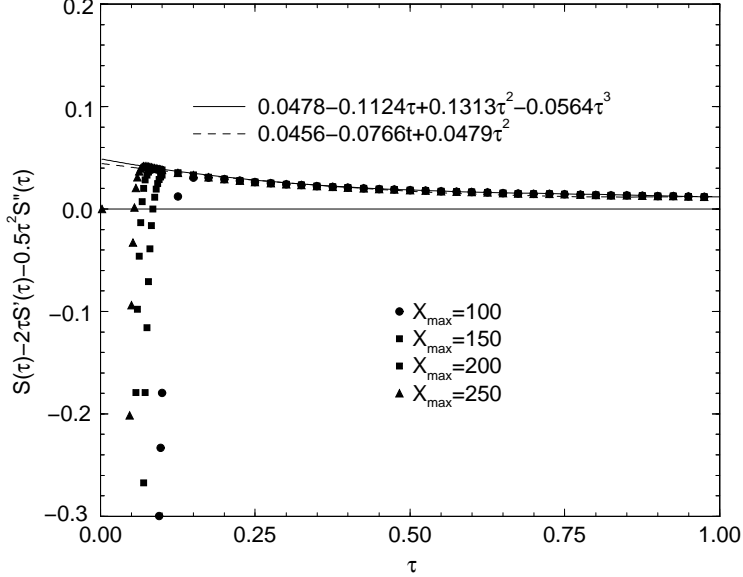Once we have the numerical value on the mode sum, the gluon vacuum contribution to FSAC

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Figure 5: $S(\tau) - 2\tau S'(\tau) + \frac{1}{2}\tau^2 S''(\tau)$ as a function of τ. The finite term of $S(\tau)$ is extracted by fitting these quantities to a cubic and quadratic curves.

can be evaluated simply as

$$a_{G,vac}^0 = -\frac{(2.10)^2}{2} \times \frac{8}{2} \times \frac{y(R)}{122\text{MeV}} \times (0.0478), \tag{59}$$

where $y(R)$ is related to $a_{B_Q}^0$ as

$$y(R) = -\frac{3m_\eta^2}{8\pi f_\eta} \frac{(1 + m_\eta R)}{[2(1 + m_\eta R) + (m_\eta R)^2](m_\eta R)^2} a_{B_Q}^0. \tag{60}$$

We have used $N_F = N_c = 3$, $\alpha_s = 2.2$, $f_0 = \sqrt{N_F/2} f_{\eta'} \sim 122\text{MeV}$ and $m_\eta = 958$ MeV.

4   Results and Conclusions

Our numerical results are given in Fig.6. Standard MIT bag parameters were used for the calculation. The quarkish component of the FSAC is given by the sum of the quark and η contributions, $a_{B_Q}^0 + a_\eta^0$ and the gluonic component by $a_{G,stat}^0 + a_{G,vac}^0$. Both increase individually as the confinement size R is increased but the sum remains small, $0 < a_{total}^0 < 0.3$ for the whole range of radii, consistent with the experiment, $a^{exp} = a^0(\infty) = 0.10_{-0.10}^{+0.17}$. It is remarkable that $a(R = 0) \simeq a(R \approx 1.5 \text{ fm})$ while each component can differ widely for the two extreme radii.

We have shown that the principal agent for the observed small FSAC in the proton in the framework of the chiral bag model is the Cheshire Cat phenomenon (CCP). It is the CCP that assures the cancellation between two contributions, one from the quarkish component and the
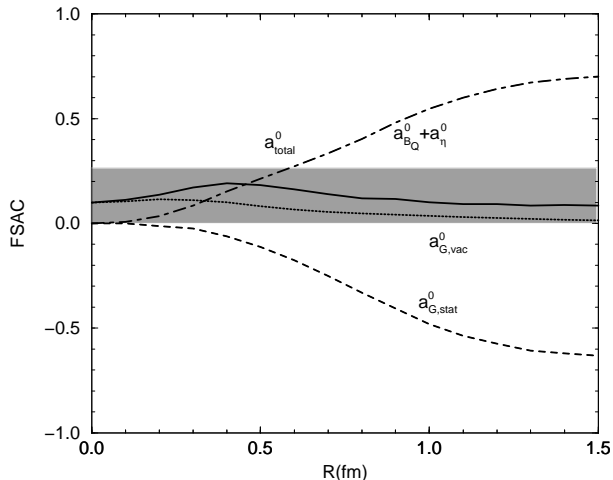
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Figure 6: Various contributions to the flavor singlet axial current of the proton as a function of bag radius and comparison with the experiment: (a) quark plus η contribution ($a^0_{B_Q} + a^0_\eta$), (b) the contribution of the static gluons due to quark source ($a^0_{G,stat}$), (c) the gluon vacuum contribution ($a^0_{G,vac}$), and (d) their sum (a^0_{total}). The shaded area corresponds to the range admitted by experiments.

