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# The dual gauge fixing property of the $S$ -matrix

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## ABSTRACT

The  $S$ -matrix is known to be independent of the gauge fixing parameter to all orders in perturbation theory. In this paper by employing the pinch technique we prove at one loop a stronger version of this independence. In particular we show that one can use a gauge fixing parameter for the gauge bosons inside quantum loops which is different from that used for the bosons outside loops, and the  $S$ -matrix is independent from both. Possible phenomenological applications of this result are briefly discussed.

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# 1 Introduction

In this paper we will discuss an interesting property of the  $S$ -matrix of gauge theories, which is easy to prove for QED, but is not at all evident for non-abelian theories such as QCD, or the electroweak  $SU(2)_L \times U(1)_Y$  model.

As a result of the quantization of a gauge theory, arbitrary gauge fixing parameters (GFP), which we will collectively denote by  $\xi$ , infest the Feynman rules used in perturbative calculations. It is well known however that even though individual Feynman diagrams are GFP-dependent, when combined to form the  $S$ -matrix element of a physical process, they give rise to GFP-independent expressions, order by order in perturbation theory [1]. It turns out that a stronger version of this GFP cancellation exists, which we will prove at one loop order.

We will separate the virtual gauge bosons of a Feynman graph into two classes: the “loop” gauge bosons, i.e. those virtual gauge bosons which appear inside the loops of a Feynman graph, and the “tree” gauge bosons, which are not part of a loop. In other words, the “loop” gauge bosons are irrigated by the virtual loop momentum we integrate over, while the “tree” gluons are not. The tree-level propagators of each gauge bosons in either class depend on  $\xi$ .

We will now go one step further and make the arbitrary replacements  $\xi \rightarrow \xi_t$  for the propagators of the tree gauge bosons and  $\xi \rightarrow \xi_l$  for the propagators of the loop gauge bosons, where  $\xi_t \neq \xi_l$ . In this way one introduces in general two entirely different gauge-fixing parameters ( $\xi_t$  and  $\xi_l$ ). Then one can show that the  $S$ -matrix is unchanged, and that it is invariant under the *separate* change :

$$\begin{aligned} (D) : \quad \xi_t &\rightarrow \xi'_t \\ \xi_l &\rightarrow \xi'_l \end{aligned} \tag{1.1}$$

or equivalently, that it is independent of *both*  $\xi_t$  and  $\xi_l$ . In particular, one can prove the

above statement *before* any momentum-integration is carried out.

To clarify the previous procedure, we consider a particular example. We start with the usual classical QCD Lagrangian density

$$\mathcal{L}_C = -\frac{1}{4}\mathcal{F}_{\mu\nu}^a\mathcal{F}^{a\mu\nu} + \bar{\psi}(i\not{D} - m)\psi , \quad (1.2)$$

which is invariant under the gauge transformations

$$A_\mu^{\prime a}(x) = U(x)A_\mu^a(x)U^{-1}(x) - [\partial_\mu U(x)]U^{-1}(x) , \quad (1.3)$$

$$\psi'(x) = U(x)\psi(x) , \quad (1.4)$$

$$U(x) = \exp(-i\omega^a(x)T^a) , \quad (1.5)$$

where  $T^a$  are the matrix representations of the SU(3) group. We then quantize  $\mathcal{L}_C$  using the gauge fixing term  $\mathcal{L}_{GF} = -\frac{1}{2\xi}(G^a)^2 = -\frac{1}{2\xi}(\partial^\mu A_\mu^a)^2$ , and the corresponding Fadeev-Popov term  $\mathcal{L}_{\Phi\Pi} = \bar{c}^a \frac{\delta G^a}{\delta \omega^b} c^b = \bar{c}^a (-\partial^\mu D_\mu^{ab}) c^b$ ,

$$\mathcal{L}_Q = \mathcal{L}_C + \mathcal{L}_{GF} + \mathcal{L}_{\Phi\Pi} . \quad (1.6)$$

Then use the Feynman rules obtained from the  $\mathcal{L}_Q$  Lagrangian density to compute the one-loop  $S$ -matrix element  $T$ , for elastic scattering of quarks  $q_1 q_2 \rightarrow q_1 q_2$ , with masses  $m_1$  and  $m_2$ . In particular, the gluon propagator reads

$$i\Delta_{\mu\nu}(q, \xi) = \frac{-i}{q^2} [g_{\mu\nu} - (1 - \xi)\frac{q_\mu q_\nu}{q^2}] , \quad (1.7)$$

and the ghost propagator

$$i\Delta_c = \frac{i}{q^2} . \quad (1.8)$$

Let us call the integration momentum  $k$ . Self-energy, vertex and box graphs will contribute respectively to the  $T_1$ ,  $T_2$  and  $T_3$  parts of the amplitude, i.e.

