

FTUV-03-0702

# Super Heavy Dark Matter Anisotropies from D-particles in the Early Universe

N. E. Mavromatos<sup>a,b</sup> and J. Papavassiliou<sup>b</sup>

<sup>a</sup> King's College London, Dept.of Physics (Theoretical Physics), Strand WC2R 2LS, U.K.

<sup>b</sup> Departamento de Física Teórica and IFIC, Universidad de Valencia-CSIC, E-46100,  
Burjassot, Valencia, Spain.

## Abstract

We discuss a way of producing anisotropies in the spectrum of superheavy Dark matter, which are due to the distortion of the inflationary space time induced by the recoil of D-particles upon their scattering with ordinary string matter in the Early Universe. We calculate such distortions by world-sheet Liouville string theory (perturbative) methods. The resulting anisotropies are found to be proportional to the average recoil velocity and density of the D-particles. In our analysis we employ a regulated version of de Sitter space, allowing for graceful exit from inflation. This guarantees the asymptotic flatness of the space time, as required for a consistent interpretation, within an effective field theory context, of the associated Bogolubov coefficients as particle number densities. The latter are computed by standard WKB methods.

*Dedicated to the memory of Ian I. Kogan, collaborator and friend*

arXiv:hep-th/0307028v1 2 Jul 2003

# 1 Introduction and Summary

A plethora of recent astrophysical data, ranging from direct evidence on the acceleration of the Universe using supernovae Ia data [1] to measurements of the cosmic microwave background anisotropies (CMB) to an unprecedented precision by WMAP [2], support strongly two important characteristics for our observable Universe [3]: (i) *inflation* [4], i.e. a phase with an exponentially expanding scale factor in the Robertson-Walker space-time seems to be an integral component of the (early) evolution of our Universe, (ii) 70% of the Universe energy content consists of a yet unknown substance, termed *dark energy*.

Inflationary dynamics is supported by the spatial flatness of the Universe, corroborated by the CMB data. In the standard field theoretic implementation such a phase requires the presence of a scalar field mode, the *inflaton* field, whose nature is still unknown. The WMAP data are still too crude [5] to determine the precise shape of its potential. The dark energy, on the other hand, may be either a cosmological *constant*, or a *relaxing to zero* component of the ‘vacuum’ energy, due to a non-equilibrium situation, e.g. a *quintessence field* [6], or in general some excitation of our Universe due to an initial cosmically catastrophic event. WMAP has measured the equation of state  $p = w\rho$ ,  $p$  being the pressure and  $\rho$  the energy density, of such a quintessence field, and found  $w < -0.78$  (for comparison the cosmological constant model has  $w = -1$ ).

In the modern context of string theory [7], the presence of a cosmological constant (de Sitter Universe) is an unwelcome feature, due to the complications produced by the existence of an event horizon when attempting to define proper asymptotic states, and thus an S-matrix [8]. Indeed, a non-trivial but constant vacuum energy density in a Friedman-Robertson-Walker (standard) cosmology will eventually dominate the evolution of the Universe, which will re-enter into an accelerating inflationary phase, thus failing to reach an asymptotically flat domain. Given that string theory, at least as we understand it today, is based on S-matrix elements, such backgrounds appear problematic from this point of view. On the other hand, relaxation or quintessence-like scenarios, despite fine-tuning drawbacks related to the shape of the scalar-mode potentials, allow the possibility of an asymptotic S-matrix, and hence they may be solutions of some versions of string theory.

Such a situation has been discussed in the context of string theory in ref. [9], and in the modern context of brane cosmology [10] in ref. [11], in a model involving colliding brane worlds. In addition, colliding-world scenarios of the “ekpyrotic” type have been discussed in the recent literature [12], where it was suggested that an inflationary phase were absent and unnecessary. This point of view appears to be in direct conflict with the above-mentioned recent evidence for inflation. Moreover, this approach has been criticized in a stringy context [13], by arguing that classical string equations of motion (conformal invariance conditions) do not lead to expanding Universes but rather to contracting ones<sup>1</sup>.

In this respect, a different point of view has been advocated in [11], where the collision of the brane worlds has been viewed as a *non-equilibrium stringy process*, quantified within

---

<sup>1</sup>This last point has been refuted, however, in [14] based on the existence of a hypothetical (non-perturbative) stringy phase-transition.

a *non-critical* (Liouville) string theory [15, 16] upon the identification of target time with the Liouville mode [17, 16]. The ‘catastrophic cosmic event’ of the collision of the brane worlds leads to a central charge deficit in the pertinent  $\sigma$ -model, which describes in a perturbative way the stringy excitations of our (brane) Universe after the collision.

An important consequence of this departure from critical string theory, and thus from the standard conformal invariance conditions used in [13], is the presence of an exponentially expanding *inflationary* phase for the four-dimensional scale factor. Moreover, such models, asymptotically lead to a current era with a relaxing to zero quintessence-like dark energy component of the Universe, scaling with the cosmic time as  $1/t^2$ <sup>2</sup>. It is important to notice that such a scaling is computed using (logarithmic [18]) conformal field theory methods. The inflationary as well as the accelerating (late) phases of the Universe in such models occur dynamically, without the introduction of extra scalar fields (inflaton or quintessence). In non-critical string theory such inflationary phases are obtained [19] upon the identification of the target time with the Liouville mode [17, 16]; the consistency of this procedure has been checked in several models so far. Here this approach is reviewed in the Appendix, for the colliding world scenario of [11].

In this work, we adopt the above scenario, and proceed further to discuss particle production during the inflationary era, in an extended brane model, where the three-dimensional brane worlds are punctured by D-particles [7]. This situation is viewed as the simplest case of intersecting branes; it may occur naturally in M-theory scenaria, where the branes are viewed as defects in a bulk space-time. It is the purpose of this article to argue that the presence of D-particles on the brane worlds<sup>3</sup> leads to anisotropies in the spectrum of the heavy particles emitted. Such anisotropies can be cosmologically relevant if the masses of the emitted particles are of order  $10^{13\pm 4}$  GeV [20].

In view of scenaria [21] in which such heavy particles may be the sources of ultra-high-energy cosmic rays, the present model provides a way of obtaining anisotropies in their spectra. Although at present there is no experimental evidence for such anisotropies [22], the situation is far from being conclusive, due to the scarcity of the available ultra-high-energy cosmic ray events. In fact, it is expected that in the near future the experimental precision will improve significantly. Should anisotropies be eventually detected, the present model could provide a possible explanation for their origin. In the contrary situation, since such anisotropies constitute a concrete prediction of the current model, one will be able of placing strict bounds on the density of (primordial) D-particles on our brane Universe, a parameter of central importance in this specific model. Moreover, this model seems

---

<sup>2</sup>We note in passing that in the model of [11] such relaxing-to-zero dark energy component at the current era can be made compatible with standard supersymmetry breaking models, with the symmetry breaking scale within the range of a few TeV.

<sup>3</sup>We note that in general the density of  $D$  particles in our brane Universe may be bounded from above by the requirement that their presence should not affect the current upper limits on the value of the cosmological constant, which stem from the requirement of the large-scale validity of Newtonian mechanics. However, supersymmetry in target space may play a subtle rôle, resulting in zero ground state energies, should one view the punctured (by D-particles) three-brane as a ground state of some (initially) supersymmetric M-theory configuration. Such issues, however, fall far beyond the scope of the present article and will not be discussed further here.

realistic from the M-theory point of view, where intersecting brane situations might be a natural possibility, with interesting cosmological implications [23], and hence we consider it as worthy of being explored further.

The structure of the article is as follows: in section 2 we present the non-critical string formalism describing the recoil of D-particles in a brane Universe, due to scattering of string matter propagating on the brane. In sections 3 and 4 we use world-sheet renormalization group methods to compute the back-reaction effects on the inflationary space time Geometry from a recoiling D-particle. We pay particular attention to discussing the range of parameters that guarantee a perturbative  $\sigma$ -model treatment. Specifically, we can compute such effects via conformal field theory methods only for times  $0 < t \ll 1/H$ , where  $H$  is the Hubble parameter (assumed constant) during inflation. As a consistency check of the approach, we find that the back-reaction distortion indeed attenuates exponentially as time progresses. In section 5 we discuss particle production in the inflationary universe, due to the presence of these D-particle-recoil-induced distortions. Using approximate (WKB) methods [20], standard in such kind of problems, we compute the associated massive particle number density by means of the relevant Bogolubov coefficient. In section 6 we discuss the particle spectrum so obtained. We find that the effect of the space-time distortion induced by the D-particle is to make the momentum spectrum for massive particles non-Gaussian, and non-thermal, with angular dependence leading to anisotropies. The estimates of [20] on the mass of such particles  $M > 10^{13}$  GeV, in order for their density to be cosmologically relevant, and thus to act as superheavy dark matter candidates, are still valid in our case. The anisotropies we find in the production spectrum will imply corresponding anisotropies in the ultra-high-energy cosmic ray spectrum, should the latter be produced by such heavy particles, according to some scenaria [21]. Conclusions and outlook are given in section 7. Finally, in an Appendix we review briefly how one can obtain inflationary space-times as consistent  $\sigma$ -model backgrounds of a non-critical string model.

## 2 Geometry fluctuations due to D-brane recoil: Non-critical String Formalism

Consider the single scattering event of a closed string with a D-particle defect embedded in a  $D$ -dimensional space time of metric  $G_{\mu\nu}$ . Long after the scattering the induced disturbance of the neighboring space time (around the recoiling defect) is determined by world-sheet methods [24, 25]. Specifically, the recoil of a D-particle, under its scattering off a (closed) string state in a metric background  $G_{\mu\nu}$  is determined, at a  $\sigma$ -model level, by the deformation vertex operator [25]:

$$V = \int_{\partial\Sigma} G_{ij} y^j(t) \Theta_\epsilon(t) \partial_n X^i \quad (1)$$

with  $\Theta_\epsilon(t)$  the regularized Heaviside ‘impulse’ operator:

$$\Theta_\epsilon(t) \sim \frac{1}{i} \int_{-\infty}^{+\infty} \frac{d\omega}{\omega} e^{i\omega t} \quad (2)$$

where  $\epsilon \rightarrow 0^+$ ,  $G_{ij}$  denotes the spatial components of the metric,  $\partial\Sigma$  denotes the world-sheet boundary,  $\partial_n$  is a world-sheet derivative normal to the boundary,  $X^i$  are  $\sigma$ -model fields obeying Dirichlet boundary conditions on the world sheet, and  $t$  is a  $\sigma$ -model field obeying Neumann boundary conditions on the world sheet, whose zero mode is the target time. The quantity  $y^i(t)$ ,  $i$  being a spatial index, denotes the trajectory of the  $D$ -particle.

This is the basic vertex deformation, which we assume to describe the motion of a  $D$ -particle in a curved geometry at least to leading order, when space-time back reaction and curvature effects are assumed weak <sup>4</sup>. Such vertex deformations may be viewed as a generalization of the flat-target-space case [24].

For times relatively long after the event, where the use of  $\sigma$ -model perturbation theory is a valid approximation, the trajectory  $y^i(t)$  will be that of free motion in the curved space-time under consideration. In the flat space-time case, this trajectory was a straight line [24], and in the more general case here it will be simply the associated *geodesic*. Let us now determine its form, which will be essential in what follows.

For our purposes we shall consider *Minkowski signature* space-time backgrounds of Robertson-Walker (RW) form:

$$ds^2 = -dt^2 + a(t)^2(dX^i)^2 \quad (3)$$

where  $a(t)$  is the RW scale factor.

