Hadron correlators and the structure of the quark propagator *

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Abstract

The structure of the quark propagator of QCD in a confining background is not known. We make an Ansatz for it, as hinted by a particular mechanism for confinement, and analyze its implications in the meson and baryon correlators. We connect the various terms in the Källen-Lehmann representation of the quark propagator with appropriate combinations of hadron correlators, which may ultimately be calculated in lattice QCD. Furthermore, using the positivity of the path integral measure for vector like theories, we reanalyze some mass inequalities in our formalism. A curiosity of the analysis is that, the exotic components of the propagator (axial and tensor), produce terms in the hadron correlators which, if not vanishing in the gauge field integration, lead to violations of fundamental symmetries. The non observation of these violations implies restrictions in the space-time structure of the contributing gauge field configurations. In this way, lattice QCD can help us analyze the microscopic structure of the mechanisms for confinement.

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1 Introduction

The structure of the quark propagator of QCD in a confining background has been a matter of debate over the years [1]. The various mechanisms for confinement hint different types of vacua and therefore different quark propagators [1, 2]. In particular the electric vortex mechanism (see Appendix A) provides us with a quark propagator with the following Källen-Lehmann representation,

$$S(x,y) = s(x,y) + v_\mu(x,y)\gamma_\mu + a_\mu(x,y)\gamma_\mu\gamma_5 + t_{\mu\nu}(x,y)\sigma_{\mu\nu} \tag{1}$$

We shall take this as an Ansatz for the structure of the quark propagator in a background to be used in the formalism of the so called QCD inequality approach.

It was realized long after QCD was formulated that one could derive some exact inequalities between hadron masses [3] and other observables [4]. The key element in deriving them is that the Euclidean fermion determinant in vector like gauge theories (such as QCD) is positive definite and so the measure

$$d\mu = Z^{-1} DA^a_\mu(x) det(i\not{D} + M) \exp{(-\frac{1}{2g^2}\int d^4x Tr F^2_{\mu\nu})} \tag{2}$$

for the A^a_μ integration obtained after integrating out the fermions is positive definite for $\Theta = 0$. Note that $\not{D} = \gamma^\mu D_\mu$, D_μ being the covariant derivative. Inequalities that hold pointwise continue to hold after integrating with respect to a positive measure. Thus any inequality among matrix elements that holds after performing the Fermi integral in a fixed background gauge field holds in the exact theory. The continuous formulation requires from an appropriate regularization scheme [5]. The great advantage of this procedure is that one sums over positive contributions weighted by a positive measure and therefore possible cancellations between different gauge configurations are not worrysome.

The aim of this paper is to analyze various consequences of Eq.(1) within the inequality approach for QCD [1]. Our interest is twofold. On the one hand we shall discuss properties of QCD, i.e., chiral symmetry realization, mass relations,..., as if the above Ansatz were the outcome of the true calculation. On the other we shall relate the

[1]Note that our analysis may be applied to any representation of the quark propagator. We have just chosen the above equation, because it is hinted by a specific mechanism of confinement and because it is sufficiently rich to allow the most general analysis.

terms in the Ansatz to hadron correlators, which can ultimately be calculated in lattice QCD. Finally we shall discuss observable consequences of the exotic terms in the Källen-Lehmann representation of the quark propagator, which imply, to avoid violation of fundamental symmetries, a strong restriction of the space-time structure of the contributing gauge field configurations.

2 The structure of the mesonic correlators

Mass inequalities have been obtained among the mesons and comparing baryons with mesons [3]. The important property in these calculations has been

$$S^{+}(x,y) = \gamma_5 S(y,x)\gamma_5 \tag{3}$$

where $S(x,y)$ is the quark propagator in a background. Our aim is to discuss mass relations also among baryons. In this case due to their current quark constituency the previous property is of no use. Some fine details of the structure of the quark propagator and of the baryon currents will be necessary to be able to address the issue. We proceed thus with the same technique in both cases, only that in the meson case we will use of this simplification.

