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QCD Duality and the Mass of the Charm Quark[1]

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Abstract

The mass of the charm quark is analyzed in the context of QCD finite energy sum rules using recent BESII e^+e^- annihilation data and a large momentum expansion of the QCD correlator which incorporates terms to order $\alpha_s^2(m_c^2/q^2)^6$. Using various versions of duality, we obtain the consistent result $m_c(m_c) = (1.37 \pm 0.09)GeV$. Our result is quite independent of the ones based on the inverse moment analysis.

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1 Introduction

Recently the BESII collaboration has presented new data on the total e^+e^- annihilation cross section above the charm threshold [1]. Data in this energy region are particularly relevant for the extraction of the charm quark mass, one of the fundamental parameters of QCD. The charm quark can be determined by comparing suitable positive moments of these data with the corresponding moments of QCD perturbation theory. This direct quark-hadron duality approach was originally applied to the charm region in Ref. [2]. A reanalysis along these lines seems indicated in view of the fact that apart from new data, enormous progress has been made in the theoretical calculation of the relevant QCD correlator in the region $q^2 \gg m_c^2$. The correlator is now known to $O(\alpha_s^2)$ and $O(m^{12}/q^{12})$ [3], so that the question of convergence can be meaningfully discussed. There exist also a result to $O(\alpha_s^3)$ for the quartic mass correction [4] which we will not consider for reasons of consistency. We believe that in the case of the charm quark mass the direct duality approach employed by us is less prone to theoretical uncertainties as the more popular one based on inverse moments [5],[6],[7],[8],[9]. It should be pointed out, that we use as phenomenological input only the new BESII data. This is because older data in this region are plagued by unknown systematical errors and appear to be mutually inconsistent. An alternative would have been to adjust the normalizations of the various data sets so that they agree for large q^2 with QCD [10].

We will manipulate our data rather on the basis of QCD duality. Using suitable linear combinations of moments the emphasis of the hadronic integral can be shifted at will to experimental regions where the data errors are small. This technique, which was originally proposed in [11], allows in many cases more accurate prediction, and supplies in addition beautiful consistency checks. This will be explained in the following.

2 Cauchy sum rule

QCD duality means that the theoretical and phenomenological information is being related by means of Cauchy sum rule

$$\int_{s_0}^{R} \frac{1}{\pi} \mathrm{Im}\Pi(s) p(s) ds = -\frac{1}{2\pi i} \oint_{|s|=R} \Pi_{QCD}(s) p(s) ds \tag{1}$$

where $\mathrm{Im}\Pi(s)$ is defined in terms of the total e^+e^- annihilation cross-section by

$$R(e^+e^- \to hadrons) = 12\pi \sum_{flavors} Q_f^2 \mathrm{Im}\Pi(s) \tag{2}$$

The experimental charm physical threshold in Eq.(1) is taken from the J/Ψ resonance mass

$$s_0 = (3.097Gev)^2 \tag{3}$$

We have included in the sum rule a polynomial weight function $p(s)$

$$p(s) = \sum a_n s^n \tag{4}$$

which makes the sum rule a linear combination of moments of Cauchy sum rules. The polynomial may be chosen in a suitable way to enhance or remove part of the phenomenological input in the calculation.

3 QCD integral

The two-point function $\Pi_{QCD}(s)$ is known to $O(\alpha_s^2)$ as a series expansion in powers of m^2/s up the sixth power,

$$\Pi_{QCD}(s) = \sum_{i=0}^{6}\sum_{j=0}^{3} A_{ij}(m,\mu)\left(\frac{m^2}{s}\right)^i \left(\ln\frac{-s}{\mu^2}\right)^j \tag{5}$$

where the coefficients $A_{ij}(m,\mu)$ may contain powers of mass logarithms $\ln(m^2/\mu^2)$. The convergence of the series is not seriously affected by the mass logarithms provided

$$\ln\frac{\mu^2}{m^2} \ll \ln\frac{\mu^2}{\Lambda^2_{QCD}}\,. \tag{6}$$

This is always the case for the scales of μ ($\sim 5GeV$) we use.