An important discussion is in due order at this point. In order for the basic conformal field theory formalism to be applicable in curved target space-times, which has been used in [25], it is *imperative* that these space-times be *at least an approximate* solution to  $\sigma$ -model conformal invariant conditions (i.e. a vacuum solution to Einstein’s equations to leading order in  $\alpha'$  expansion, where  $\alpha' = 1/M_s^2$  is the Regge slope, and  $M_s$  is the string scale). This is the case for RW backgrounds at large times  $t \rightarrow \infty$ , presented in [25], where target space curvature effects have been ignored. In the present paper we are interested in inflationary metric backgrounds. Unfortunately, such backgrounds are not vacuum solutions to any string theory, at least in an ‘unregulated’ form, because in this case they are hampered by event cosmological horizons, and hence are not compatible with a consistent  $S$ -matrix. The latter is a cornerstone of the first-quantized formulation of string theory, and hence consistent string theory backgrounds should allow for a graceful

---

<sup>4</sup>Perhaps a formally more desirable approach towards the construction of the complete vertex operator would be to start from a T-dual (Neumann) picture, where the deformation (1) should correspond to a proper Wilson loop operator of an appropriate gauge vector field. Such loop operators are by construction independent of the background geometry. One can then pass onto the Dirichlet picture by a T-duality transformation viewed as a canonical transformation from a  $\sigma$ -model viewpoint [26]. In principle, such a procedure would yield a complete form of the vertex operator in the Dirichlet picture, describing the path of a  $D$ -particle in a curved geometry. Unfortunately, such a procedure is not free from ambiguities at a quantum level [26], which are still unresolved for general curved backgrounds.

exit from an inflationary phase, and approach asymptotically a solution of the conformal invariance  $\sigma$ -model conditions. Such situations have been argued to characterize realistic non-critical string cosmologies under the identification of time with the Liouville mode [9]. As mentioned previously, for our purposes in this work we discuss briefly such a case in the Appendix, specifically the colliding brane model of [11].

Once we assume such regulated versions of de Sitter space-time, we may proceed by adopting the same spirit as in [25], i.e. concentrate in regions of time for which one may safely neglect space-time curvature effects and thus apply standard conformal field theory world-sheet methods. Contrary to the case studied in [11], in the inflationary situation, studied here, curvature effects are weak compared to  $G_{\mu\nu}$  in the time regime  $0 \ll tH \ll 1$ , where the event of the collision of the string with the D-particle is placed at  $t = 0$ . We stress once more that the left inequality is necessary for the validity of the boundary world-sheet  $\sigma$ -model perturbation theory [24].

The geodesics of a massive point particle in de Sitter space ( $a(t) = a_0 e^{Ht}$ ,  $a_0$  a constant whose size will be fixed later on) are given by:

$$y^i(t) = c_3^i - \frac{c_1^i}{|c_1|H} \left( a_0^{-2} e^{-2Ht} + \frac{2c_2}{|c_1|^2} \right)^{1/2} \quad (4)$$

where  $c_\alpha^i$ ,  $\alpha = 1, 2, 3$  are constants determined by the boundary conditions.

We shall be interested in values of  $H \sim 10^{-5} M_P$ , where  $M_P \sim 10^{19}$  GeV, is the four dimensional Planck scale, not to be confused with the string scale  $M_s$  which may be different. This is the value of the Hubble parameter during the inflation era. For the validity of our perturbative analysis we shall also be interested in times far from the end of inflation  $t \ll t_e \sim 10^9 t_P$  (where  $t_P \sim 10^{-43}$  sec is the four-dimensional Planck time), such that  $Ht \ll 1$ , where space-time curvature effects of order  $H^2 \sim 10^{-10} M_P^2$  can be safely neglected. However, as we shall discuss in the next two sections, we shall be interested in ranges of the parameters of our cosmological model such that

$$1 \gg \frac{H^2}{M_s^2} \gg Ht > 0 . \quad (5)$$

It is terms of order  $H^2/M_s^2$  that will determine the distortion of the inflationary space time due to the recoil of the D-particles. We easily see that a natural set of parameters satisfying (5) is  $M_s = 10^{-4} M_P \sim 10^{15}$  GeV (i.e. intermediate string scale), which implies that our perturbative treatment in this article is rigorously valid for times  $t < 10^3 t_P$  (beginning of inflationary period).

This will be a sufficient, and mathematically self-consistent, procedure for computing the global effects of the recoil-induced distortion of space-time, which, as we shall discuss in the next section, depend on the world-sheet violations of conformal invariance, quantified by the anomalous dimension of some operators of order  $H^2$  (in string  $M_s = 1$  units) which therefore should be kept in our analysis. As we shall show, Such space-time distortion effects diminish with time as  $e^{-4Ht}$ , and hence become even more suppressed toward the end of inflation (where  $Ht_e \sim 10^4$ ).

Working in the above regime of parameters, therefore, will guarantee : (i) the validity of the  $\sigma$ -model perturbation theory; (ii) the presence of a *smooth extension* of de Sitter space, such that asymptotically one can define flat space-time regions and a proper  $S$ -matrix. The latter requirement is necessary for a proper definition of particle production number via the Bogolubov coefficients [27, 20, 28, 29]; it may be satisfied *provided* that the constant  $a_0 \ll 1$ , for instance of order  $1/Ht_e$  in Planck units. Physically, this last condition on  $a_0$  implies that the inflationary era started at  $t = 0$  (when the recoil occurred), at which moment the spatial size of the universe  $a_0$  was considerably smaller than the Planck length. This situation corresponds physically to a smoothed version of the ‘big bang singularity’, which we adopt for our purposes here. It is worth stressing that in this way one may apply world-sheet perturbation theory at the early stages of inflation, viewing the latter [19] as an example of non-critical (Liouville)  $\sigma$ -model [15], with the identification of time with (an appropriate function) of the Liouville mode [17, 16].

We next remark that the non-relativistic nature of the D-particles, appropriate for the validity of our  $\sigma$ -model formalism [24, 25] imply small initial ( $t = 0$ ) recoil velocities, defined by:  $u^i = \frac{dy^i}{dt}|_{t=0}$ , which yields (from eq. (4)):

$$u^i = \frac{c_1^i}{|c_1|a_0^2} \left( a_0^{-2} + 2\frac{c_2}{|c_1|^2} \right)^{-1/2} \ll 1 \quad (6)$$

As mentioned earlier, in the regime of parameters we are working, one must have  $a_0 \ll 1$ , in which case non relativistic velocities  $u^i \ll 1$  are guaranteed by the condition

$$2a_0^4 c_2 \gg |c_1|^2, \quad (7)$$

which, in turn, implies

$$u^i \simeq \frac{c_1^i}{a_0^2 \sqrt{2c_2}} \quad (8)$$

An additional requirement, not usually imposed in standard inflationary scenaria of cosmology [27], is that of the *continuity* of the various results as  $H \rightarrow 0$ . As will be explained in the next section, this last requirement is motivated by the fact that, during the exponential expansion, the Hubble parameter  $H$  acts as an anomalous dimension of world-sheet operators. To be specific, in the Liouville treatment we shall impose *finiteness* of results as  $H \rightarrow 0^+$ , in order to guarantee a connection with the (world-sheet RG fixed point of) flat space-time with metric  $ds_*^2 = -(d\varphi)^2 - (dt)^2 + a_0^2(dX^i)^2$  (with  $\varphi = -t$ )<sup>5</sup>. In fact, in this approach we shall regard the inflation as a smooth interpolating RG trajectory between two asymptotically flat regimes of space-time, which are fixed points of the world-sheet of the string. What differentiate these two regimes is the size of  $a_0$ . We shall return to this point later, when we discuss the interpolating metric between such fixed points,

---

<sup>5</sup>Notice that the Liouville mode always exist in a  $\sigma$ -model path integral, but decouples from the background fields at the fixed points. However, as a target-space coordinate is always there, and thus contributes an overall normalization factor of 2 to the time component of the flat fixed point metric  $ds_*^2$ ; this guarantees the smoothness of the limit  $H \rightarrow 0^+$ .

including an inflationary era, followed by a “graceful exit” from it, so that an S-matrix can be properly defined, as expected in a string theory reaching its critical status asymptotically in time.

Requiring finiteness of  $y^i(t)$  in the limit  $H \rightarrow 0^+$  is equivalent to setting

$$c_3^i = \frac{c_1^i}{|c_1|H} \left( a_0^{-2} + \frac{2c_2}{|c_1|^2} \right) + H - \text{independent parts} . \quad (9)$$

For simplicity we assume the  $H$ -independent parts to be zero, which essentially amounts to choosing the initial position of the D-particle to be the origin of the coordinates and of our (inflationary) Universe. These considerations imply the following approximation for the geodesic (for the time  $t$  regime  $Ht \ll 1$  we are interested in):

$$y^i(t) \simeq \frac{c_1^i}{2a_0^2 H \sqrt{2c_2}} - \frac{c_1^i}{2a_0^2 H \sqrt{2c_2}} e^{-2Ht} \quad (10)$$

Thus the recoil operators for the D-particle (1) can be split in two types of  $\sigma$ -model deformations, assuming that the moment of impact of the closed string state with the D-particle defect takes place at  $t = 0$ :

$$\begin{aligned} \mathcal{C} &= \tilde{c}_i \oint_{\partial\Sigma} \Theta_\epsilon(t) \partial_n X^i \\ \mathcal{D} &= c_{0,i} \oint_{\partial\Sigma} \Theta_\epsilon(t) e^{2Ht} \partial_n X^i \end{aligned} \quad (11)$$

where  $\tilde{c}^i = -c_{0,i} \equiv -\frac{c_1^i}{2\sqrt{2c_2}H} = -a_0^2 \frac{u^i}{2H}$ . Notice that, in general, the validity of our weak coupling  $\sigma$ -model formalism requires  $\tilde{c}^i, c_0^i < 1$ , which in view of (7) is satisfied.

### 3 Renormalization Group Relevance of the recoil Deformations: Operator Product Expansion Analysis

The Liouville approach to recoil-induced space-time distortions [24, 16, 25] require the relevance of the recoil operators (11) under world-sheet renormalization group (RG) transformations; in turn, this would imply the need for “Liouville dressing”. The fact that the recoil operators are indeed RG-relevant can be confirmed by considering their Operator Product Expansion (O.P.E.) with the stress tensor of the  $\sigma$ -model, as well as among themselves.

Working with time regimes  $0 \ll Ht \ll 1$ , and also with parameters satisfying (5), which, as discussed above, guarantee the validity of our perturbative  $\sigma$ -model method, justifies the use of the following formula for the world-sheet propagator of the  $X^\mu = (t, X^i)$ ,  $i = 1, \dots, D-1$  (spatial index) coordinate fields (from now on we shall be working in string units for which  $M_s = 1$ ):

$$\langle X^\mu(z) X^\nu(z') \rangle \simeq G^{\mu\nu} \ln|z - z'| \quad (12)$$



$G^{\mu\nu}$  is the target-space metric in the  $\sigma$ -model frame, which is assumed of the form (3) with a de-Sitter type exponentially expanding scale factor  $a(t) = a_0 e^{Ht}$ . The (approximate) formula (12) will be subsequently used when computing the various OPE of the  $\sigma$ -model operators.

To begin with, the stress tensor assumes the form:

$$2T = -(\partial_z t(z))^2 + a^2(t)(\partial_z X^i)^2; \quad a(t) = a_0 e^{Ht} \quad (13)$$

We now consider the O.P.E. of  $T$  with the deformation operators  $\mathcal{C}$ ,  $\mathcal{D}$  (11).