The meson correlators in terms of meson fields are given by [2]

$$\begin{aligned}
< \sigma(x)\sigma(y) > &= -\int d\mu Tr(\gamma_5 S^{+}(x,y)\gamma_5 S(x,y)) &(4)\\
< \pi(x)\pi(y) > &= \int d\mu Tr(S^{+}(x,y)S(x,y)) &(5)\\
< \rho_\mu(x)\rho_\nu(y) > &= \int d\mu Tr(\gamma_5 S^{+}(x,y)\gamma_5\gamma_\mu S(x,y)\gamma_\nu) &(6)\\
< \alpha_\mu(x)\alpha_\nu(y) > &= -\int d\mu Tr(S^{+}(x,y)\gamma_\mu S(x,y)\gamma_\nu) &(7)\\
< \tau_{\mu\nu}(x)\tau_{\lambda\varphi}(y) > &= -\int d\mu Tr(\gamma_5 S^{+}(x,y)\gamma_5\sigma_{\mu\nu} S(x,y)\sigma_{\lambda\varphi}) &(8)
\end{aligned}$$

According to our previous discussion one should investigate the properties of these correlators for the quark propagator in the presence of a background field. The substitution of Eqs.(1) and (3) into Eqs.(4) through (8) provides us with the structure of the mesonic

[2]Since global numerical factors are of no relevance for our discussion, we later on normalize the measure $d\mu$ so that the coefficient of the scalar term is unity for the trace correlator.

correlators. With the help of Mathematica and HIP [6] the calculation is straightforward (see Appendix B). We discuss here some of the properties of the arising structures.

It was noticed some time ago [7] that the difference between the sigma and pion correlators

$$< \pi\pi > - < \sigma\sigma >= \int d\mu(|s|^2 + 2|t|^2) \tag{9}$$

could be non vanishing if anomalous structures were present in the quark propagator. In such a case chiral symmetry would be broken by the mechanism leading to these structures. In our case the electric vortex contributes both to s and $t_{\mu\nu}$ leading to the spontaneous breaking of chiral symmetry. However this contributions are proportional to the fermion mass, thus our mechanism could never explain the spontaneous breaking of chiral symmetry in a massless theory. However this is not an inconvinience, since as can be seen in ref.[9], the mass plays also a crucial role in more fundamental approaches. In the present model the restoration of chiral symmetry and deconfinement would occur at the same scale. However this statement has to be taken with precaution due to our simplified scenario. It could happen that other mechanisms, like for example instanton effects, could modify the conclusions [10].

In our formalism correlator inequalities leading to mass relations can be constructed in an explicit fashion. One can easily see that the pion has the lowest mass since the right hand sides of the following equations are positive definite

$$\begin{aligned}
< \pi\pi > - < \sigma\sigma > &= 2\int d\mu(|s|^2 + 2|t|^2) &(10)\\
< \pi\pi > -\frac{1}{4} < \rho\rho > &= \frac{1}{2}\int d\mu(|v|^2 + 3|a|^2) &(11)\\
< \pi\pi > -\frac{1}{4} < \alpha\alpha > &= 2\int d\mu(|s|^2 + \frac{1}{2}|v|^2 + \frac{3}{2}|a|^2) &(12)\\
< \pi\pi > -\frac{1}{12} < \tau\tau > &= \int d\mu(|v|^2 + |a|^2 + 3|t|^2) &(13)
\end{aligned}$$

Moreover one can isolate from the different correlators the various contributions. In particular the scalar term simply states that the axial meson has a bigger mass than the vector meson

$$< \rho\rho > - < \alpha\alpha >= 8\int d\mu|s|^2 \tag{14}$$

The rest are less instructive

$$< \pi\pi > +\frac{1}{8} < \rho\rho > +\frac{3}{8} < \alpha\alpha >= \int d\mu|v|^2 \tag{15}$$

$$< \pi\pi > -\frac{3}{8} < \rho\rho > -\frac{1}{8} < \alpha\alpha >= 2\int d\mu |a|^2 \tag{16}$$

$$< \rho\rho > - \quad < \alpha\alpha > -\frac{2}{3} < \tau\tau >= 8\int d\mu |t|^2 \tag{17}$$

The last equation, together with Eq.(14), tell us that the vector meson mass is smaller than the tensor meson mass. The remaining confirm the fact that the pion has the lowest mass among the mesons.