$$\begin{aligned} T(s, t, m_1, m_2) &= T_1(t, \xi) + T_2^{(1)}(t, m_1, \xi) + T_2^{(2)}(t, m_2, \xi) \\ &\quad + T_3(s, t, m_1, m_2, \xi) , \end{aligned} \quad (1.9)$$

where  $t = -q^2$ ,  $q = p_1 - p'_1 = p'_2 - p_2$  and  $p_i$ ,  $p'_i$  are respectively the initial and final momenta of the quarks. Label by  $\Delta_{\mu\nu}^t(q, \xi)$  the propagators of the tree gluons, and by  $\Delta_{\mu\nu}^l(k, \xi)$  the propagators of the loop gluons. Then replace  $\Delta_{\mu\nu}^t(q, \xi) \rightarrow \Delta_{\mu\nu}^t(q, \xi_t)$  and  $\Delta_{\mu\nu}^l(k, \xi) \rightarrow \Delta_{\mu\nu}^l(q, \xi_l)$ , where  $\xi_t \neq \xi_l$ , in general. The above transformation does not change the value of the  $S$ -matrix element, i.e.  $S$  is independent of *both*  $\xi_t$  and  $\xi_l$ .

Of course, instead of the  $R_\xi$  gauges, one could choose a different gauge-fixing scheme. In the case of a ghost-free non-covariant gauge such as the light-cone gauge [4] for example, the gauge fixing term is  $\mathcal{L}_{LC} = -\frac{1}{2\xi}(n_\mu A^\mu)^2$ , where  $n_\mu$  is an arbitrary four-vector, for which  $n_\mu A^\mu = 0$  and  $n^2 = 0$ ; the corresponding tree-level gluon propagator in the  $\xi \rightarrow 0$  limit is given by

$$i\Delta_{\mu\nu}(q, n) = \frac{-i}{q^2} \left[ g_{\mu\nu} - \frac{n_\mu q_\nu + n_\nu q_\mu}{n \cdot q} \right]. \quad (1.10)$$

Carrying out the corresponding replacement  $\Delta_{\mu\nu}^t(q, n) \rightarrow \Delta_{\mu\nu}^t(q, n_t)$  and  $\Delta_{\mu\nu}^l(q, n) \rightarrow \Delta_{\mu\nu}^l(q, n_l)$  we will find that the  $S$ -matrix is independent of both  $n_t$  and  $n_l$ . We call the above property, the “dual” gauge-fixing (DGF) property of the  $S$ -matrix.

The paper is organized as follows: In section 2 we prove the DGF property for the case of QED, and discuss the basic ingredients which are crucial for the proof. In section 3 we extend the proof to the case of non-Abelian gauge theories; for the case of QCD we show that the features which operate in the QED case are concealed by the conventional perturbative formulation, but they can be exposed by resorting to the systematic rearrangement of graphs dictated by the pinch technique (PT) [2], [3]. In section 4 we extend this analysis to the electroweak sector, where exactly analogous results apply. Finally, in section 5, we discuss our conclusions, and briefly present some possible applications of the DGF property in the context of the electroweak phenomenology.

## 2 The QED case

In order to understand the mechanism which enforces the DGF property at the level of the  $S$ -matrix, let us focus for a moment on QED. In QED the above property can easily be proved; this is so because the photon self-energy  $\Pi_{\mu\nu}(q)$ , the photon-electron vertex  $\Gamma_\mu$ , and the electron self-energy  $\Sigma(p)$  have the following properties (at least at one loop).

- (a)  $\Pi_{\mu\nu}(q)$  is GFP-independent, and transverse, i.e.  $q^\mu \Pi_{\mu\nu}(q) = 0$ .
- (b)  $q^\mu \Gamma_\mu(p_1, p_2) = e[\Sigma(p_1) - \Sigma(p_2)]$ , by virtue of QED the Ward identity.
- (c) the sum of the two box diagrams (direct and crossed) is GFP-independent.

From (a), (b), and (c) follows that the improper vertex  $G_\mu$ , which consists of  $\Gamma_\mu(p_1, p_2)$  and the wave function corrections to the external fermion legs, is GFP-independent, UV finite, and transverse, i.e.  $q^\mu G_\mu = 0$ .

It is now easy to see how the DGF property of the  $S$ -matrix holds in the case of QED. To begin with, the box diagrams contain only loop photons  $\Delta_{\mu\nu}^i$ , and their sum is GFP-independent, so it is invariant under the transformation (D):  $T \xrightarrow{(D)} T'$ , namely  $T_3 \equiv T'_3$ . The photon self-energy  $\Pi_{\mu\nu}$  at one loop consists of a fermion loop, so its value does not change under (D). In addition, any gauge fixing parameters stemming from  $\Delta_{\mu\nu}^t$  vanishes, because it either gets contracted with the external conserved current, or with the transverse  $\Pi_{\mu\nu}$ . So the part  $T_1$  of the  $S$ -matrix before the transformation reads

$$\begin{aligned} T_1 &= \bar{u}_1 \gamma_\rho u_1 \Delta^{\rho\mu}(q, \xi) \Pi_{\mu\nu}(q) \Delta^{\nu\sigma}(q, \xi) \bar{u}_2 \gamma_\sigma u_2 \\ &= \bar{u}_1 \gamma_\mu u_1 \left[ \frac{1}{q^2} \right] \Pi^{\mu\nu}(q) \left[ \frac{1}{q^2} \right] \bar{u}_2 \gamma_\nu u_2 . \end{aligned} \tag{2.1}$$