At tree level, the first few terms in the expansion of $\Pi_{QCD}(s)$ are given by

$$\Pi_{QCD}(s) = \frac{3}{16\pi^2}\left[\frac{20}{9} - \frac{4}{3}\ln\frac{-s}{\mu^2} + 8\frac{m^2}{s} + \left(\frac{m^2}{s}\right)^2\left(4 - 8\ln\frac{m^2}{\mu^2} + 8\ln\frac{-s}{\mu^2}\right) + ..\right]$$

We use the strong coupling constant α_s^2 and running $\overline{MS}$ mass renormalized at the scale μ. The lengthy full expression to $O(\alpha_s^2)$ and $O(m^{12}/q^{12})$ may be found in Ref.[3].

There is also a small non-perturbative contribution arising from the gluon condensate, which is known to $O(\alpha_s)$ [12]. The first few terms are

$$\Pi_{np}(s) = \left\langle\frac{\alpha_s}{\pi}GG\right\rangle\left[-\frac{1}{12}\left(\frac{m^2}{s}\right)^2 - \frac{1}{3}\left(\frac{m^2}{s}\right)^3 - \left(\frac{m^2}{s}\right)^4\left(-\frac{7}{3} - \ln\frac{m^2}{\mu^2} + \ln\frac{-s}{\mu^2}\right) + ..\right]$$

The Cauchy-Integral in Eq.(1) needs the evaluation of

$$J(k,j) = \frac{1}{2\pi i} \oint_{|s|=R} s^k \left(\ln \frac{-s}{\mu^2} \right)^j ds \tag{7}$$

for $j = 0, 1, 2, 3$ and $k = -6, -5, -4, ...$These integrals can be evaluated analytically with the result

$$J(k \neq -1, j) = \frac{1}{2\pi i} \oint_{|s|=R} s^k \left(\ln \frac{-s}{\mu^2} \right)^j ds = \frac{R^{n+1}}{n+1} \times \left[j \ln^{j-1} \frac{R}{\mu^2} - \frac{j(j-1)}{n+1} \ln^{j-2} \frac{R}{\mu^2} - \left(\frac{\pi^2}{6} - \frac{1}{(n+1)^2} \right) j(j-1)(j-2) \ln^{j-3} \frac{R}{\mu^2} \right]$$

and

$$\begin{aligned} J(k = -1, j) &= \frac{1}{2\pi i} \oint_{|s|=R} \frac{1}{s} \left(\ln \frac{-s}{\mu^2} \right)^j ds \\ &= \ln^j \frac{R}{\mu^2} - \frac{\pi^2}{6} j(j-1) \ln^{j-2} \frac{R}{\mu^2} \end{aligned}$$

See also Ref.[13]

4 Data integral

The BESII data for the total e^+e^- annihilation cross section is represented in figure 1.

The errors given by the authors of Ref. [1] distinguish between systematic and statistical errors. In our analysis this distinction is absolutely essential as the statistical errors average out almost completely in the integration process while the systematic errors prevail. The threshold of the charm continuum is at twice the D_0 mass which corresponds to $s_{0cont} = 13.9\ GeV^2$. For definiteness we discuss here only the integration up to $s_{\max} = 25 GeV^2$, the maximum value measured by BESII. If we require for duality that the data within their errors agree with QCD perturbation theory then it is seen that $s_{\max}$ may also be chosen somewhat lower. We define

$$I_1 = \int_{13.9}^{25} R_{continuum}(s) p(s) ds \tag{8}$$

From this integral over the total cross section the contribution of the light quarks must be subtracted to isolate the pure charm contribution. As $s >$