A corolary of our relations is that by studying the mesonic correlators one may be able to disentangle the existence or non-existence of anomalous terms. Therefore one should try to understand the structure of the full quark propagator *experimentally* (lattice QCD).

Once this structures have been unveiled the analysis of the various terms in the correlators becomes very rich. In first place terms with $\varepsilon_{\mu\nu\rho\sigma}$ appear which have to vanish after gluon integration if the theory is to be Poincaré invariant. This implies already a strong restriction on the space-time structure of the contributing gauge fields. However in the tensor meson correlator contributions of the form

$$\int d\mu \varepsilon_{\mu\nu\rho\sigma}(a^* \cdot v + a \cdot v^*) \tag{18}$$

$$\int d\mu \varepsilon_{\mu\nu\rho\alpha}(a^*_\alpha v_\rho \pm a_\alpha v^*_\rho) \tag{19}$$

$$\int d\mu \varepsilon_{\mu\nu\rho\alpha}(a_\rho v^*_\alpha \pm a^*_\rho v_\alpha) \tag{20}$$

arise, which are perfectly compatible with Poincaré invariance, and if non vanishing, signal the violation of CP and P invariance. If the latter are not observed, as it seems, or very small, then this will also imply further restrictions on the space-time structure of the allowed gauge field configurations.

3 The structure of the baryonic correlators

The first step in our development is the construction of the baryon currents from the constituents. We shall restrict ourselves to composite operators with baryon quantum numbers and with the least possible dimension. This leads to currents proportional to the fields without derivatives [11].

The quark fields are Dirac spinors and therefore belong to the $\mathcal{D}^{+}_{\frac{1}{2},0}$ representation of the Lorentz group, where

$$\mathcal{D}^{+}_{\frac{1}{2},0} = \mathcal{D}_{\frac{1}{2},0} \oplus \mathcal{D}_{0,\frac{1}{2}} \tag{21}$$

The baryon currents are obtained by reducing the product of three Dirac fields

$$\Psi^{fa}_{\alpha} \otimes \Psi^{gb}_{\beta} \otimes \Psi^{hc}_{\gamma} \tag{22}$$

where α, β, γ denote the spinor, a, b, c the color and f, g, h the flavor indices. The reduction is not a trivial exercise [12]. Our result, reproducing that of Dosch et al. [13], is

Proton:

$$\begin{aligned} &Au^{T}(x)\mathcal{C}\gamma_5 d(x)\gamma_\mu u_\lambda(x) + Bu^{T}(x)\mathcal{C}d(x)\gamma_5\gamma_\mu u_\lambda(x) \\ &+ Cu^{T}(x)\mathcal{C}\gamma_5\gamma_\rho d(x)(\delta^{\mu\rho} - \frac{1}{4}\gamma^\mu\gamma^\rho)u_\lambda(x) \end{aligned} \tag{23}$$

Delta^{++}:

$$Du^{T}\mathcal{C}\gamma^{\mu}u\gamma^{\nu}u_\lambda + Eu^{T}\mathcal{C}\sigma^{\mu\nu}uu_\lambda \tag{24}$$

where A,B,... are independent constants. The argument of Espriu et al. [14] is that in order to preserve the same order in momentum, C=E=0, since the projection operator to the $\mathcal{D}^{+}_{\frac{3}{2},0}$ representation depends on momentum, a statement which is certainly true in the free case. Accepting this argument one recovers Ioffe's result which can be rewritten by appropriate Fierzing as [14]

Proton:

$$u^{T}\mathcal{C}\gamma_5 du_\lambda + \xi u^{T}\mathcal{C}d\gamma_5 u_\lambda \tag{25}$$

Delta^{++}:

$$u^{T}\mathcal{C}\gamma^{\mu}uu_\lambda \tag{26}$$

with the caveat that although ξ is in principle arbitrary, in the case of chiral symmetry it has the value $\xi = -1$.