The computation of the time-dependent parts of  $T$ ,  $2T_t \equiv -(\partial_z t)^2$ , and of the space-dependent ones,  $2T_X \equiv a^2(t)(\partial_z X^i)^2$ , may be carried out separately [25]. The basic ingredients are Eq.(12), together with the standard representation (2) of the Heaviside operator, as well as the expansion

$$(\partial X^j(z))^2 \odot \partial_n X^i(w) \sim G^{ii} \frac{1}{(z-w)^2} \partial_n X^i \sim \frac{a_0^{-2} e^{-2Ht}}{(z-w)^2} \partial_n X^i, \quad (\text{no sum over } i) \quad (14)$$

The analysis is straightforward, and yields:

$$\begin{aligned} 2T_t(z) \odot \mathcal{C}(z') &\sim \frac{1}{|z-z'|^2} \left(-\frac{\epsilon^2}{2}\right) \mathcal{C}(z'), \\ 2T_X(z) \odot \mathcal{C}(z') &= \tilde{c}_i \Theta_\epsilon(t - 2H \ln|z-z'|) \frac{\partial_n X^i}{|z-z'|^2} = \tilde{c}_i \Theta_\epsilon(t) \frac{\partial_n X^i}{|z-z'|^{2-2H\epsilon}}, \\ 2T_t(z) \odot \mathcal{D}(z') &\sim \frac{1}{|z-z'|^2} \left(-\frac{\epsilon^2}{2}\right) \mathcal{D}(z'), \\ 2T_X(z) \odot \mathcal{D}(z') &= c_{0,i} \Theta_\epsilon(t - 2H \ln|z-z'|) \frac{\partial_n X^i}{|z-z'|^{2+4H^2}} e^{2Ht(z)} = \\ c_{0,i} \Theta_\epsilon(t) &\frac{\partial_n X^i}{|z-z'|^{2+4H^2-2H\epsilon}} e^{2Ht(z)}, \end{aligned} \quad (15)$$

where  $\epsilon \rightarrow 0^+$ , and we used Taylor expansion in the arguments of the Heaviside operators, and the property  $d\Theta_\epsilon(t)/dt = -\epsilon \Theta_\epsilon$ .

Thus we arrive at the following result to leading order in world-sheet divergences as  $z \rightarrow z'$ :

$$\begin{aligned} 2T(z) \odot \mathcal{C}(z') &\sim \frac{1}{|z-z'|^2} \mathcal{C}(z), \\ 2T(z) \odot \mathcal{D}(z') &\sim \frac{1}{|z-z'|^{2+4H^2}} \mathcal{D}(z), \end{aligned} \quad (16)$$

Notice that the *Minkowski signature* of the target space-time is *crucial* to the effect of yielding positive anomalous scaling dimension for the  $\mathcal{D}$  operator, proportional to  $H^2$  (notice that the operator  $\mathcal{C}$  is conformal in the limit  $\epsilon \rightarrow 0^+$ ). Due to the non-renormalizability of the world-sheet stress tensor in two dimensions,  $T$ , one infers from (16) that the operator

$\mathcal{D}$  has anomalous scaling dimension  $4H^2$  in string units ( $M_s = 1$ ), i.e.  $4H^2/M_s^2$  in arbitrary units. We therefore observe that, in the regime of parameters (5) we are working with, such terms dominate over space-time curvature effects that are neglected in (12).

For consistency, one should verify the above result by computing the O.P.E.'s of the operators  $\mathcal{C}$ ,  $\mathcal{D}$  with themselves. The computation confirms the conformal nature of  $\mathcal{C}$ , while for  $\mathcal{D}$  one obtains:

$$\mathcal{D}(z) \odot \mathcal{D}(z') \sim \frac{1}{|z - z'|^{2+4H^2}} \mathcal{D}(z') \quad (17)$$

implying its anomalous dimension  $4H^2$ , and hence its world-sheet renormalization group relevance. The appearance of the anomalous dimension implies that the recoil process has spoiled the conformal invariance of the  $\sigma$ -model, which now needs Liouville dressing [15] to restore it.

It must be noticed at this stage that above we have considered the de Sitter space-time as a valid stringy  $\sigma$ -model background, and concentrated on the effects of recoil, assuming the latter to be the only source of (boundary) world-sheet violation of conformal invariance. In the Appendix of the current article we discuss a model in which the de Sitter space-time itself is viewed as a solution of generalized conformal invariance conditions of a non-critical Liouville string [19], expressing the restoration of the conformal invariance by the Liouville mode [15]. In this more complete picture the recoil deformation constitutes one of the possible non-critical deformations of the specific Liouville  $\sigma$ -model under consideration, whose deficit  $Q^2$  causes inflation in the way explained in the Appendix. The formalism described above remains intact for this (mathematically complete) case. As will be discussed in the next section, this is so because the effects of the recoil on the space-time are such that they may be removed by means of an appropriate coordinate transformation; thus, the space-time emerging after the recoil (written in the new coordinates) retains its de-Sitter form.

## 4 Liouville Dressing and Recoil-Induced Space-Time Distortions

As discussed in [25], there are two equivalent ways of performing the Liouville dressing: one is to dress the boundary operator  $\mathcal{D}$  as it stands, and restore conformal invariance on the boundary world-sheet theory, and the other (which we shall follow here) is to write first the operator  $\mathcal{D}$  as a total world-sheet derivative bulk operator (using Stoke's theorem), and then dress the bulk operator. In particular, in this latter case we have:

$$V_{L,\text{bulk}} = \int_{\Sigma} e^{\alpha_i \varphi} \partial_{\alpha} (y_i(t) \partial^{\alpha} X^i) = c_{0,i} \int_{\Sigma} e^{\alpha_i \varphi} \partial_{\alpha} (\Theta_{\epsilon}(t) e^{2Ht} \partial^{\alpha} X^i) ,$$

$$c_{0,i} = a_0^2 \frac{u^i}{2H} , \quad \alpha_i = -\frac{Q}{2} + \sqrt{\frac{Q^2}{2} + (2 - \Delta_i)} \quad (18)$$

where  $\varphi(z)$  is the world-sheet Liouville field,  $\Delta_i$  is the conformal dimension of the bulk operator, with  $2 - \Delta_i$  its anomalous dimension, and  $\alpha_i$  is the so-called gravitational anomalous dimension. The central charge deficit  $Q^2$  is the one responsible for the initial inflationary phase, and may be estimated as follows: one may consider various scenarios for departure from criticality, as necessary for inflation in stringy  $\sigma$ -models [19]; for example, in [25] this was due to “catastrophic” cosmic events, such as the collision of two brane worlds. In such a scenario, which is discussed briefly in the Appendix, it is possible to obtain an initial *supercritical* central charge deficit (and hence a time-like Liouville mode [30]) of order

$$Q^2 = 9H^2 > 0 , \quad (19)$$

where the Hubble parameter  $H$  can be fixed in terms of other parameters in the theory; for instance, in the specific model of [25] on colliding branes,  $Q$  (and thus  $H$ ), is found to be proportional to the square of the relative velocity of the colliding branes,  $Q \propto u^2$  during the inflationary era. As we show in the Appendix, in a phase of constant  $Q$ , one obtains an inflationary de-Sitter type Universe <sup>6</sup>.

The specific normalization in (19) is imposed because, as discussed below and in the Appendix, (i) one may identify the time  $t$  with a Liouville mode  $-\varphi$  of the *supercritical*  $\sigma$ -model: the minus sign is justified below both mathematically, due to properties of the Liouville mode, and physically by the requirement of a relaxation to zero of the recoil-distorted space-time deformation. (ii), under this identification, the Liouville equation for the modes/ $\sigma$ -model background couplings [16]:

$$\ddot{g}^i + Q\dot{g}^i = -\beta^i(g) = -\mathcal{G}^{ij}\partial C[g]/\partial g^j , \quad (20)$$

where the dot denotes derivative with respect to the Liouville world-sheet zero mode  $\varphi$ , and  $\mathcal{G}^{ij}$  is an inverse Zamolodchikov metric in string theory  $\{g^i\}$  space, when applied to scalar (e.g. inflaton-like) string modes would yield standard field equations for scalar fields in de Sitter (inflationary) space-times provided the normalization (19) is valid. This would imply a “Hubble” parameter  $Q = 3H$  (notice that the gradient flow property of the  $\beta$  functions makes the analogy with the inflationary case even more profound, where the running central charge  $C[g]$  [33] plays the rôle of the inflaton potential in conventional inflationary field theory).

As discussed in the Appendix, there are consistent solutions of the generalized conformal invariance conditions with the above described desired properties <sup>7</sup>.

---

<sup>6</sup>We note in passing that cosmically catastrophic non-critical string scenarios, as the one of ref.[11], allow for a *relaxing to zero* deficit  $Q^2(t)$  in such a way that, although during the inflationary era  $Q^2$  is (for all practical purposes) constant, as in (19), eventually  $Q^2$  decreases with time so that, at the present era, one obtains compatibility with an accelerating Universe phase, as suggested by a variety of recent astrophysical observations. As already mentioned, such relaxation (quintessence-like) scenarios [25, 9] have the advantage of properly definable asymptotic states (as  $t \rightarrow \infty$ ) and string scattering  $S$ -matrix. Other scenarios for inducing de Sitter Universes in string theory may be envisaged, where the inflation space-time is obtained as a result of string loops (dilaton tadpoles) [31], but in such models a string  $S$ -matrix cannot be properly defined.

<sup>7</sup>At this stage we remark that, since in general the non-critical string scenario produces inflationary

We now remark that the relations (20) replace the conformal invariance conditions  $\beta^i = 0$  of the critical string, and serve in expressing the necessary conditions for the restoration of conformal invariance by the Liouville mode [15]. In fact, upon interpreting the latter as an extra target dimension, the conditions (20) may also be viewed as conformal invariance conditions of a (D+1)-target-space dimensional *critical*  $\sigma$ -model (D is the target dimension of the non-critical  $\sigma$ -model before Liouville dressing). In most Liouville approaches one treats the Liouville mode  $\varphi$  and time  $t$  as independent coordinates. In our approach [16, 9, 11], however, we take one step further, and provide dynamical arguments which imply the restriction in this extended (D+1)-dimensional space time of lying on a hypersurface determined by the identification  $\varphi = -t$ . This means that, as the time flows in our Universe, we are forced to lie on this D-dimensional subspace of the (D+1)-dimensional Liouville extended space time <sup>8</sup>.

From (17), (19), (18), and using that  $2 - \Delta_i = 4H^2$  one has:

$$\alpha_{\mathcal{D}} = H \tag{21}$$

Then, after performing a world-sheet integration by parts in (18), it is straightforward to obtain a non-diagonal deformation in the Liouville-extended  $(\varphi, t, X^i)$  target space metric:

$$G_{\varphi i} = -H c_{0,i} \Theta_{\epsilon}(t) e^{H(\varphi+t)}, \quad c_{0,i} = a_0^2 \frac{u^i}{2H}. \tag{22}$$

The resulting extended space-time distortion then is given by the following line element

---

phases dynamically, due to the presence of the Liouville mode [19, 9, 11], without introducing extra inflaton fields, there is no need for the requirement that the equations (20) for scalar string modes be formally identical to those of an inflaton field in conventional cosmology. The precise proportionality coefficient between  $Q^2$  and  $H^2$  is actually physically unimportant in our framework, as long as it is non-zero. Indeed, such a constant will only affect the precise expression for the Liouville anomalous dimension  $\alpha_{\mathcal{D}}$  (21) (c.f. below)  $\alpha_{\mathcal{D}} = \gamma H$ , where  $\gamma \neq 2$  for any  $Q^2 \neq 0$ , and  $\gamma = 2$  if and only if  $Q = 0$  (no central-charge deficit). Such constants  $\gamma \neq 2$  do not affect qualitatively our results and can be absorbed in normalization factors. Thus, the above normalization (19) between  $Q^2$  and  $H^2$  can actually change, and is model dependent. What is model independent (within the class of models that yield string inflation), though, is that the deficit  $Q^2$  is always proportional to the square of the Hubble parameter  $H$  appearing in the exponential of the Universe scale factor  $a(t) = a_0 e^{Ht}$ . However, it is comforting (from a phenomenological viewpoint at least) that a normalization (19), which is in agreement with conventional inflationary cosmology, as far as the inflaton field is concerned, can be found consistently in our approach.