After the transformation (D),  $T_1 \xrightarrow{(D)} T'_1$ , with  $T'_1$  given by

$$\begin{aligned} T'_1 &= \bar{u}_1 \gamma_\rho u_1 \Delta^{\rho\mu}(q, \xi_t) \Pi_{\mu\nu}(q) \Delta^{\nu\sigma}(q, \xi_t) \bar{u}_2 \gamma_\sigma u_2 \\ &= \bar{u}_1 \gamma_\mu u_1 \left[ \frac{1}{q^2} \right] \Pi^{\mu\nu}(q) \left[ \frac{1}{q^2} \right] \bar{u}_2 \gamma_\nu u_2 \\ &= T_1 . \end{aligned} \tag{2.2}$$

Notice that in the  $R_\xi$  gauges, due to current conservation, the dependence on  $\xi$  vanishes, even without using the transversality of  $\Pi_{\mu\nu}(q)$ . If instead we had been working in a non-covariant gauge, we would have to use the WI  $q^\mu\Pi_{\mu\nu}(q) = 0$  in the above equation, because the terms proportional to  $n_t^\rho q^\mu$  and  $n_t^\sigma q^\nu$  cannot be contracted with the external conserved current.

Finally, for the part  $T_2$  of the  $S$ -matrix involving the improper vertex  $G_\mu$  we have at the beginning:

$$\begin{aligned} T_2 &= \bar{u}_1 \gamma_\rho u_1 \Delta^{\rho\mu}(q, \xi) \bar{u}_2 G_\mu u_2 \\ &= \bar{u}_1 \gamma_\mu u_1 \left[ \frac{1}{q^2} \right] \bar{u}_2 G^\mu u_2 . \end{aligned} \tag{2.3}$$

On the other hand, after imposing (D):

$$\begin{aligned} T'_2 &= \bar{u}_1 \gamma_\rho u_1 \Delta^{\rho\mu}(q, \xi_t) \bar{u}_2 G_\mu u_2 \\ &= T_2 . \end{aligned} \tag{2.4}$$

Again, if we were to work in a non-covariant gauge we would need to resort to the transversality of  $G_\mu$ , i.e. use that  $q^\mu G_\mu = 0$ .

Finally, since  $T'_i = T_i$ , for  $i = 1, 2, 3$ , the  $S$ -matrix is invariant under (D).

Even though the above proof is very straightforward, it allows one to recognize the crucial ingredients which enforce the invariance under (D). They are :

(a) The fact that certain Green's functions are GFP-independent in any gauge-fixing procedure.

(b) The fact that in QED the Green's functions satisfy their naive, tree-level Ward identities, even *after* quantum corrections have been taken into account.

### 3 Non-Abelian gauge theories: The QCD case

The previous proof of the DGF property, which is very transparent in the case of QED, becomes complicated in the case of non-Abelian gauge theories (NAGT), such as QCD,

or theories with Higgs mechanism such as the  $SU(2)_L \times U(1)_Y$  electroweak sector of the standard model. The reason is that in the conventional formulation of NAGT the two crucial properties mentioned above fail to be satisfied. Regarding property (a), in NAGT the gauge boson self-energy is GFP-*dependent*, already at one loop [5], [6]. As for property (b), after quantization the tree level Ward identities are replaced by complicated Slavnov-Taylor identities, derived from the residual BRST symmetries. However, as we will explicitly illustrate, the DGF property holds *also* for these theories, at least at one-loop.

Let us first concentrate on a QCD example and examine at one-loop the  $S$ -matrix element for quark-antiquark annihilation into a pair of gluons ( $g$ ), i.e. the process  $q(p_1)\bar{q}(p_2) \rightarrow g(q_1)g(q_2)$ . This process contains both the  $gq\bar{q}$  vertex as well as the three gluon vertex at one loop. The  $S$ -matrix element is again decomposed into self-energy, vertex, and box parts,

$$T(s, t, m) = T_1(q, \xi) + T_2^f(p_1, p_2, m, \xi) + T_2^g(q_1, q_2, \xi) + T_3(s, t, m, \xi), \quad (3.1)$$

where the superscript “ $f$ ” (“ $g$ ”) in  $T_2$  refers to the two external “on-shell” fermions (gluons). Under the transformation (D), the sub-amplitudes assume the following forms:

The self energy sub-amplitude is:

$$T_1(s, \xi_t, \xi_l) = \bar{u}_1 \gamma_\alpha u_2 \Delta^{\alpha\mu}(q, \xi_t) \Pi_{\mu\nu}(q, \xi_l) \Delta^{\nu\beta}(q, \xi_t) \Gamma_{\beta\rho\sigma}^{(0)}(q, q_1, q_2) \epsilon_1^\rho \epsilon_2^\sigma \quad (3.2)$$

where  $\Gamma_{\beta\rho\sigma}^{(0)}(q, q_1, q_2)$  is the usual tree-level three-gluon vertex

$$\Gamma_{\beta\rho\sigma}^{(0)}(q, q_1, q_2) = (q - q_1)_\sigma g_{\beta\rho} + (q_1 - q_2)_\beta g_{\rho\sigma} + (q_2 - q)_\rho g_{\sigma\beta}, \quad (3.3)$$

and  $\epsilon_i^\mu$ ,  $i = 1, 2$  are the polarization vectors corresponding to the external gluon with momentum  $q_i$ ; clearly,  $q_i \cdot \epsilon_i = 0$ . The vertex parts, together with the external leg

corrections, are:

$$\begin{aligned}
T_2^f(s, m, \xi_t, \xi_l) = & \bar{u}_1 \Gamma_\alpha^{(1)}(p_1, p_2, q; \xi_l) u_2 \Delta^{\alpha\mu}(q, \xi_t) \Gamma_{\mu\rho\sigma}^{(0)}(q, q_1, q_2) \epsilon_1^\rho \epsilon_2^\sigma \\
& + \bar{u}_1 \Sigma(p_1; \xi_l) \frac{1}{\not{p}_1 - m} \gamma_\alpha u_2 \Delta^{\alpha\mu}(q, \xi_t) \Gamma_{\mu\rho\sigma}^{(0)}(q, q_1, q_2) \epsilon_1^\rho \epsilon_2^\sigma \\
& + \bar{u}_1 \gamma_\alpha \frac{1}{\not{p}_2 - m} \Sigma(p_2; \xi_l) u_2 \Delta^{\alpha\mu}(q, \xi_t) \Gamma_{\mu\rho\sigma}^{(0)}(q, q_1, q_2) \epsilon_1^\rho \epsilon_2^\sigma , \quad (3.4)
\end{aligned}$$

$$\begin{aligned}
T_2^g(s, \xi_t, \xi_l) = & \bar{u}_1 \gamma_\alpha u_2 \Delta^{\alpha\mu}(q, \xi_t) \Gamma_{\mu\rho\sigma}^{(1)}(q, q_1, q_2; \xi_l) \epsilon_1^\rho \epsilon_2^\sigma \\
& + \bar{u}_1 \gamma_\alpha u_2 \Delta^{\alpha\mu}(q, \xi_t) \Gamma_{\mu\beta\sigma}^{(0)}(q, q_1, q_2) \Delta^{\beta\nu}(q_1, \xi_t) \Pi_{\nu\rho}(q_1; \xi_l) \epsilon_1^\rho \epsilon_2^\sigma \\
& + \bar{u}_1 \gamma_\alpha u_2 \Delta^{\alpha\mu}(q, \xi_t) \Gamma_{\mu\rho\beta}^{(0)}(q, q_1, q_2) \Delta^{\beta\nu}(q_2, \xi_t) \Pi_{\nu\sigma}(q_2; \xi_l) \epsilon_1^\rho \epsilon_2^\sigma , \quad (3.5)
\end{aligned}$$

Finally, the box is given by:

$$T_3(s, t, m, \xi_l) = B(p_1, p_2, q_1, q_2, m, \xi_l)_{\rho\sigma} \epsilon_1^\rho \epsilon_2^\sigma . \quad (3.6)$$

To prove that the  $S$ -matrix element is independent of both  $\xi_t$  and  $\xi_l$  we proceed as follows:

The first step is to show that the dependence on  $\xi_l$  cancels regardless of what one chooses for  $\xi_t$ . To this end we employ the PT. The PT rearranges the Feynman diagrams by appropriately exploiting the following two elementary Ward identities, satisfied by the tree level  $gf\bar{f}$  and  $ggg$  vertices respectively:

$$k_\mu \gamma^\mu \equiv \not{k} = (\not{k} + \not{p} - m) - (\not{p} - m) , \quad (3.7)$$

$$k^\mu \Gamma_{\mu\nu\alpha}^{(0)}(k, p - k, p) = (p - k)^2 t_{\nu\alpha}(p - k) - p^2 t_{\nu\alpha}(p) , \quad (3.8)$$

where  $t_{\alpha\beta}(q) = g_{\alpha\beta} - q_\alpha q_\beta / q^2$  is the usual transverse projector.