The calculation of the correlators gives for the proton

$$< P_\mu(x)\bar{P}_\nu(y) > = <>_{11} + \xi(<>_{12} + <>_{21}) + \xi^2 <>_{22} \tag{27}$$

where

$$<>_{11} \;=\; \int d\mu\{(\gamma_5 S(x,y)\mathcal{C}S^{T}(x,y)\mathcal{C}S(x,y)\gamma_5)_{\mu\nu} - Tr(S(x,y)\mathcal{C}S^{T}(x,y)\mathcal{C})(\gamma_5 S(x,y)\gamma_5)_{\mu\nu}\}$$

$$
\begin{aligned}
<>_{12} &= \int d\mu\{(S(x,y)\mathcal{C}S^T(x,y)\mathcal{C}\gamma_5 S(x,y)\gamma_5)_{\mu\nu} - Tr(S(x,y)\mathcal{C}S^T(x,y)\mathcal{C}\gamma_5)(S(x,y)\gamma_5)_{\mu\nu}\} \\
<>_{21} &= \int d\mu\{(\gamma_5 S(x,y)\gamma_5\mathcal{C}S^T(x,y)\mathcal{C}S(x,y))_{\mu\nu} - Tr(S(x,y)\gamma_5\mathcal{C}S^T(x,y)\mathcal{C})(\gamma_5 S(x,y))_{\mu\nu}\} \\
<>_{22} &= \int d\mu\{(S(x,y)\gamma_5\mathcal{C}S^T(x,y)\mathcal{C}\gamma_5 S(x,y))_{\mu\nu} - Tr(S(x,y)\gamma_5\mathcal{C}S^T(x,y)\mathcal{C}\gamma_5)S(x,y)_{\mu\nu}\}
\end{aligned} \tag{28}
$$

and for the Delta^{++}

$$
\begin{aligned}
< \Delta^\alpha_\mu(x)\bar{\Delta}^\beta_\nu(y) >= \int d\mu\{2(S(x,y)\gamma^\beta\mathcal{C}S^T(x,y)\mathcal{C}\gamma^\alpha S(x,y))_{\mu\nu} \\
- Tr(S(x,y)\gamma^\beta\mathcal{C}S^T(x,y)\mathcal{C}\gamma^\alpha)S(x,y)_{\mu\nu}
\end{aligned} \tag{29}
$$

We next take the equation for the propagator Eq.(1) into the equations of the baryonic correlators. The calculation can be easily performed with Mathematica and HIP [6] but the result is too messy to be shown here (in Appendix C we show some of the terms for the proton). We proceed to discuss certain features which are relevant.

Let us discuss the mass relations. The diagonal correlators are given by

$$
\frac{1}{4} < P\bar{P} >= \int d\mu\{s(s^2 - \frac{1}{2}v^2 + \frac{1}{2}a^2 - 2t^2) + \varepsilon_{\mu\nu\lambda\varphi}a_\mu v_\nu t_{\lambda\varphi}\} \tag{30}
$$

and

$$
\frac{1}{16} < \Delta\bar{\Delta} >= \int d\mu\{s(s^2 + v^2 - a^2 + \frac{2}{3}t^2) + \frac{2}{3}\varepsilon_{\mu\nu\lambda\varphi}a_\mu v_\nu t_{\lambda\varphi}\} \tag{31}
$$

It is immediate to realize that the terms on the right hand side are individually not of definite sign and therefore the possibility of obtaining adequate relations leading to mass inequalities diminishes considerably. An appropriate choice which leads after repeated use of Hölder's and Schwarz' inequalities to a meaningful bound is

$$
|\frac{1}{6} < P\bar{P} > -\frac{1}{16} < \Delta\bar{\Delta} > | \leq | < \pi\pi > |^{\frac{3}{2}} \tag{32}
$$

and therefore the following bound arises

$$
m_{Baryon} \geq \frac{3}{2}m_\pi \tag{33}
$$

We cannot say however which of the two baryons is the heaviest, since all meaningful bounds imply modulus of differences of the correlators like in Eq.(32).

If one looks at the full correlators, anomalous terms (pseudoscalar, axial and tensor) appear both in the the proton and the delta correlators. The tensor terms should vanish after integration if Poincaré invariance is to hold. The pseudoscalar and axial, if not vanishing after integration, signal a violation of CP and P in the strong interaction [8].