<sup>8</sup>For instance, in the work of [11], involving brane-world collisions as a source of departure from criticality, this restriction was imposed because the potential of massive particles, in an effective field theory context, was found to be proportional to  $\cosh(t + \varphi)$ , which is thus minimized at  $\varphi = -t$ . In fact, such an opposite flow of the Liouville mode as compared to that of target time may be given a deeper mathematical interpretation. It may be viewed as a consequence of a specific treatment of the area constraint in non-critical (Liouville)  $\sigma$ -models [17, 16], which involves the evaluation of the Liouville mode path integral via an appropriate steepest-descent contour. In this way one obtains a ‘breathing’ world-sheet evolution, in which the world-sheet area starts from a very large value (infrared cutoff), shrinks to a very small one (ultraviolet cutoff), and then inflates again towards very large values (infrared cutoff). Such a situation may then be interpreted as a world-sheet ‘bounce’ at the infrared, implying, following the reasoning of ref. [32], that the physical flow of target time is opposite to that of the world-sheet scale (Liouville zero mode).

(for times  $t \gg 0$ , i.e. long after the scattering event):

$$ds^2 = -(d\varphi)^2 - (dt)^2 + e^{2Ht} \left( a_0^2 (dX^i)^2 + 2H c_{0,i} e^{H\varphi} d\varphi dX^i \right) , \quad c_{0,i} = a_0^2 \frac{u^i}{2H} . \quad (23)$$

Upon the identification  $\varphi = -t$  one then obtains an expression for the distorted space-time due to D-particle recoil, for  $t > 0$ :

$$ds^2 = -2(dt)^2 + e^{2Ht} \left( a_0^2 (dX^i)^2 + 2H c_{0,i} e^{-Ht} d\varphi dX^i \right) \quad (24)$$

This result is consistent with our perturbative (world-sheet) treatment of inflation, which we consider to start at  $t = 0$ , in the sense that as  $Ht$  becomes larger the distortion effects become considerably smaller (exponentially suppressed). Thus, the distortion effects will be washed out completely by the exponential expansion, as expected. However, as we shall discuss in the next section, global effects due the presence of D-particles will leave their trace through small angular modifications in the particle production,

To study such effects, we first observe that the distorted space-time (24) can be cast in a conventional de Sitter inflationary form

$$ds^2 = -(dt')^2 + e^{\sqrt{2}Ht'} (a_0^2 d\tilde{X}^i)^2 \quad (25)$$

upon changing coordinates  $(t, X^i) \rightarrow (t', \tilde{X}^i)$ :

$$\begin{aligned} t' &= \sqrt{2}t - \frac{H}{2} |c_{0,i}|^2 e^{-2Ht} = \sqrt{2}t - \frac{a_0^4}{8H} |u^i|^2 (e^{-2Ht} - 1) , \\ \tilde{X}^i &= X^i + a_0^2 \frac{u^i}{2H} (e^{-Ht} - 1) \end{aligned} \quad (26)$$

where smoothness of the limit  $H \rightarrow 0^+$  has been guaranteed by imposing appropriate boundary conditions. Thus, one arrives at the following two space-time geometries, before and after the D-particle collision:

$$\begin{aligned} (ds_{\text{IN}})^2 &= -2(dt)^2 + a_0^2 (dX^i)^2 , \\ (ds_{t>0})^2 &= -(dt')^2 + a(t')^2 (d\tilde{X}^i)^2 , \end{aligned} \quad (27)$$

where then IN vacuum occurs at  $t \rightarrow -\infty$ , while  $(t', \tilde{X}^i)$  occur for  $t \geq 0$  and are given in (26). In the next section we will use the above modification in the two geometries in order to compute the corresponding particle production: one will seek an appropriate (smooth) interpolating function  $a(t)$  between the IN flat space-time (which is assumed to occur from  $t = -\infty$  till  $t = 0$ ) and the OUT space-time, which should include inflation in the regime  $0 < Ht \ll 1$  consistent with our  $\sigma$ -model considerations (we stress again that the moment of impact of the string with the D-particle is at  $t = 0$ ). Before closing this section we would like to mention that the effects of D-brane recoil in flat background space-time have been considered in [34], and a non-thermal, non-isotropic particle production has also been demonstrated in that case.

## 5 Particle Production in D-particle-recoil-Distorted Inflationary Universe

For the benefit of the reader we would like first to outline the general method for computing particle production in Robertson-Walker space-times [20].

Consider for definiteness a *massive* scalar field  $X$ , of mass  $M_X$ , on a curved Robertson-Walker (RW) background, in the conformal frame  $(\eta, X^i)$  which is defined as [27]:

$$ds^2 = a^2(\eta) \left( -(d\eta)^2 + (dX^i)^2 \right) \quad (28)$$

This frame is related to the RW (cosmological comoving) frame  $(t, X^i)$  used in the previous sections by:

$$d\eta = \frac{dt}{a(t)} \quad (29)$$

For the inflationary case of  $a(t) = a_0 e^{Ht}$ , we have that the corresponding conformal time is given by

$$\eta = -\frac{1}{H} \left( \frac{1}{a_0} e^{-Ht} - 1 \right) \quad (30)$$

where the integration constant has been chosen such that  $\eta \rightarrow t$  as  $H \rightarrow 0$ .

One may expand the field  $X$  in Fourier modes  $h_k$ , defined through the Klein-Gordon equation that  $X$  obeys in such metric backgrounds. In particular, in the comoving frame  $(t, X^i)$ :

$$X = \int \frac{d^3k}{(2\pi)^{3/2} a(t)} \left[ a_k e^{i\vec{k}\cdot\vec{X}} h_k(t) + a_k^\dagger e^{-i\vec{k}\cdot\vec{X}} h_k^*(t) \right] \quad (31)$$

where the vectors denote spatial components of four vectors. In the conformal time frame one arrives at the following equation (stemming from the Klein Gordon equation):

$$h_k''(\eta) + \omega_k^2(\eta) h_k(\eta) = 0 \quad (32)$$

where the prime denotes differentiation with respect to  $\eta$ , and the energy (positive frequency) is given by  $\omega_k = +\sqrt{k^2 + M^2 C(\eta)}$ , where  $C(\eta) = a^2 (1 + \text{interactions})$  is an effective scale factor [20] including the effects of interaction of the  $X$  field with the inflaton background *etc.*

After casting  $h_k$  in the form

$$h_k = \frac{\alpha_k}{\sqrt{2\omega}} e^{-i \int^\eta \omega_k d\eta'} + \frac{\beta_k}{\sqrt{2\omega}} e^{i \int^\eta \omega_k d\eta'} \quad (33)$$

one obtains a system of differential (first order) equations for  $\alpha_k, \beta_k$ , which are Bogolubov [27] coefficients connecting the two vacua (IN and OUT) satisfying  $\alpha_k^2 - \beta_k^2 = 1$ :

$$\begin{aligned} \alpha'_k &= \frac{\omega'_k}{2\omega_k} \beta_k \exp \left( -2i \int_{\eta_p}^\eta \omega_k(\eta') d\eta' \right) \\ \beta'_k &= \frac{\omega'_k}{2\omega_k} \alpha_k \exp \left( -2i \int_{\eta_p}^\eta \omega_k(\eta') d\eta' \right) \end{aligned} \quad (34)$$

As mentioned earlier, for a consistent interpretation of  $|\beta_k|^2$  as a particle density, it is *imperative* [29] that, asymptotically in the (conformal) time  $\eta \rightarrow \pm\infty$ , the space-time be *flat*. This is so because, it is only in this case that the modes used in the expansion of the field  $X$  can be put in a suitable relation to the known modes of Minkowski space-time. This will be assumed in what follows, which will necessitate viewing the inflationary epoch of the Universe only as a specific era, occurring in a space-time which interpolates smoothly (in  $\eta$  time) between two asymptotically flat regions. From our world-sheet (Liouville  $\sigma$ -model) point of view this means that the inflationary (non-conformal) space-time background connects (in the sense of a running along a world-sheet renormalization group (RG) trajectory) two asymptotic fixed points of the RG flow (flat space-time regions). Upon our identification of the RG flow with the actual target-space time flow of the  $\sigma$ -model [16], the RG evolution becomes in this way a cosmological evolution, implying graceful exit from the de Sitter (inflationary) era, and thus allowing for the definition of an S-matrix.

In our considerations in this article we shall use some simplified interpolating metrics appearing in the literature, which may not be exact solutions of (Liouville) string theory. Their main advantage is that they can be treated analytically. For more detailed studies within the context of realistic (Liouville) string models, allowing for a graceful exit from inflation, see [9]; such metrics, however, can only be studied numerically.

Assuming the above, one can apply WKB (essentially derivative expansion) methods [20, 28] to approximate the Bogolubov coefficient  $\beta_k$  as [20]:

$$\beta_k \simeq \int d\eta \frac{\omega'_k}{2\omega_k} \exp\left(-2i \int_{\eta_p}^{\eta} \omega_k(\eta') d\eta'\right) \quad (35)$$

to leading order, with the boundary conditions  $\alpha_k(\eta_p) = 1$ ,  $\beta_k(\eta_p) = 0$ . In deriving the approximate expression of Eq.(35) one has to make sure that the conditions for the validity of the WKB approximation are satisfied at all times; this in turn leads to restrictions, which are discussed in detail in ref. [20], where we refer the interested reader.

To evaluate the integral approximately, one can apply a steepest-descent method, by complexifying the time  $\eta$ , and then approximating the value of the integral by a sum of values of the integrand at the branch-cut points of a steepest-descent contour, which can be appropriately constructed. The starting points  $\tilde{\eta}_j$  of the branch-cut in the complex  $\eta$  plane are defined as:

$$\omega_k(\tilde{\eta}_j) = 0 = k^2 + M^2 C(\eta_j) \quad (36)$$

There are two approximations involved [20]: one is the above-mentioned approximation of the integral over  $\eta$  in (35) as a sum over branch-cut contributions, and the second is a derivative expansion of the integral over  $\eta'$  in the exponent near each branch-cut point. To lowest order in derivatives one may write:

$$\int_{\eta_p}^{\eta} \omega_k(\eta') d\eta' \simeq \int_{\eta_p}^{\tilde{\eta}_j} \omega_k(\eta') d\eta' + \frac{2M_X}{3} \sqrt{C'(\tilde{\eta}_j)} \delta^{3/2} + \dots \quad (37)$$

with  $\delta \equiv \eta - \tilde{\eta}_j$ .

