Effective gluon mass and infrared fixed point in QCD

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Abstract. We report on a special type of solutions for the gluon propagator of pure QCD, obtained from the corresponding non-linear Schwinger-Dyson equation formulated in the Feynman gauge of the background field method. These solutions reach a finite value in the deep infrared and may be fitted using a massive propagator, with the crucial characteristic that the effective "mass" employed depends on the momentum transfer. Specifically, the gluon mass falls off as the inverse square of the momentum, as expected from the operator-product expansion. In addition, one may define a dimensionless quantity, which constitutes the generalization in a non-Abelian context of the universal QED effective charge. This strong effective charge displays asymptotic freedom in the ultraviolet whereas in the low-energy regime it freezes at a finite value, giving rise to an infrared fixed point for QCD.

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A plethora of theoretical and phenomenological studies spanning more than two decades have corroborated the possibility of describing the infrared (IR) sector of QCD in terms of an effective gluon mass (for an extended list of references see [1]). According to this picture, even though the gluon is massless at the level of the fundamental Lagrangian, and remains massless to all order in perturbation theory, the non-perturbative QCD dynamics generate an effective, momentum-dependent mass, without affecting the local $SU(3)_c$ invariance, which remains intact [2].

The most standard way for studying such a non-perturbative effect in the continuum is the (appropriately truncated) Schwinger-Dyson equation (SDE) for the gluon propagator $\Delta_{\mu\nu}(q)$, defined (in the Feynman gauge) as

$$\Delta_{\mu\nu}(q) = -i\left[\mathrm{P}_{\mu\nu}(q)\Delta(q^2) + \frac{q_\mu q_\nu}{q^4}\right], \qquad \mathrm{P}_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}. \tag{1}$$

Specifically, one looks for solutions having $\Delta(q^2)$ reaching finite (non-vanishing) values in the deep infrared, that may be fitted by "massive" propagators of the form $\Delta^{-1}(q^2) = q^2 + m^2(q^2)$. The crucial characteristic is that $m^2(q^2)$ is not "hard", but depends non-trivially on the momentum transfer q^2. When the renormalization-group logarithms are properly taken into account, one obtains in addition the non-perturbative generalization of $g^2(q^2)$, the QCD running coupling (effective charge). The presence of $m^2(q^2)$ in the argument of $g^2(q^2)$ tames the Landau singularity associated with the perturbative β function, and the resulting effective charge is asymptotically free in the ultraviolet (UV), "freezing" at a finite value in the IR.

The running of $m^2(q^2)$ is of central importance for the self-consistency of this approach, mainly because the value of $\Delta^{-1}(0)$ is determined by integrals involving $\Delta(q^2)$, $m^2(q^2)$, and $g^2(q^2)$ over the entire range of (Euclidean) momenta. The UV convergence of these integrals depends crucially on how $m^2(q^2)$ behaves as $q^2 \to \infty$. If $m^2(q^2)$ drops off asymptotically faster than a logarithm, then $\Delta^{-1}(0)$ is finite. This, in turn, is crucial because the finiteness of $\Delta^{-1}(0)$ guarantees essentially the renormalizability of QCD.

In earlier studies of *linear* SDE [2, 1] the $m^2(q^2)$ obtained drops in the deep UV as an inverse power of a logarithm. The main result reported in this talk is the existence of a new type of solutions for $m^2(q^2)$ that drop asymptotically as an *inverse power* of momentum (multiplied by logarithms) [3].

These solutions are found in the study of *nonlinear* SDE, in the framework defined from the combination of the Pinch Technique (PT) [2, 4, 5] and the Feynman gauge of the Background Field Method (BFM) [6, 7], known as PT-BFM truncation scheme [1]. One of the most powerful features of the PT-BFM formalism is that, by virtue of the Abelian Ward identities satisfied by the various vertices, gluonic and ghost contributions are *separately* transverse, within *each* order in the "dressed-loop" expansion [1]. This, in turn, allows one to truncate the series meaningfully, by considering only the diagrams ($\mathbf{a_1}$) and ($\mathbf{a_2}$) shown in Fig.1, (no ghosts included), without compromising the transversality of the answer.

