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Power corrections in models with extra dimensions

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Abstract. We critically revisit the issue of power-law running in models with extra dimensions. The general conclusion is that, in the absence of any additional physical principle, the power-corrections tend to depend strongly on the details of the underlying theory.

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1 Introduction

The power-law running of couplings has been considered as one of the most characteristic predictions of models with extra dimensions [1], allowing the exciting possibility of an early unification [2]. Even though there is no doubt that power-law corrections will appear in such theories, their precise physical interpretation merits further scrutiny [3,4].

The basic argument in favor of power-law running is $\overline{\mathrm{MS}}$ inspired. In a general renormalization scheme satisfying decoupling the β function assumes the form

$$\beta = \sum_n \beta_0 f\left(\frac{\mu}{M_n}\right) \tag{1}$$

with μ the renormalization scale, β_0 the contribution of a single mode, and $f(\mu/M) \to 0 \quad \mu \ll M$ and $f(\mu/M) \to 1 \quad \mu \gg M$. In particular, in the case of the $\overline{\mathrm{MS}}$ the function $f(\mu/M)$ is chosen to be the step-function, $f(\mu/M) \equiv \theta(\mu/M - 1)$, in order to enforce decoupling. Theories with δ extra compact dimensions contain an infinite tower of Kaluza-Klein (KK) modes with masses

$$M_n^2 = \left(n_1^2 + n_2^2 + \cdots + n_\delta^2\right) M_c^2, \tag{2}$$

where $M_c = 1/R_c$ is the compactification scale. Then, the naive way of generalizing the $\overline{\mathrm{MS}}$ in the presence of an infinite number of such modes is simply

$$\beta = \sum_{n<\mu/M_c} \beta_0 \approx \beta_0 \int_{n<\mu/M_c} d\Omega_\delta n^{\delta-1} dn$$
$$= \beta_0 \frac{1}{\Gamma(1+\delta/2)} \left(\pi \frac{\mu^2}{M_c^2}\right)^{\delta/2},$$

giving rise to a β which just counts the number of active modes, i.e. lighter than μ. But this generalization is ambiguous, because the $\overline{\mathrm{MS}}$ scheme does not satisfy decoupling. Instead, decoupling must be imposed by hand every time a threshold is crossed [5]. Therefore, in the presence of an infinite number of thresholds the result becomes extremely dependent on the prescription used.

Of course, particles decouple naturally and smoothly in the Vacuum Polarization Function (VPF), because of unitarity (optical theorem). In [2] the VPF of the photon was calculated in the presence of the tower of fermionic KK modes. However, the VPF was computed at $Q^2 = 0$, a fact which obscures the relation with the optical theorem. In addition, a cutoff Λ (in proper time) was used. The cutoff was eventually identified with the sliding scale and the result used to compute the β function. It is easy to convince oneself however, that the above procedure is equivalent to using the function $f(\Lambda/M) \equiv \exp\left(-M_n^2/\Lambda^2\right)$ to decouple the KK modes. So, the resulting β-function reads

$$\beta = \sum_n \beta_0 e^{-\frac{M_n^2}{\Lambda^2}} \approx \beta_0 \left(\pi \frac{\Lambda^2}{M_c^2}\right)^{\delta/2},$$

and μ was chosen by hand to satisfy $\mu^\delta = \Gamma(1+\delta/2)\Lambda^\delta$ in order to reproduce the $\overline{\mathrm{MS}}$ inspired result.
Thus, even though the VPF is used, the introduction of a hard cutoff is not any better conceptually than the direct use of a sharp step function for decoupling the modes: one gets a smooth β function because one puts in by hand a smooth function to decouple the KK modes. Because of the very sharp step-like decoupling, these two ways of decoupling KK modes lead to a finite result for any number of extra dimensions.

The physical decoupling function $f(\mu/M)$ that is really obtained from the VPF can be approximated [6] by the simple expression $f(\mu/M) = \mu^2/(\mu^2 + 5M^2)$. Substituting it in Eq.(1), we see that the sum over all KK modes converges only for one extra dimension, but is badly divergent for several extra dimensions. The extra infinities one finds when summing all the KK modes are just the manifestation of the non-renormalizability of the underlying uncompactified theory; the latter is non-renormalizable simply because the gauge coupling has dimension $1/M^{\delta/2}$.