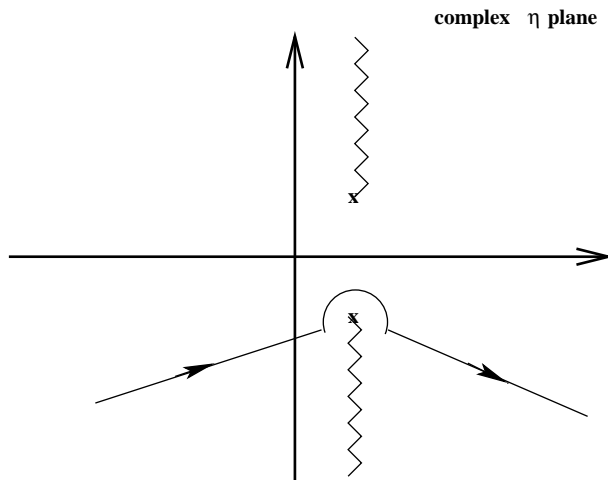


Figure 1: *Steepest descent contour and branch cut structure for the integral (35).*

In this way one may *approximate* the coefficient  $\beta_k$  (35) as a sum over branch cut points:

$$\beta_k \simeq \sum_j \mathcal{U}_j, \quad \mathcal{U}_j \equiv \exp\left(-2i \int_{\eta_p}^{\tilde{\eta}_j} \omega_k(\eta') d\eta'\right) \int_{C_j} \frac{d\delta}{\delta} \exp\left(-\frac{4i}{3} M_X \sqrt{C'(\tilde{\eta}_j)} \delta^{3/2} + \dots\right) \quad (38)$$

where  $C_j$  is a steepest descent contour. In the examples considered in [20], and also here, the appropriate contour is the one that encompasses the branch cuts that lie in the lower half plane (see fig. 1). This is associated with the fact that one chooses the sign  $\sqrt{C'(\eta)} > 0$ , due to the positive frequency requirement; the branch cuts have been chosen to go towards  $\pm\infty$ , and a steepest descent contour *must* have an “incoming” and an “outgoing” direction [20].

An important approximation in the WKB analysis of [20], which as we shall see remains valid in our case here, is that *only one* pole dominates the integral (38). Let the value of  $\tilde{\eta}_j$  in this dominant pole be split in real ( $r$ ) and imaginary ( $\mu$ ) parts:  $\eta = r + i\mu$ . This, then, yields

$$|\beta_k|^2 \simeq \exp\left(-4 \left[ \frac{(k/a_{\text{eff}}(r))^2}{M_X \sqrt{H_{\text{eff}}^2(r) + R_{\text{eff}}(r)/6}} + \frac{M_X}{\sqrt{H_{\text{eff}}^2(r) + R_{\text{eff}}(r)/6}} \right]\right) \quad (39)$$

up to irrelevant proportionality constants of order  $\mathcal{O}(1)$ , with  $a_{\text{eff}} = \sqrt{C}$ ,  $H_{\text{eff}}$  the effective Hubble parameter, and  $R_{\text{eff}}(r)$  the effective scalar curvature (which in our case may be neglected). The density of the  $X$  particles [20], created at time  $t$  in comoving frame, is given by

$$n_X(t) = \int_0^\infty \frac{dk}{2\pi^2 a^3(t)} k^2 |\beta_k|^2 \quad (40)$$



If we now assume, as in [20], that most of the support of the integral comes from  $(k/M_X)^2 \ll 1$ , where the  $k$ -dependence of  $r$  may be neglected, we arrive at

$$n_X \simeq \frac{a_{\text{eff}}^2(r)}{8\pi^{3/2}a^3(t)} \exp\left(\frac{-4M_X}{\sqrt{H_{\text{eff}}^2(r) + R_{\text{eff}}(r)}/6}\right) \times \left(\frac{M_X}{4}\sqrt{H_{\text{eff}}^2(r) + R_{\text{eff}}(r)}/6\right)^{3/2} \quad (41)$$

The above considerations pertain to the standard RW cosmology, including a de Sitter phase, provided one connects the latter smoothly to asymptotically flat regions [29].

After these general considerations, we now turn to estimate the effects on the particle production due to the distortion in the underlying metric induced by the initial D-brane recoil. To accomplish that we must first determine the modifications produced to the corresponding Bogolubov coefficient  $\beta_k$ , due to the presence of the terms proportional to  $u^i$  in the coordinate transformation of (26).

Keeping only terms linear in  $u^i$ , and introducing the conformal time  $\eta$ , (26) becomes

$$\begin{aligned} t' &= \sqrt{2}t \\ \tilde{X}^i &= X^i - \frac{1}{2}a_0^2 u^i \eta \end{aligned} \quad (42)$$

Then, under the influence of the recoil, the corresponding Bogolubov coefficient, to be denoted by  $\beta_k^{(u)}$ , reads

$$\beta_k^{(u)} \simeq \int d\eta \frac{\omega'_k}{2\omega_k} \exp\left(-2i \int_{\eta_p}^{\eta} \omega_k(\eta') d\eta'\right) \exp\left[-\frac{i}{2} a_0^2 (u_i k^i) \eta\right] \quad (43)$$

Under the same WKB assumptions, the branch cut contributions  $\mathcal{U}_j$  of (38) are now modified to

$$\begin{aligned} \mathcal{U}_j^{(u)} &= \exp\left(-2i \int_{\eta_p}^{\tilde{\eta}_j} \omega_k(\eta') d\eta' - \frac{i}{2} a_0^2 (u_i k^i) \tilde{\eta}_j\right) \\ &\times \int_{c_j} \frac{d\delta}{\delta} \exp\left(-\frac{4i}{3} M_X \sqrt{C'(\tilde{\eta}_j)} \delta^{3/2} - \frac{i}{2} a_0^2 (u_i k^i) \delta \dots\right) \end{aligned} \quad (44)$$

Notice at this point that the additional contribution due to the recoil does not alter the branch-cut structure of our problem, which is still determined by (36) and by the sign of  $\sqrt{C'(\eta)}$ ; the latter may still be chosen positive as in [20]. It turns out that, under the assumptions of [20], the additional term in the integral over  $\delta$  does not change the result of [20]. This is so because the value is determined solely by the angular integration, since the steepest descent contour evades the beginning of the branch point in such a way that [20]  $\delta = \epsilon e^{i\theta}$ ,  $\epsilon \rightarrow 0^+$  and  $\theta \in (-\frac{2\pi}{3}, \frac{4\pi}{3})$ , along the semicircle around it (c.f. figure 1). On the other hand, the imaginary part of  $\tilde{\eta}_j$  in the first exponential gives a non-vanishing contribution, whereas its real part contributes only a phase; the latter does not contribute in the physically relevant  $|\beta_k|^2$ . Thus, setting  $\tilde{\eta}_j = \tilde{r}_j + i\tilde{\mu}_j$ , and assuming again one dominant pole we have

$$|\beta_k^{(u)}|^2 = |\beta_k|^2 \exp\left[a_0^2 (u_i k^i) \tilde{\mu}\right] \quad (45)$$

In the case that  $C(\eta)$  is purely of inflationary origin, i.e.  $C(\eta) = (H\eta - 1)^{-2}$ , the solution of (36) gives  $\tilde{\eta}^\pm = H^{-1}(1 \pm iM/|k|)$ ; choosing the lower half plane, we have that  $\tilde{\mu} = -H^{-1}(M/|k|)$ . Then, setting  $u_i k^i = |u||k| \cos \theta$ , where  $\theta$  is the angle between the velocity of the original D-particle and the emitted (produced) particle, (45) yields a  $k$ -independent correction term to  $|\beta_k^{(u)}|^2$  of the form

$$|\beta_k^{(u)}|^2 = |\beta_k|^2 \exp\left(-a_0^2 |u| H^{-1} M \cos \theta\right) \quad (46)$$

which in turn modifies the particle number production from (40) to

$$n_X^{(u)}(t) = n_X(t) \exp\left(-a_0^2 |u| H^{-1} M \cos \theta\right) \quad (47)$$

Notice that in the absence of inflation, i.e. if  $H \rightarrow 0$ , the additional effect vanishes exponentially. The above result seems to persist once one takes into account interpolating metrics, with asymptotically flat regions, as required for the proper interpretation of the square of the Bogolubov coefficient  $|\beta_k|^2$  as particle number. For example, consider the RW metric with effective scale factor [35]

$$C(\eta) = A + B \frac{H\eta - 1}{\sqrt{(H\eta - 1)^2 + 1}} \quad (48)$$

which, for appropriate choice of the parameters ( $A = -B > 0$ ) contains a region of standard inflationary evolution, i.e.  $1/(H\eta - 1)^2$ , for  $(H\eta - 1)^2 \gg 1$ , and interpolates between asymptotically flat metrics. To ensure that the inflationary phase of this interpolating metric coincides with our time regime  $0 \ll Ht \ll 1$  for the validity of the  $\sigma$ -model perturbation theory, one must impose the condition  $a_0 \ll 1$  in the relation (30) connecting the conformal time  $\eta$  with the Robertson-Walker time  $t$ . In practical terms,  $a_0$  can be taken to be two to three orders of magnitude less than unity (in Planck units), yielding  $|H\eta - 1| \sim 10^3$  during inflation, for times  $0 < Ht \ll 1$ . It goes without saying that the metric (48) is used only for illustrative purposes, and is not directly related to any realistic string model. At present, within the context of the non-critical string approach to inflation, interpolating metrics including inflation at an early stage of their evolution have only been obtained numerically for realistic (Liouville) string theories [9].

The branch cut structure of the metric (48) leads again to two branch cuts, with the (relevant) one lying in the lower half plane being given by:  $\tilde{\eta} = \frac{1}{H} - i \frac{\sqrt{k^2 + AM^2}}{H|k|}$ . Then, the corresponding  $|\beta_k^{(u)}|^2$  displays a  $k$ -dependence,

$$|\beta_k^{(u)}|^2 = |\beta_k|^2 \exp\left(-a_0^2 |u| H^{-1} \sqrt{k^2 + AM^2} \cos \theta\right) \quad (49)$$

If  $k^2$  can be neglected next to  $M^2$ , which is what one usually assumes, then the results with and without interpolating metrics coincide.

Before turning into the discussion of the possible physical implications of the above results, we would like to comment on an additional point. Returning to (18), we see that

in the case of a critical Liouville string, i.e for  $Q^2 = 0$ , the corresponding  $\alpha_{\mathcal{D}}$  would be  $\alpha_{\mathcal{D}} = 2H$  instead of the one given in (21). In that case, the corresponding  $\tilde{X}^i$  in (26) would then become

$$\begin{aligned}\tilde{X}^i &= X^i + a_0^2 \frac{u^i}{2H} (e^{-2Ht} - 1) \\ &= X^i + a_0^2 u^i \eta \left( \frac{H\eta}{2} - 1 \right)\end{aligned}\quad (50)$$

Then, the contribution due to the recoil in the expressions of (44) would be

$$\mathcal{U}_j^{(u)} = \exp \left( -2i \int_{\eta_p}^{\tilde{\eta}_j} \omega_k(\eta') d\eta' + iL_{\tilde{\eta}_j}^{(u)} \right) \int_{\mathcal{C}_j} \frac{d\delta}{\delta} \exp \left( -\frac{4i}{3} M_X \sqrt{C'(\tilde{\eta}_j)} \delta^{3/2} + iL_{\delta}^{(u)} \dots \right) \quad (51)$$

where the  $\delta$ -dependent factor  $L_{\delta}^{(u)}$  and the  $\delta$ -independent factor  $L_{\tilde{\eta}_j}^{(u)}$  are given by

$$\begin{aligned}L_{\delta}^{(u)} &= a_0^2 (u_i k^i) \left[ \frac{H}{2} \delta^2 + (H\tilde{\eta}_j - 1)\delta \right] \\ L_{\tilde{\eta}_j}^{(u)} &= a_0^2 (u_i k^i) \tilde{\eta}_j \left( \frac{H\tilde{\eta}_j}{2} - 1 \right)\end{aligned}\quad (52)$$

Again, only the imaginary parts of  $L_{\tilde{\eta}_j}^{(u)}$ , to be denoted by  $\Im m L_{\tilde{\eta}_j}^{(u)}$ , give a non-vanishing contribution, whereas its real part contributes only a phase, i.e.

$$\mathcal{U}_j^{(u)} = \mathcal{U}_j \exp \left( -\Im m L_{\tilde{\eta}_j}^{(u)} \right) \quad (53)$$

and after setting again  $\tilde{\eta}_j = \tilde{r}_j + i\tilde{\mu}_j$  we have (assuming again one dominant pole)

$$\mathcal{U}_j^{(u)} = \mathcal{U}_j \exp \left[ -a_0^2 (u_i k^i) \tilde{\mu} (H\tilde{r} - 1) \right] \quad (54)$$

Then for the purely inflationary part of the metric,  $\tilde{\eta}^{\pm} = H^{-1}(1 \pm iM/|k|)$  and therefore  $H\tilde{r} - 1 = 0$ ; thus, when the string is critical, the bulk of the effect from the recoil vanishes, as it should. This result persists as long as  $\tilde{r} = H^{-1}$ , as is the case of the interpolating metric of (48). We consider this as a non-trivial self-consistency check of both our method and of the integration techniques developed in [20].