In order to reduce the algebraic complexity of the problem, we perform one additional approximation, dropping the longitudinal terms from the gluon propagators inside the integrals, i.e. we set $\Delta_{\alpha\beta} \to -ig_{\alpha\beta}\Delta$. Omitting these terms does not interfere with the transversality of the resulting propagator, provided that one drops, at the same time, the longitudinal pieces in the WI of Eq.(4) [1, 3].

After these steps, the scalar function, $\Delta^{-1}(q^2) = q^2 + i\Pi(q^2)$, (where $\Pi(q^2)$ is the gluon-self energy given by the diagrams ($\mathbf{a_1}$) and ($\mathbf{a_2}$) in Fig.1) can be written as

$$
\begin{aligned}
i\mathrm{P}_{\mu\nu}(q)\Delta^{-1}(q^2) = i\mathrm{P}_{\mu\nu}(q)\, q^2 \; &- \; \frac{C_{\mathrm{A}} g^2}{2} \int [dk]\, \widetilde{\Gamma}_{\mu}^{\alpha\beta} \Delta(k) \widetilde{\mathrm{I\Gamma}}_{\nu\alpha\beta} \Delta(k+q) \\
&+ \; 4 C_{\mathrm{A}} g^2 g_{\mu\nu} \int [dk]\, \Delta(k)\,,
\end{aligned} \tag{2}
$$

where the tree-level vertex $\widetilde{\Gamma}_{\mu\alpha\beta}$ appearing in (2) is given by

$$
\widetilde{\Gamma}_{\mu\alpha\beta}(q,p_1,p_2) = (p_1 - p_2)_\mu g_{\alpha\beta} + 2q_\beta g_{\mu\alpha} - 2q_\alpha g_{\mu\beta}\,, \tag{3}
$$

and $\widetilde{\mathrm{I\Gamma}}_{\nu\alpha'\beta'}$ represents the full three-gluon vertex.

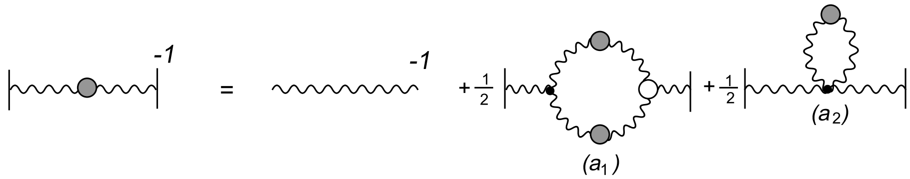

FIGURE 1. The gluonic "one-loop dressed" contributions to the SDE.

As a next step we will employ the "gauge technique" [8], expressing $\widetilde{\mathrm{I\!\Gamma}}$ as a functional of Δ, in such a way as to satisfy (by construction) the all-order Ward identity

$$q^{\mu}\widetilde{\mathrm{I\!\Gamma}}_{\mu\alpha\beta}(q,p_1,p_2) = i[\Delta^{-1}_{\alpha\beta}(p_1) - \Delta^{-1}_{\alpha\beta}(p_2)]\,, \tag{4}$$

characteristic of the PT-BFM. Specifically, we propose the following form for the vertex [3]

$$\widetilde{\mathrm{I\!\Gamma}}^{\mu\alpha\beta} = L^{\mu\alpha\beta} + T_1^{\mu\alpha\beta} + T_2^{\mu\alpha\beta}\,, \tag{5}$$

with

$$\begin{aligned}
L^{\mu\alpha\beta}(q,p_1,p_2) &= \widetilde{\Gamma}^{\mu\alpha\beta}(q,p_1,p_2) + ig^{\alpha\beta}\,\frac{q^{\mu}}{q^2}\,[\Pi(p_2) - \Pi(p_1)]\,,\\
T_1^{\mu\alpha\beta}(q,p_1,p_2) &= -i\frac{c_1}{q^2}\left(q^{\beta}g^{\mu\alpha} - q^{\alpha}g^{\mu\beta}\right)[\Pi(p_1) + \Pi(p_2)]\,,\\
T_2^{\mu\alpha\beta}(q,p_1,p_2) &= -ic_2\left(q^{\beta}g^{\mu\alpha} - q^{\alpha}g^{\mu\beta}\right)\left[\frac{\Pi(p_1)}{p_1^2} + \frac{\Pi(p_2)}{p_2^2}\right].
\end{aligned} \tag{6}$$

