FTUV-06-02xx

Quantum Decoherence and Neutrino Data

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arXiv:hep-ph/0603028v1 3 Mar 2006

Abstract

In this work we perform global fits of microscopic decoherence models of neutrinos to all available current data, including LSND and KamLAND spectral distortion results. In previous works on related issues the models used were supposed to explain LSND results by means of quantum gravity induced decoherence. However those models were purely phenomenological without any underlying microscopic basis. It is one of the main purposes of this article to use detailed microscopic decoherence models with complete positivity, to fit the data.The decoherence in these models has contributions not only from stochastic quantum gravity vacua operating as a medium, but also from conventional uncertainties in the energy of the (anti)neutrino beam. All these contributions lead to oscillation-length independent damping factors modulating the oscillatory terms from which one obtains an excellent fit to all available neutrino data, including LSND and Kamland spectral distortion. The fit is much superior to all earlier ones. It appears that the results of the fit are most naturally interpreted as corresponding to conventional energy uncertainties. This represents a radical departure from previous analyses where the neutrino data (including LSND but not Kamland spectral distortion) were regarded as evidence for quantum gravity decoherence.

PACS numbers: 14.60.Pq, 04.60.-m

I. INTRODUCTION

The theory of Quantum Gravity (QG) is still elusive. In some theoretical models, the phenomenon of space-time 'foam', invoked by J.A. Wheeler [1], may be in place; according to this picture the singular microscopic fluctuations of the metric, give the ground state of QG the structure of a 'stochastic medium'. The medium has the profound effect of leading to decoherence of quantum matter as it propagates.This may have experimentally observable consequences in principle [2].

One of the basic effects of decoherence is the presence of damping factors in front of the oscillatory terms. However, one should be very careful when interpreting decoherence effects, if observed in an experiment, because ordinary matter can easily 'fake' decoherence effects, especially the damping exponents [3, 4]. For instance, uncertainties in the energy of a neutrino beam [3], which are associated with ordinary physics, and have nothing to do with 'fuzziness' of space time, do reproduce a damping exponent similar to that encountered in Lindblad decoherence models [5]. Of course, stochastic quantum gravity effects can induce such uncertainties in the energy beam, and hence contribute to the damping exponent themselves [2], but such effects are usually subleading. Thus, one should know the energy of the beam with high precision in order to eliminate 'fake' decoherence effects and probe quantum gravity effects sufficiently well.

In ref. [6], henceforth referred to as I, we have attempted to fit the available neutrino data, including LSND results [7], using phenomenological decoherent models with mixing in all three generations of neutrinos. Such fits extended earlier similar attempts to study decoherence with two-generation neutrino models [8]. In I it was seriously entertained that the decoherence might be attributable to environmental entanglement with the quantum gravity foamy vacuum, and could be distinguished from ordinary-matter-induced decoherence [2].

A simplified model, of Lindblad type [5] has been used for the fit, following earlier work in [9]. The model of [9] involved a phenomenological diagonal decoherence matrix, and in the fit of I, decoherence was assumed to be dominant *only* in the antineutrino sector. This assumption was made in order to fit the LSND results [7] pointing to significant $\overline{\nu}_\mu \leftrightarrow \overline{\nu}_e$ oscillations, but no significant evidence of oscillations in the particle sector. In this way a fit was made to a three generation model with the LSND "anomalous" result, without introducing a sterile neutrino. The possibility of strong CPT violation in the decoherence

sector, allowed for an equality of neutrino and antineutrino mass differences in agreement with atmospheric and solar neutrino data.

The particular choice of [6], which yielded an extremely good fit to all available neutrino data involved mixed energy dependence for the (antineutrino-sector) decoherence coefficients, some of which were proportional to the neutrino energies E, while the rest were inversely proportional to it, $\propto 1/E$. In I, the coefficients proportional to $1/E$ were interpreted as describing ordinary matter effects, whilst those proportional to E were assumed to correspond to *genuine* quantum-gravity effects. The latter increase with the energy of the the (anti)neutrino is consistent with a larger back reaction effect on quantum space-time and hence with a larger decoherence.

The strong difference assumed in I between the decoherence coefficients of the particle and antiparticle sectors, although not incompatible with a breakdown of CPT at a fundamental level [2], appears at first sight somewhat curious, and in fact is unlike any other case of decoherence in other sensitive particle probes, like neutral mesons, examined in the past [10]. There, the oscillations between particle and antiparticle sectors, necessitate a common decoherence environment between mesons and antimesons. If one accepts the Universality of gravity, then, the sample point of I seems incompatible with this property. Moreover, there are two more problematic points of the fit in I,which were already discussed in that reference. The first point concerns the complete positivity of the model. In [9] the *ad hoc* diagonal form of the decoherence matrix, used in I, was postulated , without a discussion of the necessary conditions required in the Lindblad approach to guarantee complete positivity. Indeed the particular choice of the decoherence parameters of I, did *not* lead to positive definite probabilities for the *entire* regime of the parameter space of the model, although the probabilities were positive definite for the portion of the parameter space appropriate for the various neutrino experiments used for the fit. Specifically, it was found in I, that with the particular choice of the decoherence parameters in the (antineutrino sector), one obtains positive-definite transition probabilities for energies restricted to $E > \mathcal{O}(1 \text{ MeV})$. The second, and more important point, is that the choices of decoherence parameters of I were good for all the neutrino experiments available at the time, but *unfortunately* it could not reproduce the spectral distortion observed by the KamLAND experiment [11], whose first results came out simultaneously with the results of I.

The aim of this article is two fold. One is to rectify the above points, and present a

novel fit, using as in I the simplified three-generation Lindblad model of decoherence of [9], but crucially amended so as to respect the general conditions among the coefficients necessary to guarantee complete positivity in the *entire* parameter space. We shall show below, that it is possible to find such a consistent Lindblad model consistently, for which the fit to all available neutrino data is excellent, including the spectral distortion seen by KamLAND, and the LSND results [7]. This substantially extended decoherence model (in comparison to I), constitutes therefore the first mathematically consistent three-generation neutrino decoherence model of Lindblad type which fits all the available data, including spectral distortion seen by KamLAND.

The second and more significant aim was to give a microscopic and physically motivated model which would fit into the general scheme of the linear Lindblad decoherence. In this way the constraints obtained from the phenomenological fit can be examined to check consistency with values that can be deemed reasonable for phenomena originating from quantum gravity. Such a comparison has not been done before and leads to a major shift in our views concerning decoherence due to quantum gravity. In fact, as we shall argue below, the most natural explanation of the fit seems to be provided by energy uncertainties in the (anti)neutrino beam, due to conventional physics. Several microscopic quantum space-time models, that we have examined in this work, yield too small effects to reproduce the result of the fit.

In the present work, the decoherence parameters in the model are *assumed* to be the same in *both neutrino and antineutrino sectors*, consistent with the above-mentioned universal property of a quantum-gravity environment. In this sense, we assume that the LSND result is correct in both channels, although their observed excess of $\overline{\nu}_e$ events is not corroborated (at the same level at least) in the neutrino channel. This is to be contrasted with the approach of I, where following [12], only the evidence in the antineutrino sector was considered.

The structure of this article is the following: in section 2 we review the basic theory of Lindblad decoherence, and specify the conditions for complete positivity in the type of model for decoherence used in [9] and in I. We examine the limitations imposed on the parameter space of the model in I in order to guarantee complete positivity, and then we construct a modified model, in which one obtains positive definite transition probabilities for the entire regime of the parameter space. The decoherence implies exponential damping with time (oscillation length), which violates microscopic time irreversibility, irrespective of CP

properties of neutrinos, and hence CPT violation. In order to agree with the experimental results of KamLAND on spectral distortions [11] we require such exponential damping factors to imply a modulation in front of some of the oscillatory terms giving rise to a modification in the (survival) transition probabilities of order per mil. We obtain in section 3 stringent constraints on the exponents of the damping factors. The sample point that fits all available data, including LSND,is discussed in detail in section 3. This model-point corresponds to an exponent of the decoherent damping factors which is independent of the oscillation length. An attempt to explain such a result in terms of microscopic models of stochastic space time foam is given in section 4. However, as we show there, explanations based on conventional physics such as the uncertainties in the (anti)neutrino beam energy, are definitely much more plausible. The present data when interpreted in terms of a microscopic model make quantum gravity an unlikely candidate for the origin of decoherence (claimed to be observed in our fits). Finally, conclusions and outlook are presented in section 5.

II. LINDBLAD DECOHERENCE AND TRANSITION PROBABILITIES: A REVIEW

In this section we present the details of the calculation for transition probabilities of three generations of neutrinos, where complete positivity is maintained within the (linear) Lindblad approach [5]. In this framework the general evolution equation of the ρ density matrix, representing a (spinless) neutrino state reads :

$$\partial_t \rho = L[\rho] \tag{2.1}$$

where there are conditions on the decoherence contribution to L which guarantee complete positivity of the probabilities as they evolve in time. The spin of the neutrino will not play an important rôle in constraining the decoherence sector by comparing with experimental data, and hence we shall present a formalism based on scalar particles. Detailed studies of Dirac and spinless neutrinos have been performed in [13]; as explained there the inclusion of spin does not affect qualitatively the main decoherence effects which is the damping of oscillation probabilities. In section 4, where we attempt to interpret the time (i.e. oscillation length) dependence and order of magnitude of the decoherence parameters, we shall present a more detailed discussion on the results of [13].