other from the gluonic component in the particular way the separation is made. For a small bag radius, both components are small, so the net FSAC being small is immediate. This is consistent with the observation that in the limit that $R \to 0$, we recover the skyrmion description which gives a vanishing FSAC at the leading order, modified by matter fields at the next order. At large bag radius which leads to the MIT bag model, both the quarkish contribution and the gluonic contribution are large but they cancel. Our assertion is that this cancellation is caused by the CCP. We should however recall that the separation between the quarkish component and the gluonic component we adopted in (14) and (15) is entirely arbitrary although the sum is unique. *Whether the separate component by itself is large or small has no physical meaning. Only the total does.* Different separations would lead to different scenarios leading to the same small value. It is plausible that in some limit – unknown to us – the FSAC would be exactly zero with the finite nonzero value indicating a departure from this limit. Understanding this limit would allow a unique separation of the components.

One of the principal results of this paper is that *it is possible to have a nonzero value for the FSAC at $R = 0$ and is of the same size as at large R*[4]. While the effect of the surface color anomaly term is generally small for all radii, the finite nonzero value of FSAC for $R = 0$ is assured by the surface boundary term. Thus the violation of the CCP observed in the previous calculations at $R = 0$ [6, 7] is neatly eliminated by the color anomaly boundary condition. More importantly, the *monopole* structure of the color electric field previously proposed is found to be required for the sign that comes with the important static gluonic contribution from the quark

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[4]The reason for this nonzero value is intimately connected with the CCP, since it is the finite part of the gluon mode sum which normalizes the value of this contribution at the origin. Moreover the color boundary condition provides us with a decreasing η field contribution which changes softly as a function of R.

source. We believe that this cancellation is a manifestation in the bag scenario of the recently discovered one for QCD [21]. The MIT configuration would strongly violate the CCP. We are thus led to the conclusion that the CCP *requires* the monopole configuration for the color electric field. Whether or not this configuration leaves undisturbed other – successful – phenomenology was discussed in [6].

In calculating the gluonic Casimir effect, we *made* the *ab initio* assumption that the CCP holds, an assumption which is expected to be valid for small bag radius. We then extend it, in accordance with the CCP, to all bag radii. We can justify this only a posteriori by showing that the CCP assumption is consistent with what one gets out. Note however that the gluonic Casimir effect is most significant for small R where it is needed for the CCP and plays little role for large R. Thus our assumption is validated. It would of course be more satisfying if one could obtain the CCP as an output of the formalism, not put in as an input. Such a calculation is in progress.

We should mention a caveat left unspecified in the text in regularizing this Casimir contribution. Since $a_{G,vac}^0$ vanishes when the η field is removed, the so-called "vacuum contribution" is duly subtracted in what we have computed. However we have also explicitly subtracted quadratic and linear divergences appearing from the mode sum by resorting to a procedure used in the past in most of Casimir-type calculations [29] which as far as we know, is physically reasonable but has not yet been rigorously justified from first principles. The same caveat applies to our calculation as it does to others. The finite term we have obtained might therefore be subject to additional finite corrections. In this paper we have invoked the Cheshire Cat Principle to ignore such corrections in $a_{G,vac}^0$. We hope that the calculation in progress [18] will eliminate this ambiguity.

Given the caveat mentioned above and the approximations used, our result can at best be qualitative. A better treatment (such as a more realistic gauge coupling constant running with the bag size, a more accurate calculation of $a_{G,vac}^0$ etc.) might modify the result quantitatively. Even so, we believe it to be quite robust that the overall FSAC is small, $\stackrel{<}{\sim} 0.3$ and that it is more or less independent of the confinement size.

Acknowledgments

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