Therefore, higher dimension operators are needed as counterterms, and one is naturally led to the effective field theories (EFT) .

2 Extra dimensions and EFT

The general rules of continuum EFT (to be distinguished from the "Wilsonian" type [7]), may be summarized as follows [5]: (i) Virtual momenta in loops run up to infinity; (ii) heavy particles are removed from the spectrum at low energies; (iii) effects of heavy particles are absorbed in the coefficients of higher-dimensional operators; (iv) regularization and renormalization are necessary; (v) the use of dimensional regularization and the $\overline{\mathrm{MS}}$ scheme is advantageous, because in that case there is no mixing between operators of different dimensionality.

We next proceed to use the continuum EFT at the level of the extra-dimensional (uncompactified) theory. Clearly, the virtual momenta associated with the extra dimensions run up to infinity, as in point (i) above. However, at the level of the 4-d (compactified) theory they are KK masses and one is supposed to keep only particles lighter than the relevant scale. Thus, truncating the KK tower by introducing a large (but otherwise arbitrary) cutoff N_s amounts to cutting-off the momenta of the uncompactified theory. Identifying N_s with a physical cutoff gives illusion of predictivity, but is plagued with ambiguities [8]. Even when using cutoffs one has to add counterterms from higher order operators, absorb the cutoff and express the result in terms of a series of unknown coefficients. Therefore, in order to define a genuine non-cutoff continuum EFT framework, we must keep *all* KK modes, or, equivalently, study how they decouple *all of them at once*.

3 Computing the VPF in a toy model

We will follow the strategy outlined above in the context of a simple toy model [4]. Consider a theory with one fermion and one photon in $4+\delta$ dimensions, with the extra dimensions compactified on a torus of equal radii $R_c \equiv 1/M_c$. The Lagrangian reads

$$\mathcal{L}_\delta = -\frac{1}{4}F^{MN}F_{MN} + i\bar\psi\gamma^M D_M\psi + \mathcal{L}_{\mathrm{ct}} \tag{3}$$

where $M = 0, \cdots, 3, \cdots, 3+\delta$, $\mu = 0, \cdots 3$, $D_M = \partial_M - ie_D A_M$, and e_D is the coupling in $D = 4+\delta$ dimensions. It has dimension $[e_D] = 1/M^{\delta/2}$. After compactification, the four-dimensional dimensionless gauge coupling, e_4, and e_D are related by the compactification scale: $e_4 = e_D\left(\frac{M_c}{2\pi}\right)^{\delta/2}$. The part of the spectrum relevant to our purposes is: (i) one massless photon; (ii) $2^{[\delta/2]}$ massless Dirac fermions; (iii) a tower of massive Dirac fermions with masses given by Eq.(2). In addition, the counterterm Lagrangian is given by $\mathcal{L}_{\mathrm{ct}} = \frac{\kappa_1}{M_s}D_M F^{MK}D^N F_{NK} + \cdots$, where the ellipses denote operators of higher dimensionality.

We next compute the VPF at the level of the compactified theory; at one-loop it is given by

$$\Pi^{\mu\nu}(q) = ie_4^2\sum_n \int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left\{\gamma^\mu\frac{1}{\not k - M_n}\gamma^\nu\frac{1}{\not k + \not q - M_n}\right\}.$$

From gauge invariance, $\Pi^{\mu\nu}(q) = \left(q^2 g^{\mu\nu} - q^\mu q^\nu\right)\Pi(q)$. In order to exploit dimensional regularization techniques, we add and subtract the contribution of VPF in the uncompactified space: $\Pi(q) = [\Pi(q) - \Pi_{\mathrm{uc}}(q)] + \Pi_{\mathrm{uc}}(q)$. This allows us to trade off the divergent sum for a divergent integral, which may be computed using the standard results of dimensional regularization. Indeed, one may verify that $[\Pi(q) - \Pi_{\mathrm{uc}}(q)]$ is in fact UV and IR finite (and can be evaluated numerically), whereas the expression