With the above in mind, we commence our analysis with a theorem due to Gorini, Kossakaowski and Sudarshan [14] on the structure of L, the generator of a quantum dynamical semi-group [5, 14]. For a non-negative matrix c_{kl} (i.e. a matrix with non-negative eigenvalues) such a generator is given by

$$L[\rho] = -i[H,\rho] + \frac{1}{2}\sum_{k,l} c_{kl}\left([F_k\rho, F_l^\dagger] + [F_k, \rho F_l^\dagger]\right), \tag{2.2}$$

where $H = H^\dagger$ is a hermitian Hamiltonian, $\{F_k, k = 0, ..., n^2-1\}$ is a basis in $M_n(\mathbf{C})$ such that $F_0 = \frac{1}{\sqrt{n}} I_n$, $\mathrm{Tr}(F_k) = 0 \forall k \neq 0$ and $\mathrm{Tr}(F_i^\dagger F_j) = \delta_{ij}$. In our application we can take $F_i = \frac{\Lambda_i}{2}$ (where Λ_i are the Gell-Mann matrices) and satisfy the Lie algebra $[F_i, F_j] = i\sum_k f_{ijk}F_k, (i = 1, ...8)$ f_{ijk} being the standard structure constants, antisymmetric in all indices.

Without a microscopic model, in the three generation case, the precise physical significance of the decoherence matrix cannot be fully understood. In this work we shall consider the simplified case in which the matrix $C \equiv (c_{kl})$ is *assumed* to be of the form

$$C = \begin{pmatrix} c_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & c_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_{33} & 0 & 0 & 0 & 0 & c_{38} \\ 0 & 0 & 0 & c_{44} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_{77} & 0 \\ 0 & 0 & c_{38} & 0 & 0 & 0 & 0 & c_{88} \end{pmatrix} \tag{2.3}$$

As stated above, positivity can be guaranteed if and only if the matrix C is positive and hence has non-negative eigenvalues. We have also taken C to be symmetric. A similar simplification has been used in [9] and in I, to yield an economic decoherence model which can be compared with experimental data. However, as discussed in detail in [13], such special choices can be realised for models of the propagation of neutrinos in models of stochastically fluctuating environments [15], where the decoherence term corresponds to an appropriate double commutator involving operators that entangle with the environment.The quantum-gravity space time foam may in principle behave as one such stochastic environment [2, 10, 13], and it is this point of view, that we will critically examine in this work. In section 4, we

shall discuss the viability of the interpretation of the fit in terms of such microscopic models of space-time foam.

Since the F_i are Hermitian, we can rewrite the expression for $L[\rho]$ as

$$L[\rho] \equiv -i[H,\rho] + \mathcal{D}[\rho] = -i[H,\rho] + \frac{1}{2}\sum_{k,l} c_{kl}\left([F_k\rho, F_l] + [F_k, \rho F_l]\right) \tag{2.4}$$

After standard manipulations, we may write the non-Hamiltonian decoherence part $\mathcal{D}[\rho]$ as

$$\mathcal{D}[\rho] = \frac{1}{4}\left([F_k,[\rho,F_l]] + \{F_k,[\rho,F_l]\} - [F_l,[F_k,\rho]] + \{F_l[F_k,\rho]\}\right) + \frac{1}{2}\{\rho,[F_k,F_l]\} \tag{2.5}$$

On using the expansion $\rho = \sum_i \rho_i F_i$, this expression can be written

$$\begin{aligned}\mathcal{D}[\rho] = \frac{\rho_i c_{kl}}{4}\Bigg(&-f_{ilm}f_{kmj}F_j + if_{ilm}(\frac{1}{3}\delta_{km} + \frac{1}{2}d_{kmj}F_j) + f_{kim}f_{lmj}F_j \\ &+ if_{kim}(\frac{1}{3}\delta_{lm} + \frac{1}{2}f_{lmj}F_j) + i2f_{klm}(\frac{1}{3}\delta_{im} + \frac{1}{2}d_{imj}F_j)\Bigg)\end{aligned} \tag{2.6}$$

We note that the only terms which contribute are $\frac{\rho_i c_{kl}}{4}\left(-f_{ilm}f_{kmj} + f_{kim}f_{lmj}\right)F_j$.

We follow the basic notation [5, 9] and express the time evolution of the density matrix as

$$\dot{\rho}_k = \sum_j (\sum_i h_i f_{ijk} + \mathcal{D}_{kj})\rho_j = \sum_j M_{kj}\rho_j \tag{2.7}$$

where we have

$$\mathcal{D}_{ij} = \sum_{k,l,m} \frac{c_{kl}}{4}\left(-f_{ilm}f_{kmj} + f_{kim}f_{lmj}\right). \tag{2.8}$$

Using the values of the structure constants f_{ijk} of the SU(3) group, appropriate to the three generation case being examined here, we arrive at:

$$\begin{aligned}
\mathcal{D}_{11} &= -\frac{1}{2}\left(c_{22} + c_{33} + \frac{1}{4}(c_{44} + c_{55} + c_{66} + c_{77})\right) \\
\mathcal{D}_{22} &= -\frac{1}{2}\left(c_{11} + c_{33} + \frac{1}{4}(c_{44} + c_{55} + c_{66} + c_{77})\right) \\
\mathcal{D}_{33} &= -\frac{1}{2}\left(c_{11} + c_{22} + \frac{1}{4}(c_{44} + c_{55} + c_{66} + c_{77})\right) \\
\mathcal{D}_{44} &= -\frac{1}{2}\left(c_{55} + \frac{1}{4}(c_{11} + c_{22} + c_{33} + c_{66} + c_{77} + 3c_{88}) + \frac{\sqrt{3}}{2}c_{38}\right) \\
\mathcal{D}_{55} &= -\frac{1}{2}\left(c_{44} + \frac{1}{4}(c_{11} + c_{22} + c_{33} + c_{66} + c_{77} + 3c_{88}) + \frac{\sqrt{3}}{2}c_{38}\right)
\end{aligned}$$

$$
\begin{aligned}
\mathcal{D}_{66} &= -\frac{1}{2}\left(c_{77}+\frac{1}{4}(c_{11}+c_{22}+c_{33}+c_{44}+c_{55}+3c_{88})-\frac{\sqrt{3}}{2}c_{38}\right)\\
\mathcal{D}_{77} &= -\frac{1}{2}\left(c_{66}+\frac{1}{4}(c_{11}+c_{22}+c_{33}+c_{44}+c_{55}+3c_{88})-\frac{\sqrt{3}}{2}c_{38}\right)\\
\mathcal{D}_{88} &= -\frac{3}{8}(c_{44}+c_{55}+c_{66}+c_{77})\\
\mathcal{D}_{83} &= \mathcal{D}_{38} = -\frac{\sqrt{3}}{8}(c_{44}+c_{55}-c_{66}-c_{77})
\end{aligned}
\tag{2.9}
$$

or conversely,

$$
\begin{aligned}
c_{11} &= \frac{1}{3}\mathcal{D}_{88}+\mathcal{D}_{11}-\mathcal{D}_{22}-\mathcal{D}_{33}\\
c_{22} &= -\mathcal{D}_{11}+\frac{1}{3}\mathcal{D}_{88}+\mathcal{D}_{22}-\mathcal{D}_{33}\\
c_{33} &= \frac{1}{3}\mathcal{D}_{88}-\mathcal{D}_{11}-\mathcal{D}_{22}+\mathcal{D}_{33}\\
c_{44} &= -\mathcal{D}_{55}+\mathcal{D}_{44}-\frac{2}{\sqrt{3}}\mathcal{D}_{38}-\frac{2}{3}\mathcal{D}_{88}\\
c_{55} &= \mathcal{D}_{55}-\mathcal{D}_{44}-\frac{2}{3}\sqrt{3}\mathcal{D}_{38}-\frac{2}{3}\mathcal{D}_{88}\\
c_{66} &= -\mathcal{D}_{77}+\mathcal{D}_{66}+\frac{2}{\sqrt{3}}\mathcal{D}_{38}-\frac{2}{3}\mathcal{D}_{88}\\
c_{77} &= \mathcal{D}_{77}-\mathcal{D}_{66}+\frac{2}{\sqrt{3}}\mathcal{D}_{38}-\frac{2}{3}\mathcal{D}_{88}\\
c_{88} &= -\frac{2}{3}\mathcal{D}_{55}-\frac{2}{3}\mathcal{D}_{77}-\frac{2}{3}\mathcal{D}_{66}-\frac{2}{3}\mathcal{D}_{44}+\mathcal{D}_{88}+\frac{1}{3}\mathcal{D}_{11}+\frac{1}{3}\mathcal{D}_{22}+\frac{1}{3}\mathcal{D}_{33}\\
c_{38} &= -\frac{1}{\sqrt{3}}\mathcal{D}_{55}+\frac{1}{\sqrt{3}}\mathcal{D}_{77}+\frac{1}{\sqrt{3}}\mathcal{D}_{66}-\frac{1}{\sqrt{3}}\mathcal{D}_{44}+\frac{2}{3}\mathcal{D}_{38}
\end{aligned}
\tag{2.10}
$$

The simplified form of the c_{ij} matrix given in (2.3) implies a matrix L_{ij} of the form:

$$
L = \begin{pmatrix}
\mathcal{D}_{11} & -\Delta_{12} & 0 & 0 & 0 & 0 & 0 & 0\\
\Delta_{12} & \mathcal{D}_{22} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \mathcal{D}_{33} & 0 & 0 & 0 & 0 & \mathcal{D}_{38}\\
0 & 0 & 0 & \mathcal{D}_{44} & -\Delta_{13} & 0 & 0 & 0\\
0 & 0 & 0 & \Delta_{13} & \mathcal{D}_{55} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \mathcal{D}_{66} & -\Delta_{23} & 0\\
0 & 0 & 0 & 0 & 0 & \Delta_{23} & \mathcal{D}_{77} & 0\\
0 & 0 & \mathcal{D}_{83} & 0 & 0 & 0 & 0 & \mathcal{D}_{88}
\end{pmatrix}
\tag{2.11}
$$

where we have used the notation $\Delta_{ij} = \frac{m_i^2 - m_j^2}{2p}$.