Then, substituting Eqs.(3) and (5) into (2), introducing $q^2 \equiv x$, $k^2 \equiv y$, and defining the renormalization-group invariant quantity $d(q^2) = g^2\Delta(q^2)$, we arrive at

$$d^{-1}(x) = K'x + \tilde{b}\sum_{i=1}^{8}\widehat{A}_i(x) + d^{-1}(0)\,, \tag{7}$$

with

$$\begin{aligned}
\widehat{A}_1(x) &= -\left(1+\frac{6c_2}{5}\right)x\int_x^{\infty} dy\, y\,\mathcal{L}^2(y)d^2(y)\,,\\
\widehat{A}_2(x) &= \frac{6c_2}{5}x\int_x^{\infty} dy\,\mathcal{L}(y)d(y)\,,\\
\widehat{A}_3(x) &= -\left(1+\frac{6c_2}{5}\right)x\,\mathcal{L}(x)d(x)\int_0^{x} dy\, y\,\mathcal{L}(y)d(y)\,,\\
\widehat{A}_4(x) &= \left(-\frac{1}{10}-\frac{3c_2}{5}+\frac{3c_1}{5}\right)\int_0^{x} dy\, y^2\,\mathcal{L}^2(y)d^2(y)\,,\\
\widehat{A}_5(x) &= -\frac{6}{5}\Big(1+c_1\Big)\mathcal{L}(x)d(x)\int_0^{x} dy\, y^2\,\mathcal{L}(y)d(y)\,,\\
\widehat{A}_6(x) &= \frac{6c_2}{5}\int_0^{x} dy\, y\,\mathcal{L}(y)d(y)\,,\\
\widehat{A}_7(x) &= \frac{2}{5}\mathcal{L}(x)\,\frac{d(x)}{x}\int_0^{x} dy\, y^3\,\mathcal{L}(y)d(y)\,,\\
\widehat{A}_8(x) &= \frac{1}{5x}\int_0^{x} dy\, y^3\,\mathcal{L}^2(y)d^2(y)\,.
\end{aligned} \tag{8}$$

The renormalization constant K' is fixed by the condition $d^{-1}(\mu^2) = \mu^2/g^2$, (with $\mu^2 \gg \Lambda^2$), and $\mathcal{L}(q^2) \equiv \tilde{b}\ln(q^2/\Lambda^2)$, where Λ is QCD mass scale. Due to the poles contained in the Ansatz for $\widetilde{\rm I\!\Gamma}$, $d^{-1}(0)$ does not vanish, and is given by the (divergent) expression

$$d^{-1}(0)=\frac{3\tilde{b}}{5\pi^2}\left[2(1+c_1)\int d^4k\,\mathcal{L}(k^2)\,d(k^2)-(1+2c_1)\int d^4k\,k^2\,\mathcal{L}^2(k^2)\,d^2(k^2)\right], \quad (9)$$

which can be made finite using dimensional regularization, and assuming that $m^2(q^2)$ drops sufficiently fast in the UV [1].

In order to determine the asymptotic behavior that Eq.(7) predicts for $m^2(x)$ at large x, we perform the following replacements in the r.h.s. of (8)

$$x\mathcal{L}(x)d(x)\to 1, \quad \mathcal{L}(x)d(x)\to 1/x, \quad \mathcal{L}(y)d(y)=\tilde{\Delta}(y), \quad \tilde{\Delta}(y)=\frac{1}{y+m^2(y)}. \quad (10)$$

Next, use the identity $y\tilde{\Delta}(y)=1-m^2(y)\tilde{\Delta}(y)$ in all $\widehat{A}_i(x)$, keeping only terms linear in m^2 . Then separate all contributions that go like x from those that go like m^2 on both sides, and match them up [9]. This gives rise to two independent equations, one for the "kinetic" term, which simply reproduces the asymptotic behavior $x\ln x$ on both sides, and an equation for the terms with $m^2(x)$, given by

$$\begin{aligned} m^2(x)\ln x \;&=\; \mathcal{C}-a_1\int_x^\infty dy\,m^2(y)\tilde{\Delta}(y)+\frac{a_2}{x}\int_0^x dy\,y m^2(y)\tilde{\Delta}(y) \\ &\quad +\frac{a_3}{x^2}\int_0^x dy\,y^2 m^2(y)\tilde{\Delta}(y)+a_4 x\int_x^\infty dy\,m^2(y)\tilde{\Delta}^2(y)\,, \qquad (11)\end{aligned}$$

with

$$a_1=\frac{6}{5}(1+c_2-c_1)\,, \quad a_2=\frac{4}{5}+\frac{6c_1}{5}\,, \quad a_3=-\frac{2}{5}\,, \quad a_4=1+\frac{6c_2}{5}\,, \qquad (12)$$

and

$$\mathcal{C}\equiv\tilde{b}^{-1}d^{-1}(0)+a_1\int_0^\infty dy\,m^2(y)\tilde{\Delta}(y)\,. \qquad (13)$$