Before carrying out any calculations, we first let the longitudinal momenta supplied by the gluon propagators or the trilinear gluon vertices trigger the above WI. The inverse propagators thus generated will either vanish on shell or cancel (pinch) an internal fermionic or bosonic propagator inside the loop. As a result of these cancellations, parts



from the vertex or box graphs will emerge, which will have the same kinematic structure as the self-energy graphs. The final step of casting these expressions into the desired form of the self-energy graphs as in  $T_1$ , is to recognize that a tree-level gluon propagator must be attached at the point where pinching took place. For this purpose unity is inserted in the form of a propagator times its inverse, using the following elementary identity which holds for any gauge fixing procedure (covariant, non-covariant, etc.)

$$\begin{aligned}
g_\alpha^\beta &= \Delta_{\alpha\mu}(q; \xi_t) [\Delta^{-1}]^{\mu\beta}(q; \xi_t) = \Delta_{\alpha\mu}(q; \xi_t) [-q^2 t^{\mu\beta}] + \dots \\
&= \Delta_{\alpha\mu}^{-1}(q; \xi_t) \Delta^{\mu\beta}(q; \xi_t) = [-q^2 t_{\alpha\mu}] \Delta^{\mu\beta}(q; \xi_t) + \dots
\end{aligned} \tag{3.9}$$

where the ellipses denote terms that will vanish when contracted either with  $\bar{u}_1 \gamma_\alpha u_2$  or  $\Gamma_{\beta\rho\sigma}^{(0)}(q, q_1, q_2) \epsilon_1^\rho \epsilon_2^\sigma$ . The  $q^2 t_{\alpha\mu}$  factor will be part of the pinch expression and it is manifestly gauge independent. It is important to emphasize that no  $\xi_l$  dependences have been introduced in this step. Subsequently, the pinch parts extracted from the vertex and box graphs are allotted to the usual self-energy graphs, in order to define a new effective one-loop self-energy for the gluon. As has been shown by explicit calculations in a wide variety of gauges [2],[7],[8] (non-covariant, covariant, background), and recently by rigorous arguments based on analyticity, unitarity, and BRST symmetry [9] this rearrangement suffices to cancel all dependence on  $\xi_l$  inside the loop integrals. The crucial point is that the  $\xi_l$ -cancellations takes place in a kinematically distinct way, i.e. one ends up with propagator, vertex, and box-like structures, which are *individually* independent of  $\xi_l$ . Thus after the PT rearrangement the sub-amplitudes assume the form:

$$T_1(q, \xi_t) = \bar{u}_1 \gamma_\alpha u_2 \Delta^{\alpha\mu}(q, \xi_t) \widehat{\Pi}_{\mu\nu}(q) \Delta^{\nu\beta}(q, \xi_t) \Gamma_{\beta\rho\sigma}^{(0)}(q, q_1, q_2) \epsilon_{1\rho} \epsilon_{2\sigma} , \tag{3.10}$$

$$\begin{aligned}
T_2^f(p_1, p_2, m, \xi_t) &= \bar{u}_1 \widehat{\Gamma}_\alpha^{(1)}(p_1, p_2, q) u_2 \Delta^{\alpha\mu}(q, \xi_t) \Gamma_{\mu\rho\sigma}^{(0)}(q, q_1, q_2) \epsilon_1^\rho \epsilon_2^\sigma \\
&\quad + \bar{u}_1 \widehat{\Sigma}(p_1) \frac{1}{\not{p}_1 - m} \gamma_\alpha u_2 \Delta^{\alpha\mu}(q, \xi_t) \Gamma_{\mu\rho\sigma}^{(0)}(q, q_1, q_2) \epsilon_1^\rho \epsilon_2^\sigma \\
&\quad + \bar{u}_1 \gamma_\alpha \frac{1}{\not{p}_2 - m} \widehat{\Sigma}(p_2) u_2 \Delta^{\alpha\mu}(q, \xi_t) \Gamma_{\mu\rho\sigma}^{(0)}(q, q_1, q_2) \epsilon_1^\rho \epsilon_2^\sigma ,
\end{aligned} \tag{3.11}$$

the vertex parts together with the corrections for the external legs are

$$\begin{aligned}
T_2^g(q_1, q_2, \xi_t, \xi_l) &= \bar{u}_1 \gamma_\alpha u_2 \Delta^{\alpha\mu}(q, \xi_t) \widehat{\Gamma}_{\mu\rho\sigma}^{(1)}(q, q_1, q_2) \epsilon_1^\rho \epsilon_2^\sigma \\
&+ \bar{u}_1 \gamma_\alpha u_2 \Delta^{\alpha\mu}(q, \xi_t) \Gamma_{\mu\beta\sigma}^{(0)}(q, q_1, q_2) \Delta^{\beta\nu}(q_1, \xi_t) \widehat{\Pi}_{\nu\rho}(q_1) \epsilon_1^\rho \epsilon_2^\sigma \\
&+ \bar{u}_1 \gamma_\alpha u_2 \Delta^{\alpha\mu}(q, \xi_t) \Gamma_{\mu\rho\beta}^{(0)}(q, q_1, q_2) \Delta^{\beta\nu}(q_2, \xi_t) \widehat{\Pi}_{\nu\sigma}(q_2) \epsilon_1^\rho \epsilon_2^\sigma , \quad (3.12)
\end{aligned}$$

and finally the box-like contributions

$$T_3(p_1, p_2, q_1, q_2, m) = \widehat{B}(p_1, p_2, q_1, q_2, m)_{\rho\sigma} \epsilon_1^\rho \epsilon_2^\sigma . \quad (3.13)$$

The hatted quantities in the above expressions denote the PT effective Green's functions, which are manifestly independent of  $\xi_l$ ; their exact closed expressions have been reported elsewhere [3], and are not important for the subsequent analysis.