The corresponding eigenvalues are:

$$
\begin{aligned}
\lambda_1 &= \frac{1}{2}[(\mathcal{D}_{11}+\mathcal{D}_{22}) - \sqrt{(\mathcal{D}_{22}-\mathcal{D}_{11})^2 - 4\Delta_{12}^2}] \equiv \frac{1}{2}[(\mathcal{D}_{11}+\mathcal{D}_{22}) - \Omega_{12}] \\
\lambda_2 &= \frac{1}{2}[(\mathcal{D}_{11}+\mathcal{D}_{22}) + \sqrt{(\mathcal{D}_{22}-\mathcal{D}_{11})^2 - 4\Delta_{12}^2}] \equiv \frac{1}{2}[(\mathcal{D}_{11}+\mathcal{D}_{22}) + \Omega_{12}] \\
\lambda_3 &= \frac{1}{2}[(\mathcal{D}_{33}+\mathcal{D}_{88}) - \sqrt{(\mathcal{D}_{33}-\mathcal{D}_{88})^2 + 4\mathcal{D}_{38}^2}] \equiv \frac{1}{2}[(\mathcal{D}_{11}+\mathcal{D}_{22}) - \Omega_{38}] \\
\lambda_4 &= \frac{1}{2}[(\mathcal{D}_{44}+\mathcal{D}_{55}) - \sqrt{(\mathcal{D}_{44}-\mathcal{D}_{55})^2 - 4\Delta_{13}^2}] \equiv \frac{1}{2}[(\mathcal{D}_{44}+\mathcal{D}_{55}) - \Omega_{13}] \\
\lambda_5 &= \frac{1}{2}[(\mathcal{D}_{44}+\mathcal{D}_{55}) + \sqrt{(\mathcal{D}_{44}-\mathcal{D}_{55})^2 - 4\Delta_{13}^2}] \equiv \frac{1}{2}[(\mathcal{D}_{44}+\mathcal{D}_{55}) + \Omega_{13}] \\
\lambda_6 &= \frac{1}{2}[(\mathcal{D}_{66}+\mathcal{D}_{77}) - \sqrt{(\mathcal{D}_{66}-\mathcal{D}_{77})^2 - 4\Delta_{23}^2}] \equiv \frac{1}{2}[(\mathcal{D}_{66}+\mathcal{D}_{77}) - \Omega_{23}] \\
\lambda_7 &= \frac{1}{2}[(\mathcal{D}_{66}+\mathcal{D}_{77}) + \sqrt{(\mathcal{D}_{66}-\mathcal{D}_{77})^2 - 4\Delta_{23}^2}] \equiv \frac{1}{2}[(\mathcal{D}_{66}+\mathcal{D}_{77}) + \Omega_{23}] \\
\lambda_8 &= \frac{1}{2}[(\mathcal{D}_{33}+\mathcal{D}_{88}) + \sqrt{(\mathcal{D}_{33}-\mathcal{D}_{88})^2 + 4\mathcal{D}_{38}^2}] \equiv \frac{1}{2}[(\mathcal{D}_{11}+\mathcal{D}_{22}) + \Omega_{38}].
\end{aligned}
\tag{2.12}
$$

The probability of a neutrino of flavor ν_α, created at time $t = 0$, being converted to a flavor ν_β at a later time t, is calculated in the Lindblad framework [5, 9] to be

$$
P_{\nu_\alpha \to \nu_\beta}(t) = \mathrm{Tr}[\rho^\alpha(t)\rho^\beta] = \frac{1}{3} + \frac{1}{2}\sum_{i,j,k} e^{\lambda_k t} D_{ik} D^{-1}_{kj} \rho^\alpha_j(0)\rho^\beta_i \ . \tag{2.13}
$$

where the matrix $\mathbf{D}$ and its inverse are

$$
\mathbf{D} = \begin{pmatrix}
\frac{\lambda_1-\mathcal{D}_{22}}{\Delta_{12}} & \frac{\lambda_2-\mathcal{D}_{22}}{\Delta_{12}} & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\lambda_3-\mathcal{D}_{33}}{\mathcal{D}_{38}} & 0 & 0 & 0 & 0 & \frac{\lambda_8-\mathcal{D}_{33}}{\mathcal{D}_{38}} \\
0 & 0 & 0 & \frac{\lambda_4-\mathcal{D}_{55}}{\Delta_{13}} & \frac{\lambda_5-\mathcal{D}_{55}}{\Delta_{13}} & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\lambda_6-\mathcal{D}_{77}}{\Delta_{23}} & \frac{\lambda_7-\mathcal{D}_{77}}{\Delta_{23}} & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}. \tag{2.14}
$$

and

$$\mathbf{D}^{-1} = \begin{pmatrix} -\frac{\Delta_{12}}{\Omega_{12}} & \frac{\lambda_2-\mathcal{D}_{22}}{\Omega_{12}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\Delta_{12}}{\Omega_{12}} & -\frac{\lambda_1-\mathcal{D}_{22}}{\Omega_{12}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\mathcal{D}_{38}}{\Omega_{38}} & 0 & 0 & 0 & 0 & \frac{\lambda_8-\mathcal{D}_{33}}{\Omega_{38}} \\ 0 & 0 & 0 & -\frac{\Delta_{13}}{\Omega_{13}} & \frac{\lambda_5-\mathcal{D}_{55}}{\Omega_{13}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\Delta_{13}}{\Omega_{13}} & -\frac{\lambda_4-\mathcal{D}_{55}}{\Omega_{13}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\Delta_{23}}{\Omega_{23}} & \frac{\lambda_7-\mathcal{D}_{77}}{\Omega_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\Delta_{23}}{\Omega_{23}} & -\frac{\lambda_6-\mathcal{D}_{77}}{\Omega_{23}} & 0 \\ 0 & 0 & \frac{\mathcal{D}_{38}}{\Omega_{38}} & 0 & 0 & 0 & 0 & -\frac{\lambda_3-\mathcal{D}_{33}}{\Omega_{38}} \end{pmatrix}. \tag{2.15}$$

It will be sufficient to look explicitly at the $k = 1$ and $k = 2$ terms in the sum of the right hand side of (2.13), since by block symmetry the other terms will be of the same form.

For the $k = 1$ term we have:

$$e^{\lambda_1 t}\left(D_{i1}D^{-1}_{1j}\rho^\alpha_j(0)\rho^\beta_i\right) = e^{\lambda_1 t}\left(\rho^\alpha_1\rho^\beta_1 D_{11}D^{-1}_{11} + \rho^\alpha_2\rho^\beta_2 D_{21}D^{-1}_{12} + \rho^\alpha_2\rho^\beta_1 D_{11}D^{-1}_{12} + \rho^\alpha_1\rho^\beta_2 D_{21}D^{-1}_{11}\right)$$

$$= e^{\frac{(\mathcal{D}_{11}+\mathcal{D}_{22})t}{2}} e^{\frac{-\Omega_{12}t}{2}}\left[\rho^\alpha_1\rho^\beta_1\frac{-\mathcal{D}_{11}+\mathcal{D}_{22}+\Omega_{12}}{2\Omega_{12}} + \rho^\alpha_2\rho^\beta_2\frac{-\mathcal{D}_{22}+\mathcal{D}_{11}+\Omega_{12}}{2\Omega_{12}} + \rho^\alpha_2\rho^\beta_1\frac{\Delta_{12}}{\Omega_{12}} + \rho^\alpha_1\rho^\beta_2\frac{-\Delta_{12}}{\Omega_{12}}\right] \tag{2.16}$$

Likewise for $k = 2$ we have

$$e^{\lambda_2 t}\left(D_{i2}D^{-1}_{2j}\rho^\alpha_j(0)\rho^\beta_i\right) = e^{\lambda_2 t}\left(\rho^\alpha_1\rho^\beta_1 D_{12}D^{-1}_{21} + \rho^\alpha_2\rho^\beta_2 D_{22}D^{-1}_{22} + \rho^\alpha_2\rho^\beta_1 D_{12}D^{-1}_{22} + \rho^\alpha_1\rho^\beta_2 D_{22}D^{-1}_{21}\right)$$

$$= e^{\frac{(\mathcal{D}_{11}+\mathcal{D}_{22})t}{2}} e^{\frac{\Omega_{12}t}{2}}\left[\rho^\alpha_1\rho^\beta_1\frac{-\mathcal{D}_{22}+\mathcal{D}_{11}+\Omega_{12}}{2\Omega_{12}} + \rho^\alpha_2\rho^\beta_2\frac{(-\mathcal{D}_{22}+\mathcal{D}_{11}-\Omega_{12})}{2\Omega_{12}} + \rho^\alpha_2\rho^\beta_1\frac{-\Delta_{12}}{\Omega_{12}} + \rho^\alpha_1\rho^\beta_2\frac{\Delta_{12}}{\Omega_{12}}\right] \tag{2.17}$$

Upon combining equations (2.17) and (2.18) we obtain:

$$(2.17) + (2.18) = e^{(\mathcal{D}_{11}+\mathcal{D}_{22})\frac{t}{2}}\left[\left(\rho^\alpha_1\rho^\beta_1 + \rho^\alpha_2\rho^\beta_2\right)\left(\frac{e^{-\Omega_{12}\frac{t}{2}} + e^{\Omega_{12}\frac{t}{2}}}{2}\right) + \left(\frac{2\Delta_{12}(\rho^\alpha_1\rho^\beta_2 - \rho^\alpha_2\rho^\beta_1) + \Delta\mathcal{D}_{21}(\rho^\alpha_1\rho^\beta_1 - \rho^\alpha_2\rho^\beta_2)}{\Omega_{12}}\right)\left(\frac{e^{-\Omega_{12}\frac{t}{2}} - e^{\Omega_{12}\frac{t}{2}}}{2}\right)\right] \tag{2.18}$$

As mentioned earlier, by block symmetry we can see that the other terms will be of the same form.