Now, the important point to appreciate is that, in order for (11) to have solutions vanishing in the UV, it is necessary to be sure that the constant term on the r.h.s. vanishes, i.e. $\mathcal{C}=0$. Since we know that $d^{-1}(0)$ and the integral appearing in the r.h.s. of Eq.(13) are manifestly positive quantities, it follows immediately that the $\mathcal{C}$ will be zero if and only if $a_1<0$. Notice that Eq.(13) restricts the range of allowed values of the parameters c_1 and c_2 through Eq.(12). In addition, and more importantly, it constrains the momentum dependence of $m^2(x)$ in the IR and intermediate regimes to be such that both terms on the r.h.s of Eq.(13) cancel against each other.

Assuming that Eq.(13) is satisfied, it can be shown that Eq.(11) admits the following asymptotic solutions for $m^2(x)$ [3],

$$m_1^2(x)=\lambda_1^2(\ln x)^{-(1+\gamma_1)}\,, \qquad m_2^2(x)=\frac{\lambda_2^4}{x}(\ln x)^{\gamma_2-1}\,, \qquad (14)$$

where λ_1 and λ_2 are two mass-scales, and $\gamma_1 = -a_1$, $\gamma_2 = a_2$.

The first type of solutions, $m_1^2(x)$, are familiar from studying linearized versions of Eq.(2), see for example [2, 1]. The second type of solutions, $m_2^2(x)$, displaying power-law running, are particularly interesting because they are derived for the first time in the context of SDE. The possibility of an effective gluon mass dropping in the UV as an *inverse power of the momentum* was first conjectured in [2], and was subsequently obtained in the context of the operator-product expansion [10]; there the resulting gluon self-energy was identified as the effective gluon mass, leading to the relation $m^2(x) \sim \langle G^2 \rangle / x$, where $\langle G^2 \rangle$ is the dimension four gauge-invariant gluon condensate.

Which of the two types of solution will be actually realized depends on the details of the three-gluon vertex, $\widetilde{\rm I\!\Gamma}$, and more specifically on the values of the parameters c_1 and c_2. Our numerical analysis reveals that the sets of values for c_1 and c_2 giving rise to logarithmic running belong to an interval that is disjoint and well-separated from that producing power-law running. In what follows we will focus our attention on the latter type of solutions. In Fig.2 we present typical solutions for the $d(q^2)$, $m^2(q^2)$ and the effective charge $\alpha(q^2) = g^2(q^2)/4\pi$.