$$\Pi^{MN}_{\mathrm{uc}}(q) = ie_D^2\int\frac{d^{4+\delta}k}{(2\pi)^{4+\delta}}\mathrm{Tr}\left\{\gamma^M\frac{1}{\not k}\gamma^N\frac{1}{\not k + \not q}\right\} \tag{4}$$

contains all divergent contributions. In particular, setting $\Pi^{MN}_{\mathrm{uc}}(q) = \left(q^2 g^{MN} - q^M q^N\right)\Pi_{\mathrm{uc}}(q)$, we have that

$$\Pi_{\mathrm{uc}}(Q) = \frac{e_4^2}{2\pi^2}\frac{\pi^{\delta/2}\,\Gamma^2(2+\frac{\delta}{2})}{\Gamma(4+\delta)}\Gamma\left(-\frac{\delta}{2}\right)\left(\frac{Q^2}{M_c^2}\right)^{\delta/2}. \tag{5}$$

The extra divergences that cannot be canceled by the wave-function renormalization are to be absorbed in the operator $D_M F^{MK} D^N F_{NK}$. In the limit $Q^2 \ll M_c^2$, we finally obtain [4]

$$\Pi^{(\delta)}(Q) = \frac{e_4^2}{2\pi^2}\left(a_0^{(\delta)} - \underbrace{\frac{1}{6}\log\left(\frac{Q^2}{M_c^2}\right)}_{ordinary\ running} + a_1^{(\delta)}\frac{Q^2}{M_c^2} + \cdots\right)$$

with

δ	1	2	3
$a_0^{(\delta)}$	-0.335	-0.159	-0.094
$a_1^{(\delta)}$	-0.110	0.183	0.298

Notice that the coefficients $a_1^{(\delta)}$ can be affected by non-calculable contributions from higher dimension operators.

Using the above VPF we define a sort of "effective charge"

$$\frac{1}{\alpha_{\mathrm{eff}}(Q)} \equiv \frac{1}{\alpha_4}\left(1 + \Pi^{(\delta)}(Q)\right). \tag{6}$$

To determine the relation between e_4 and the QED coupling, we identify our effective charge at some low energy scale (for instance $Q^2 = m_Z^2 \ll M_c^2$) with the QED coupling

$$\frac{1}{\alpha_{\mathrm{eff}}(m_Z)} \approx \frac{1}{\alpha_4} + \frac{2}{\pi}a_0^{(\delta)} - \frac{2}{3\pi}\log\left(\frac{m_Z}{M_c}\right). \tag{7}$$

The relation between the QED coupling $\alpha_{\mathrm{eff}}(m_Z)$ and α_4 contains only logarithmic running. This is the only matching we can reliably compute without knowing the physics beyond M_s. Notice also that, in this EFT framework, the gauge coupling does not run above the compactification

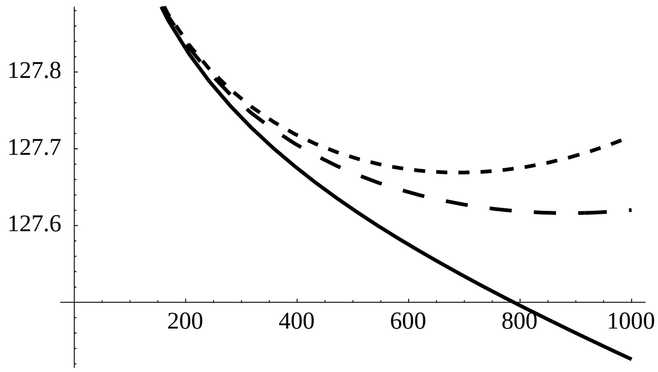

Fig. 1. The effective coupling as a function of Q, for $\delta = 1$ (solid), $\delta = 2$ (long-dashed), and $\delta = 3$ (short-dashed)

scale. That is what happens in χPT [9], when using dimensional regularization : f_π does not run, it just renormalizes higher dimensional operators.