We thus obtain for the relevant probability:

$$
\begin{aligned}
P_{\nu_\alpha\to\nu_\beta}(t) \;=\; & \frac{1}{3}+\frac{1}{2}\Bigg\{\Bigg[\left(\rho_1^\alpha\rho_1^\beta+\rho_2^\alpha\rho_2^\beta\right)\left(\frac{e^{-\Omega_{12}\frac{t}{2}}+e^{\Omega_{12}\frac{t}{2}}}{2}\right) \\
& + \left(\frac{2\Delta_{12}(\rho_1^\alpha\rho_2^\beta-\rho_2^\alpha\rho_1^\beta)+\Delta\mathcal{D}_{21}\left(\rho_1^\alpha\rho_1^\beta-\rho_2^\alpha\rho_2^\beta\right)}{\Omega_{12}}\right)\left(\frac{e^{-\Omega_{12}\frac{t}{2}}-e^{\Omega_{12}\frac{t}{2}}}{2}\right)\Bigg]e^{(\mathcal{D}_{11}+\mathcal{D}_{22})\frac{t}{2}} \\
& + \Bigg[\left(\rho_4^\alpha\rho_4^\beta+\rho_5^\alpha\rho_5^\beta\right)\left(\frac{e^{-\Omega_{13}\frac{t}{2}}+e^{\Omega_{13}\frac{t}{2}}}{2}\right) \\
& + \left(\frac{2\Delta_{13}\left(\rho_4^\alpha\rho_5^\beta-\rho_5^\alpha\rho_4^\beta\right)+\Delta\mathcal{D}_{54}\left(\rho_4^\alpha\rho_4^\beta-\rho_5^\alpha\rho_5^\beta\right)}{\Omega_{13}}\right)\left(\frac{e^{-\Omega_{13}\frac{t}{2}}-e^{\Omega_{13}\frac{t}{2}}}{2}\right)\Bigg]e^{(\mathcal{D}_{44}+\mathcal{D}_{55})\frac{t}{2}} \\
& + \Bigg[\left(\rho_6^\alpha\rho_6^\beta+\rho_7^\alpha\rho_7^\beta\right)\left(\frac{e^{-\Omega_{23}\frac{t}{2}}+e^{\Omega_{23}\frac{t}{2}}}{2}\right) \\
& + \left(\frac{2\Delta_{23}(\rho_6^\alpha\rho_7^\beta-\rho_7^\alpha\rho_6^\beta)+\Delta\mathcal{D}_{76}\left(\rho_6^\alpha\rho_6^\beta-\rho_7^\alpha\rho_7^\beta\right)}{\Omega_{23}}\right)\left(\frac{e^{-\Omega_{23}\frac{t}{2}}-e^{\Omega_{23}\frac{t}{2}}}{2}\right)\Bigg]e^{(\mathcal{D}_{66}+\mathcal{D}_{77})\frac{t}{2}} \\
& + \Bigg[\left(\rho_3^\alpha\rho_3^\beta+\rho_8^\alpha\rho_8^\beta\right)\left(\frac{e^{-\Omega_{38}\frac{t}{2}}+e^{\Omega_{38}\frac{t}{2}}}{2}\right) \\
& + \left(\frac{2\mathcal{D}_{38}(\rho_3^\alpha\rho_8^\beta-\rho_8^\alpha\rho_3^\beta)+\Delta\mathcal{D}_{83}\left(\rho_3^\alpha\rho_3^\beta-\rho_8^\alpha\rho_8^\beta\right)}{\Omega_{38}}\right)\left(\frac{e^{-\Omega_{38}\frac{t}{2}}-e^{\Omega_{38}\frac{t}{2}}}{2}\right)\Bigg]e^{(\mathcal{D}_{33}+\mathcal{D}_{88})\frac{t}{2}}.
\end{aligned}
$$

Above we have used the notation that $\Delta\mathcal{D}_{ij} = \mathcal{D}_{ii} - \mathcal{D}_{jj}$. We have assumed that $2|\Delta_{ij}| < |\Delta\mathcal{D}_{ij}|$ with the consequence that $\Omega_{ij},\ ij = 12, 13, 23$ is imaginary. However, $\Omega_{38} = \sqrt{(\mathcal{D}_{33}-\mathcal{D}_{88})^2+4\mathcal{D}_{38}^2}$ will be real. Thus, the final expression for the probability reads

$$
\begin{aligned}
P_{\nu_\alpha\to\nu_\beta}(t) \;=\; & \frac{1}{3}+\frac{1}{2}\Bigg\{\Bigg[\rho_1^\alpha\rho_1^\beta\cos\left(\frac{|\Omega_{12}|t}{2}\right)+\left(\frac{\Delta\mathcal{D}_{21}\rho_1^\alpha\rho_1^\beta}{|\Omega_{12}|}\right)\sin\left(\frac{|\Omega_{12}|t}{2}\right)\Bigg]e^{(\mathcal{D}_{11}+\mathcal{D}_{22})\frac{t}{2}} \\
& + \Bigg[\rho_4^\alpha\rho_4^\beta\cos\left(\frac{|\Omega_{13}|t}{2}\right)+\left(\frac{\Delta\mathcal{D}_{54}\rho_4^\alpha\rho_4^\beta}{|\Omega_{13}|}\right)\sin\left(\frac{|\Omega_{13}|t}{2}\right)\Bigg]e^{(\mathcal{D}_{44}+\mathcal{D}_{55})\frac{t}{2}} \\
& + \Bigg[\rho_6^\alpha\rho_6^\beta\cos\left(\frac{|\Omega_{23}|t}{2}\right)+\left(\frac{\Delta\mathcal{D}_{76}\rho_6^\alpha\rho_6^\beta}{|\Omega_{23}|}\right)\sin\left(\frac{|\Omega_{23}|t}{2}\right)\Bigg]e^{(\mathcal{D}_{66}+\mathcal{D}_{77})\frac{t}{2}} \\
& + \Bigg[\left(\rho_3^\alpha\rho_3^\beta+\rho_8^\alpha\rho_8^\beta\right)\cosh\left(\frac{\Omega_{38}t}{2}\right)
\end{aligned}
$$

$$+ \left(\frac{2\mathcal{D}_{38}(\rho_3^\alpha \rho_8^\beta - \rho_8^\alpha \rho_3^\beta) + \Delta\mathcal{D}_{83}\left(\rho_3^\alpha \rho_3^\beta - \rho_8^\alpha \rho_8^\beta\right)}{\Omega_{38}} \right) \sinh\left(\frac{\Omega_{38} t}{2}\right) \Bigg] e^{(\mathcal{D}_{33}+\mathcal{D}_{88})\frac{t}{2}} \ . \tag{2.19}$$

On using the relations

$$\begin{aligned}
\rho_0^\alpha &= \sqrt{\frac{2}{3}} \\
\rho_1^\alpha &= 2Re(U_{\alpha 1}^* U_{\alpha 2}) \\
\rho_2^\alpha &= -2Im(U_{\alpha 1}^* U_{\alpha 2}) \\
\rho_3^\alpha &= |U_{\alpha 1}|^2 - |U_{\alpha 2}|^2 \\
\rho_4^\alpha &= 2Re(U_{\alpha 1}^* U_{\alpha 3}) \\
\rho_5^\alpha &= -2Im(U_{\alpha 1}^* U_{\alpha 3}) \\
\rho_6^\alpha &= 2Re(U_{\alpha 2}^* U_{\alpha 3}) \\
\rho_7^\alpha &= -2Im(U_{\alpha 2}^* U_{\alpha 3}) \\
\rho_8^\alpha &= \sqrt{\frac{1}{3}} \left(|U_{\alpha 1}|^2 + |U_{\alpha 2}|^2 - 2|U_{\alpha 3}|^2\right) .
\end{aligned}$$

we can readily see that for a real U matrix (i.e. no CP violating phases) the relevant probabilities are bounded. Indeed, there is no danger of the cosh and sinh terms blowing up with time t, as we always have $\Omega_{38} < \mathcal{D}_{33} + \mathcal{D}_{88}$. We can see this by checking that $(\mathcal{D}_{33}-\mathcal{D}_{88})^2+4\mathcal{D}_{38}^2 < \mathcal{D}_{33}^2+\mathcal{D}_{88}^2+2\mathcal{D}_{33}\mathcal{D}_{88}$, $-2\mathcal{D}_{33}\mathcal{D}_{88}+4\mathcal{D}_{38}^2 < 2\mathcal{D}_{33}\mathcal{D}_{88}$, and $\mathcal{D}_{38}^2 < \mathcal{D}_{33}\mathcal{D}_{88}$, which are automatically satisfied, as can be readily checked from the relevant expressions (2.9).