The second step in the proof is to observe that the new effective one-loop Green's functions constructed via the PT in the first step satisfy their respective *tree-level* WI. It is important to emphasize that these classical WI are now valid even after the one-loop quantum corrections have been taken into account. This is to be contrasted to the complicated Slavnov-Taylor identities that the one-loop Green's functions usually satisfy. One can easily verify for the PT Green's functions that :

$$q^\mu \widehat{\Pi}_{\mu\nu}(q) = 0 , \quad (3.14)$$

$$q^\mu \widehat{\Gamma}_\mu(q, p_1, p_2) = g \left[ \widehat{\Sigma}(p_1) - \widehat{\Sigma}(p_2) \right] , \quad (3.15)$$

$$q^\mu \widehat{\Gamma}_{\mu\rho\sigma}(q, q_1, q_2) = g \left[ \widehat{\Pi}_{\rho\sigma}(q_1) - \widehat{\Pi}_{\rho\sigma}(q_2) \right] , \quad (3.16)$$

where  $g$  is the gauge coupling. Consequently, the improper vertices  $\widehat{G}_\mu$  and  $\widehat{G}_{\mu\rho\sigma}$  which contain the corrections to the external fermion or gluon legs are transverse;  $q^\mu \widehat{G}_\mu = 0$ ,  $q^\mu \widehat{G}_{\mu\rho\sigma} = 0$ . Using the above property, it is now straightforward to show that the residual  $\xi_t$  dependence cancels within each sub-amplitude, and that the  $S$ -matrix element is independent of  $\xi_t$ . At this point it is important to note that the key element to the

proof has been the PT rearrangement, which transforms the ordinary sub-amplitudes  $T_i$  to hatted ones,  $\widehat{T}_i$ , *without* mixing the “loop” GFP  $\xi_l$  with the “tree” GFP  $\xi_t$ . Exactly as in QED, the new sub-amplitudes consist of one-loop Green’s functions which are independent of  $\xi_l$ , and satisfy their tree level WI; this last property in turn eliminates all remaining  $\xi_t$  dependences.

## 4 The Electroweak sector

The previous arguments can be generalized to the case of a non-Abelian theory with tree-level symmetry breaking, such as the  $SU(2)_L \times U(1)_Y$  electroweak model. Even though the equivalent proof is technically more involved, mainly because in the electroweak sector the currents are not conserved, and the presence of additional unphysical degrees of freedom (such as the would-be Goldstone bosons) complicates matters considerably, the conceptual issues remain the same. One needs to construct effective Green’s functions which are manifestly GFP-independent, and, in addition, they satisfy tree-level Ward identities, even at one loop. Both of these requirements can be satisfied when one resorts to the PT rearrangement of the  $S$ -matrix [10].

Let us concentrate on the  $S$ -matrix element of a charged four-fermion process, and work in the renormalizable  $R_\xi$  class of gauges. We consider the scattering  $i_u i_{\bar{d}} \rightarrow f_u f_{\bar{d}}$ , where  $i$  and  $f$  are the initial and final  $SU(2)$  fermion doublets respectively, with masses  $m_{\{i\}} = m_u, m_d$  and  $m_{\{f\}} = M_u, M_d$ , and momenta  $p_u, p_d$  and  $l_u, l_d$ , where  $q = p_u - p_d = l_d - l_u$ . The  $S$ -matrix element consists again of the sub-amplitudes  $T_1(s; \xi_j)$ ,  $T_2^i(s, m_{\{i\}}; \xi^j)$ ,  $T_2^f(s, m_{\{f\}}; \xi^j)$  and  $T_3(s, t, m_{\{i\}}, m_{\{f\}}; \xi^j)$ ; they depend explicitly on the gauge fixing parameters  $\xi^j$ , where  $j = W, Z, \gamma$ . We now show that by replacing  $\xi \rightarrow \xi_t$  outside of the loops (there is only one gauge parameter outside the loops, namely  $\xi = \xi_W$ ) and  $\xi^j \rightarrow \xi_l^j$  inside the loops, for all  $\xi_j$ , the  $S$ -matrix that consists of the sum  $T_1 + T_2 + T_3$  remains

unchanged. After the above replacement the amplitudes read :

$$\begin{aligned}
T_1(t; \xi_t, \xi_l^j) &= J_{W\alpha} \Delta_W^{\alpha\mu}(q, \xi_t) \Pi_{\mu\nu}^W(q, \xi_l^j) \Delta_W^{\nu\beta}(q, \xi_t) J_{W\beta}^+ \\
&+ J_\phi \Delta_\phi(q, \xi_t) \Pi_\nu^-(q, \xi_l^j) \Delta_W^{\nu\beta}(q, \xi_t) J_{W\beta}^+ \\
&+ J_{W\alpha} \Delta_W^{\alpha\mu}(q, \xi_t) \Pi_\mu^-(q, \xi_l^j) \Delta_\phi(q, \xi_t) J_\phi^+ \\
&+ J_\phi \Delta_\phi(q, \xi_t) \Pi^\phi(q, \xi_l^j) \Delta_\phi(q, \xi_t) J_\phi^+ \tag{4.1}
\end{aligned}$$