4 Conclusion

Within a plausible scenario for confinement [17, 18] we have discussed the most general possible structure of the quark propagator. Using a current description for the hadrons we have calculated their correlators, limiting ourselves to those corresponding to low lying hadrons [3]. Our calculation has remained qualitative because we have not been able, despite many efforts, to find a solvable model á la Schwinger. However we have laid down the formalism for a quantitative analysis via lattice calculations. Furthermore the formalism is independent of the particular structure chosen for the quark propagator.

We have recalculated some of the correlator inequalities leading to mass relations and chiral symmetry realization in an effort to show the contribution to these observables arising from the so called anomalous terms. In this way we have related these terms to hadron-hadron correlators which are in principle calculable.

We have shown explicitly that the appearence of anomalous terms is not a peculiarity of the formalism, but a quite general phenomena of Schwinger's equations. The existence of pseudoscalar, axial and tensor terms has important implications from the physical point of view. They may be instrumental in describing the realization of chiral symmetry [7] and moreover might lead to observable consequences associated with the violation of discrete symmetries [8].

To eliminate these anomalous components from the fermion propagators before gluon integration one needs structureless color fields. The elimination through the integration requires high gluon entropy. The observed feature of none or very small violation of discrete symmetries by the strong interaction implies necessarily a strong dynamical restriction on the possible confinement mechanisms. It would be very advisable to have, no matter how naive, a solvable model that would instruct us on how these facts restrict the structure of the color fields. In this way we could forsee questions to be asked to the more exact, but less intuitive lattice calculations. In the meantime we have to resort to qualitative features and hopefully *experimental* observations.

[3]Recently [16] similar techniques have been applied to heavy quark baryons.

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Appendices

A The structure of the quark propagator in an electric vortex

The confinement phase of QCD can be understood as a coherent plasma of monopoles [17, 19]. This phase allows for electric vortices and therefore color charges are confined. In spite the appeal of this proposal no one has been able to use this characterization for realistic quantitative calculations in continuum QCD. However a topological description of confinement has arisen [20] which has been implemented [21, 22] and searched for in in lattice calculations [18].

Let us accept this appealing scenario and assume that a hadron is represented simply by two *opposite*, i.e. N and $\bar{N}$, N being the number of colors, color charges connected by a color electric vortex. This scenario is no more than the dual of the confinement mechanism in QED once monopoles are included in the theory. In this case the monopole and antimonopole are confined by a magnetic vortex [17, 23]. Thus a hadron consists of a string like configuration between two opposite charges, which in the case of (non exotic) mesons are a quark and an antiquark, and in the case of (non exotic) baryons a quark and a diquark. This *hadronic* configurations exist on top of a highly non perturbative vacuum, the monopole plasma, that can be understood as a very disordered (large entropy) system of color magnetic flux tubes [24]. This latter description of the vacuum, as a disordered system, motivates our second assumption, namely that local magnetic effects will disappear in the averaging process,i.e., on a global scale where observable effects take place. Thus from the observational point of view the role of the complicated non perturbative vacuum is just to allow for the confinement scheme sketched above. We do not take into account other non perturbative mechanisms arising from a more complete description of the vacuum, e.g., instantons, which might contribute also to some of the effects we shall discuss.

In order to obtain the structure of the quark propagator in the vecinity of a color electric vortex we repeat the construction of the Nielsen-Olesen vortex [23] for the dual fields obtaining an electric vortex of the form

$$\vec{E} = \hat{z} f(x_1, x_2) \tag{34}$$

The color indices saturate with the appropriate choice of the color structure of the non abelian vortices and drop out of the calculation.

In Schwinger's [25] proper time method the equations of motion become

$$\frac{d\pi_1}{ds} = g\frac{\partial f(x_1,x_2)}{\partial x_1}\,\sigma_{34} \quad ; \quad \frac{dx_1}{ds} = 2\pi_1 \tag{35}$$

$$\frac{d\pi_2}{ds} = g\frac{\partial f(x_1,x_2)}{\partial x_2}\,\sigma_{34} \quad ; \quad \frac{dx_2}{ds} = 2\pi_2 \tag{36}$$

$$\frac{d\pi_3}{ds} = -2gf(x_1,x_2)\;\pi_4 \quad ; \quad \frac{dx_3}{ds} = 2\pi_3 \tag{37}$$

$$\frac{d\pi_4}{ds} = 2gf(x_1,x_2)\;\pi_3 \quad ; \quad \frac{dx_4}{ds} = 2\pi_4 \tag{38}$$