We are now ready to discuss the fit to the experimental data. This is done in the next section.

III. FITTING THE EXPERIMENTAL DATA

In order to check the viability of our simplified scenario, we have performed a χ^2 comparison (as opposed to a χ^2 fit) to SuperKamiokande sub-GeV and multi GeV data (the 40 data points that are shown in Figure 1), CHOOZ data (15 data points), KamLAND (13 data points, shown in Figure 2) and LSND (1 datum), for a sample point in the vast parameter space of our extremely simplified version of decoherence models. Rather than performing

a χ^2-fit (understood as a run over all the parameter space to find the global minimum of the χ^2 function) we have selected (by "eye" and not by χ) a point which is not optimised to give the best fit to the existing data. Instead, our sample point must be regarded as a local minimum around a starting point chosen by an educated guess. It follows then that it may be quite possible to find a better fitting point through a complete (and highly time consuming) scan over the whole parameter space.

To simplify the analysis and gain intuition concerning the rather cumbersome expressions for the transition probabilities, we have imposed,

$$\begin{aligned} \mathcal{D}_{11} = \mathcal{D}_{22} \ , \qquad & \mathcal{D}_{44} = \mathcal{D}_{55} \ , \\ \mathcal{D}_{66} = \mathcal{D}_{77} \ , \qquad & \mathcal{D}_{33} = \mathcal{D}_{88} \ , \\ \mathcal{D}_{38} = \mathcal{D}_{83} \ &= \ 0 \ . \end{aligned} \tag{3.1}$$

implying a diagonal $\mathcal{D}$-matrix.

Later on we shall set some of the $\mathcal{D}_{ii}$ to zero. Furthermore, we have also set the CP violating phase of the KMS matrix to zero, so that all the mixing matrix elements become real.

With these assumptions, the complicated expression for the transition probability (2.19) simplifies to:

$$\begin{aligned} P_{\nu_\alpha \to \nu_\beta}(t) \ &= \ \frac{1}{3} + \frac{1}{2} \left\{ \rho_1^\alpha \rho_1^\beta \cos\left(\frac{|\Omega_{12}|t}{2}\right) e^{\mathcal{D}_{22}t} \right. \\ &+ \ \rho_4^\alpha \rho_4^\beta \cos\left(\frac{|\Omega_{13}|t}{2}\right) e^{\mathcal{D}_{55}t} + \rho_6^\alpha \rho_6^\beta \cos\left(\frac{|\Omega_{23}|t}{2}\right) e^{\mathcal{D}_{66}t} \\ &+ \ \left. \left(\rho_3^\alpha \rho_3^\beta + \rho_8^\alpha \rho_8^\beta\right) \cosh\left(\frac{\Omega_{38}t}{2}\right) e^{\mathcal{D}_{33}t} \right\} \end{aligned} \tag{3.2}$$

for both, neutrino and antineutrino sectors.

As indicated by the state-of-the-art analysis, masses and mixing angles are selected to have the values

$$\begin{gathered} \Delta m_{12}^2 = 7 \cdot 10^{-5} \ \mathrm{eV}^2, \\ \Delta m_{23}^2 = 2.5 \cdot 10^{-3} \ \mathrm{eV}^2, \\ \theta_{23} = \pi/4, \quad \theta_{12} = .45, \quad \theta_{13} = .05 \end{gathered}$$

.

For the decoherence parameters we find

$$\mathcal{D}_{33} = \mathcal{D}_{66} = 0 \ , \qquad \mathcal{D}_{11} = \mathcal{D}_{22} = \mathcal{D}_{44} = \mathcal{D}_{55} = -\frac{1.3 \cdot 10^{-2}}{L}, \tag{3.3}$$

in units of 1/km with $L = t$ the oscillation length. The $1/L$-behaviour of $\mathcal{D}_{11}$, implies oscillation-length independent Lindblad exponents. We shall attempt to interpret this result in the next section.

The complete positivity of the case defined by (3.1) and (3.3) is guaranteed; this follows from the fact that the solutions for c_{ij} in terms of $\mathcal{D}_{\ell k}$ (cf. (2.11)) is such that the only non-zero elements are $c_{88} = c_{38}/\sqrt{3}$. Such a C-matrix has only non-negative eigenvalues

$$C - \text{matrix eigenvalues} = (0, 0, 0, 0, 0, 0, 0, -8\,\mathcal{D}_{11}/3) \tag{3.4}$$

In summary we have introduced only one new parameter, a new degree of freedom, by means of which we shall try to explain all the available experimental data. It is important to stress that the inclusion of one new degree of freedom by itself does not guarantee that all the experimental observations can be accounted for. Indeed for situations without decoherence the addition of a sterile neutrino (which introduces four new degrees of freedom -excluding CP violating phases) seemed to be insufficient for matching *all* available experimental data, at least in CPT conserving situations [16].

In order to test our model with this decoherence parameter, we have calculated the zenith angle dependence of the ratio "observed-events/(expected-events in the no oscillation case)", for muon and electron atmospheric neutrinos, for the sub-GeV and multi-GeV energy ranges, when mixing is taken into account. Since matter effects are important for atmospheric neutrinos, we have implemented them through a two-shell model, where the density in the mantle (core) is taken to be roughly 3.35 (8.44) gr/cm^3, and the core radius is taken to be 2887 km.

The results are shown in Fig. 1, where, for the sake of comparison, we have also included the experimental data. As can be easily seen the agreement is remarkable.

As bare eye comparisons can be misleading, we have also calculated the χ^2 value for each of the cases, defining the atmospheric χ^2 as

$$\chi^2_{\text{atm}} = \sum_{M,S} \sum_{\alpha=e,\mu} \sum_{i=1}^{10} \frac{(R^{\text{exp}}_{\alpha,i} - R^{\text{th}}_{\alpha,i})^2}{\sigma^2_{\alpha i}} \quad . \tag{3.5}$$

Here $\sigma_{\alpha,i}$ are the statistical errors, the ratios $R_{\alpha,i}$ between the observed and predicted signal can be written as

$$R^{\text{exp}}_{\alpha,i} = N^{\text{exp}}_{\alpha,i}/N^{\text{MC}}_{\alpha,i} \tag{3.6}$$

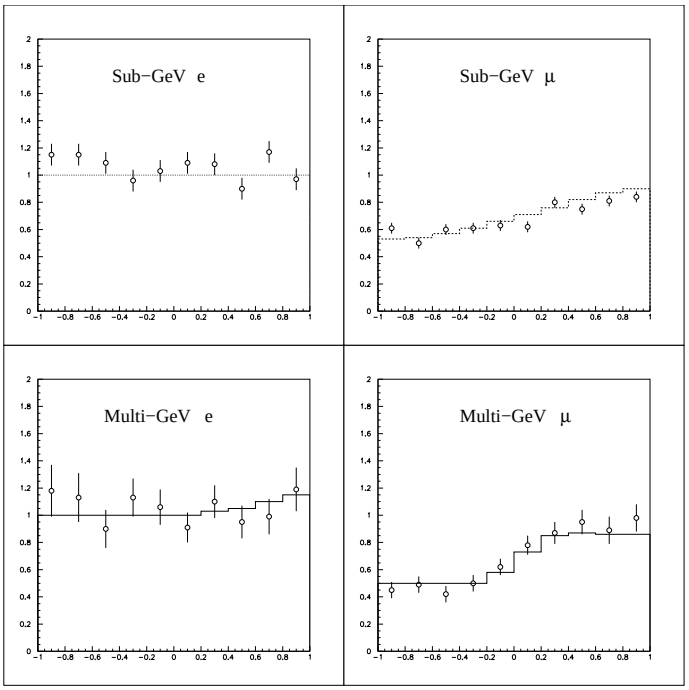

FIG. 1: Decoherence fit. The dots correspond to SK data.

(with α indicating the lepton flavor and i counting the different bins, ten in total) and M, S stand for the multi-GeV and sub-GeV data respectively. For the CHOOZ experiment we used the 15 data points with their statistical errors, where in each bin we averaged the probability over energy. For the KamLAND experiment, their 13 data points have been used for a fixed distance of $L_0 = 180$ km, as if all antineutrinos detected in KamLAND were due to a single reactor at this distance and plotted in figure 2 while for LSND one datum has been included.

The results are summarised in Table 1, where we present the χ^2 comparison for the model in question and the standard scenario (calculated with the same program).

From the table it becomes clear that our simplified version of decoherence in both neutrino and antineutrino sectors can easily account for all the available experimental information, including LSND. It is important to stress once more that our sample point was not obtained through a scan over all the parameter space, but by an educated guess, and therefore plenty of room is left for improvements. On the other hand, for the mixing-only/no-decoherence scenario, we have taken the best fit values of the state of the art analysis and therefore no significant improvements are expected.