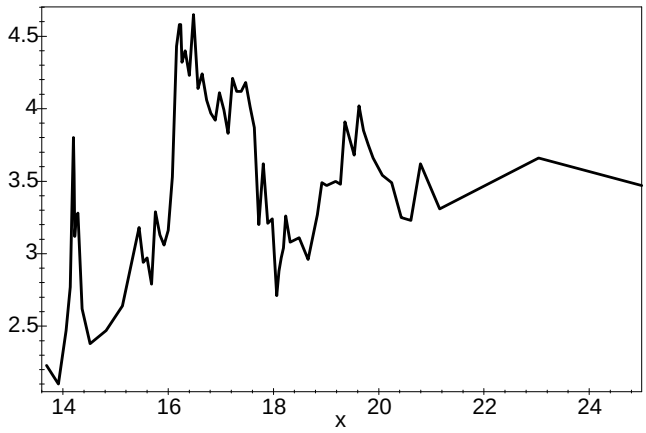

Figure 1: $R(e^+e^- \to hadrons)$ from BES data as a function of $x = (cm - energy)^2$.

$13.9GeV^2$ it is perfectly safe to use QCD perturbation theory for contribution of the light flavors. We use the four loop result for the massless correlator

$$I_2 = 2\int_{13.9}^{25} \left(1 + \frac{\alpha_s}{\pi} + ...\right) p\left(x\right) dx \tag{9}$$

where terms up to $O(\alpha_s^4)$ may be found in refs. [14], [15],[16]. We use here the highest known order in α_s because I_2 may be considered as an experimental input. The charm continuum contribution is then given by

$$I_{cont} = I_1 - I_2 \tag{10}$$

Finally we have to take into account the two J/Ψ and Ψ' charmonium resonances below the continuum threshold

$$I_{res} = 9\pi(137.04)^2 \left(m_1\Gamma_1 p\left(m_1^2\right) + m_2\Gamma_2 p\left(m_2^2\right)\right) \tag{11}$$

where the resonance masses and widths, are given by

$$\begin{aligned} m_1 &= 3.0969GeV \\ \Gamma_1 &= (5.26 \pm 0.37) \times 10^{-6}GeV \\ m_2 &= 3.6860GeV \\ \Gamma_2 &= (2.12 \pm 0.18) \times 10^{-6}GeV. \end{aligned} \tag{12}$$

The complete hadronic contribution to the sum rule is sum of these two integrals

$$I_{charm} = I_{cont} + I_{res} \tag{13}$$

5 Polynomial weight functions

There is much freedom in the choice of the polynomial in the duality relation, Eq.(1). In this note we will use the polynomial to reduce the importance of the continuum contribution relative to the well established sub-threshold resonances. We impose the following conditions:

The polynomial $p(s) = 1$ at $s = m^2_{J/\Psi}$, it vanishes at the end of the integration range ($s = 25 Gev^2$), and it is small in the continuum region. To be specific we choose an n-th order polynomial $p_n(s) = a_0 + a_1 s + .. + a_n s^n$ and determine the coefficients by the constraints

$$\begin{aligned} p_n(25) &= 0 \\ p_n(m^2_{J/\Psi}) &= 1 \\ \int_{3.9}^{25} s^k p_k(s) ds &= 0, \quad k = 0, 1, .., n-1 \end{aligned} \tag{14}$$

For a 3-degree polynomial we obtain, for example

$$\begin{aligned} p_3(s) &= 7.933\,787\,5 - 1.209\,157\,911 s \\ &\quad +6.015\,360\,076 \times 10^{-2} s^2 \\ &\quad -9.792\,537\,729 \times 10^{-4} s^3 \end{aligned} \tag{15}$$

This result is plotted in Fig.2.

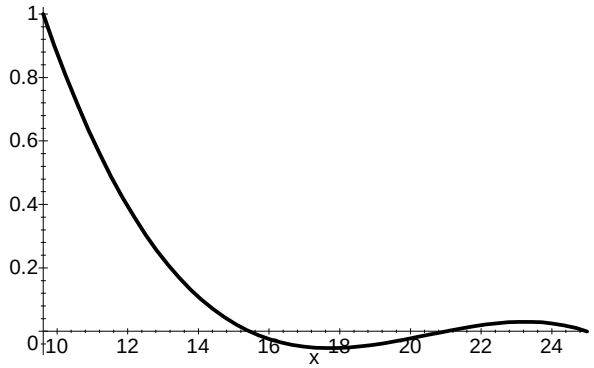

Figure 2: Polynomial fit vanishing at the continuum range $(13.9 - 25) GeV^2$.