## 6 Analysis of the Particle Spectrum

In this section we will explore some of the possible physical consequences of the spectrum (49), incorporating the back-reaction effects of the D-particle recoil onto the inflationary space-time. First of all, the spectrum is *non thermal*, due to the presence of the  $\cos\theta$  term. This will induce anisotropies into the spectrum of the emitted Dark matter particles (c.f. figure 2), which, in principle, may be observable.

As discussed in [20], and is valid also in our case, the produced particles will be cosmologically abundant, and thus relevant, i.e. will have densities of order one, if and only if

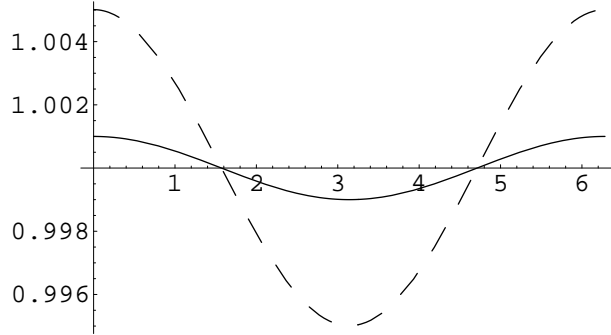


Figure 2: *The ratio of the Bogolubov coefficient (49) over the unperturbed one  $u = 0$  (39) versus the angle  $\theta \in [0, 2\pi]$  for two characteristic momentum scales  $k_1$  (dashed) and  $k_2$  (continuous), with  $k_2 \gg k_1$  (in units of  $H$  during inflation).*

their masses are *at least* of order of the Hubble parameter during inflation,  $H \sim 10^{-5} M_P$ . Thus, as suggested in [20], such heavy particles can be candidates for Dark matter. This conclusion survives in our case, given that the bulk effect of particle production is still due to the exponential expansion of the Universe (the distortions due to D-particle recoil are sub-dominant effects). Therefore the anisotropies in our case, inferred from (49), will in general imply anisotropies in the superheavy dark matter spectrum, which could have phenomenological significance.

The effects leading to (49) stem from single scattering events between a string and a D-particle. In general, one encounters a distribution of D-particles in the Early Universe, characterized by a density  $\rho_D$ . Assuming for simplicity uniform densities, one can incorporate the effects of such ensembles of D-particles by replacing  $u$  in the metric (23) by  $\rho_D \bar{u}$ , where the bar indicates average quantities over the ensemble. In addition, one should integrate (49) over all possible angles  $\theta$ . Setting

$$z_k = a_0^2 |u| H^{-1} \sqrt{k^2 + AM^2}, \quad \bar{z}_k \equiv a_0^2 |\bar{u}| H^{-1} \sqrt{k^2 + AM^2}, \quad (55)$$

the integral over angles, for uniform D-particle densities and distributions, can be performed analytically [36], and yields:

$$|\beta_k^{(\bar{u})}|^2 = |\beta_k|^2 \int_0^{2\pi} d\theta \exp(-\bar{z}_k \rho_D \cos \theta) = 2\pi |\beta_k|^2 I_0(\bar{z}_k \rho_D), \quad (56)$$

where  $|\beta_k|^2$  is the unperturbed inflation spectrum (39) of [20], and  $I_0(z)$  is the modified Bessel function, which in our case can be expressed in terms of the Bessel function  $\mathcal{J}_0(z)$  as:  $I_0(z) = \mathcal{J}_0(e^{i\frac{\pi}{2}} z)$ . For small arguments, as is the case above due to the smallness of  $\bar{u}$ ,

we may use the perturbative expansion [36]:

$$I_0(x) = \sum_{k=0}^{\infty} \frac{x^{2k}}{2^{2k} k! \Gamma(k+1)}, \quad (57)$$

which to order  $\bar{u}^2$  yields from (56):

$$|\beta_k^{(\bar{u})}|^2 = |\beta_k|^2 \left( 1 + \frac{1}{4} \rho_D^2 \bar{z}_k^2 + \dots \right) \quad (58)$$

Another quantity of interest is the total number of emitted particles, from a single scattering event of a string colliding with a D-particle. This is obtained upon integrating (49) over momentum scale vectors  $d^3k = k^2 dk \sin\theta d\theta d\phi$ , where we selected the axis of the recoil velocity  $\vec{u}$  to coincide with the  $k_z$  axis. The result is (c.f. (40)):

$$n_{X(t)}^u = \int_0^\infty \frac{d^3k}{(2\pi)^3 a^3(t)} |\beta_k^{(u)}|^2 = \int_0^\infty \frac{k^2 dk}{2\pi^2 a^3(t)} \frac{1}{z_k} \sinh(z_k) |\beta_k|^2 ; \quad (59)$$

Expanding for small  $u$ , and keeping terms up to order  $u^2$ , we obtain:

$$n_{X(t)}^u - n_{X(t)} \simeq \frac{1}{6} a_0^4 |u|^2 H^{-2} \int_0^\infty dk k^2 (k^2 + AM^2) |\beta_k|^2 \simeq \frac{A}{6} a_0^4 |u|^2 \frac{M^2}{H^2} n_X(t) \quad (60)$$

where  $n_X(t)$  is the unperturbed total particle density ( $u = 0$ ) (41), and in the last (approximate) equality on the right-hand-side of (60) we have considered for simplicity the case of low momenta  $k \ll M$ .

## 7 Conclusions and Outlook

In this article we have presented a brane-inspired model of anisotropies (49) in the spectrum of superheavy (masses  $M > 10^{13}$  GeV) Dark matter particles. This may also imply an anisotropic spectrum of ultra-high-energy cosmic rays, should the latter be the product of decay of such superheavy particles, according to some scenaria [21]. Although at present there seems to be no experimental evidence for such anisotropies [22], however, the experimental situation is far from being conclusive, mainly due to the scarcity of the ultra-high-energy cosmic ray events available to date. The situation is about to be improved significantly in the near future, whereby the launch of experiments such as the Piere Auger will improve the available statistics by several orders of magnitude (provided the ultra-high-energy cosmic ray events are actually there).

On the other hand, if anisotropies are not detected in such improved facilities, one should be able to place strict upper bounds on parameters of models like the one presented here, for instance on the quantity  $\bar{u}\rho_D$  entering the spectrum (49), (58). Such bounds, if combined with progress in the theoretical understanding of D-brane theory and cosmology, may be instrumental in seriously constraining non-critical string theory models of the type employed here.

We have arrived at the above conclusions using perturbative  $\sigma$ -model conformal field theory methods, which we expect to be applicable at some regime of the parameters of an inflationary Universe. We are fully aware of the possibility that non-perturbative string Physics may be responsible for the inflationary background, on which our calculations have been based, and we have therefore also provided arguments on how one could tackle the problem using perturbative  $\sigma$ -model methods, at the cost, however, of departing from critical string theory. We demonstrated the mathematical consistency of such an approach, at least within a given time range. In fact, the non-critical string theory approach to inflation has many phenomenologically desirable features, including the possibility of accelerating phases of the Universe [9], and thus non-zero dark energy components in agreement with standard (TeV scale) supersymmetry breaking scenaria [11].

There are many avenues for improving the results presented in this article. First of all, one should use realistic string Universe brane models, by taking into account the standard model fields on the brane and/or including supersymmetry and its breaking in a consistent way (e.g. through compactification of extra dimensions in five-branes on magnetized tori, as in [11]: such models are known to have instabilities, but in a cosmological context this may be a desirable feature that needs to be explored). Second, one should improve on the effective field theory computation of particle production presented in this article by using as interpolating metrics realistic space-time metrics obtained from the above non-critical string models. At present the situation is only known numerically [9], but we may not be far from obtaining some analytic results, given the increasing recent interest towards issues like inflationary physics in the context of string theory from various viewpoints [37, 35].

Finally, in this context, we stress the preliminary conclusions of [35], on the potentially important rôle of strings at the end of inflation towards reheating and other related issues. It would be interesting to place the last issue in the framework of non-critical strings discussed here. As argued above, such a framework appears to provide a viable treatment of inflationary dynamics within the context of perturbative string theory, with the potential of providing us with experimentally testable (in principle) predictions.

## Acknowledgements

This paper is dedicated to the memory of Ian I. Kogan, collaborator and friend. The work of N.E.M. is partially supported by a visiting professorship at the University of Valencia (Spain), Department of Theoretical Physics, and by the European Union (contract HPRN-CT-2000-00152). The work of J.P. is supported by the Grants BFM2002-000568, FPA2002-00612, and GV01-94.

# Appendix: Inflation as a Liouville String $\sigma$ -model Background

## A Concrete Example of Non-critical Strings: Colliding Brane Worlds

In this Appendix we discuss briefly how an inflationary space-time may be derived as a consistent background of a non-critical string theory. Although the approach is general, and can be applied to many non-critical string models, for definiteness we shall concentrate here on one particular case [11], where the non-criticality is induced by the collision of two brane worlds.

In what follows we shall first discuss the basic features of this scenario, and then proceed to demonstrate explicitly the emergence of inflationary space-times from such situations. Following [11], we consider two five-branes of type II string theory, in which the extra two dimensions have been compactified on tori. In one of the branes (assumed to be the hidden (not our) world) the torus is magnetized, with a magnetic field intensity  $\mathcal{H}$ . Initially our world is compactified on a normal torus, without magnetic fields, and the two branes are assumed on collision course in the bulk, with a small velocity  $v \ll 1$ . The collision produces a non-equilibrium situation, which results in electric current transfer from the hidden to the visible brane. This causes the (adiabatic) emergence of magnetic fields in our world.

The associated instabilities that accompany such magnetized-tori compactifications are not a problem in the context of cosmological scenarios we are discussing here. In fact, as discussed in [11], the collision may also produce decompactification of the extra toroidal dimensions, which however takes place at a rate much slower than any other rate in the problem. As discussed in [11], this is important for guaranteeing an asymptotic equilibrium situation and a proper definition of an S-matrix for the stringy excitations on the observable world.