For all energies we can write for the effective coupling:

$$\frac{1}{\alpha_{\rm eff}(Q)} \approx \frac{1}{\alpha_{\rm eff}(m_Z)} + \frac{1}{\alpha_4}\left(\Pi^{(\delta)}(Q) - \Pi^{(\delta)}(m_Z)\right).$$

As seen in Fig.1, at low energies it displays the standard logarithmic running, whereas at $Q^2/M_c^2 \approx 1$ the behavior deviates dramatically from the logarithmic running. However, for $Q > M_c$ this effective charge cannot be interpreted anymore as the running charge since it could contain physics from higher dimension operators.

We finally comment on possible pitfalls related to the cavalier interpretation of hard cutoffs in terms of physical masses, especially if the theory that gives meaning to those masses is not known [8]. Cutoffs can give an indication of the presence of power corrections, but the coefficients of these corrections cannot be computed without knowing the details of the full theory. Results obtained through the use of hard cutoffs hint to the appearance of contributions which go as $(M_s/M_c)^\delta$, where M_s is some scale related to the onset of new physics; however, the coefficients multiplying these corrections are not reliably determined.

To illustrate this point with an example, let us calculate the expression of Eq.(4) using a cutoff Λ:

$$\begin{aligned}
\Pi^{(1)}_{\rm uc}(Q) &= \frac{e_4^2}{2\pi^2}\left(-\frac{3\pi^2 Q}{64 M_c} + \frac{\sqrt{\pi}Q^2}{15 M_c} + \frac{\sqrt{\pi}\Lambda}{3M_c}\right)\\
\Pi^{(2)}_{\rm uc}(Q) &= \frac{e_4^2}{2\pi^2}\left(\frac{\pi\Lambda^2}{6M_c^2} + \frac{\pi Q^2}{30 M_c^2}\left[\log(Q^2/\Lambda^2) + \gamma - \frac{77}{30}\right]\right)\\
\Pi^{(3)}_{\rm uc}(Q) &= \frac{e_4^2}{2\pi^2}\left(\frac{5\pi^3 Q^3}{768 M_c^3} - \frac{\pi^{3/2}Q^2\Lambda}{15M_c^3} + \frac{\pi^{3/2}\Lambda^3}{9M_c^3}\right). \qquad (8)
\end{aligned}$$

We note that the terms independent of the cutoff are the same as those obtained by using dimensional regularization, Eq.(5), whereas the additional pieces depend strongly on the cutoff. Let us next assume that the "new physics" is due to the presence of an additional fermion in our $4+\delta$ dimensional theory, whose mass satisfies $M_f \gg M_c$, and let us compute its effects on the coupling constant for $M_c \ll Q^2 \ll M_f$: We have

$$\begin{aligned}
\Pi^{(\delta)}_f(Q) &= \frac{e_4^2}{2\pi^2}\left(\frac{\pi}{M_c}\right)^{\delta/2}\Gamma(-\delta/2)\\
&\times \int dx x(1-x)\left(M_f^2 + x(1-x)Q^2\right)^{\delta/2} \qquad (9)
\end{aligned}$$

Expanding for $Q^2 \ll M_f$ and integrating on x we obtain

$$\Pi^{(\delta)}_f(Q) \approx \frac{e_4^2}{2\pi^2}\left(\sqrt{\pi}\frac{M_f}{M_c}\right)^{\delta}\Gamma\left(-\frac{\delta}{2}\right)\left(\frac{1}{6} + \frac{\delta}{60}\frac{Q^2}{M_f^2}\right). \qquad (10)$$

Evidently, when integrating out the heavy fermion one receives power corrections to the gauge coupling (also higher dimension operators), even if the dimensional regularization is employed. However, the coefficients are completely different from those obtained in Eq.(8) using a hard cutoff.

4 Conclusions

We have argued that continuum EFT, with dimensional regularization and the $\overline{\rm MS}$ scheme, provides a self-consistent framework for computing in models with extra dimensions. The running of the coupling that can be computed reliably within this framework is only logarithmic. Additional power corrections are expected, but cannot be computed without knowing the details of the complete theory in which the D dimensional theory is embedded (for example, extra-dimensional GUT). Thus, the requirement of coupling unification opens a window to physics much beyond the compactification scale.

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