It is obvious from the figure that the continuum data will almost cancel when integrated with this polynomial.

6 Results

From the duality sum rule Eq.(1) we obtain with the help of Eqs.(8-13) and Eqs.(5, 7)

$$I_{charm} = -\sum_{k=0}^{n}\sum_{i=0}^{6}\sum_{j=0}^{3} a_k A_{ij}(m_c,\mu)\left(m_c^2\right)^i J(k-i,j) \tag{16}$$

which can be solved for m_c.

We present here predictions for simple duality, i.e. $p(s) = 1$, and for the 3rd-degree polynomial above. The unknown m_c in Eq.(16) is the running charm mass at a scale μ., which we fix to be $\mu = 5GeV$, the relevant scale of the problem.

For the coupling constant α_s we take as an input its value at the mass of the tau lepton [17]

$$\alpha_s(m_\tau) = 0.345 \pm 0.020 \tag{17}$$

with $m_\tau = 1.777GeV$. After appropriate matching [18] from 3 to 4 flavors this corresponds to

$$\alpha^{4f}(5GeV) = 0.224 \pm 0.013 \tag{18}$$

For simple duality, we plot in Fig.3 the contribution of the QCD integral (lhs of Eq.(16)), at tree level and first and second order in the strong coupling, as a function of the charm quark mass m_c.

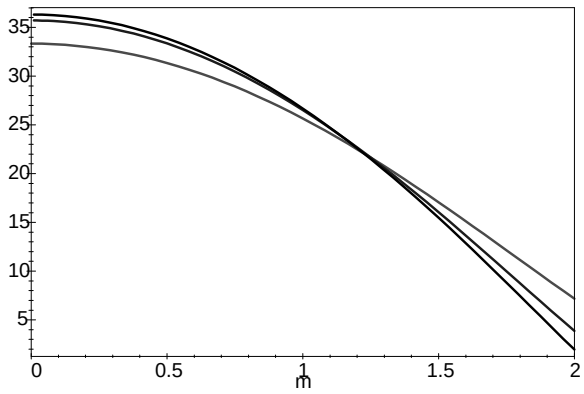

Figure 3: QCD integral without polynomial weight as a function of mass. The three curves (from top to bottom at $m = 2$) represent tree, first and second order calculations in the strong coupling constant.

For the Data integral we have the continuum contribution $I_{cont} = 14.06GeV^2$ and the resonance contribution $I_{res} = 12.80GeV^2$ with their sum being the total charm data integral

$$I_{charm} = 26.86GeV^2 \tag{19}$$

Solving Eq.(16) for this value of charm data, the results for the mass of the charm quark at different orders in the perturbative expansion are: $m_c^{(0)} = 0.916GeV$, $m_c^{(1)} = 0.980GeV$ and $m_c^{(2)} = 0.990GeV$. We see that the convergence of the QCD asymptotic expansion is extremely good. The main source of uncertainties come from the strong coupling constant and data. The result that we quote with this approach is

$$m_c(\mu = 5GeV) = (0.99 \pm 0.01_{asymp} \pm 0.01_{\alpha_s} \pm 0.04_{res} \pm 0.10_{cont})GeV \tag{20}$$

The asymptotic uncertainty comes from the difference of two- and three-loop results used for the QCD correlators. The result of Eq. 20 corresponds to an invariant mass [19]

$$m_c(m_c) = (1.40 \pm 0.11)GeV \tag{21}$$

In this final result the errors have been added quadratically.