The collision of the two branes implies, for a short period after it, when the branes are at most a few string scales apart, the exchange of open string excitations stretching between the branes with their ends attached on them. As argued in [11], the exchange of such pairs of open strings in the type II string theory considered in that work result in an excitation energy on the visible world. The latter may be estimated by computing the corresponding scattering amplitude of the two branes, using string-theory world-sheet methods [38]. Essentially the time integral for the relevant potential yields the scattering amplitude. Such estimates involve the computation of appropriate world-sheet annulus diagrams, due to the existence of open string pairs in type II string theory. This implies the presence of “spin factors” as proportionality constants in the scattering amplitude, which are expressed in terms of Jacobi  $\Theta$  functions. For small brane velocities  $v \ll 1$  we are considering in our case, the appropriate spin structures start at *quartic order* in  $v$ , as a result of mathematical properties of the Jacobi functions [38]. This in turn implies [11] that the resulting excitation energy on the brane world is of order  $V = \mathcal{O}(v^4)$ , which may be thought of as an initial (approximately constant) value of a *supercritical* central-charge deficit for the non-critical  $\sigma$ -model describing stringy excitations on the observable world

after the collision:

$$Q^2 = \mathcal{O}(v^4) > 0 \quad (61)$$

The supercriticality of the model is essential [30] for a time-like signature of the Liouville mode.

As we discuss below, such constant  $Q^2$  can produce inflation with a scale factor varying exponentially with the Robertson-Walker time:  $a(t) \sim e^{Qt}$  (up to proportionality numerical constants in the exponent, of order one, which can be fixed by normalization, c.f. below). The duration of the inflationary era is therefore of order  $t_* \sim 1/Q$ , which matches the conventional cosmology estimates of  $10^9 t_P$  for non-relativistic brane velocities  $v^2 \sim 10^{-9}$ . These small velocities are consistent with our perturbative  $\sigma$ -model formalism of recoil [24] we follow here.

For times long after the collision, the central charge deficit is no longer constant but relaxes with time  $t$ . In the approach of [11] such a relaxation has been computed by means of logarithmic conformal field theory world-sheet methods [18, 24], taking into account recoil (in the bulk) of the observable-world brane and the identification of target time with the (zero mode of the) Liouville field. This late-time varying deficit  $Q^2(t)$  has been identified [11] with a ‘quintessence-like’ dark energy density component of our world:

$$\Lambda(t) \sim \frac{R^2(\mathcal{H}^2 + v^2)^2}{t^2} \left(\frac{M_s}{M_P}\right)^4 M_P^4, \quad (62)$$

where  $R$  is the compactification radius. In our model, for the validity of the  $\sigma$ -model perturbative formalism the constraints (5) apply, which lead naturally to  $M_s \sim 10^{-4} M_P$ .

We next remark that the presence of the magnetic field  $\mathcal{H}$  is responsible for a breaking of target-space supersymmetry [39], due to the fact that bosons and fermions on the brane worlds couple differently to  $\mathcal{H}$ . The resulting mass difference between bosonic and fermionic string excitations for our problem, where the magnetic field is turned on adiabatically, is found to be [11]:

$$\Delta m_{\text{string}}^2 \sim 2\mathcal{H} \cosh(\epsilon\varphi + \epsilon t) \Sigma_{45} \quad (63)$$

where  $\Sigma_{45}$  is a standard spin operator on the plane of the torus, and  $\epsilon \rightarrow 0^+$  is the regulating parameter of the Heaviside operator (2) appearing in the D-brane recoil formalism [24]. From (63) we observe that the formalism selects dynamically a Liouville mode which flows opposite to the target time  $\varphi = -t$ , as a result of minimization of the effective field-theoretic potential of the various stringy excitations.

By choosing appropriately  $\mathcal{H}$  we may thus arrange for the supersymmetry breaking scale to be of order of a few TeV. It turns out that the so-chosen magnetic field contribution is subdominant, as compared with the velocity contribution  $v^2 \sim 10^{-9}$ , in the expression for the current era dark energy (62). The model, therefore, is capable of reproducing naturally a current value of the dark energy (i.e. for  $t \sim 10^{60} t_P$ ) compatible with observations [1, 2], provided one chooses relatively large compactification radii  $R \sim 10^{17} \ell_P \sim 10^{-18}$  m, which are common in modern string theories<sup>9</sup>. However, we cannot claim at this stage that this

---

<sup>9</sup>For models where the compactification involves higher-dimensional manifolds than tori a volume factor



(toy) model is free from fine tuning, since the final asymptotic value of the central charge deficit has been arranged to vanish, by an appropriate choice of various constants appearing in the problem [11]. This is required by the assumption that our non-critical string system relaxes asymptotically in time to a critical string. In the complete model, the identification of the Liouville field with target time [16, 24] would define the appropriate renormalization-group trajectory, which hopefully would pick up the appropriate asymptotic critical string state dynamically. This still remains to be seen analytically in realistic models, although it has been demonstrated numerically for some stringy models in [9]. Nevertheless, the current toy example is sufficient in providing a non-trivial, and physically relevant, concrete example of an inflationary Universe in the context of Liouville strings, a demonstration of which we now turn to.

### Inflation from Liouville Strings

Consider a non-critical  $\sigma$ -model in metric ( $G_{\mu\nu}$ ), antisymmetric tensor ( $B_{\mu\nu}$ ), and dilaton backgrounds ( $\Phi$ ), with the following  $\mathcal{O}(\alpha')$   $\beta$ -functions ( $\alpha'$  the Regge slope) [40]:

$$\begin{aligned}\beta_{\mu\nu}^G &= \alpha' \left( R_{\mu\nu} + 2\nabla_\mu \partial_\nu \Phi - \frac{1}{4} H_{\mu\rho\sigma} H_\nu^{\rho\sigma} \right) , \\ \beta_{\mu\nu}^B &= \alpha' \left( -\frac{1}{2} \nabla_\rho H_{\mu\nu}^\rho + H_{\mu\nu}^\rho \partial_\rho \Phi \right) , \\ \tilde{\beta}^\Phi &= \beta^\Phi - \frac{1}{4} G^{\rho\sigma} \beta_{\rho\sigma}^G = \frac{1}{6} (C - 26)\end{aligned}\tag{64}$$

where Greek indices are four-dimensional, including target-time components  $\mu, \nu, \dots = 0, 1, 2, 3$  on the D3 brane worlds of ref. [11], and  $H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$  is the field strength.

We consider the following representation of the four-dimensional field strength in terms of a pseudoscalar (axion-like) field  $b$ :

$$H_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\sigma} \partial^\sigma b\tag{65}$$

where  $\epsilon_{\mu\nu\rho\sigma}$  is the four-dimensional antisymmetric symbol.

Choose the linear in time axion background [30]:

$$b = b(t) = \beta t , \quad \beta = \text{constant}\tag{66}$$

which yields a constant field strength with spatial indices only:  $H_{ijk} = \epsilon_{ijk}\beta$ ,  $H_{0jk} = 0$ . This implies that this background is a conformal solution of the full four-dimensional antisymmetric tensor  $\mathcal{O}(\alpha')$   $\beta$ -function.

We also consider a dilaton background linear in time  $t$  [30]

$$\Phi(t, X) = \text{const} + (\text{const})'t\tag{67}$$

---

$R^n$ , with  $n > 2$  the number of extra dimensions, appears in (62), and thus in such cases the compactification radii are significantly smaller.

This background does not contribute to the antisymmetric tensor and metric  $\beta$ -functions of (64).

Suppose, now, that only the metric is a non conformal background, due to some initial quantum fluctuations or catastrophic events, like the collision of two brane worlds discussed above and in [25], which result in the presence of an initial central charge deficit  $Q^2$  (61), constant at early stages after the collision. Let

$$G_{ij} = e^{\kappa\varphi + Hct} \eta_{ij} \ , \quad G_{00} = e^{\kappa'\varphi + Hct} \eta_{00} \quad (68)$$

where  $t$  is the target time,  $\varphi$  is the Liouville mode,  $\eta_{\mu\nu}$  is the four-dimensional Minkowski metric, and  $\kappa, \kappa', c$  are constants to be determined.

According to the discussion in the text, following (20), we require:

$$Q = -3H \quad (69)$$

which partially stems from [17, 16]

$$\varphi = -t \quad (70)$$

This restriction will be imposed dynamically [25, 16] only at the end of our computations. Initially, one should treat  $\varphi, t$  as independent target space components.

The Liouville-dressing induces [15]  $\sigma$ -model terms of the form  $\int_{\Sigma} R^{(2)} Q \varphi$ , where  $R^{(2)}$  is the world-sheet curvature. Such terms lead to non-trivial contributions to the dilaton background in the (D+1)-dimensional space-time  $(\varphi, t, X^i)$ :

$$\Phi(\varphi, t, X^i) = Q \varphi + (\text{const})' t + \text{const} \quad (71)$$

Upon choosing  $(\text{const})' = Q$ , we observe that (70) implies a constant dilaton background.

We now consider the Liouville-dressed equations (20) for the  $\beta$ -functions of the metric and antisymmetric tensor fields (64). For constant dilatons that we assume, the dilaton equation yields no independent information, apart from expressing the dilaton  $\beta$  function in terms of the central charge deficit as usual. For the axion background (66) only the metric yields a non-trivial constraint (we work in units of  $\alpha' = 1$  for convenience):

$$G''_{ij} + Q G'_{ij} = -R_{ij} + \frac{1}{2} \beta^2 G_{ij} \quad (72)$$

where the prime indicates differentiation with respect to the (world-sheet zero mode of the) Liouville mode  $\varphi$ , and  $R_{ij}$  is the (non-vanishing) Ricci tensor of the (non-critical)  $\sigma$ -model with coordinates  $(t, \vec{x})$ :  $R_{00} = 0$ ,  $R_{ij} = \frac{c^2 H^2}{2} e^{(\kappa - \kappa')\varphi}$ .

One should also take into account the temporal ( $t$ ) equation for the metric tensor (for the antisymmetric backgrounds this is identically zero):

$$G''_{00} + Q G'_{00} = -R_{00} = 0 \quad (73)$$

where the vanishing of the Ricci tensor stems from the specific form of the background (68).

Moreover we seek metric backgrounds in Robertson-Walker inflationary form (de Sitter):

$$G_{00} = -1, \quad G_{ij} = e^{2Ht} \eta_{ij} \quad (74)$$

Then, from (74), (68), (67),(66) and (69), and imposing (70) at the end, we do observe that there is a consistent solution with:

$$Q = -3H = -\kappa', \quad c = 3, \quad \kappa = H, \quad \beta^2 = 5H^2 \quad (75)$$

It is in such backgrounds that we consider the (back reaction) effects of recoiling D-particles, by employing again Liouville dressing techniques and the identification [25, 24] (70). As discussed in the text, we consider the effects only for times  $0 < Ht \ll 1$  for the validity of our  $\sigma$ -model perturbation theory. As a consistency check of our approach, we find that the identification (70), with opposite flows of Liouville and target time fields, implies the decay of the back reaction effects as the time elapses (c.f. (24),(26)).