It is possible to obtain first integrals of these equations, i.e.,

$$\frac{d}{ds}(\pi_1^2+\pi_2^2) = g\frac{df(x_1,x_2)}{ds}\,\sigma_{34} \tag{39}$$

$$\frac{d}{ds}(\pi_3^2+\pi_4^2) = 0 \tag{40}$$

that lead to a hamiltonian for the evolving quasi particle

$$\mathcal{H}(s) = \pi^2(0) - g\sigma_{34} f(x_1(0), x_2(0)) \tag{41}$$

The first of Schwinger's equations reads

$$i\partial_s (x(s)'|x(0)'') = (x(s)'|\mathcal{H}|x(0)'') \tag{42}$$

Using Eq.(41) we can rewrite Eq.(42) as

$$i\partial(\tilde{x}(s)'|\tilde{x}(0)'') = (\tilde{x}(s)'|\pi^2(0)|\tilde{x}(0)'') \tag{43}$$

just by rotating in spin space as

$$|x(s)) = e^{-ig\sigma_{34} f(x_1(0)'', x_2(0)'')s}|\tilde{x}(s)) \tag{44}$$

Equation (43) corresponds to that of a spinless field and can be integrated formally together with the remaining equations of Schwinger [25] leading to [9]

$$D^A(x', x'', m) = \int_0^\infty ds \int_{x(0)=x'}^{x(s)=x''} [dx^\mu] \exp(-i \int_0^s ds (\frac{dx^\mu}{ds})^2)$$

$$\exp(-im^2 s) \mathcal{P}(\exp(i \int_{x(0)=x'}^{x(s)=x''} A_\mu dx^\mu)) \tag{45}$$

In particular the last factor is a consequence of the additional proper time equations [25]. One obtains the fermionic propagator from Eq.(45) in a straightforward fashion

$$S^A(x', x'', m) = \int_0^\infty ds \int_{x(0)=x'}^{x(s)=x''} [dx^\mu](\gamma_\mu \frac{dx^\mu}{ds} + m) \exp(-i \int_0^s ds (\frac{dx^\mu}{ds})^2)$$

$$\exp(-im^2 s) \exp(ig\sigma_{34} f(x'', x') s) \mathcal{P}(\exp(i \int_{x(0)=x'}^{x(s)=x''} A_\mu dx^\mu)) \tag{46}$$

Since $\sigma_{34}^2 = 1$ the spin phase becomes

$$e^{ig\sigma_{34} f(x'', x')s} = \cos(gf(x'', x')s) + i\sigma_{34} \sin(gf(x'', x')s) \tag{47}$$

Furthermore

$$\gamma_\mu \sigma_{34} = i(\delta_{\mu 3}\gamma_4 - \delta_{\mu 4}\gamma_3) + \varepsilon_{34\mu\nu}\gamma_5\gamma_\nu \tag{48}$$

Thus the fermion propagator will contain besides the conventional scalar and vector terms, axial and tensor terms.

B Some Meson correlators

In this appendix we show some of the non trivial meson correlators, in particular the vector and axial mesons, since the scalar and pseudoscalar meson correlators are given in the text and since the tensor meson correlator is too messy, we shall only show some of its anomalous terms.

For the vector meson we have

$$\begin{aligned} < \rho_\mu \rho_\nu > \; &= \int d\mu \{ \delta_{\mu\nu}(|s|^2 + |v|^2 - |a|^2 + 2|t|^2) - (v_\mu v_\nu^* + v_\mu^* v_\nu) \\ &+ (a_\mu a_\nu^* + a_\mu^* a_\nu) + \varepsilon_{\mu\nu\lambda\varphi}(a_\lambda^* v_\varphi + a_\lambda v_\varphi^*) \\ &2i(s^* t_{\mu\nu} - s t_{\mu\nu}^*) + 4(t_{\mu\lambda} t_{\lambda\nu}^* + t_{\nu\lambda} t_{\lambda\mu}^*) \} \end{aligned} \tag{49}$$