As we have seen, the decoherence effects suffered by our model, are just an overall sup-

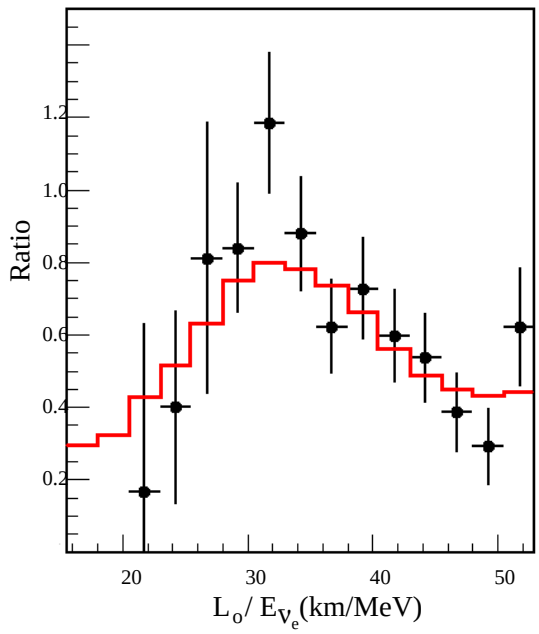

FIG. 2: Ratio of the observed $\overline{\nu}_e$ spectrum to the expectation versus L_0/E for our decoherence model. The dots correspond to KamLAND data

χ^2	decoherence	standard scenario
SK sub-GeV	38.0	38.2
SK Multi-GeV	11.7	11.2
Chooz	4.5	4.5
KamLAND	16.7	16.6
LSND	0.	6.8
TOTAL	70.9	77.3

TABLE I: χ^2 obtained for our model and the one obtained in the standard scenario for the different experiments calculated with the same program.

pression on some of the oscillatory terms modifying the transition/survival probabilities at the per mil level (to account precisely for LSND, a per mil evidence) and therefore, no effect is expected (or found) in the oscillation dominated physics, where the level of precision is at the percent level, at most. We are guaranteed then to have an excellent agreement with solar data, as long as we keep the relevant mass difference and mixing angle within the LMA-I

region, something which we certainly did. Thus, there is no need to include these data on our fit.

At this point, a word of warning is in order. Although from the table, it seems that the decoherence model we are presenting here and the standard no-decoherence scenario provide equally good a fit, i.e. while the former has a $\chi^2/\mathbf{DOF} = 70.9/63$ the latter has a $\chi^2/\mathbf{DOF} = 77.3/64$, both quite "acceptable" from the statistical point of view, one must remember that only the decoherence model can explain the LSND result. This fact, however, gets blurred in the total χ^2 because LSND is represented by only one experimental point with a poor precision.

Before closing this section, it is worth revisiting the models of I, in order to understand in the above context, the failure of complete positivity in certain regions of the parameter space. In that case, the following restrictions on the decoherence matrix (which was also diagonal, as in the case (3.1) above) had been imposed [6]:

$$\begin{aligned}
\mathcal{D}_{11} &= \mathcal{D}_{22} = \mathcal{D}_{44} = \mathcal{D}_{55} = -2 \cdot 10^{-18} \cdot E = -A \ , \\
\mathcal{D}_{66} &= \mathcal{D}_{77} = \mathcal{D}_{33} = \mathcal{D}_{88} = -10^{-24}/E = -B \ , \\
\mathcal{D}_{38} &= \mathcal{D}_{83} = 0 \ ,
\end{aligned} \tag{3.7}$$

leading to a solution for the c-matrix $c_{38} = \frac{2}{3}\sqrt{3}A - \frac{2}{3}\sqrt{3}B,\ c_{55} = \frac{2}{3}B,\ c_{44} = \frac{2}{3}B,\ c_{88} = \frac{2}{3}A,\ c_{66} = \frac{2}{3}B,\ c_{22} = \frac{2}{3}B,\ c_{33} = -4/3\,B + 2\,A,\ c_{77} = \frac{2}{3}B,\ c_{11} = \frac{2}{3}B$ (all other matrix entries zero) such that the pertinent eigenvalues $(-2\,B + \frac{8}{3}\,A, \frac{2}{3}\,B, \frac{2}{3}\,B, \frac{2}{3}\,B, \frac{2}{3}\,B, \frac{2}{3}\,B, \frac{2}{3}\,B, \frac{2}{3}\,B)$ which are not positive for arbitrary values of A and B. The positivity condition can be obtained by demanding positivity of the first eigenvalue, i.e. $2 \cdot 10^{-24}/E < \frac{16}{3} \cdot 10^{-18} \cdot E$, where E is in units of GeV, which leads to the condition $E > \mathcal{O}(1\ \text{MeV})$ which was the condition found in I.

IV. ATTEMPT AT INTERPRETING THE FIT

The microscopic origin of the "observed" decoherence effects, according to our fit above ((3.3),(3.4)), may not be unique. In fact there can be many contributions to the decoherence-induced (oscillation-independent) damping (3.4), which modulates the oscillatory terms, arising from a variety of effects, ranging from microscopic quantum-space time fluctuations ('stochastic quantum gravity foam' [1]), to ordinary matter effects, e.g. uncertainties in the

energy and oscillation length of the (anti)neutrino beam. It is the purpose of this section to attempt and disentangle these very different in nature contributions, and in particular to estimate their plausible order in terms of microscopic theoretical models and see which one gives the order specified by the fit.

To understand the results (3.3),(3.4), in connection with either stochastically fluctuating quantum-gravity space-time-foam models [2, 13], or energy-uncertainty driven decoherence [3], it suffices to restrict our discussion to the simplified (but phenomenologically realistic) case of three neutrino families, but with dominant mixing only between 12, 23 [13], with mixing angles $\theta_{12} = \theta_{23} = \theta$, and $\theta_{13} = 0$. The three generation case with full mixing does not affect qualitatively the form of the damping exponents used to fit of the oscillation experiments, and hence we are free to use results on theoretical models from this simpler case, in order to interpret qualitatively the above result.

A. Stochastic Quantum-Gravity Models

We commence our analysis with models of neutrinos propagating in a quantum-gravity ground state. To this end, consider the propagation of such a neutrino system, in a medium whose density stochastically fluctuates. For our purposes the medium is taken to be a quantum space time foam [1], with fluctuating densities of charged black-hole/anti-black-hole pairs produced by the vacuum and being absorbed by it within Planckian time scales [17]. Such a case, will not produce any vacuum charge on average, but the associated density fluctuations will produce vacuum fluctuations in electron currents with which electron neutrinos will interact. An inherent CPT violation may result in asymmetries between particle antiparticle sectors, as far as the appropriate interactions of the (anti)neutrinos with these currents are concerned.

Such asymmetries can produce a bias in flavour of, say, the electron current fluctuations in the space-time foam vacuum, with the result that a "gravitationally-induced" MSW [18] effect is in place [17], with small contributions to the standard oscillations due to bare mass differences between neutrinos, that could be due to non-gravitational physics [21]. The precise reason why a preference to electron current fluctuations as opposed to positron ones cannot be answered at this stage, since a microscopic model of quantum space time foam is lacking. However, the situation is not incompatible with the intrinsic CPT violation (to

be precise *microscopic time irreversibility*, unrelated to CP properties, according to which the generator of time reversal operations is ill-defined) characterising such problems, where a proper scattering matrix cannot be defined [19].

In such a case, the evolution equation of the density matrix ρ of the neutrino probe involves a time-reversal (CPT) breaking decoherence matrix of a double commutator form [15],

$$\partial_t \rho = i[\rho, H_{eff}] - \Omega^2 [H_I, [H_I, \rho]] \tag{4.1}$$

where $\langle n(r) n(r') \rangle = \Omega^2 n_0^2 \delta(r - r')$ denote the stochastic (Gaussian) fluctuations of the density of the medium and $H_{eff} = H_0 + H_I$, H_0 being the standard Hamiltonian, and H_I an MSW-type interaction [15, 18]. This double-commutator decoherence is a specific case of Lindblad evolution, of the type considered in previous sections, with a C-matrix of the form:

$$C = \begin{pmatrix} h_1^2 & 0 & h_1 h_3 & 0 & 0 & 0 & 0 & h_1 h_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ h_1 h_3 & 0 & h_3^2 & 0 & 0 & 0 & 0 & h_3 h_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ h_1 h_8 & 0 & h_3 h_8 & 0 & 0 & 0 & 0 & h_8^2 \end{pmatrix} \tag{4.2}$$

with $h_1 = (a_{\nu_e} - a_{\nu_\mu}) \sin(2\theta)$, $h_3 = (a_{\nu_e} - a_{\nu_\mu}) \cos(2\theta)$, and $h_8 = \frac{(a_{\nu_e} - a_{\nu_\mu})}{\sqrt{3}}$. This matrix indeed has positive eigenvalues for real h_i. The difference $a_{\nu_e} - a_{\nu_\mu}$ is proportional to the average density of the medium n_0.

We note at this stage that, for gravitationally-induced MSW effects (due to, say, black-hole foam models as in [17]), one may write

$$\Delta a_{e\mu} \equiv a_{\nu_e} - a_{\nu_\mu} \propto G_N n_0 \tag{4.3}$$

with $G_N = 1/M_P^2$, $M_P \sim 10^{19}$ GeV, the four-dimensional Planck scale. This gravitational coupling replaces the weak interaction Fermi coupling constant G_F in the conventional MSW effect. This is the case we shall be interested in for the purposes of this work.