$$\begin{aligned}
T_2(t, m_{\{i\}}, m_{\{f\}}; \xi_t, \xi_l^j) &= \Gamma_\alpha^{W^- i_u \bar{i}_d}(-q, p_u, -p_d; \xi_l^j) \Delta_W^{\alpha\beta}(q, \xi_t) J_{W\beta}^+ \\
&+ \Gamma^{\phi^- i_u \bar{i}_d}(-q, p_u, -p_d; \xi_l^j) \Delta_\phi(q, \xi_t) J_\phi^+ \\
&+ J_{W\alpha} \Delta_W^{\alpha\beta}(q, \xi_t) \Gamma_\beta^{W^+ \bar{f}_u f_d}(q, -l_u, l_d; \xi_l^j) \\
&+ J_\phi \Delta_\phi(q, \xi_t) \Gamma^{\phi^+ \bar{f}_u f_d}(q, -l_u, l_d; \xi_l^j) \tag{4.2}
\end{aligned}$$

$$+ \text{external leg corrections} \tag{4.3}$$

$$T_3(s, t, m_u, m_d, M_u, M_d; \xi_l^j) \equiv B(s, t, m_u, m_d, M_u, M_d; \xi_l^j) \tag{4.4}$$

We now use the PT to rearrange the above amplitudes by employing the tree level Ward identity of the vertex  $W f \bar{f}'$

$$k_\mu \gamma^\mu P_L \equiv \not{k} P_L = S_i^{-1}(t+k) P_L - P_R S_j^{-1}(t) + m_i P_L - m_j P_R \tag{4.5}$$

where  $k_\mu$  is a loop integration momentum and  $t = p, l$  is one of the external momenta. As in the QCD case, the action of the first term in Eq.(4.5) is to cancel the fermion propagator of the loop, while the second vanishes on shell. Such  $k_\mu$  momenta are provided inside the loops by the three gauge boson vertices, the longitudinal parts of the gauge boson propagators, and by the gauge-scalar-scalar vertices. This procedure allows us to extract from the box amplitude  $T_3$  pieces, which exhibit either the propagator-like structure of  $T_1$  or the vertex-like structure of  $T_2$ , depending on how many internal fermion propagators have been cancelled [11]. Similarly from the vertex amplitude  $T_2$  we extract the parts that have the propagator-like structure of  $T_1$ . These pinch parts are appended to the

relevant amplitudes and define the new  $\hat{T}_1$ ,  $\hat{T}_2$  and  $\hat{T}_3$ ; they can be obtained from the expressions of Eq.(4.1) - Eq.(4.4) by substituting  $\Pi \rightarrow \hat{\Pi}$ ,  $\Gamma \rightarrow \hat{\Gamma}$ ,  $B \rightarrow \hat{B}$ . Again, as has been verified by explicit calculations [10], all  $\xi_l$  dependence in the above amplitudes has cancelled, and this has happened completely independently of what  $\xi_t$  is.

We now show that the  $\xi_t$  dependence also cancels in these amplitudes. This final cancellation is enforced by a set of Ward identities that the new hatted, manifestly  $\xi_l$  independent, Green's functions satisfy. The PT self energy functions have been shown to satisfy the following WI [10]

$$\begin{aligned} q^\mu \hat{\Pi}_{\mu\nu}^W(q) \mp iM_W \hat{\Pi}_\nu^\pm(q) &= 0 , \\ q^\mu \hat{\Pi}_\mu^\pm(q) \pm iM_W \hat{\Pi}^\phi(q) &= 0 , \\ q^\mu q^\nu \hat{\Pi}_{\mu\nu}^W(q) - M_W^2 \hat{\Pi}^\phi(q) &= 0 , \end{aligned} \quad (4.6)$$

while the PT vertices satisfy

$$q^\mu \hat{\Gamma}_\mu^{W^- d\bar{u}}(q, k, -k-q) + iM_W \hat{\Gamma}^{\phi^- d\bar{u}}(q, k, -k-q) = \frac{g}{\sqrt{2}} \left[ \hat{\Sigma}^d(k) P_L - P_R \hat{\Sigma}^u(k+q) \right] , \quad (4.7)$$

and

$$q^\mu \hat{\Gamma}_\mu^{W^+ u\bar{d}}(q, k, -k-q) - iM_W \hat{\Gamma}^{\phi^+ d\bar{u}}(q, k, -k-q) = \frac{g}{\sqrt{2}} \left[ \hat{\Sigma}^u(k) P_L - P_R \hat{\Sigma}^d(k+q) \right] . \quad (4.8)$$