The contribution of the gluon condensate is completely negligible

Our second approach consist in plug-in the 3rd degree polynomial in Eq.(15) into Eq.(16) in order to minimize the contribution from the charm continuum data, and therefore minimize the error involved in these data. As before we plot in Fig.4 the QCD integral as a function of m_c for different orders in the perturbative expansion

The contribution from the continuum data is now $I_{cont} = -0.176Gev^2$, whereas the resonance contribution, practically from J/Ψ resonance, is $I_{res} = 9.341GeV^2$. The complete charm data contribution is then

$$I_{charm} = 9.165GeV^2 \tag{22}$$

With this value we solve again Eq.(16) at different orders in the perturbative expansion of α_s with the results: $m_c^{(0)} = 1.153GeV$, $m_c^{(1)} = 0.991GeV$, $m_c^{(2)} = 0.927GeV$. We see that, although we have eliminate the uncertainty coming from continuous data, the price we pay is that the asymptotic expansion does not converge so nicely as before, but still good enough to make a sensible prediction for the mass of the charm quark. We have

$$m_c(\mu = 5GeV) = (0.93 \pm 0.06_{asymp} \pm 0.015_{\alpha_s} \pm 0.014_{res})GeV \tag{23}$$

This corresponds to an invariant mass [19]

$$m_c(m_c) = (1.34 \pm 0.08)GeV \tag{24}$$

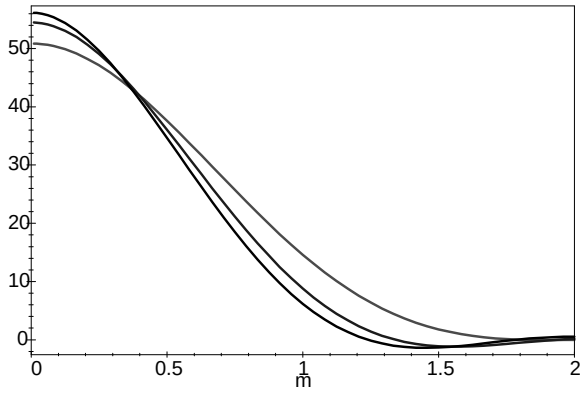

Figure 4: QCD integral with polynomial weight as a function of mass. The three curves (from top to bottom at $m = 1$) represent tree, first and second order calculations in the strong coupling constant.

The influence of the gluon condensate is in this approach $\sim 0.3\%$ for $\langle \alpha_s GG/\pi \rangle \sim .024 GeV^4$, still negligible.

The result we find is perfectly compatible, within error-bars, with the one we found above by the simple duality approach. This agreement constitutes a non trivial confirmation of the duality ansatz. Averaging the results of both approaches, we finally find

$$m_c(m_c) = (1.37 \pm 0.09) GeV. \tag{25}$$

Our value for $m_c(m_c)$ appears to be slightly bigger than the ones given by alternative QCD sum rule methods. With our normalization point the latter results read $m_c(m_c) = (1.29 \pm 0.05) GeV$[7] and $m_c(m_c) = (1.28 \pm 0.06) GeV$[8]. Although these values agree, within error bars, with ours, it should be kept in mind that these authors use older and lower values of the QCD coupling constant, so that the error-bar quoted should really be larger.

7 Conclusions

In this letter we have analyzed the mass of the charm quark in the context of QCD finite energy sum rules. In the phenomenological side of the sum rule we use recent BESII e^+e^- data, whereas in the theoretical side we employ the a large momentum expansion of QCD vector correlator function up to $O(\alpha_s^2)$ and $O(m_c^{12}/q^{12})$. Two approaches are considered. The first one uses simple Cauchy sum rule for the correlator. The second one includes a

polynomial in the sum rule to minimize the contribution of the continuum data. The results from both approaches are nicely compatible with each other. Whereas with the first approach suffers from a substantial uncertainty arising from the continuum data, the second one shifts this uncertainty to QCD asymptotic expansion. The results allow a nice consistency check of QCD duality assumption. More precise results need either better data or further terms in QCD asymptotic expansion.

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