In such a case the density fluctuations Ω^2 are therefore assumed small compared to other quantities present in the formulae, and an expansion to leading order in Ω^2 is in place.

Following then the standard analysis, outlined above, one obtains the following expression for the neutrino transition probability $\nu_e \leftrightarrow \nu_\mu$ in this case, to leading order in the small parameter $\Omega^2 \ll 1$:

$$
\begin{aligned}
P_{\nu_e \to \nu_\mu} \quad &= \\
& e^{-\Delta a_{e\mu}^2 \Omega^2 t (1+\frac{\Delta_{12}^2}{4\Gamma}(\cos(4\theta)-1))} \sin(t\sqrt{\Gamma}) \cos^2(\theta) \sin^2(2\theta) \Delta a_{e\mu}^2 \Omega^2 \Delta_{12}^2 \left(\frac{3\sin^2(2\theta)\Delta_{12}^2}{4\Gamma^{5/2}} - \frac{1}{\Gamma^{3/2}} \right) \\
& - e^{-\Delta a_{e\mu}^2 \Omega^2 t (1+\frac{\Delta_{12}^2}{4\Gamma}(\cos(4\theta)-1))} \cos(t\sqrt{\Gamma}) \cos^2(\theta) \sin^2(2\theta) \frac{\Delta_{12}^2}{2\Gamma} \\
& - e^{-\frac{\Delta a_{e\mu}^2 \Omega^2 \Delta_{12}^2 t \sin^2(2\theta)}{\Gamma}} \cos^2(\theta) \frac{(\Delta a_{e\mu} + \cos(2\theta)\Delta_{12})^2}{2\Gamma} + \frac{1}{2}\cos^2(\theta)
\end{aligned}
\tag{4.4}
$$

where $\Gamma = (\Delta a_{e\mu}\cos(2\theta) + \Delta_{12})^2 + \Delta a_{e\mu}^2 \sin^2(2\theta)$, $\Delta_{12} = \frac{\Delta m_{12}^2}{2p}$.

From (4.4) we easily conclude that the exponents of the damping factors due to the stochastic-medium-induced decoherence, are therefore of the generic form, for $t = L$, the oscillation length (in units of $c = 1$):

$$
\text{exponent} \sim \Delta a_{e\mu}^2 \Omega^2 t \left(1 + \frac{\Delta_{12}^2(\cos(4\theta) - 1)}{4\Gamma} \right) \tag{4.5}
$$

The reader should note at this stage that, in the limit $\Delta_{12} \to 0$, which could characterise the situation in [6], where the space-time foam effects on the induced neutrino mass difference are the dominant ones, the damping factor is of the form $\text{exponent}_{\text{gravitational MSW}} \sim \Omega^2 (\Delta a_{e\mu})^2 L$, with the precise value of the mixing angle θ not affecting the leading order of the various exponents. However, in that case, as follows from (4.4), the overall oscillation probability is suppressed by factors proportional to Δ_{12}^2, and, hence, the stochastic gravitational MSW effect [17], although in principle capable of inducing mass differences for neutrinos, however does not suffice to produce the bulk of the oscillation probability, which is thus attributed to conventional flavour physics.

In what follows, therefore, we assume the case where $\Delta_{12} \gg \Delta a_{e\mu}$, and this is the case we shall compare with the results of our experimental fit above. The result of the fit (3.3),(3.4), then, implies that the above decoherence-induced damping exponent (4.5) is independent of L and actually we have, to leading order in $\Delta a_{e\mu}/\Delta_{12} \ll 1$ (re-instating dimensions of $\hbar, c$):

$$
\Omega^2 (\Delta a_{e\mu})^2 \left(1 + \frac{\cos(4\theta) - 1}{4} \right) \cdot L \sim 2.56 \times 10^{-19} \text{ GeV} \cdot \text{km}. \tag{4.6}
$$

This in turn implies that in this specific model of foam, the density fluctuations of the space-time charged black holes is such that, for maximal mixing, say, $\theta = \pi/4$ assumed

for concreteness, and for $L \sim 180$ Km, as appropriate for the KamLAND experiment, the decoherence damping factor is $\mathcal{D} = \Omega^2 G_N^2 n_0^2 \sim 2.84 \cdot 10^{-21}$ GeV, if the result of the fit is due exclusively to this effect (note that the mixing angle part does not affect the order of the exponent). Smaller values are found for longer L, such as in atmospheric neutrino experiments.

The independence of the relevant damping exponent from the oscillation length, then, implied by our fit above, may be understood as follows in this context: In the spirit of [17], the quantity $G_N n_0 = \xi \frac{\Delta m^2}{E}$, where $\xi \ll 1$ parametrises the contributions of the foam to the induced neutrino mass differences, according to our discussion above. Hence, the damping exponent becomes in this case $\xi^2 \Omega^2 (\Delta m^2)^2 \cdot L/E^2$. Thus, for oscillation lengths L we have $L^{-1} \sim \Delta m^2/E$, and one is left with the following estimate for the dimensionless quantity $\xi^2 \Delta m^2 \Omega^2/E \sim 1.3 \cdot 10^{-2}$. This implies that the quantity Ω^2 is proportional to the probe energy E. In principle, this is not an unreasonable result, and it is in the spirit of [17], since back reaction effects onto space time, which affect the stochastic fluctuations Ω^2, are expected to increase with the probe energy E. However, due to the smallness of the quantity $\Delta m^2/E$, for energies of the order of GeV, and $\Delta m^2 \sim 10^{-3}$ eV^2, we conclude (taking into account that $\xi \ll 1$) that Ω^2 in this case would be unrealistically large for a quantum-gravity effect in the model.

We remark at this point that, in such a model, we can in principle bound independently the Ω and n_0 parameters by looking at the modifications induced by the medium in the arguments of the oscillatory functions of the probability (4.4), that is the period of oscillation. Unfortunately this is too small to be detected in the above example, for which $h_i \ll \Delta_{12}$.

The result of the fit, however, may be interpreted more generally, as implying independent of the oscillation length L exponents in the decoherence exponential suppression factors in front of the oscillatory terms in the transition probabilities. In this sense, the bound (3.3),(3.4) determined by the fit above, can be applied to other stochastic decoherence models, for instance the one discussed in [13], in which one averages over random (Gaussian) fluctuations of the background space-time metric over which the neutrino propagates.

In such an approach, one considers merely the Hamiltonian of the neutrino in a stochastic metric background. This is one contribution to decoherence, since other possible non-Hamiltonian (like the Lindblad terms above) interactions of the neutrino with the foam are ignored. In this case, one obtains transition probabilities with exponential damping factors

in front of the oscillatory terms, but now the scaling with the oscillation length (time) is quadratic [13], consistent with time reversal invariance of the neutrino Hamiltonian. For instance, for the two generation case, which suffices for our qualitative purposes in this work, we have:

$$\begin{aligned}
&\langle e^{i(\omega_1-\omega_2)t}\rangle = e^{i\frac{\left(z_0^+-z_0^-\right)t}{k}} e^{-\frac{1}{2}\left(-i\sigma_1 t\left(\frac{(m_1^2-m_2^2)}{k}+V\cos 2\theta\right)\right)} \times \\
&e^{-\frac{1}{2}\left(\frac{i\sigma_2 t}{2}\left(\frac{(m_1^2-m_2^2)}{k}+V\cos 2\theta\right)-\frac{i\sigma_3 t}{2}V\cos 2\theta\right)} \times \\
&e^{-\left(\frac{(m_1^2-m_2^2)^2}{2k^2}(9\sigma_1+\sigma_2+\sigma_3+\sigma_4)+\frac{2V\cos 2\theta(m_1^2-m_2^2)}{k}(12\sigma_1+2\sigma_2-2\sigma_3)\right)t^2}
\end{aligned} \tag{4.7}$$

where k is the neutrino energy, σ_i , $i = 1, \ldots 4$ parameterise appropriately the stochastic fluctuations of the metric in the model of [13], $\Upsilon = \frac{Vk}{m_1^2-m_2^2}$, $|\Upsilon| \ll 1$, and $k^2 \gg m_1^2$, m_2^2, and

$$\begin{aligned}
z_0^+ &= \frac{1}{2}\left(m_1^2 + \Upsilon(1+\cos 2\theta)(m_1^2-m_2^2) + \Upsilon^2(m_1^2-m_2^2)\sin^2 2\theta\right) \\
z_0^- &= \frac{1}{2}\left(m_2^2 + \Upsilon(1-\cos 2\theta)(m_1^2-m_2^2) - \Upsilon^2(m_1^2-m_2^2)\sin^2 2\theta\right) \ .
\end{aligned} \tag{4.8}$$

Note that the metric fluctuations-σ_i induced modifications of the oscillation period, as well as exponential $e^{-(\ldots)t^2}$ time-reversal invariant damping factors [13]. The latter is attributed to the fact that in this approach, only the Hamiltonian terms are taken into account (in a stochastically fluctuating metric background), and as such time reversal invariance $t \to -t$ is not broken explicitly. But there is of course decoherence, and the associated damping.