and for the axial meson

$$
\begin{aligned}
< \alpha_\mu \alpha_\nu > &= \int d\mu \{ \delta_{\mu\nu}(-|s|^2 + |v|^2 - |a|^2 - 2|t|^2) - (v_\mu v_\nu^* + v_\mu^* v_\nu) \\
&\quad + (a_\mu a_\nu^* + a_\mu^* a_\nu) + \varepsilon_{\mu\nu\lambda\varphi}(a_\lambda^* v_\varphi + a_\lambda v_\varphi^*) \\
&\quad - 2i(s^* t_{\mu\nu} - s t_{\mu\nu}^*) - 4(t_{\mu\lambda} t_{\lambda\nu}^* + t_{\nu\lambda} t_{\lambda\mu}^*) \}
\end{aligned} \tag{50}
$$

We notice that in these cases the CP and P violating terms must vanish after integration if Poincaré invariance is to hold.

The tensor meson contains the following terms which are not forced to vanish after integration by Poincaré invariance

$$
\begin{aligned}
< \tau_{\mu\nu} \tau_{\lambda\varphi} > &= \int d\mu \{ \varepsilon_{\mu\nu\lambda\varphi}(a \cdot v^* + a^* \cdot v) + \varepsilon_{\mu\nu\lambda\eta}(a_\varphi^* v_\eta - a_\varphi v_\eta^*) \\
&\quad + \varepsilon_{\mu\lambda\varphi\eta}(a_\nu^* v_\eta + a_\nu v_\eta^*) - \varepsilon_{\mu\nu\varphi\eta}(a_\lambda^* v_\eta - a_\lambda v_\eta^*) \\
&\quad - \varepsilon_{\nu\lambda\varphi\eta}(a_\mu^* v_\eta + a_\mu v_\eta^*) - \varepsilon_{\nu\lambda\varphi\eta}(a_\eta^* v_\mu + a_\eta v_\mu^*) \\
&\quad + \varepsilon_{\mu\nu\varphi\eta}(a_\eta^* v_\lambda - a_\eta v_\lambda^*) + \varepsilon_{\mu\lambda\varphi\eta}(a_\eta^* v_\nu + a_\eta v_\nu^*) \\
&\quad - \varepsilon_{\mu\nu\lambda\eta}(a_\eta^* v_\varphi - a_\eta v_\varphi^*) + \ldots \}
\end{aligned} \tag{51}
$$

C Some anomalous terms of the proton correlator

In the chiral limit ($\xi = -1$) the following anomalous terms arise in the proton correlator

$$
\begin{aligned}
< P\bar{P} > &= \int d\mu \{ i a_\alpha v_\beta t_{\beta\alpha} \gamma_5 + [(\frac{1}{2}s^2 + v^2 + a^2 + 2t^2) a_\alpha \\
&\quad - 2(a \cdot v) v_\alpha + 4 t_{\alpha\lambda} t_{\lambda\varphi} a_\varphi - i \varepsilon_{\alpha\beta\lambda\varphi} t_{\beta\lambda} v_\varphi] \gamma_5 \gamma_\alpha \\
&\quad + [\frac{i}{2} s \varepsilon_{\alpha\beta\lambda\varphi} a_\lambda v_\varphi + \frac{1}{2} \varepsilon_{\beta\lambda\varphi\sigma}(a_\alpha t_{\lambda\varphi} v_\sigma + a_\lambda t \alpha\sigma v_\varphi + a_\lambda t_{\sigma\varphi} v_\alpha) \\
&\quad + \frac{1}{2} \varepsilon_{\alpha\lambda\varphi\sigma}(a_\lambda t_{\sigma\beta} v_\varphi + a_\beta t \varphi\lambda v_\sigma + a_\lambda t_{\varphi\sigma} v_\beta) + (s^2 + \frac{1}{2} v^2 - \frac{1}{2} a^2 + 2t^2) t_{\alpha\beta} \\
&\quad + a_\alpha a_\lambda t_{\lambda\beta} + a_\beta a_\lambda t_{\alpha\lambda} + v_\beta v_\lambda t_{\beta\lambda} + v_\alpha v_\lambda t_{\lambda\alpha}] \sigma_{\alpha\beta} + \ldots \}
\end{aligned} \tag{52}
$$

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