Using the elementary decomposition

$$\Delta_i^{\mu\nu}(q, \xi_j) = U_j^{\mu\nu}(q) - \frac{q^\mu q^\nu}{M_j^2} \Delta_s(q, \xi_j) , \quad (4.9)$$

where  $j = W, Z, \gamma$ , we observe that all the remaining  $\xi_t$  dependence is carried by the propagators of the unphysical scalars. Recalling the current relations:

$$q_\mu J_W^\mu = -iM_W J_\phi , \quad q_\mu J_W^{\mu\dagger} = iM_W J_\phi^\dagger , \quad (4.10)$$

it is easy to observe that by virtue of the WI of Eq.(4.6), Eq.(4.7) and Eq.(4.8) this residual  $\xi_t$  dependence cancels. Finally, as advocated, the amplitudes  $\hat{T}_1$ ,  $\hat{T}_2$  and  $\hat{T}_3$  are independent of *both*  $\xi_l$  and  $\xi_t$ .

## 5 Discussion and Conclusions

The analysis presented in the previous sections shows rather transparently the mechanism responsible for the dual gauge cancellations of the S-matrix. In summary, the one-loop Feynman diagrams of an  $S$ -matrix reorganize themselves systematically via the PT algorithm, which relies on the full exploitation of tree-level WI. At the end of the PT algorithm all gauge dependences inside loops has cancelled, giving rise to effective GFP-independent Green's functions . These one-loop effective Green's functions satisfy their tree-level WI, which in turn enforce the elimination of all remaining gauge dependences, appearing outside of the loops. Consequently, one can freely choose *different* gauge parameters  $\xi_l$  and  $\xi_t$ , to gauge fix the bosonic propagators appearing inside and outside of quantum loops, respectively.

It would be interesting to understand this dual choice of gauges at a more formal level; this is however beyond our power at this point. The only known context where such a dual choice of gauge fixing parameters can be field-theoretically justified is the Background Field Method (BFM) [12] . In the BFM framework the gauge field is split into two pieces, a “background” field (which corresponds to the field we call “tree” in this paper) and a “quantum” field (corresponding to our “loop” gauge field). It turns out that the background and quantum fields can be gauge-fixed using to completely independent gauge fixing terms, which in turn, introduce two independent gauge fixing parameters,  $\xi_C$  and  $\xi_Q$ . The gauge fixing procedure is chosen in such a way as to retain the original gauge invariance for the background fields; consequently,  $n$ -point functions involving background fields satisfy naive, tree-level WI to all orders in perturbation theory. By virtue of this last property one can show that in the BFM formulation the  $S$ -matrix is independent of both  $\xi_C$  and  $\xi_Q$  [13]. The analysis presented in this paper however precisely points to the fact that the DGF property holds *regardless* of the gauge fixing procedure used to quantize the theory. Indeed, nowhere throughout the paper have we resorted to the BFM

formalism. From this point of view, the DGF should be regarded as a general property of the  $S$ -matrix, rather than a property linked to some sophisticated gauge fixing procedure.

It is plausible that the DGF property holds true to all orders in perturbation theory; so far we can only show its validity at the one loop level since the PT has thus far been implemented only at one loop.

We believe that the PT in general, and the DGF property in particular, will be very useful in the implementation of automatic codes for calculating one-loop cross sections [14]. The advantages of writing one-loop amplitudes in a manifestly gauge independent way, as dictated by the PT, are numerous :

(i) All UV divergences reside in the self energy functions only, while the improper vertices are UV finite.

(ii) In the self energies, bosonic and fermionic contributions are treated in an equal footing. Furthermore, the PT self energies can be Dyson resummed, giving rise to the running couplings of the theory [2],[15]. In addition, their imaginary parts provide the natural regulator for resonant amplitudes, i.e. amplitudes containing unstable particles [9], [15].

(iii) Since each class of diagrams is rendered gauge parameter independent analytically, large gauge cancellations, which may significantly slow down the numerical computations, are thus avoided. A characteristic example is the unitarity of the process  $e^+e^- \rightarrow W^+W^-$ . In this case, the contributions to the cross section of the electromagnetic and weak dipole moment form factors of the  $W$ , stemming from the conventional vertex graphs, grow monotonically with the momentum transfer  $s$  [16]; it is only after the appropriate contributions from box diagrams have been identified by the PT and added to the vertex that one arrives at expressions for the form factors which respect unitarity [17]. Even though all such pieces exist in the  $S$ -matrix anyway, the advantage of carrying out the cancellations analytically, before resorting to numerical integrations, is obvious.

(iv) As far as one-loop calculations are concerned, the DGF property results in the following simplification. For the tree bosons one is free to use the unitary gauge ( $\xi_t \rightarrow \infty$ ) while for the loop bosons one can use the Feynman gauge for example ( $\xi_t = 1$ ). The advantage is two-fold: since only physical particles appear in the unitary gauge, the number of diagrams is significantly reduced, while, at the same time, manifest renormalizability is still retained, because the loop integrals are evaluated in the Feynman gauge.

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