## References

- [1] B. P. Schmidt *et al.*, *Astrophys. J.* **507** (1998) 46 [arXiv:astro-ph/9805200]; S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], *Astrophys. J.* **517** (1999) 565 [arXiv:astro-ph/9812133]; J. P. Blakeslee *et al.*, *Astrophys. J.* **589** (2003) 693 [arXiv:astro-ph/0302402]; A. G. Riess *et al.*, *Astrophys. J.* **560** (2001) 49 [arXiv:astro-ph/0104455].
- [2] C. L. Bennett *et al.*, arXiv:astro-ph/0302207; D. N. Spergel *et al.*, arXiv:astro-ph/0302209; H. V. Peiris *et al.*, arXiv:astro-ph/0302225.
- [3] For recent reviews, in light of WMAP data, see: J. Garcia-Bellido, arXiv:hep-ph/0303153; J. R. Ellis, arXiv:astro-ph/0304183 and arXiv:astro-ph/0305038, and references therein.
- [4] A. H. Guth, *Phys. Rev. D* **23** (1981) 347; A. D. Linde, *Phys. Lett. B* **108** (1982) 389.
- [5] W. H. Kinney, E. W. Kolb, A. Melchiorri and A. Riotto, arXiv:hep-ph/0305130;
- [6] S. M. Carroll, *Living Rev. Rel.* **4** (2001) 1 [arXiv:astro-ph/0004075], and references therein.
- [7] J. Polchinski, *String Theory*. Vols. 1,2 (Cambridge University Press (U.K.) 1998).
- [8] S. Hellerman, N. Kaloper and L. Susskind, *JHEP* **0106** (2001) 003 [arXiv:hep-th/0104180]; W. Fischler, A. Kashani-Poor, R. McNees and S. Paban, *JHEP* **0107** (2001) 003 [arXiv:hep-th/0104181]; J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, arXiv:hep-th/0105206.

- [9] G. A. Diamandis, B. C. Georgalas, N. E. Mavromatos, E. Papantonopoulos and I. Pappa, *Int. J. Mod. Phys. A* **17** (2002) 2241 [arXiv:hep-th/0107124]; G. A. Diamandis, B. C. Georgalas, N. E. Mavromatos and E. Papantonopoulos, *Int. J. Mod. Phys. A* **17** (2002) 4567 [arXiv:hep-th/0203241].
- [10] P. Binetruy, C. Deffayet and D. Langlois, *Nucl. Phys. B* **565** (2000) 269 [arXiv:hep-th/9905012]; For a comprehensive review see: D. Langlois, *Prog. Theor. Phys. Suppl.* **148** (2003) 181 [arXiv:hep-th/0209261], and references therein.
- [11] E. Gravanis and N. E. Mavromatos, *Phys. Lett. B* **547** (2002) 117 [arXiv:hep-th/0205298]; see also N. E. Mavromatos, *Quantum gravity, cosmology, (Liouville) strings and Lorentz invariance*, sec 4.2 [arXiv:hep-th/0210079] (*Proc. Beyond the Desert 2002* (Oulu Finland, June 2002), in press).
- [12] J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, *Phys. Rev. D* **64** (2001) 123522 [arXiv:hep-th/0103239].
- [13] R. Kallosh, L. Kofman, A. D. Linde and A. A. Tseytlin, *Phys. Rev. D* **64** (2001) 123524 [arXiv:hep-th/0106241]; A. Linde, arXiv:hep-th/0205259.
- [14] J. Khoury, B. A. Ovrut, N. Seiberg, P. J. Steinhardt and N. Turok, *Phys. Rev. D* **65** (2002) 086007 [arXiv:hep-th/0108187].
- [15] F. David, *Mod. Phys. Lett.* **A3**, 1651 (1988); J. Distler and H. Kawai, *Nucl. Phys.* **B321**, 509 (1989); see also: N. E. Mavromatos and J. L. Miramontes, *Mod. Phys. Lett. A* **4** (1989) 1847; E. D'Hoker and P. S. Kurzepa, *Mod. Phys. Lett. A* **5** (1990) 1411; I. R. Klebanov, I. I. Kogan and A. M. Polyakov, *Phys. Rev. Lett.* **71** (1993) 3243 [arXiv:hep-th/9309106].
- [16] J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, *Phys. Lett. B* **293** (1992) 37 [arXiv:hep-th/9207103], for a review see: arXiv:hep-th/9311148, *Erice lectures*, Sub-nuclear series, Vol. 31, 1 (World Sci. 1994).
- [17] I. I. Kogan, *Prepared for Particles & Fields 91: Meeting of the Division of Particles & Fields of the APS, Vancouver, Canada, 18-22 Aug 1991*; *Phys. Lett. B* **265** (1991) 269.
- [18] Partial list of references: V. Gurarie, *Nucl. Phys. B* **410** (1993) 535 [arXiv:hep-th/9303160]; M. A. Flohr, *Int. J. Mod. Phys. A* **11** (1996) 4147 [arXiv:hep-th/9509166] and arXiv:hep-th/0111228; A. Bilal and I. I. Kogan, *Nucl. Phys. B* **449** (1995) 569 [arXiv:hep-th/9503209]. J. S. Caux, I. I. Kogan and A. M. Tselik, *Nucl. Phys. B* **466** (1996) 444 [arXiv:hep-th/9511134]; M. R. Gaberdiel and H. G. Kausch, *Nucl. Phys. B* **477** (1996) 293 [arXiv:hep-th/9604026]; M. R. Rahimi Tabar, A. Aghamohammadi and M. Khorrami, *Nucl. Phys. B* **497** (1997) 555 [arXiv:hep-th/9610168]; I. I. Kogan and A. Lewis, *Nucl. Phys. B* **509** (1998) 687 [arXiv:hep-th/9705240]. I. I. Kogan, *Phys. Lett. B* **458** (1999) 66 [arXiv:hep-th/9903162]. I. I. Kogan and J. F. Wheeler, *Phys.*

- Lett. B **486** (2000) 353 [arXiv:hep-th/0003184]. I. I. Kogan and A. Nichols, Int. J. Mod. Phys. A **17** (2002) 2615 [arXiv:hep-th/0107160]; Y. Ishimoto, Nucl. Phys. B **619** (2001) 415 [arXiv:hep-th/0103064]. J. Fjelstad, J. Fuchs, S. Hwang, A. M. Semikhatov and I. Y. Tipunin, Nucl. Phys. B **633** (2002) 379 [arXiv:hep-th/0201091]; N. E. Mavromatos and R. J. Szabo, JHEP **0301** (2003) 041 [arXiv:hep-th/0207273]; I. Bakas and K. Sfetsos, Nucl. Phys. B **639** (2002) 223 [arXiv:hep-th/0205006]; K. Sfetsos, arXiv:hep-th/0305109.
- [19] J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Mod. Phys. Lett. A **10** (1995) 1685 [arXiv:hep-th/9503162].
- [20] D. J. Chung, Phys. Rev. D **67** (2003) 083514 [arXiv:hep-ph/9809489].
- [21] J. R. Ellis, G. B. Gelmini, J. L. Lopez, D. V. Nanopoulos and S. Sarkar, Nucl. Phys. B **373** (1992) 399; P. Gondolo, G. Gelmini and S. Sarkar, Nucl. Phys. B **392** (1993) 111 [arXiv:hep-ph/9209236]. V. Berezhinsky, M. Kachelriess and A. Vilenkin, Phys. Rev. Lett. **79** (1997) 4302 [arXiv:astro-ph/9708217]. V. A. Kuzmin and V. A. Rubakov, Phys. Atom. Nucl. **61** (1998) 1028 [Yad. Fiz. **61** (1998) 1122] [arXiv:astro-ph/9709187]; M. Birkel and S. Sarkar, Astropart. Phys. **9** (1998) 297 [arXiv:hep-ph/9804285]; G. Gelmini and A. Kusenko, Phys. Rev. Lett. **84** (2000) 1378 [arXiv:hep-ph/9908276]; J. L. Crooks, J. O. Dunn and P. H. Frampton, Astrophys. J. **546** (2001) L1 [arXiv:astro-ph/0002089]; S. Sarkar, arXiv:hep-ph/0005256.
- [22] L. A. Anchordoqui, C. Hojvat, T. P. McCauley, T. C. Paul, S. Reucroft, J. D. Swain and A. Widom, arXiv:astro-ph/0305158; C. Hojvat, T. P. McCauley, S. Reucroft and J. D. Swain, arXiv:astro-ph/0305206.
- [23] See, for instance: J. Garcia-Bellido, arXiv:astro-ph/0306195, and references therein.
- [24] I. I. Kogan and N. E. Mavromatos, Phys. Lett. B **375** (1996) 111 [arXiv:hep-th/9512210]; I. I. Kogan, N. E. Mavromatos and J. F. Wheeler, Phys. Lett. B **387**, 483 (1996) [arXiv:hep-th/9606102]; J. R. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Int. J. Mod. Phys. A **12**, 2639 (1997) [arXiv:hep-th/9605046]; N. E. Mavromatos and R. J. Szabo, Phys. Rev. D **59**, 104018 (1999) [arXiv:hep-th/9808124]; JHEP **0110** (2001) 027 [arXiv:hep-th/0106259].
- [25] E. Gravanis and N. E. Mavromatos, JHEP **0206**, 019 (2002).
- [26] H. Dorn and H. J. Otto, Phys. Lett. B **381**, 81 (1996); Nucl. Phys. Proc. Suppl. **56B**, 30 (1997); J. Borlaf and Y. Lozano, Nucl. Phys. B **480**, 239 (1996); Y. Lozano, Mod. Phys. Lett. A **11**, 2893 (1996); G. Amelino-Camelia and N. E. Mavromatos, Phys. Lett. B **422**, 101 (1998).
- [27] See, for instance: N.D. Birrell and P.C.W. Davies, *Quantum Field in Curved Space Times* (Cambridge Univ. Press, Cambridge (U.K.) 1982).

- [28] S. Biswas, A. Shaw and P. Misra, *Gen. Rel. Grav.* **34** (2002) 665; S. Biswas and I. Chowdhury, arXiv:gr-qc/0207058.
- [29] S. Hossenfelder, D. J. Schwarz and W. Greiner, *Class. Quant. Grav.* **20** (2003) 2337 [arXiv:gr-qc/0210110].
- [30] I. Antoniadis, C. Bachas, J. R. Ellis and D. V. Nanopoulos, *Phys. Lett. B* **211** (1988) 393; *Nucl. Phys. B* **328** (1989) 117; *Phys. Lett. B* **257** (1991) 278.
- [31] W. Fischler and L. Susskind, *Phys. Lett. B* **171** (1986) 383; *Phys. Lett. B* **173** (1986) 262.
- [32] S. Coleman, *Aspects of Symmetry*, pp. 278 sec. 2.4 (Cambridge University Press 1985).
- [33] A. B. Zamolodchikov, *JETP Lett.* **43** (1986) 730 [*Pisma Zh. Eksp. Teor. Fiz.* **43** (1986) 565].
- [34] J. R. Ellis, P. Kanti, N. E. Mavromatos, D. V. Nanopoulos and E. Winstanley, *Mod. Phys. Lett. A* **13** (1998) 303 [arXiv:hep-th/9711163].
- [35] S. S. Gubser, arXiv:hep-th/0305099.
- [36] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series and Products*, pp. 517, sec 3.915(4) (ed. by A. Jeffrey, Academic Press Inc., New York 1994).
- [37] E. J. Martinec, arXiv:hep-th/0305148; B. C. Da Cunha and E. J. Martinec, arXiv:hep-th/0303087; A. Strominger and T. Takayanagi, arXiv:hep-th/0303221; N. R. Constable and F. Larsen, *JHEP* **0306** (2003) 017 [arXiv:hep-th/0305177].
- [38] C. Bachas, *Phys. Lett. B* **374** (1996) 37 [arXiv:hep-th/9511043].
- [39] C. Bachas, arXiv:hep-th/9503030.
- [40] R. R. Metsaev and A. A. Tseytlin, *Nucl. Phys. B* **293** (1987) 385.