We, then, observe that the result of the fit above, (3.3),(3.4), implying L-independent exponents in the associated damping factors due to decoherence, may also apply to this case, implying for the damping exponent:

$$\left(\frac{(m_1^2-m_2^2)^2}{2k^2}(9\sigma_1+\sigma_2+\sigma_3+\sigma_4) + \frac{2V\cos 2\theta(m_1^2-m_2^2)}{k}(12\sigma_1+2\sigma_2-2\sigma_3)\right)t^2 \sim 1.3\cdot 10^{-2} \ . \tag{4.9}$$

Ignoring subleading MSW effects V, for simplicity, and considering oscillation lengths $t = L \sim \frac{2k}{(m_1^2-m_2^2)}$, we then observe that the independence of the length L result of the experimental fit, found above, may be interpreted, in this case, as bounding the stochastic fluctuations of the metric to $9\sigma_1+\sigma_2+\sigma_3+\sigma_4 \sim 1.3.\cdot 10^{-2}$. This is too large to be a quantum gravity effect, which means that the L^2 contributions to the damping due to stochastic fluctuations of the metric, as in the model of [13] above, cannot be the explanation of the fit.

B. Conventional Explanation: Energy Uncertainties

The reader's attention is called at this point to the fact that such time-reversal invariance decoherence may also be due to ordinary uncertainties [3] in energies and/or oscillation lengths, which are unrelated to quantum gravity effects. For instance, consider the ordinary oscillation formula for neutrinos, with a mixing matrix U,

$$\begin{aligned} &P_{\alpha\beta} = P_{\alpha\beta}(L,E) = \\ &\delta_{\alpha\beta} - 4\sum_{a=1}^{n}\sum_{\beta=1,a<b}^{n} \mathrm{Re}\left(U^*_{\alpha a}U_{\beta a}U_{\alpha b}U^*_{\beta b}\right)\sin^2\left(\frac{\Delta m^2_{ab}L}{4E}\right) - \\ &2\sum_{a=1}^{n}\sum_{b=1,a<b}^{n} \mathrm{Im}\left(U^*_{\alpha a}U_{\beta a}U_{\alpha b}U^*_{\beta b}\right)\sin\left(\frac{\Delta m^2_{ab}L}{2E}\right) \end{aligned} \tag{4.10}$$

where $\alpha,\beta = e,\mu,\tau,...,\ a,b = 1,2,...n,\ \Delta m^2_{ab} = m^2_a - m^2_b$.

In general there are uncertainties in the energy E in the production of a ν (and/or $\overline{\nu}$)-wave, and also in the oscillation length. As a result of these uncertainties one has to average the oscillation probability (4.10) over the L/E dependence.

Considering a Gaussian average [3],

$$\langle P\rangle = \int_{-\infty}^{\infty} \mathrm{d}x\ P(x)\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\ell)^2}{2\sigma^2}}$$

$\ell \equiv \langle x\rangle$, $\sigma = \sqrt{\langle (x-\langle x\rangle)^2}$, $x = L/4E$, and approximating $\langle L/E\rangle \simeq \langle L\rangle/\langle E\rangle$ we obtain

$$\begin{aligned} &\langle P_{\alpha\beta}\rangle = \delta_{\alpha\beta} - \\ &2\sum_{a=1}^{n}\sum_{\beta=1,a<b}^{n} \mathrm{Re}\left(U^*_{\alpha a}U_{\beta a}U_{\alpha b}U^*_{\beta b}\right)\left(1-\cos(2\ell\Delta m^2_{ab})e^{-2\sigma^2(\Delta m^2_{ab})^2}\right) \\ &-2\sum_{a=1}^{n}\sum_{b=1,a<b}^{n} \mathrm{Im}\left(U^*_{\alpha a}U_{\beta a}U_{\alpha b}U^*_{\beta b}\right)\sin(2\ell\Delta m^2_{ab})e^{-2\sigma^2(\Delta m^2_{ab})^2} \end{aligned} \tag{4.11}$$

Notice the exponential damping factors due to the fluctuations σ.

In fact, as discussed in [3], there are two kinds of bounds for σ: A *Pessimistic:* one, according to which $\sigma \simeq \Delta x \simeq \Delta\frac{L}{4E} \leq \frac{\langle L\rangle}{4\langle E\rangle}\left(\frac{\Delta L}{\langle L\rangle} + \frac{\Delta E}{\langle E\rangle}\right)$ and an *Optimistic:* $\sigma \leq \frac{\langle L\rangle}{4\langle E\rangle}\left([\frac{\Delta L}{\langle L\rangle}]^2 + [\frac{\Delta E}{\langle E\rangle}]^2\right)^{1/2}$.

In our case, where we consider long baseline experiments, the uncertainties in the oscillation length L are negligible, and hence the two cases degenerate to a single expression for $\sigma = \frac{\langle L\rangle}{4\langle E\rangle}\frac{\Delta E}{E}$.

The damping exponent, then, in (4.11), arising from the uncertainties in the energy of the (anti)neutrino beam, becomes

$$2\sigma^2(\Delta m^2)^2 = 2\frac{\langle L\rangle^2}{(4\langle E\rangle)^2}\left(\frac{\Delta E}{E}\right)^2(\Delta m^2)^2 \ . \tag{4.12}$$

As mentioned above, for oscillation lengths we have $L\Delta m^2/2E \sim \mathcal{O}(1)$, and hence, the result of the best fit (3.3),(3.4), implying independence of the damping exponent on L (irrespective of the power of L), yields an uncertainty in energy of order

$$\frac{\Delta E}{E} \sim 1.6 \cdot 10^{-1} \tag{4.13}$$

if one assumes that this is the principal reason for the result of the fit. This is not an unreasonable result, thus implying that the result of the fit may be interpreted as being due to ordinary physics associated with uncertainties in the energy of the neutrino beam.

The important difference from the stochastic fluctuations of gravity medium, discussed above, lies on the fact that the period of oscillation is not affected (4.11) by the above averaging procedure, in contrast to the stochastic gravity cases (4.4) and (4.7), and thus in principle the effects can be disentangled. However, in general these latter corrections are small, and beyond the sensitivity of the current experiments. Nevertheless, as we have seen, some effects, such as the time-reversal symmetric stochastic fluctuations of the background metric (4.7), can be already disentangled straightforwardly by their order as compared with the above energy-uncertainty effects due to ordinary physics.

The precise energy and length dependence of the damping factors is an essential step in order to determine the microscopic origin of the induced decoherence and disentangle genuine new physics effects [2] from conventional effects, which as we have seen above may also contribute to decoherence-like damping [4]. For instance, as we discussed above, some genuine quantum-gravity effects, such as the stochastic fluctuations of the space time, are expected to increase in general with the energy of the probe [2], as a result of back reaction effects on space-time geometry, in contrast to ordinary-matter-induced 'fake' CPT violation and 'decoherence-looking' effects, which decrease with the energy of the probe [4]. At present, the sensitivity of the experiments is not sufficient to unambiguously determine the microscopic origin of the decoherence effects, as we have seen above, but we think that in the near future, when experiments involving both higher energy and precision become available, one would be able to arrive at definite conclusions on this important issue. Thus,

phenomenological analyses like ours are of value and should be actively pursued, in our opinion, in the future, not only in neutrino physics but also in other sensitive probes of quantum mechanics, such as neutral mesons.

V. CONCLUSIONS AND OUTLOOK

In this work we have presented a complete analysis of three-generation neutrino transition probabilities, which include decoherence effects with guaranteed positivity. In our opinion this is the first complete, mathematically consistent, example of a Lindblad-decoherence model for neutrinos with full three generation mixing.

We have shown that decoherence effects can account for all available neutrino data, including LSND results even in a minimalistic scenario (with only one new parameter, which parametrises all the decoherence effects). Contrary to other approaches in the literature, using sterile neutrinos [16], and following the spirit of our earlier work [6, 17], we have attempted to interpret the LSND results not by means of oscillations, but as a decoherence effect inducing damping in the oscillatory terms, which is also present (as a per-mil-ish additional suppression) in other neutrino experiments as well.

The specific oscillation length L dependence of the single decoherence parameter, implying L-independence of the corresponding damping exponents, could in principle find a natural explanation in some theoretical models of stochastic quantum gravity. However, its order of magnitude seems incompatible with this possibility, at least in the concrete space-time foam microscopic models considered here, since it would imply quantum-gravity effects unrealistically large.

On the other hand, the result of our fit can find a natural explanation in terms of ordinary physics. It could be due, for instance, to uncertainties in the energy beam of (anti)neutrinos, and indeed this scenario seems to provide the most natural explanation of our fit.

We now remark that quantum-gravity contributions could indeed be present, and lead to similar damping in oscillatory terms, but their suppressed order of magnitude would imply that they could only be probed at higher energies. For instance, high-energy neutrinos detected from distant supernovae, may probe these issues further, since they will increase significantly the sensitivity to genuine quantum gravity effects [20], and thus may probe the induced changes in the damping as well as the oscillation period, as discussed in this work.

It goes without saying that there is much more work to be done, both theoretical and experimental, before definite conclusions are reached on this important issue, but we believe that neutrino (astro)physics will provide a very sensitive probe of new physics, including quantum gravity, in the not-so-distant future.

Acknowledgements

G.B. is partially supported by the spanish MEC and FEDER under FPA2005-01678 and by Generalitat Valenciana under GV05/267. The work of N.E.M. is partially supported by funds made available by the European Social Fund (75%) and National Resources (25%) - EPEAEK B - PYTHAGORAS (Greece). G.B. and N.E.M. would like to thank M. Baldo-Ceolin for the invitation to the *III International Workshop on Neutrino Oscillations in Venice*, 7-10 February 2006, Venice (Italy), where preliminary results of this work have been presented (N.E.M).

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