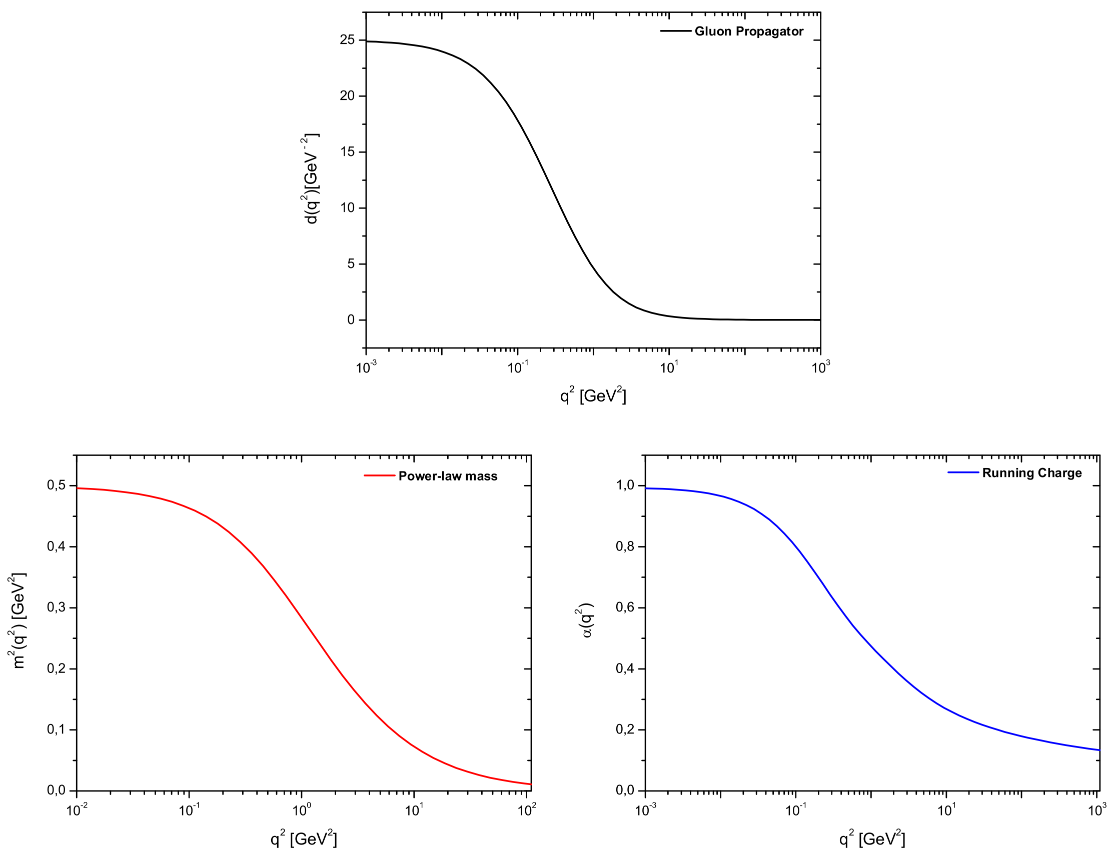

FIGURE 2. Upper panel: the numerical solution for $d(q^2)$. Lower panels: Left: dynamical mass with power-law running, for $m_0^2 = 0.5\ \text{GeV}^2$ and $\rho = 1.046$ in Eq.(17). Right: the running charge, $\alpha(q^2) = g^2(q^2)/4\pi$, fitted by Eqs.(15) and (16).

The way to extract from $d(q^2)$ the corresponding $m^2(q^2)$ and $g^2(q^2)$ is by casting the numerical solutions shown in Fig.2 into the form

$$d(q^2) = \frac{g^2(q^2)}{q^2+m^2(q^2)}, \quad g^2(q^2) = \left[\tilde{b}\ln\left(\frac{q^2+f(q^2,m^2(q^2))}{\Lambda^2}\right)\right]^{-1}, \tag{15}$$

with

$$f(q^2,m^2(q^2)) = \rho_1 m^2(q^2) + \rho_2 \frac{m^4(q^2)}{q^2+m^2(q^2)} + \rho_3 \frac{m^6(q^2)}{[q^2+m^2(q^2)]^2}, \tag{16}$$

where ρ_1, ρ_2, and ρ_3 are fitting parameters.

The functional form used for the running mass is

$$m^2(q^2) = \frac{m_0^4}{q^2+m_0^2}\left[\ln\left(\frac{q^2+\rho m_0^2}{\Lambda^2}\right)\Big/\ln\left(\frac{\rho m_0^2}{\Lambda^2}\right)\right]^{\gamma_2-1}. \tag{17}$$

In the deep UV Eq.(17) goes over to $m_2^2(q^2)$, whereas at $q^2=0$ it reaches the finite value $m^2(0)=m_0^2$. Note that $f(q^2,m^2(q^2))$ is such that $f(0,m^2(0))>0$; as a result the perturbative Landau pole in the running coupling is tamed, and $g^2(q^2)$ reaches a finite positive value at $q^2=0$, leading to an *infrared fixed point* [2, 11, 12].

To summarize our results, from a gauge-invariant SDE for the gluon propagator we have derived an integral equation that describes the running of the effective gluon mass in the UV, and have demonstrated that, depending on the values of two basic parameters appearing in the three-gluon vertex, one finds solutions that drop as inverse powers of a logarithm of q^2, or much faster, as an inverse power of q^2. Moreover, we have extracted an asymptotically free effective (running) charge, that freezes in the low-momentum region, implying the existence of a IR fixed point for QCD.

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